Voting Choice

Andrey Malenko
University of Michigan, CEPR and ECGI

Nadya Malenko
University of Michigan, CEPR and ECGI

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Voting Choice

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Andrey Malenko
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Abstract

Traditionally, fund managers cast votes on behalf of investors whose capital they manage. Recently, this system has come under intense debate given the growing concentration of voting power among a few asset managers and disagreements over environmental and social issues. Major fund managers now offer their investors a choice: delegate their votes to the fund or cast votes themselves ("voting choice"). This paper studies the implications of voting choice for investor welfare. If the reason for offering voting choice is that investors have different preferences, then investors may retain their voting rights excessively, inefficiently prioritizing their private preferences over information. As a result, investors on aggregate are not always better off if voting choice is offered to them. In contrast, if the reason for offering voting choice is that investors have information about the proposal that the fund manager does not have, then voting choice is generally efficient, increasing investor welfare. However, if information collection is costly, voting choice may lead to coordination failure, resulting in less informed voting outcomes.

Keywords: voting, delegation, voting choice, pass-through voting, ESG, index funds, aggregation of information, heterogeneous preferences, externalities

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Andrey Malenko
Associate Professor of Finance
Stephen M. Ross School of Business, University of Michigan
701 Tappan Street, R5412,
Ann Arbor, MI 48109, United States
phone: +1 734 763-4233
e-mail: amalenko@umich.edu

Nadya Malenko*
Associate Professor of Finance
Stephen M. Ross School of Business, University of Michigan
701 Tappan Street
Ann Arbor, MI 48109, United States
e-mail: nmalenko@umich.edu

*Corresponding Author
Voting Choice*

Andrey Malenko† Nadya Malenko‡

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Abstract

Traditionally, fund managers cast votes on behalf of investors whose capital they manage. Recently, this system has come under intense debate given the growing concentration of voting power among a few asset managers and disagreements over environmental and social issues. Major fund managers now offer their investors a choice: delegate their votes to the fund or cast votes themselves (“voting choice”). This paper studies the implications of voting choice for investor welfare. If the reason for offering voting choice is that investors have different preferences, then investors may retain their voting rights excessively, inefficiently prioritizing their private preferences over information. As a result, investors on aggregate are not always better off if voting choice is offered to them. In contrast, if the reason for offering voting choice is that investors have information about the proposal that the fund manager does not have, then voting choice is generally efficient, increasing investor welfare. However, if information collection is costly, voting choice may lead to coordination failure, resulting in less informed voting outcomes.

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†University of Michigan, CEPR, and ECGI. Email: amalenko@umich.edu.
‡University of Michigan, NBER, CEPR, and ECGI. Email: nmalenko@umich.edu.
1 Introduction

The tremendous growth of institutional investors, particularly large passive funds, has drawn attention to their increasingly important role in corporate governance. Major investment advisors such as BlackRock, Vanguard, and State Street have become among the largest shareholders in many publicly traded firms (Lewellen and Lewellen, 2022). In this role, they cast votes on behalf of millions of investors and have substantial voting power, making them pivotal in important corporate votes (Bebchuk and Hirst, 2019; Brav et al., 2022). At the same time, shareholder disagreements over voting issues are becoming increasingly prevalent given the growth in environmental and social (E&S) issues that appear on companies’ agendas.

These trends have generated an intense debate about whether asset managers are in the best position to vote their investors’ shares.\(^1\) Do funds’ votes represent their investors’ preferences? Are their votes sufficiently informed? This debate has become especially heated in the context of E&S issues and has led to several important institutional and regulatory developments. In October 2021, BlackRock announced the “Voting Choice” program, which gives its investors the choice: delegate their votes to BlackRock, as had been the default before, or exercise their voting rights themselves (so-called “pass-through voting”). As of September 2022, investors representing 25% of BlackRock’s eligible assets had chosen to cast their own votes. While BlackRock initially offered this choice only to institutional clients, it soon expanded the program to some retail investors, and Vanguard and State Street followed suit.\(^2\) Regulators have been considering a more drastic change: The INvestor Democracy is EXpected (INDEX) Act, introduced in the Senate in May 2022, aims to require passively managed funds to collect voting instructions from all of their individual investors and vote according to them.\(^3\)

When is delegating voting to the fund manager beneficial for a fund’s investor, and when

\(^1\)See “BlackRock Walks a Political Tightrope on Climate Issues,” The Wall Street Journal, Oct. 9, 2022. According to Reuters (May 22, 2023), large asset managers’ “influential votes have drawn much criticism, both from activists urging them to push portfolio companies harder on issues such as climate change or workforce diversity and, this year, from right-wing U.S. politicians who say the firms focus too much on ESG matters.” See Lund (2018) and Fisch and Schwartz (2023) for the legal debate about the role of asset managers in voting.


\(^3\)Specifically, the Act proposes that the fund “cannot vote without instructions from fund investors, except for routine matters” if it holds more than 1% of a firm’s shares (https://www.sullivan.senate.gov/download/indexact_051722). An identical bill was introduced in the House of Representatives in July 2022.
will the investor prefer to cast his own vote? Do investors benefit from having the choice between delegating and voting themselves? Does such “voting choice” dominate the two other extremes – the fund voting all its investors’ shares and all investors voting themselves? These are the questions we explore in this paper.

In our model, the fund manager owns the firm on behalf of the fund’s investors. There is a proposal up for a vote, whose value depends on the unknown state. The fund manager gets a signal about the state and casts the votes that are delegated to her. Under complete delegation (which corresponds to the system that has been in place until recently), all investors delegate their voting rights to the fund manager, so she votes all the shares and effectively controls the voting outcome. Under mandatory pass-through voting (which corresponds to the system proposed by the INDEX Act), all investors cast their own votes. Finally, under voting choice, all investors independently decide whether to delegate their votes to the fund manager or to vote themselves. Investors may prefer to retain their votes for two reasons: to vote according to their own preferences (rather than the preferences of the fund manager) or to use their private information about the proposal (rather than to rely on the fund manager’s information).

Our analysis demonstrates that whether voting choice increases investor welfare crucially depends on whether investors retain their votes because of their private preferences or their private information. To show this, we consider two scenarios: in the first, fund investors have heterogeneous preferences but no private information, whereas in the second, investors have aligned preferences but receive conditionally independent private signals about the proposal.

In the setting with heterogeneous preferences, the value of the proposal to each investor depends on an uncertain common value and on an investor-specific private value. The fund manager gets a private signal about the common value and votes to maximize the expected welfare of her investors, knowing that their private values are drawn from some distribution centered around zero. Each fund investor faces the following trade-off when deciding whether to delegate his vote to the fund manager. On the one hand, delegation is valuable because the fund manager is more informed about the state, but on the other hand, the fund manager’s preferences generally differ from those of the investor. In equilibrium, each investor delegates his vote only if his preferences are sufficiently aligned with those of the fund manager.

If the fund manager has one investor only, having voting choice always benefits the investor,

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4 More specifically, under the “Voting Choice” program, the fund’s investors have a choice between delegating their vote to the fund manager, casting their own vote, and voting according to a custom policy offered by a proxy advisor that best aligns with their preferences. In our paper, for simplicity, we analyze the last two options together and treat both of them as the investor casting his own vote (since for both options, the investor’s vote closely represents the investor’s preferences).
and his welfare is maximized under this system. Intuitively, the investor chooses to retain his voting rights and not delegate only when it is in his interest to do so. With multiple investors, however, the question of whether voting choice improves investor welfare is more nuanced because of a collective action problem: when an investor decides whether to retain his voting rights or delegate voting to the fund manager, he trades off the costs and benefits of delegation for himself, but ignores the externalities imposed by his choice on other investors.

In particular, an investor’s decision to delegate his vote affects other investors in two ways. First, the decision is made based on the fund manager’s information about the state, whereas the investor’s vote would be uninformed. This force, which we refer to as the “information effect,” imposes a positive externality on other investors. Because investors only internalize the benefits of a more informed decision on the value of their own shares, but not on the value of other investors’ shares, the information effect leads to excessive retention of voting rights by investors and inefficiently little delegation. Second, the decision is made according to the fund manager’s preferences rather than the delegating investor’s preferences. This “preference effect” benefits investors aligned with the fund manager’s vote and hurts investors misaligned with the fund manager’s vote. We show that even though investors’ preferences are on average unbiased, the preference effect on aggregate hurts other investors. Intuitively, this is because the investor’s delegation decision only matters when the vote is split, and since the fund manager votes a block of shares, a split vote implies that more investors oppose the fund manager’s vote than support it. Because the investor does not internalize this overall negative externality of his delegation decision, the preference effect leads to excessive delegation of voting rights.

Which of the two effects dominates and whether voting choice is ultimately beneficial depends on the distribution of investors’ preferences, and the heterogeneity of preferences in particular. However, the impact of preference heterogeneity varies significantly depending on the exact form of heterogeneity. Suppose first that the preferences of many moderate investors become stronger. We show that in this case, the information effect can dominate, i.e., there is insufficient delegation under voting choice. Intuitively, if many investors have strong preferences regarding the proposal, then the probability of delegation is low, since all such investors prioritize their private values over information and retain their votes. Interestingly, this means that as preference heterogeneity increases in this way, voting choice does not necessarily become

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5 This echoes the concern about pass-through voting expressed by Fisch and Schwartz (2023), who write: “Even if fund investors could be nudged to vote, there are reasons to question whether their votes would be informed. ... Pass-through voting ... fails to account for the significant loss of sophistication, expertise, and efficiency that institutional intermediaries provide.”

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more beneficial and may become dominated by complete delegation. There are two opposite effects. On the one hand, the preferences of investors who do not delegate become stronger, which favors voting choice over complete delegation. On the other hand, because the information effect is more likely to dominate, there is more underutilization of the fund manager’s information, and requiring delegation of all votes to the fund helps correct this inefficiency.

Another way for investors’ preferences to become more heterogeneous is for the tails of the distribution to become heavier: moderate investors (those that mostly care about the common value) remain moderate, whereas the preferences of extreme investors become more extreme. We show that in this case, the preference effect eventually dominates, i.e., there is excessive delegation under voting choice. Intuitively, moderate investors continue to delegate their votes, so the fund’s information is not excessively underutilized. Instead, the key concern is insufficient aggregation of investors’ preferences due to the negative externality that delegating investors impose on those with extreme private values. As a result, voting choice is preferred to complete delegation. Moreover, because even voting choice features excessive delegation and insufficient aggregation of investors’ preferences, mandatory pass-through voting dominates both voting choice and complete delegation if the tails of the distribution are heavy enough. Together, these results highlight that greater heterogeneity in investors’ preferences does not necessarily make voting choice more attractive, and the type of preference heterogeneity matters a lot.

In practice, the fund manager’s preferences may be misaligned with those of the average investor (Zytnick, 2022; Li, Naaraayanan, and Sachdeva, 2023). For example, the fund manager may be reluctant to vote against management to avoid jeopardizing business ties with the company (Davis and Kim, 2007; Cvijanovic, Dasgupta, and Zachariadis, 2016) or may be excessively biased towards E&S issues (as some critics of large asset managers have alleged). We show that if the fund manager is biased, vote delegation has another, indirect, effect: it changes the fund manager’s voting behavior. When all votes are delegated to the fund manager, she is more likely to vote in line with her information (rather than in line with her private value from the proposal) compared to the system with voting choice. This is because the fund manager cares not only about her private value, but also about the welfare of her current investors, and her concern about fund investors’ welfare constrains her opportunistic behavior more when all votes are delegated to her. We refer to this as the “incentive effect” of delegation. Each individual investor, however, does not internalize the incentive effect, which may also lead to insufficient delegation under voting choice.

Overall, when investors have heterogeneous preferences, voting choice generally results in
either excessive or insufficient delegation and is often dominated by complete delegation or mandatory pass-through voting. The conclusions are very different when investors have heterogeneous information but are aligned in their preferences. Then, equilibrium with voting choice achieves the efficient level of delegation, one that maximizes expected investor welfare. As a result, voting choice dominates both complete delegation and mandatory pass-through voting. Intuitively, since fund investors have the same preferences and incur no costs of delegating or casting their votes, their interests are fully aligned, and hence there are no inefficiencies in equilibrium (McLennan, 1998).

However, even in this case, voting choice can lead to coordination failure and decrease investor welfare relative to complete delegation. This is because investors and the fund manager optimally decide whether to become informed about voting issues (Iliev, Kalodimos, and Lowry, 2021), and the extent of delegation changes their incentives to become informed. In particular, we extend the baseline model, in which players are endowed with information, to a setting where the fund manager and fund investors choose the quality of their private information at a cost. If investors’ preferences are aligned, voting choice is still beneficial, in the sense that investor welfare in the best equilibrium under voting choice is weakly higher than under complete delegation. At the same time, voting choice creates equilibrium multiplicity because of a feedback loop: fewer votes delegated to the fund lead the fund manager to acquire less precise information, which, in turn, leads fund investors to delegate even fewer votes to her. This introduces potential coordination failure: If investors do not delegate their votes, the fund manager has no incentives to become informed, so the equilibrium with no delegation and no information acquisition by the fund manager is self-fulfilling, even if it is collectively in the interest of all investors to delegate voting and information acquisition to the fund.

Our results have several policy implications. The case of uninformed investors with heterogeneous preferences can capture the scenario in which the fund’s clients are small institutional investors or retail investors voting on E&S proposals. Then, the comparison between voting choice and complete delegation depends on the distribution of investors’ preferences and the quality of the fund manager’s information. Moreover, greater heterogeneity of investors’ preferences does not necessarily make voting choice more desirable, as it may lead to excessive retention of voting rights and underutilization of the fund manager’s information. What matters is why preferences become more heterogeneous: do most investors become more concerned about E&S issues, or do only extreme investors become more extreme.\footnote{Another concern about offering voting choice to retail investors is that these investors may not participate}
In contrast, the case of privately informed investors with aligned preferences can describe the scenario in which the fund’s clients are relatively large institutional investors focused on profit maximization. Voting choice in this case can achieve the optimal level of delegation and dominate both the status quo system and the system proposed by the INDEX Act, but it is important to ensure investor coordination on the efficient equilibrium. Despite this, voting choice has not been actively discussed or universally offered to funds’ clients until recently, even though it could have been an efficient solution for governance proposals, which have been common on voting ballots for years.

Finally, the scenario in which the fund’s clients are institutional investors with different ideologies, e.g., as in voting on E&S proposals, is likely to combine both cases, and whether voting choice improves investor welfare depends on their relative importance. In particular, voting choice can make investors worse off if they are not very informed about the financial benefits of the proposals and the heterogeneity in their preferences is not too large.

**Related literature.** The literature on shareholder voting examines how efficiently voting aggregates shareholders’ heterogeneous information and preferences. The contribution of our paper is to study the delegation of voting rights. We show that whether investors have heterogeneous preferences or heterogeneous information is crucial for the optimal level of delegation. Bar-Isaac and Shapiro (2020) analyze strategic voting with a blockholder and small shareholders. Our result that voting choice is beneficial in the homogeneous-preference setting is related to their result that the blockholder may optimally abstain on part of his votes: in both cases, this helps improve information aggregation by avoiding over-reliance on one signal. Malenko and Malenko (2019) study shareholders’ choice between buying information from a proxy advisor and acquiring their own signals. Their setting features homogenous preferences and a proxy advisor that maximizes profits from information sale, whereas our focus is on the trade-off between aggregating heterogeneous preferences and making an informed decision, and

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7For example, Maug (1999), Bond and Eraslan (2010), Levit and Malenko (2011), Meirowitz and Pi (2022), and Bouton et al. (2023) focus on the aggregation of heterogeneous information, whereas Van Wesep (2014), Cvijanovic, Groen-Xu, and Zachariadis (2020), Levit, Malenko, and Maug (2022, 2023), and Meirowitz, Pi, and Ringgenberg (2023) focus on the aggregation of heterogeneous preferences. See Brav, Malenko, and Malenko (2023) for a survey of the literature.
the fund manager cares about the welfare of her investors.\(^8\)

Our paper also contributes to the literature on delegation, started by Holmstrom (1984).\(^9\) The trade-off faced by the principal in this literature is similar to that faced by fund investors in our setting with heterogeneous preferences: the benefit of delegation is that the agent (fund manager) is more informed, whereas the cost is that the agent’s preferences are misaligned with those of the principal. The key difference in that in our paper, multiple principals (fund investors) decide about delegating authority to one agent, without internalizing the effect of their delegation decisions on each other. Thus, our paper is an example of “common agency” (Bernheim and Whinston, 1986), but in a delegation context. The delegation literature has also studied delegation with externalities (e.g., Alonso, Dessein, and Matouschek, 2008 and 2015; Rantakari, 2008) but has analyzed very different problems: in these papers, one principal decides on delegation to multiple agents, and agents do not interact through voting.

2 Model

Consider a firm that is fully owned by a fund. The fund has \(N\) clients (fund investors) with equal ownership. We normalize the number of shares in the firm to \(N\), so that each fund investor owns exactly one share in the firm via the fund manager.

There is a proposal up for a vote. Investors’ preferences regarding the proposal consist of a common value component and a private value component. The common value component is \(u(d, \theta)\), where \(d \in \{0, 1\}\) is the decision to accept \((d = 1)\) or reject \((d = 0)\) and \(\theta \in \{0, 1\}\) is the state of the world. Function \(u(d, \theta)\) is given by

\[
\begin{align*}
    u(1, \theta) &= \begin{cases} 
    1 & \text{if } \theta = 1, \\
    -1 & \text{if } \theta = 0,
    \end{cases} \\
    u(0, \theta) &= 0.
\end{align*}
\]

In other words, approving the proposal increases (decreases) common value if \(\theta = 1\) \((\theta = 0)\), while rejecting the proposal and maintaining the status quo leaves value unchanged. The ex-ante probability that the proposal increases common value is \(\Pr(\theta = 1) = \frac{1}{2}\).

\(^8\)Levit and Tsoy (2022), Ma and Xiong (2021), Malenko, Malenko, and Spatt (2022), and Matsusaka and Shu (2021) endogenize the quality of proxy advisors’ recommendations, and Buechel, Mechтенberg, and Wagner (2023) highlight that their recommendations can trigger more independent research by shareholders.

Investor $i$’s utility from the proposal depends on common value $u(d, \theta)$ and the investor’s private value, captured by the preference parameter $x_i \in (-\infty, \infty)$, as follows:

$$v(x_i, d, \theta) = u(d, \theta) + dx_i. \tag{1}$$

Each investor’s private value is an independent and identically distributed draw from distribution $F(\cdot)$ with density $f(\cdot)$, which is symmetric around zero. Heterogeneity in shareholders’ preferences and views and its effect on voting outcomes has been widely documented (e.g., Bolton et al., 2020; Bubb and Catan, 2022; Li, Maug, and Schwartz-Ziv, 2022).

We assume that the fund maximizes the expected welfare of his investors: $x = 0$ for the fund manager, i.e., her preferences capture the expected preferences of fund investors. For example, the fund manager may care about the fund’s AUM and hence does not want to make decisions that decrease her investors’ welfare. We relax this assumption in Section 3.

The information structure is as follows. The fund manager observes a private signal $s \in \{0, 1\}$ about state $\theta$ with precision $p \in [\frac{1}{2}, 1]$:

$$\Pr(s = 1|\theta = 1) = \Pr(s = 0|\theta = 0) = p, \tag{2}$$

and each fund investor observes a private signal $\sigma_i \in \{0, 1\}$ with precision $\pi \in [\frac{1}{2}, 1]$:

$$\Pr(\sigma_i = 1|\theta = 1) = \Pr(\sigma_i = 0|\theta = 0) = \pi. \tag{3}$$

All signals are independent conditional on the state.

We assume that investors do not abstain from voting. If voting is costly, pass-through voting raises another potential concern: it may involve insufficient participation of retail investors. For now, we abstract from costly participation and abstention to focus on the mechanisms that apply even to institutional investors, which rarely abstain.

In our analysis, we will be interested in comparing the following three voting systems:

**Complete delegation to the fund.** The fund manager votes on behalf of all $N$ investors. This corresponds to the proxy voting system that has been the status quo until recently, before major asset managers introduced voting choice for their clients.

**Mandatory pass-through voting.** There is no delegation of votes to the fund manager, and all investors are required to vote themselves. This best corresponds to the voting system proposed under the INDEX Act.
Voting choice. Given the realization of his private value $x_i$, each investor $i$ decides whether to delegate his vote to the fund manager or to vote himself. These delegation decisions are made simultaneously and non-cooperatively. Then, investors and the fund manager observe their signals about the state, the fund votes on behalf of investors that delegated their votes, and each investor that did not delegate casts his own vote. This corresponds to the system in which asset managers give voting choice to all their clients.

As we show below, the comparison between these three systems crucially depends on whether investors have heterogeneous private signals about the proposal or whether they have heterogeneous preferences about it. To show this distinction most clearly, we separately consider the following two settings:

1. Heterogeneous preferences. In this setting, the private signals of investors are fully uninformative, i.e., $\pi = \frac{1}{2}$. Thus, from each investor’s point of view, the trade-off in the decision to keep his voting right is as follows. On the one hand, by delegating the voting decision to the fund manager, the investor ensures that his vote is more informed, since the fund manager gets an informative signal about the state and the investor is uninformed. On the other hand, by not delegating the voting decision, the investor ensures that his vote reflects his preference. For example, in the context of an environmental proposal, a fund investor can be less informed about its financial impact on the firm than the fund manager but may nevertheless decide to not delegate voting if he has a weaker environmental preference than the fund manager. This trade-off between information and preference misalignment is familiar from the delegation literature (e.g., Dessein, 2002).

2. Heterogeneous information. In this setting, investors get informative private signals about the state, i.e., $\pi > \frac{1}{2}$, and all investors have the same preferences, i.e., $x_i$ in (1) equals zero with certainty. Many existing models of strategic voting belong to this class (e.g., Austen-Smith and Bank, 1996; Feddersen and Pesendorfer, 1998).

A possible interpretation of these two settings is the following. The setting with heterogeneous preferences can capture the scenario in which the fund’s clients are retail investors or small institutional investors. Arguably such investors are unlikely to have significant private information about the value of the proposal given their small stakes in the firm. At the same time, each investor may have preferences that differ from those of other investors: for example, investors may have different preferences regarding E&S issues. The setting with heterogeneous information can capture the scenario in which the fund manager’s clients are relatively
large institutional investors focused on profit maximization (for example, BlackRock manages money on behalf of many institutional investors): these are often sophisticated investors that plausibly possess private information about the value of the proposal. Of course, in reality, investors can have both heterogeneous preferences and private information at the same time. For tractability, we do not consider this case. We conjecture that such a model will have effects present in the models that feature heterogeneous preferences and heterogeneous information separately, and which effect will dominate will depend on their relative strength.

Note that if the fund has one investor only, then voting choice always weakly benefits the investor: under voting choice, the investor optimally decides whether to delegate or not, so having the option to decide cannot make the investor worse off. With multiple investors, however, the delegation decision that is privately optimal from an individual investor’s perspective may not be optimal from the aggregate investor welfare perspective because of the externalities it imposes on other investors. As a result, the question of whether voting choice improves expected investor welfare is now non-trivial. We next analyze this question in each of the two settings: with heterogeneous preferences and heterogeneous information.

3 Heterogeneous preferences

Suppose investors have heterogeneous preferences but no private information about the state. We first characterize the equilibrium under the three regimes described above.

Equilibrium under complete delegation to the fund. Since the fund manager maximizes the expected welfare of her investors and their private values are on average unbiased ($\mathbb{E} [x_i] = 0$), she votes for the proposal if and only if she gets a positive signal about its common value. The expected common value of each investor is then given by

$$\Pr (\theta = 1) \Pr (s = 1|\theta = 1) - \Pr (\theta = 0) \Pr (s = 1|\theta = 0) = p - \frac{1}{2},$$

and the expected welfare of all investors is $N \left( p - \frac{1}{2} \right)$.

Equilibrium under mandatory pass-through voting. Since all investors are uninformed about common value, each investor simply votes in the direction of his private value: in favor (against) the proposal if $x_i > 0$ ($x_i < 0$).
**Equilibrium with voting choice.** Because the game is symmetric, we focus on equilibria in which investors with private values $x_i$ and $-x_i$ follow the same delegation strategy. Consider investor $i$ with private value $x_i$ deciding whether to vote himself or delegate his vote to the fund. If the investor delegates, the fund manager votes on his behalf, so the investor’s vote is cast for the proposal only if the fund manager’s signal is positive, $s = 1$. If the investor does not delegate, he votes $v = 1 \{x_i > 0\}$. Without loss of generality, consider $x_i > 0$. The investor understands that his vote matters only when he is pivotal. By symmetry, the pivotal state does not reveal any information about the state. If the investor delegates, his expected payoff in the pivotal state is

$$\Pr (s = 1) (\mathbb{E} [u (1, \theta) | s = 1] + x_i) = \frac{1}{2} (2p - 1 + x_i).$$

If the investor does not delegate, his expected payoff in the pivotal state is $x_i$. Hence, the investor delegates if and only if

$$\frac{1}{2} (2p - 1 + x) \geq x_i \Leftrightarrow 2p - 1 \geq x_i.$$ 

The case $x_i \leq 0$ is analogous by symmetry. Hence, investor $i$ delegates his vote if and only if

$$|x_i| \leq 2p - 1. \quad (5)$$

This strategy reflects the standard trade-off between information and bias in the delegation literature (e.g., Dessein, 2002): the benefit of delegating the vote is that the fund manager is more informed, but if the investor’s private value is sufficiently extreme, he prefers not to delegate since his preferences differ substantially from those of the fund manager.

**Investors’ welfare.** Notice that the equilibrium under all three systems takes the following form: investors delegate voting to the fund manager if and only if $|x_i| \leq \hat{x}$ for some cutoff $\hat{x}$. The case of complete delegation corresponds to $\hat{x} = \infty$, mandatory pass-through voting corresponds to $\hat{x} = 0$, and voting choice corresponds to $\hat{x} = 2p - 1$. To compare these systems, it is useful to characterize the expected welfare of fund investors, $U (\hat{x})$, as a function of any given cutoff $\hat{x} \in [0, \infty]$.

**Lemma 1.** Suppose that each investor delegates voting to the fund manager if and only if
\[ |x_i| \leq \hat{x} \text{ for some cutoff } \hat{x}. \] Then the expected investor welfare, \( U(\hat{x}) \), satisfies

\[
\frac{U(\hat{x})}{N} = \left( 2 \sum_{k=\frac{N+1}{2}}^{N} P(F(\hat{x}), N, k) - 1 \right) \left( p - \frac{1}{2} \right) + P(F(\hat{x}), N - 1, \frac{N - 1}{2}) \int_{\hat{x}}^{\infty} x f(x) \, dx, \tag{6}
\]

where \( P(z, N, k) = \frac{N!}{k!(N-k)!} z^k (1-z)^{N-k} \). The first component of (6) increases in \( \hat{x} \), whereas the second component decreases in \( \hat{x} \).

Equation (6) shows that investor welfare is the sum of two components. The first component captures the expected common value of investors, which is determined by the probability of making the decision that maximizes common value (accepting the proposal if \( \theta = 1 \) and rejecting the proposal if \( \theta = 0 \)). This component is the product of two terms. The term \( p - \frac{1}{2} \) coincides with (4) and captures the expected common value if the decision were made according to the fund manager’s signal. This term is multiplied by the probability that the overall voting outcome coincides with the vote of the fund manager, and this probability depends on the delegation strategy \( \hat{x} \): the more delegation there is (the higher \( F(\hat{x}) \)), the higher is this term. For example, under complete delegation (\( \hat{x} \to \infty \)), this term converges to one and the entire first component of (6) converges to the expression (4), whereas under mandatory pass-through voting, this term equals zero. The second component in (6) captures the expected private value of investors and depends on the extent to which the decision reflects investors’ preferences. This component is larger if there is less delegation, i.e., \( \hat{x} \) is smaller.

Lemma 1 thus illustrates the trade-off between the costs and benefits of delegation. On the one hand, more delegation (higher \( \hat{x} \)) increases the probability that the decision maximizes investors’ common value from the proposal. On the other hand, more delegation increases the chances that the decision does not reflect correctly the preferences of investors. The optimal \( \hat{x} \) that maximizes expected investor welfare trades off these two effects.

The optimal level of delegation.

To compare complete delegation, mandatory pass-through voting, and voting choice, it is useful to compare the delegation cutoffs under these systems (\( \infty, 0, \) and \( 2p - 1 \), respectively) to the delegation cutoff that maximizes investor welfare. We therefore analyze the following problem.

Suppose we could optimally choose the cutoff \( \hat{x}^* \in [0, \infty] \) (such that any investor would delegate his vote if and only if \( |x_i| \leq \hat{x}^* \)) to maximize expected investor welfare. If \( \hat{x}^* > 2p - 1 \),
the equilibrium with voting choice features underdelegation: it would be optimal to delegate more votes than what happens in equilibrium. If \( \hat{x}^* < 2p - 1 \), the equilibrium with voting choice features overdelegation: it would be optimal to delegate fewer votes.

How does the optimal \( \hat{x}^* \) compare to \( 2p - 1 \) (the equilibrium with voting choice)? To understand the intuition, consider the trade-off in increasing the delegation cutoff from \( \hat{x} \) to \( \hat{x} + \varepsilon \) for a small \( \varepsilon \). This change only matters if there is an investor with a private value satisfying \( |x| \in (\hat{x}, \hat{x} + \varepsilon) \): otherwise, the increase in the cutoff does not change any investors’ delegation decisions and hence any of the votes. Since \( \varepsilon \) is infinitesimal, we can focus on the case in which only one investor has a private value \( x \) in this interval. Consider an investor with such private value, e.g., \( -(\hat{x} + \varepsilon) < x < -\hat{x} \). If the investor did not delegate his vote, he would vote against the proposal (as his expectation of the common value is zero and his private value is negative). The increase in the delegation cutoff from \( \hat{x} \) to \( \hat{x} + \varepsilon \) induces the investor to delegate his vote, which changes the investor’s vote in situations where he would vote differently from the fund manager, i.e., in situations where the fund manager’s signal is positive. What are the effects of this change in the investor’s vote on overall investor welfare? Importantly, it only matters if the investor’s vote is pivotal, i.e., the same number of votes are cast in the direction of the fund manager’s vote (for the proposal) and in the opposite direction (against the proposal). Changing the voting outcome from being against to being in line with the fund manager’s vote has effect on both common values and private values of other investors:

1. **Information effect** (effect on common values): The decision is now made according to the fund manager’s information. This effect increases the common value from the proposal and thereby benefits all \( N \) investors.

2. **Preference effect** (effect on private values): The decision is now made according to the preferences of investors who like the proposal (in line with the fund manager’s positive vote) and against the preferences of investors who dislike the proposal. The preferences of delegating investors are on average zero: \( \mathbb{E} [x \mid x \in (-\hat{x}, \hat{x})] = 0 \). However, the preferences of non-delegating investors are, conditional on a split vote, on average negative. To see this, note that the fund manager casts a block of votes delegated to her, all in favor of the proposal. Hence, conditional on the pivotal event, it must be that out of the non-delegated votes (with \( |x| > \hat{x} \)), there are more investors who dislike the proposal and vote against it than investors who like the proposal, so that the combined votes against counteract the block of favorable votes cast by the fund manager. Thus, on average, conditional on the pivotal event, the effect
on the private values of all investors is negative. Put simply, the investor’s delegation decision only matters when the vote is split, and since the fund manager votes a block of shares, a split vote implies that more investors oppose the fund manager’s vote than support it.

If only the information effect were present, the investor’s decision to delegate voting to the fund would impose a positive externality on other investors, so voting choice would feature underdelegation relative to the optimal level of delegation. If only the preference effect were present, the externality from delegation would be on average negative, so voting choice would feature overdelegation. The combination of these two effects implies that generally, there can be both underdelegation and overdelegation.

Which of the two effects dominates depends on the distribution of investors’ preferences \( F \). As preferences become more heterogeneous, two effects happen simultaneously. First, more investors have strong preferences regarding the proposal and prioritize them over information, so they prefer to cast their own votes rather than delegate them to the fund manager. While this is individually optimal for each investor, this is suboptimal for investors as a whole because the information of the fund manager is underutilized. This effect strengthens the positive externality and leads to underdelegation. The second effect is that as investors with strong preferences become even more concerned about their private values, the preference externality becomes stronger as well, leading to overdelegation.

To isolate the two effects, it is useful to consider a change in the distribution that keeps investors’ private values constant in the middle of the distribution (i.e., moderate investors remain moderate), but varies the preferences of investors in the tails. This limits the information effect (as moderate investors continue to delegate their votes to the fund), resulting in the following comparative statics.

**Proposition 1.**

(i) Consider distribution \( G \), symmetric around zero, such that \( G(x) = F(x) \) for all \( x \in [-\hat{x}^*(F), \hat{x}^*(F)] \) but \( G(x | x \geq \hat{x}^*(F)) \) dominates \( F(x | x \geq \hat{x}^*(F)) \) in the sense of first-order stochastic dominance. Then, the optimal delegation cutoff for \( G, \hat{x}^*(G) \), is lower than \( \hat{x}^*(F) \).

(ii) Consider a class of symmetric around zero distributions that coincide with \( F \) for \( x \in [1 - 2p, 2p - 1] \) but differ in the tails, \( \int_{x_{2p-1}}^{\infty} x f(x) \, dx \). If the tails are sufficiently heavy (\( \int_{x_{2p-1}}^{\infty} x f(x) \, dx \) is sufficiently high), there is overdelegation in equilibrium. In contrast, if the tails are sufficiently thin (\( \int_{x_{2p-1}}^{\infty} x f(x) \, dx \) is sufficiently low), there is underdelegation.
Intuitively, the importance of the tails captures the severity of the negative externality. If the tails are heavy, then the preferences of non-delegating investors are, on average, very strong. When each individual investor delegates his vote to the fund manager, the negative externality she imposes on non-delegating investors is very large. At the same time, there are still many investors with moderate preferences, who prioritize information over their private values and delegate to the fund manager, so the fund manager’s information is not strongly underutilized. As a result, the negative externality dominates, so there is overdelegation.

In general, however, as investors’ preferences become more heterogeneous, either of the two effects can dominate, which has important implications for the optimality of voting choice vis-a-vis two other proxy voting systems.

**Comparison of voting choice to the two other systems**

We now use our results to compare the welfare of investors under the three voting systems. Recall that if the fund had only one investor, having a choice could never be detrimental, so the system with voting choice would dominate both complete delegation and mandatory pass-through voting. In contrast, with multiple investors, the comparison between voting choice and the two other systems is not obvious.

We first ask whether the introduction of voting choice increases investor welfare relative to complete delegation, which has been the status quo until recently. Interestingly, it is not necessarily the case that if investors’ preferences become more heterogeneous, voting choice is more likely to dominate complete delegation. As the next example shows, it is possible that an increase in preference heterogeneity: (1) results in lower expected investor welfare under voting choice than when preferences are less heterogeneous; (2) results in investors being better off under complete delegation of voting to the fund, even though investors are better off under voting choice when preferences are less heterogeneous.

**Example 1.** Consider $N = 5$, $p = 0.75$, and two distributions of investor preferences, $F$ and $G$, symmetric around zero, with densities

\[
\begin{align*}
f(x) & \begin{cases} 
0.3 & x \in [-0.5, 0.5] \\
1.75 & x \in [-0.7, 0.5] \cup [0.5, 0.7]
\end{cases} \\
g(x) & \begin{cases} 
0.1 & x \in [-0.5, 0.5] \\
2.25 & x \in [-0.7, 0.5] \cup [0.5, 0.7]
\end{cases}
\end{align*}
\]
Distribution $G$ is a mean-preserving spread of distribution $F$, so $G$ features higher heterogeneity in investors’ preferences. Under voting choice, the equilibrium delegation cutoff is $\hat{x} = 2p - 1 = \frac{1}{2}$, regardless of the distribution. Expected per-share investor welfare under voting choice (expression (6) for $\hat{x} = \frac{1}{2}$) is 0.2564 for distribution $F$ and 0.2475 for distribution $G$. Thus, an increase in preference heterogeneity decreases investor welfare under voting choice. Moreover, complete delegation of voting to the fund results in expected per-share investor welfare (expression (6) for $\hat{x} = \infty$) of $p - \frac{1}{2} = 0.25$, so investors prefer voting choice over complete delegation under distribution $F$, but prefer complete delegation over voting choice under distribution $G$.\(^{10}\)

Intuitively, greater heterogeneity of investors’ preferences has two effects. First, the tails of preferences become more important, as highlighted in Proposition 1. But second, as more investors’ preferences become stronger, they choose to delegate less to the fund manager, i.e., $F (2p - 1)$ declines. When the information effect dominates the preference effect, so that there is underdelegation in equilibrium, an increase in preference heterogeneity further exacerbates the underdelegation problem and underutilization of the fund manager’s information, decreasing investor welfare under voting choice. Requiring all votes to be delegated to the fund manager could then be preferred, so as to alleviate this inefficiency. This is exactly what happens in Example 1: the probability that an investor voluntarily delegates voting to the fund falls from 30\% under distribution $F$ to 10\% under distribution $G$. As a result, complete delegation to the fund dominates voting choice under greater heterogeneity of investors’ preferences, even though voting choice was preferred under lower heterogeneity.

If, however, the probability of delegation remains high and preferences become more heterogeneous in the tails, then the underutilization of the fund manager’s information is not a concern, and the key goal is to ensure the aggregation of investors’ preferences. This corresponds to the perturbation of the distribution introduced in Proposition 1, which increases the importance of the tails but keeps the middle of the distribution unchanged. As the next result shows, if tails become sufficiently heavy, voting choice dominates complete delegation. Moreover, at some point, both voting choice and complete delegation become dominated by mandatory pass-through voting.

\(^{10}\)Mandatory pass-through voting is, in this example, dominated by both voting choice or complete delegation for both $F$ and $G$. Specifically, the expected per-share investor welfare under mandatory pass-through voting (expression (6) for $\hat{x} = 0$) is 0.0928 for $F$ and 0.1059 for $G$. 

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Proposition 2. Consider a class of symmetric around zero distributions that coincide with $F$ for $x \in [1 - 2p, 2p - 1]$ but differ in the tails, $\int_{2p-1}^{\infty} xf(x) \, dx$.

(i) If the tails are sufficiently heavy ($\int_{2p-1}^{\infty} xf(x) \, dx$ is sufficiently high), voting choice results in higher expected investor welfare than complete delegation. If the tails are sufficiently thin ($\int_{2p-1}^{\infty} xf(x) \, dx$ is sufficiently low), complete delegation results in higher expected investor welfare than voting choice.

(ii) If the tails are sufficiently heavy, mandatory pass-through voting results in higher expected investor welfare than either voting choice or complete delegation.

Intuitively, the advantage of voting choice over complete delegation is that it aggregates the preferences of investors with strong realizations of private values. If the tails of preference distribution are thin, this advantage of voting choice is not very important. Instead, the equilibrium under voting choice features underdelegation and underutilizes the fund manager’s information (see Proposition 1). Complete delegation of voting uses the fund manager’s information more efficiently, leading to the result that full delegation is better for expected investor welfare than voting choice.

In contrast, if the tails of preference distribution are sufficiently heavy, then aggregation of investors’ preferences becomes an important concern, and voting choice results in higher expected investor welfare than complete delegation. Moreover, recall from Proposition 1 that if the tails of the distribution are sufficiently heavy, equilibrium under voting choice features overdelegation: investors do not internalize that by delegating their votes, they impose a negative externality on other investors with extreme preferences. In this case, although voting choice allows some aggregation of investors’ preferences (as opposed to no aggregation of preferences under complete delegation), it still features inefficiently little aggregation. Full aggregation of investors’ preferences can be achieved by requiring mandatory pass-through voting. While this happens at a cost of not using the fund manager’s information, this cost is dominated by its benefits if the tails are important enough.

The comparison of Propositions 1, 2, and Example 1 shows that greater heterogeneity in preferences could have very different effects depending on the source of heterogeneity. A chance in the distribution of preferences in Example 1 corresponds to the situation where some investors with moderate preferences became investors with strong preferences. For example, in the context of E&S proposals, many investors who previously mostly cared about the common
value, now become more concerned about E&S issues. In contrast, the perturbation analyzed in Proposition 1 corresponds to the situation where investors with strong E&S preferences become even more extreme, but investors with moderate preferences remain moderate.

### 3.1 Biased fund manager and the “incentive effect”

We now relax the assumption that the fund manager only cares about the welfare of her investors. Instead, suppose that the fund manager’s utility equals the expected per-share utility of her investors plus additional utility $w$ (private value) that she gets if the proposal is accepted. The private value $w$ is a random draw from distribution $H(\cdot)$ with support $[\underline{w}, \bar{w}]$, where $\underline{w} \geq 0$ and $\bar{w} \in [0, \infty]$.

When fund investors make their delegation decisions, they know the distribution $H$ (and hence know that the manager is biased towards the proposal) but do not know the realization of $w$. For example, investors know that the fund manager is on average supportive of ESG proposals, but the exact private value she gets from a certain proposal is proposal- and firm-specific and hence is unknown. The case $w = \bar{w} = 0$ corresponds to an unbiased fund manager analyzed up to now. For simplicity, we assume that when voting, the fund manager does not observe the realized number of votes that were delegated to her, so she votes based on the conjectured delegation strategies of investors (in equilibrium, the fund manager has rational expectations about their delegation strategies), as well as based on her signal $s$ and her private value $w$ from the proposal.

We characterize the equilibrium in the appendix and only describe the key effects of the manager’s bias here. Suppose the fund manager receives a positive signal about the proposal, $s = 1$. Since she is biased towards the proposal and the expected private value of her investors is zero, she finds it optimal to vote in favor of the proposal, in line with the signal. If the fund manager receives a negative signal, $s = 1$, she may still find it optimal to vote for the proposal if her private value $w$ is large enough. In equilibrium, there is a cutoff $w^*$ such that the fund manager votes against the proposal upon observing a negative signal if and only if $w \leq w^*$.

When fund investors decide whether to delegate their votes to the fund or not, they anticipate that upon receiving a negative signal, the fund manager votes against with probability $\alpha = H(w^*)$. In the appendix, we show that given $\alpha$, investors’ delegation decisions are char-

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11 The case where the manager is biased against the proposal is analogous by symmetry.

12 The assumption that the fund manager does not observe the realized number of votes delegated to her is not needed if the manager is unbiased ($\underline{w} = \bar{w} = 0$), but greatly simplifies the analysis when the manager is biased and investors’ delegation strategies are thus asymmetric around zero. We believe that this assumption is not important for the conceptual results.
acterized by two cutoffs, \( x_l \) and \( x_h \), \( x_l < 0 < x_h \), such that investor \( i \) delegates his vote to the fund manager if and only if \( x_i \in [x_l, x_h] \), where

\[
\begin{align*}
x_l &= -\frac{\alpha \left(2p - 1\right)}{2 - \alpha}, \\
x_h &= 2p - 1.
\end{align*}
\]

(7) (8)

If \( \alpha = 1 \), i.e., the fund manager always votes in line with the signal (which is the case when she is unbiased, \( w = \bar{w} = 0 \)), then investors’ delegation strategy is symmetric around zero: \( -x_l = x_h = 2p - 1 \), exactly as in (5). For any \( \alpha < 1 \), investors’ delegation strategy is asymmetric: \( -x_l < x_h \). Intuitively, knowing that the fund manager is biased towards the proposal, investors with \( x_i > 0 \) (positive private value from the proposal) are generally satisfied with the fund manager’s voting decisions and only choose to retain their votes if their bias towards the proposal is very strong. In contrast, investors with \( x_i < 0 \) (negative private value from the proposal) disagree with the fund manager’s voting decisions and choose to retain their votes even if their bias is moderate. In the extreme case when \( \alpha = 0 \) and the fund manager’s vote is completely uninformative, we have \( x_l = 0 \): every investor with a negative \( x_i \) does not delegate and votes against the proposal.\(^\text{13}\)

To compare voting choice and complete delegation, we derive the expected welfare of fund investors, \( U(x_l, x_h, \alpha) \) for any \( x_l, x_h, \) and \( \alpha \). The proof of Lemma 1 shows that it is given by

\[
\frac{U(x_l, x_h, \alpha)}{N} = \left( \sum_{k=-\frac{N-1}{2}}^{\frac{N}{2}} P(F(-x_l), N, k) + \sum_{k=-\frac{N+1}{2}}^{\frac{N}{2}} P(F(x_h), N, k) - 1 \right) \cdot \alpha \left( p - \frac{1}{2} \right) \\
+ \left( \frac{2 - \alpha}{2} \right) \left( -\int_{-x_l}^{x_l} x f(x) \, dx \right) P\left( F(-x_l), N - 1, \frac{N - 1}{2} \right) \\
+ \alpha \left( \int_{x_h}^{\infty} x f(x) \, dx \right) P\left( F(x_h), N - 1, \frac{N - 1}{2} \right).
\]

(9)

This expression reflects the two effects familiar from the discussion of Lemma 1. The first term in (9) captures the expected common value of investors: the term \( \alpha \left( p - \frac{1}{2} \right) \) is the expected common value if the decision were made according to the fund manager’s vote, and this term is multiplied by the probability that the overall voting outcome coincides with the fund manager’s vote.\(^\text{13}\) Since the fund manager always votes in favor in this case, investors with a positive private value are indifferent between delegating and not (in both cases, their vote is always cast in favor of the proposal), and the vote outcome under the delegation strategy \([x_l, x_h] = [0, 2p - 1]\) is exactly the same as the vote outcome when no investor delegates his vote.
vote. For fixed $x_l, x_h$, if the fund manager is more biased and votes for the proposal more often ($\alpha$ is lower), the common value of investors is lower. The second component of investor welfare, represented by the sum of the second and third terms in (9), captures the expected private value of investors. As in the case of an unbiased manager, delegation has two direct effects on investor welfare: for any fixed $\alpha$, more delegation (lower $x_l$ and higher $x_h$) increases the common value component, but decreases the private value component.

Expression (9) shows that when the fund manager is biased, delegation also has an indirect effect on investor welfare through $\alpha$: the fund manager’s voting strategy, and in particular the probability of voting against upon receiving a negative signal, depends on investors’ delegation strategy $(x_l, x_h)$. In other words, delegation changes the fund manager’s incentives to vote in line with her signal. We refer to this indirect effect as the “incentive effect” of delegation.

To understand the incentive effect, recall that the fund manager gets $w > 0$ if the proposal is accepted. If this were the only factor that mattered for the fund manager, she would always vote for the proposal, i.e., the cutoff $w^*$ would be infinite and $\alpha$ would be zero. However, the fund manager also cares about the expected welfare of her investors, which encourages her to vote in line with her signal if $w$ is not too large. We argue that this incentive effect is stronger when all votes are delegated to the fund manager, i.e., $(x_l, x_h) = (-\infty, \infty)$, compared to the case when only investors with $x_i \in (x_l, x_h)$ delegate their votes. In other words, the fund manager votes more informatively and in a less biased way when she casts all investors’ votes than when investors have voting choice and may choose to retain their votes:

**Lemma 2.** The fund manager is more likely to vote according to her signal under complete delegation than under voting choice.

To understand this result, note that the fund manager only cares about the scenario in which her vote is pivotal, i.e., when the votes of investors who do not delegate their votes are split. In this scenario, the number of investors with positive private values from the proposal ($x_i > 0$) equals the number of investors with negative private values ($x_i < 0$). While these two groups of investors have the same size, the intensity of their preferences is different: as shown above, knowing that the fund manager is biased towards the proposal, investors biased towards the proposal will only retain their votes if their bias is very strong ($x_i > x_h$), whereas investors biased against the proposal will generally retain their votes even if their bias is moderate ($x_i < x_l$, where $-x_l < x_h$). Hence, conditional on a split vote among non-delegating

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investors, the average bias of the fund’s investors is positive, which induces the fund manager to vote in favor of the proposal. In some sense, the optimal response of fund investors to the fund manager’s bias further exacerbates this bias given the fund manager’s concern about her investors’ welfare. More specifically, as the proof of Lemma 2 shows, under complete delegation, the fund manager votes against if and only if \( w \leq 2p - 1 \), whereas under voting choice, she votes against if \( w \leq w^* \), where \( w^* < 2p - 1 \).

4 Heterogeneous information

Next, we consider the case in which all fund investors and the fund manager have the same preferences \((x_i = 0, w = 0)\) but investors have private signals \(\sigma_i\) with precision \(\pi > \frac{1}{2}\) about the proposal. We assume that \(p > \pi\), i.e., signal \(s\) of the fund manager is more precise than the signal of each investor, so that there is a reason to delegate votes to her. If \(p < \pi\), the problem would be trivial: no investor would delegate voting to the fund manager if voting choice were offered and giving investors voting choice would be optimal.\(^\text{14}\)

Consider a symmetric equilibrium in which each investor delegates voting to the fund manager with probability \(q_d\). With probability \(1 - q_d\), the investor votes based on his own signal. For a given \(q_d\), we can calculate the investor’s value from delegating and not delegating, \(V_{nd}(q_d)\) and \(V_d(q_d)\), assuming that each other investor delegates with probability \(q_d\):

\[
V_{nd}(q_d) = \left(\pi - \frac{1}{2}\right)(p\Omega_1(q_d) + (1 - p)\Omega_0(q_d)),
\]

\[
V_d(q_d) = \frac{1}{2}(p\Omega_1(q_d) - (1 - p)\Omega_0(q_d)),
\]

where \(\Omega_1(q_d)\) and \(\Omega_0(q_d)\) are the probabilities that investor \(i\) is pivotal when \(s = \theta\) (the fund manager’s signal is correct) and \(s \neq \theta\), respectively:

\[
\Omega_1(q_d) = P\left(q_d + (1 - q_d)\pi, N - 1, \frac{N - 1}{2}\right),
\]

\[
\Omega_0(q_d) = P\left((1 - q_d)\pi, N - 1, \frac{N - 1}{2}\right).
\]

\(^{14}\)The system in which all investors vote themselves would dominate the system where the fund manager casts all the votes for two reasons. First, each individual investor’s signal is more informative than that of the fund manager. Second, since investors’ signals are conditionally independent, such system features the “wisdom of the crowd” effect, whereby the idiosyncratic errors in individual investors’ signals cancel out in the aggregate vote outcome.
In equilibrium, \( q_d \) is such that each investor is indifferent between delegating his vote to the fund manager and voting himself, \( V_{nd}(q_d) = V_d(q_d) \). Denote the equilibrium value by \( q^*_d \).

The next result shows that unlike the case of heterogeneous preferences (where the equilibrium generally features either under- or over-delegation), the voting choice equilibrium under homogeneous preferences implements the optimal level of delegation – one that maximizes expected investor welfare:

**Proposition 3.** The equilibrium probability of delegation \( (q^*_d) \) coincides with the optimal probability of delegation, i.e., one that maximizes expected investor welfare. Voting choice thus dominates both complete delegation and mandatory pass-through voting.

The intuition for this result is that fund investors have common interests: their preferences are fully aligned, and they also do not incur any private costs when they decide to delegate their votes to the fund or to vote themselves. Hence, this result is related to the general property of common interest games that a strategy profile that maximizes the combined ex-ante utility of the players is a Nash equilibrium (McLennan, 1998). In our context it means that the efficient combination of investors’ private signals and the fund manager’s signal is sustained in equilibrium. And, since voting choice features the optimal level of delegation, it dominates any other voting system, including complete delegation and mandatory pass-through voting.

It is useful to compare Proposition 3 with the results of Malenko and Malenko (2019), who consider shareholders’ choice between relying on the information from the proxy advisor and relying on their own private information. In their paper, shareholders rely on the proxy advisor either too much or too little, depending on the precision of the proxy advisor’s signal. In contrast, in this paper, shareholders’ choice between relying on the fund manager’s signal vs. their own signals implements the efficient level of delegation. The reason for these different conclusions is the following. In our setting, information is costless, whereas in Malenko and Malenko (2019), information is costly to acquire: relying on the proxy advisor’s recommendations can be thought of as delegating voting to the proxy advisor, but at a cost equal to the advisor’s fee. As a result of the information acquisition costs, the game in Malenko and Malenko (2019) is not a common interests game, so inefficiencies arise. We next analyze how introducing costly information acquisition changes our conclusions.
4.1 Endogenous information acquisition

In the baseline model, fund investors and the fund manager are endowed with information. In reality, the quality of private signals is a result of the information collection process, and it may change depending on who casts the vote. Hence, we consider an extension in which information about the common value needs to be acquired at a cost.

The timeline is as follows. At date 1, the fund manager chooses precision \( p \in \left[ \frac{1}{2}, 1 \right] \) of her signal at a per-share cost \( c(p) \), and simultaneously each investor chooses whether to delegate his vote to the fund. Each investor who does not delegate chooses precision \( \pi \in \left[ \frac{1}{2}, 1 \right] \) of his private signal \( \sigma_i \) at cost \( \gamma(\pi) \) (of course, only investors who choose not to delegate will have incentives to acquire information). We assume that \( c(p) \) is a twice differentiable function satisfying \( c\left(\frac{1}{2}\right) = 0, \ c'(p) > 0, \lim_{p \to \frac{1}{2}^-} c'(p) = 0, \lim_{p \to 1} c'(p) = \infty, \) and \( c''(p) > 0 \). Since \( c(p) \) is the per-share cost, the total cost of information acquisition by the fund manager is \( Nc(p) \). We assume that investors’ cost function \( \gamma(\pi) \) satisfies the same properties as \( c(p) \). At date 2, all agents observe their private signals, and the votes are cast. Since the model is symmetric in states and signals, we look for equilibria that are symmetric around the state.

Let \( q_d \) denote the probability with which an investor delegates voting to the fund manager. Since the fund manager controls a block of shares, there are multiple events in which her vote is pivotal for the outcome. To make the analysis tractable, we make two assumptions. First, the fund manager cannot split votes, i.e., she votes all shares in the same direction, which corresponds to the observed voting practices of asset managers.\(^{15}\) Second, the fund manager casts votes before learning the exact number of investors that delegate (i.e., she knows \( q_d \) but not the realization \( K \in [0, N] \) of investors that delegated). These assumptions are made for simplicity, and we do not think that they are critical for the insights in this section.

In this setting, there is a new externality that an investor’s delegation decision has: it affects information acquisition by the fund manager. Less frequent delegation reduces the incentives of the fund manager to acquire precise information. Intuitively, if the fund manager expects to vote on behalf of fewer investors, her vote matters less, so she has weaker incentives to become informed. This creates a feedback loop between investors’ delegation decisions and the fund manager’s information acquisition. If an investor expects other investors to delegate voting to the fund manager with a very high probability, he expects the fund manager to engage in information acquisition (since the fund is expected to control many votes) and the fund’s vote to largely determine the vote outcome. Hence, the benefits from unilaterally exercising

\(^{15}\)Bar-Isaac and Shapiro (2020) analyze strategic voting when the blockholder can split his votes.
voting choice are very low, since the likelihood of a pivotal outcome is very low and information acquisition is costly. At the other extreme, if an investor expects all other investors to vote themselves, he rationally concludes that the fund manager will not engage in costly information production, and thus the benefits from delegating votes are low. This feedback loop leads to multiple equilibria, which are characterized by the next result.

**Proposition 4.** The set of equilibria under voting choice is as follows.

1. There always exists an equilibrium in which all investors delegate voting to the fund manager, \( q_d = 1 \). In this case,  \( \pi = \frac{1}{2} \) and  \( p = c^{f-1}(f) \).

2. There always exists an equilibrium in which no investor delegates voting to the fund manager, \( q_d = 0 \). In this case,  \( p = \frac{1}{2} \) and  \( \pi \) is given by (27) in the appendix.

3. There can exist an equilibrium in which some investors delegate voting and some vote themselves: \( q_d \in (0, 1) \). In this case,  \( \pi \in (\frac{1}{2}, 1) \) and  \( p \in (\frac{1}{2}, 1) \). Equilibrium parameters \( q_d, \pi, \) and  \( p \) satisfy (22), (23), and (26) in the appendix.

The case in which voting choice is not permitted is equivalent to the equilibrium with \( q_d = 1 \) in the proposition. Thus, if the same equilibrium is played under voting choice, then voting choice makes no difference. In contrast, if one of the other two types of equilibria are played, then voting choice changes investor welfare. If investors can coordinate on the equilibrium with the highest investor welfare, voting choice is weakly beneficial, echoing the conclusion of the baseline model without information acquisition. However, if one is sceptical about such efficient coordination and is worried about coordination failure, then voting choice can lead to worse outcomes. The next proposition shows this result formally:

**Proposition 5.** Let \( \tau \in (\underline{\tau}, \bar{\tau}) \) be the parameter of the cost function \( \gamma(\pi, \tau) \) that satisfies: (i) for a fixed \( \tau \), \( \gamma(\pi, \tau) \) as a function of \( \pi \) satisfies all properties of the cost function introduced above; (ii) for any \( \pi \in (\frac{1}{2}, 1) \) and  \( \tau \in (\underline{\tau}, \bar{\tau}) \), \( \frac{\partial^2}{\partial \pi \partial \tau} \gamma(\pi, \tau) > 0 \); (iii) \( \lim_{\tau \to \underline{\tau}} \frac{\partial}{\partial \tau} \gamma(\pi, \tau) = 0 \) and \( \lim_{\tau \to \bar{\tau}} \frac{\partial}{\partial \tau} \gamma(\pi, \tau) = \infty \) for any \( \pi \in (\frac{1}{2}, 1) \). Then:

1. if \( \tau \) is sufficiently low, there is an equilibrium under voting choice that Pareto dominates the equilibrium under complete delegation in the sense of delivering both higher investor welfare and higher expected utility to the fund manager.
2. If $\tau$ is sufficiently high, there is an equilibrium under voting choice that is Pareto-inferior to the equilibrium under complete delegation in the sense of delivering both lower investor welfare and lower expected utility to the fund manager.

Intuitively, if $\tau$ is low, investors’ information acquisition technology is efficient compared to that of the fund manager. In this case, the equilibrium in which all investors collect their own signals and vote based on them (which, as Proposition 4 shows, always exists under voting choice) leads to more informed voting outcomes than if all votes were delegated to the fund and it acquired relatively imprecise information. This is especially so because investors’ private signals are conditionally independent, allowing for the “wisdom of the crowd” effect, whereas under complete delegation, the voting outcome entirely depends on the fund’s signal. In contrast, if $\tau$ is high enough, the fund’s information acquisition technology is more efficient than that of investors, so delegation of voting leads to more informed decisions than if investors were acquiring information privately and voting based on it. However, under voting choice, there is a possibility of coordination failure: if none of the votes are delegated to the fund, the fund will not invest in information acquisition, even if it were efficient and relatively cheap to do so. Such an equilibrium exists under voting choice (Proposition 4) and features lower investor welfare than the equilibrium in which voting choice is not offered.

5 Conclusion

The growing concentration of voting power among several large asset managers, combined with the increasing prevalence of E&S-related proposals and investor disagreements over them, has generated a heated debate and concerns about large asset managers’ voting power. These concerns have led to several policy proposals, including the INDEX Act, and have encouraged major fund managers to offer “voting choice” to their clients. Under voting choice, investors of the fund can choose whether to delegate their votes to the fund or to exercise their voting rights themselves. The goal of this paper is to study the implications of voting choice for investor welfare and to analyze the effectiveness of related policy proposals.

We show that whether voting choice is beneficial for investors crucially depends on whether investors retain their votes because of their private preferences or their private information. If the reason for offering voting choice is that investors have heterogeneous preferences, but investors are uninformed about the value of the proposal, then the equilibrium under voting choice is generally inefficient: it features either too little or too much delegation. Greater
heterogeneity in investors’ preferences does not necessarily make voting choice more desirable, as it may lead to excessive retention of voting rights by investors and underutilization of the fund manager’s information. Moreover, the form of preference heterogeneity matters: depending on whether moderate investors, or only the more extreme investors, become more concerned about their private values, voting choice can either dominate or be dominated by complete delegation, and it may even be inferior to mandatory pass-through voting.

In contrast, if the reason for offering voting choice is that investors have information about the proposal that the fund manager does not have, but all investors’ preferences are aligned, then voting choice is efficient: the equilibrium level of delegation is the one that maximizes investor welfare. Hence, voting choice offers an improvement over both complete delegation and mandatory pass-through voting proposed by the INDEX Act. However, if information acquisition is costly, voting choice can also lead to coordination failure: if too few votes are delegated to the fund, the fund has weak incentives to acquire information, which discourages delegation even further and may result in insufficiently informed voting outcomes.
References


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Appendix

Proof of Lemma 1.

We derive the expected welfare of investors for the more general case, which also applies to the case of a biased fund manager. Suppose each investor delegates his vote to the fund manager if and only if \( x_i \in (x_l, x_h) \), and the fund manager votes against the proposal upon receiving a negative signal with probability \( \alpha \). We derive the expected welfare of fund investors, \( U(x_l, x_h, \alpha) \) for any possible \( x_l, x_h \), and \( \alpha \). The case of an unbiased fund manager corresponds to \( x_l = -x_h \) and \( \alpha = 1 \). Denote \( v_{FM} \) the vote of the fund manager: \( v_{FM} = 1 \) (\( v_{FM} = 0 \)) corresponds to voting for (against) the proposal.

First, consider the expected welfare of investors conditional on \( v_{FM} = 1 \). In this case, the fund manager votes for the proposal, and hence a randomly drawn investor with preference parameter \( x \) votes for the proposal either if he delegates his vote to the fund manager, i.e., \( x_l \leq x \leq x_h \), or if he does not delegate but his private value is \( x > x_h \). Hence, a randomly drawn investor votes in favor if and only if if \( x \geq x_l \), i.e., with probability \( \Pr(\theta = 1|v_{FM} = 1) = 1 - F(x_l) \).

The proposal is accepted if at least \( \frac{N+1}{2} \) votes are cast in favor, and conditional on \( k \) votes in favor, the common value to each of \( N \) investors is

\[
2 \Pr(\theta = 1|v_{FM} = 1) - 1 = \frac{2 \left( 1 - \frac{(1-p)\alpha}{2} \right)}{2 - \alpha} - 1 = \frac{2 - 2\alpha + 2p\alpha - 2 + \alpha}{2 - \alpha} = \frac{\alpha (2p - 1)}{2 - \alpha},
\]

whereas the sum of all investors’ private values is \( k \mathbb{E}[x|x \geq x_l] + (N - k) \mathbb{E}[x|x < x_l] \), where the first term comes from \( k \) investors who vote for (with \( x_i \geq x_l \)) and the second term comes from \( N - k \) investors who vote against (with \( x_i < x_l \)). If the proposal is rejected, then all investors’ common values and private values are zero. Hence, the expected investor welfare in this case is given by

\[
U(x_l, x_h, \alpha|v_{FM} = 1) = \sum_{k=\frac{N+1}{2}}^{N} \frac{N!}{k!(N-k)!} \left( 1 - F(x_l) \right)^k F(x_l)^{N-k} \left( \frac{N \alpha (2p - 1)}{2 - \alpha} + k \mathbb{E}[x|x \geq x_l] \right),
\]

where

\[
\mathbb{E}[x|x \geq x_l] = \frac{1}{1-F(x_l)} \int_{x_l}^{\infty} x f(x) \, dx,
\]

\[
\mathbb{E}[x|x < x_l] = \frac{1}{F(x_l)} \int_{-\infty}^{x_l} x f(x) \, dx.
\]
Hence,
\[
U (x_l, x_h, \alpha \mid v_{FM} = 1) = \left( \sum_{k=\frac{N+1}{2}}^{N} P \left( 1 - F(x_l) \right) N, k \right) \frac{N^{\alpha(2p-1)}}{2-\alpha} \\
+ N \left( \int_{-x_l}^{x_l} x f (x) \, dx \right) \left( \sum_{k=\frac{N+1}{2}}^{N} \frac{(N-1)!}{(k-1)!(N-k)!} \left( 1 - F(x_l) \right)^{k-1} F(x_l)^{N-k} \right) \\
+ N \left( \int_{-\infty}^{-x_l} x f (x) \, dx \right) \left( \sum_{k=\frac{N+1}{2}}^{N} \frac{(N-1)!}{k!(N-k)!} \left( 1 - F(x_l) \right)^k F(x_l)^{N-k-1} \right).
\]

Note that
\[
\int_{x_l}^{\infty} x f (x) \, dx = \int_{-x_l}^{-x_l} x f (x) \, dx + \int_{-\infty}^{x_l} x f (x) \, dx = \int_{-\infty}^{\infty} x f (x) \, dx = - \int_{-\infty}^{x_l} x f (x) \, dx,
\]
and hence,
\[
U (x_l, x_h, \alpha \mid v_{FM} = 1) = \left( \sum_{k=\frac{N+1}{2}}^{N} P \left( 1 - F(x_l) \right) N, k \right) \frac{N^{\alpha(2p-1)}}{2-\alpha} \\
- N \left( \int_{-\infty}^{x_l} x f (x) \, dx \right) \left( \sum_{k=\frac{N+1}{2}}^{N} \frac{(N-1)!}{(k-1)!(N-k)!} \left( 1 - F(x_l) \right)^{k-1} F(x_l)^{N-k} \right) \\
+ N \left( \int_{-\infty}^{x_l} x f (x) \, dx \right) \left( \sum_{k=\frac{N+1}{2}}^{N} \frac{(N-1)!}{k!(N-k)!} \left( 1 - F(x_l) \right)^k F(x_l)^{N-k-1} \right).
\]

Consider the last two terms:
\[
N \left( \int_{-\infty}^{x_l} x f (x) \, dx \right) \left( \sum_{k=\frac{N+1}{2}}^{N} \frac{(N-1)!}{(k-1)!(N-k)!} \left( 1 - F(x_l) \right)^{k-1} F(x_l)^{N-k} \right) \\
- N \left( \int_{-\infty}^{x_l} x f (x) \, dx \right) \left( \sum_{k=\frac{N+1}{2}}^{N} \frac{(N-1)!}{k!(N-k)!} \left( 1 - F(x_l) \right)^k F(x_l)^{N-k-1} \right)
\]
\[
= N \left( \int_{-\infty}^{x_l} x f (x) \, dx \right) \left( -P \left( 1 - F(x_l), N-1, \frac{N-1}{2} \right) \left( 1 - F(x_l) \right)^{N-1} F(x_l)^{N-2} \right) \\
- N \left( \int_{-\infty}^{x_l} x f (x) \, dx \right) P \left( 1 - F(x_l), N-1, \frac{N-1}{2} \right).
\]

Therefore,
\[
\frac{U(x_l, x_h, \alpha \mid v_{FM} = 1)}{N} = \left( \sum_{k=\frac{N+1}{2}}^{N} P \left( 1 - F(x_l) \right) N, k \right) \frac{\alpha(2p-1)}{2-\alpha} \\
- \left( \int_{-\infty}^{x_l} x f (x) \, dx \right) P \left( 1 - F(x_l), N-1, \frac{N-1}{2} \right).
\]

Second, consider the expected welfare of investors conditional on \(v_{FM} = 0\). In this case, the fund manager votes against the proposal, and hence a randomly drawn investor votes against the proposal either if he delegates his vote to the fund manager, i.e., \(x_l \leq x \leq x_h\), or if he does not delegate but his private value is \(x < x_l\). Hence, a randomly drawn investor votes for the proposal if and only if \(x \geq x_h\), i.e., with probability \(\Pr(x \geq x_h) = 1 - F(x_h)\). The proposal is accepted if at least \(\frac{N+1}{2}\) votes are cast in favor, and conditional on \(k\) votes in favor, the common value to each of \(N\) investors is
\[
2 \Pr(\theta = 1 \mid v_{FM} = 0) - 1 = 2 \Pr(\theta = 1 \mid s = 0) - 1 \\
= 2(1-p) - 1 = 1 - 2p,
\]

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whereas the sum of all investors’ private values is \( k \mathbb{E} [x|x \geq x_h] + (N - k) \mathbb{E} [x|x < x_h] \), where the first term comes from \( k \) investors who vote for (with \( x_i \geq x_h \)) and the second term comes from \( N - k \) investors who vote against (with \( x_i < x_h \)). If the proposal is rejected, then all investors’ common values and private values are zero. Hence, the expected investor welfare in this case is given by

\[
U (x_l, x_h, \alpha | v_{FM} = 0) = \sum_{k=\frac{N+1}{2}}^{N} \frac{N!}{k! (N-k)!} (1 - F (x_h))^k F (x_h)^{N-k} \left[ N(1-2p) + k \mathbb{E} [x|x \geq x_h] + (N-k) \mathbb{E} [x|x < x_h] \right].
\]

Repeating the derivations for \( v_{FM} = 1 \), we get

\[
\frac{U(x_l, x_h, \alpha | v_{FM}=0)}{N} = \left( \sum_{k=\frac{N+1}{2}}^{N} P \left( 1 - F (x_h), N, k \right) \right) (1 - 2p) - \left( \int_{-\infty}^{x_h} x f (x) \, dx \right) P \left( 1 - F (x_h), N - 1, \frac{N-1}{2} \right)
\]

\[
= \left( \sum_{k=\frac{N+1}{2}}^{N} P \left( 1 - F (x_h), N, k \right) \right) (1 - 2p) + \left( \int_{x_h}^{\infty} x f (x) \, dx \right) P \left( 1 - F (x_h), N - 1, \frac{N-1}{2} \right).
\]

Finally, we combine these cases together to get the unconditional expected investor welfare:

\[
\frac{U(x_l, x_h, \alpha)}{N} = \left( 2 - \alpha \right) \left( \frac{\sum_{k=\frac{N+1}{2}}^{N} P \left( 1 - F (x_l), N, k \right) \frac{\alpha(2p-1)}{2-\alpha}}{\sum_{k=0}^{N} P \left( 1 - F (x_l), N, k \right)} \right) \left( \int_{-\infty}^{x_l} x f (x) \, dx \right) P \left( 1 - F (x_l), N - 1, \frac{N-1}{2} \right)
\]

\[
+ \frac{\alpha}{2} \left( \sum_{k=\frac{N+1}{2}}^{N} P \left( 1 - F (x_h), N, k \right) (1 - 2p) + \left( \int_{x_h}^{\infty} x f (x) \, dx \right) P \left( 1 - F (x_h), N - 1, \frac{N-1}{2} \right) \right).
\]

Using the expression for \( P (y, N, k) \), note that

\[
\sum_{k=\frac{N+1}{2}}^{N} P \left( 1 - F (x_l), N, k \right) = \sum_{k=0}^{\frac{N-1}{2}} P \left( 1 - F (x_l), N, k \right) = 1 - \sum_{k=\frac{N+1}{2}}^{N} P \left( 1 - F (x_l), N, k \right)
\]

and that \( P \left( 1 - F (x_l), N - 1, \frac{N-1}{2} \right) = P \left( 1 - F (x_l), N - 1, \frac{N-1}{2} \right) \). Hence, we can simplify (10) as

\[
\frac{U(x_l, x_h, \alpha)}{N} = \left( \sum_{k=\frac{N+1}{2}}^{N} P \left( F (-x_l), N, k \right) \frac{\alpha(2p-1)}{2} \right) \left( \int_{-\infty}^{x_l} x f (x) \, dx \right) P \left( F (-x_l), N - 1, \frac{N-1}{2} \right)
\]

\[
- \left( 1 - \sum_{k=\frac{N+1}{2}}^{N} P \left( F (x_h), N, k \right) \right) \left( \frac{\alpha(2p-1)}{2} \right) + \frac{\alpha}{2} \left( \int_{x_h}^{\infty} x f (x) \, dx \right) P \left( F (x_h), N - 1, \frac{N-1}{2} \right).
\]
or equivalently,
\[
\frac{U(x_l, x_h, \alpha)}{N} = \left( \sum_{k=\frac{N+1}{2}}^{N} P(F(-x_l), N, k) + \sum_{k=\frac{N+1}{2}}^{N} P(F(x_h), N, k) - 1 \right) \frac{\alpha(2p-1)}{2} - \left( \frac{2-\alpha}{2} \right) \left( \int_{-\infty}^x x f(x) dx \right) P(F(-x_l), N - 1, \frac{N-1}{2}) + \frac{\alpha}{2} \left( \int_{x_h}^{\infty} x f(x) dx \right) P(F(x_h), N - 1, \frac{N-1}{2}).
\]
(13)

When \(-x_l = x_h = \hat{x}\) and \(\alpha = 1\), (13) becomes (6), which completes the proof.

**Proof of Proposition 1.** Consider the derivative of the investor’s expected utility in \(\hat{x}\):

\[
2 \left( \sum_{k=\frac{N+1}{2}}^{N} P' (G(\hat{x}), N, k) \right) \left( p - \frac{1}{2} \right) + P'(G(\hat{x}), N - 1, \frac{N-1}{2}) \int_{\hat{x}}^{\infty} x g(x) dx = \hat{x}P(G(\hat{x}), N - 1, \frac{N-1}{2}).
\]
(14)

When \(F\) is substituted with \(G\), this derivative equals zero when evaluated at \(\hat{x} = \hat{x}^*(F)\) by the first-order condition. Evaluating (14) at \(\hat{x}^*(F)\):

\[
2 \left( \sum_{k=\frac{N+1}{2}}^{N} P' (G(\hat{x}^*(F)), N, k) \right) \left( p - \frac{1}{2} \right) + P'(G(\hat{x}^*(F)), N - 1, \frac{N-1}{2}) \int_{\hat{x}^*(F)}^{\infty} x g(x) dx - \hat{x}^* (F) P(G(\hat{x}^*(F)), N - 1, \frac{N-1}{2})
\]
\[
= 2 \left( \sum_{k=\frac{N+1}{2}}^{N} P' (F(\hat{x}^*(F)), N, k) \right) \left( p - \frac{1}{2} \right) + P'(F(\hat{x}^*(F)), N - 1, \frac{N-1}{2}) \int_{\hat{x}^*(F)}^{\infty} x g(x) dx - \hat{x}^* (F) P(F(\hat{x}^*(F)), N - 1, \frac{N-1}{2})
\]
\[
= P'(F(\hat{x}^*(F)), N - 1, \frac{N-1}{2}) \int_{\hat{x}^*(F)}^{\infty} x (g(x) - f(x)) dx < 0.
\]

The first equality holds because \(G(\hat{x}^*(F)) = F(\hat{x}^*(F))\). The second equality holds because the derivative is zero at \(\hat{x} = \hat{x}^*(F)\) for distribution \(F\). Finally, the inequality holds because \(P'(q, N - 1, \frac{N-1}{2}) < 0\) for \(q > \frac{1}{2}\) and \(\int_{\hat{x}^*(F)}^{\infty} x (g(x) - f(x)) dx > 0\) by first-order stochastic dominance. Therefore, \(\hat{x}^*(G) < \hat{x}^*(F)\). An analogous argument applies if \(G(x|x \geq \hat{x}^*(F))\) is dominated by \(F(x|x \geq \hat{x}^*(F))\) in the sense of first-order stochastic dominance.

**Proof of Proposition 2.** To prove part (i), we subtract the expected investor welfare under complete delegation \((2p-1)\) from (6) for \(\hat{x} = 2p - 1\) and divide by \(P(F(2p - 1), N - 1, \frac{N-1}{2})\):

\[
\Delta = -D(F(2p - 1))(2p - 1) + \left( \int_{2p-1}^{\infty} x f(x) dx \right),
\]
(15)

where

\[
D(q) = \frac{\sum_{k=0}^{\frac{N-1}{2}} P(q, N, k) - P(q, N - 1, \frac{N-1}{2})}{P(q, N - 1, \frac{N-1}{2})}
\]

Voting choice dominates full delegation if and only if \(\Delta > 0\). It follows that \(\Delta > 0\) if and only if \(\int_{2p-1}^{\infty} x f(x) dx > D(F(2p - 1))(2p - 1)\).

We next prove part (ii). By part (i), voting choice results in higher expected investor welfare than complete delegation when \(\int_{2p-1}^{\infty} x f(x) dx\) is sufficiently high, so it is sufficient to prove
that mandatory pass-through voting results in higher expected investor welfare than voting choice. The difference in the two expected utilities is given by:

\[
P(\frac{1}{2}, N - 1, \frac{N-1}{2}) (\int_0^\infty x f(x) \, dx) - \left(2 \sum_{k=\frac{N+1}{2}}^N P(F(2p-1), N, k) - 1\right) (p - \frac{1}{2}) - P(F(2p-1), N - 1, \frac{N-1}{2}) \left(\int_{2p-1}^\infty x f(x) \, dx\right)
\]

\[
> \left(\int_{2p-1}^\infty x f(x) \, dx\right)^2
\]

where the first inequality follows from \(\int_0^\infty x f(x) \, dx > \int_{2p-1}^\infty x f(x) \, dx\). Since \(P(\frac{1}{2}, N - 1, \frac{N-1}{2}) > P(q, N - 1, \frac{N-1}{2})\) for any \(q > \frac{1}{2}\) and \(F(2p-1) > 0\), the expression in the last two lines of (16) is strictly positive if \(\int_{2p-1}^\infty x f(x) \, dx\) is sufficiently high.

**Characterizing the equilibrium under voting choice for the case of a biased fund manager.**

Suppose that conditional on \(s = 0\), the fund manager votes against the proposal with probability \(\alpha\). Denote \(v_{FM}\) the fund manager’s vote, where \(v_{FM} = 0\) \((v_{FM} = 1)\) corresponds to a vote in favor (against) the proposal. By Bayes’ rule,

\[
Pr(\theta = 0|v_{FM} = 0) = \Pr(\theta = 0|s = 0) = p
\]

\[
Pr(\theta = 1|v_{FM} = 1) = \frac{Pr(v_{FM} = 1|\theta = 1)}{Pr(v_{FM} = 1|\theta = 1) + Pr(v_{FM} = 1|\theta = 0)}
\]

\[
= \frac{pPr(v_{FM} = 1|s = 1) + (1 - p) Pr(v_{FM} = 1|s = 0)}{Pr(v_{FM} = 1|s = 1) + Pr(v_{FM} = 1|s = 0)}
\]

\[
= \frac{p + (1 - p)(1 - \alpha)}{2 - \alpha} = \frac{1 - (1 - p)\alpha}{2 - \alpha} \in \left[\frac{1}{2}, p\right].
\]

In particular, if \(\alpha = 1\), \(Pr(\theta = 1|v_{FM} = 1) = p\), and if \(\alpha = 0\), \(Pr(\theta = 1|v_{FM} = 1) = \frac{1}{2}\).

We solve for the equilibrium in three steps.

1. **Investors’ delegation decisions as a function of the fund manager’s strategy \(\alpha\).**

Consider investor \(i\) with preference parameter \(x_i\), who is deciding whether to delegate his vote to the fund manager or not. The investor’s delegation decision only matters when the investor’s vote is pivotal, so the investor optimally conditions his decision on the information that is true in the event of him being pivotal (we denote this event by \(Piv_i\)). If the investor delegates, he gets

\[
Pr(v_{FM} = 1|Piv_i) (\mathbb{E}[u(1, \theta)|v_{FM} = 1, Piv_i] + x_i).
\]
We next calculate \( \Pr (v_{FM} = 1 | Piv_i) \) and \( \mathbb{E} [u(1, \theta) | v_{FM} = 1, Piv_i] \) given \( x_l \) and \( x_h \). Note that

\[
\Pr (v_{FM} = 1 | Piv_i) = \frac{\Pr (Piv_i | v_{FM} = 1) \Pr (v_{FM} = 1)}{\Pr (Piv_i | v_{FM} = 1) \Pr (v_{FM} = 1) + \Pr (Piv_i | v_{FM} = 0) \Pr (v_{FM} = 0)},
\]

where

\[
\Pr (v_{FM} = 1) = \frac{1}{2} + \frac{1}{2} (1 - \alpha) = 1 - \frac{\alpha}{2},
\]

\[
\Pr (v_{FM} = 0) = \frac{\alpha}{2},
\]

and

\[
\Pr (Piv_i | v_{FM} = 1) = C_{N-1}^{N-1} \left( (1 - F(x_l)) F(x_l) \right)^{\frac{N-1}{2}},
\]

\[
\Pr (Piv_i | v_{FM} = 0) = C_{N-1}^{N-1} \left( (1 - F(x_h)) F(x_h) \right)^{\frac{N-1}{2}}.
\]

Hence,

\[
\Pr (v_{FM} = 1 | Piv_i) = \frac{((1 - F(x_l)) F(x_l))^{\frac{N-1}{2}} \left( 1 - \frac{\alpha}{2} \right)}{((1 - F(x_l)) F(x_l))^{\frac{N-1}{2}} \left( 1 - \frac{\alpha}{2} \right) + ((1 - F(x_h)) F(x_h))^{\frac{N-1}{2}} \frac{\alpha}{2}}
\]
\[
= \frac{1 - \frac{\alpha}{2}}{1 - \frac{\alpha}{2} + \left( \frac{1 - F(x_h) F(x_h)}{(1 - F(x_l)) F(x_l)} \right)^{\frac{N-1}{2}} \frac{\alpha}{2}}.
\]

Denoting

\[
\left( \frac{(1 - F(x_h)) F(x_h)}{(1 - F(x_l)) F(x_l)} \right)^{\frac{N-1}{2}} = K \in [0, \infty],
\]

we get

\[
\Pr (v_{FM} = 1 | Piv_i) = \frac{2 - \alpha}{2 + (K - 1) \alpha}.
\]

Next,

\[
\mathbb{E} [u(1, \theta) | v_{FM} = 1, Piv_i] = 2 \Pr (\theta = 1 | v_{FM} = 1, Piv_i) - 1 = 2 \Pr (\theta = 1 | v_{FM} = 1) - 1
\]
\[
= 2 \frac{1 - (1 - p) \alpha}{2 - \alpha} - 1 = \frac{\alpha (2p - 1)}{2 - \alpha} - 1,
\]

where the second equality is due to the fact that only the fund manager has a signal informative about \( \theta \), so state-relevant information from \( Piv_i \) is subsumed by \( v_{FM} = 1 \). Hence, the payoff of investor \( i \) from delegation is:

\[
\frac{2 - \alpha}{2 + (K - 1) \alpha} \left( \frac{\alpha (2p - 1)}{2 - \alpha} + x_i \right).
\]

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The payoff of investor $i$ from not delegating and voting for the proposal is

$$
\mathbb{E}[u(1, \theta) | P_i v_i] + x_i = \Pr(v_{FM} = 1 | P_i v_i) \mathbb{E}[u(1, \theta) | P_i v_i, v_{FM} = 1] + \Pr(v_{FM} = 0 | P_i v_i) \mathbb{E}[u(1, \theta) | P_i v_i, v_{FM} = 0] + x_i
$$

$$
= \frac{2 - \alpha}{2 + (K - 1) \alpha} \frac{\alpha (2p - 1)}{2 - \alpha} + \frac{K \alpha}{2 + (K - 1) \alpha} \mathbb{E}[u(1, \theta) | s = 0] + x_i
$$

$$
= \frac{2 - \alpha}{2 + (K - 1) \alpha} \frac{\alpha (2p - 1)}{2 - \alpha} \frac{(2p - 1)}{2 + (K - 1) \alpha} + x_i
$$

$$
= \frac{\alpha (2p - 1)}{2 + (K - 1) \alpha} \frac{K \alpha (2p - 1)}{2 + (K - 1) \alpha} + x_i = \frac{(1 - K) \alpha (2p - 1)}{2 + (K - 1) \alpha} + x_i.
$$

The investor’s payoff from not delegating and voting against is zero.

It follows that an investor’s delegation decision is characterized by two cutoffs, $x_l$ and $x_h$, such that investor $i$ delegates his vote to the fund manager if and only if $x_i \in [x_l, x_h]$. In particular, for $x_i = x_l$, the investor must be indifferent between delegating and voting against, which gives

$$
\frac{2 - \alpha}{2 + (K - 1) \alpha} \frac{\alpha (2p - 1)}{2 - \alpha} + x_l = 0,
$$

and for $x_i = x_h$, the investor must be indifferent between delegating and voting for, which gives

$$
\frac{2 - \alpha}{2 + (K - 1) \alpha} \frac{\alpha (2p - 1)}{2 - \alpha} + x_h = \frac{(1 - K) \alpha (2p - 1)}{2 + (K - 1) \alpha} + x_h.
$$

From (17), we immediately get

$$
x_l = -\frac{\alpha (2p - 1)}{2 - \alpha}.
$$

From (18), we get

$$
\left(1 - \frac{2 - \alpha}{2 + (K - 1) \alpha}\right) x_h = \frac{2 - \alpha}{2 + (K - 1) \alpha} \frac{\alpha (2p - 1)}{2 - \alpha} - \frac{(1 - K) \alpha (2p - 1)}{2 + (K - 1) \alpha} = K x_h - (2p - 1) = K (2p - 1)
$$

$$
x_h = 2p - 1.
$$

Hence, the delegation region is $[x_l, x_h] = \left[-\frac{\alpha (2p - 1)}{2 - \alpha}, 2p - 1\right]$.

2. Characterizing the voting strategy of the fund manager

Suppose the fund manager gets signal $s = 0$ and expects each investor to delegate if $x_i \in [x_l, x_h]$. Recall that the fund manager’s utility equals the expected per-share utility of her $N$ clients plus a constant $w > 0$ if $d = 1$ is implemented. Then, if the fund manager votes in favor, $v_{FM} = 1$, her expected utility is:
\[
\frac{U(x_l, x_h|v_{FM} = 1, s = 0)}{N} + w \Pr (d = 1|v_{FM} = 1).
\]

By analogy with the derivation in the proof of Lemma 1,
\[
\frac{U(x_l, x_h|v_{FM}=1,s=0)}{N} = \left( \sum_{k=\frac{N+1}{2}}^{N} P(F(-x_l), N, k) \right) (1 - 2p) - \left( \int_{-\infty}^{x_l} x f(x) \, dx \right) P(F(-x_l), N - 1, \frac{N-1}{2})
\]
and \( \Pr (d = 1|v_{FM} = 1) = \sum_{k=\frac{N+1}{2}}^{N} P(F(-x_l), N, k) \). Hence, the fund manager’s expected utility from voting in favor is
\[
\left( \sum_{k=\frac{N+1}{2}}^{N} P(F(-x_l), N, k) \right) (1 - 2p + w) - \left( \int_{-\infty}^{x_l} x f(x) \, dx \right) P(F(-x_l), N - 1, \frac{N-1}{2}).
\]

If the fund manager chooses \( v_{FM} = 0 \), her expected utility is:
\[
\frac{U(x_l, x_h|v_{FM} = 0, s = 0)}{N} + w \Pr (d = 1|v_{FM} = 0).
\]

By analogy with the derivation in the proof of Lemma 1,
\[
\frac{U(x_l, x_h|v_{FM}=0,s=0)}{N} = \frac{U(x_l, x_h|v_{FM}=0)}{N} = \left( 1 - \sum_{k=\frac{N+1}{2}}^{N} P(F(x_h), N, k) \right) (1 - 2p) + \left( \int_{x_h}^\infty x f(x) \, dx \right) P(F(x_h), N - 1, \frac{N-1}{2})
\]
and \( \Pr (d = 1|v_{FM} = 0) = \sum_{k=\frac{N+1}{2}}^{N} P(1 - F(x_h), N, k) = 1 - \sum_{k=\frac{N+1}{2}}^{N} P(F(x_h), N, k) \). Hence, the fund manager’s expected utility from voting against is
\[
\left( 1 - \sum_{k=\frac{N+1}{2}}^{N} P(F(x_h), N, k) \right) (1 - 2p + w) + \left( \int_{x_h}^\infty x f(x) \, dx \right) P(F(x_h), N - 1, \frac{N-1}{2}).
\]

The fund manager finds it optimal to vote against upon \( s = 0 \) if and only if
\[
\left( \sum_{k=\frac{N+1}{2}}^{N} P(F(-x_l), N, k) \right) (1 - 2p + w) - \left( \int_{-\infty}^{x_l} x f(x) \, dx \right) P(F(-x_l), N - 1, \frac{N-1}{2}) \leq \left( 1 - \sum_{k=\frac{N+1}{2}}^{N} P(F(x_h), N, k) \right) (1 - 2p + w) + \left( \int_{x_h}^\infty x f(x) \, dx \right) P(F(x_h), N - 1, \frac{N-1}{2})
\]
or equivalently,
\[
\left( \sum_{k=\frac{N+1}{2}}^{N} P(F(-x_l), N, k) + \sum_{k=\frac{N+1}{2}}^{N} P(F(x_h), N, k) - 1 \right) (1 - 2p + w) \leq \left( \int_{x_h}^\infty x f(x) \, dx \right) P(F(x_h), N - 1, \frac{N-1}{2}) + \left( \int_{-\infty}^{x_l} x f(x) \, dx \right) P(F(-x_l), N - 1, \frac{N-1}{2}).
\]

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Hence, the cutoff \( w^* \) on informative voting is given by

\[
- \left( \int_{-\infty}^{x_l} x f(x) \, dx \right) P \left( F(-x_l), N - 1, \frac{N-1}{2} \right) - \left( \int_{x_h}^{\infty} x f(x) \, dx \right) P \left( F(x_h), N - 1, \frac{N-1}{2} \right) \\
= (2p - 1 - w^*) \left( \sum_{k=0}^{N} P \left( F(-x_l), N, k \right) + \sum_{k=0}^{N} P \left( F(x_h), N, k \right) - 1 \right),
\]

which defines \( w^* \) as a function of \( x_l \) and \( x_h \): \( w^*(x_l, x_h) \).

3. Finding the equilibrium \( \alpha \) as the fixed point

Finally, we note that in equilibrium, \( x_l \) and \( x_h \) are functions of \( \alpha \) given above, and \( \alpha = \Pr(w \leq w^*) = H(w^*(x_l, x_h)) \). Hence, the equilibrium \( \alpha \) is found as the solution to the fixed point equation:

\[
H^{-1}(\alpha) = w^* \left( \frac{\alpha (2p - 1)}{2 - \alpha}, 2p - 1 \right),
\]

which completes the characterization of the equilibrium under voting choice.

Proof of Lemma 2.

Consider the cutoff \( w^* \) on informative voting defined by (19). Under complete delegation, \( x_h = \infty \) and \( x_l = -\infty \). Hence, the left-hand side of (19) equals zero, and thus, for the right-hand side to also equal zero, \( w^* = 2p - 1 \). Under voting choice, \( -x_l < x_h \), and given the symmetry of \( F \),

\[
- \int_{-\infty}^{x_l} x f(x) \, dx = \int_{x_h}^{\infty} x f(x) \, dx > \int_{x_h}^{\infty} x f(x) \, dx.
\]

In addition, since \( -x_l > 0 \), we have \( \frac{1}{2} < F(-x_l) < F(x_h) \). The function \( P(z, N - 1, \frac{N-1}{2}) \) is decreasing in \( z \) for \( z > \frac{1}{2} \) because \( z(1-z) = 1-2z < 0 \) for \( z > \frac{1}{2} \). Thus, \( P(F(-x_l), N - 1, \frac{N-1}{2}) > P(F(x_h), N - 1, \frac{N-1}{2}) \), and hence the left-hand side of (19) is strictly positive. For the right-hand side to also be positive, \( w^* < 2p - 1 \), which implies less informative voting than under complete delegation.

Proof of Proposition 3. The equilibrium probability of an investor being pivotal:

\[
p \Omega_1(q^*_d) + (1-p) \Omega_0(q^*_d) = \frac{1}{2\pi - 1} \left( p \Omega_1(q^*_d) - (1-p) \Omega_0(q^*_d) \right).
\]

Malenko and Malenko (2019) show that holding the probability of pivotal constant, investor welfare is maximized when \( \Omega_1 = \Omega_0 \). Simplifying this expression, we get:

\[
p \left( 2\pi - 2 \right) \Omega_1(q^*_d) + (1-p) \left( 2\pi - 1 \right) \Omega_0(q^*_d) + (1-p) \Omega_0(q^*_d) = 0
\]

\[
(1-p) \pi \Omega_0(q^*_d) = p (1-\pi) \Omega_1(q^*_d) \iff \Omega_1(q^*_d) = \frac{\Omega_0(q^*_d)}{\Omega_0(q^*_d)} = \frac{(1-p) \pi}{p (1-\pi)}.
\]

Next consider the optimal \( q_d \), which maximizes expected investor welfare. Expected investor
welfare is given by
\[ \sum_{k = \frac{N+1}{2}}^{N} (p P (p_a, N, k) + (1 - p) P (p_d, N, k)) - \frac{1}{2}, \]
where
\[ p_a = q_d + (1 - q_d) \pi, \]
\[ p_d = (1 - q_d) \pi. \]

We can think about maximization over \( p_a \) and \( p_d \) subject to the constraint:
\[ 1 - \frac{p_d}{\pi} = \frac{p_a - \pi}{1 - \pi} \iff \pi (1 - \pi) - p_d (1 - \pi) = \pi (p_a - \pi) \]
\[ \iff \pi - p_d + \pi p_d = \pi p_a \iff \pi p_a + (1 - \pi) p_d = \pi. \]

Consider the following optimization problem:
\[
\max_{p_a, p_d} \sum_{k = \frac{N+1}{2}}^{N} (p P (p_a, N, k) + (1 - p) P (p_d, N, k))
\]
\[ \text{s.t. } \pi p_a + (1 - \pi) p_d \leq \pi \]

The Lagrangian is:
\[ \mathbf{L} = \sum_{k = \frac{N+1}{2}}^{N} (p P (p_a, N, k) + (1 - p) P (p_d, N, k)) + \lambda (\pi - \pi p_a - (1 - \pi) p_d), \]
and the first-order conditions are:
\[ \sum_{k = \frac{N+1}{2}}^{N} P_p (p_a, N, k) = \lambda \frac{\pi}{p}, \]
\[ \sum_{k = \frac{N+1}{2}}^{N} P_p (p_d, N, k) = \lambda \frac{1 - \pi}{1 - p}. \]

Since \( p > \pi \), we have \( \frac{\pi}{p} < 1 \) and \( \frac{1 - \pi}{1 - p} > 1 \). Therefore, \( p_a > p_d \). Dividing the two first-order conditions by each other:
\[ \frac{\sum_{k = \frac{N+1}{2}}^{N} P_p (p_a, N, k)}{\sum_{k = \frac{N+1}{2}}^{N} P_p (p_d, N, k)} = \frac{\pi (1 - p)}{p (1 - \pi)}. \]

Recall that the equilibrium satisfies:
\[ \frac{P (p_a, N - 1, \frac{N-1}{2})}{P (p_d, N - 1, \frac{N-1}{2})} = \frac{(1 - p) \pi}{p (1 - \pi)}. \]
Recall also that \( P(p, N, k) = C_N^k p^k (1 - p)^{N-k} \) and \( P_p(p, N, k) = P(p, N, k) \frac{k-Np}{p(1-p)} \), so
\[
\sum_{k=N+1}^{N} P_p(p, N, k) = \sum_{k=N+1}^{N} \frac{N!}{k!(N-k)!} p^{k-1} (1-p)^{N-k-1} (k-Np).
\]

From Malenko and Malenko (2019):
\[
L'(x) = \sum_{k=N+1}^{N} P_x(x, N, k) = -x(1-x) \left( \sum_{k=0}^{N} P_x(x, N, k) \right)
\]
\[
= -\frac{1}{x(1-x)} \left( \sum_{k=0}^{N} k P(x, N, k) - N x \sum_{k=0}^{N} P(x, N, k) \right).
\]

Note that \( \sum_{k=0}^{N} P(x, N, k) = I_{1-x} \left( \frac{N+1}{2}, \frac{N+1}{2} \right) \), where \( I_x(a, b) \) is the regularized incomplete beta function. In addition, according to the proof of Auxiliary Lemma A1,
\[
\sum_{k=0}^{N} k P(x, N, k) = N x I_{1-x} \left( \frac{N+1}{2}, \frac{N-1}{2} \right) = N x \left[ I_{1-x} \left( \frac{N+1}{2}, \frac{N+1}{2} \right) - \frac{(1-x)^{N+1}}{N} \right],
\]
where \( B(a, b) \) is the beta function. Hence,
\[
x (1-x) L'(x) = N x \frac{(1-x)^{N+1}}{B(N+1, N-1)} = \frac{(1-x)^{N+1}}{B(N+1, N-1)} = N x \left( 1-x \right) P \left( x, N-1, \frac{N-1}{2} \right)
\]
\[
L'(x) = N P \left( x, N-1, \frac{N-1}{2} \right).
\]

Therefore,
\[
\sum_{k=N+1}^{N} P_p(p_a, N, k) = \sum_{k=N+1}^{N} P_p(p_d, N, k) = \frac{P(p_a, N-1, \frac{N-1}{2})}{P(p_d, N-1, \frac{N-1}{2})}.
\]

Hence, the equilibrium under voting choice implements the level of delegation that is optimal for investors as a whole.

**Proof of Proposition 4.**

We argue that on equilibrium path, if the fund manager engages in costly information acquisition (\( \pi > \frac{1}{2} \)), then she will always vote according to her signal. To see this, suppose, in contradiction, that the fund manager plays a mixed strategy for some signal realization. By symmetry, she also plays the mixed strategy for the other signal realization. However, this implies that conditional on signal realization and being pivotal, the fund manager is indifferent between the two actions \( d \in \{0, 1\} \). This implies that information has no value to the fund manager, so she is better off deviating and not acquiring the signal in the first place.

Let \( q_d \) denote the probability with which an investor delegates voting to the fund manager. Suppose that fund investors expect that the fund manager acquires a signal with precision \( p \)
and votes according to her signal. Consider an investor who expects that each other investor delegates with probability \( q_d \) and those who do not delegate acquire signals with precision \( \tilde{\pi} \). At the end of this proof, we show that the investor’s values from delegating voting to the fund and not delegating and acquiring a signal with precision \( \tilde{\pi} \) are given, respectively, by

\[
V_{d} (\tilde{\pi}, p, q_d) = \frac{1}{2} (p \Omega ((q_d + (1 - q_d) \tilde{\pi}) - (1 - p) \Omega ((1 - q_d) \tilde{\pi})) ,
\]

\[
V_{nd} (\pi, \tilde{\pi}, p, q_d) = \left( \pi - \frac{1}{2} \right) (p \Omega ((q_d + (1 - q_d) \tilde{\pi}) + (1 - p) \Omega ((1 - q_d) \tilde{\pi})) - \gamma (\pi) ,
\]

where \( \Omega (p) = P \left( \frac{q}{N-1}, \frac{N-1}{2} \right) \) is the probability that the votes of \( N-1 \) agents are split equally when each investor votes “for” with probability \( q \) and the votes are independent across investors. The intuition for (20)–(21) comes from the fact that an investor’s vote makes a difference only if his vote is pivotal for the outcome, which happens if the other \( N-1 \) votes are split equally. Consider investor \( i \) who has not delegated. Since the fund manager’s signal and the signals of other investors are conditionally independent of investor \( i \)’s signal, investor \( i \)’s gross value from his signal equals the probability that his vote is pivotal times the value of his signal in that event. The latter equals \( \pi - \frac{1}{2} \) since acquiring signal with precision \( \tilde{\pi} \) changes the probability of a correct decision in the pivotal event from \( \frac{1}{2} \) to \( \pi \). The former equals \( p \Omega ((q_d + (1 - q_d) \tilde{\pi}) + (1 - p) \Omega ((1 - q_d) \tilde{\pi}) \), reflecting the fact that there are two possible events: the fund manager gets a correct signal (with probability \( p \)) and the fund manager gets an incorrect signal (with probability \( 1 - p \)). In the former case, each investor votes correctly if he delegates voting to the fund manager (with probability \( q_d \)) or if he does not delegate voting but gets a correct private signal (with probability \( (1 - q_d) \tilde{\pi} \)). In the latter case, each investor votes correctly only if he does not delegate voting but gets a correct private signal (with probability \( (1 - q_d) \tilde{\pi} \)).

In equilibrium, \( \pi \) must satisfy the first-order optimality condition and the beliefs of investors must be consistent with their actual strategies, \( \tilde{\pi} = \pi \), which yields:

\[
P \Omega ((q_d + (1 - q_d) \tilde{\pi} + (1 - p) \Omega ((1 - q_d) \tilde{\pi}) = \gamma' (\pi)
\]

In addition, in equilibrium, if \( q_d \in (0, 1) \), then \( q_d \) must be such that each investor is indifferent between delegating and not delegating, i.e., \( V_{d} (\pi, p, q_d) = V_{nd} (\pi, \tilde{\pi}, p, q_d) \):

\[
\frac{1}{2} (p \Omega ((q_d + (1 - q_d) \tilde{\pi}) - (1 - p) \Omega ((1 - q_d) \tilde{\pi})) \]

\[
= \left( \pi - \frac{1}{2} \right) (p \Omega ((q_d + (1 - q_d) \tilde{\pi}) + (1 - p) \Omega ((1 - q_d) \tilde{\pi})) - \gamma (\pi) .
\]

Finally, consider the fund manager’s information acquisition problem. Suppose the fund manager expects that each investor delegates voting with probability \( q_d \) and that investors who do not delegate acquire signals with precision \( \pi \). The objective of the fund manager is

\[
\max_{p} fN \left( p - \frac{1}{2} \right) \Pr (Piv_{FM} \mid q_d, \pi) - Nc (p) ,
\]
where
\[
\Pr (Piv_{FM}|q_d, \pi) = \sum_{k=0}^{N} P(q_d, N, k) \left( \sum_{m=\frac{N+1}{2}-k}^{N+1+k} P(\pi, N - k, m) \right)
\]  

(25)
is the probability that the fund manager’s vote is pivotal. Intuitively, when the fund manager is pivotal, acquiring the signal with precision \( p \) changes the probability of a correct decision from \( \frac{1}{2} \) to \( p \), which increases the value of all shares by \( N \left(p - \frac{1}{2}\right)\), and the fund manager captures fraction \( f \) of this increase. Taking the first-order condition yields the equilibrium choice of precision \( p \):
\[
f \Pr (Piv_{FM}|q_d, \pi) = c’(p) .
\]  

(26)

We next summarize all symmetric equilibria.

(a) Equilibrium with \( q_d = 1 \). Consider a potential equilibrium in which \( q_d = 1 \). If \( q_d = 1 \), then the fund manager is pivotal with certainty. Hence, (24) reduces to \( \max_{p} f \left(p - \frac{1}{2}\right) - c(p) \), implying \( p = c^{-1}(f) \). Consider a deviation of investor \( i \) to not delegating. Since she expects each other investor to delegate with certainty, she expects to be pivotal with probability zero. Hence, deviation yields a payoff of zero, if she does not acquire information, or negative, if she does. Hence, deviation is not profitable, and thus \( q_d = 1 \) is indeed an equilibrium, and it always exists.

(b) Equilibrium with \( q_d = 0 \). Consider a potential equilibrium in which \( q_d = 0 \). If \( q_d = 0 \), then \( \Pr (Piv_{FM}|q_d, \pi) = 0 \), and therefore the solution to (24) is \( p = \frac{1}{2} \), i.e., the fund manager does not acquire information. Therefore, deviation to delegation to the fund manager yields an investor a payoff of zero. In contrast, not deviating yields the investor a payoff of
\[
\left(\pi - \frac{1}{2}\right) \Omega(\pi) - \gamma(\pi) = \max_x \left( x - \frac{1}{2}\right) \Omega(\pi) - \gamma(x) > 0.
\]

Hence, deviation is unprofitable. Finally, (22) with \( q_d = 0 \) implies that \( p \) is given by
\[
\Omega(\pi) = \gamma’(\pi) .
\]  

(27)

Hence this is indeed an equilibrium, and it always exists.

(c) Equilibrium with \( q_d \in (0, 1) \). Such an equilibrium exists for a non-empty set of parameters, which can be shown by constructing a numerical example.

We conclude the proof by deriving the values of information (20)–(21) and (24). The first two values are very similar to the derivations in Malenko and Malenko (2019) (see Section C of their Online Appendix). For completeness, we repeat these derivations here. For brevity, we omit the dependence of expectations on \( \tilde{\nu}, p \), and \( q_d \) in these derivations.

1. Value of not delegating and acquiring a signal with precision \( p \), (21). Investor \( i \)’s vote makes a difference only if \( \sum_{j \neq i} v_j = \frac{N-1}{2} \). Conditional on \( \sigma_i = \theta \) and on being pivotal, his utility from the signal is \( \frac{1}{2} \mathbb{E}[u(1, \theta)|\sigma_i = 1, PIV_i] \). Similarly, conditional on being pivotal and his
private signal being $\sigma_0 = 0$, the investor’s utility from the signal is $-\frac{1}{2} \mathbb{E} [u(1, \theta) | \sigma_0 = 0, PIV_i]$. Overall, the investor’s gross (i.e., excluding costs) value of acquiring a signal with precision $\pi$ is

$$
\Pr (\sigma_0 = 1) \Pr (PIV_i | \sigma_0 = 1) \frac{1}{2} \mathbb{E} [u(1, \theta) | \sigma_0 = 1, PIV_i] \\
- \Pr (\sigma_0 = 0) \Pr (PIV_i | \sigma_0 = 0) \frac{1}{2} \mathbb{E} [u(1, \theta) | \sigma_0 = 0, PIV_i].
$$

By the symmetry of the model, $\Pr (PIV_i | \sigma_0 = 1) = \Pr (PIV_i | \sigma_0 = 0)$ and $\mathbb{E} [u(1, \theta) | \sigma_0 = 1, PIV_i] = -\mathbb{E} [u(1, \theta) | \sigma_0 = 0, PIV_i]$, so we get

$$
\frac{1}{2} \Pr (PIV_i | \sigma_0 = 1) \mathbb{E} [u(1, \theta) | \sigma_0 = 1, PIV_i] \\
= \frac{1}{2} \Pr (PIV_i | \sigma_0 = 1) (\Pr (\theta = 1 | \sigma_0 = 1, PIV_i) - \Pr (\theta = 0 | \sigma_0 = 1, PIV_i)) = (\pi - \frac{1}{2}) \Pr (PIV_i),
$$

where

$$
\Pr (PIV_i) = \Pr (PIV_i | \theta = 1) = p \Pr (PIV_i | s = 1, \theta = 1) + (1 - p) \Pr (PIV_i | s = 0, \theta = 1) \\
= pP \left( q_d + (1 - q_d) \pi, N - 1, \frac{N - 1}{2} \right) + (1 - p) P \left( (1 - q_d) \pi, N - 1, \frac{N - 1}{2} \right).
$$

Adding the cost of information acquisition yields expression (21).

2. Value of delegation (20). For brevity, we omit the dependence of expectations on $\tilde{\pi}$, $p$, and $q_d$. As before, investor $i$’s vote makes a difference only if $\sum_{j \neq i} v_j = \frac{N - 1}{2}$. Conditional on $s = 1$ and on being pivotal, his payoff from delegation is $\frac{1}{2} \mathbb{E} [u(1, \theta) | s = 1, PIV_i]$. Similarly, conditional on $s = 0$ and on being pivotal, investor $i$’s utility from delegation is $-\frac{1}{2} \mathbb{E} [u(1, \theta) | s = 0, PIV_i]$. Overall, the investor’s value from delegation is

$$
\Pr (s = 1) \Pr (PIV_i | s = 1) \frac{1}{2} \mathbb{E} [u(1, \theta) | s = 1, PIV_i] \\
- \Pr (s = 0) \Pr (PIV_i | s = 0) \frac{1}{2} \mathbb{E} [u(1, \theta) | s = 0, PIV_i].
$$

By the symmetry of the model, $\Pr (PIV_i | s = 1) = \Pr (PIV_i | s = 0)$ and $\mathbb{E} [u(1, \theta) | s = 1, PIV_i] = -\mathbb{E} [u(1, \theta) | s = 0, PIV_i]$, so we get

$$
\frac{1}{2} \Pr (PIV_i | s = 1) \mathbb{E} [u(1, \theta) | s = 1, PIV_i] \\
= \frac{1}{2} \Pr (PIV_i | s = 1) (\Pr (\theta = 1 | s = 1, PIV_i) - \Pr (\theta = 0 | s = 1, PIV_i)) \\
= \frac{1}{2} \Pr (PIV_i | s = 1, \theta = 1) \pi - \frac{1}{2} \Pr (PIV_i | s = 1, \theta = 0) (1 - \pi).
$$

Note that $\Pr (PIV_i | s = 1, \theta = 1) = P \left( q_d + (1 - q_d) \pi, N - 1, \frac{N - 1}{2} \right)$ and $\Pr (PIV_i | s = 1, \theta = 0) = P \left( (1 - q_d) \pi, N - 1, \frac{N - 1}{2} \right)$, which yields expression (20).

3. Value of information for the fund manager (24). Conditional on $s = 1$ and on being pivotal, the fund manager’s utility from signal with precision $p$ is $\frac{1}{2} fN \mathbb{E} [u(1, \theta) | s = 1, PIV_{FM}]$. Similarly, conditional on being pivotal and the signal being $s = 0$, the fund manager’s utility from the signal is $-\frac{1}{2} fN \mathbb{E} [u(1, \theta) | s = 0, PIV_{FM}]$. Overall, the fund manager’s gross (i.e., excluding costs) value of acquiring a signal with precision $p$ is

$$
\Pr (s = 1) \Pr (PIV_{FM} | s = 1) \frac{1}{2} fN \mathbb{E} [u(1, \theta) | s = 1, PIV_{FM}] \\
- \Pr (s = 0) \Pr (PIV_{FM} | s = 0) \frac{1}{2} fN \mathbb{E} [u(1, \theta) | s = 0, PIV_{FM}].
$$

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By the symmetry of the model, \( \Pr(P_{IVFM}|s=1) = \Pr(P_{IVFM}|s=0) \) and \( \mathbb{E}[u(1,\theta)|s=1,P_{IVFM}]=-\mathbb{E}[u(1,\theta)|s=0,P_{IVFM}] \), so we get
\[
\frac{1}{2} \Pr(P_{IVFM}|s=1) \mathbb{E}[u(1,\theta)|s=1,P_{IVFM}]
= \frac{1}{2} \Pr(P_{IVFM}|s=1) (\Pr(\theta=1|s=1,P_{IVFM}) - \Pr(\theta=0|s=1,P_{IVFM})) = (p - \frac{1}{2}) \Pr(P_{IVFM}).
\]

It remains to calculate \( \Pr(P_{IVFM}) \). Consider an event that \( k \) investors out of \( N \) delegated to the fund manager. The probability of this event is \( P(q_d,N,k) \). In this event, the fund manager is pivotal if the number \( m \) of \( N - k \) votes that investors cast themselves is between \( \frac{N+1}{2} - k \) and \( \frac{N-1}{2} + k \). The probability of one of these events occurring is \( \sum_{m=\frac{N+1}{2}-k}^{\frac{N-1}{2}+k} P(\pi,N-k,m) \).

Combining, we get (24).

**Proof of Proposition 5.** We prove the proposition by comparing investor welfare and the fund manager’s utility in equilibrium with \( q_d = 0 \) and \( q_d = 1 \). If \( q_d = 1 \), investor welfare per-share equals
\[
\mathbb{E}[u(1,\theta)|d] = \frac{1}{2} \mathbb{E}[s=1|\theta=1] - \frac{1}{2} \mathbb{E}[s=1|\theta=0] = c^{\tau-1}(f) - \frac{1}{2},
\]
and the fund manager’s utility is
\[
N \max_p \left( f(p - \frac{1}{2}) - c(p) \right).
\]
If \( q_d = 0 \), investor welfare per-share equals
\[
\Pr(\theta=1) \sum_{k=\frac{N+1}{2}}^{N} P(\pi,N,k) - \Pr(\theta=0) \sum_{k=\frac{N+1}{2}}^{N} P(1-\pi,N,k) - \gamma(\pi,\tau)
= \frac{1}{2} \sum_{k=\frac{N+1}{2}}^{N} P(\pi,N,k) - \frac{1}{2} \sum_{k=\frac{N+1}{2}}^{N} P(1-\pi,N,N-k) - \gamma(\pi,\tau)
= \sum_{k=\frac{N+1}{2}}^{N} P(\pi,N,k) - \frac{1}{2} - \gamma(\pi,\tau),
\]
where \( \pi : \Omega(\pi) = \frac{\partial}{\partial \tau} \gamma(\pi,\tau) \), and where we used \( \sum_{k=0}^{N} P(1-\pi,N,k) = 1 \). The fund manager’s utility is
\[
N f \left( \sum_{k=\frac{N+1}{2}}^{N} P(\pi,N,k) - \frac{1}{2} \right).
\]

As \( \tau \to \tau, \frac{\partial}{\partial \tau} \gamma(\pi,\tau) \) approaches zero for any \( \pi < 1 \). Therefore, \( \pi \) approaches one as \( \tau \to \tau \). Hence, investor welfare approaches \( \frac{1}{2} > c^{\tau-1}(f) - \frac{1}{2} \) and the fund manager’s utility approaches \( N f \frac{1}{2} > N \max_p \left( f(p - \frac{1}{2}) - c(p) \right) \), which proves the first part of the proposition. As \( \tau \to \tau \), \( \frac{\partial}{\partial \tau} \gamma(\pi,\tau) \) approaches infinity for any \( \pi > \frac{1}{2} \). Therefore, \( \pi \) approaches \( \frac{1}{2} \) as \( \tau \to \tau \). Hence, investor welfare and the fund manager’s utility approach zero, which proves the second part of the proposition.
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