

Ownership, Investment and Governance: The Costs and Benefits of Dual Class Shares

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Abstract

In this paper we show that dual-class shares can be an answer to agency conflicts rather than a result of agency conflicts. When a firm issues voting shares to raise funds, an incumbent manager's control rights are diluted. This increases the risk that an incumbent could lose control of the firm and therefore, could lose the associated benefits of control. Thus, the incumbent may forgo positive NPV investments in an effort to maximize his expected wealth. Non-voting shares allow a firm to raise funds without diluting manager's control rights; hence, it can alleviate the underinvestment problem. But non-voting shares facilitate entrenchment and therefore, reduces value-enhancing takeover activities. Also, non-voting shares dilute dividends per share. We obtain conditions under which the benefit of using non-voting shares, that is, higher firm value due to higher investment outweighs the entrenchment and dividend dilution costs. Others have shown that deviations from "one share-one vote" can be optimal, but our study is the first to integrate the dualclass decision into the rich body of research on capital structure and underinvestment.

Keywords: Blockholders, Controlling shareholders, Dual-class shares, Hostile takeovers, Ownership structure, Private benefits of control, Non-voting shares, Shareholder welfare, Takeover defenses, Underinvestment, Voting rights.

JEL Classifications: G32, G34, G38, K20

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Abstract

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1 Introduction

In recent years, equalizing the voting power and influence of common shares has become a touchstone of good governance and shareholder democracy in much of the world. Corporate charter
provisions that explicitly limit the rights of minority shareholders are completely antithetical to
this viewpoint, though they widely exist world wide.¹ Thus, it is not surprising that dual-class
provisions, which create a second class of common stock with reduced voting power, have come
under fire. More generally, activist shareholders have voiced serious concern about the disenfranchisement of investors holding inferior classes of shares.² The corporate democracy movement has
led policy makers, most vocal among them a high-level group of EU company law experts and
Indian corporate activists, to warn of the threats posed by dual-class ownership structure.

Theoretical literature on dual-class shares consisting of papers like Grossman and Hart (1988), Harris and Raviv (1988) and Ruback (1988), have analyzed non-voting equity in the context of control contests and find that dual-class share structure produces a negative shareholder wealth effect. These papers trace this negative wealth effect to the unbundling of voting rights and cash flow rights – arguing that the unbundled votes can act as an anti-takeover device. When the likelihood of a successful takeover is significantly diminished, it allows managers to deviate from taking actions that enhance shareholders' wealth. For example, Shleifer and Wolfenzon (2002) show that firms with weaker shareholders' protection have lower valuation, which is consistent with investors taking into account the fact that some profits are likely to be diverted. As the market for corporate control weakens as a disciplining mechanism, it provides managers with greater latitude to extract private benefits.

We argue for a more nuanced view of the role of dual-class shares: Although a dual-class struc-

¹Companies like Berkshire Hathaway, Blackstone Group, Clearwire, Dolby, Echostar, Facebook, Ford, Fox News, Google, MasterCard, News Corp, Rosetta Stone, VISA, VMWare, and WebMD have multiple classes of shares. Dualclass structures are widely used in countries like Brazil, Canada, Denmark, Finland, Germany, Italy, Norway, Sweden, and Switzerland. In Canada 5 to 6% of listed companies, like Metro, Bombardier, Gluskin Sheff, Air Canada, Exfo, Cossette, and Celestica, have multiple classes of shares.

²Institutional Shareholder Services (ISS) recently recommended that dual-class structures be eliminated entirely for all newly listed companies. Also, ISS wants corporate laws to be changed to require sunset provisions for companies with dual class structure, such that all shares will revert to common shares after a pre-specified time, unless a majority of inferior-class shareholders vote to reaffirm the dual-class structure.

ture weakens the incentives associated with the market for corporate control, it helps to mitigate the problem of underinvestment resulting from the non-contractibility of a firm's investment policy. For example, a scale-expanding investment project generally requires sizable issue of new shares. If a manager is forced to issue voting shares, then these newly issued votes dilute a manager's proportional vote power and severely impede his ability to resist future takeover attempts. Thus, a rational manager owning voting shares and extracting private benefits has an incentive to reject such scale-expanding investments. Wruck (1989) finds average management-controlled holdings fall by 1.5% around the time of a typical private equity sale, while public offers are likely to dilute manager voting rights much more, given the typical larger size of SEOs. On the other hand, newly issued non-voting shares do not affect the manager's control rights and thus, do not discourage value maximizing investments.³

However, an incumbent, who has the option to use non-voting shares, will not necessarily use non-voting shares to fund all new investments. This is because issuance of non-voting shares also comes at a cost: Non-voting shares dilute the cash flow rights of all existing shareholders, including the incumbent, as more non-voting shares relative to voting shares must be issued to raise same level of investible funds. For example, Faccio and Masulis (2005) find incentives to use non-voting shares to fund an acquisition are strongest when the target firm's ownership is concentrated and a bidder's controlling shareholder has an intermediate level of voting power – a range where he is most vulnerable to a loss of control under a stock-financed acquisition.

Similarly, an incumbent who has no option but to use voting shares to fund new investments will not necessarily underinvest. This is because as a shareholder he bears part of the cash flow loss if he forgoes positive NPV projects and this cash flow loss may outweigh his control-related benefits from underinvestment. Thus, an incumbent manager faces a clear trade-off when choosing what type of security to issue to fund new investment projects: dilution of the manager's control

³According to the Library of Canadian Parliament, "Undeniably, some of the best-performing companies in Canada have multiple-voting shares. Thus, not all shareholders are concerned with the voting rights attached to a share. They may be more interested in the potential of sharing the company's wealth or trading on future prospects by buying cheaper, subordinated shares." Also, according to Barry Reiter of Bennett Jones LLP, "some dual-class firms are created to favor Canadian ownership in strategic or culturally sensitive fields. Many foreign investors have happily bought into structures of this sort."

rights versus dilution of the manager's cash flow rights against the cost of forgoing positive NPV investments.

To add yet another wrinkle to the decision process, we observe that outside shareholders, who are assumed to have approval rights, also face a trade-off: the expected costs of enhanced entrenchment of the incumbent against a higher firm value due to higher investments. If the outside shareholders allow the incumbent to issue both voting and non-voting shares, then the incumbent invests in all available positive NPV projects; but outside shareholders also know that when this additional investment is funded using non-voting shares, the incumbent manager is relatively more entrenched because his private benefits play an enhanced role in potential takeover contests. Hence, future takeover attempts are made more costly, which results in lower expected takeover premiums and, consequently, lower firm values. However, if outside shareholders force the incumbent to use only voting shares to fund new investments, then he may forgo some positive NPV projects, which is also costly for the outside shareholders. For example, DeAngelo and DeAngelo (1985) indicate that there can be an underinvestment problem if the managers are faced with increasing risk of losing control when they fund new projects with voting equity.

Taking into consideration all of these possibilities, we propose that differences in investment opportunity sets may help to explain the considerable variation in the effects of dual-class share issues both within and among firms. Others have shown that deviations from "one share-one vote" can be optimal, but our paper is the first to integrate the dual-class decision (heretofore viewed simply as of concern in the control literature) into the rich body of research on capital structure and underinvestment. We focus specifically on the firm's decision to forgo positive NPV investment opportunities.⁴

⁴For example, Burkart et al. (1998) show that a dual-class share structure is optimal if the benefits of a higher likelihood of tender offers outweigh the costs of less efficient tender offers; Burkart and Raff (2012) show that even active boards destroy acquisition opportunities for rival managers, thus forcing all firms to pay higher incentives to incumbents; Blair et al. (1989) show that a market for votes can increase efficiency during control contests in the presence of taxes; Neeman and Orosel (2006) show that a contest for votes in addition to a contest for shares can have efficiency advantages; Sercu and Vinaimont (2008) demonstrate with the help of simulations that one share-one vote is never optimal from an entrepreneur's point of view; and more recently, Edmans (2009) and Edmans and Manso (2011) show that shareholders who hold non-voting shares can exert influence through the threat of exit. Also, Brav and Mathews (2011) show that separate vote trading can improve overall efficiency. In contrast, Masulis et al. (2011) show that control can also be maintained by pyramid structured business groups, which may act as substitute for nonvoting

In a control context, the problem of underinvestment differs considerably from standard underinvestment scenarios: In a control context, underinvestment results from a manager's fear of diluting his control rights – which reduces the likelihood of retaining control of the firm. For example, Masulis et al. (2009) find that in quite a few firms with dual-class shares, the largest shareholder does not have a majority of the voting rights; however, if the manager owns an insignificant block of voting shares or owns a large voting block well in excess of 50%, then there is no scope for significant control dilution and hence, there is no underinvestment problem. Thus, only when manager voting power is in an intermediate range is there an underinvestment problem.

One possible solution to underinvestment problem is to issue debt rather than equity. However, debt does not solve the underinvestment problem, because it carries with it the risk of bankruptcy. Issuing more debt can require stricter covenants, which in turn risks a loss of control for the incumbent manager if a covenant is violated. The creditor often demand that the managers be replaced before agreeing to either a covenant revision or debt restructuring. For example, DeAngelo and DeAngelo (1985) find evidence that dual-class firms infrequently resort to increasing leverage to retain control. Instead, dual class firms seek to keep leverage low, consistent with their desire to minimize the risk of creditors taking control of the firm. The issuance of non-voting stock to fund new investment does not result in a dilution of a manager's control rights and has none of the adverse effects of debt covenants.

A dual-class structure can be particularly helpful for a smaller firm facing large, profitable investment opportunities, enabling it to significantly increase shareholder wealth by not passing up the profitable investments and thereby improve the firm's overall economic efficiency. However, as mentioned earlier the costs associated with issuing non-voting equity limit its effectiveness in solving the underinvestment problem.⁶

shares. In a recent paper Laux (2012) shows that suboptimal vesting periods in CEO incentive compensations can induce myopic investment behavior similar to ours.

⁵Underinvestment and its causes have been studied in a number of papers: Debt-induced underinvestment has been considered by Galai and Masulis (1976), Henderson (1993), Myers (1977), and Berkovitch and Kim (1990). Myers and Majluf (1984) and Cooney and Kalay (1993) derive conditions under which an undervalued firm forgoes positive NPV investments.

⁶If shareholders collectively make the firms investment decisions, then underinvestment is no longer an issue. However, this solution would undercut efficient investment decision making and it would be next to impossible to prevent

Our model predicts that high growth firms, rather than firms with lots of assets in place, are more likely to use dual-class shares and this prediction is consistent with empirical findings in Lehn et al. (1990) and Dimitrov and Jain (2006). Our analysis produces three new testable implications. First, restrictions on issuance can reduce shareholder value. Such restrictions can lead to severe underinvestment and may at times outweigh the positive value effects that voting shares have on control contests. Next, we show that the likelihood of issuing non-voting shares is initially increasing and then decreasing in a manager's share holding. If a manager's holding is small, then there is no scope for control dilution; thus, the manager fully invest even using voting shares. If his holding size is large, then he bears a large part of the cost of underinvestment and dividend dilution. Hence, it is more likely that his expected cost of control dilution is outweighed by the benefits of full investment even using voting shares. Lastly, we show that the investment benefits of non-voting shares are more pronounced in countries with relatively weaker institutions and inefficient legal systems. Effective institutional monitoring and legal systems act as exogenous mechanisms that help to deter excessive control rents. If these mechanisms are weak, then losing control of the firm is more costly to a manager; hence, forcing a manager to use voting shares to fund new investments may lead to significant underinvestment.

The remainder of this article is organized as follows: In Section 2 we present the basic environment. In Section 3 we analyze the potential takeover contest and the effect of investment financing on the likelihood of potential takeover. In Section 4, we characterize the underinvestment problem. We analyze the effect of underinvestment on outside shareholders as well as on the incumbent manager's welfare. Possible extensions are discussed in Section 5. Conclusions are presented in Section

competitors from gaining access to important proprietary information. If less stringent information requirements are imposed, the manager can ensure underinvestment by withholding crucial information from shareholders. Also, if it is possible to directly contract with a manager across all states of the world, again underinvestment can not be avoided. Such a contractual solution would require all a firms investment opportunities to be known to its shareholders. In addition, it would require investment opportunities to be verifiable - imposing verification costs on shareholders.

⁷There are few more empirical implications that have been partly or completely tested in prior papers. For example, our model corroborates that, conditional on the same level of assets and investment activities, a dual-class firm is valued less than a comparable single-class firm – a prediction partly tested in Claessens et al. (2002), Boone and Mulherin (2007) and Gompers et al. (2010). Also, our model shows that dual-class firm is less likely to become a takeover target, but conditional on a takeover the premium paid for the voting shares is likely to be higher. Papers like Seligman (1986), Jarrell and Poulsen (1988), Ambrose and Megginson (1992), Smart and Zutter (2003) and Krishnan and Masulis (2011) have empirically tested these implications.

6. Some of the more cumbersome results and an extensive numerical exercise are delegated to the appendix.

2 Model Preliminaries

The model considers a firm that faces an investment opportunity. Our firm is a typical publicly traded firm with sizable insider holding. Initially, our firm has only one class of shares – the "commons." Each common share has equal claim to cash flows as well as equal voting rights. We assume that the shareholders are able to influence, through simple majority votes, broad corporate objectives and polices such as changes in the board of directors, changes in control of the firm, and the menu of securities that the firm can issue to raise new capital. We highlight four players in our model – (i) the incumbent manager (I), (ii) outside shareholders, (iii) potential new investors, and (iv) the manager of a potential rival firm (R).

The incumbent is the one who searches for new investment opportunities, does the initial evaluation, and decides what investment projects to undertake. Like Jensen and Meckling (1976), Myers (1977), Cooney and Kalay (1993) and Zwiebel (1996) papers, we assume that the incumbent maximizes the market value of the firm as well as the private benefits he derives from being in control. In addition, we assume that the incumbent is a "block" shareholder of the firm, such that the incumbent owns β fractions of the existing N commons (or voting shares).⁸

There can be two types of "closely held" ownership structures: The incumbent has a large minority block, which exceeds that of any other shareholders; or the incumbent owns absolute majority of the votes. Initially, we consider the case where the incumbent has a large minority block which exceeds that of any other shareholder; that is, $0 < \beta < 1/2$ implying that the incumbent has effective control rather than absolute control.⁹ The remaining $1 - \beta$ fraction of the common shares

⁸Although only about 20% of the major exchange-listed public firms are closely held in the United States, a vast majority of U.S. corporations are closely held. Also, a study of top 27 stock markets finds that only 36% of the largest publicly traded firms are widely held – that is, there is no single shareholder controlling more than 20% of the total votes. Most large publicly traded firms (64%) have a controlling shareholder, which may be a family (30%), the state (19%), or another firm (15%). Among smaller companies the share of closely held firms is even higher. For detailed discussion see, for example, La Porta et al. (1999) and Claessens et al. (1999).

⁹ All of our results can be reproduced if we consider the case where the incumbent has absolute control; that is,

are diffusely held by outside shareholders. Each individual outside shareholder wants to maximize the value of his holdings.

The incumbent manager needs to issue equity to raise investment funds. Potential new investors are the ones who buy the securities that the firm issues, if any, to finance a new investment project. We do not restrict the existing outside shareholders from purchasing the newly issued securities, although we do assume that the incumbent manager is wealth constrained and cannot buy enough newly issued shares to keep his ownership fraction constant. Thus, if the firm invests by issuing common shares, the incumbent's ownership fraction declines. Also, we rule out any kind of preemptive rights.

The final player is the rival manager, representing the rival firm. The rival manager, if he values our firm higher than the incumbent, offers to buy the firm. We rule out a "manager-rival negotiated" takeover: The only way to acquire the firm is through the market, in an open market purchase of at least 50% plus voting shares. All participants are risk-neutral and the discount rate is zero; all securities have prices equal to their expected payoff.

The temporal evolution of events is as follows: Shareholders decide on the types of securities that the firm can issue to finance the new investment opportunity. Next, the incumbent decides the level of investment, x, and if x > 0 issues securities to finance the new investment. A rival arrives, and if he can take over the firm, he bids for the firm and gains control. The actual investment is undertaken and subsequently, the firm is liquidated in the final period and the public value is paid out to the investors as a dividend. The manager-in-control obtains the private benefits. The quality of the rival is uncertain at the beginning of the scenario, but is revealed at the time of his arrival. The figure below depicts the timeline described above:

2.1 New Project

The project generates public value for the shareholders of the firm and a private benefit that accrues to the firm's manager. The realized value of the project is $x + a_i P(x) + \varepsilon_x$. The random variable, he owns just about majority votes ($\approx 50\%$ +). We develop this case as a numerical example in Appendix 7.

 ε_x , is uniformly distributed over the interval $(-\sigma_x, +\sigma_x)$, with a mean zero and variance $\sigma_x^2/3$. P(x) is a concave function, differentiable everywhere with a unique maximum at \bar{x} . Thus, the maximized expected value of the new project is $\bar{x} + a_i P(\bar{x})$.

The parameter a_i is a measure of the manager-in-control's ability to generate cash flows from the new project. Henceforth, we call the parameter a_i "public quality" of the manager-in-control at the end of the investment process, where the manager-in-control is either the incumbent (I) or the potential rival manager (R). We assume that the public quality of the incumbent is common knowledge and $a_I \in [0, 1]$. Initially, the potential rival manager's public quality is unknown; thus, we assume that a_R is a random variable drawn from a uniform distribution with support 0 and 1. The lowest public quality manager is the one with $a_i = 0$, and the resulting NPV of the new project is 0. The highest public quality manager is the one with $a_i = 1$, and the resulting NPV of the new project is P(x).

Also, we assume that the manager-in-control (whether incumbent or rival) can appropriate some benefits that are not shared by outside shareholders – a private benefits of control. This private benefit is not verifiable; otherwise, if it is verifiable, it will be relatively easy for outside shareholders to stop the manager from appropriating it. The realized value of the private benefit is $B_i = b_i \alpha a_i P(x)$, where i = I, R. The parameter b_i measures the manager-in-control's ability to convert one unit of NPV into his private benefit. Henceforth, we will call the parameter b_i "ability to extract private benefits" of the manager-in-control. We avoid the problem of over investment

by assuming that private benefits are also maximized at \bar{x} . Like public quality, we assume that incumbent's ability to extract private benefits, $b_I \in [0, 1]$, is common knowledge and the potential rival manager's ability to extract private benefits b_R is a random variable drawn from a uniform distribution with support 0 and 1. The parameter α is an inverse measure of the ineffectiveness of "other outside private benefit monitoring mechanisms" that can help, directly or indirectly, shareholders to access and/or prevent managers from converting public value into excessive private benefits. For example, an effective income/consumption/wealth disclosure system for government tax purposes and/or efficient legal system can act as an additional deterrence to extracting excessive private benefits. The higher the value of α , the higher the ineffectiveness of such outside monitoring mechanisms. An $\alpha < 1$ implies that the other monitoring mechanisms in place are such that even the most "villainous" managers, $b_i = 1$, cannot "convert" the entire NPV into private benefits. See, for example, Dyck and Zingales (2004) and Nenova (2003) for detailed discussions on estimates of private benefits.

2.2 Firm Value

We normalize the initial value of the firm, $V_0 = 0$; hence, the present value of all the cash flows generated from the new investment is the only source of future dividends for the shareholders adjusted for the private benefit of control. What are the costs of private benefits? There is a direct loss to the shareholders – a dollar worth of private benefit is a dollar less for the outside shareholders – we call it "value effect" of private benefits. There is also an important indirect loss to the shareholders: Private benefit does not directly reduce value of the firm, but allows the manager to use the private benefits to stall potential value-enhancing takeovers activities – we call it entrenchment effect of private benefits. In both the cases, these private benefits are direct gains for the manager. We focus on both these effects of private benefits.¹¹

 $^{^{10}}$ Otherwise, ability to issue nonvoting shares to fund new investments may encourage managers to invest in negative NPV projects that help to enhance their private benefits.

¹¹In Section 7 of the paper, we develop an extensive numerical example that deals only the "entrenchment effect" of private benefits on investment decision. Also, in a different paper titled "Strategic Underinvestment and Ownership Structure of a Firm" we formally developed the case that deals with only the entrenchment effect of private benefits on new investments.

If the firm invests in this new project and the manager-in-control at the liquidation date is of (a_i, b_i) type, then the expected firm value, denoted by FV_i , is

$$FV_i = \text{Investment} + \text{NPV} - \text{Private Benefits} = x + a_i P(x) - b_i \alpha a_i P(x) = x + a_i (1 - b_i \alpha) P(x), (1)$$

where i=I,R. We assume \tilde{a}_R and \tilde{b}_R are independent random variables. Outside shareholders want to maximize the firm value; hence, they want a manager who is best in terms of public quality and has the least ability to convert shareholders' value into private benefits; that is, a manager with $a_i=1$ and $b_i=0$. If $a_i=1$ and $b_i=0$, then the expected firm value, x+P(x) for any level of investment x, is maximized. If $a_i=1$ and $b_i=1$, that is, the manager-in-control is the best manager in terms of public quality, but also extracts the most private benefits; hence, the resultant firm value is $x+(1-\alpha)P(x)$, which is strictly less than x+P(x) unless $\alpha=0$ implying that other outside mechanisms to deter private benefits are perfect. Thus, if $1>\alpha>0$, then there must exist a rival manager with the same/lower public quality and lower ability to extract private benefits than the incumbent such that $a_R(1-b_R\alpha)>(1-\alpha)$. This is important! This condition implies that if such a rival takes control of the firm, then he can generate a higher firm value than the incumbent can. For example, using an average estimate of 14% shareholder wealth appropriation by the manager from Dyck and Zingales (2004), we find that a rival with public quality, $a_R=0.62$ and ability to extract private benefits, $b_R=0.5$, can generate higher cash flows than the incumbent with $a_I=0.75$ and $b_I=1$.

3 Potential Control Contest

The potential control contest is a critical stage of the process. To gain control of the firm, the rival has to offer outside shareholders a greater amount for their shares than the incumbent. If the rival cannot offer a greater amount, he does not bid and the incumbent retains control. If the rival can offer a greater amount, then he pays shareholders an amount slightly higher than what the incumbent can offer and the firm comes under rival's control. Initially, we assume that the

incumbent does not tender in a control contest.¹² In Subsection 5.1 we relax this assumption and allow the manager to tender if a rival makes an offer. We show that qualitatively results are similar if the incumbent is allowed to tender in the control contest.

Also, at this stage we introduce additional notation: A superscript $j \in \{0, 1\}$ on variables indicates the value of the variable if the firm issues new shares with j votes per-share; e.g., j = 0 corresponds to non-voting shares, while j = 1 corresponds to conventional voting shares. Let

- n^j = number of new shares issued if j-vote shares are issued to finance the investment;
- ϕ^{j} = probability of no takeover, if j-vote shares are issued to finance the investment;
- V_D^j =public value per-share if j-vote shares are issued to finance the investment; and
- V_{vote}^{j} = value of a pure vote claim if j-vote shares are issued to finance the investment.
- V_k^j =value of k-vote share when j-vote shares are issued to finance the new investment.

If the new investment is financed using voting shares, only one type of share is outstanding and their value is denoted by V_1^1 . If non-voting shares are issued to fund new investments, then two different types of share are outstanding and their values are given by V_1^0 , for the old voting shares, and V_0^0 , for the newly issued non-voting shares. The value of the voting shares is equal to the value of the dividend received plus the value of the vote, while the value of the non-voting shares is equal to just the value of the dividend received. Thus, value of the voting shares if new shares are also voting shares, value of the voting shares if new shares are non-voting shares and value of the newly issued non-voting shares are

$$V_1^1 = V_D^1 + V_{vote}^1, \quad V_1^0 = V_D^0 + V_{vote}^0, \quad \text{and } V_0^0 = V_D^0$$
 (2)

respectively. We assume that the existing voting shares and the non-voting shares get equal dividends; but the per-share dividend is different if newly issued shares have voting rights as opposed

¹²This is because firm insiders' stock sells are subject to restrictions by supervisory authorities and these restrictions may severely affect the manager's ability to tender in a control contest. For example, the incumbent manger may hold "restricted voting" shares, which will create hurdles for tendering in a control contest.

to being non-voting, because of the lower price of non-voting shares. Our analysis is restricted to two types of securities – non-voting shares and "commons" or voting shares. The effect of multiple classes of securities and the problem of optimal security design (the "best" combination of dividend and vote) are not formally addressed here. A discussion later in the Section 5 of the paper briefly addresses these issues. That is, $V_D^1 \neq V_D^0$. This is because the number of new shares that the firm has to issue to finance x dollars of investment, $n^j = \frac{x}{V_j^j}$, depends on the type of security issued.

Proposition 1. The number of new non-voting shares, n^0 , issued to raise x dollars is always at least weakly greater than the number of voting shares, n^1 , needed to be issued to raise the same dollar amount.

Proof. Follows directly from Equation
$$(2)$$
.

By design, voting shares and non-voting shares have equal dividends. Since the value of vote is nonnegative (vote premium is similar to option premium), the value of one voting share, which is equal to the expected value of dividend plus the value of vote, has to be at least weakly greater than value of one non-voting share, which is just the expected value of dividend. See, for example, Smith and Amoako-Adu (1995) for detailed discussion on relative prices of voting versus non-voting shares. Hence, the number of voting shares issued to raise x dollars is at least weakly less than the number of non-voting shares issued to raise x dollars.

3.1 The Decision Problems

Both the incumbent manager and the outside shareholders are assumed to be interested in maximizing their expected wealth. For the manager the decision variable is the level of investment, x. Given that the manager does not tender his shares to the rival, this level is equivalent to

$$\max_{x} W_{I}(x) = \max_{x} \{ \beta N V_{D}^{j}(x) + \phi^{j} b_{I} \alpha a_{I} P(x) \}.$$
 (3)

The objective function above has two parts: The first part is related to the public value of the firm and reflects the fact that the manager is similar to any other shareholder. The second part is related to the private benefit of incumbency, and is realized only if the manager retains control of the firm. The solution to the manager's problem gives the manager's optimal response to restrictions on the type of security that the firm can issue.

Let \hat{x}^j be the solution to the manager's optimization problem given that he issues j—vote shares to finance the investment. Outside shareholders maximize the value of their shares, picking the type of security that the manager can issue and taking the manager's optimal response as given. Thus, the decision problem of the outside shareholders is

$$\max_{j=0,1} V_1^j \left(\hat{x}^j \right). \tag{4}$$

To solve the two optimization problems, given by Equations (3) and (4), we need (i) the probability that there is no takeover, (ii) value of the dividend, and (iii) value of the vote, for the case where the firm issues voting shares and for the case where the firm issues non-voting shares to fund new investments.

3.2 Potential Control Payoffs

A change in control occurs when the rival can offer a higher per-share price to the outside voting shares than the incumbent. The incumbent can retain control if he can offer higher price for the outside voting shares than the rival. We assume that the rival will only bid if he is sure to win. We separately consider the cases of voting shares and non-voting shares. The question that the rival asks: What is the maximum price the incumbent is willing to pay to buy the outside voting shares? Obviously, the incumbent has to pay the public value per share to the outside voting shareholders; also, he is likely to throw in a significant part of the present value of all his private benefits to stop the rival from gaining control of the firm. Thus, the maximum price the incumbent is willing to

offer for the outside voting shares is

$$\frac{\text{Public value of the firm under incumbent's control}}{\text{Number of outstanding cash flow claims}} + \frac{\text{Incumbent's total private benefits}}{\text{Number of outstanding outside votes}}$$
(5)

Incumbent can retain control of the firm, only if the number generated by Equation (5) is weakly greater than the potential rival's maximum offer. If voting shares are used to finance the investment, this condition is equivalent to

$$\frac{FV_I}{N+n^1} + \frac{b_I \alpha a_I P(x)}{(1-\beta)N+n^1} \geqslant \frac{FV_R}{N+n^1} + \frac{b_R \alpha a_R P(x)}{(1-\beta)N+n^1}.$$
 (6)

The first term on the LHS of Equation (6) describe the per-share public value that is generated with the incumbent in control. The second term on the LHS is related to the incumbent's private benefits: Denominator of the second term is smaller because private benefits are distributed only to the outside voting shareholders since the incumbent does not tender. The RHS terms are related to the public and private benefits per-share generated under the rival. Substituting Equation (1) into (6) and simplifying we obtain

$$a_I \left(1 + \alpha \kappa^1 b_I \right) \geqslant a_R \left(1 + \alpha \kappa^1 b_R \right),$$
 (7)

where $\kappa^1 = \frac{N\beta}{(1-\beta)N+n^1}$. If non-voting shares are issued to finance new investments, then the incumbent retains control of the firm if

$$\frac{FV_I}{N+n^0} + \frac{b_I \,\alpha \,a_I \,P(x)}{(1-\beta) \,N} \geqslant \frac{FV_R}{N+n^0} + \frac{b_R \,\alpha \,a_R \,P(x)}{(1-\beta) \,N}. \tag{8}$$

The private benefits are distributed only to those outside shareholders who own voting shares. Simplifying Equation (8) we get

$$a_I \left(1 + \alpha \kappa^0 b_I \right) \geqslant a_R \left(1 + \alpha \kappa^0 b_R \right),$$
 (9)

where $\kappa^0 = \frac{N\beta + n^0}{(1-\beta)N}$. 13 After simplifications, Equations (7) and/or (9) can be expressed as

$$b_R \leqslant \underline{b}_R^j = \frac{1}{\alpha \kappa^j} \left(\frac{a_I}{a_R} - 1 \right) + b_I \frac{a_I}{a_R}, \quad j = 0, 1,$$
 (10)

where \underline{b}_R^j is the lowest value of b_R such that takeover is not possible. Given that $\underline{b}_R^j \in [0, 1]$, we simplify and rearrange Equation (10) to define \underline{a}_R^j and \overline{a}_R^j :

$$\bar{a}_R^j = a_I (1 + \alpha \,\kappa^j \,b_I) \quad \text{and} \quad \underline{a}_R^j = \frac{a_I (1 + \alpha \,\kappa^j \,b_I)}{1 + \alpha \,\kappa^j} = \frac{\bar{a}_R^j}{1 + \alpha \,\kappa^j}.$$
 (11)

The likelihood of takeover and the role of private benefits in a takeover contest depends on \bar{a}_R^j , \underline{a}_R^j and \underline{b}_R^j . Proposition below formally states these observations.

Proposition 2. (i) Rivals with public quality a_R higher than \bar{a}_R^j can gain control of the firm regardless of their ability to extract private benefits (i.e., even if $b_R = 0$); whereas (ii) rivals with public quality lower than \underline{a}_R^j cannot gain control of the firm, even if they have the highest possible ability to extract private benefits (i.e., even if $b_R = 1$).

Proof. Directly follows from Equations (10) and (11).
$$\Box$$

If the rival's public quality is significantly higher (lower) than the public quality of the incumbent, then the control contest will be decided based only on the public quality of the contestants, and their private qualities will not play any role in the control contest. If the rival manager's public quality is high $(a_R > \bar{a}_R^j)$, then the rival manager gains control of the firm; whereas, if the rival manager's public quality is low $(a_R < \underline{a}_R^j)$, then the incumbent retains control of the firm. Ability to extract private benefits plays a role in the control contest only when the rival's public quality takes on intermediary values, $a_R \in [\underline{a}_R^j, \bar{a}_R^j]$. Rival managers with public quality, $a_R \in [\underline{a}_R^j, \bar{a}_R^j]$

¹³From Equations (7) and (9) it is clear that it is not possible for any rival to takeover the firm from an incumbent whose public quality, $a_I = 1$ and ability to extract private benefits, $b_I = 1$. This is because the incumbent can always offer at least as much as any rival and hence, keep control of the firm. Corporate control market fails to work – firm value is lower than what could have been under a host of rivals! Also, this result is independent of the type of security incumbent uses to fund new investments.

¹⁴There can be many (a_I, b_I) combinations such that $\bar{a}_R^j > 1$. This simply implies that there exists no rival who can takeover the firm based only on his public quality, a_R .

and ability to extract private benefits, $b_R \in [\underline{b}_R^j, 1]$ can rest control of the firm from the incumbent. These control regions, based on the potential rival's public quality and his ability to extract private benefits are depicted in Figure 1.

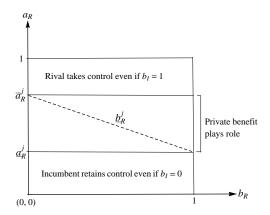


Figure 1: This figure depicts critical control regions as a function of potential rival's public quality, a_R and ability to extract private benefits, b_R . Private benefits do not play any role in the takeover contest if the potential rival's public quality belongs to either $[\bar{a}_R^j, 1]$ or $[0, \underline{a}_R^j]$. If the potential rival's public quality is drawn from the range $[\underline{a}_R^j, \bar{a}_R^j]$, then only the private benefits of the rival manager plays a role in the takeover contest. If the potential rival's ability to extract private benefits, $b_R > \underline{b}_R^j$ (dashed line), then the incumbent loses control of the firm; otherwise, incumbent retains control.

3.3 Effect of Investment on Control Contest

Next, consider the effects of increasing investment, x, on \underline{a}_R^j , \bar{a}_R^j , and \underline{b}_R^j . These bounds determine the outcome of the control contest: Likelihood that incumbent retains control of the firm after investment depends on the level of investment. We show in Appendix B that $\partial n^j/\partial x > 0$, $\forall j$ implies that $\frac{\partial \kappa^0}{\partial x} > 0$ and $\frac{\partial \kappa^1}{\partial x} < 0$. Thus,

$$\frac{\partial}{\partial x}\bar{a}_R^j = \alpha a_I b_I \frac{\partial \kappa^j}{\partial x} \begin{cases} > 0 & \text{if } j = 0\\ < 0 & \text{if } j = 1, \end{cases}$$
(12)

and

$$\frac{\partial}{\partial x} \underline{a}_R^j = \frac{\partial}{\partial \kappa^j} \underline{a}_R^j \times \frac{\partial \kappa^j}{\partial x} = -\frac{a_I (1 - b_I) \alpha}{(1 + \alpha \kappa^j)^2} \times \frac{\partial \kappa^j}{\partial x} \begin{cases} < 0 & \text{if } j = 0 \\ > 0 & \text{if } j = 1 \end{cases}$$
(13)

We formally state these results in the Propositions 3 and 4 below.

Proposition 3. (i) If the incumbent issues voting shares to fund new investments, then the set of rivals who can take over the firm using only their public quality, $[\bar{a}_R^1, 1]$ increases; whereas (ii) if the incumbent issues non-voting shares to fund new investments, then the set of rivals who can take over the firm using only their public quality, $[\bar{a}_R^0, 1]$, decreases.

Proof. Follows directly from Equation (12). \Box

If the incumbent uses voting shares to fund the new investments, then the larger the investment, x, the larger is the likelihood that a rival can gain control of the firm regardless of his ability to extract private benefits (i.e., even if $b_R = 0$). An incumbent with relatively high ability to extract private benefits, b_I , will be worried! As he issues more and more voting shares, his private benefits become less and less useful in the control contest! This is because issuing new voting shares shift the relative weights from vote premium to dividends in any control contest. When voting shares are issued, the vote premium is divided among a larger number of outside vote holders, $(1 - \beta)N + n^1$, and as a result the per-share vote premium falls. In contrast, if the incumbent issues non-voting shares to raise funds for investments, the role of private benefits remains unchanged; the incumbent can use his private benefits to buy just the existing outside votes, $(1 - \beta)N$.

Proposition 4. (i) If the incumbent issues voting shares to fund new investments, then the region over which he retains control of the firm regardless of the rival's ability to extract private benefits, $[0, \underline{a}_R^1]$, increases; whereas (ii) if the incumbent use non-voting shares, then the region over which he retains control of the firm regardless of rival's ability to extract private benefits, $[0, \underline{a}_R^0]$, decreases.

Proof. Follows directly from Equation (13).

Intuition is similar; if the incumbent uses voting shares, then the role of his private benefits in the control contest gets reduced (region II gets smaller). On the other hand, if the incumbent uses non-voting shares, then the role of his private benefits in the control contest increases (region II expands). Panel A and B of Figure 2 depict the results stated in Propositions 3 and 4.

The effect of increasing investment, x, on the likelihood of incumbent's retaining control is not unambiguous: $Region\ I$, where incumbent loses control for sure, expands. But $Region\ III$, where

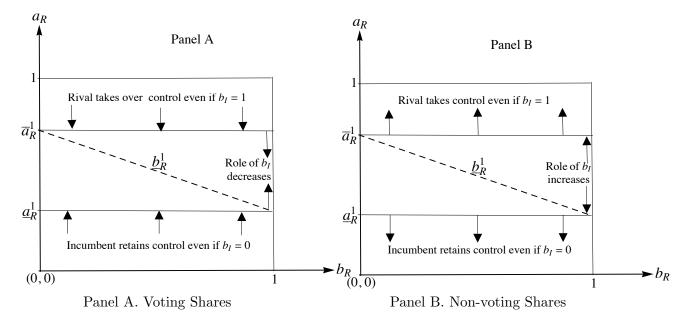


Figure 2: Effect of increasing x on \bar{a}_R^j , \underline{a}_R^j and \underline{b}_R^j . Panel A depicts the case when the incumbent issues voting shares to fund new investments. Panel B depicts the case when the incumbent issues non-voting shares to fund new investments. In both cases, arrows show the direction of movement of the upper bound, \bar{a}_R^j , and lower bound \underline{a}_R^j , as x, increases. In panel A, as x increases region II shrinks implying that private benefit plays a relatively lesser role in the control contest when investments are finance using voting shares. In panel B, as x increases region II expands implying that private benefit plays a more important role in the control contest when investments are finance using non-voting shares.

incumbent retains control for sure, expands too. Thus, the net effect of increasing x on probability of incumbent retaining control, ϕ^j depends on incumbent's public and private qualities relative to the average public and private qualities of a potential rival.

3.4 Probability of Control

The probability of the incumbent's retaining control of the firm after issuing j-th type shares is

$$\phi^{j} = \int_{0}^{\underline{a}_{R}^{j}} da_{R} + \int_{a_{R}^{j}}^{\bar{a}_{R}^{j}} \int_{0}^{\underline{b}_{R}^{j}} db_{R} da_{R} \quad \text{where } j = 0, 1.$$
 (14)

The first term in Equation (14) is the region where the potential rival's public quality is very low. In this region the rival has no hope of gaining control regardless of his ability to extract private benefits. The second term is the region where the rival's public quality is such that the incumbent retains control if the rival's ability to extract private benefits is lower than \underline{b}_{R}^{j} ; otherwise, the rival

gains control. There is one more range $[\bar{a}_R^j, 1]$, but the incumbent has no hope of retaining control over this range, regardless of the ability to extract private benefits of the rival. Integrating the expression in Equation (14) and further simplifying, we get the probability of the incumbent's retaining control,

$$\phi^{j}(a_{I}, b_{I}, \beta, N, \alpha, x) = a_{I}(1 + b_{I} k^{j} \alpha) \frac{\log(1 + k^{j} \alpha)}{k^{j} \alpha} = a_{I} \times \phi^{j'}(b_{I}, \beta, N, \alpha, x).$$
(15)

We use chain rule to differentiate Equation (15) with respect to x and obtain

$$\frac{\partial \phi^{j}(x)}{\partial x} = a_{I} \times \frac{\partial \phi^{j'}(x)}{\partial k^{j}} \times \frac{\partial k^{j}}{\partial x} = a_{I} \times \frac{\frac{k^{j}(1 + b_{I} k^{j} \alpha)}{1 + k^{j} \alpha} - \frac{\log(1 + k^{j} \alpha)}{\alpha}}{(k^{j})^{2}} \times \frac{\partial k^{j}}{\partial x}.$$
 (16)

Thus, the public quality of the manager only has "level effect" on the likelihood of the incumbent manager retaining control, given that he invests x. For any given value of a_I change in the likelihood of incumbent retaining control with investment is function of the incumbent's ability to extract private benefits, b_I . For $b_I \geqslant 1/2$, $\frac{\partial \phi^j(x)}{\partial k^j}$ is nonnegative. For all $b_I < 1/2$, $\frac{\partial \phi^j(x)}{\partial k^j}$ is strictly negative. Hence, the sign of $\frac{\partial \phi^j(x)}{\partial x}$ depends on the sign of $\frac{\partial k^j}{\partial x}$ and value of b_I . Thus,

$$\frac{\partial \phi^{1}(x)}{\partial x} = \frac{\partial \phi^{1}(x)}{\partial k^{1}} \times \frac{\partial k^{1}}{\partial x} \begin{cases}
< 0 & \text{if } b_{I} \geqslant 1/2 \\
> 0 & \text{if } b_{I} < 1/2.
\end{cases}$$
(17)

Similarly,

$$\frac{\partial \phi^{0}(x)}{\partial x} = \frac{\partial \phi^{0}(x)}{\partial k^{0}} \times \frac{\partial k^{0}}{\partial x} \begin{cases} > 0 & \text{if } b_{I} \geqslant 1/2\\ < 0 & \text{if } b_{I} < 1/2 \end{cases} \tag{18}$$

Propositions 5 and 6 describe the relationships among the likelihood of retaining control of the firm, the ability to extract private benefits of the incumbent, and the level of investment.

Proposition 5. Suppose that the incumbent's ability to extract private benefits is greater than an average rival's ability to extract private benefits; that is, $b_I \ge 1/2$. (i) If the incumbent issues voting shares to fund new investments, then the likelihood of incumbent retaining control of the firm, ϕ^1 ,

decreases as the level of investment, x, increases; whereas, if the incumbent issues non-voting shares to fund new investments, then the likelihood of incumbent retaining control of the firm, ϕ^0 , increases as the level of investment, x, increases.

Proof. Follows directly from Equation (17).

Given the public quality of the incumbent, a_I , if the incumbent's ability to extract private benefits, b_I , is greater than an average rival's ability to extract private benefit, $E(b_R) = 1/2$, then using voting shares to fund new investments reduces the incumbent's ability to retaining control of the firm after the investments. This is because voting shares make dividends as opposed to vote premium relatively more important in a control contest. On the other hand, non-voting shares make vote premium as opposed to dividends relatively more important in a control contest. Hence, an incumbent who has relatively weak ability to extract private benefits, $b_I < 1/2$ will prefer voting shares to fund new investments to shift weight away from vote premium towards dividends.

Proposition 6. Suppose that the incumbent's ability to extract private benefits is lower than an average rival's ability to extract private benefits; that is, $b_I < 1/2$. (i) If the incumbent issues voting shares to fund new investments, then the likelihood of incumbent retaining control of the firm, ϕ^1 , increases as the level of investment, x, increases; whereas, if the incumbent issues non-voting shares to fund new investments, then the likelihood of incumbent retaining control of the firm, ϕ^0 , decreases as the level of investment, x, increases.

Proof. Follows directly from Equation (18). \Box

The main intuition from Proposition 5 and 6 are depicted in the Figure 3 below. 15

3.5 Value of One Dividend Claim

The value of a pure dividend claim is equal to the expected dividend that the holder of the claim gets. This value depends on the manager-in-control's public quality and the type of security issued

¹⁵As x increases so does n^j – not linear but strictly monotonic relationship; hence, we use n^j instead of x.

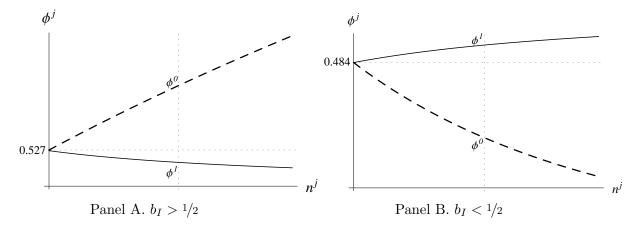


Figure 3: This figure depicts the likelihood that the incumbent retains control of the firm, ϕ^j , as a function of the size of the new equity issue, n^j . The solid black lines, in both panels A and B, correspond to the case where the new investment is financed using voting shares. The dashed lines and dotted lines, in both panels A and B, correspond to the case where the new investment is financed using non-voting shares.

to finance the new investments. Thus, the value of the per-share pure dividend claim is

$$V_D^j = \phi^j \frac{FV_I}{N + n^j} + \int_{\underline{a}_R^j}^{\overline{a}_R^j} \int_{\underline{b}_R^j}^1 \frac{FV_R}{N + n^j} \, db_R \, da_R + \int_{\overline{a}_R^j}^1 \int_0^1 \frac{FV_R}{N + n^j} \, db_R \, da_R, \tag{19}$$

where j=0, 1. The first term is the probability that the incumbent retains control times the pershare public value of the firm under the incumbent. The second and third terms give the expected dividend under the rival. The second term is generated by a rival of intermediate public quality and relatively high $(> \underline{b}_R^j)$ ability to extract private benefits. The third term is generated by rivals of very high public quality, who can take over the firm regardless of their ability to extract private benefits. If ϕ^j decreases, then $1 - \phi^j$ increases and vice versa.

Given our assumption that all available projects are positive NPV projects, the level of dividend increases with the level of investment, x. If x increases, either it is more likely for the incumbent to retain control or it is more likely for the rival to wrest control from the incumbent. If non-voting shares are used to fund the new investment, then the average dividend per-share can increase less than if voting shares are used. This is because the number of new non-voting shares required to raise x is greater than the number of voting shares; hence, each dividend claim gets diluted.

3.6 Value of One Vote Claim

as

Voting rights matter because they allow stockholders to have a say in who runs the company and how it is run.¹⁶ Voting power arises, especially at badly managed companies, when a challenge is mounted against the incumbent either from within (activist stockholders) or from outside (hostile acquisitions).¹⁷

The value of a pure vote claim is related to the extraction of private benefits from the rival in the form of a vote premium. To obtain an expression for the value of the vote, we classify rival managers into one of three types: The first type represents rivals who cannot gain control of the firm because they have very low public quality $(a_R < \underline{a}_R^j)$. If this type of rival is drawn, no private benefit is extracted and the value of the vote is zero. Next consider the rivals who can gain control of the firm without having to pay out any of their private benefit – those with very high public quality $(a_R > \overline{a}_R^j)$. Again, it is not necessary for the rival manager to give up any of his private benefits. Hence, the private benefit is extracted only when a rival manager is of an intermediate type $(\underline{a}_R^j < a_R^j < \overline{a}_R^j)$. The rival needs to pay "takeover" premium that us greater than the dividend it will produce; hence, the payoff to the vote claim when the firm issues voting shares can be written

$$\begin{cases}
\frac{FV_I}{N+n^1} + \frac{B_I}{N(1-\beta)+n^1} - \frac{FV_R}{N+n^1} & \text{if } \underline{a}_R^1 \leqslant a_R \leqslant \overline{a}_R^1 \\
0 & \text{otherwise.}
\end{cases} (20)$$

Similarly, the payoff to the vote claim when the firm issues non-voting shares is

$$\begin{cases}
\frac{FV_I}{N+n^0} + \frac{B_I}{N(1-\beta)} - \frac{FV_R}{N+n^0} & \text{if } \underline{a}_R^0 \leqslant a_R \leqslant \overline{a}_R^0 \\
0 & \text{otherwise,}
\end{cases} \tag{21}$$

¹⁶Zingales (1995b) and Nenova (2003) estimate value of the vote based on the price difference of shares with disparate voting rights of firms that have both classes publicly traded. They find that the value of the vote is positive and varies across countries. See also Smart et al. (2008) for implications of "vote" on IPO valuation.

¹⁷Institutional investors' benign neglect of different voting share classes at Google is rationalized by the fact that they think the company is well managed and that control is therefore worth little or nothing. There is a kernel of truth to this statement: The expected value of control (and voting rights) is greater in badly managed companies than in well managed ones. However, if you are an investor for the long term, you have to worry about whether managers who are perceived as good managers today could be perceived otherwise in a few years.

The value of the vote is simply the expectation of these values,

$$V_{vote}^{1} = \left(\frac{FV_{I}}{N+n^{1}} + \frac{B_{I}}{N(1-\beta)+n^{1}}\right) \int_{\underline{a}_{R}^{1}}^{\overline{a}_{R}^{1}} \int_{\underline{b}_{R}^{1}}^{1} db_{R} da_{R} - \int_{\underline{a}_{R}^{1}}^{\overline{a}_{R}^{1}} \int_{\underline{b}_{R}^{1}}^{1} \frac{FV_{R}}{N+n^{1}} db_{R} da_{R}$$
 (22)

and

$$V_{vote}^{0} = \left(\frac{FV_{I}}{N + n^{0}} + \frac{B_{I}}{N(1 - \beta)}\right) \int_{\underline{a}_{R}^{0}}^{\overline{a}_{R}^{0}} \int_{\underline{b}_{R}^{0}}^{1} db_{R} da_{R} - \int_{\underline{a}_{R}^{0}}^{\overline{a}_{R}^{0}} \int_{\underline{b}_{R}^{0}}^{1} \frac{FV_{R}}{N + n^{0}} db_{R} da_{R}.$$
 (23)

Using expressions (2), we can derive the value of voting shares when the new shares issued are voting, as well as when the new shares issued are non-voting shares. Next, we depict the type of manager who will underinvest if he is forced to finance investment using voting shares.

4 Entrenchment and Investment

The manager chooses the investment level to maximize his expected wealth. There are three terms in the manager's objective function that depend on the level of investment chosen: The value of the dividend, the probability of retaining control, and the private benefits of control. The value of the dividend increases with investment as we consider only positive NPV opportunities. By design, the private benefits of control also increase with investment. We also see, from Equations (17) and (18), that the likelihood that the incumbent retains control of the firm depends primarily on two things: Level of investment (x) and ability of the incumbent to extract private benefits (b_I) .

4.1 Low Quality Managers and the Control Contest

First, we turn to the issue of economic efficiency already established in the existing literature. Grossman and Hart (1988) and other studies have found that non-voting shares allow control of the firm to remain in or pass to the hands of inferior managers, lowering economic efficiency. We show that it is true that non-voting shares allow inferior managers to win control contests.¹⁸

¹⁸A statement on economic efficiency, though, requires analysis of a trade-off between the costs of underinvestment and the cost of inefficiently managed firms. This process requires assumptions regarding the ability of other firms to undertake projects that the firm under consideration has forgone. We leave this aspect of the problem to future

Our result is similar to their result. The difference between the voting shares and the non-voting shares is that non-voting shares cause the private benefits of managers to have a larger impact on the outcome of the control contest. Consider a rival with ability to extract private benefits higher than the incumbent's ability to extract private benefits, that is, $b_R > b_I$. Nonvoting shares favor this rival in a control contest, making it easier for him to gain control of the firm; that is, he can gain control for lower values of a_R , values for which he would lose the control contest if the firm had financed its investment using voting shares. Similarly, if $b_R < b_I$; that is, if the incumbent's ability to extract private benefits is greater than that of the rival, non-voting shares would favor the incumbent in a control contest, making it easier for him to retain control of the firm. That is, an incumbent can keep control of the firm for lower levels of a_I , values for which he would lose control of the firm if the investment had been financed using voting shares. The proposition below formalizes this result.

Proposition 7. The minimum public quality required for an incumbent manager to retain control of the firm is lower in firms financed with dual-class shares.

Proof. See the proof stated in Section 8.2.

The fact that a manager of lower public quality may gain control of firms will be an important consideration for market regulators. If other mechanisms can be used to discipline managers, the cost of this problem will be small. Moyer et al. (1992) find that alternative monitoring mechanisms emerge in firms after the issuance of dual-class shares.¹⁹ This is an additional issue that will make the costs and benefits of dual-class shares difficult to evaluate.

research.

¹⁹Hollinger International presents a good example of the negative effects of dual-class shares. Former CEO Conrad Black controlled all of the company's class-B shares, which gave him 30% of the equity and 73% of the voting power. He ran the company as if he were the sole owner, exacting huge management fees, consulting payments, and personal dividends. Hollinger's board of directors was filled with Black's friends who were unlikely to forcefully oppose his authority. Holders of publicly traded shares of Hollinger had almost no power to make any decisions in terms of executive compensation, mergers and acquisitions, board construction poison pills, or anything else for that matter. Hollinger's financial and share performance suffered under Black's control.

4.2 Incumbent's Ownership and Underinvestment

Next, consider the case in which the manager does not own any existing shares in the firm; that is, $\beta = 0$. Suppose investments are financed by issuing voting shares; because the incumbent has no shareholding in the firm, there is no possibility of dilution in his control rights and thus, the likelihood of the incumbent retaining control of firm is not affected by the level of investments. In fact, the likelihood that the incumbent retains control depends only on his public quality, a_I . Thus, the incumbent's objective function is strictly increasing in x. Since underinvestment is not an issue, there is no need to allow the incumbent to issue non-voting shares. The proposition below formalizes this result.

Proposition 8. When investments are financed by issuing voting shares and the incumbent does not own any equity in the firm (i.e., $\beta = 0$), then the incumbent invests in all available positive NPV projects.

Proof. See the proof stated in Section 8.3.

This is a counter-intuitive result: When the incumbent owns part of the firm's equity, he bears part of the cost of underinvestment. The larger the incumbent's ownership, β , the larger his share of the cost of underinvestment. Thus, it seems if $\beta=0$, then the incumbent cares only about the probability of retaining control and his private benefits! But Proposition 8 shows that if $\beta=0$, then the incumbent always invests in all available positive NPV projects. From Equation (6), we see that if $\beta=0$, then it does not matter whether the incumbent pays more dividends or pays some as vote premium, because the number of outside votes, $N+n^1$, is equal to number of dividend claims, $N+n^1$. Hence, the control contest depends only on how much cash flow the incumbent generates vis-á-vis the rival. Thus, he invests \bar{x} to maximize the cash flows and consequently, maximize his chance of retaining control.

4.3 Investment using Voting and Non-voting Shares

Next, we consider the firm's financing decision and its effect on the incumbent's likelihood of retaining control of the firm after the new investments are completed. From Equations (17) and (18) we know that the ability to extract private benefits of the incumbent, b_I , determines whether the probability of retaining control, ϕ^j , increases or decreases with the level of investment. Given any level of public quality of the incumbent, a_I , we can isolate two types of incumbent: The incumbent with relatively high ability to extract private benefits, $1/2 \le b_I \le 1$; and the incumbent with relatively low ability to extract private benefits, $0 \le b_I < 1/2$.

If the incumbent has relatively high ability to extract private benefits and uses voting shares to fund the new investments, then two implications arise for the incumbent's objective function: First, his expected private benefit can decrease because the likelihood that he retains control of the firm decreases as x increases. But new investment increases his share of the dividend, $N \beta V_D^1$, as well as his level of actual private benefit, $b_I \alpha a_I P(x)$. Hence, the level he invests depends on the net effect of increasing x on his expected wealth. This result is formalized in the proposition below.

Proposition 9. If voting shares are issued to fund new investments, then the necessary conditions for the incumbent manager to forgo some positive NPV projects are (i) $\beta > 0$ and (ii) $b_I \geqslant 1/2$.

Proof. See the proof stated in Section 8.4.

Even if the expected private benefit of the incumbent falls as he invests more using voting shares, the higher value of the dividend claim resulting from the increased investment can increase the incumbent's objective function. Not only that, by our design, the level of private benefit also increases with the level of investment; thus, if the negative impact of investment on the likelihood of retaining control outweighs the positive effects on dividends and private benefits, then only will the incumbent underinvest. Next, we derive the sufficiency condition for underinvestment.

Proposition 10. When voting shares are issued to fund new investments and the incumbent manager owns some equity in the firm $(\beta > 0)$, then the incumbent manager forgoes some positive NPV

projects if his ability to extract private benefits, b_I , is weakly greater than \hat{b}_I , where

$$\hat{b}_{I} = \min \left(\frac{\left(1 - \beta\right) \left(\frac{2(1-\beta)^{2}(1-(1-\alpha)\beta)\log\left(1 + \frac{\alpha\beta}{1-\beta}\right)}{2-\beta(4-\beta(\alpha+(1-\alpha)(2-\alpha)\beta))} - \alpha\beta \right)}{\alpha^{2}\beta^{2}}, 1 \right).$$

Proof. See the proof stated in Section 8.5.

The condition provided in the Proposition 10 is a sufficient condition for underinvestment: The incumbent manager will always forgo some positive NPV investments if he is forced to use voting shares to fund the new investments and his ability to extract private benefits is greater than \hat{b}_I . The manager may forgo some positive NPV projects for even smaller values of his ability to extract private benefits, $\frac{1}{2} \leqslant b_I \leqslant \hat{b}_I$.

One can interpret \hat{b}_1 as a proxy for the likelihood of underinvestment: Incumbent managers with private qualities greater than \hat{b}_I are sure to forgo some positive NPV investments. As \hat{b}_I gets bigger and approaches 1, it gets less and less likely that we will find an incumbent with ability to extract private benefits greater than \hat{b}_I ; hence, it also becomes less likely that the incumbent will underinvest. As a shareholder the incumbent bears part of the costs of underinvestment. The larger his ownership, the larger the his share of the costs; thus, the condition given in the Proposition 10 depends on β . If the manager owns 8% of the equity in the firm and a relatively effective outside private benefit monitoring mechanism is in place (i.e., $\alpha = 0.20$), then the incumbent forgoes some positive NPV projects only if $b_I > 0.8689$ – implying that if we collect a sample of firms with 8% managerial ownership located in countries with reasonably effective private benefit monitoring mechanisms, then we should expect to find some underinvestment in 13.11% of these firms. Figure 4 depicts the likelihood that the incumbent will underinvest, $1 - \hat{b}_I$, as a function of an incumbent's ownership, β , and ineffectiveness of outside private benefit monitoring mechanisms, α .

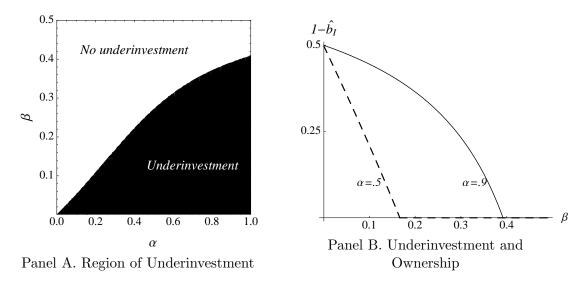


Figure 4: This figure depicts the likelihood that the incumbent manager underinvesting, $1 - \hat{b}_I$ as function of his share holdings, β and ineffectiveness of outside private benefit monitoring mechanisms, α . In panel A the incumbent forgoes some positive NPV projects for all parameter-combinations generating the shaded region of the figure. In panel B we plot the the likelihood that the incumbent manager underinvesting as function of β and for two different value of α .

4.4 Welfare of Existing Shareholders and the Incumbent Manager

So far we have obtained conditions under which investment increases or decreases if voting shares or non-voting shares are issued to raise funds. Increased investment financed issuing non-voting shares is not always in the best interests of either outside shareholders or the incumbent. There are costs to issuing non-voting equity. These costs are considered here.

Since investors who buy non-voting shares are not entitled to any of the private benefits paid as vote premium in a control contest, they are only willing to pay a lower per-share price for non-voting shares than for otherwise comparable voting shares. This means that a larger number of non-voting shares, n^0 , relative to voting shares, n^1 , have to be issued to finance a given level of investment, x, reducing the per-share dividend that is available to the existing shareholders. This is what we call the "dividend dilution" effect. Also, issuance of non-voting shares decreases the likelihood of a successful takeover bid – which we call the "entrenchment" effect. Thus, holding constant the extracted private benefits, a drop in the likelihood of a bid reduces the value of the voting rights. Though there is an offsetting gain – sharing of the extracted private benefits as vote premium among a smaller number shareholders. To summarize, the issuance of non-voting shares

affects existing shareholders and the manager in three ways: (i) Lower per-share dividends; (ii) a lower probability of a change in control; and, (iii) a higher per-share takeover premium conditional on a takeover.

Remember, the value of the firm for any incumbent type (a_I, b_I) and new investments, $x \in (0, \bar{x}]$, financed using voting shares, $FV(x^1 = x) \geqslant FV(x^0 = x)$, the value of the same firm for the same level of new investments financed using non-voting shares. Thus, the existing outside shareholders will voluntarily allow the incumbent to issue non-voting shares only if the level of investment, $\bar{x} - \Delta \bar{x}$, using voting shares leads to a firm value, $FV(x^1 = \bar{x} - \Delta \bar{x}) < FV(x^0 = \bar{x})$, the full investment firm value using non-voting shares. Next proposition presents conditions under which the value of existing voting shares is higher if new non-voting shares are used to fund new investments.

Proposition 11. For all $b_I \geqslant \hat{b}_I$ and \bar{x} such that $n^0(\bar{x}) \leqslant N$, existing outside shareholders prefer investment financed using non-voting shares if

$$1 - \frac{P(x)}{P(\bar{x})} \geqslant \frac{a_I^2 \alpha \, b_I(2 - \alpha)(2 + b_I(2 \, \alpha \, \beta + \alpha) - 2 \, \beta)}{2 \, a_I(1 - \beta)^2 \, (1 - \alpha \, b_I) - (2 - \alpha)(a_I - a_I \, \beta(1 - \alpha \, b_I))^2 + (2 - \alpha)(1 - \beta)^2}.$$

Proof. See the proof stated in Section 8.6.

The result above gives the level of underinvestment needed before the existing shareholders voluntarily allow the incumbent manager to raise funds by issuing non-voting shares. The LHS of the above inequality is a measure of the shareholders' loss due to underinvestment, while the RHS is a measure of the costs related to the issuance of non-voting shares. It is optimal for outside shareholders to allow the manager to issue non-voting shares only when the gains realized from reduced underinvestment outweigh the costs of issuing non-voting shares. If $a_I = 0.5$, $b_I = 0.75$, $\alpha = 0.2$, and $\beta = 0.05$, outside shareholders will find the issuance of non-voting shares to finance an investment optimal, even if underinvestment is just about 6.8% and likelihood of underinvestment, $1 - \hat{b}_I$ is equal to 27%.

Next, we consider the incumbent's expected wealth. Use of non-voting shares to fund the new investments lowers the per-share dividend and thus, affects the incumbent's wealth negatively. If

 $b_I \geqslant 1/2$, then non-voting shares lower the probability of a successful takeover and thus, non-voting shares help to increase expected wealth of the incumbent.²⁰ Given the incumbent's type (a_I, b_I) and his objective function, underinvestment will occur if the incumbent is forced to issue only voting shares to fund new investment such that $W_I(x^1 = \bar{x}) < W_I(x^0 = \bar{x})$ and there exists a $\Delta \bar{x}$ such that $W_I(x^1 = \bar{x} - \Delta \bar{x}) = W_I(x^0 = \bar{x})$. The propositions below provide results on the types of managers who are better off if the firm issues non-voting stock.

Proposition 12. For all $1/2 \le b_I \le 1$ and \bar{x} such that $n^0(\bar{x}) \le N$, the incumbent prefers investment financed by non-voting equity if $b_I \ge \hat{b}_I$, where

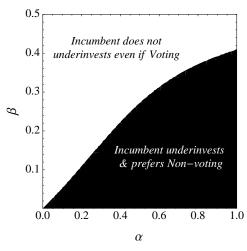
$$\hat{\hat{b}}_I = \frac{(1-\beta)\left((2-\alpha)\alpha\beta^2(1+\beta) + 2(1-\beta)^2\left((1+\beta)\log\left(1+\frac{\alpha\beta}{1-\beta}\right) - \beta\log\left(1+\frac{\alpha(1+\beta)}{1-\beta}\right)\right)\right)}{\alpha\beta(1+\beta)\left(4(1-\beta)^2\log\left(1+\frac{\alpha(1+\beta)}{1-\beta}\right) - (2-\alpha)\alpha\beta(1+\beta) - 4(1-\beta)^2\log\left(1+\frac{\alpha\beta}{1-\beta}\right)\right)}.$$

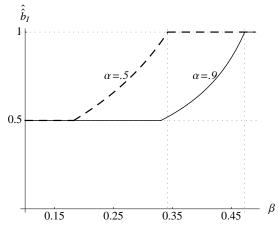
Proof. See the proof stated in Section 8.7.

From Equation (18) we see that investment using non-voting shares increases the likelihood that the incumbent retains control of the firm if $b_I \geqslant \frac{1}{2}$. From Proposition 12 we know that the incumbent is better off if non-voting shares can be issued and $b_I \geqslant \hat{b}_I$. The divergence exists because of the cost related to dividend dilution associated with non-voting shares. If non-voting shares are issued, then the aggregate dividend is divided among $N + n^0$ shares, which is strictly greater than $N + n^1$ shares if voting shares are issued. If $\beta = 0.2$ and $\alpha = 0.4$, the manager prefers non-voting shares if b_I is greater than 0.59, even though the incumbent's likelihood of retaining control increases with investment for all $b_I \geqslant 0.5$. For all $\beta > 0.22$ and for all $\alpha \leqslant 0.2$ the value of $\hat{b}_I \geqslant 1$, implying that the incumbent never likes non-voting shares.

From Propositions 10, 11, and 12 we can derive some interesting observations. Consider the case when $b_I \in [1/2, 1]$. It is easy to show for all β and α such that \hat{b}_I and \hat{b}_I are within the range 1/2 and 1, that $\hat{b}_I > \hat{b}_I$. We depict the comparable values of \hat{b}_I and \hat{b}_I in Figure 5 below. Thus, we can divide the range of the incumbent's private qualities into four parts: $[0, 1/2), [1/2, \hat{b}_I), [\hat{b}_I, \hat{b}_I)$, and

²⁰The increase in the takeover premium does not affect the manager because we have assumed that he does not tender in a takeover.





Panel A. Region of Underinvestment and Non-voting Shares

Panel B. Underinvestment, Non-voting Shares and Ownership

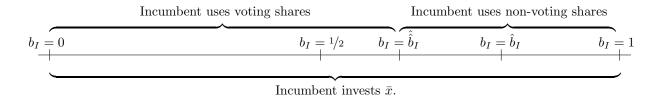
Figure 5: This figure depicts parameter values such that the incumbent prefers non-voting shares. In panel A we find that for parameter values such that the likelihood of underinvestment is positive, $1 - \hat{b}_I > 0$, we find $\hat{b}_I > \hat{b}_I$. This implies that for all parameter values such that the manager underinvests, the manager also prefers financing with non-voting shares. In panel B we depict the minimum b_I such that the incumbent prefers non-voting shares as a function of his ownership fraction, β . As incumbents holding size increases, the incumbent is less likely to prefer non-voting shares.

 $[\hat{b}_I, 1]$. If the incumbent is forced to use only voting shares, then the incumbent with $b_I \in [\hat{b}_I, 1]$ underinvests.²¹ Otherwise, he invests in all available positive NPV projects. If the incumbent is given a choice of using either voting shares or non-voting shares, then the incumbent always invests in all available positive NPV projects. The manager will use voting shares to fund all investments, if his ability to extract private benefits $b_I \in [\hat{b}_I, 1]$. The manager will use non-voting shares to fund all investments, if his ability to extract private benefits $b_I \in [\hat{b}_I, 1]$. Figure 7 below depicts these b_I ranges.

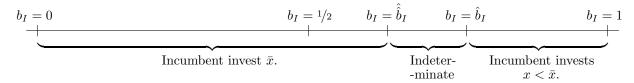
The final question that we answer in this subsection is in what instances we will observe firms issuing non-voting shares. The answer depends on the balance of power between the manager and the shareholders. If shareholders have the upper hand and can force the manager to issue a particular type of security, the condition given in Proposition 11 will determine when the firm will issue non-voting shares. In contrast, if shareholders can only specify a menu of securities, then conditions in Proposition 10 and Proposition 12 will both have to be satisfied before the firm issues

²¹If the incumbent's $b_I \in [\hat{b}_I, \hat{b}_I]$, he may or may not under invest. This range is indeterminate as we are only able to solve sufficient condition of underinvestment as well as incumbent's choice of voting vs. non-voting shares.

Allowed to use either voting shares or non-voting shares



Forced to use only voting shares



non-voting shares. The next section discusses extensions to our model. It also looks at the effect of relaxing some of our initial assumptions.

5 Extensions

In this section we consider three related issues. First we allow the incumbent to tender his holdings in a control contest. This is important because it helps to make the incumbent more entrenched. Second we address the issuance of shares with fewer than one vote per-share. The reason that these may be useful is that they would have lower costs related to dividend dilution than zero-vote shares. Firms in many countries allow firms to issue multiple classes of shares with different voting rights just like dual class shares. Third, we discuss the costs and benefits of multiple classes of shares.

5.1 Entrenchment and Investment when the Incumbent Tenders

A change in control occurs when the rival manager can offer a higher per-share value to the outside shareholders than the incumbent. In our initial setup we assumed that the incumbent does not tender his shares in the control contest. We now relax this assumption and allow the manager to tender. If the manager tenders his shares in a control contest, then the rival's private benefit is divided over a larger number of shares, N + n as opposed to $(1 - \beta)N + n$ outside voting shares when the manager is not able to tender. This puts the rival at a disadvantage relative to the incumbent.²² The probability of a takeover can be obtained by considering the cases of non-voting shares and voting shares separately. The incumbent can retain control if he can offer more for the shares than the rival. If voting shares are used to finance the investment, this contest is equivalent to

$$\frac{x + a_I P(x)}{N + n^1} - \frac{b_I \alpha a_I P(x)}{N + n^1} + \frac{b_I \alpha a_I P(x)}{(1 - \beta) N + n^1} \geqslant \frac{x + a_R P(x)}{N + n^1} - \frac{b_R \alpha a_R P(x)}{N + n^1} + \frac{b_R \alpha a_R P(x)}{N + n^1}.$$
(24)

The first two terms on the LHS of Equation (6) are the per-share public value that is generated with the incumbent in control. The third term on the LHS is related to the incumbent's private value. The denominator is smaller here than in the first two terms because the private benefit is distributed only to the outside shareholders. The RHS terms are related to the public and private benefit per-share generated under the rival. The private benefits of the potential rival is divided among all the shareholders of the firm. Simplifying Equation (24) we obtain

$$a_I \left(1 + \alpha \kappa^1 b_I \right) \geqslant a_R,$$
 (25)

where $\kappa^1 = \frac{N\beta}{(1-\beta)N+n^1}$. From Equation (11) we know that $\bar{a}_R^1 = a_I \left(1 + \alpha \kappa^1 b_I\right)$. Thus, the incumbent retains control of the firm if $a_R \in [0, \bar{a}_R^1]$. The ability to extract private benefits of the potential rival plays no role in the control contest, whereas the ability to extract private benefits of the incumbent plays a significant role in the control contest. When the incumbent can tender, the range of the potential rival's qualities over which the incumbent retains certain control, $[0, \bar{a}_R^1]$, is much higher than $[0, \underline{a}_R^1]$, the range when the incumbent does not tender. Since the range over which the incumbent certainly loses control, $[\bar{a}_R^1, 1]$, remains the same, the likelihood that

²²See, for example, Burkart et al. (1998) for more discussions.

the incumbent retains control of the firm after investing x, say $\hat{\phi}^1(x)$, is greater than $\phi^1(x)$, the likelihood that the incumbent retains control of the firm after investing x, when the incumbent is not able to tender.

What happens to the incumbent's likelihood of retaining control of the firm if x increases? Because $\frac{\partial \kappa^1}{\partial x}$ is negative, $\frac{\partial \hat{\phi}^1}{\partial x}$ is also negative. Hence, the incumbent's likelihood of retaining firm control decreases as the incumbent raises the level of investment.

If non-voting shares are issued to finance the investment and if the manager can tender his shares in a control contest, the rival's private benefit is divided over a larger number of shares, N rather than $(1 - \beta)N$. Then the incumbent retains control if

$$\frac{x + a_I P(x)}{N + n^0} - \frac{b_I \alpha a_I P(x)}{N + n^0} + \frac{b_I \alpha a_I P(x)}{(1 - \beta) N} \geqslant \frac{x + a_R P(x)}{N + n^0} - \frac{b_R \alpha a_R P(x)}{N + n^0} + \frac{b_R \alpha a_R P(x)}{N}. \quad (26)$$

The private value is distributed to all shareholders who own voting shares. The holders of the non-voting shares do not get a share of the private value because they cannot affect the outcome of the control contest. Simplifying Equation (26) we get

$$a_I \left(1 + \alpha \kappa^0 b_I \right) \geqslant a_R \left(1 + \alpha \hat{\kappa}^0 b_R \right),$$
 (27)

where $\kappa^0 = \frac{N\beta + n^0}{(1-\beta)N}$ and $\hat{\kappa}^0 = \frac{n^0}{N}$. Also note that $\kappa^0 - \hat{\kappa}^0 = \frac{\beta(N+n^0)}{(1-\beta)N} > 0$. This gives

$$\hat{a}_R^0 = a_I (1 + \alpha \kappa^0 b_I) = \bar{a}_R^0 \text{ and } \hat{\underline{a}}_R^0 = \frac{a_I (1 + \alpha \kappa^0 b_I)}{1 + \alpha \hat{\kappa}^0} = \frac{\bar{a}_R^0}{1 + \alpha \hat{\kappa}^j} > \underline{a}_R^0.$$
 (28)

From Equation (28), we see that the range of the potential rival's qualities over which the incumbent retains certain control, $[0, \hat{\underline{a}}_R^0]$, is larger than $[0, \underline{a}_R^0]$, rival's qualities over which the incumbent retains control when the incumbent is not able to tender. Since the range over which the incumbent loses control, $[\bar{a}_R^0, 1]$, remains the same in this case also, the incumbent's likelihood of retaining control of the firm for a given level of investment, $\hat{\phi}^0(x)$, is at least weakly greater when the manager can tender, relative to the likelihood of retaining control of the firm for the same level of investment,

 $\phi^0(x)$, when the manager cannot tender. This is formally stated in the proposition below.

Proposition 13. Regardless of the type of financing of the issue, voting or non-voting shares, the ability the incumbent to extract private benefits, b_I , plays a relatively more decisive role in the control contest and makes the incumbent more entrenched, if the incumbent can tender his shares in the control contest.

Proof. Follows directly from Equations (25) or (28).
$$\Box$$

Like our initial setup, these bounds determine the outcome of the control contest: Likelihood that the incumbent retains control of the firm after investment depends on the level of investment, and the incumbent can choose the size of x. Because the number of new shares needed to finance the investment increases as the size of the investment increases, $\frac{\partial n^0}{\partial x} > 0$, then $\frac{\partial \hat{\kappa}^0}{\partial x} > 0$. Thus,

$$\frac{\partial}{\partial x}\,\hat{\underline{a}}_{R}^{0} = \frac{\partial}{\partial n^{0}}\,\hat{\underline{a}}_{R}^{j} \times \frac{\partial n^{0}}{\partial x} = -\frac{a_{I}N\alpha(1-\beta-\frac{b_{I}}{2}(1-\alpha\beta))}{(N+n^{0}\alpha)^{2}(1-\beta)} \times \frac{\partial n^{0}}{\partial x} < 0. \tag{29}$$

Thus, the implications of increasing investments on the incumbent manager's likelihood of retaining control of the firm remains qualitatively similar to the case when the manager is not able to tender.

5.2 Optimal vote-dividend combination

The optimal vote-dividend combintion can be thought of in two different ways. The first is to consider shares that have 1 unit of dividend and θ votes, and to find the optimal value of θ . The second is to allow the firm to simultaneously issue both voting and non-voting shares. We first consider θ -vote shares.

The optimality of θ -vote shares, with $0 < \theta < 1$, will depend on the size of the investment opportunity available to the firm. For a class of shares, the vote will have value only if a sufficient mass of votes of that class exist so that these shares can be used by the manager to block a takeover. This means that managers will issue θ -vote shares only if the investment opportunity is large enough that $\beta N + n \theta \geqslant \frac{1}{2} (N + n \theta)$.

The reasoning captured by the above inequality is as follows: Consider a firm with two classes of shares, one-vote and θ -vote, outstanding. Suppose $n \theta$ is small so that the above inequality is not fulfilled. In this case the manager has no incentive to bid for this θ -vote class of shares; blocking the rival requires the manager to bid for the voting shares. The rival has no incentive to bid for the θ -vote shares either. The outcome of the control contest is determined solely by the owners of the voting shares. This causes the vote to have zero value in the θ -vote shares, giving the manager no incentive to issue θ -vote shares.

If \bar{x} is small, $\theta = 0$ is likely to be optimal. This is because the number of shares that are issued is going to be small for small \bar{x} , and the total number of votes held by shareholders in that class will be insufficient to meet the above condition. Our model has assumed that shareholders are homogeneous. Heterogeneity among shareholders may result in cases where θ -vote shares may become optimal even when \bar{x} is small.

Allowing firms to simultaneously issue both non-voting and voting shares will increase the set of firms that find it optimal to issue dual-class shares. This assertion is based on the following line of reasoning: Existing one-vote shareholders prefer non-voting shares when the level of underinvestment is high; that is, $1 - \frac{P(x)}{P(\bar{x})} \geqslant g(\alpha, \beta, \kappa^0, b_I, a_I)$. From Equation (P11-5) we know that the RHS of the above inequality is an increasing function of κ^0 which itself is an increasing function of $n^0(\bar{x})$. Hence, as $n^0(\bar{x})$ decreases, the outside shareholders will find it optimal to allow the manager to finance investments using non-voting shares, even for low levels of underinvestment. If the investments are partly financed using voting shares and this analysis is carried out over the remaining projects, the relevant $n^0(\bar{x})$ will have a smaller value, implying that the existing shareholders, the owners of the voting shares, would be more willing to allow managers to issue non-voting shares. In this case the existing shareholders could allow the manager the choice of issuing voting shares or a mix of θ -voting shares per non-voting share issued.

5.3 Multiple classes of shares

We considered a firm that issues only two classes of shares: Voting and non-voting shares. One logical extension to this model is to consider multiple classes of shares. Is it optimal either for the manager or for existing shareholders to issue multiple classes of shares? Shares that give their owners fractional voting rights can be considered. Thus, the firm could simultaneously issue shares with θ_0 , θ_1 , θ_2 , and θ_3 votes (an example can be $\theta_0 = 0, \theta_1 = 0.33, \theta_2 = 0.5$, and $\theta_3 = 1$). In the above framework, the shares with fractional votes will be issued if the fractional votes have value. The fractional votes will have value if there is a sufficient mass of each of these classes of shares outstanding so that the rival is forced to buy them to take control of the firm.

The manager can raise the cost of a takeover for the rival by issuing multiple classes of shares. This does not mean that it is optimal for the manager to issue multiple classes of shares. The manager bears a cost when he issues multiple classes of shares. This cost is in the form of lower dividends. The existing shareholders are likely to find multiple classes of shares detrimental to their interest. As the number of classes of shares increases the probability of a change in control is likely to decrease very quickly. The compensating factor, investment, is unlikely to go up fast enough to increase the value of the shares held by outside shareholders. Thus, multiple classes of shares are unlikely to be optimal for old shareholders.

6 Conclusions

This study provides a theoretical justification for easing the prohibitions on the issuance of dualclass shares, which has recently been proposed or enacted in a number of developed and developing countries. We analyze the decision problem of a firm with a set of available positive NPV projects. we show that if a firm requires outside equity financing to undertake profitable investment projects, then in some cases managers will find separation of the voting and dividend claims optimal. Raising equity capital has two effects: (i) The value of the firm increases as positive NPV projects are undertaken and (ii) the proportion of the firm's shares owned by the manager decreases, increasing the likelihood that the manager loses control of the firm. Thus a manager, who values control because of private benefits, may find it optimal to forgo some positive NPV projects. Non-voting shares enables the manager to finance the investment without diluting his control, and thus increasing his chances of losing control of the firm, which in turn increases the manager's willingness to undertake all positive NPV projects.

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7 Appendix A

7.1 A Numerical Example

Consider a firm that has a public value of \$2.00, generates a private value for the incumbent of \$0.20 and has 100 shares outstanding. The value of the existing firm, both public and private, is the same under both the incumbent and the rival manager. The incumbent owns 50 shares in the firm and the incumbent is wealth constrained. Given our assumption that the incumbent owns half the shares in the firm, there is a zero probability of a change in control of the firm, ($\phi = 1$), without the incumbent's consent.

The expected value of the incumbent's stake in the firm is the sum of the expected public value of the shares that he owns, plus the expected private benefits of control; that is, the value of the incumbent's stake in the firm is $\$1.20 \ (= 1/2 \times \$2.00 + \phi \times 0.20 = \$1.00 + 1.0 \times 0.2)$. The value of the shares owned by outside shareholders is the probability of the incumbent's retaining control times the public value of the firm under the incumbent, plus the probability of the rival's gaining control

times the price paid by the rival. Thus, the value of the shares owned by the existing outside shareholders is \$1.00.

To keep the numerical example simple, we assume that the incumbent has to choose from three discrete investment levels: invest nothing, invest \$1.00, or invest \$2.00. If the incumbent invests nothing, there is no addition to the value of the firm and no new shares are issued. If the incumbent invest \$1.00 or \$2.00, the resulting value of the firm, additional public and private value generated under the incumbent and a rival manager are summarized in Table I below.

Table I

Initial number of shares outstanding 100; incumbent manager owns 50							
Existing Firm Value		Investment Opportunity					Number of New Shares
Public Value	Private Value	Investment	nt Incumbent Manager		Rival Manager		Issued
			Addition to Public Value	Addition to Private Value	Addition to Public Value	Addition to Private Value	
2.00	0.20	0.00	0.00	0.00	0.00	0.00	0.00
2.00	0.20	1.00	1.10	0.06	1.18	0.00	50.00
2.00	0.20	2.00	2.12	0.07	2.20	0.00	100.00

Figure 6: This table summarizes the value of the existing firm and the additional public and private value created by the new investments under the incumbent and the rival manager. The first row depicts the no investment case; the second and the third rows depict the cases where the incumbent invests \$1.00 and \$2.00.

Investment in the projects adds to the public value of the firm and also to the private benefits of the manager-in-control at the end of the investment horizon. We assume that the rival manager is strictly better than the incumbent: The rival manager can generate public value that is higher than the sum of the public and private value that the incumbent can generate. For example, if the incumbent invests \$1.00 and he is the manager-in-control at the end of the investment horizon, then the public value is \$1.10 and his private benefit is \$0.06, giving an aggregate value \$1.16. Whereas, if the incumbent invests \$1.00 and the rival manager is the manager-in-control at the end of the investment horizon, then the public value is \$1.18 > \$1.10 + \$0.06.

If non-voting equity is used to finance the investment, the incumbent's proportional ownership of the control rights (votes) remains at 50% and the incumbent retains the ability to prevail in all control contests. If voting shares are issued to finance the investment, the incumbent's proportional ownership of the control rights drops to either 33% (= $^{50}/_{150}$) or 25% (= $^{50}/_{200}$) depending on the level of investment. We assign probabilities $\phi^1(x=1) = 0.95$ and $\phi^1(x=2) = 0.79$. These probabilities are essentially the ability of the incumbent to prevail in a control contest when he owns 33% and 25% of the voting shares. Table II summarizes this information.

The expected value of the incumbent's stake in the firm is the sum of the expected public value of the shares that he owns plus the expected private benefits of control. The expected public value of a share in the firm is the probability that the incumbent retains control times the public value of the firm under the incumbent, plus the probability that the rival manager gains control times the public value of the firm under the rival manager. For investment level of \$1 the expected public value is equal to the expected NPV under the incumbent plus the expected NPV under the rival

Table II

Voting Rights Ownership and Probability of Retaining Control under Different Types of Equity Financing and Different Investment Levels				
Voting Shares Issued		Finance New Investment	Nonvoting Shares Issued to Finance New Investment	
Investments	Managerial Ownership of Voting Rights	Probability of Retaining Control	Managerial Ownership of Voting Rights	Probability of Retaining Control
0.00	50.00%	1.00	50.00%	1.00
1.00	33.00%	0.95	50.00%	1.00
2.00	25.00%	0.79	50.00%	1.00

Figure 7: The first half of the table shows the likelihood of retaining control if voting shares are used to fund the new investment. The second half of the table shows the likelihood of retaining control if non-voting shares are used to fund the new investment.

manager; that is, $2 + (0.95 \times (1.1 - 1) + 0.05 \times (1.18 - 1)]$ or 2.104. The expected private benefit extracted by the incumbent is the private benefit of control times the probability of remaining in control. For investment level of \$1.00 the expected private benefit is 0.95×0.26 or 0.247. Therefore, the expected value of the incumbent's stake if he invests \$1.00 is $0.5 \times (2.104) + 0.247$ or 1.299. For the investment level \$1.00, the shareholders' expected wealth is 0.5×2.104 or 1.052. If the incumbent has a choice regarding the type of equity to issue to finance the project, the incumbent will issue non-voting equity to invest \$1.00 and the expected value of the shares owned by existing outside shareholders is \$1.05. For investment level of \$2.00, the expected public value of the firm is $2 + 0.79 \times (2.12 - 2) + 0.21 \times (2.20 - 2)$ or 2.1368. The expected private benefit extracted by the incumbent is 0.79×0.27 or 0.2133. Therefore, the expected value of the incumbents stake for investment level of \$2.00 is $0.5 \times 2.1368 + 0.2133$ or 1.2817. The expected welfare of the outside shareholders is 0.5×2.1368 or 1.0684. Table III summarizes this information.

Table III

Payoff of Incumbent Manager and Outside Shareholders' under Different Types of Equity Financing and Different Investment Levels					
	Voting Shares Issued to	Finance New Investment	Nonvoting Shares Issued to Finance New Investment		
Investments	Manager's Payoff	Outside Shareholders' Payoff	Manager's Payoff	Outside Shareholders' Payoff	
0.00	1.2000	1.0000	1.2000	1.0000	
1.00	1.2990	1.0520	1.3100	1.0500	
2.00	1.2817	1.0684	1.3300	1.0600	

Figure 8: The first half of the table shows the expected payoff of the incumbent and outside shareholders when voting shares are issued to finance the investment, and the second half of the table shows payoff of the incumbent and outside shareholders when non-voting shares are issued to finance the investment. Outside shareholders always want the manager to invests in all positive NPV projects. But if forced to use voting shares, then it is optimal for the incumbent to invest \$1.00 rather than \$2.00 given his payoff from investing \$1.00 is \$1.2990 which is strictly greater than \$1.2817 – incumbent's payoff from investing \$2.00.

The incumbent's expected wealth is maximized at an investment level of \$1.00 when the investment is finance using voting shares and at an investment level of \$2.00 when the investment is financed using non-voting shares. The decision made by the existing outside shareholders is related

to the type of shares that the firm can issue to finance the new investment. If the incumbent is required to finance the investment by issuing voting shares, the incumbent will invest \$1.00 and the expected value of the shares owned by existing outside shareholders is \$1.052. From the table it can be seen that there are situations in which it is value increasing for outside shareholders to allow the incumbent to issue non-voting shares to finance investments. This increases the outside shareholders' wealth from \$1.052 to \$1.06. This is true regardless of the fact that non-voting shares are likely to entrench the incumbent and prevent better rivals from taking over the firm. The difference in the value of the shares owned by the existing outside shareholders when voting and non-voting shares are used to finance the investment is a cost of entrenchment (for investment level \$1, the costs entrenchment is \$1.052 - \$1.05 = \$0.002) per dollar of investment.

Allowing the manager to issue non-voting shares will raise the value of the shares owned by existing outside shareholders when the loss in value from underinvestment is larger than the loss in value from entrenchment. Examples of this situation are firms that have many growth opportunities and firms in relatively new industries. For firms that have relatively few growth opportunities the above result is unlikely to hold. In these firms underinvestment is less likely to be a problem and will have a smaller negative impact on the value of the firm.

Does a contractual solution to the underinvestment problem work? Often it may be possible to make a side payment to the manager to induce him to undertake the investment. This alternative would require the outside shareholders to compensate the manager for the decrease in expected wealth associated with an investment of \$2 financed using one-vote shares. In the case presented above, contractual solution does not work. Increase in the outside shareholders expected wealth, \$1.0684 - \$1.052 = \$0.0164, if investment level goes up from \$1.00 to \$2.00, is smaller than the drop in the incumbent's expected wealth, \$1.2817 - \$1.299 = -\$0.0173. Hence, cross-subsidization will not be feasible.

8 Appendix B

8.1 Basic Results

Before we present the proofs of the propositions, we first show some basic results that we will use repeatedly. By definition we have

$$\kappa^{0}(x) = \frac{\beta N + n^{0}(x)}{(1 - \beta)N} \text{ and } \kappa^{1}(x) = \frac{\beta N}{(1 - \beta)N + n^{1}(x)},$$
 (BR-1)

where $n^j(x) = 0$ at x = 0, and $n^j(x) > 0$, $\forall x \in (0, \bar{x}]$, and j = 0, 1. Also, $\frac{\partial n^j(x)}{\partial x} > 0$. Thus,

$$\kappa^0(0) = \kappa^1(0) = \frac{\beta}{1-\beta}.$$
(BR-2)

For x > 0 we get

$$\kappa^{0}(x) = \frac{\beta}{1-\beta} + \frac{n^{0}(x)}{(1-\beta)N} > \frac{\beta}{1-\beta} = \kappa^{0}(0),$$

If $n^0(\bar{x}) \leq N$, then

$$\kappa^0(x) \leq \frac{\beta}{1-\beta} + \frac{1}{(1-\beta)}.$$
 (BR-3)

$$\kappa^{1}(x) = \frac{\beta}{1-\beta} - \frac{\beta n^{1}(x)}{(1-\beta)N[(1-\beta)N + n^{1}]} < \frac{\beta}{1-\beta} = \kappa^{1}(0).$$
 (BR-4)

Thus $\kappa^0(x) \geqslant \kappa^1(x) \ \forall x \in [0, \bar{x}]$. Differentiating $\kappa^0(x)$ and $\kappa^1(x)$ with respect to x we get

$$\frac{\partial \kappa^0}{\partial x} = \frac{\partial}{\partial x} \left(\frac{N \beta + n^0}{(1 - \beta)N} \right) = \frac{1}{(1 - \beta)N} \frac{\partial n^0}{\partial x} > 0.$$
 (BR-5)

and

$$\frac{\partial \kappa^1}{\partial x} = \frac{\partial}{\partial x} \left(\frac{N \beta}{(1-\beta)N + n^1} \right) = \frac{-1}{((1-\beta)N + n^1)^2} \frac{\partial n^1}{\partial x} < 0.$$
 (BR-6)

8.1.1 Dividend claims

We can simplify Equation (19) and rewrite as

$$(N+n^{j})V_{D}^{j} = x + \phi^{j} \times a_{I}(1-\alpha b_{I})P(x) + \int_{\underline{a}_{R}^{j}}^{\overline{a}_{R}^{j}} \int_{\underline{b}_{R}^{j}}^{1} (a_{R}(1-\alpha b_{R})P(x)) db_{R} da_{R} + \int_{\overline{a}_{R}^{j}}^{1} \int_{0}^{1} (a_{R}(1-\alpha b_{R})P(x)) db_{R} da_{R}.$$
(BR-7)

Integrating the third term of the above expression we get

$$\int_{\bar{a}_{P}^{j}}^{1} \int_{0}^{1} \left(a_{R} (1 - \alpha \, b_{R}) P(x) \right) db_{R} \, da_{R} = \frac{2 - \alpha}{4} \left(1 - (\bar{a}_{R}^{j})^{2} \right) P(x). \tag{BR-8}$$

Integrating the second term of the above expression we get

$$\int_{\underline{a}_R^j}^{\overline{a}_R^j} \int_{\underline{b}_R^j}^1 (a_R(1 - \alpha \, b_R) P(x)) \, db_R \, da_R = A(x, \, \kappa^j) + B(x, \, \kappa^j), \tag{BR-9}$$

where
$$A(x, \kappa^{j}) = \frac{(\bar{a}_{R}^{j})^{2} (2 + (4 - 5\alpha)\kappa^{j} + \alpha(6 - \alpha)(\kappa^{j})^{2})P(x)}{4\kappa^{j}(1 + \alpha\kappa^{j})}$$
 and $B(x, \kappa^{j}) = \frac{\bar{a}_{R}^{j}P(x)}{2\alpha\kappa^{j}}\ln(1 + \alpha\kappa^{j})$.

8.1.2 Value of existing voting shares when voting shares are issued

Using Equations (19) and (22) we can express total value of all voting shares when voting shares are issued to finance the new projects:

$$(N+n^1)V_1^1 = (N+n^1)V_D^1 + (N+n^1)V_{\text{vote}}^1$$
(BR-10)

$$(N+n^{1})V_{1}^{1} = \int_{0}^{\overline{a}_{R}^{1}} \int_{0}^{1} FV_{I} db_{R} da_{R} + \int_{\overline{a}_{R}^{1}}^{1} \int_{0}^{1} FV_{R} db_{R} da_{R} + \int_{\underline{a}_{R}^{1}}^{\overline{a}_{R}^{1}} \int_{\underline{b}_{R}^{1}}^{1} \frac{(N+n^{1})B_{I}}{(1-\beta)N+n^{1}} db_{R} da_{R}$$

$$= x + \frac{1}{2}P(x) \left(\left(a_{I}(1-\alpha b_{I}) + (1+\kappa^{1}) \left(1 - \frac{\ln(1+\alpha \kappa^{1})}{\alpha \kappa^{1}} \right) \bar{a}_{R}^{1} \right) + \frac{1}{2}P(x) \left(1 - \alpha/2 \right) \left(1 - (\bar{a}_{R}^{1})^{2} \right).$$
(BR-11)

If the firm issues voting stock to finance the investment, and because all new securities are issued at zero expected profit, $n^1V_1^1 = x$, we get

$$N V_1^1 = \frac{1}{2} P(x) \left(\left(a_I (1 - \alpha b_I) + (1 + \kappa^1) \left(1 - \frac{\ln(1 + \alpha \kappa^1)}{\alpha \kappa^1} \right) \bar{a}_R^1 + (1 - \alpha/2) (1 - (\bar{a}_R^1)^2) \right).$$
(BR-12)

8.1.3 Value of existing voting shares when non-voting shares are issued

If non-voting shares are used to finance the investment, then using Equations (19) and (22), the value of an existing voting share is

$$(N+n^{0})V_{1}^{0} = (N+n^{0})V_{D}^{0} + (N+n^{0})V_{\text{vote}}^{0}$$

$$= \frac{P(x)}{2} \left((a_{I}(1-\alpha b_{I}) + \left((1+\kappa^{0}) - \frac{1}{1-\beta} \right) \left(1 - \frac{\ln(1+\alpha \kappa^{0})}{\alpha \kappa^{0}} \right) \bar{a}_{R}^{0} \right)$$

$$+ \frac{P(x)}{2} (1-\alpha/2) \left(1 - (\bar{a}_{R}^{0})^{2} \right).$$
(BR-13)

8.2 Proof of Proposition 7

Using Equations (7) and (9) we get the incumbent's minimum public quality required to win the control costest: If financed using voting shares, it is

$$\underline{a}_{I}^{1} \geqslant \frac{a_{R} \left(1 + \alpha \kappa^{1} b_{R} \right)}{\left(1 + \alpha \kappa^{1} b_{I} \right)}, \tag{P7-1}$$

and if financed using non-voting shares, it is

$$\underline{a}_{I}^{0} \geqslant \frac{a_{R} \left(1 + \alpha \kappa^{0} b_{R} \right)}{\left(1 + \alpha \kappa^{0} b_{I} \right)}.$$
(P7-2)

We need to show that $\underline{a}_I^0 \leqslant \underline{a}_I^1$. Thus, we obtain

$$\underline{a}_{I}^{0} - \underline{a}_{I}^{1} = -\frac{a_{R} \alpha \left(b_{I} - b_{R}\right) \left(\kappa^{0} - \kappa^{1}\right)}{\left(1 + \alpha \kappa^{1} b_{I}\right) \left(1 + \alpha \kappa^{0} b_{I}\right)} \leqslant 0. \tag{P7-3}$$

From Equation (BR-1) we know that $\kappa^0 - \kappa^1 > 0$. Thus, if $b_I \geqslant b_R$ then $\underline{a}_I^0 < \underline{a}_I^1$. Hence, the proof.

8.3 Proof of Proposition 8

The manager chooses the investment level to maximize his objective function. We show that the first derivative of the manager's objective function evaluated at \bar{x} is nonnegative, which implys that the incumbent invests \bar{x} .

When investments financed using voting shares and $\beta = 0$, then from Equation (11) we get $\underline{a}_R^1 = \overline{a}_R^1 = a_I$. This is because $\kappa^1 = \frac{\beta N}{(1-\beta)N+n^1} = 0$. Thus,

$$\phi^{1}(x) = \int_{0}^{\underline{a}_{R}^{1}} da_{R} + \int_{\underline{a}_{R}^{1}}^{\bar{a}_{R}^{1}} \int_{0}^{\underline{b}_{R}^{1}} db_{R} da_{R} = \int_{0}^{a_{I}} da_{R} = a_{I}.$$
 (P8-1)

Incumbent manager's objective function if he finances the new investment issuing j-type shares, say MO^{j} , is

$$MO^{j}(x) = \beta N V_{D}^{j}(x) + \phi^{j}(x) b_{I} \alpha a_{I} P(x).$$
 (P8-2)

Differentiating the incumbent's objective function with respect to x gives

$$\frac{\partial MO^{j}(x)}{\partial x} = \beta N \frac{\partial V_{D}^{j}(x)}{\partial x} + b_{I} \alpha a_{I} \left(\frac{\partial \phi^{j}(x)}{\partial x} P(x) + \phi^{j}(x) \frac{\partial P(x)}{\partial x} \right). \tag{P8-3}$$

Substituting j = 1 and $\beta = 0$ we get

$$\frac{\partial MO^{1}(x)}{\partial x} = b_{I} \alpha a_{I} \left(\frac{\partial \phi^{1}(x)}{\partial x} P(x) + \phi^{1}(x) \frac{\partial P(x)}{\partial x} \right). \tag{P8-4}$$

But $\frac{\partial \phi^1(x)}{\partial x}|_{\beta=0} = \frac{\partial a_I}{\partial x} = 0$. Hence,

$$\frac{\partial MO^{1}(x)}{\partial x}|_{\beta=0} = b_{I} \alpha a_{I} \phi^{1}(x) \frac{\partial P(x)}{\partial x}.$$
 (P8-5)

Since by definition $\frac{\partial P(x)}{\partial x} > 0$ for all $x \in [0, \bar{x})$ and $\frac{\partial P(x)}{\partial x} = 0$ for $x = \bar{x}$, it must be the case that $\frac{\partial MO^1(x)}{\partial x} > 0$ for all $x \in [0, \bar{x})$ and $\frac{\partial MO^1(x)}{\partial x} = 0$ for $x = \bar{x}$. Thus, the incumbent manager invests in all available positive NPV projects. Hence, the proof.

From Equation (17) we know that if voting shares are issued and $b_I < 1/2$, then $\frac{\partial \phi^1(x)}{\partial x} > 0$ for all x. Hence, the sufficient condition for the incumbent to invest \bar{x} using voting shares is $b_I < 1/2$. Similarly, from Equation (18) we know that if non-voting shares are issued and $b_I \geqslant 1/2$, then $\frac{\partial \phi^0(x)}{\partial x} > 0$ for all x. Hence, the sufficient condition for the incumbent to invest \bar{x} using non-voting shares is $b_I \geqslant 1/2$. Hence, the proof.

8.4 Proof of Proposition 9

Differentiating Equation P8-2, substituting j=1 and further simplifying we obtain

$$\frac{\partial MO^{1}(x)}{\partial x} = \beta N \frac{\partial V_{D}^{1}(x)}{\partial x} + b_{I} \alpha a_{I} \left(\frac{\partial \phi^{1}(x)}{\partial x} P(x) + \phi^{1}(x) \frac{\partial P(x)}{\partial x} \right). \tag{P9-1}$$

Let us consider following situations:

- 1. $\beta = 0$ and $b_I \in [0, 1]$;
- 2. $\beta > 0$ and $b_I \in [0, 1/2)$; and,
- 3. $\beta > 0$ and $b_I \in [1/2, 1]$.

We have shown in Proposition 9 that regardless of the incumbent manager's ability to extract private benefits, the incumbent will fund all available positive NPV projects if $\beta = 0$. Hence, $\beta > 0$ is a necessary condition for underinvestment. If $\beta > 0$, then $\frac{\partial \phi^1(x)}{\partial x} < 0$ if $b_I \geqslant 1/2$ and $\frac{\partial \phi^1(x)}{\partial x} > 0$ if $b_I < 1/2$.

Thus, if $b_I < 1/2$, then all three, $\frac{\partial V_D^1(x)}{\partial x}$, $\frac{\partial P(x)}{\partial x}$, and $\frac{\partial \phi^1(x)}{\partial x}$ in Equation P9-1, are increasing in all $x \in [0, \bar{x}]$; hence, $\frac{\partial MO^1(x)}{\partial x}$ is increasing in x. If $b_I \geqslant 1/2$, then $\frac{\partial V_D^1(x)}{\partial x}$ and $\frac{\partial P(x)}{\partial x}$ are still increasing in x, but $\frac{\partial \phi^1(x)}{\partial x}$ is decreasing in x. Thus, only way $\frac{\partial MO^1(x)}{\partial x}$ can be negative is when $\frac{\partial \phi^1(x)}{\partial x} < 0$. But the only way $\frac{\partial \phi^1(x)}{\partial x} < 0$ is if $\beta > 0$ and $b_I \geqslant 1/2$. Hence, the proof.

8.5 Proof of Proposition 10

Using Equation (??) we find that the total value of voting shares,

$$N V_1^1 = \frac{1}{2} P(x) \left(\left(a_I (1 - \alpha b_I) + (1 + \kappa^1) \left(1 - \frac{\ln(1 + \alpha \kappa^1)}{\alpha \kappa^1} \right) \bar{a}_R^1 + (1 - \alpha/2) \left(1 - (\bar{a}_R^1)^2 \right) \right), \tag{P10-1}$$

and the total value of vote is

$$N V_{\text{vote}}^1 = \frac{P(x)}{2} (1 + \kappa^1) \left(\frac{1 - \ln(1 + \alpha \kappa^1)}{\alpha \kappa^1} \right) \bar{a}_R^1.$$
 (P10-2)

Hence, the total value of the dividend claim is

$$N V_D^1 = N V_1^1 - N V_{\text{vote}}^1 = \frac{1}{2} P(x) \left(\left(a_I (1 - \alpha b_I) + (1 - \alpha/2) \left(1 - (\bar{a}_R^1)^2 \right) \right) \right). \tag{P10-3}$$

Thus, MO^1 can be expressed as

$$MO^{1} = \beta N V_{D}^{1} + \phi^{1} b_{I} \alpha a_{I} P(x) = P(x) \left(\frac{\beta}{2} \left((a_{I}(1 - \alpha b_{I}) + (1 - \alpha/2) (1 - (\bar{a}_{R}^{1})^{2})) \right) + \phi^{1} b_{I} \alpha a_{I} \right).$$
(P10-4)

Let $A_{p11} = \frac{\beta}{2} \left((a_I (1 - \alpha b_I) + (1 - \alpha/2) (1 - (\bar{a}_R^1)^2)) \right) + \phi^1 b_I \alpha a_I$. Differentiating Equation (P10-4) with respect to x and rearranging terms, we have

$$\frac{\partial MO^{1}}{\partial x} = A_{p11}(\kappa^{1}) \frac{\partial P(x)}{\partial x} + P(x) \frac{\partial A_{p11}(\kappa^{1})}{\partial k^{1}} \frac{\partial \kappa^{1}}{\partial x}.$$
 (P10-5)

From Equation (P10-5) we know that $\frac{\partial MO^1}{\partial x} < 0$ at $x = \bar{x}$ implies that $\frac{\partial A_{p11}(\kappa^1)}{\partial k^1} \frac{\partial \kappa^1}{\partial x} < 0$ because $\frac{\partial P(x)}{\partial x} = 0$ at $x = \bar{x}$. Also, $\frac{\partial A_{p11}(\kappa^1)}{\partial k^1} \frac{\partial \kappa^1}{\partial x} < 0$ when $\frac{\partial \kappa^1}{\partial x} < 0$ implies that $\frac{\partial A_{p11}(\kappa^1)}{\partial k^1} > 0$. Differentiating

 A_{p11} with respect to κ^1 and rearranging terms, we have

$$\frac{\partial A_{p11}(\kappa^1)}{\partial \kappa^1} = \frac{\alpha(1 + b_I \alpha \kappa^1)}{\kappa^1(1 + \alpha \kappa^1)} + \frac{b_I \alpha \log(1 + \alpha \kappa^1)}{\kappa^1} - \frac{(1 + b_I \alpha \kappa^1) \log(1 + \alpha \kappa^1)}{(\kappa^1)^2} - \left(1 - \frac{\alpha}{2}\right) \alpha \beta(1 + b_I \alpha \kappa^1). \tag{P10-6}$$

For $\frac{\partial A_{p11}(\kappa^1)}{\partial \kappa^1} > 0$, we need

$$\frac{\alpha(1 + b_I \alpha \kappa^1)}{\kappa^1(1 + \alpha \kappa^1)} + \frac{b_I \alpha \log(1 + \alpha \kappa^1)}{\kappa^1} > \frac{(1 + b_I \alpha \kappa^1) \log(1 + \alpha \kappa^1)}{(\kappa^1)^2} + \left(1 - \frac{\alpha}{2}\right) \alpha \beta(1 + b_I \alpha \kappa^1). \tag{P10-7}$$

Solving for b_I such that the condition in Equation (P10-7) is satisfied,

$$b_{I} > \frac{\frac{2\log(1+\alpha\kappa^{1})}{(\kappa^{1})^{2}} + \alpha(2-\alpha)\beta - \frac{2\alpha}{\kappa^{1}(1+\alpha\kappa^{1})}}{\alpha^{2}\left(\frac{2}{1+\alpha\kappa^{1}} - \alpha(2-\alpha)\beta\kappa^{1}\right)}.$$
 (P10-8)

We do not know the exact value of κ^1 , but we do know the RHS of expression (P10-8) is strictly increasing in κ^1 . Hence, we replace κ^1 by its maximum value $\frac{\beta}{1-\beta}$, and get the sufficient condition for underinvestment as follows:

$$b_{I} > \frac{\left(1 - \beta\right) \left(\frac{2(1 - \beta)^{2} \left(1 - \left(1 - \alpha\right)\beta\right) \log\left(1 + \frac{\alpha\beta}{1 - \beta}\right)}{2 - \beta\left(4 - \beta\left(\left(1 - \alpha\right)\left(2 - \alpha\right)\beta + \alpha\right)\right)} - \alpha\beta\right)}{\alpha^{2} \beta^{2}}.$$
(P10-9)

8.6 Proof of Proposition 11

Consider the values of b_I for which the manager invests in all available positive NPV projects if non-voting equity is used to finance the investment. Assume that the manager invests some x if voting equity is used to finance the investment. We have to obtain conditions such that $V_1^0(\bar{x}) \geq V_1^1(x)$. Substituting for $V_1^0(\bar{x})$ and $V_1^1(x)$ from Equations (??) and (BR-12), we get

$$V_{1}^{0}(\bar{x}) - V_{1}^{1}(x) = \frac{a_{I}(1 - \alpha b_{I})}{2} \left(P(\bar{x}) - P(x) \right) + \frac{2 - \alpha}{4} \left(1 - a_{I}^{2} - a_{I}^{2} b_{I} \alpha \kappa^{0} (2 + b_{I} \alpha \kappa^{0}) \right) P(\bar{x})$$

$$- \frac{2 - \alpha}{4} \left(1 - a_{I}^{2} - a_{I}^{2} b_{I} \alpha \kappa^{1} (2 + b_{I} \alpha \kappa^{1}) \right) P(x) - \frac{P(x)}{2} (1 + \kappa^{1}) \left(1 - \frac{\ln(1 + \alpha \kappa^{1})}{\alpha \kappa^{1}} \right) \bar{a}_{R}^{1}$$

$$+ \frac{P(\bar{x})}{2} \left((1 + \kappa^{0}) - \frac{1}{1 - \beta} \right) \left(1 - \frac{\ln(1 + \alpha \kappa^{0})}{\alpha \kappa^{0}} \right) \bar{a}_{R}^{0}$$
(P11-1)

But $\bar{a}_R^0 - \bar{a}_R^1 = b_I \alpha a_I(\kappa^0 - \kappa^1) > 0$ and $P(\bar{x}) - P(x) > 0 \ \forall x \neq \bar{x}$. Thus, ignoring these terms we substitute maximum value of κ^1 and minimum value of κ^0 , as both are strictly increasing in κ , and

simplifying we get

$$\left((1 + \kappa^0) - \frac{1}{1 - \beta} \right) \left(1 - \frac{\ln(1 + \alpha \kappa^0)}{\alpha \kappa^0} \right) - (1 + \kappa^1) \left(1 - \frac{\ln(1 + \alpha \kappa^1)}{\alpha \kappa^1} \right)$$

$$= \frac{(1 - \beta) \log\left(\frac{\alpha \beta}{1 - \beta} + 1\right) - \alpha \beta^2 - \beta (1 - \beta)^2 \log\left(\frac{\alpha}{1 - \beta} + 1\right)}{\alpha (1 - \beta) \beta} > 0, \tag{P11-2}$$

for all $\alpha \in [0, 1]$ and for all $\beta \in (0, 1/2)$. Using this result we can ignore these terms and rewrite Equation P11-1 as

$$V_{1}^{0}(\bar{x}) - V_{1}^{1}(x) = \frac{a_{I}(1 - \alpha b_{I})}{2} \left(P(\bar{x}) - P(x) \right) + \frac{2 - \alpha}{4} \left(1 - a_{I}^{2} - a_{I}^{2} b_{I} \alpha \kappa^{0} (2 + b_{I} \alpha \kappa^{0}) \right) P(\bar{x})$$

$$- \frac{2 - \alpha}{4} \left(1 - a_{I}^{2} - a_{I}^{2} b_{I} \alpha \kappa^{1} (2 + b_{I} \alpha \kappa^{1}) \right) P(x). \tag{P11-3}$$

After simplifying and rearranging we get

$$1 - \frac{P(x)}{P(\bar{x})} \geqslant \frac{\frac{2-\alpha}{4} \left(\left(1 - a_I^2 - a_I^2 b_I \alpha \kappa^1 (2 + b_I \alpha \kappa^1) \right) - \left(1 - a_I^2 - a_I^2 b_I \alpha \kappa^0 (2 + b_I \alpha \kappa^0) \right) \right)}{\frac{2-\alpha}{4} \left(1 - a_I^2 - a_I^2 b_I \alpha \kappa^1 (2 + b_I \alpha \kappa^1) \right) + \frac{a_I (1 - \alpha b_I)}{2}}.$$
(P11-4)

The expression, say $J_{p11}^1 = \frac{2-\alpha}{4} \left(1 - a_I^2 - a_I^2 b_I \alpha \kappa^1 (2 + b_I \alpha \kappa^1)\right)$, is a decreasing function of κ^1 and the entire expression is an increasing function of J_{p11}^1 . Thus, we substitute minimum value of κ^1 into the above expression. Similarly, the expression, say $J_{p11}^0 = \frac{2-\alpha}{4} \left(1 - a_I^2 - a_I^2 b_I \alpha \kappa^0 (2 + b_I \alpha \kappa^0)\right)$, is a decreasing function of κ^0 , but the entire expression is a decreasing function of J_{p11}^0 . Thus, substituting maximum value of $\kappa^0 = \frac{\beta}{1-\beta} + \frac{1}{1-\beta}$ and simplifying we get

$$1 - \frac{P(x)}{P(\bar{x})} \geqslant \frac{a_I^2 \alpha b_I (2 - \alpha)(2 + b_I (2 \alpha \beta + \alpha) - 2 \beta)}{2 a_I (1 - \beta)^2 (1 - \alpha b_I) - (2 - \alpha)(a_I - a_I \beta (1 - \alpha b_I))^2 + (2 - \alpha)(1 - \beta)^2}.$$
 (P11-5)

8.7 Proof of Proposition 12

We prove this proposition in two parts: First we hold the investment level fixed and obtain conditions under which the manager is better off if the investment is financed using non-voting shares; then, we show that the manager remains better off if the investment level is increased. From Proposition 11 we know that $MO^j = A_{p1j} P(x)$, where j stands for the type of shares issued. To show that $MO^0 \geqslant MO^1$, we need to obtain conditions under which $A(k^0) \geqslant A_{p11}$. Substituting for A_{p10} and A_{p11} from Equations and simplifying we get

$$\frac{\beta}{2} \left(a_I (1 - \alpha b_I) + (1 - \alpha/2) \left(1 - (\bar{a}_R^0)^2 \right) \right) + \phi^0 b_I \alpha a_I > \frac{\beta}{2} \left((1 - \alpha/2) \left(1 - (\bar{a}_R^1)^2 \right) \right) + \phi^1 b_I \alpha a_I.$$
(P12-1)

Rearranging the terms we get

$$\phi^{0} b_{I} \alpha a_{I} - \phi^{1} b_{I} \alpha a_{I} > \frac{\beta}{2} \left((1 - \alpha/2) \left(1 - (\bar{a}_{R}^{1})^{2} \right) \right) - \frac{\beta}{2} \left((1 - \alpha/2) \left(1 - (\bar{a}_{R}^{0})^{2} \right) \right). \tag{P12-2}$$

After substituting for \bar{a}_R^j from Equation (11) and ϕ^j from Equation (15) and on further simplification,

$$4 \kappa^{1} (1 + \alpha b_{I} \kappa^{0}) \log(1 + \alpha \kappa^{0}) - 4 \kappa^{0} (1 + \alpha b_{I} \kappa^{1}) \log(1 + \alpha \kappa^{1}) - (2 - \alpha) \beta \kappa^{0} \kappa^{1} (\kappa^{0} - \kappa^{1}) (2 + \alpha b_{I} (\kappa^{0} + \kappa^{1})) > 0.$$
(P12-3)

Next, we solve for b_I such that the above inequality holds:

$$b_{I} \geqslant \hat{b}_{I} = \frac{(2 - \alpha)\alpha\beta(\kappa^{0} - \kappa^{1}) + 4\frac{1}{\kappa^{0}}\log(1 + \alpha\kappa^{0}) - 4\frac{1}{\kappa^{1}}\log(1 + \alpha\kappa^{1})}{\alpha(4\log\frac{1 + \alpha\kappa^{1}}{1 + \alpha\kappa^{0}} + (2 - \alpha)\alpha\beta(\kappa^{0} - \kappa^{1})(\kappa^{1} + \kappa^{0}))}.$$
 (P12-4)

As κ^1 increases for \hat{b}_I , so we substitute the maximum value of $\kappa^1 = \frac{\beta}{1-\beta}$ into the expression. But \hat{b}_I is also an increasing function of κ^0 and unfortunately κ^0 is unbounded:

$$\kappa^{0} = \frac{\beta N + n^{0}}{(1 - \beta)N} = \frac{\beta}{1 - \beta} + \frac{n^{0}}{(1 - \beta)N}.$$
 (P12-5)

If we assume that \bar{x} is such that $n^0(\bar{x}) \leq N$, (i.e., if new shares issued are no greater than the existing number of shares), then the maximum value of $\kappa^0 = \frac{\beta}{1-\beta} + \frac{1}{1-\beta}$. Substituting maximum value of $\kappa^1 = \frac{\beta}{1-\beta}$ and assumed maximum value of $\kappa^0 = \frac{\beta}{1-\beta} + \frac{1}{1-\beta}$ we obtain

$$\hat{\hat{b}}_{I} = \frac{(1-\beta)\left((2-\alpha)\alpha\beta^{2}(1+\beta) + 2(1-\beta)^{2}\left((1+\beta)\log\left(1+\frac{\alpha\beta}{1-\beta}\right) - \beta\log\left(1+\frac{\alpha(1+\beta)}{1-\beta}\right)\right)\right)}{\alpha\beta(1+\beta)\left(4(1-\beta)^{2}\log\left(1+\frac{\alpha(1+\beta)}{1-\beta}\right) - (2-\alpha)\alpha\beta(1+\beta) - 4(1-\beta)^{2}\log\left(1+\frac{\alpha\beta}{1-\beta}\right)\right)}.$$
(P12-6)

Equation P12-6 ensures that the manager is better off if the level of investment remains the same. The second part of the proof requires us to show that the manager is better off if the level of investment increases. This requires that $MO^0(x_2) > MO^0(x_1)$, where $x_2 > x_1$. But we know that if $b_I \geqslant \frac{1}{2}$, then $\frac{\partial MO^0(x)}{\partial x} > 0$. Hence, $\hat{b}_I \geqslant \frac{1}{2}$ is sufficient condition for full investment. If $b_I \geqslant \hat{b}_I$, then the incumbent is better off if he is allowed to use non-voting shares to fund new projects.

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