The Wall Street Walk when Blockholders Compete for Flows

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Journal of Finance forthcoming

ABSTRACT

Effective monitoring by equity blockholders is important for good corporate governance. A prominent theoretical literature argues that the threat of block sale ("exit") can be an affective governance mechanism. Many blockholders are money managers. We show that when money managers compete for investor capital, the threat of exit loses credibility, weakening its governance role. Money managers with more skin in the game will govern more successfully using exit. Allowing funds to engage in activist measures ("voice") does not alter our qualitative results. Our results link widely prevalent incentives in the ever-expanding money management industry to the nature of corporate governance.

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Equity blockholders in publicly traded corporations who are dissatisfied with the actions of company management can sell their blocks—the so-called "Wall Street Walk". A growing theoretical literature starting with Admati and Pfleiderer (2009) and Edmans (2009) argues that the Wall Street Walk can be an effective form of governance. The exit of a blockholder will typically depress the stock price, punishing management whenever executive compensation is linked to the market price of equity. Thus, faced with a credible threat of exit, management will be reluctant to underperform. Admati and Pfleiderer argue that when blockholders observe managers underperforming, it is in their own interest to exit early before information about the manager's underperformance becomes public. This makes exit a credible threat which ameliorates managerial underperformance and enhances firm value. Edmans argues that informed institutional trading enhances the informational efficiency of the firm's equity in the secondary market, enabling myopic managers to make better investment decisions.

The theoretical literature on exit treats the blockholder as a profit-maximizing principal: She acts as an individual owner of an equity block would. In contrast, a significant proportion of equity blocks is held by delegated portfolio managers who manage money for others (for example mutual funds, hedge funds, etc).[1] This matters because money managers often face short-term incentives that may drive them to behave in ways that do not aid corporate governance. For example, the EU Corporate Governance Green Paper notes (2011):

> It appears that the way asset managers' performance is evaluated... encourages asset managers to seek short-term benefits... The Commission believes that short-term incentives... may contribute significantly to asset managers' short-termism, which probably has an impact on shareholder apathy.

An important reason why money managers may not take a long-term view is that their investors chase short-term performance, generating well-documented short-term flow-performance relationships.[2] In this paper we build on Admati and Pfleiderer (2009) to study how the presence of (endogenous) short-term flow-performance relationships affects the ability of delegated blockholders to govern via the threat of exit. Our key observation is that when funds differ in stock picking ability, exit may be informative about the fund manager's skill and thus affect investor flows. We show that this signalling role of exit impairs its disciplinary potential. The perverse effect of flow-performance relationships operates through both linear assets-under-management (AUM) fees and convex performance fees (carried interest or "carry"), the two components of standard "two and twenty" contracts. Thus, common contractual arrangements in money management foster endogenous short-termism and impair the effectiveness of exit. We show that offsetting, long-term, incentives arise from the degree to which the fund manager self-invests in her fund: Whether a money manager can successfully govern via the threat of exit depends on the degree to which she has skin in the game.

We analyze a three-date model with many funds, investors, and firms. Each fund uses a combination of investor capital and proprietary resources (self-investment) to hold a block in a firm. Funds are compensated via a combination of AUM fees and carried interest. At the *initial* date, each firm manager takes actions that affect firm performance. Each fund can observe whether the manager of the company in which she owns a block underperforms and may then sell the block at the *interim* date before the market learns about managerial actions. At the *final* date, uncertainty resolves and consumption occurs.

Funds differ in their ability as stock pickers. Funds that are good stock pickers are more likely to invest in companies with better corporate governance. In such companies, management is less likely to underperform, making blockholder exit less likely to be necessary. Investors are able to observe the returns generated by all funds at the interim date, make inferences about stock picking ability, and allocate their money accordingly. Such inferences are relevant because, following the interim date, funds have access to further investment opportunities, the quality of which are again determined by their stock picking skills: Good stock pickers have access to better opportunities. Accordingly, investors rationally use the interim performance of funds to make capital allocation decisions.

Suppose that a fund—after acquiring a block in a company—observes that management is underperforming. The fund can either sell her block in the underperforming company at the interim date (that is, exit) or wait until the final date. If she sells early, she may be able to hide her trade behind market noise and sell her block at a price not reflecting the full reduction in value implied by management underperformance. If she waits and sells later, she will liquidate her block at a lower price. Thus, to the extent that the fund cares directly about her portfolio value due to her self-investment, she will be inclined to exit.

However, the fund may also be concerned about inferences made in the short-term by investors, which may affect her payoffs via endogenous flows. If she sells the block early, she will hurt her short-term return relative to other funds, causing an earlier loss of investor flow. In contrast, if she does not sell her block early—and some other funds do—her short-term return relative to others will be improved. She will not only keep her own investors, but she may actually attract some investors from other funds (who may have sold their underpeforming blocks early and thus underperformed). Of course, there will be a price to pay later in terms of lowered liquidation value. But in the meanwhile, the fund will earn AUM fees by retaining her own investors and attracting new ones. Selling the block early also reduces carried interest: Not only will a sale reduce *current* carry by lowering today's marked-to-market portfolio value, but, due to endogenous outflows an early sale will also reduce fund size, limiting access to new investments and reducing *future* carry. Thus, given flow-performance relationships, the presence of AUM fees and carried interest discourages funds from exiting.

Our main result (Proposition 2) formalizes this trade-off. We show that when delegated blockholders do not have sufficient self-investment and good and bad funds are sufficiently different (so that investors chase performance), the threat of exit cannot be credible in equilibrium. The applied implication of our result is that funds' ability to govern via exit will be determined by the relative strength of contractual incentives and self-investment. For a given compensation contract, we also show (Proposition 5) that there is always a level of self-investment high enough to induce the fund to behave identically to Admati and Pfleiderer's principal blockholder even when faced with flow-performance relationships. We couch our applied discussion in terms of two prominent classes of money managers: Mutual funds and hedge funds.

Mutual funds are distinctive in that—for regulatory reasons—they do not charge carried interest. Further, mutual fund managers invest very little in their funds: According to Khorana, Servaes, and Wedge (2007) 57% of mutual fund managers do not self-invest and—for those who do—average self-investment is 0.04% of assets under management. Proposition 2 applies directly to mutual funds with no self-investment at all. Further, simple calculations based on our model show that self-investment on the order of 10^{-4} of assets under management is insufficient to overcome flow-based disincentives to exit.

Hedge funds differ from mutual funds in that they charge both AUM fees and carried interest but—as general partners in limited partnerships—their managers invest significantly in their own funds. Building on the characterization in Proposition 5, our calculations show that, for reasonable ranges of parameters, hedge funds with self-investment of around 10% of assets under management—a number consistent with the literature—will successfully govern via the threat of exit, which is in keeping with evidence in Edmans, Fang, and Zur (2013).

Thus, with respect to the efficacy of exit as a governance mechanism, our analysis offers a reassuring view of hedge funds but raises concerns about mutual funds. Such concerns must, of course, be interpreted in the context of the limits of our stylized game theoretic model. To examine the robustness of our negative result on mutual funds, we discuss a number of variations of the model. The key driver of perverse incentives in our model is short-term performance evaluation by investors. We find that introducing uncertainty over short-term performance evaluation improves behaviour: Mutual funds who may not be evaluated by all of their investors in the interim period will be better incentivized to exit (Proposition 3). Thus, our negative results are most salient when the evaluation horizon is short relative to the resolution of uncertainty about firm value. Holding fixed short-term evaluation, we also ask whether the presence of long-term evaluation affects our results. Long-term evaluation may improve incentives to exit when fund portfolios are opaque and feature multiple investments. Then, funds may be able to camouflage losses from exiting an underperforming block by better performances in other investments. Mutual funds face reporting requirements that reveal detailed portfolio holdings to investors at quarterly frequencies. Such disclosure requirements may have the undesirable side-effect of making mutual funds less effective in using exit to govern. Finally, while stock selection is a natural way to model skill for mutual fund managers, for theoretical completeness we examine whether a different definition of ability may alter our results. In particular, we consider what happens when funds differ in their ability to observe managerial underperformance at the interim date. We find that competition for flow still has a negative effect on mutual fund behaviour: Instead of exiting too little, mutual funds now exit too much. While this changes our specific empirical prediction, it does not alter our economic message: Excessive exit also reduces discipline, and thus competition for flow again reduces the effectiveness of exit when used by mutual fund blockholders.

The growing empirical literature on exit as a governance mechanism[3] has not, to date, focussed on the impact of blockholder compensation. The literature nevertheless provides findings that may be interpreted through the lens of our model. Parrino et al. (2003) were the first to empirically investigate the role of exit as a governance mechanism. They showed that the degree to which institutions use exit may depend on their type. Using the CDA/Spectrum classification of institutions (into Bank Trusts, Insurance Companies, Independent Investment Advisors, Investment Companies, and Others) they found that, for the years 1982 to 1993, investment companies used exit less than bank trusts. While the legal nature of the CDA/Spectrum classification is hard to interpret, a mutual fund typically appears as an investment company under this classification. This finding is then broadly consistent with our model: Mutual funds are likely to face more performance-chasing by clients and have lower proprietary ownership than bank trusts.[4]

In contrast to the empirical literature on exit, there is established variation on the different degrees to which different types of institutional investors use *other* governance tools such as behind the scenes engagement with management, jawboning, etc.—collectively referred to as "voice"—to discipline management and deliver shareholder value. A growing body of empirical papers provides evidence that hedge funds produce substantial gains to shareholders of target companies by using voice (see, for example, Brav et al. (2008), Klein and Zur (2009), and Becht, Franks, and Grant (2010)). In contrast it is commonly observed that mutual funds do not use voice to a similar degree. For example, Kahan and Rock (2007) argue that mutual funds do not typically sponsor shareholder proposals, do not uniformly use proxy voting to improve corporate governance, and do not even seem to make significant demands to management during "behind-the-scenes" negotiations. The "silence" of mutual funds is also evident from the survey of Gillan and Starks (2007), who list the roles of different institutional investors in using voice since the 1930s.

Our results linking blockholder incentives with the effectiveness of exit may provide a basis for interpreting the empirical evidence on institutional voice. The link arises from the fact that shareholder voice is usually not legally binding on the company's management. As a result, it is sometimes asserted that the threat of exit *supports* shareholders' voice. This idea dates back at least to Hirschman (1970, p. 82), who writes: "The chances for voice to function effectively...are appreciably strengthened if voice is backed up by the threat of exit."

Motivated by Hirschman's complementarity hypothesis we extend our model to incorporate active monitoring and ask whether exit and voice can be complementary to each other. We allow blockholding funds to use voice if they realize that their portfolio firm cannot be disciplined via the threat of exit alone. Voice takes the form of costly proposals for changes in business strategy that make it more attractive for managers to make better choices. We show that there exists a class of firms for which exit and voice are complementary: Managers heed blockholder voice if and only if it is backed up by a credible threat of exit if voice is ignored (Proposition 6). For such firms, only those funds that can credibly threaten to exit will use voice. Thus, our results provide one way to interpret the empirical regularity that mutual funds are less vocal than hedge funds.[5]

At a theoretical level, our analysis relates most directly to the relatively recent literature that shows that the threat of exit is, in itself, a governance mechanism. Apart from the papers of Admati and Pfleiderer (2009) and Edmans (2009), this literature includes the work of Edmans and Manso (2011) who consider the trade-off between voice and exit and solve for the number of blockholders which maximizes firm value. In recent work, Levit (2014) shows that voice and exit can be complementary because the option to exit enhances the efficacy of communication between the informed blockholder and the firm's manager. In contrast to these papers, which treat the blockholder as a principal, we focus on the delegated nature of blockholding. This new literature on exit, as well as our work, builds on a large theoretical literature on the role of blockholders in corporate governance.[6] That literature also treats blockholders as principals and focuses on their incentives to monitor. While some papers within that literature have considered the trade-off between voice and exit (for example, Kahn and Winton (1998), Maug (1998), Mello and Repullo (2004)) they do not focus on exit as a governance mechanism in itself. Like us, Goldman and Strobl (2011) study the impact of fund managers' incentives on blockholder monitoring. In contrast to us, they take fund managers' short-termism as given and examine its impact on firm investment policy.

Our paper also has a familial connection to the growing literature on the financial equilibrium implications of the career concerns of funds (for example, Dasgupta and Prat (2008), Dasgupta, Prat, and Verardo (2011), or Guerrieri and Kondor (2012)). These papers establish a link between fund managers' flow-motivations and the equilibrium prices, returns, and volume of assets they trade. In contrast, we focus on the implications of funds' flow-motivations on the nature of corporate governance in firms in which they hold equity blocks.

The rest of the paper is organised as follows. In section I we introduce the governance problem. Section II reviews Admati and Pfleiderer's result on exit as a governance mechanism when the blockholder is a principal. In section III we enrich the analysis by introducing delegated blockholding. Section IV presents our central result while sections V and VI examine its applied implications. In section VII we extend our model to incorporate active monitoring. Section VIII concludes.

I. The Governance Problem

We consider a continuum of unit measure of publicly traded all-equity financed firms. Some of these firms are characterized by an agency problem as described below. We ask how changes in the ownership structure—the presence of blockholders of different types—can influence the nature of corporate governance in the firm. The underlying model of the firm facing agency problems is a slight variation of that of Admati and Pfleiderer (2009).[7]

There are three dates ($t = 0, 1, 2$). Each firm is run by a manager. A measure $A \in (0, 1)$ of firms is characterized by a moral hazard problem. In each of these firms, at $t = 0$ the manager chooses action $a \in \{0, 1\}$. If he chooses action $a = 0$ the resulting firm value at $t = 2$ is $v_{\mathrm{H}} > 0$. If he chooses $a = 1$, the value of the firm is $v_{\mathrm{L}} = v_{\mathrm{H}} - \Delta v$ for some $\Delta v \in (0, v_{\mathrm{H}})$. Thus $a = 1$ is undesirable and—following Admati and Pfleiderer—we refer to it as the "perverse action". The manager is tempted to choose $a = 1$ because, by doing so, he receives a stochastic private benefit $\tilde{\beta} \sim F$, where F is strictly increasing and differentiable on $[0, \bar{\beta}]$. $\tilde{\beta}$ is privately observed by the manager and is never revealed to others. All cash flows of the firm become public at $t = 2$ when consumption occurs.

We assume, following Admati and Pfleiderer, that each manager's contractual payoff depends on his firm's market prices at $t = 1$ and $t = 2$. Denoting by P_1 and P_2 the market price of a given firm at $t = 1$ and $t = 2$ respectively, the manager's payoff when he takes action 0 is $\omega_1 P_1 + \omega_2 P_2$, where $\omega_1 > 0$ and $\omega_2 > 0$ represent the sensitivities of managerial compensation to, respectively, the short- and the long-term market value. If the manager instead takes action 1, his payoff is $\omega_1 P_1 + \omega_2 P_2 + \beta$, where $\beta \geq 0$ is the realized value of $\tilde{\beta}$.

Prices P_1 and P_2 for any given firm are set by a risk-neutral market maker on the basis of all available public information. In addition to firms' equity, there is also another asset in the economy which we refer to as the index asset, representing a broad benchmark. We assume this asset is in infinitely elastic supply and normalize its gross return to unity.

The complementary set of firms of measure $1 - A$—while identical to the firms described above in all other ways—is free of agency problems, that is in each such firm the manager has a degenerate action space: $a \in \{0\}$. Thus, each such firm is worth v_{H}.

Each firm is owned by many small passive direct shareholders as well as by a large blockholder. The identity of the blockholder will change across different variants of our model. In the initial baseline case, which is essentially identical to Admati and Pfleiderer's[8], the blockholder is a principal, and we think of her as a large private blockholding investor. In the core of our paper—motivated by the significant degree of blockholding by institutional asset managers in Anglo-Saxon financial systems—we instead model the blockholder as a fund who acts on behalf of a continuum of identical investors.

In all variants of our model, the blockholder is initially unaware of whether her firm is characterized by an agency problem or not but is able to observe the action chosen by the manager of her firm at $t = 0$, and is able to sell her stake in the firm at $t = 1$ in response. Because the blockholder's potential sales are based on her observation of the manager's action, which in turn affects firm value, the price at the interim date ($t = 1$) will be affected by the trading decision of the blockholder. This, in turn, will affect the payoffs of the manager, generating the core corporate governance mechanism. If the blockholder can credibly threaten to exit when the manager takes action 1, thus lowering the firm's traded price at $t = 1$, the resulting reduction in the manager's payoff can induce him to take the perverse action less often, thus reducing the agency costs and increasing the value of the firm.

It is useful at the outset to outline the incidence of the perverse action (in a firm with agency problems) in the *absence* of a blockholder. In such a setting, since small shareholders are passive (implicitly, they have neither the skill nor the incentive to acquire private information about the manager's actions) the price of the firm at $t = 1$ is insensitive to the manager's actions. Accordingly, the manager compares his rent from taking the perverse action $\beta + \omega_1 P_1 + \omega_2 v_{\mathrm{L}}$ with that of taking the non-perverse action $\omega_1 P_1 + \omega_2 v_{\mathrm{H}}$. He thus takes the perverse action if and only if $\beta \geq \omega_2 \Delta v =: \beta_{\text{No-L}}$. To rule out the trivial case where agency problems never matter, we assume throughout that $\bar{\beta} > \omega_2 \Delta v$.

In what follows, we consider whether the presence of different types of blockholders can reduce the incidence of the manager's perverse action. We begin with the important benchmark case in which the blockholder acts as a principal.

II. The Blockholder as Principal: Governance via Exit

In Admati and Pfleiderer (2009), the blockholder is a principal and cares only about the liquidating value of her position. She may, however, face a liquidity shock at $t = 1$ with probability $\theta \in (0,1)$ which forces her to immediately liquidate her position. The market maker does not observe the liquidity shock. We can now state and prove our (minor) variant of Admati and Pfleiderer's result here.

PROPOSITION 1. ***(Admati and Pfleiderer)*** *In the unique equilibrium, the blockholder chooses to exit at* $t = 1$ *whenever the manager chooses* $a = 1$. *There exists a* $\beta_{\mathrm{L}} \in \left(\beta_{\mathrm{No-L}}, \infty\right)$ *such that, unless the firm is free of agency problems, the manager chooses* $a = 1$ *if and only if* $\beta \geq \beta_{\mathrm{L}}$.

This and all other proofs are in the appendix. The intuition is as follows. When the blockholder observes that the manager has chosen $a = 1$, she realizes that, at $t = 2$, when information becomes public the firm's value will be v_{L}. If she has not been hit by the liquidity shock she has the choice to hold her block until $t = 2$ and get v_{L}, or to sell at $t = 1$. Of course, her sale at $t = 1$ will lower the price of the block, because her trade may reflect private information. However, because the market maker assigns positive probability to the sale being induced by the blockholder's liquidity shock, the loss in value from the early sale will be smaller than the loss from holding until $t = 2$. Thus, the blockholder will exit at $t = 1$, lowering P_1. Knowing this, the manager will hesitate to take the perverse action: The blockholder's action reduces the payoff to the manager from choosing $a = 1$ via a lower interim price P_1, which makes him relatively reluctant to do so. Thus, the equilibrium is characterised by a cutoff β_{L} such that the manager takes the perverse action if and only if $\beta \geq \beta_{\mathrm{L}}$, where $\beta_{\mathrm{L}} > \beta_{\text{No-L}}$. The increase in the threshold for taking the perverse action from $\beta_{\text{No-L}}$ to β_{L} embodies the disciplining role of the threat of exit. We now turn to the case where the blockholder is not a principal, but an agent.

III. The Blockholder as Agent: A Model

We now consider the case where the blockholder is a delegated portfolio manager. We refer to such a blockholder as a fund (F) and assume that there is a continuum of funds of equal measure to that of firms. Funds and their investors are essential to each other: Investors without fund managers and fund managers without money to manage can only invest in the index asset.[9] For simplicity, we do not allow investors or funds to lever up. Each fund enters the model holding a block in one firm.

The initial financing of the block derives from two sources. A fraction $\alpha \in [0,1)$ is directly financed by the fund manager and represents self-investment i.e., "skin in the game". The remainder is financed by a continuum of identical small investors of measure $1-\alpha$. A distinct continuum of small investors finances each fund. A manager's skin in the game or any investment proceeds from it cannot be moved to a different fund. We do not model the sources of self-investment but examine its consequences. We argue below that there is considerable variation in the observed levels of self-investment across different types of funds. Thus we treat α as a parameter that captures a relevant source of cross-sectional variation.

A blockholding fund, like the principal blockholder of the previous section, can observe the firm's manager's action at $t=0$, and can choose whether to exit at $t=1$ or to hold until $t=2$. To match the liquidity shock of Admati and Pfleiderer (2009), we assume that each fund is hit by a liquidity shock at $t=1$ with probability $\theta \in (0,1)$ which forces her to liquidate her holding at $t=1$ and shut down after returning the value of liquidated funds to investors and consuming any fees (specified below) payable at $t=1$. The fund's liquidity shock is not observed by the market maker. Shocks are *iid* across funds. Thus, as in Admati and Pfleiderer (2009), θ is a proxy for secondary market liquidity.

As discussed in the introduction, an important strand of the empirical literature has documented that investors chase performance across funds of different ability, generating funds' competition for investor flow. In order to incorporate concerns for flow, we augment the model by adding two ingredients.

First, we assume a degree of heterogeneity across funds, which affects their relative desirability as agents from the perspective of investors. Blockholding funds differ in their stock picking ability; this affects both their ability to select firms in which to hold blocks at $t=0$ and their access to new investments at $t=1$. There are two types of funds: good ($\tau^{\mathrm{F}}=\mathrm{G}$) and bad ($\tau^{\mathrm{F}}=\mathrm{B}$), where $\Pr(\tau^{\mathrm{F}}=\mathrm{G})=\gamma_{\mathrm{F}}$. As is standard in experts models, we assume that funds do not know their own type. Blocks held by good funds are free of agency problems (that is, blocks in firms in which managers have a degenerate action space: $a \in \{0\}$) with probability $\gamma_{\mathrm{M}}^{\mathrm{G}} \leq 1$, while those held by bad funds are free of agency problems with probability $\gamma_{\mathrm{M}}^{\mathrm{B}} \in \left(0, \gamma_{\mathrm{M}}^{\mathrm{G}}\right)$. By the law of large numbers, for consistency with Section I, it follows that the measure of firms with agency problems is: $A := 1 - \left[\gamma_{\mathrm{F}}\gamma_{\mathrm{M}}^{\mathrm{G}} + (1-\gamma_{\mathrm{F}})\,\gamma_{\mathrm{M}}^{\mathrm{B}}\right].$

At $t = 1$ funds with money to manage have access to new investment opportunities. A fund with $\tau^{\mathrm{F}} = \mathrm{G}$ can generate gross returns $R_{\mathrm{G}} > 1$ at $t = 2$ per dollar invested between $t = 1$ and $t = 2$. In contrast, a fund with $\tau^{\mathrm{F}} = \mathrm{B}$ can generate gross returns $R_{\mathrm{B}} < 1$ at $t = 2$ per dollar invested between $t = 1$ and $t = 2$.

Second, we introduce a hiring and replacement process between investors and funds. Each investor at $t = 0$ is matched to a fund who holds a block on his behalf. He does not know the type of the fund that he is matched to. At $t = 1$ he can update his inference about the fund's type by observing the value of the fund's portfolio. The investor also observes the portfolio values of all other funds and can make all relevant inferences. After such observation, the investor may either retain or fire his fund. The investor who fires his fund may then invest in one or more alternative funds or invest directly in the index asset.

To conclude the model description, we describe the payoffs to investors and funds. The payoffs to fund investors are as follows. Consider an investor who invests i_t at date t in a fund, and let I_t be the total investment in that fund at date t. That investor is entitled to a date $t+1$ payoff of $\frac{i_t}{I_t}V_{t+1}$ where V_{t+1} represents the date $t+1$ market value of the investment I_t. New and old investors are treated symmetrically.[10]

Investors also pay fees to their funds. We model fees in the form of "two and twenty" contracts involving assets under management (AUM) fees, given by a fraction $w \in [0, 1)$ of committed capital and a carried interest given by a fraction $\phi \in [0, 1)$ of any positive investment profits generated between t and $t + 1$. In particular, a fund receiving a total investment of I_t at date t receives an AUM fee of wI_t at t and also a carry of $\phi\left[V_{t+1} - I_t\right]^{+}$ at $t + 1$ where $[x]^{+} := \max\{x, 0\}$.

Each investor pays fees in proportion to his investment in the fund. If at date t he invests i_t in a fund which receives a total date-t investment of I_t, he pays wi_t at t and at

$t+1$ pays a fraction $\frac{i_t}{I_t}$ of the total carried interest paid to the fund at date $t+1$. All statements made about fees here apply to all investment in the fund including the fund's self-investment. Thus, effectively, the fund pays fees to herself on her proprietary investment, though of course these fees net out of the fund's payout and do not affect her incentives. For simplicity, we assume that fees are paid out of pocket, that is, not deducted from assets under management. In what follows, we shall typically use Θ to denote the full set of model parameters. For any subset $S \subset \Theta$ we use $\Theta \backslash S$ to denote the set of parameters excluding those in S.

IV. The Failure of Governance via Exit

Is it feasible for delegated blockholders to credibly threaten managers with exit conditional on a perverse action being taken? We find that:

PROPOSITION 2. *For R_{G} and $\gamma_{\mathrm{M}}^{\mathrm{G}}$ sufficiently large and for any $w \in [0,1)$ and $\phi \in [0,1)$ there exists $\bar{\alpha}(\Theta, w, \phi) \in (0,1)$ such that for all $\alpha < \bar{\alpha}(\Theta, w, \phi)$ it cannot be an equilibrium for any fund to choose to sell if and only if $a = 1$.*

The formal argument, which is detailed in the appendix, proceeds as follows. We first establish conditions under which, if a fund adopts a strategy of selling the block at $t = 1$ if and only if she observes that the manager has taken the perverse action, then that fund's investors choose to retain her services at $t = 1$ if and only if the fund has *not* sold at $t = 1$. We then establish conditions under which such a retention strategy on the part of investors induces the fund *not* to sell at $t = 1$ even if she has observed the manager taking the perverse action. This then establishes a set of conditions under which it is impossible for the fund to sell (in equilibrium) at $t = 1$ if and only if she observes the perverse action.

Our result characterizes conditions under which the threat of exit cannot be credibly utilized in equilibrium. A key condition is that α is not too large. The parameter α represents a fund manager's self-investment (or "skin in the game") as a fraction of initial assets under management. Admati and Pfleiderer's principal blockholder can be represented by $\alpha = 1$

(and $w = \phi = 0$). In their model, the blockholder exits in equilibrium. In ours, for $\alpha < \bar{\alpha}$ exit cannot arise in equilibrium. Since $\bar{\alpha} < 1$, our result does not contradict theirs. Indeed, we show later (Proposition 5) that for sufficiently large α it *is* an equilibrium for funds to exit when the manager takes the perverse action.

Professional money managers typically run funds that are much larger than their personal stakes. This makes the small-α case of significant interest. For small α, how does the presence of incentives embedded in $w > 0$ and $\phi > 0$ affect the fund's ability to exit?

The incentives of funds must be understood in the context of investor behaviour. Under the conditions of Proposition 2, investors rationally chase performance. When $\gamma_{\mathrm{M}}^{\mathrm{G}}$ is sufficiently high, good funds are sufficiently likely to invest in companies with no agency problems. When an investor infers upon observing the high marked-to-market value of their fund's $t = 1$ portfolio that his fund has not exited (which implies, given the fund's proposed strategy, that $a = 0$), he infers that his fund is likely to be good. Since R_{G} is high, it is in his best interest to remain invested in his fund. Instead, when an investor infers from the observation of a low portfolio value at $t = 1$ that his fund has chosen to exit (i.e., $a = 1$), he realizes that his fund is sufficiently likely to be bad. Given that $R_{\mathrm{B}} < 1$, if such a fund were to undertake further investments at $t = 1$ these would generate a lower expected return than the index asset, so that the investor would prefer to directly invest in the index asset.[11] If, on the other hand, the fund were to invest in the index asset at $t = 1$, the investor who retains the fund would be paying fees for investments he could undertake himself. Thus, in either case, the investor withdraws his capital from a fund that exits, and reallocates his capital to one or more funds that do *not* exit.[12]

Earning a fraction of assets under management ($w > 0$) creates an incentive for funds to maximize their size. If investors chase performance, funds will compete for flow, because each dollar of additional money to manage earns them an additional fee of w. As discussed above, funds that exit lose flow because they are fired by their initial clients and do not receive any inflow from other investors at $t = 1$. In contrast, funds that do not exit not only retain their initial clients but also receive *additional* inflow from the original clients of funds that have exited. Thus, earning assets under management fees makes it *less* attractive for the fund to exit, ceteris paribus.

Earning a carry ($\phi > 0$) also discourages funds from exiting. Not selling the block when the firm's management has taken the perverse action allows the fund to (temporarily) enjoy a higher portfolio value which results in a higher $t = 1$ carry. Of course, the firm's management's actions will lead to a lowered block value at $t = 2$, leading to *losses* for the fund on the block position if retained. However, since the carry only applies to the positive part of profits, such future losses are *not* costly from the perspective of the carry. Denoting by P_0, P_1^{ns}, and P_1^{s} the initial price of the block and its interim price conditional on no sale and on sale at $t = 1$ respectively, we show in the proof of Proposition 2 that $P_1^{\mathrm{ns}} > P_0 > P_1^{\mathrm{s}} > v_{\mathrm{L}}$. Thus, exiting at $t = 1$ earns a fund an immediate carry of $\phi\,[P_1^{\mathrm{s}} - P_0]^+ = 0$, after which it no longer has money to manage and thus there is no further carry to be earned. Not exiting at $t = 1$ earns the fund an immediate carry of $\phi\,[P_1^{\mathrm{ns}} - P_0]^+ > 0$. In addition, future carry may also be non-negative: By not exiting, the fund earns inflow and may choose to invest in new investment opportunities at $t = 1$ despite the fact that they have negative expected net returns, because the carry allows her to enjoy the upside only.

The argument above also demonstrates that there is a subtle interaction between the effect of competition for flow and the carry on the funds' incentives: Exit is costly from the point of view of a fund's carry because exiting funds not only do not earn a current carry (because $P_1^{\mathrm{s}} < P_0$) but also do not earn a *future* carry (since they lose their investors as a result of outflows). In the (counterfactual) absence of any flow performance relationship, the disincentive to exit due to carry would be reduced. For example, if $\gamma_{\mathrm{M}}^{\mathrm{G}} = \gamma_{\mathrm{M}}^{\mathrm{B}}$, so that exit was uninformative about ability, then investors would (rationally) not make negative inferences from exit by their fund at $t = 1$. In that case, funds that observe $a = 1$ can still be retained in equilibrium if they exit and will (rationally) choose to invest in new investment opportunities and thus earn future carry upon exiting. Thus, exit is less costly to carry without competition for flow than with. In other words, competition for flow and carry incentives complement each other in reducing a fund's incentive to exit.

Countervailing incentives stem from the fund's self-investment α. A fund who observes $a = 1$ faces a choice between two options. She may hold the block, be retained by her investors and receive inflow from new investors, earn an positive current carry, but suffer from having to liquidate the block at a lower price at $t = 2$ which lowers the value of her self-investment. Further, any attempt by this fund to enhance future carry by investing available capital in new opportunities at $t = 1$ will also be costly to self-investment, because when $\gamma_{\mathrm{M}}^{\mathrm{G}}$ is high the new investments have a negative expected net return conditional on observing $a = 1$. Alternatively, the fund may sell the block early, be fired by her investor and lose assets-under-management fees for the second period, earn zero (current and future) carry, but obtain a higher return on her self-invested capital because of a higher block liquidation price. How the fund behaves depends on the relative sizes of w, ϕ, and α. For any given w and ϕ, if α is small, the option not to exit is more attractive. This destroys the equilibrium incentives of funds to exit underperforming blocks.[13]

There is considerable variation in the types of money management vehicles available to investors. At one end of the spectrum are regulated retail vehicles such as mutual funds. At the other are (relatively) unregulated and nimble institutions such as hedge funds. What unites the two ends of the spectrum in the context of our model are flow performance relationships. There is a wealth of evidence that flow-based rewards for good performance are experienced by both mutual fund managers (for example, Brown, Harlow, and Starks (1996), Chevalier and Ellison (1997)) and hedge funds managers (for example, Agarwal, Daniel, and Naik (2009), Lim, Sensoy, and Weisbach (2013)). The similarities end there. Two key differences between mutual funds and hedge funds are of immediate relevance to our model. First, there is a significant difference in the fee structure of mutual funds and hedge funds: Hedge funds charge a carry to their investors (typically around 20%) while mutual funds do not. Second, it is well known that mutual funds typically feature less managerial self-investment than hedge funds. Since competition for flow, the carry (ϕ) and self-investment (α) are all key interrelated driving features of Proposition 2, our model may have implications for the degree to which mutual funds and hedge funds can govern via the threat of exit. Other differences between mutual funds and hedge funds, for example, differences in the incidence of "lock up" provisions or differences in opacity, may also be indirectly relevant to our analysis. Thus, to explore the applied implications of our findings, we split the next steps of our analysis into a discussion of mutual funds and hedge funds.

V. Mutual Funds

The 1970 amendment to the Investment Companies Act of 1940 prohibits mutual funds from charging asymmetric performance fees. As a result, mutual funds almost universally charge only flat assets-under-management fees (Elton, Gruber, and Blake (2003)). In the context of our model, therefore, mutual funds are motivated purely by a combination of their assets under management fee, $w > 0$, and by managerial self-investment, $\alpha \geq 0$, if any. The former gives rise to flow-motivations. The latter endows direct profit-motivations. The relative size of w vs α determines the relative degree of flow motivation. Proposition 2 implies that those mutual funds which have little managerial self-investment, that is, those that are principally flow motivated, will not be able to credibly threaten to exit in equilibrium.

The available evidence suggests that mutual fund managers have very limited self-investment. For example, Khorana, Servaes, and Wedge (2007) document that 57% of mutual fund managers have no self-investment, and are thus purely flow motivated. In the Khorana, Servaes, and Wedge (2007) sample, the average managerial self-investment in those funds that are not purely flow motivated is 0.04%. Increased reporting requirements has given rise to greater availability of information on mutual fund managers' self investment in recent years, but the degree of self-investment remains very low. For example, according to a Morningstar report (Kinnel (2011)) as of 2011, even in the style category with *highest* managerial investment—domestic equity funds—47% of funds had zero managerial investment and 88% of funds had managerial investment of under \$1 million. Given the large size of equity mutual funds, managerial investments of less than \$1 million are likely to represent a trivial fraction of assets under management.[14]

Thus, the data indicate that about half of mutual funds are *purely* flow motivated ($\alpha = 0$) and that the average mutual fund is *principally* flow motivated ($\alpha \sim 10^{-4}$). Proposition 2 applies immediately to purely flow motivated mutual funds and suggests that they will not be effective in using exit as a governance device. For those funds that are not purely flow motivated, simple calculations based on our model suggest that for $\alpha \sim 10^{-4}$ funds will not exit for any feasible set of parameters. Details can be found in the appendix. It is worth noting that our calculations are *conservative*: In our calculations we fix $\alpha = 0.01$, two orders of magnitude larger than the average self-investment of non-purely flow motivated managers. Thus, the calculations support our conclusion with significant margin for error to account for relevant factors that may not feature in the model.

Mutual funds own over 20% of corporate equity in the US, hold blocks of non-trivial size in a majority of large US corporations (Davis and Yoo (2003)) and are the main investment vehicle for retail investors. A conclusion suggesting that half or more of such funds are unable to effectively use exit for governance is, therefore, a matter for significant concern. In the remainder of this section, therefore, we dig deeper into our result and the factors driving mutual fund behaviour.

Our characterization of mutual fund behaviour is driven by the fact that they compete for flow. Our baseline model only considers a (endogenously generated) *short-term* flow-performance relationship. To further our understanding, we first examine whether our finding is robust to the presence of long-term flows.

A. Long-term flows

We investigate the effect of long-term vs short-term flows via two complementary approaches. First, we mute short-term evaluation by introducing investors who do not evaluate performance at $t = 1$. Second, we hold fixed the (endogenous) flow-performance relationship at $t = 1$ and *add* a flow-performance relationship at $t = 2$.

A.1. Reducing short-term flows

It is possible that at the time when the fund has an opportunity to sell an underperforming block, she realizes that she may not face scrutiny from all of her own investors before uncertainty about firm value is resolved. How would this affect the relative desirability of exit? To examine this question in the simplest possible manner we change the investor base of a single fund, leaving the other funds unchanged. In particular, we assume that a proportion $\iota \in (0, 1)$ of a *given* fund's investors are *inattentive*—that is, they do not evaluate the fund at $t = 1$. We do not change the nature of the other funds' investors. We show that as the measure of inattentive investors increases, exit becomes more attractive to a fund that has observed $a = 1$. Since our result is stated for mutual funds, we set $\phi = 0$.

PROPOSITION 3. *For R_{G} and $\gamma_{\mathrm{M}}^{\mathrm{G}}$ sufficiently large and for any $w \in [0, 1)$ and $\iota \in [0, 1)$ there exists $\bar{\alpha}(\Theta, w, \iota) \in (0, 1)$ such that for all $\alpha < \bar{\alpha}(\Theta, w, \iota)$ it cannot be an equilibrium for any fund to choose to sell if and only if she observes $a = 1$. The bound $\bar{\alpha}(\Theta, w, \iota)$ is decreasing in ι.*

In other words, for a fund with more inattentive investors, exit fails for a *smaller* range of α. The intuition is that—as the measure of inattentive investors increases—the fund faces a lower threat of a loss of flow due to exit. Ceteris paribus, this increases the incentives to exit. This implies that our critique of exit is most relevant when the frequency of investor evaluation is high relative to the frequency of resolution of uncertainty about the firm's value.

A.2. Adding a long-term flow performance relationship

We now consider the possibility of simultaneous short-term and long-term evaluation. To introduce long-term flow we add new investors at $t = 2$ who can observe the fund's returns at $t = 1$ and $t = 2$ and hire any surviving fund. If funds follow the strategy of exiting if and only if they observe $a = 1$, these investors have the choice of hiring either funds that have *not* sold at $t = 1$ (having observed $a = 0$) and funds that *have* voluntarily sold at $t = 1$ (having observed $a = 1$). We are interested in whether the presence of such investors can change the $t = 1$ actions of the latter fund, that is, one who has observed $a = 1$. Consider such a fund. If the fund exits, as shown in the proof of Proposition 2 and informally discussed above, at $t = 1$ investors update their beliefs to $\underline{\gamma}_{\mathrm{F}} < \gamma_{\mathrm{F}}$. Of course, now the *new* investors will observe the fund's portfolio value at $t = 2$. However, such observation does not change inferences about the fund, since a sale at $t = 1$ perfectly predicts the portfolio value at $t = 2$. This is for two reasons: First, the fund sells only if $a = 1$, which means that the block value is v_{L}. Second, upon observing that she had chosen the wrong firm to hold a block in, the fund rationally downgrades her beliefs about her own ability and — since $\phi = 0$ — invests any funds available to her between $t = 1$ and $t = 2$ in the index asset, which provides a type-independent return. Thus, at $t = 2$ the posterior attached to any fund that *did* sell at $t = 1$ remains $\underline{\gamma}_{\mathrm{F}} < \gamma_{\mathrm{F}}$. However, the new investors have the option of hiring instead funds that did not sell at $t = 1$ and for whom the investment return between $t = 1$ and $t = 2$ has turned out to be R_{G}. Clearly, these latter funds are good. If we now make the natural assumption (consistent with the analysis above) that the continuation expected returns for these new investors at $t = 2$ are increasing in the fund's type, then they will only invest in funds that have *not* exited. Thus, for those funds who observe $a = 1$ at $t = 0$, there is no added incentive to exit at $t = 1$ introduced by the prospect of future long-term flow. In other words, adding a $t = 2$ flow-performance relationship to our model does not weaken the negative result of Proposition 2.

This stark result must be qualified for applied purposes. Our model has one firm per fund portfolio between $t = 0$ and $t = 1$, delivering a simple relationship between block prices and portfolio returns. In reality, fund managers invest in multiple assets. Then investors' inferences about their fund must be filtered through the performance of the rest of the fund's portfolio. A block liquidation would still lower the value of the fund's portfolio potentially generating negative inferences, but the inference problem would be complex. In particular, key differences may arise if the fund also has private information on the $t = 1$ returns or holdings of some of these other assets. The fund manager may then camouflage the negative return impact of a block sale with the high performance of the other assets in her portfolio and thus not suffer from outflows upon exiting at $t = 1$. This is even more relevant in the presence of long-term flows. Camouflaged exit at $t = 1$ enables the fund to avoid the price hit associated with liquidating the block at v_L at $t = 2$. If there is an increasing flow performance relationship at $t = 2$, avoiding this price hit may bring additional benefits to the fund. Of course, an offsetting force is that the fund may be able to camouflage the $t = 2$ price hit as well, though it is conceivable that the larger $t = 2$ price hit may be harder to camouflage.

A full analysis of this issue would require modeling investor inferences with multiple assets, microfounding camouflage by the fund, and examining the relative desirability of early vs late camouflage. While this is beyond the scope of the current paper, we believe that it represents an interesting direction for future research. Such investigation also holds the promise of relevant regulatory implications: Mutual funds have relatively transparent portfolios in comparison to other fund managers due to quarterly reporting requirements. The discussion here suggests that such reporting requirements may have the unintended consequence of making mutual funds reluctant to exit underperforming blocks, reducing their effectiveness as company stewards.

B. *Equilibrium without voluntary exit*

To complement our central non-existence result (Proposition 2), we now show that there exists an equilibrium in which principally flow motivated mutual funds never choose to exit.[15] For funds to choose *not* to exit in equilibrium, exit must come at sufficient cost in terms of flow to the fund. Yet, if voluntary exit is an off-equilibrium event—so that block sales arise *only* from liquidity shocks—the block liquidation price does *not* reflect the possibility of exit due to $a = 1$ and thus exit *cannot* be inferred by investors from returns, rendering the flow cost of exit meaningless. To constructively deal with this issue we introduce a *small* measure $\epsilon > 0$ of funds who are *non-strategic* and exit whenever they observe $a = 1$. The existence of such funds implies that informative exit *will* arise in equilibrium. We then construct an equilibrium where, given the presence of such non-strategic funds, no strategic fund will choose to exit. As $\epsilon \to 0$, the limiting equilibrium is characterized by the complete absence of voluntary exit.[16]

PROPOSITION 4. *For ϵ sufficiently small and for $\gamma_{\mathrm{M}}^{\mathrm{G}}$ and R_{G} sufficiently large, for any $w \in [0, 1)$ and $\phi \in [0, 1)$ there exists $\widehat{\alpha}(\Theta, w, \epsilon) \in (0, 1)$ such that if $\alpha \leq \widehat{\alpha}(\Theta, w, \phi, \epsilon)$, there is an equilibrium in which*

(i) The investor chooses to fire his fund if she sells at $t = 1$ and retains her otherwise;

(ii) A strategic fund never chooses to sell at $t = 1$ regardless of the action chosen by the manager.

The intuition is similar to that of Proposition 2. First, high $\gamma_{\mathrm{M}}^{\mathrm{G}}$ and R_{G} generate an increasing flow-performance relationship at $t = 1$. Thus, as before, funds lose existing clients at $t = 1$ if they exit and retain existing clients and garner additional flow from new investors if they do not. However, not exiting when $a = 1$ will come at direct cost to the manager via the self-invested component of the fund. Accordingly, funds will not exit if self-investment is low.

If we let $\epsilon \to 0$, it is easy to see that $\lim_{\epsilon \to 0} \widehat{\alpha}(\Theta, w, \phi, \epsilon) > 0$. In this limit all funds are strategic and the behaviour of the manager in equilibrium is identical to that of the manager of a firm in which there is *no* blockholder (as in the benchmark section I): The manager will choose the perverse action $a = 1$ if and only if $\beta \geq \beta_{\text{No-L}} = \omega_2 \Delta v$. This is because the blockholder never chooses to exit in equilibrium in the limit, and is thus "inactive," imposing no discipline on the manager.

To conclude our analysis of mutual funds, we examine a different definition of ability.

C. Could exit be a good signal of managerial ability?

In our baseline model, investors who infer that their fund exited at $t = 1$ lower their opinion of the fund's ability. Exit at $t = 1$ suggests that this fund was a poor stock picker and thus unlikely to generate high returns between $t = 1$ and $t = 2$. The empirical literature provides ample evidence of cross-sectional differences in skill amongst mutual funds when measured via metrics that are related to stock selection ability (for example Cremers and Petajisto (2009), Fama and French (2010)). Recent survey evidence provides more specific support for our modeling choice. In their survey of activism by institutional investors, McCahery, Sautner, and Starks (2014) ask respondents about the factors that are important to them in deciding whether to sell an underperforming block. Motivated by our paper, they augmented the set of survey answers to include the consideration that clients make negative inferences about stock selection ability if the institutional investor exits. They find that 25% of investors state that such inferences are "important" or "very important" factors affecting the exit threat. The only factors outweighing the importance of client inferences are the essential considerations that must affect the exit decision of any blockholder: block size, liquidity, etc. Thus, we believe that stock selection skill is the natural way to model ability differences in our context, and use this as the metric in our baseline model.

However, from a theoretical perspective, it is certainly conceivable to construct alternative models in which funds differ, instead, in their ability to spot perverse behaviour ex post. In such models, it is possible for exit to be a *positive* signal, because—since there is no question of ex ante information—exit simply signals to investors that the exiting fund knows that management is acting suboptimally. Are our results robust to such a modification?

We argue that—as in our baseline model—the flow-motivations of mutual funds would again interfere with their ability to effectively discipline management via exit. If exit is a good signal of ability, principally flow motivated funds exit *excessively*, that is, they sometimes exit not because the manager has taken a perverse action but because they wish to attract or retain flows. But this would again weaken the disciplining effect of the threat of exit: Knowing that he may be punished not just for bad but also sometimes for good choices, the manager may be less inclined to behave. This intuition is formalized in a simple model in the Internet Appendix. In that model, funds are distinguished by the quality of their information about the internal working of firms in which they hold blocks. Firms are heterogeneous in the degree to which they suffer from agency problems, with differences arising from the extent of private benefits that the management can extract by effort avoidance. We show that when funds are principally flow motivated, excessive exit will arise—and thus limit the disciplinary effect of exit—exactly for those firms in which the moral hazard problem is most severe, because it is for these firms that exit will be endogenously viewed as a positive signal of ability on the part of the fund.

To summarize, using an alternative definition of ability centered around ex post observation of management modifies the specific empirical content of our result: Instead of exiting *too little* (as implied by Proposition 2), principally flow motivated mutual funds will exit *too much*. But the *economic* content of our results is unchanged: In both cases, competition for flow weakens the ability of mutual funds to discipline management via the threat of exit.

VI. Hedge funds

We now consider the efficacy of exit as a governance mechanism when the blockholder is a hedge fund. How do the incentives identified in Section IV affect whether hedge funds can credibly govern via the threat of exit? Given our prior analysis of mutual funds, our discussion is best couched in comparative terms. As discussed above, like mutual funds, hedge funds also face flow performance relationships. Indeed, Lim, Sensoy, and Weisbach (2013) document that rewards from future flows matter four times as much for the average hedge fund as compensation arising from explicit compensation. Thus, like mutual funds, the need to compete for flow may negatively affect the ability of hedge funds to credibly threaten to exit. In addition, unlike mutual funds, hedge funds charge a convex carry. As we showed in Section IV, the possibility of earning a carry—particularly when combined with the need to compete for flow—further weakens the ability of funds to credibly threaten to exit. Thus, on the basis of these two factors, it may seem that hedge funds are *less* likely than mutual funds to successfully use exit to govern. However, *unlike* mutual funds, hedge fund managers invest significantly in their investment funds, and self-investment *enhances* the incentives to exit. Will hedge funds behave similarly to principally flow motivated mutual funds or will their higher degree of self-investment induce them to exit?

For the latter type of behaviour to be a theoretical possibility, we need to show that for sufficiently high self-investment, blockholders *would* choose to exit despite competing for flow and earning a carry. Thus, we extend the arguments used to prove Proposition 2 to show that funds that compete for flow and charge a carry *will* nevertheless exit *in equilibrium* if their degree of self-investment is high enough.

PROPOSITION 5. *For R_{G} and $\gamma_{\mathrm{M}}^{\mathrm{G}}$ sufficiently large and for any $w \in [0,1)$ and $\phi \in [0,1)$ there exists $\alpha' (\Theta, w, \phi) \in (0,1)$ such that for all $\alpha > \alpha' (\Theta, w, \phi)$, there is an equilibrium in which*

(i) Each investor chooses to fire his fund if she sells at $t = 1$ and retains her otherwise;

(ii) The fund chooses to sell at $t = 1$ whenever the manager chooses $a = 1$.

In equilibrium investors chase flow and punish funds who do exit. Thus exit comes at a flow and carry cost, but not exiting diminishes the value of the manager's self-investment. If the manager has sufficient skin in the game, then she will exit despite the flow and carry costs implied by her action. It is worth noting that in this equilibrium the fund behaves exactly as the principal blockholder of Admati and Pfleiderer (2009). Thus, the manager behaves as in Section II: He chooses the perverse action $a = 1$ if and only if $\beta \geq \beta_{\mathrm{L}} > \beta_{\text{No-L}} = \omega_2 \Delta v$.

While Proposition 5 establishes the existence of equilibria in which funds with sufficient self-investment will exit despite the disincentives created by their flow and carry motivations, the question of whether the result has bearing on hedge fund behaviour is a *quantitative* matter. Hedge funds typically charge assets under management fees of around 2% ($w = 0.02$) and carry fees of around 20% ($\phi = 0.2$). As Fung and Hsieh (1999) point out, hedge funds "are typically organized as limited partnerships, in which the investors are limited partners and the managers are general partners [who invest] a significant proportion of their personal wealth into the partnership." While it is therefore clear that hedge fund managers self-invest, there is little available data on the size of such self-investment. Agarwal, Daniel, and Naik (2009) estimate a *lower bound* on self investment by computing the skin in the game implied *only* by a reinvestment of fees (assuming no initial investment by the hedge fund manager) and find $\alpha \geq 0.071$ with a standard deviation of 0.145. Papers calibrating hedge fund parameters typically assume that α is between 0.1 (Hodder and Jackwerth (2007)) and 0.2 (He and Krishnamurthy (2013)). Guided by this, we assume in our computations that $\alpha = 0.1$. We normalize v_{H} to 1 and allow v_{L} to vary in the set $\left\{0, \frac{1}{3}, \frac{2}{3}\right\}$ representing different levels of long-term equity value destruction due to perverse action choices by managers. We then ask whether there exist parameters $\left(\theta, \gamma_{\mathrm{F}}, \gamma_{\mathrm{M}}^{\mathrm{G}}, \gamma_{\mathrm{M}}^{\mathrm{B}}, R_{\mathrm{G}}, R_{\mathrm{B}}\right)$ which imply that hedge funds would exit.

Given the high dimensionality of the relevant parameter set, it is difficult to visually depict when hedge funds will exit. We therefore first depict the case in which $\gamma_{\mathrm{M}}^{\mathrm{G}} \rightarrow 1$, so that good hedge funds are extremely capable. The structure of our model implies that, when $\gamma_{\mathrm{M}}^{\mathrm{G}} \rightarrow 1$, for any given (w, ϕ, α) whether a fund will exit when investors compete for flow is determined by (θ, A) where A represents the measure of firms with agency problems. This makes it feasible to transparently depict the set of parameters for which hedge funds will exit. The (θ, A) that induce the fund to exit for $(w, \phi, \alpha) = (0.02, 0.2, 0.1)$ are shown below (shaded regions) in Figure 1, Panels (a)-(c) (θ is plotted along the x-axis and A along the y-axis). We then provide examples of parameter constellations with $\gamma_{\mathrm{M}}^{\mathrm{G}} < 1$ such that hedge funds will exit for (θ, A) pairs consistent with Figure 1. These further examples, as well a detailed description of our computational procedure, can be found in the appendix.

[**Insert Figure 1 here**]

Figure 1 delivers an intuitive message. As the three panels illustrate, hedge fund exit is viable exactly when θ and A are not large. This is both intuitive and relevant from an applied perspective. It is intuitive because a high θ implies that flows are large (because the set of investors who are forced to liquidate as a result of shocks and find new funds is increasing in θ).[17] Further since A is decreasing in γ_{F}, the measure of good funds, a low A is commensurate with a high proportion of able funds. But the flow motivations of funds in our model are a result of funds' *career concerns*, and these are usually most potent when most agents are bad and only a few are good (see, for example, Dasgupta and Prat (2008)). It is relevant from an applied perspective because hedge funds typically invest in less liquid stocks (low θ) and may have a fairly talented manager pool (low A).

The decreasing incidence of exit moving from left to right across panels (a) to (c) can be understood as follows. The key force incentivising hedge funds to exit is that not exiting is costly to the manager's self-invested funds, and this cost is increasing in the degree of value destroyed by the choice of the perverse action. Thus, when the manager's perverse action choices do not destroy much value ($v_{\mathrm{L}} = 2/3$) exit becomes more challenging to sustain. To the extent that the percentage value decrease for equity holders caused by any given incidence of perverse action choice by company management is likely to be increasing in firm leverage, our findings suggest that hedge fund exit may be more salient for more leveraged firms.

Our microfounded, game-theoretic model is clearly not ideally suited for formal calibration exercises. Thus, it would be unreasonable to ask that these calculations be accepted as *literal* representations of reality. However, we believe that these computations represent a conservative approach. In our model, and therefore in our computations, all investors chase performance at $t = 1$ and the hedge fund's portfolio at $t = 1$ is composed of a single asset—the block—making the inference problem relatively simple for the investors. In reality hedge fund investors are often subject to lock-up periods and hedge funds own multiple assets and are opaque. Both of these factors are likely to enhance their incentives to exit. Locked-up investors cannot take money out of hedge funds even if they observe exit. The presence of lock-ups is likely to enhance the incentives of a hedge fund to exit conditional on observing $a = 1$ as it reduces the flow cost that such an action entails. Opacity is also likely to enhance hedge funds' incentives to exit in the presence of multiple investments. Opaque hedge funds, with complex assets that are not easy to mark to market, may well have private information about the value of these assets and thus may be able to camouflage the price impact of exit at $t = 1$. Then, as already discussed in Section V.A.2, they may be more inclined to exit. It is worth noting that for an opaque hedge fund with multiple investments the effect of carry on exit may weaken. Not exiting still creates an artificial price increase at $t = 1$ thus enhancing $t = 1$ carry. However, if the rest of the portfolio is likely to generate a positive carry at $t = 2$ (and exit at $t = 1$ can be camouflaged, thus avoiding outflows) then exiting at $t = 1$ may increase $t = 2$ carry because it eliminates $t = 2$ losses from holding the underperforming block. Taking all these factors into account, it is reasonable to conclude hedge funds may be more willing to exit in reality than is implied by our model. Accordingly, our calculations only represent a conservative estimate and can form a basis for concluding that that hedge funds *would* effectively use exit even allowing for some significant margin of error to account for the stylized aspects of the model. It is worth noting that the empirical findings of Edmans, Fang, and Zur (2013) suggest that hedge funds do successfully use exit to govern.

VII. Exit, Voice, and Money Managers

We now consider the possibility of active monitoring (the use of "voice") by delegated blockholders. In a setting with both active and passive monitoring, it is necessary to take a view on the interaction of the two. The existing literature has typically thought of exit and voice as being substitutes. Two important contributions to this literature include Kahn and Winton (1998) and Maug (1998). Kahn and Winton (1998) argue that the incentives of blockholders to speculate or intervene may be in conflict. They theoretically delineate conditions under which a blockholder will choose to intervene or exit. Maug (1998) also considers the choice between exit and voice, and argues that—in addition to aiding exit—liquidity can also enhance voice by facilitating block formation. In contrast, in our analysis, we are guided by an additional source of interaction between active and passive monitoring outlined in Hirschman (1970). Hirschman argued that exit and voice are potentially *complementary* governance mechanisms: The existence of the threat of exit makes blockholder voice worth listening to. If voice and exit and complementary, do our results on the effect of funds' compensation on exit correspond to different ability and willingness to use voice? We consider this question next.

Recall our baseline model with a fund with α high enough to satisfy the conditions of Proposition 5. For firms with agency problems in which $\beta \leq \beta_{\mathrm{L}}$ the existence of the threat of exit, by itself, prevents perverse behaviour by the manager at no cost to the fund (since the threat of exit is not executed for these firms in equilibrium). However, for firms with $\beta > \beta_{\mathrm{L}}$, the perverse action cannot be prevented by the threat of exit, and the fund must engage in costly exit in equilibrium. Consider the following modification of the model. Imagine that following block formation, at $t = 0$, the fund learns whether the type of the firm is such that the threat of exit alone will discipline the manager, that is, the fund learns whether $\beta > \beta_{\mathrm{L}}$, before the manager makes his action choice. In case $\beta > \beta_{\mathrm{L}}$, could the fund be tempted to use voice to discipline management?

We model voice as follows. If the fund learns that the threat of exit alone is insufficient, she can make a proposal for a series of operational and financial remedies (for example changes in business strategy) to the firm. Formulating the proposal comes at effort cost e to the fund. The proposal may be accepted or rejected by the manager. If accepted, the resulting change in business strategy leads the manager to relinquish the perverse action (that is, choose $a = 0$) and yields him non-pecuniary benefits, $\rho \in (0, \beta_{\mathrm{L}} - \omega_2 \Delta v)$, over and above his normal compensation from choosing $a = 0$. The cost e is sunk regardless of whether the manager accepts or rejects the proposal. Our formulation for voice can be interpreted as follows: The change in business strategy generates a reduction in the effort cost for the manager for choosing $a = 0$, which translates into an increase in benefits for choosing $a = 0$.[18] Our formulation for voice is consistent with the description of active monitoring by hedge funds given by Brav et al. (2008), who argue that activist hedge funds propose an array of strategic, operational, and financial remedies. For simplicity, we assume that the voice pre-game described here is unobservable to investors and the market.

PROPOSITION 6. *For $\gamma_{\mathrm{M}}^{\mathrm{G}}$ and R_{G} large enough*[19] *and e small enough, for $\beta \in [\beta_{\mathrm{L}}, \beta_{\mathrm{L}} + \rho]$:*

1. *For $\alpha > \alpha'(\Theta, w, \phi)$, there exists an equilibrium in which funds successfully use voice to prevent the perverse action (and thus avoid exit).*

2. *For $\alpha < \widehat{\alpha}(\Theta, w, \phi, 0)$, there exists an equilibrium in which funds do not use voice.*

Here $\widehat{\alpha}(\Theta, w, \phi, 0) = \lim_{\epsilon \to 0} \widehat{\alpha}(\Theta, w, \phi, \epsilon)$ where $\widehat{\alpha}(\Theta, w, \phi, \epsilon)$ is defined in Proposition 4 and $\alpha'(\Theta, w, \phi)$ is defined in Proposition 5.

Thus, there exist equilibria of our model which jointly identify a class of firms for which the threat of exit and voice are complementary in generating good governance, because blockholders will use voice if and only if they can credibly threaten to exit. Intuitively, the manager's payoff from ignoring voice depends on whether the fund exits or not if voice is ignored, and is higher when the fund does not exit than when she does. This reduces the reward required to induce the manager to choose $a = 0$ when the fund uses voice. Indeed, for $\rho \in (0, \beta_{\mathrm{L}} - \omega_2 \Delta v)$ blockholders' voice will never induce the manager to choose $a = 0$ over $a = 1$ if he knows that the fund will not exit. This is not true when he instead anticipates that the fund will exit if voice is ignored. This implies that for low cost e, funds with high α will use voice backed by the threat of exit. The use of voice reduces the range of β for which the manager takes the perverse action from $\beta \geq \beta_{\mathrm{L}}$ to $\beta \geq \beta_{\mathrm{L}} + \rho$, thereby making voice an additional corporate governance instrument. In contrast, funds with low α, being unable to credibly threaten to exit, never induce the manager to take $a = 0$ through voice if $\rho \in (0, \beta_{\mathrm{L}} - \omega_2 \Delta v)$ and thus rationally refrain from paying the costs of using voice.

The required conditions on α above are identical to those already introduced in Propositions 4 and 5. Our results on voice, in turn, may be interpreted in terms of mutual funds and hedge funds. Accordingly, this result provides one potential explanation—based on the interaction between voice and exit—for the empirical regularity that hedge funds use voice and produce significant gains for shareholders in target companies (Brav et al. (2008), Becht, Franks, and Grant (2010)), while mutual funds choose to remain silent and do not deliver similar gains (Karpoff (2001), Barber (2007), and Kahan and Rock (2007)).

VIII. Conclusions

Blockholders are often seen as a solution to problems arising from the separation of ownership and control in publicly traded corporations. Admati and Pfleiderer (2009) show that the threat of exit can be an effective form of corporate governance when the blockholder is a profit-maximizing principal. Motivated by the prevalence of equity blocks that are held by delegated portfolio managers, we analyze whether agency frictions arising from delegated portfolio management may affect the ability of blockholders to govern through exit.

We show that when investors chase performance, funds will be reluctant to exit underperforming blocks. The reason is that when funds are differentiated by stock selection ability, exit is informative about skill and endogenously generates outflows. The perverse incentives generated by competition for flow operates through both AUM fees and carried interest, the components of standard "two and twenty" compensation contracts. Offsetting incentives are provided by proprietary investment in funds which provide direct exposure to portfolio value. Thus, for a given two and twenty contract, a fund will be successful in using exit to govern only if she has sufficient skin in the game. Our results suggest that mutual funds—whose managers self-invest very little—may be less successful in governing via the threat of exit than hedge funds, whose managers self-invest significantly.

While no systematic attempt has been made to empirically connect the type of money manager with the effectiveness of exit, some existing empirical results are consistent with our theoretical prediction. In contrast, a significant empirical literature connects the type of asset manager to the effectiveness of blockholder voice. We provide theoretical support for this literature by demonstrating the potential complementarity between exit and voice: The threat of exit determines the effectiveness of voice, implying that funds will little skin in the game will be unlikely to be vocal activists.

Our analysis examines the interplay of two distinct agency problems: Between the managers and equity holders of firms on the one hand, and between delegating investors and their portfolio managers on the other. Both of these problems are ubiquitous. Our results show that the two agency problems may interact in crucial ways: The existence of the latter may undermine traditional solutions to the former. Thus our findings emphasise the potential importance of frictions arising from the delegation of portfolio management to the effectiveness of corporate governance.

Appendix A. Proofs

Proof of Proposition 1: We first compute the manager's strategy and then the blockholder's. If the manager chooses $a = 1$, he knows that the blockholder will sell her shares at $t = 1$ at P_1^{s} and that $P_2 = v_{\mathrm{L}}$. Thus his expected utility is

$$\beta + \omega_1 P_1 + \omega_2 P_2 = \beta + \omega_1 P_1^{\mathrm{s}} + \omega_2 v_{\mathrm{L}}.$$

If he does not choose $a = 1$, he knows that the blockholder will sell her shares at $t = 1$ only for liquidity reasons—which occurs with probability θ—and that the market price is P_1^{ns}; he also knows that $P_2 = v_{\mathrm{H}}$. Thus his expected utility is

$$\omega_1 P_1 + \omega_2 P_2 = \omega_1(\theta P_1^{\mathrm{s}} + (1-\theta)P_1^{\mathrm{ns}}) + \omega_2 v_{\mathrm{H}}. \tag{A.1}$$

Hence, the manager's strategy is

$$s_{\mathrm{M}}(\beta) = \begin{cases} 1 & \text{if } \beta - \omega_1(1-\theta)(P_1^{\mathrm{ns}} - P_1^{\mathrm{s}}) - \omega_2 \Delta v \geq 0 \\ 0 & \text{otherwise.} \end{cases} \tag{A.2}$$

Since $\beta - \omega_1(P_1^{\mathrm{ns}} - P_1^{\mathrm{s}}) - \omega_2 \Delta v$ is increasing in β, the manager's best response will be characterised by a cutoff point β_{L}, such that he takes the perverse action for any $\beta \geq \beta_{\mathrm{L}}$, where the cutoff is equal to the fixed point of the following equation:

$$\beta_{\mathrm{L}} = \omega_1(1-\theta)(P_1^{\mathrm{ns}}(\beta_{\mathrm{L}}) - P_1^{\mathrm{s}}(\beta_{\mathrm{L}})) + \omega_2 \Delta v.$$

The cutoff point β_{L} is unique if $P_1^{\mathrm{ns}}(\beta_{\mathrm{L}}) - P_1^{\mathrm{s}}(\beta_{\mathrm{L}})$ is decreasing in β_{L}. To establish this, we compute $P_1^{\mathrm{s}}(\beta_{\mathrm{L}})$ and $P_1^{\mathrm{ns}}(\beta_{\mathrm{L}})$ as functions of β_{L}. When the blockholder sells her shares, the market does not know whether it is for liquidity or speculative reasons and hence

$$P_1^{\mathrm{s}}(\beta_{\mathrm{L}}) = v_{\mathrm{L}} + \Delta v \frac{\theta \left[1 - A\mathbb{P}(\tilde{\beta} \geq \beta_{\mathrm{L}})\right]}{\theta + (1-\theta)A\mathbb{P}(\tilde{\beta} \geq \beta_{\mathrm{L}})}. \tag{A.3}$$

If the blockholder does not sell, the market infers that the manager has not taken the perverse action and that the value of the firm is v_{H}. Hence,

$$P_1^{\mathrm{ns}}(\beta_{\mathrm{L}}) = v_{\mathrm{H}}.$$

Since $\mathbb{P}(\tilde{\beta} \geq \beta_{\mathrm{L}})$ is decreasing in β_{L}, it is immediate that

$$P_1^{\mathrm{ns}}(\beta_{\mathrm{L}}) - P_1^{\mathrm{s}}(\beta_{\mathrm{L}}) = \Delta v \frac{A\mathbb{P}(\tilde{\beta} \geq \beta_{\mathrm{L}})}{\theta + (1-\theta)A\mathbb{P}(\tilde{\beta} \geq \beta_{\mathrm{L}})}.$$

is decreasing in β_{L} establishing the uniqueness of β_{L}.

To conclude the characterization of the manager's strategy, we now show that $\beta_{\mathrm{L}} > \beta_{\text{No-L}}$. It is immediate from the previous expression that $P_1^{\mathrm{ns}}(\beta_{\mathrm{L}}) - P_1^{\mathrm{s}}(\beta_{\mathrm{L}}) \geq 0$. It follows that $\beta_{\mathrm{L}} \geq \beta_{\text{No-L}}$. To show that the inequality is strict, we first assume that $\beta_{\mathrm{L}} = \beta_{\text{No-L}}$. Since $\beta_{\text{No-L}} = \omega_2 \Delta v$, this implies that $\mathbb{P}(\tilde{\beta} \geq \beta_{\mathrm{L}}) = \mathbb{P}(\tilde{\beta} \geq \omega_2 \Delta v) = 1 - F(\omega_2 \Delta v) = 0$. But, since $\bar{\beta} > \omega_2 \Delta v$ and F is strictly increasing, we have that $F(\omega_2 \Delta v) < 1$, a contradiction. Thus, $\beta_{\mathrm{L}} > \beta_{\text{No-L}}$.

The blockholder's incentives are immediate. If she observes that the manager has chosen $a = 1$, if she does not exit she will receive v_{L} while if she does she will receive $P_1^{\mathrm{s}}(\beta_{\mathrm{L}}) > v_{\mathrm{L}}$ (immediately from equation (A.3)). Thus she exits. If she observes that the manager has chosen $a = 0$, if she does not exit she will receive v_{H} while if she does she will receive $P_1^{\mathrm{s}}(\beta_{\mathrm{L}}) < v_{\mathrm{H}}$. Thus she does not exit.

Proof of Proposition 2: We focus here on firms whose managers face an agency problem and we attempt to construct an equilibrium in which each fund's strategy involves the use of exit as in Admati and Pfleiderer, that is for those funds not hit by a liquidity shock:

$$s_{\mathrm{F}}(a) = \begin{cases} \mathrm{ns} & \text{if } a = 0 \\ \mathrm{s} & \text{if } a = 1. \end{cases} \tag{A.4}$$

We first construct each manager's best response to such a strategy. Since the putative equilibrium behaviour of the fund in (A.4) is identical to the blockholder's strategy in Proposition 1, it follows immediately that the manager will follow the strategy in (A.2). Note that (the unique) β_{L} does not depend on the returns on new investments R_{G} and R_{B}.

We now proceed to compute the best response of an investor and delineate conditions under which, given the fund's strategy, it is a best response for each investor in the fund to follow the strategy

$$s_{\mathrm{I}}(a^{\mathrm{F}}) = \begin{cases} \text{replace with a fund that has not sold} & \text{if } a^{\mathrm{F}} = \mathrm{s} \\ \text{retain} & \text{otherwise,} \end{cases} \tag{A.5}$$

as long as all other investors also do so.

We begin with two preliminaries. First, we compute the measure of flows implied by the investors' strategy. In this equilibrium at $t = 1$, a measure $(1-\alpha)(1-\theta)A\mathbb{P}(\tilde{\beta} \geq \beta_{\mathrm{L}})$ of investors voluntarily liquidates the funds that have sold and optimally replaces them with funds that have not sold. In addition, a measure $(1-\alpha)\theta$ of investors are matched with funds who have been hit by a liquidity shock and are thus looking to replace them with funds that have not sold. Funds that have not sold are in measure $(1-\theta)\left(1 - A\mathbb{P}(\tilde{\beta} \geq \beta_{\mathrm{L}})\right)$. The investors from the funds that have sold will be distributed proportionally to the funds that have not sold. Thus each fund that has not sold will get a proportion

$$\frac{(1-\alpha)\left[\theta + (1-\theta)A\mathbb{P}(\tilde{\beta} \geq \beta_{\mathrm{L}})\right]}{(1-\theta)\left(1 - A\mathbb{P}(\tilde{\beta} \geq \beta_{\mathrm{L}})\right)}$$

of new investors. How much money does each new investor bring to a fund that has not sold?

Denote the block price at $t = 0$ by P_0. Given the initial mix of investment in the fund this means that the fund manager contributed αP_0 and each of a continuum of measure $1-\alpha$ investors contributed P_0 each, giving rise to a total investment of $\alpha P_0 + (1-\alpha)P_0 = P_0$. Conditional on (voluntary or forced) liquidation, the market value of the initial investment of P_0 at $t = 1$ is P_1^{s}, and according to the payoff rules specified, each small investor receives $\frac{P_0}{P_0} P_1^{\mathrm{s}} = P_1^{\mathrm{s}}$. Thus the total amount of wealth that each fund will have to invest in new investments is:

$$\frac{(1-\alpha)\left[\theta + (1-\theta) A \mathbb{P}(\tilde{\beta} \geq \beta_{\mathrm{L}})\right]}{(1-\theta)\left(1 - A\mathbb{P}(\tilde{\beta} \geq \beta_{\mathrm{L}})\right)} P_1^{\mathrm{s}} =: K_\alpha. \tag{A.6}$$

Second, we compute the expected return to new investments conditional on the two possible managerial actions that the fund may observe. Conditional on $a = 1$, the posterior probability that the fund is good is:

$$\underline{\gamma}_{\mathrm{F}} = \mathbb{P}(\tau^{\mathrm{F}} = \mathrm{G} \,|\, a = 1) = \frac{\gamma_{\mathrm{F}}(1-\gamma_{\mathrm{M}}^{\mathrm{G}})}{\gamma_{\mathrm{F}}(1-\gamma_{\mathrm{M}}^{\mathrm{G}}) + (1-\gamma_{\mathrm{F}})(1-\gamma_{\mathrm{M}}^{\mathrm{B}})} < \gamma_{\mathrm{F}}. \tag{A.7}$$

Thus the expected per dollar return from the new investments is: $\underline{R} = \underline{\gamma}_{\mathrm{F}} R_{\mathrm{G}} + (1 - \underline{\gamma}_{\mathrm{F}}) R_{\mathrm{B}}$. Since $\underline{\gamma}_{\mathrm{F}}$ is decreasing in $\gamma_{\mathrm{M}}^{\mathrm{G}}$, the $\lim_{\gamma_{\mathrm{M}}^{\mathrm{G}} \to 1} \underline{\gamma}_{\mathrm{F}} = 0$ and $R_{\mathrm{B}} < 1$, there exists a $\underline{\gamma}_{\mathrm{M}}^{\mathrm{G}}\left(R_{\mathrm{G}}; R_{\mathrm{B}}, \gamma_{\mathrm{F}}\right) \in (0,1)$ such that if

$$\gamma_{\mathrm{M}}^{\mathrm{G}} > \underline{\gamma}_{\mathrm{M}}^{\mathrm{G}}\left(R_{\mathrm{G}}; R_{\mathrm{B}}, \gamma_{\mathrm{F}}\right), \tag{A.8}$$

then $\underline{R} < 1$.

In contrast, conditional on $a = 0$ the posterior probability that the fund is good is:

$$\widehat{\gamma}_{\mathrm{F}}^{\mathrm{sep}} = \mathbb{P}(\tau^{\mathrm{F}} = \mathrm{G} \,|\, a = 0) = \frac{\gamma_{\mathrm{F}}\left[\gamma_{\mathrm{M}}^{\mathrm{G}} + (1-\gamma_{\mathrm{M}}^{\mathrm{G}})(1 - \mathbb{P}(\tilde{\beta} \geq \beta_{\mathrm{L}}))\right]}{1 - A\mathbb{P}(\tilde{\beta} \geq \beta_{\mathrm{L}})} > \gamma_{\mathrm{F}}. \tag{A.9}$$

Thus the expected per dollar return from the new investments is:

$$\widehat{R}_{\mathrm{sep}} = \widehat{\gamma}_{\mathrm{F}}^{\mathrm{sep}} R_{\mathrm{G}} + (1 - \widehat{\gamma}_{\mathrm{F}}^{\mathrm{sep}}) R_{\mathrm{B}} = R_{\mathrm{B}} + \Delta R \frac{\gamma_{\mathrm{F}}\left[\gamma_{\mathrm{M}}^{\mathrm{G}} + (1-\gamma_{\mathrm{M}}^{\mathrm{G}})(1 - \mathbb{P}(\tilde{\beta} \geq \beta_{\mathrm{L}}))\right]}{1 - A\mathbb{P}(\tilde{\beta} \geq \beta_{\mathrm{L}})}.$$

Then, $\widehat{R}_{\mathrm{sep}} > 1$ if

$$R_{\mathrm{G}} > \underline{R}_{\mathrm{G},1}(\gamma_{\mathrm{M}}^{\mathrm{G}}; \Theta). \tag{A.10}$$

Investors may either infer from returns that their fund has not sold (the portfolio value is P_1^{ns}) or that the fund has sold (the portfolio value is P_1^{s}). Conditional on the fund selling, the investor knows either that the fund has experienced a liquidity shock (and hence is shutting down) or that the fund has chosen to sell (and thus remains open for potential investment). Consider the latter investor first. This investor may:

1. Retain the fund (i.e., leave the proceeds received from the fund with the fund).

2. Fire the fund, after paying the carry at $t = 1$, and invest in the index asset:

$$P_1^{\text{s}} - \phi \left(P_1^{\text{s}} - P_0\right)^+ .$$

3. Fire the fund and invest in one or more funds that have not sold:

$$\underbrace{\frac{P_1^{\text{s}}}{P_1^{\text{ns}} + K_\alpha}}_{\text{expected payoff}} \Big[v_{\text{H}} + K_\alpha \widehat{R}_{\text{sep}} - \underbrace{\phi \left(v_{\text{H}} + K_\alpha \widehat{R}_{\text{sep}} - \left(P_1^{\text{ns}} + K_\alpha\right)\right)^+}_{\text{carry at } t=2} \Big] - \underbrace{\phi \left(P_1^{\text{s}} - P_0\right)^+}_{\text{carry at } t=1} - \underbrace{w P_1^{\text{s}}}_{\text{AUM fee at } t=1} .$$

$\frac{P_1^{\text{s}}}{P_1^{\text{ns}}+K_\alpha}(v_{\text{H}} + K_\alpha \widehat{R}_{\text{sep}})$ represents, the expected dollar amount he receives from the new fund: since he has fired his original fund and received P_1^{s}, he has a claim to $\frac{P_1^{\text{s}}}{P_1^{\text{ns}}+K_\alpha}$ of the new fund's payoff. The payoff of a fund that has not sold is $v_{\text{H}} + K_\alpha \widehat{R}_{\text{sep}}$ since the fund keeps her block and invests the additional wealth that she receives, K_α, in the new opportunity which returns $\widehat{R}_{\text{sep}}$. Further, the $t = 1$ contingent fee is $\phi \left(P_1^{\text{s}} - P_0\right)^+$ because the investor pays a fraction $\frac{P_0}{P_0}$ (his dollar investment divided by the total dollar value of the block at $t = 0$) of a total fee of $\phi \left(P_1^{\text{s}} - P_0\right)^+$, and the contingent fee at $t = 2$ is $\frac{P_1^{\text{s}}}{P_1^{\text{ns}}+K_\alpha} \phi \left(v_{\text{H}} + K_\alpha \widehat{R}_{\text{sep}} - \left(P_1^{\text{ns}} + K_\alpha\right)\right)^+$ because the investor pays a fraction $\frac{P_1^{\text{s}}}{P_1^{\text{ns}}+K_\alpha}$ of a total fee of $\phi \left(v_{\text{H}} + K_\alpha \widehat{R}_{\text{sep}} - \left(P_1^{\text{ns}} + K_\alpha\right)\right)^+$.

It is immediate that (1) is dominated by (2): Under (A.8) the fund can at best generate index returns so, by retaining the fund, the investor would pay fees to undertake investments that he can access directly. Equilibrium requires that the payoff to (3) is higher than the payoff to (2), which can be ensured by

$$\widehat{R}_{\text{sep}} \geq 1 + \frac{w(v_{\text{H}} \ + K_\alpha)}{(1-\phi)K_\alpha}. \tag{A.11}$$

Since $\widehat{R}_{\text{sep}}$ is increasing in R_{G} and $\lim_{R_{\text{G}}\to\infty} \widehat{R}_{\text{sep}} = \infty$, there exists a $\underline{R}_{\text{G},2}(\gamma^{\text{G}}_{\text{M}};\Theta) \in \mathbb{R}_{++}$ such that if

$$R_{\text{G}} \ \geq \underline{R}_{\text{G},2}(\gamma^{\text{G}}_{\text{M}};\Theta) \tag{A.12}$$

then inequality (A.11) holds.

Investors whose funds shut down have options (2) and (3) above but not option (1). Under (A.11), they will prefer option (3).

Consider an investor who has inferred that her fund has not sold. This investor may:

1. Retain his fund and receive:

$$\frac{P^{\text{ns}}_1}{P^{\text{ns}}_1 + K_\alpha}\left[\left(v_{\text{H}} + K_\alpha \widehat{R}_{\text{sep}}\right) - \phi\left(v_{\text{H}} + K_\alpha \widehat{R}_{\text{sep}} - (P^{\text{ns}}_1 + K_\alpha)\right)^+\right] - \phi\,(P^{\text{ns}}_1 - P_0)^+ - wP^{\text{ns}}_1.$$

2. Fire the fund and invest in index:

$$P^{\text{ns}}_1 - \phi\,(P^{\text{ns}}_1 - P_0)^+ .$$

3. Fire the fund and invest his money in one or more funds that have not sold:[20]

$$\frac{P^{\text{ns}}_1}{P^{\text{ns}}_1 + K_\alpha}\left[\left(v_{\text{H}} + K_\alpha \widehat{R}_{\text{sep}}\right) - \phi\left(v_{\text{H}} + K_\alpha \widehat{R}_{\text{sep}} - (P^{\text{ns}}_1 + K_\alpha)\right)^+\right] - \phi\,(P^{\text{ns}}_1 - P_0)^+ - wP^{\text{ns}}_1.$$

Clearly, the investor is indifferent between (1) and (3) and prefers either to (2) as long as condition (A.12) holds.

Thus, we have established that under conditions (A.8), (A.10) and (A.12), given the fund's strategy in (A.4) each investor will best respond by (A.5). We now show that if each investor follows (A.5) then there are conditions under which the fund will choose not to follow (A.4).

Suppose that a fund observes a $= 0$. If she chooses to hold the block she is retained by her investor and receives inflows of K_α:

$$(1-\alpha)\,\phi\,(P_1^{\mathrm{ns}} - P_0)^+ + w\,(P_1^{\mathrm{ns}} + K_\alpha) - w\alpha P_1^{\mathrm{ns}} + \\ + \frac{(1-\alpha)\,P_1^{\mathrm{ns}} + K_\alpha}{P_1^{\mathrm{ns}} + K_\alpha}\phi\left(v_{\mathrm{H}} + K_\alpha \widehat{R}_{\mathrm{sep}} - (P_1^{\mathrm{ns}} + K_\alpha)\right)^+ + \frac{\alpha P_1^{\mathrm{ns}}}{P_1^{\mathrm{ns}} + K_\alpha}\left(v_{\mathrm{H}} + K_\alpha \widehat{R}_{\mathrm{sep}}\right).$$

If she exits she gets $(1-\alpha)\,\phi\,(P_1^{\mathrm{s}} - P_0)^+ + \alpha P_1^{\mathrm{s}}$. Thus, she clearly prefers to retain.

Suppose that the fund observes a $= 1$. If she sells, she is fired and gets

$$\Pi_{\mathrm{s}} := (1-\alpha)\,\phi\,(P_1^{\mathrm{s}} - P_0)^+ + \alpha P_1^{\mathrm{s}}.$$

If she does not sell she retains investor capital and chooses optimally whether to invest in the new opportunities or not. The reason that the fund may be tempted to invest in the new opportunities despite the fact that, given (A.8), conditional on $a = 1$ such investment has negative net returns is that she earns a convex carry. Thus, by not selling the fund gets $\max\left\{\Pi_{\mathrm{ns}}^{\mathrm{I}}, \Pi_{\mathrm{ns}}^{\mathrm{NI}}\right\}$, where

$$\Pi_{\mathrm{ns}}^{\mathrm{NI}} := (1-\alpha)\,\phi\,(P_1^{\mathrm{ns}} - P_0) + w\,(P_1^{\mathrm{ns}} + K_\alpha) - w\alpha P_1^{\mathrm{ns}} + \frac{\alpha P_1^{\mathrm{ns}}}{P_1^{\mathrm{ns}} + K_\alpha}\,(v_{\mathrm{L}} + K_\alpha),$$

is the payoff from not selling and not investing in the new opportunities and

$$\Pi_{\mathrm{ns}}^{\mathrm{I}} := (1-\alpha)\,\phi\,(P_1^{\mathrm{ns}} - P_0) + w\,(P_1^{\mathrm{ns}} + K_\alpha) - w\alpha P_1^{\mathrm{ns}} + \frac{\alpha P_1^{\mathrm{ns}}}{P_1^{\mathrm{ns}} + K_\alpha}\,(v_{\mathrm{L}} + K_\alpha \underline{R}) + \\ + \frac{(1-\alpha)\,P_1^{\mathrm{ns}} + K_\alpha}{P_1^{\mathrm{ns}} + K_\alpha}\underline{\gamma}_{\mathrm{F}}\ \phi\Big(v_{\mathrm{L}} + K_\alpha R_{\mathrm{G}} - P_1^{\mathrm{ns}} - K_\alpha\Big)^+$$

is the payoff from not selling and investing in the new opportunities. Note that if she does not invest in the new opportunities she earns no carry at $t = 2$ because the returns between

$t=1$ and $t=2$ are $v_{\mathrm{L}}+K_\alpha-(P_1^{\mathrm{ns}}+K_\alpha)$, which are negative since $P_1^{\mathrm{ns}}-P_0>0$ and $v_{\mathrm{L}}-P_1^{\mathrm{ns}}<0$. The fund will adopt strategy (A.4) if and only if:

$$\Pi_{\mathrm{s}} \geq \max\left\{\Pi_{\mathrm{ns}}^{\mathrm{I}}, \Pi_{\mathrm{ns}}^{\mathrm{NI}}\right\}. \tag{A.13}$$

Let us first compare Π_{s} with $\Pi_{\mathrm{ns}}^{\mathrm{NI}}$. $\Pi_{\mathrm{s}} \geq \Pi_{\mathrm{ns}}^{\mathrm{NI}}$ if

$$(1-\alpha)\,\phi\,(P_1^{\mathrm{s}}-P_0)^{+}+\alpha P_1^{\mathrm{s}} \geq (1-\alpha)\,\phi\,(v_{\mathrm{H}}-P_0)+w\,(v_{\mathrm{H}}+K_\alpha)-w\alpha v_{\mathrm{H}}+\frac{\alpha v_{\mathrm{H}}}{v_{\mathrm{H}}+K_\alpha}\,(v_{\mathrm{L}}+K_\alpha)\,.$$

Given the equilibrium strategies, the price of the block at $t=0$ is given by $P_0=v_{\mathrm{L}}+\Delta v\,[1-A\mathbb{P}\,(\beta\geq\beta_{\mathrm{L}})]$ which is greater than P_1^{s} (given by equation (A.3)), thus $(1-\alpha)\,\phi\,(P_1^{\mathrm{s}}-P_0)^{+}=0$. In addition, again using the proof of Proposition 1, $P_1^{\mathrm{ns}}=v_{\mathrm{H}}$. Thus, rearranging, the fund will exit if

$$\alpha\left(v_{\mathrm{H}}\;(\phi+w)+P_1^{\mathrm{s}}-\phi P_0-\frac{v_{\mathrm{H}}\,(v_{\mathrm{L}}+K_\alpha)}{v_{\mathrm{H}}+K_\alpha}\right)-wK_\alpha \geq wv_{\mathrm{H}}\;+\phi\,(v_{\mathrm{H}}-P_0)\,. \tag{A.14}$$

Define

$$f(\alpha):=v_{\mathrm{H}}\;(\phi+w)+P_1^{\mathrm{s}}-\phi P_0-\frac{v_{\mathrm{H}}\;(v_{\mathrm{L}}+K_\alpha)}{v_{\mathrm{H}}+K_\alpha}-\frac{wK_\alpha}{\alpha}.$$

Now it is easy to see that

$$\lim_{\alpha\to 0}\alpha f(\alpha)=-wK_{\alpha=0}<0<wv_{\mathrm{H}}\;+\phi\,(v_{\mathrm{H}}-P_0)\,,$$

and, since $\lim_{\alpha\to 1}K_\alpha=0$, that

$$\lim_{\alpha\to 1}\alpha f(\alpha)=v_{\mathrm{H}}\;(\phi+w)+P_1^{\mathrm{s}}-\phi P_0-v_{\mathrm{L}}=wv_{\mathrm{H}}+\phi\,(v_{\mathrm{H}}-P_0)+P_1^{\mathrm{s}}-v_{\mathrm{L}}>wv_{\mathrm{H}}+\phi\,(v_{\mathrm{H}}-P_0)\,.$$

Thus, continuity implies that there exists an $\bar\alpha\in(0,1)$ such that $\bar\alpha f\,(\bar\alpha)=wv_{\mathrm{H}}+\phi\,(v_{\mathrm{H}}-P_0)$.

Computing the derivative of $f\,(\alpha)$ gives:

$$f'(\alpha)=-\frac{v_{\mathrm{H}}K'_\alpha\Delta V}{[v_{\mathrm{H}}+K_\alpha]^2}-\frac{\alpha wK'_\alpha-wK_\alpha}{\alpha^2}.$$

Since $K_\alpha > 0$ and $K'_\alpha < 0$, $f'(\alpha) > 0$. Thus, for any α such that $f(\alpha) > 0$, $\alpha f(\alpha)$ is increasing in α. Now note that since $\bar{\alpha} f(\bar{\alpha}) > 0$, it follows that $f(\bar{\alpha}) > 0$. Then, for any $\widehat{\alpha} < \bar{\alpha}$, one of two possible statements must be true. Either $f(\widehat{\alpha}) > 0$, in which case $\alpha f(\alpha)$ is increasing in α for all $\alpha \in [\widehat{\alpha}, \bar{\alpha}]$ and thus $\widehat{\alpha} f(\widehat{\alpha}) < \bar{\alpha} f(\bar{\alpha}) = w v_{\mathrm{H}} + \phi(v_{\mathrm{H}} - P_0)$. Or $f(\widehat{\alpha}) \leq 0$, in which case $\widehat{\alpha} f(\widehat{\alpha}) \leq 0 < w v_{\mathrm{H}} + \phi(v_{\mathrm{H}} - P_0)$. Thus, for any $\alpha < \bar{\alpha}$, $\Pi_{\mathrm{s}} < \Pi_{\mathrm{ns}}^{\mathrm{NI}}$. For any $\alpha < \bar{\alpha}$, one of the following statements must be true:

1. Either $\Pi_{\mathrm{ns}}^{\mathrm{NI}} > \Pi_{\mathrm{ns}}^{\mathrm{I}}$. If so $\max\left\{\Pi_{\mathrm{ns}}^{\mathrm{I}}, \Pi_{\mathrm{ns}}^{\mathrm{NI}}\right\} = \Pi_{\mathrm{ns}}^{\mathrm{NI}}$ and since $\Pi_{\mathrm{s}} < \Pi_{\mathrm{ns}}^{\mathrm{NI}}$ the fund will not exit.

2. Or $\Pi_{\mathrm{ns}}^{\mathrm{NI}} < \Pi_{\mathrm{ns}}^{\mathrm{I}}$. If so, $\max\left\{\Pi_{\mathrm{ns}}^{\mathrm{I}}, \Pi_{\mathrm{ns}}^{\mathrm{NI}}\right\} = \Pi_{\mathrm{ns}}^{\mathrm{I}} > \Pi_{\mathrm{ns}}^{\mathrm{NI}} > \Pi_{\mathrm{s}}$. Thus, the fund will not exit.■

Proof of Proposition 3: We only state here the section of the proof that deviates from that of Proposition 2. Since each fund is measure zero, a change in the investor base in one fund does not change the incentives of any other fund. Similarly, for any investor within the chosen fund, it is unnecessary to compute the best response again: If the investor is attentive, his incentives are identical to those of the investors in the baseline model, whereas if the investor is inattentive, he is inactive. To demonstrate that there is no equilibrium in which any fund chooses to sell if and only if she observes $a = 1$, we simply show that for small enough α the fund that observes $a = 1$ will fail to exit. Suppose that the fund observes $a = 1$. Since $\phi = 0$ the fund will not invest in new opportunities at $t = 1$ upon observing $a = 1$. If she sells, she is fired and gets $\alpha P_1^{\mathrm{s}} + (1-\alpha)\iota w P_1^{\mathrm{s}}$, since she is retained by the inattentive investors. If she does not sell she gets:

$$w\left(P_1^{\mathrm{ns}} + K_\alpha\right) - w\alpha P_1^{\mathrm{ns}} + \frac{\alpha P_1^{\mathrm{ns}}}{P_1^{\mathrm{ns}} + K_\alpha}\left(v_{\mathrm{L}} + K_\alpha\right).$$

Thus, the fund will adopt strategy (A.4) if

$$\alpha P_1^{\mathrm{s}} + (1-\alpha)\iota w P_1^{\mathrm{s}} \geq w\left(P_1^{\mathrm{ns}} + K_\alpha\right) - w\alpha P_1^{\mathrm{ns}} + \frac{\alpha P_1^{\mathrm{ns}}}{P_1^{\mathrm{ns}} + K_\alpha}\left(v_{\mathrm{L}} + K_\alpha\right).$$

Collecting terms involving α and setting $P_1^{\text{ns}} = v_{\text{H}}$, this can be expressed as:

$$\alpha \left(v_{\text{H}} w + P_1^{\text{s}} (1 - \iota w) - \frac{v_{\text{H}} (v_{\text{L}} + K_\alpha)}{v_{\text{H}} + K_\alpha} \right) - w K_\alpha \geq w \left(v_{\text{H}} - \iota P_1^{\text{s}} \right). \tag{A.15}$$

Following steps identical to the proof of Proposition 2, it is clear that there exists $\bar{\alpha} \in (0, 1)$ such that the inequality above obtains with equality and that for all $\alpha < \bar{\alpha}$ the above inequality cannot be satisfied. To see that the bound $\bar{\alpha}$ is decreasing in ι, implicitly differentiate the equality obtained by setting $\alpha = \bar{\alpha}$ in (A.15) with respect to ι, which gives:

$$\frac{d\bar{\alpha}}{d\iota} \left(v_{\text{H}} w + P_1^{\text{s}} (1 - \iota w) - \frac{v_{\text{H}} (v_{\text{L}} + K_{\bar{\alpha}})}{v_{\text{H}} + K_{\bar{\alpha}}} - \alpha v_{\text{H}} \frac{(v_{\text{H}} - v_{\text{L}}) K'_{\bar{\alpha}}}{(v_{\text{H}} + K_{\bar{\alpha}})^2} - w K'_{\bar{\alpha}} \right) = -(1 - \alpha) w P_1^{\text{s}}$$

from which the result follows immediately since $w v_{\text{H}} + P_1^{\text{s}} (1 - \iota w) - \dfrac{v_{\text{H}} (v_{\text{L}} + K_{\bar{\alpha}})}{v_{\text{H}} + K_{\bar{\alpha}}} > 0$ and $K'_{\bar{\alpha}} < 0$.■

Proof of Proposition 4: We construct a perfect Bayesian equilibrium in which the action of the fund who observes the perverse action is the same as the action of the fund who observes the non-perverse action, except for a small measure ϵ of non-strategic funds who sell whenever they observe $a = 1$. We then study the limit of this equilibrium. In the limit, as $\epsilon \to 0$, no fund exits.

The structure of the proof is similar to that of Proposition 2. We sketch the proof here, highlighting only the points of departure from that argument.

Let's start with the manager's strategy. The manager's expected utility if he chooses $a = 1$ is

$$\beta + \omega_1 \left[(\theta + (1 - \theta)\epsilon) P_1^{\text{s}} + (1 - \theta)(1 - \epsilon) P_1^{\text{ns}} \right] + \omega_2 v_{\text{L}}.$$

This is because he knows that at time 1 the fund is going to sell either for liquidity reasons (which occur with probability θ) or if she is non-strategic (which occurs with probability ϵ). The manager's expected utility from $a = 0$ is the same as that in Proposition 2 (see equation

(A.1)). As before, the manager's strategy will be characterized by a threshold β_{pool} which is now implicitly defined by:

$$\beta_{\text{pool}} = \omega_1(1-\theta)\epsilon(P_1^{\text{ns}}(\beta_{\text{pool}}) - P_1^{\text{s}}(\beta_{\text{pool}})) + \omega_2 \Delta v, \tag{A.16}$$

where

$$P_1^{\text{ns}}(\beta_{\text{pool}}) = v_{\text{L}} + \Delta v \frac{1 - A\mathbb{P}(\tilde{\beta} \geq \beta_{\text{pool}})}{1 - \epsilon A\mathbb{P}(\tilde{\beta} \geq \beta_{\text{pool}})}$$

and

$$P_1^{\text{s}}(\beta_{\text{pool}}) = v_{\text{L}} + \Delta v \frac{\theta[1 - A\mathbb{P}(\tilde{\beta} \geq \beta_{\text{pool}})]}{\theta + (1-\theta)\epsilon A\mathbb{P}(\tilde{\beta} \geq \beta_{\text{pool}})}.$$

LEMMA 1. *For ϵ sufficiently small, β_{pool} is unique.*

Proof of Lemma 1: We want to show that the function on the RHS of equation (A.16) has at most one fixed point on $[0, \bar{\beta}]$ for small ϵ. Since the vertical intercept of the function on the RHS of equation (A.16) is strictly positive ($\omega_2 \Delta v$) and the function is everywhere differentiable, as long as the slope of such function is less then 1, it cannot cross the function on the LHS of equation (A.16) twice. Consider the limiting case $\epsilon \to 0$ and appeal to continuity to obtain the result for $\epsilon > 0$,

$$\lim_{\epsilon \to 0} \omega_1(1-\theta)\epsilon \frac{\partial}{\partial \beta_{\text{pool}}}(P_1^{\text{ns}}(\beta_{\text{pool}}) - P_1^{\text{s}}(\beta_{\text{pool}})) < 1.$$

Note that

$$\frac{\partial P_1^{\text{ns}}(\beta_{\text{pool}})}{\partial \beta_{\text{pool}}} = -\frac{\Delta v A\mathbb{P}(\beta \geq \beta_{\text{pool}})'(1-\epsilon)}{[1 - \epsilon A\mathbb{P}(\tilde{\beta} \geq \beta_{\text{pool}})]^2}$$

is continuous in β_{pool}, and that

$$\lim_{\epsilon \to 0} \frac{\partial P_1^{\text{ns}}(\beta_{\text{pool}})}{\partial \beta_{\text{pool}}} = -\Delta v A\mathbb{P}(\beta \geq \beta_{\text{pool}})'.$$

And,

$$\frac{\partial P_1^{\text{s}}(\beta_{\text{pool}})}{\partial \beta_{\text{pool}}} = -\frac{\Delta v \theta A\mathbb{P}(\beta \geq \beta_{\text{pool}})'[\theta + (1-\theta)\epsilon]}{[\theta + (1-\theta)\epsilon A\mathbb{P}(\tilde{\beta} \geq \beta_{\text{pool}})]^2}$$

is continuous and

$$\lim_{\epsilon \to 0} \frac{\partial P_1^{\mathrm{s}}(\beta_{\mathrm{pool}})}{\partial \beta_{\mathrm{pool}}} = -\Delta v \theta A \mathbb{P}(\beta_{\mathrm{pool}})'.$$

Now, since

$$\lim_{\epsilon \to 0} \left(\frac{\partial P_1^{\mathrm{ns}}(\beta_{\mathrm{pool}})}{\partial \beta_{\mathrm{pool}}} - \frac{\partial P_1^{\mathrm{s}}(\beta_{\mathrm{pool}})}{\partial \beta_{\mathrm{pool}}} \right)$$

is bounded and for $\epsilon = 0$, $P_1^{\mathrm{ns}}(\beta_{\mathrm{pool}}) - P_1^{\mathrm{s}}(\beta_{\mathrm{pool}}) = 0$, then for ϵ small, $\omega_1(1-\theta)\epsilon \frac{\partial}{\partial \beta_{\mathrm{pool}}}(P_1^{\mathrm{ns}}(\beta_{\mathrm{pool}}) - P_1^{\mathrm{s}}(\beta_{\mathrm{pool}}))$ is close to zero, and β_{pool} is unique. This concludes the proof of the lemma. □

We now proceed to compute the best response of an investor and delineate conditions under which, given the fund's strategy, it is a best response for such an investor to follow the strategy in (A.5).

We use a similar argument to that in the proof of Proposition 2 and find that, in this equilibrium, at $t = 1$ funds that have not sold get inflows of

$$\frac{\theta + \epsilon(1-\theta) A \mathbb{P}(\tilde{\beta} \geq \beta_{\mathrm{pool}})}{(1-\theta)\left(1 - \epsilon A \mathbb{P}(\tilde{\beta} \geq \beta_{\mathrm{pool}})\right)} (1-\alpha) P_1^{\mathrm{s}} =: k_\alpha.$$

First note that if a fund observes the manager choose $a = 1$, she updates her beliefs about her own type to be as in equation (A.7). Thus, under condition (A.8), the expected per dollar return from new investments is $\underline{R} < 1$.

Suppose, instead, that the fund observes $a = 0$. She updates her beliefs about her type to be:

$$\widehat{\gamma}_{\mathrm{F}}^{\mathrm{pool}} = \frac{\gamma_{\mathrm{F}} \left[\gamma_{\mathrm{M}}^{\mathrm{G}} + (1 - \gamma_{\mathrm{M}}^{\mathrm{G}})(1 - \mathbb{P}(\tilde{\beta} \geq \beta_{\mathrm{pool}})) \right]}{1 - A \mathbb{P}(\tilde{\beta} \geq \beta_{\mathrm{pool}})} > \gamma_{\mathrm{F}}.$$

Thus the expected per dollar returns from the new investments is:

$$\widehat{R}_{\mathrm{pool}} = \widehat{\gamma}_{\mathrm{F}}^{\mathrm{pool}} R_{\mathrm{G}} + (1 - \widehat{\gamma}_{\mathrm{F}}^{\mathrm{pool}}) R_{\mathrm{B}} = R_{\mathrm{B}} + \Delta R \frac{\gamma_{\mathrm{F}} \left[\gamma_{\mathrm{M}}^{\mathrm{G}} + (1 - \gamma_{\mathrm{M}}^{\mathrm{G}})(1 - \mathbb{P}(\tilde{\beta} \geq \beta_{\mathrm{pool}})) \right]}{1 - A \mathbb{P}(\tilde{\beta} \geq \beta_{\mathrm{pool}})}.$$

Then, $\widehat{R}_{\text{pool}} > 1$ if

$$R_{\text{G}} > \underline{R}^{\text{pool}}_{\text{G},1}(\gamma^{\text{G}}_{\text{M}}; \Theta). \tag{A.17}$$

Investors may either infer from returns that their fund has not sold (the portfolio value is P_1^{ns}) or that the fund has sold (the portfolio value is P_1^{s}). Conditional on the fund selling, the investor knows either that the fund has experienced a liquidity shock (and hence is shutting down) or that the fund has chosen to sell (and thus remains open for potential investment). Consider first the latter investor who observes that his portfolio value is P_1^{s} at $t = 1$. This investor faces the same set of choices as the investor in the proof of Proposition 2 who has inferred that his fund sold (listed there as (1), (2), and (3)). However, a difference is that in this equilibrium the investor believes that by investing in funds that have not sold (option (3)) the lower bound on his expected returns is $\bar{R}^{\text{ns}}_{\text{pool}}$ instead of $\widehat{R}_{\text{sep}}$ where

$$\begin{aligned}\bar{R}^{\text{ns}}_{\text{pool}} &= \mathbb{P}(a = 1 \,|\, a^{\text{F}} = \text{ns})\underline{R} + \mathbb{P}(a = 0 \,|\, a^{\text{F}} = \text{ns})\widehat{R}_{\text{pool}} = \\ &= \frac{(1-\epsilon)A\mathbb{P}(\tilde{\beta} \geq \beta_{\text{pool}})}{1 - \epsilon A\mathbb{P}(\tilde{\beta} \geq \beta_{\text{pool}})}\underline{R} + \frac{1 - A\mathbb{P}(\tilde{\beta} \geq \beta_{\text{pool}})}{1 - \epsilon A\mathbb{P}(\tilde{\beta} \geq \beta_{\text{pool}})}\widehat{R}_{\text{pool}}.\end{aligned}$$

This is because funds that have not sold may have either observed $a = 0$ and thus will optimally invest at $t = 1$ at an expected return of $\widehat{R}_{\text{pool}}$, or they may have observed $a = 1$, in which case they can either invest in new investments at an expected return of $\underline{R}$ or in the index asset at an expected return of 1. Since $\underline{R} < 1$, $\bar{R}^{\text{ns}}_{\text{pool}}$ constitutes a lower bound on the investor's expected return.

As in the proof of Proposition 2, the investor prefers (2) to (1); he also prefers (3) to (2), if

$$\bar{R}^{\text{ns}}_{\text{pool}} \geq 1 + \frac{w(P_1^{\text{ns}} + k_\alpha)}{(1-\phi)k_\alpha}. \tag{A.18}$$

Since $\bar{R}^{\text{ns}}_{\text{pool}}$ is increasing in R_{G} and $\lim_{R_{\text{G}} \to \infty} \bar{R}^{\text{ns}}_{\text{pool}} = \infty$, there exists a $\underline{R}^{\text{pool}}_{\text{G},2}(\gamma^{\text{G}}_{\text{M}}; \Theta) \in \mathbb{R}_{++}$ such that if

$$R_{\text{G}} \geq \underline{R}^{\text{pool}}_{\text{G},2}(\gamma^{\text{G}}_{\text{M}}; \Theta), \tag{A.19}$$

then inequality (A.18) holds. Thus, under condition (A.19) the investor who fires the fund, invests in a fund that has not sold.

The investor whose fund is hit by a liquidity shock, is identical to the investor above, except that he lacks option (1). Thus, again, this investor invests with a fund that has not sold.

Finally, we compute the best response of an investor who has inferred on the basis of the portfolio value of his fund that his fund has not sold at $t = 1$. This investor has the same options as the investor in the proof of Proposition 2 who has inferred that his fund did not sell. However, here in computing the payoffs to the respective options, we replace v_{H} by P_1^{ns} and $\widehat{R}_{\mathrm{sep}}$ by $\bar{R}_{\mathrm{pool}}^{\mathrm{ns}}$. Thus, as before, the investor prefers option (1) as long as

$$\frac{k_\alpha \left(\bar{R}_{\mathrm{pool}}^{\mathrm{ns}} - 1\right)}{P_1^{\mathrm{ns}} + k_\alpha} \geq \frac{w}{1 - \phi}.$$

This is satisfied as long as inequality (A.19) holds.

Thus, we have established that under conditions (A.8), (A.17), and (A.19), given the funds' strategy described above, each investor will best respond by (A.5). It remains to show that the non-naive fund will choose not to sell regardless of observing $a = 0$ or $a = 1$. Suppose that a fund observes $a = 0$. If she chooses to hold the block she is retained by her investor, receives inflows of k_α and gets:

$$\begin{aligned}
&(1-\alpha)\,\phi\left(P_1^{\mathrm{ns}} - P_0\right)^+ + w\left(P_1^{\mathrm{ns}} + k_\alpha\right) - w\alpha P_1^{\mathrm{ns}} + \\
&+ \frac{(1-\alpha)\,P_1^{\mathrm{ns}} + k_\alpha}{P_1^{\mathrm{ns}} + k_\alpha}\phi\left(v_{\mathrm{H}} + k_\alpha \hat{R}_{\mathrm{pool}} - \left(P_1^{\mathrm{ns}} + k_\alpha\right)\right)^+ + \frac{\alpha P_1^{\mathrm{ns}}}{P_1^{\mathrm{ns}} + k_\alpha}\left(v_{\mathrm{H}} + k_\alpha \hat{R}_{\mathrm{pool}}\right).
\end{aligned}$$

If she exits she gets $(1-\alpha)\,\phi\left(P_1^{\mathrm{s}} - P_0\right)^+ + \alpha P_1^{\mathrm{s}}$. Thus, she clearly prefers to hold.

Suppose that the fund observes $a = 1$. If she sells, she is fired and gets

$$\Pi_{\mathrm{s}} := (1-\alpha)\,\phi\left(P_1^{\mathrm{s}} - P_0\right)^+ + \alpha P_1^{\mathrm{s}}.$$

If she does not sell she retains investor capital and chooses optimally whether to invest in the new opportunities or not. The reason that the fund may be tempted to invest in the new opportunities despite the fact that, given (A.8), conditional on $a = 1$ such investment has negative net returns, is that she earns a convex carry. Thus, by not selling the fund gets $\max\left\{\Pi_{\mathrm{ns}}^{\mathrm{I}}, \Pi_{\mathrm{ns}}^{\mathrm{NI}}\right\}$, where

$$\Pi_{\mathrm{ns}}^{\mathrm{NI}} := (1-\alpha)\,\phi\left(P_1^{\mathrm{ns}} - P_0\right) + w\left(P_1^{\mathrm{ns}} + k_\alpha\right) - w\alpha P_1^{\mathrm{ns}} + \frac{\alpha P_1^{\mathrm{ns}}}{P_1^{\mathrm{ns}} + k_\alpha}\left(v_{\mathrm{L}} + k_\alpha\right)$$

and

$$\begin{aligned}\Pi_{\mathrm{ns}}^{\mathrm{I}} := &(1-\alpha)\,\phi\left(P_1^{\mathrm{ns}} - P_0\right) + w\left(P_1^{\mathrm{ns}} + k_\alpha\right) - w\alpha P_1^{\mathrm{ns}} + \frac{\alpha P_1^{\mathrm{ns}}}{P_1^{\mathrm{ns}} + k_\alpha}\left(v_{\mathrm{L}} + k_\alpha \underline{R}\right) + \\ &+ \frac{(1-\alpha)\,P_1^{\mathrm{ns}} + k_\alpha}{P_1^{\mathrm{ns}} + k_\alpha}\underline{\gamma}_{\mathrm{F}}\phi\Big(v_{\mathrm{L}} + k_\alpha R_{\mathrm{G}} - P_1^{\mathrm{ns}} - k_\alpha\Big)^+.\end{aligned}$$

Note that if she does not invest in new investment opportunities she does not earn a carry at $t = 2$. In fact, the returns between $t = 1$ and $t = 2$ are $\left(v_{\mathrm{L}} + k_\alpha - \left(P_1^{\mathrm{ns}} + k_\alpha\right)\right)$, which are negative since $P_1^{\mathrm{ns}} - P_0 > 0$ and $v_{\mathrm{L}} - P_1^{\mathrm{ns}} < 0$. The fund will not sell if and only if:

$$\Pi_{\mathrm{s}} < \max\left\{\Pi_{\mathrm{ns}}^{\mathrm{I}}, \Pi_{\mathrm{ns}}^{\mathrm{NI}}\right\}.$$

Let us first compare Π_{s} with $\Pi_{\mathrm{ns}}^{\mathrm{NI}}$. $\Pi_{\mathrm{s}} < \Pi_{\mathrm{ns}}^{\mathrm{NI}}$ if:

$$(1-\alpha)\,\phi\left(P_1^{\mathrm{s}} - P_0\right)^+ + \alpha P_1^{\mathrm{s}} \leq (1-\alpha)\,\phi\left(P_1^{\mathrm{ns}} - P_0\right) + w\left(P_1^{\mathrm{ns}} + k_\alpha\right) - w\alpha P_1^{\mathrm{ns}} + \frac{\alpha P_1^{\mathrm{ns}}}{P_1^{\mathrm{ns}} + k_\alpha}\left(v_{\mathrm{L}} + k_\alpha\right).$$

Given the equilibrium strategies, the price of the block at $t = 0$ is given by $P_0 = v_{\mathrm{L}} + \Delta v\left[1 - A\mathbb{P}\left(\beta \geq \beta_{\mathrm{pool}}\right)\right]$ which is greater than P_1^{s}. Thus, rearranging, the fund will not exit if

$$\alpha\left(P_1^{\mathrm{ns}}\left(\phi + w\right) + P_1^{\mathrm{s}} - \phi P_0 - \frac{P_1^{\mathrm{ns}}\left(v_{\mathrm{L}} + k_\alpha\right)}{P_1^{\mathrm{ns}} + k_\alpha}\right) - w k_\alpha \leq w P_1^{\mathrm{ns}} + \phi\left(P_1^{\mathrm{ns}} - P_0\right). \tag{A.20}$$

Define

$$h(\alpha) := P_1^{\mathrm{ns}}(\phi + w) + P_1^{\mathrm{s}} - \phi P_0 - \frac{P_1^{\mathrm{ns}}(v_{\mathrm{L}} + k_\alpha)}{P_1^{\mathrm{ns}} + k_\alpha} - \frac{wk_\alpha}{\alpha}.$$

Now it is easy to see that

$$\lim_{\alpha \to 0} \alpha h(\alpha) = -wk_{\alpha=0} < 0 < wP_1^{\mathrm{ns}} + \phi\left(P_1^{\mathrm{ns}} - P_0\right),$$

and, since $\lim_{\alpha \to 1} k_\alpha = 0$, that

$$\lim_{\alpha \to 1} \alpha h(\alpha) = P_1^{\mathrm{ns}}\ (\phi + w) + P_1^{\mathrm{s}} - \phi P_0 - v_{\mathrm{L}} = wP_1^{\mathrm{ns}} + \phi\left(P_1^{\mathrm{ns}} - P_0\right) + P_1^{\mathrm{s}} - v_{\mathrm{L}} > wP_1^{\mathrm{ns}} + \phi\left(P_1^{\mathrm{ns}} - P_0\right).$$

Thus, continuity implies that there exists an $\widehat{\alpha} \in (0,1)$ such that $\widehat{\alpha} h(\widehat{\alpha}) = wP_1^{\mathrm{ns}} + \phi\left(P_1^{\mathrm{ns}} - P_0\right)$. Computing the derivative of $h(\alpha)$:

$$h'(\alpha) = -\frac{P_1^{\mathrm{ns}} k'_\alpha\left(P_1^{\mathrm{ns}} - v_{\mathrm{L}}\right)}{\left[P_1^{\mathrm{ns}}\ + k_\alpha\right]^2} - \frac{\alpha w k'_\alpha - wk_\alpha}{\alpha^2}.$$

Since $k_\alpha > 0$ and $k'_\alpha < 0$, we have $h'(\alpha) > 0$. Thus, for any α such that $h(\alpha) > 0$, $\alpha h(\alpha)$ is increasing in α. In particular, since $h(\widehat{\alpha}) > 0$, $\alpha h(\alpha)$ is increasing in α for $\alpha = \widehat{\alpha}$. For any $\alpha' < \widehat{\alpha}$, one of two possible statements may be true. Either $h(\alpha') > 0$, in which case $\alpha h(\alpha)$ is increasing in α for all $\alpha \in [\alpha', \widehat{\alpha}]$ and thus $\alpha' h(\alpha') < \widehat{\alpha} h(\widehat{\alpha}) = wP_1^{\mathrm{ns}} + \phi\left(P_1^{\mathrm{ns}} - P_0\right)$. Or $h(\alpha') \leq 0$, in which case $\alpha' h(\alpha') \leq 0 < wP_1^{\mathrm{ns}} + \phi\left(P_1^{\mathrm{ns}} - P_0\right)$. Thus, for any $\alpha < \widehat{\alpha}$, $\Pi_{\mathrm{s}} < \Pi_{\mathrm{ns}}^{\mathrm{NI}}$. For any $\alpha < \widehat{\alpha}$, one of the following statements must be true:

1. Either $\Pi_{\mathrm{ns}}^{\mathrm{NI}} > \Pi_{\mathrm{ns}}^{\mathrm{I}}$. If so $\max\left\{\Pi_{\mathrm{ns}}^{\mathrm{I}}, \Pi_{\mathrm{ns}}^{\mathrm{NI}}\right\} = \Pi_{\mathrm{ns}}^{\mathrm{NI}}$ and since $\Pi_{\mathrm{s}} < \Pi_{\mathrm{ns}}^{\mathrm{NI}}$ the fund will not exit.

2. Or $\Pi_{\mathrm{ns}}^{\mathrm{NI}} < \Pi_{\mathrm{ns}}^{\mathrm{I}}$. If so, $\max\left\{\Pi_{\mathrm{ns}}^{\mathrm{I}}, \Pi_{\mathrm{ns}}^{\mathrm{NI}}\right\} = \Pi_{\mathrm{ns}}^{\mathrm{I}} > \Pi_{\mathrm{ns}}^{\mathrm{NI}} > \Pi_{\mathrm{s}}$. Thus, the fund will not exit.■

Proof of Proposition 5: The proof builds directly on that of Proposition 2 where we showed that a fund that has observed $a = 1$ will exit if and only if

$$\Pi_{\mathrm{s}} \geq \max\left\{\Pi_{\mathrm{ns}}^{\mathrm{I}}, \Pi_{\mathrm{ns}}^{\mathrm{NI}}\right\}, \tag{A.21}$$

where Π_{s}, $\Pi_{\mathrm{ns}}^{\mathrm{I}}$, and $\Pi_{\mathrm{ns}}^{\mathrm{NI}}$ are as defined in the proof of Proposition 2. We first observe that there exists $\tilde{\alpha} \in (0,1)$ such that for all $\alpha > \tilde{\alpha}$, $\max\left\{\Pi_{\mathrm{ns}}^{\mathrm{I}}, \Pi_{\mathrm{ns}}^{\mathrm{NI}}\right\} = \Pi_{\mathrm{ns}}^{\mathrm{NI}}$. To see this, note that for any α

$$\frac{\alpha P_1^{\mathrm{ns}}}{P_1^{\mathrm{ns}} + K_\alpha}\left(v_{\mathrm{L}} + K_\alpha\right) > \frac{\alpha P_1^{\mathrm{ns}}}{P_1^{\mathrm{ns}} + K_\alpha}\left(v_{\mathrm{L}} + K_\alpha \underline{R}\right),$$

since $\underline{R} < 1$. Consider now the term

$$\phi\Big(v_{\mathrm{L}} + K_\alpha R_{\mathrm{G}} - P_1^{\mathrm{ns}} - K_\alpha\Big)^+ = \phi\Big(K_\alpha\left(R_{\mathrm{G}} - 1\right) - \Delta v\Big)^+.$$

Since $K_\alpha \to 0$ monotonically as $\alpha \to 1$, for any R_{G}, there exists some $\tilde{\alpha} < 1$ such that $K_\alpha\left(R_{\mathrm{G}} - 1\right) - \Delta v < 0$ for $\alpha > \tilde{\alpha}$ and thus

$$\phi\Big(K_\alpha\left(R_{\mathrm{G}} - 1\right) - \Delta v\Big)^+ = 0.$$

Thus, for $\alpha > \tilde{\alpha}$, $\max\left\{\Pi_{\mathrm{ns}}^{\mathrm{I}}, \Pi_{\mathrm{ns}}^{\mathrm{NI}}\right\} = \Pi_{\mathrm{ns}}^{\mathrm{NI}}$. Now, let us check when $\Pi_{\mathrm{ns}}^{\mathrm{NI}} < \Pi_{\mathrm{s}}$. In the proof of Proposition 2, we showed that this was equivalent to checking when $\alpha f(\alpha) \geq w v_{\mathrm{H}} + \phi\left(v_{\mathrm{H}} - P_0\right)$ where

$$f(\alpha) := v_{\mathrm{H}}\left(\phi + w\right) + P_1^{\mathrm{s}} - \phi P_0 - \frac{v_{\mathrm{H}}\left(v_{\mathrm{L}} + K_\alpha\right)}{v_{\mathrm{H}} + K_\alpha} - \frac{w K_\alpha}{\alpha},$$

and that there exists an $\bar{\alpha} \in (0,1)$ such that $\bar{\alpha} f(\bar{\alpha}) = w v_{\mathrm{H}} + \phi\left(v_{\mathrm{H}} - P_0\right)$. Since $f'(\alpha) > 0$ and $f(\bar{\alpha}) > 0$, $\alpha f(\alpha)$ is increasing for $\alpha \geq \bar{\alpha}$. Thus, for all $\alpha \in [\bar{\alpha}, 1)$, $\alpha f(\alpha) \geq w v_{\mathrm{H}} + \phi\left(v_{\mathrm{H}} - P_0\right)$. For $\alpha > \max\left(\bar{\alpha}, \tilde{\alpha}\right) := \alpha'$ the fund exits.■

Proof of Proposition 6: We consider the following pre-game to the exit game: If voice is not used or if voice is used and ignored, then the exit game begins with choices and payoffs to the fund and manager as in the baseline. If voice is used and the manager accepts he must choose $a = 0$ (and receives at $t = 2$ the normal payoff augmented by ρ), and the exit game begins in which the fund can decide whether to exit or not. In this proof, we reuse two thresholds defined in the proofs of Propositions 4 and 5: $\widehat{\alpha}(\Theta, w, \phi, 0) = \lim_{\epsilon \to 0} \widehat{\alpha}(\Theta, w, \phi, \epsilon)$ and $\alpha'\left(\Theta, w, \phi\right)$ respectively, which we denote $\widehat{\alpha}$ and α' for brevity. We consider two cases: $\alpha > \alpha'$ and $\alpha < \widehat{\alpha}$.

Let $\alpha > \alpha'$. We prove the existence of an equilibrium in which the fund uses voice and the manager ignores voice if $\beta > \beta_{\mathrm{L}} + \rho$ and accepts voice otherwise. We solve the game by backward induction.

Suppose voice is not used or used and ignored. In either case, the exit game begins and the manager's payoffs are as in the baseline model. It is easy to construct an equilibrium in which the fund chooses to exit if and only if $a = 1$. Investors don't know about the voice pre-game, and thus (for $\gamma_{\mathrm{M}}^{\mathrm{G}}$ and R_{G} high enough) behave identically to the baseline. Thus, if a fund observes $a = 1$, she faces the same choice as in the baseline and exits since $\alpha > \alpha'$. If the fund observes $a = 0$, what will she do? If the fund exits she is fired, earning αP_1^{s}. If she does not exit, she is retained, and if she then invests in the index asset she earns

$$\chi \equiv (1-\alpha)\,\phi\,(P_1^{\mathrm{ns}} - P_0)^+ + w(1-\alpha)P_1^{\mathrm{ns}} + wK_\alpha + \frac{\alpha P_1^{\mathrm{ns}}}{P_1^{\mathrm{ns}} + K(\alpha)}\,(v_{\mathrm{H}} + K_\alpha)$$

Clearly, she will not exit. Thus, the fund chooses to exit if and only if $a = 1$, and the manager chooses $a = 1$ because $\beta \geq \beta_{\mathrm{L}}$ as in the baseline. Thus, for $\alpha > \alpha'$, if voice is not used or used and ignored, the manager chooses $a = 1$ and the fund exits. If voice is used and accepted, then the manager chooses $a = 0$. The fund's problem is then identical to the case above in which she observes $a = 0$. Thus, she chooses not to exit.

Given this, we now check the strategies in the pre-game. Let's check the manager's strategy first. The fund uses voice and the manager is faced with the option to accept or reject the proposal. If he accepts the fund's proposal he has to choose $a = 0$, the fund does not exit, and he gets $\omega_1(\theta P_1^{\mathrm{s}} + (1-\theta)P_1^{\mathrm{ns}}) + \omega_2 v_{\mathrm{H}} + \rho$. If he ignores voice he then chooses $a = 1$, the fund exits, and he gets $\beta + \omega_1 P_1^{\mathrm{s}} + \omega_2 v_{\mathrm{L}}$. Thus, the manager chooses to accept if and only if

$$\omega_1(\theta P_1^{\mathrm{s}} + (1-\theta)P_1^{\mathrm{ns}}) + \omega_2 v_{\mathrm{H}} + \rho \geq \beta + \omega_1 P_1^{\mathrm{s}} + \omega_2 v_{\mathrm{L}}, \text{ i.e. if } \beta \leq \beta_{\mathrm{L}} + \rho.$$

Let's now check the fund's strategy. If the fund does not use voice, the manager chooses $a = 1$, the fund exits, is fired, and earns αP_1^{s}. If, the fund uses voice, if $\beta \in (\beta_{\mathrm{L}}, \beta_{\mathrm{L}} + \rho]$, the manager accepts and the fund gets at least $\chi - e$, but otherwise the manager rejects and chooses $a = 1$ and the fund gets $\alpha P_1^{\mathrm{s}} - e$. So, the fund loses by using voice when $\beta > \beta_{\mathrm{L}} + \rho$ and gains when $\beta \in (\beta_{\mathrm{L}}, \beta_{\mathrm{L}} \ + \rho]$. Since the losses are on the order of e, and the gains are not, and e can be as small as desired, there exists an e small enough such that the fund uses voice.

Let $\alpha < \widehat{\alpha}$. We shall show that there exists an equilibrium in which the fund does not use voice. We solve the game by backward induction. For this case, since we wish to utilize the existence of an equilibrium without voluntary exit, as before we allow for the existence of an ϵ-measure of non-strategic funds and consider the limiting case in which $\epsilon \to 0$.

Suppose voice is not used or used and ignored. In either case, the exit game begins and the manager's payoffs are identical to the baseline model. It is easy to construct an equilibrium in which the fund never exits. Investors don't know about the voice pre-game, and thus (for $\gamma_{\mathrm{M}}^{\mathrm{G}}$ and R_{G} high enough) will behave identically to the baseline model. Thus, if a fund observes $a = 1$, she faces the same choice as in the baseline model, and does not exit since $\alpha < \widehat{\alpha}$. If the fund observes $a = 0$, what will she do? If the fund exits she is fired, earning αP_1^{s}. If she does not exit, she is retained and if she then invests in the index asset, she earns

$$(1-\alpha)\,\phi\,(P_1^{\mathrm{ns}} - P_0)^+ + w(1-\alpha)P_1^{\mathrm{ns}} + wk_\alpha + \frac{\alpha P_1^{\mathrm{ns}}}{P_1^{\mathrm{ns}} + k_\alpha}\,(v_{\mathrm{H}} + k_\alpha)$$

Clearly, she will not exit. Thus, given that the fund never exits, the manager will choose $a = 1$ whenever $\beta > \beta_{\text{No-L}}$ as in the baseline. Since $\beta > \beta_{\mathrm{L}} > \beta_{\text{No-L}}$, he chooses $a = 1$ and the fund does not exit. If voice is used and accepted, then the manager chooses $a = 0$. The fund's problem is then identical to the case above in which she observes $a = 0$. Thus, she chooses not to exit.

Given this, we now check the strategies in the pre-game. Let's check the manager's strategy first. The fund does not use voice. Thus, the manager has no decision to make in the pre-game on the equilibrium path. Let's now check the fund's strategy. If no voice is used, the manager chooses $a = 1$ and the fund does not exit and earns $\max\left\{\Pi_{\mathrm{ns}}^{\mathrm{I}}, \Pi_{\mathrm{ns}}^{\mathrm{NI}}\right\}$ where $\Pi_{\mathrm{ns}}^{\mathrm{I}}$ and $\Pi_{\mathrm{ns}}^{\mathrm{NI}}$ are defined in the proof of Proposition 4. If the fund uses voice, then the manager has (off equilibrium) the option to accept or reject it. If he accepts, he has to choose $a = 0$ and gets $\omega_1(\theta P_1^{\rm s} + (1-\theta)P_1^{\rm ns}) + \omega_2 v_{\rm H} + \rho$. If he ignores voice, he goes on to choose $a = 1$ and, since the fund does not exit, he earns $\beta + \omega_1(\theta P_1^{\rm s} + (1-\theta)P_1^{\rm ns}) + \omega_2 v_{\rm L}$. He therefore prefers to ignore voice as long as $\rho < \beta - \omega_2 \Delta v$. Since by assumption $\rho < \beta_{\rm L} - \omega_2 \Delta v$, $\rho < \beta - \omega_2 \Delta v$ for all $\beta > \beta_{\rm L}$. Thus, the manager ignores voice and chooses $a = 1$ and then the fund does not exit and receives $\max\left\{\Pi_{\rm ns}^{\rm I}, \Pi_{\rm ns}^{\rm NI}\right\} - e$. Thus the fund does not use voice.■

Appendix B. Numerical Examples

Here we provide details for the computations referred to in Sections V and VI. In Section V we argued that Proposition 2 applies to mutual funds with self-investment levels consistent with the data. In Section VI, we argued that there exist model parameters for which Proposition 5 applied to hedge funds with self-investment levels consistent with the data. Propositions 2 and 5 share sufficient conditions on $R_{\rm G}$ and $\gamma_{\rm M}^{\rm G}$ (given by conditions (A.8), (A.10) and (A.12)). Conditions (A.10) and (A.12) are implied by:

$$R_{\rm G} \ \geq \frac{1}{\widehat{\gamma}_{\rm F}} - \frac{1-\widehat{\gamma}_{\rm F}}{\widehat{\gamma}_{\rm F}} R_{\rm B} + \frac{w(v_{\rm H} + K_\alpha)}{\widehat{\gamma}_{\rm F}(1-\phi)K_\alpha} \tag{B.1}$$

where $\widehat{\gamma}_{\rm F}$ is defined in (A.9), and (A.8) can be expressed as:

$$\gamma_{\rm M}^{\rm G} \geq 1 - \frac{(1-\gamma_{\rm F})(1-\gamma_{\rm M}^{\rm B})(1-R_{\rm B})}{\gamma_{\rm F}(R_{\rm G}-1)}. \tag{B.2}$$

Throughout our examples we set the AUM fee $w = 0.02$ and assume that β is uniformly distributed on [0,1]. We normalize $v_{\rm H} = 1$. In order to take no view on the relative sensitivity of executive compensation to long-term vs short-term stock prices, we set $\omega_1 = \omega_2 = 1/2$. For mutual funds with set $\phi = 0$ while for hedge funds we set $\phi = 0.2$. We choose α as appropriate below. We carry out our computations using Mathematica.

A Mutual Funds

Since $\phi = 0$ for mutual funds, the relevant condition (A.13) determining whether the fund will exit reduces to:

$$\alpha P_1^{\mathrm{s}} \geq w(1-\alpha)v_{\mathrm{H}} + wK_\alpha + \frac{\alpha v_{\mathrm{H}}}{v_{\mathrm{H}} + K_\alpha}(v_{\mathrm{L}} + K_\alpha) \tag{B.3}$$

It is easy to see that, for a given $v_{\mathrm{H}}, v_{\mathrm{L}}, \alpha, w, \phi, \omega_1$ and ω_2 whether inequality (B.3) is satisfied depends on the values of A and θ. To obtain a highly conservative estimate, we set $\alpha = 0.01$. Then inequality (B.3) becomes

$$0.01 P_1^{\mathrm{s}} \geq 0.0198 + 0.02K_\alpha + \frac{0.01}{1 + K_\alpha}(v_{\mathrm{L}} + K_\alpha).$$

Since $P_1^{\mathrm{s}} < 1$ and K_α and v_{L} are both positive this inequality is never satisfied. This means that whenever investors chase performance, funds with $\alpha = 0.01$ will never exit for *any* (θ, A). Given this, it only remains to check the two conditions (B.1) and (B.2) which jointly guarantee performance chasing. Since the bound on R_{G} given by condition (B.1) is a function of $\gamma_{\mathrm{M}}^{\mathrm{G}}$ and, in turn, the bound on $\gamma_{\mathrm{M}}^{\mathrm{G}}$ given by condition (B.2) is a function of R_{G}, we use a guess and verify procedure. First, we guess a $\gamma_{\mathrm{M}}^{\mathrm{G}}$ and find a lower bound on R_{G}. Then, we choose an R_{G} consistent with that bound and substitute it into condition (B.2) and check whether our initial choice of $\gamma_{\mathrm{M}}^{\mathrm{G}}$ satisfies this condition. For example, consider the following two parameterizations, where the interpretation should account for the fact that the required conditions are sufficient and not necessary for our results. For $v_{\mathrm{L}} = 2/3$, $\gamma_{\mathrm{F}} = 0.7$, $\gamma_{\mathrm{M}}^{\mathrm{B}} = 0.4$, $\gamma_{\mathrm{M}}^{\mathrm{G}} = 0.9$, $R_{\mathrm{B}} = 0.9$, and $\theta = 0.4$, (B.1) reduces to $R_{\mathrm{G}} \geq 1.076$ i.e., the required return from the best mutual funds is less than 10%. Substituting $R_{\mathrm{G}} = 1.076$ in (B.2) gives $\gamma_{\mathrm{M}}^{\mathrm{G}} \geq 0.662$, which is clearly satisfied with $\gamma_{\mathrm{M}}^{\mathrm{G}} = 0.9$. Alternatively, for $v_{\mathrm{L}} = 1/3$, $\gamma_{\mathrm{F}} = 0.5$, $\gamma_{\mathrm{M}}^{\mathrm{B}} = 0.6$, $\gamma_{\mathrm{M}}^{\mathrm{G}} = 0.75$, $R_{\mathrm{B}} = 0.9$, and $\theta = 0.5$, (B.1) reduces to $R_{\mathrm{G}} \geq 1.157$. Substituting $R_{\mathrm{G}} = 1.157$ in (B.2) gives $\gamma_{\mathrm{M}}^{\mathrm{G}} \geq 0.745$, which is satisfied by $\gamma_{\mathrm{M}}^{\mathrm{G}} = 0.75$.

B Hedge Funds

As discussed above, for hedge funds we set $\alpha = 0.1$. We consider $v_{\mathrm{L}} \in [0, 1/3, 2/3]$. For hedge funds, our numerical exercise is substantially more challenging. It is easiest to appreciate this by comparison to mutual funds. For mutual funds, since $\phi = 0$, the exit condition (B.3) was fully determined by (θ, A). This meant that we could first check whether a particular (θ, A) pair satisfied the exit condition. We could then check—via the guess a verify procedure described above—whether there existed $\left(\gamma_{\mathrm{F}}, \gamma_{\mathrm{M}}^{\mathrm{G}}, \gamma_{\mathrm{M}}^{\mathrm{B}}, R_{\mathrm{G}}, R_{\mathrm{B}}\right)$ consistent with that (θ, A) that satisfied the interrelated conditions (B.1) and (B.2). When $\phi > 0$, the exit condition (A.21) depends on the full 6-tuple $\left(\theta, \gamma_{\mathrm{F}}, \gamma_{\mathrm{M}}^{\mathrm{G}}, \gamma_{\mathrm{M}}^{\mathrm{B}}, R_{\mathrm{G}}, R_{\mathrm{B}}\right)$ giving rise to a six-dimensional numerical problem, which is further complicated by the fact that for each candidate 6-tuple, we have to simultaneously check (i) whether $\Pi_{\mathrm{ns}}^{\mathrm{NI}} > \Pi_{\mathrm{ns}}^{\mathrm{I}}$ or vice versa; (ii) whether $\max\left(\Pi_{\mathrm{ns}}^{\mathrm{NI}}, \Pi_{\mathrm{ns}}^{\mathrm{I}}\right) < \Pi_{\mathrm{s}}$ or vice versa; and (iii) whether (B.1) and (B.2) are satisfied. For this general case, it is challenging to depict the set of parameters for which hedge funds would exit. In order to provide easily interpretable pictorial depiction of our results, we first consider the case in which $\gamma_{\mathrm{M}}^{\mathrm{G}} \to 1$. This radically simplifies matters because now it is immediate that $\Pi_{\mathrm{ns}}^{\mathrm{NI}} > \Pi_{\mathrm{ns}}^{\mathrm{I}}$ because when $\gamma_{\mathrm{M}}^{\mathrm{G}} \to 1$ the fund that observes $a = 1$ will conclude that she is bad, and thus will not be tempted by the carry to invest in new opportunities at $t = 1$. Since $\Pi_{\mathrm{ns}}^{\mathrm{NI}}$ depends only on (θ, A) the six-dimensional search problem is reduced to a two-dimensional one. In Figure 1 in Section VI, we plot the set of (θ, A) that give rise to exit. For $\gamma_{\mathrm{M}}^{\mathrm{G}} \to 1$ (B.2) is automatically satisfied, and for each such (θ, A), it is easy to see that there exist R_{G} that satisfy (B.1).

However, we do *not* require $\gamma_{\mathrm{M}}^{\mathrm{G}} \to 1$ in order to obtain reasonable parameters for which hedge funds will exit. Below, we present three different examples with $\gamma_{\mathrm{M}}^{\mathrm{G}} < 1$ for which (i) $\max\left(\Pi_{\mathrm{ns}}^{\mathrm{NI}}, \Pi_{\mathrm{ns}}^{\mathrm{I}}\right) < \Pi_{\mathrm{s}}$, so that hedge funds exit, (ii) $\Pi_{\mathrm{ns}}^{\mathrm{NI}} > \Pi_{\mathrm{ns}}^{\mathrm{I}}$ and (iii) (B.1) and (B.2) are satisfied. We fix $R_{\mathrm{B}} = \gamma_{\mathrm{F}} = 0.8$ throughout. The three examples are: (a) $v_{\mathrm{L}} = 0$, $A = 0.2$, $\gamma_{\mathrm{M}}^{\mathrm{G}} = 0.95$, $\gamma_{\mathrm{M}}^{\mathrm{B}} = 0.2$, $\theta = 0.3$, and $R_{\mathrm{G}} = 1.14$; (b) $v_{\mathrm{L}} = 1/3$, $A = 0.1$, $\theta = 0.3$, $\gamma_{\mathrm{M}}^{\mathrm{G}} = 0.97, \gamma_{\mathrm{M}}^{\mathrm{B}} = 0.62$, and $R_{\mathrm{G}} = 1.141$ (c) $v_{\mathrm{L}} = 2/3$, $A = 0.01$, $\theta = 0.2$, $\gamma_{\mathrm{M}}^{\mathrm{G}} = 0.999, \gamma_{\mathrm{M}}^{\mathrm{B}} = 0.954$, and $R_{\mathrm{G}} = 1.214$. Note that since $\Pi_{\mathrm{ns}}^{\mathrm{NI}} > \Pi_{\mathrm{ns}}^{\mathrm{I}}$, these examples are consistent with Figure 1, and in each case the (θ, A) selected belongs to the shaded region in the relevant panel of

Figure 1. Further, note that the requirements on returns generated by the best hedge funds are not particularly demanding: R_{G} ranges between 14% and 21%.

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Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's web site:

Appendix S1: Internet Appendix.

Notes

[1]Institutional money managers hold over 70% of US equity (for example, Gillan and Starks (2007)), and a significant measure of these holdings is quite concentrated. For example, Hawley and Williams (2007) show that, in 2005, the hundred largest US institutions owned 52% of publicly held equity. Gopalan (2008) notes that in 2001 almost 60% of NYSE-listed firms had an institutional blockholder with at least 5% equity ownership.

[2]See, for example, Brown, Harlow, and Starks (1996), Chevalier and Ellison (1997, 1999) for mutual funds, Agarwal, Daniel, and Naik (2009) and Lim, Sensoy, and Weisbach (2013) for hedge funds.

[3]See, for example, Gopalan (2008), Bharat, Jayaraman, and Nagar (2013), and Helwege, Intintoli, and Zhang (2012).

[4]Holderness, Kroszner and Sheehan (1999) estimate that the average equity ownership of officers and directors in the Finance, Insurance, and Real Estate sector was 17.4% in 1995.

[5]Needless to say, there may well be many reasons why mutual funds are not effective users of voice, such as, for example, business ties with portfolio firms (see Davis and Kim (2007)).

[6]See, for example, Grossman and Hart (1980), Shleifer and Vishny (1986), Admati, Pfleiderer, and Zechner (1994), Burkart, Gromb, and Panunzi (1997), Bolton and von Thadden (1998), Tirole (2001), Noe (2002), and Faure-Grimaud and Gromb (2004).

[7]We build on Admati and Pfleiderer's Model B. This is the version of the model in which they show exit to be most effective as a governance mechanism. In other variants of their model, they show that—even when the blockholder is a principal—exit has potentially less desirable effects. We wish to take as a starting point the version of their model that gives exit its best chance as a governance mechanism and still show (see Proposition 2 below) that agency frictions arising from the delegation of portfolio management can reduce its effectiveness.

[8]We vary their model slightly by replacing stochastic agency costs by stochastic private benefits to managers and by introducing a set of firms that are free of a agency problems.

[9]We refer to *funds* and *fund managers* interchangeably throughout. In other words, we do not consider potential incentive conflicts between funds and the managers they employ, focusing only on those between funds and their investors.

[10]Note that V_{t+1} is not necessarily identical to I_{t+1} because, as the hiring and replacement process described above indicates, a fund may experience inflows or outflows at date $t+1$.

[11] Funds with little self-investment may be tempted to invest in new opportunities at $t = 1$, even if the expected return is negative, if they earn a convex carry, since they benefit from the upside (if the ex post return is R_{G}) without suffering from the downside (if the ex post return is R_{B}).

[12] Proposition 2 requires that, fixing $\Theta \backslash \{\gamma_{\mathrm{M}}^{\mathrm{G}}, R_{\mathrm{G}}\}$, $\gamma_{\mathrm{M}}^{\mathrm{G}}$ and R_{G} are large. Since $\gamma_{\mathrm{M}}^{\mathrm{B}}$, $R_{\mathrm{B}} \in \Theta \backslash \{\gamma_{\mathrm{M}}^{\mathrm{G}}, R_{\mathrm{G}}\}$, this implies non-trivial ability differences across funds. This is sufficient but not necessary for performance-chasing. For example, if $\gamma_{\mathrm{M}}^{\mathrm{G}}$ is only slightly greater than $\gamma_{\mathrm{M}}^{\mathrm{B}}$, investors would still prefer funds who have not exited (and thus have generated higher $t = 1$ returns) and may re-allocate funds to better performers, chasing performance. However, since $\gamma_{\mathrm{M}}^{\mathrm{G}}$ is close to $\gamma_{\mathrm{M}}^{\mathrm{B}}$, exit is not very informative about ability. Thus, relative to the baseline, investors would be less keen (i) to fire their current fund if she exits (because now it is no longer clear that the exiting fund generates a return dominated by the index asset) and (ii) to invest in a new fund that has not exited (because exiting and non-exiting funds are not very different). Now, even small transaction costs for switching funds may preclude performance chasing.

[13] Proposition 2 considers the case in which blockholders punish managers *non*-stochastically for choosing $a = 1$. A careful reader may wonder whether there are equilibria in which the fund punishes the manager with *arbitrarily* high probability for choosing $a = 1$. While threats involving mixed strategies are, in our view, of limited applied relevance, we can show that even stochastic punishment fails without sufficient self-investment. A formal result is stated and proved in the Internet Appendix.

[14] It is not difficult to obtain a back of the envelope estimate of an upper bound on α implied by these figures. According to the ICI 2013 Factbook there were 4,544 equity mutual funds with total assets of $5215.26 billion, giving an average fund size of around $1.1 billion. Thus, the Morningstar numbers suggest that for 88% of equity mutual funds $\alpha < 0.0009$.

[15] Since we model competition for investor flow as a reputational cheap talk game, it is possible to sustain many possible equilibria with arbitrarily chosen off-equilibrium beliefs. A full characterization of the equilibrium set—even if it were feasible—would represent a digression and distract from the applied points we wish to make. Instead, in Propositions 4 and 5 we characterize equilibria with the minimal and maximum achievable amount of voluntary exit.

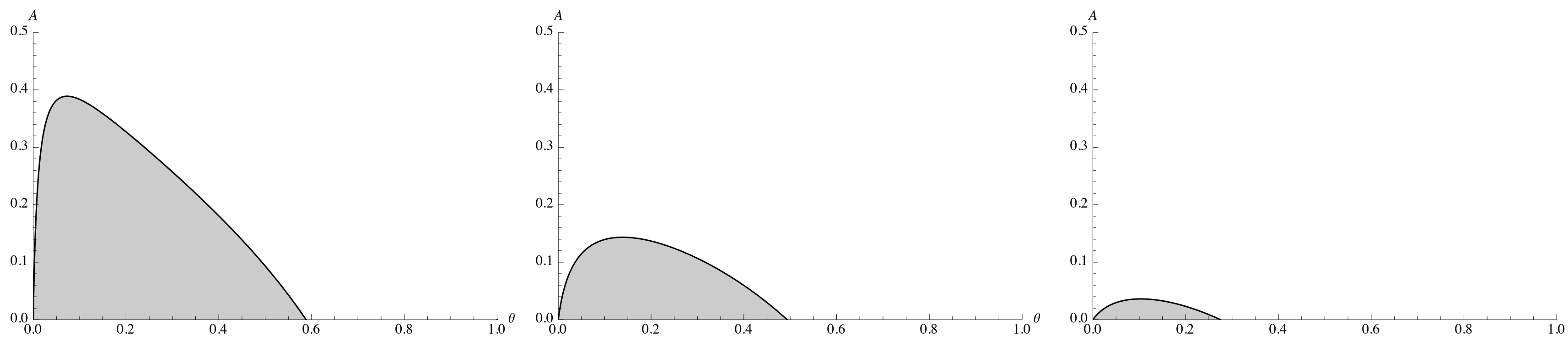
[16] From a theory standpoint, it would have been preferable to dispense with non-strategic funds and characterize the infimum of mixed equilibria with decreasing probabilities of exit. Unfortunately, it is challenging to characterize mixed equilibria in our model. The key reason is that a higher exit probability has an ambiguous effect on prices: It lowers exit prices for informational reasons (exit is more likely to be due to $a = 1$) but raises them for disciplinary reasons (higher firm value with more exit) introducing a non-monotonicity into the model.

[17]The non-monotonicity in θ can be understood as follows: When $\theta \to 0$, the exit price at $t = 1$ becomes fully revealing, weakening the incentives to exit.

[18]Our formal model of voice can be re-interpreted as one in which the use of voice results in a decrease in the manager's private benefits from choosing $a = 1$.

[19]In particular, large enough to jointly satisfy the conditions for Proposition 4 (for $\epsilon \to 0$) and Proposition 5.

[20]Since any individual investor is infinitesimal, we assume that individual withdrawals generate no price impact at the level of the fund.



(a) $v_{\mathrm{H}} = 1, v_{\mathrm{L}} = 0$ **(b)** $v_{\mathrm{H}} = 1, v_{\mathrm{L}} = 1/3$ **(c)** $v_{\mathrm{H}} = 1, v_{\mathrm{L}} = 2/3$

Figure 1: Parameters for which hedge funds use exit.
Panels (a), (b), and (c) depict (θ, A) constellations for which hedge funds use exit for different values of $(v_{\mathrm{H}}, v_{\mathrm{L}})$.