Polarization, Purpose and Profit

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Keywords: Labor Markets, Job Design, Compensating Differentials, Socially Responsible Investment, Polarization

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Abstract

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1 Introduction

Many workers want their jobs to have a higher purpose (e.g., "changing the world," "saving the planet," "helping people," "promoting diversity and equality," etc). Some also care about what the job does and how it is done (e.g., how sustainable their jobs are, how socially responsible the company is, etc.). Purpose, sustainability, social responsibility, and working conditions in general (e.g., flexible working arrangements, health and safety, etc.) are all examples of nonpecuniary job attributes that may be valuable to workers.

An empirical literature shows that workers are willing to pay for desirable job attributes. Sorkin (2018) shows that *compensating differentials* (i.e., wage premiums or discounts that compensate workers for negative or positive nonpecuniary job attributes) account for two-thirds of the firm component of the variance of earnings.[1] Some of these desirable attributes are a consequence of firms' social and environmental decisions or, more generally, their social responsibility stances. Krueger, Metzger, and Wu (2023) find that workers earn nine percent lower wages in firms that operate in more sustainable sectors. In a field experiment, Colonnelli et al. (2023) find that job applicants value ESG characteristics at about ten percent of average wages, which is more than what applicants value most other nonwage amenities.[2] There is also significant heterogeneity in workers' preferences for nonpecuniary job attributes. In a review article, Cassar and Meier (2018) conclude that "*not everyone cares about having a meaningful job (...) heterogeneity in preferences for meaning is substantial.*"[3]

We present a model in which firms choose the characteristics of their jobs. Firms com-

[1]Further evidence of compensating differentials can be found in Stern (2004), Mas and Pallais (2017), Focke, Maug, and Niessen-Ruenzi (2017), Wiswall and Zafar (2018), Sockin (2022), and Ouimet and Tate (2022), among others.

[2]Hedblom, Hickman, and List (2019) find that advertising as a CSR firm increases job application rates by 24%. Similarly, Cen, Qiu, and Wang (2022) find that CSR investments improve employee retention.

[3]Krueger et al. (2023) find that about half of survey participants are willing to accept a wage cut to work for a more environmentally sustainable firm. Colonnelli et al. (2023) document that job applicants' ESG preferences vary with education, ethnic background, and political leanings. Hedblom et al. (2019) find that heterogeneous preferences for CSR cause workers to vary by their propensity to select different jobs.

pete for workers who have a taste for a nonpecuniary job attribute. We call this attribute *s-quality*. *S*-quality may refer to job purpose or meaning, job sustainability, the ES/CSR attributes of the job, working conditions, or any other positive job attribute that has the following two features. First, while all workers prefer high *s*-quality jobs, workers vary in their willingness to pay for such jobs. Second, some investors (e.g., socially responsible investors) may also have a preference for investing in high *s*-quality firms.

Our model is based on Rosen's (1986) "equalizing differences" framework. The model has no frictions: competition is perfect, information is symmetric, capital is plentiful, risk sharing is perfect, and there are no agency problems, incentive issues, or financial constraints. We make these assumptions not for realism but, instead, to show that the results are theoretically robust. Thus, the model can be used as a benchmark to assess whether frictions are needed to explain existing or future evidence. Similar to models of the assignment of heterogeneous workers to firms, jobs, or tasks (see, e.g., Tinbergen (1956), Sattinger (1993), and Garicano and Rossi-Hansberg (2006)), our model considers the efficient allocation of workers to (endogenously) different firms. Similar to models of sustainable investment in which investors have preferences for some nonpecuniary characteristics of their portfolio firms (see, e.g., Heinkel, Kraus, and Zechner (2001), Pástor, Stambaugh, and Taylor (2021), and Pedersen, Fitzgibbons, and Pomorski (2021)), our model also considers the efficient allocation of heterogeneous investors to firms. Thus, our model integrates firms' real and financial sides in a simple competitive assignment framework.

The model is as follows. Some entrepreneurs develop or acquire technologies that allow them to design jobs of varying *s*-quality levels. We call the firms that use such technologies *flexible firms*. Other entrepreneurs own *inflexible firms*, which are firms that cannot change the *s*-quality of their jobs. Firms compete for workers by offering contracts specifying a wage and an *s*-quality level. While all flexible firms are initially identical, they can differentiate themselves by adopting technologies associated with different *s*-quality levels. A high-quality job is expensive for the firm. For example, if workers prefer jobs that are environmentally sustainable, the firm may choose to adopt low-emission technologies even when they are not cost-efficient.

Technological flexibility allows a firm to tailor its job characteristics to the preferences of its workers. Thus, flexibility is a lever that a firm can use to increase the surplus from a match. Naturally, this lever will have the most impact for workers with the most extreme types. For example, while some workers are willing to accept significant wage cuts to work for more sustainable firms, others may be willing to accept jobs in low-sustainability firms in exchange for high wages. Thus, flexible firms can create more value by designing jobs for workers with extreme preferences. Our main result is that, in equilibrium, flexible firms become polarized: they cater to workers with extreme preferences. That is, flexible firms end up hiring workers with either strong or weak preferences for s-quality. By contrast, inflexible firms have no choice but to hire workers with moderate preferences.

While increasing s-quality is costly, compensating differentials imply that wages fall with s-quality. Thus, profit and purpose do not always conflict. We show that technological flexibility implies that a firm's profit potential is U-shaped in s-quality. Thus, firms with very high or very low s-quality levels have higher value-added (i.e., profit plus wages).[4]

To consider the determinants of polarization, we solve a parameterized version of the model. We show that firms are more polarized when they become more efficient at producing s-quality. Polarization also increases with labor market concentration and with the dispersion in worker preferences for s-quality. More polarized sectors have higher profits, lower average wages, and a lower labor share of value added.

After modeling the labor market, we introduce financial markets. Entrepreneurs (i.e., those who initially own firms) can choose to sell shares of their firms to outside investors. There are two types of investors: profit-driven investors and socially responsible investors. Profit-driven investors care only about the financial return on their shares. Socially re-

[4]Most papers in the sustainable investment literature assume that CSR/ESG qualities come at the expense of firm cash flows. A notable exception is Pedersen, Fitzgibbons, and Pomorski (2021), who argue that ESG may be positively related to firm profits. Similarly, Edmans (2011) argues that employee satisfaction may be positively associated with long-run cash flows.

sponsible investors are willing to sacrifice some financial gains to invest in companies with high s-quality levels. Socially responsible investors may care about job quality directly because they prefer to invest in companies offering better job conditions. They may also care about job quality indirectly if they share some of their employees' values, such as a concern for sustainability or environmental responsibility.

In equilibrium, profit-driven investors buy shares in firms where workers have either very strong or very weak preferences for s-quality, while socially responsible investors invest in companies where workers have moderate preferences for s-quality. At first glance, this result is counterintuitive. Why wouldn't a socially responsible investor buy shares in companies where workers strongly support social responsibility? The reason is that firms where workers have extreme preferences for s-quality have more *profit potential* than firms where workers are more moderate. This profit potential attracts profit-driven investors, who have a comparative advantage in investing in high-profit companies. These investors chase returns and, ultimately, earn zero abnormal returns due to competition. They crowd out socially responsible investors, who have a comparative advantage in investing in low-profit firms.

The model delivers several empirical predictions. The model predicts that firms should be on average more valuable in sectors with high cross-sectional polarization in s-quality levels (e.g., ESG, sustainability, or similar scores). Such sectors should also display high (expected) stock returns. More polarized sectors should also have lower average wages and labor shares. Within a sector, all else constant, wages decrease with s-quality. Thus, polarization in s-quality should be positively related to wage polarization.

The model also generates cross-section relationships between employee satisfaction, firm value, and stock returns. While the link between employee satisfaction and stock returns does not need to be monotonic, the model implies that firms with the highest levels of employee satisfaction also deliver the highest returns. Similarly, firms with the lowest levels of employee satisfaction have the lowest returns. Edmans (2011) shows evidence that employee satisfaction is positively related to stock returns. His explanation is that the market does not fully recognize the value of intangibles. Our model provides an alternative explanation that does not require any friction or mispricing. This is not to say that frictions cannot explain some (or even all) of the evidence. Rather, the model illustrates that a link between employee satisfaction and stock returns can arise even if there are no frictions. Edmans, Pu, Zhang, and Li (2023) show that the positive link between employee satisfaction and stock returns is stronger in countries with flexible labor markets. This finding is also consistent with our model of competition in a frictionless labor market.

Our model predicts firm polarization as an equilibrium outcome. Polarization may occur for any characteristic that employees value. An emerging empirical literature study firm polarization in social and political stances. Di Giuli and Kostovetsky (2014) find an association between stakeholders' political views and firms' CSR policies. Conway and Boxell (2023) show that firms' public stances on controversial social issues align with the preferences of their consumers and employees. Giannetti and Wang (2023) show that heterogeneity in corporate cultures explains differences in corporate reactions to heightened public attention to gender equality. Colonnelli, Pinho Neto, and Teso (2022), Fos, Kempf, and Tsoutsoura (2023), and Duchin et al. (2023) analyze some of the economic consequences of firm political polarization.

In Section 2, we present our model of the labor market and derive our main findings. In Section 3, we present additional empirical predictions after solving a parameterized version of the model. Section 4 introduces outside investors. Section 5 endogenizes the number of firms by allowing for entry. We offer a brief review of the related theoretical literature in Section 6. Section 7 concludes. All proofs not in the text are in the Appendix. The Internet Appendix presents several extensions and generalizations.

2 Model

2.1 Technology and Preferences

We consider an economy with a continuum of agents of two types: *entrepreneurs* and *workers*, with masses F and L, respectively. Each entrepreneur owns one firm. There are two types of firms, $\iota \in \{0,1\}$. We call such types *sectors*. Each sector has a continuum of mass F_ι of firms. Thus, F_0 and F_1 are also the masses of entrepreneurs in each sector. Entrepreneurs are pure profit-maximizers. Each firm can hire one worker. If a firm of type ι employs one worker, it generates revenue $y_\iota > 0$. A firm can choose its s-quality level, $s \in [\underline{s}_\iota, \bar{s}_\iota]$, which we also call the *s-attribute*, at cost $c_\iota(s)$ to the firm. We can interpret s as the choice of a technology that generates $y_\iota - c_\iota(s)$ as earnings before wages. We assume that $c'_\iota > 0, c''_\iota > 0$, and $c_\iota(0) = c'_\iota(0) = 0$, the latter being an Inada condition to avoid corner solutions. A firm's profit is thus $\pi_\iota(s,w) = y_\iota - c_\iota(s) - w$, where w is the wage per worker. An entrepreneur always has the option of shutting down her firm and receiving zero profit. For simplicity, we impose no constraints on w; the qualitative results are unchanged if w is constrained to be non-negative (alternatively, we can interpret our analysis as the case in which non-negative wage constraints do not bind).

From now on we set $y_0 = y_1 =: y$ and $c_0(s) = c_1(s) =: c(s)$, unless explicitly noted otherwise. This assumption is inconsequential for our core results, but it simplifies the notation and helps with the intuition by eliminating most of the heterogeneity across sectors. The only remaining difference between the two sectors is the flexibility of their production technologies. We assume that Sector 1 is more flexible than Sector 0: $[\underline{s}_0, \bar{s}_0] \subset (\underline{s}_1, \bar{s}_1)$. To simplify the analysis, for the remainder of the paper, we assume that Sector 0 is completely inflexible: $\underline{s}_0 = \bar{s}_0 =: s_0$, while Sector 1 is perfectly flexible, that is, $\underline{s}_1 = 0$ and $\bar{s}_1 = \infty$. Thus, we refer to Sector 1 as the *flexible sector* and Sector 0 as the *inflexible sector*. Our interpretation is that each s indexes a production technology, with technologies with higher s delivering lower earnings before wages, $y - c(s)$. While Sector 1 firms can choose any technology $s \geq 0$ they want, Sector 0 firms are stuck with technology s_0. To make sure that inflexible firms always prefer to operate, we assume that $y \geq c(s_0)$.

Labor supply in the economy is inelastic. To keep the analysis general, we consider the number of firms as exogenous for most of the paper (except Section 5). We make the following parametric assumption:

Assumption 1. $F_1 \leq L < F_0 + F_1$.

That is, we assume that workers are in short supply relative to the overall number of jobs in the economy. We focus our discussion on the more interesting case, in which there is an active inflexible sector (i.e., $F_1 < L$). However, our results also hold for $F_1 = L$. In Section 5, we endogenize the number of firms by allowing for costly entry into either sector.[5]

A firm may offer contract (s, w) to a worker of type $i \in \{1, ..., n\}$. A type-i worker has a quasi-concave utility $u^i(s, w)$ over wages and the s-attribute. For simplicity of exposition, we assume $u^i(s, w) = \varepsilon_i + \alpha_i s + (1 - \alpha_i)w$, where $\alpha_i \in (0, 1)$ measures the worker's relative taste for the s-attribute and $\varepsilon_i > 0$ is an exogenous (monetary) endowment. To save on notation, we let the utility function "absorb" the endowment parameter, which is equivalent to setting $\varepsilon_i = 0$. We normalize the "unemployment contract" to $(0, 0)$, thus workers of any type have zero utility when unemployed.

The linearity of preferences simplifies the analysis but is not necessary for the results. In the Internet Appendix, we show that our results hold for (quasi-concave) utility functions of the form $u^i(s, w) = f(g_1(\alpha_i)h_1(s, w) + g_2(1 - \alpha_i)h_2(s, w))$, provided some conditions on the curvature of $g_1(.)$ and $g_2(.)$ hold. This family of functions includes most of the commonly used utility functions, such as Cobb-Douglas, CES, quasi-linear utilities, and many others.

Workers are heterogeneous in their preferences for the s-attribute. There are $n \geq 3$ types of workers, with $\alpha \in \{\alpha_1, ..., \alpha_n\}$, with $\alpha_i < \alpha_{i+1}$. Let p_i denote the proportion of type i in the population. That is, $p_i L$ is the mass of workers of type i. We initially work

[5] In the Internet Appendix, we also consider the alternative case in which workers are in excess supply: $L \geq F_0 + F_1$.

with a finite number of types to keep the equilibrium conditions simple and intuitive. Later, we extend the analysis to a continuum of worker types. Although we assume that all workers are equally productive, a natural extension—not pursued here—is to consider different correlation structures between α and worker productivity.[6]

2.2 Benchmark: Efficient Contracts

In this subsection, we characterize the set of efficient contracts between a worker and a flexible firm. Such contracts serve as a benchmark for assessing the efficiency properties of the equilibrium contracts that we will describe in the next subsection.

Suppose a flexible firm matches with a worker of type i. Suppose the firm (i.e., the entrepreneur) offers contract (s, w) to the worker. Let $\pi(s, w) = y - w - c(s)$ denote the firm's profit under this contract. If the worker accepts to work for the flexible firm, her utility is $u^i(s, w) = \alpha_i s + (1 - \alpha_i)w$. We use $\underline{u} \geq 0$ to denote the worker's outside utility if she does not accept the contract (she either works for another firm or stays unemployed). Similarly, let $\underline{\pi}$ denote the firm's outside profit (the firm either hires another worker or shuts down).

To characterize the efficient contract set, we solve the following Nash bargaining problem:

$$\begin{aligned} &\max_{s,w} \omega \left[f(u^i(s, w)) - f(\underline{u}) \right] + (1 - \omega)(\pi(s, w) - \underline{\pi}) \\ &\text{s.t. } u^i(s, w) \geq \underline{u} \text{ and } \pi(s, w) \geq \underline{\pi} \end{aligned} \tag{1}$$

where $\omega \in [0, 1]$ and $f(.)$ is some strictly increasing and strictly concave function. Any Pareto-efficient contract (s, w) is a solution to (1) for some ω. Thus, changing ω allows us to trace the Pareto set of all efficient contracts.[7] The first-order conditions for solving (1)

[6]For example, Colonnelli et al. (2023) find that workers with stronger preferences for ESG tend also to be more qualified.

[7]The reason for using a concave transformation of $u^i(s, w)$ is to allow for interior solutions. If we don't transform $u^i(s, w)$, for any given ω, w will adjust to make at least one constraint bind and the solution to (1) would not trace the whole Pareto frontier as we change ω.

imply:

$$\frac{\alpha_i}{1-\alpha_i} = c'(s_i^*). \tag{2}$$

The left-hand side of (2) is the worker's marginal rate of substitution between s and w. In an efficient allocation, this rate must equal the marginal cost of producing s, which is the right-hand side of (2). Thus, the efficient quantity of s is at a tangency between a given indifference curve and some isoprofit, and is unique for a given worker type: $s_i^* = h(\alpha_i) := c'^{-1}\left(\frac{\alpha_i}{1-\alpha_i}\right)$. The uniqueness of s_i^* results from two properties of technology and preferences: (i) the profit function is quasi-linear and (ii) the worker's utility is linear. While this uniqueness is a convenient feature, it does not drive our main results. In the Internet Appendix, we show how to solve the model with preferences that do not imply a unique s for each α_i.

A flexible firm can always offer the same contract as that of an inflexible firm, but the converse is not true. Thus, because $F_1 < L$, it is socially optimal for all flexible firms to operate and produce s_i^*. Pareto-efficiency alone does not impose further conditions. Therefore, there are multiple efficient allocations. In general, an allocation is efficient if and only if (i) all workers are employed, (ii) all flexible firms employ workers, and (iii) a flexible firm that employs a type-i worker offers s_i^*.

In the Nash bargaining problem in (1), suppose that the firm has all the bargaining power and the worker's outside option is to work for an inflexible firm under contract (w_0, s_0). That is, assume that $\omega = 0$ and $\underline{u} = \alpha_i s_0 + (1-\alpha_i)w_0 \geq 0$. Then, the Nash bargaining problem reduces to

$$v(\alpha_i) := \max_{s,w} \pi(s,w) \quad \text{s.t. } u^i(s,w) \geq u^i(w_0, s_0). \tag{3}$$

The value function $v(\alpha_i)$ is the maximum profit a flexible firm could extract from a worker of type α_i whose outside option is to work for an inflexible firm. We thus call $v(\alpha_i)$ the *profit potential*. The profit potential is the actual profit that a monopolist firm would enjoy if matched with a worker of type α_i. We then have the following result:

Proposition 1 (Profit Potential). *The profit potential $v(\alpha_i)$ is strictly U-shaped in α_i.*

This result is economically meaningful. It implies that flexible firms create more surplus when they match with workers with extreme preferences. To understand the intuition, note that the flexible technology is a real option: it allows firms to create value by adapting to the preferences of their workers. The value of the option increases with the distance between the default position (i.e., the inflexible-sector contract) and the flexible contract. Workers with intermediate preferences for the s-attribute have the lowest surplus because the flexible sector cannot improve much upon the inflexible contract. The extreme types, on the other hand, value flexibility more. Thus, the flexible sector creates more value by catering to the preferences of the extreme types.

The shape of the profit potential function is the main force behind our results. Because s_i^* is increasing in α_i, Proposition 1 implies that the profit potential is also U-shaped in "purpose," i.e., s_i^*. Intuitively, by offering jobs with higher s-quality, the firm pays higher direct costs but can also pay lower wages. We observe a U-shaped pattern because the firm can create (and thus extract) more surplus when matched with workers with extreme preferences. We note that this result is robust to different assumptions on preferences and technology. In particular, preferences do not need to be linear in (s, w). As we elaborate in the Internet Appendix, under some conditions on how α_i affects utility, any quasi-concave utility over (s, w) implies that $v(\alpha_i)$ is U-shaped. For some non-linear preferences, we also do not need $c(s)$ to be strictly convex.

2.3 Competitive Equilibrium

We now consider a competitive equilibrium involving all firms and workers. We can think of the model as a location game in which each contract (s, w) on the plane $\Re^+ \times \Re$ is a feasible location. In a competitive equilibrium, a Walrasian auctioneer chooses a set $\Gamma \subseteq \Re^+ \times \Re$. Then each firm chooses a location in Γ that maximizes its profit. Note that flexible firms can locate anywhere in Γ, but inflexible firms can only locate in points

$(s_0, w) \in \Gamma$. Workers also choose their location (i.e., they apply for a job) by maximizing their utility over the set of contracts in Γ. For an allocation to be an equilibrium, labor demand in each location must equal labor supply.

Consider an equilibrium in which a worker of type i chooses contract (s, w). If $s \neq s_i^*$ (as given by (2)), the worker and the firm could renegotiate the contract so that both are better off. Thus, in equilibrium, if a worker of type i chooses to locate at (s, w), where a flexible firm is also located, then we must have $s = s_i^*$. In addition, all agents of type i working for flexible firms must have the same w_i.[8] We conclude that there are at most n flexible-sector contracts (s_i^*, w_i) that could be accepted by some workers in equilibrium, where (s_i^*, w_i) is the contract *intended* for worker $i \in \{1, ..., n\}$. In the inflexible sector, in equilibrium, all firms must choose the same location, (s_0, w_0), because for a given s_0 the profit is strictly decreasing in w_0. Thus, without loss of generality, we can write Γ as $\{(s_0, w_0), (s_1^*, w_1), ..., (s_n^*, w_n)\}$. That is, Γ is a set of $n + 1$ contracts, one for each s-quality in $(s_0, s_1^*, ..., s_n^*)$.

Because the vector $(s_0, s_1^*, ..., s_n^*)$ is fixed, choosing Γ is equivalent to choosing a wage vector, $\mathbf{w} = (w_0, ..., w_n)$, where w_0 is the wage in the inflexible sector and $(w_1, ..., w_n)$ are the wages intended for each s_i^* location in the flexible sector. When referring to such wages, we call each index $j \in \{0, 1, ..., n\}$ a *market*. Let

$$A(\mathbf{w}) := \arg \max_{j \in \{0,1,...,n\}} \pi(s_j^*, w_j) \text{ subject to } \pi(s_j^*, w_j) \geq 0. \tag{4}$$

That is, $A(\mathbf{w})$ is the set of indices $j \in \{0, 1, ..., n\}$ representing the markets that offer the highest (non-negative) profit to firms given the vector of wages $\mathbf{w}$. We can then define the

[8] Suppose there are two locations, (s_i, w_i) and (s_i, w_i'), with $w_i' < w_i$. Then all firms that demand a worker of type i would would choose location (s_i, w_i') and no worker would be employed at (s_i, w_i).

flexible firms' *labor demand correspondence* for market j as

$$D_j(\mathbf{w}) := \begin{cases} F_1 & \text{if } \{j\} = A(\mathbf{w}) \\ 0 & \text{if } \{j\} \not\subseteq A(\mathbf{w}) \\ [0, F_1] & \text{if } \{j\} \subset A(\mathbf{w}), \end{cases} \tag{5}$$

where $\subset$ denotes a proper subset. That is, if only market j offers the highest profit, all F_1 firms will demand workers of type j. If market j is not a profit-maximizing market, labor demand in that market will be zero. If there are multiple markets with the same maximal profit, firms will be indifferent among these markets. Similarly, define the labor demand correspondence for inflexible firms as

$$D_0(w_0) := \begin{cases} F_0 & \text{if } \pi(s_0, w_0) > 0 \\ 0 & \text{if } \pi(s_0, w_0) < 0 \\ [0, F_0] & \text{if } \pi(s_0, w_0) = 0. \end{cases} \tag{6}$$

Note that $D_0(w_0)$ denotes the inflexible firms' labor demand, while $D_0(\mathbf{w})$ is the flexible firms' demand in market $j = 0$. That is, flexible firms can also offer the inflexible firms' contract if they wish.

Define

$$B_i(\mathbf{w}) := \arg \max_{j \in \{0,1,...,n\}} u^i(s_j^*, w_j). \tag{7}$$

That is, $B_i(\mathbf{w})$ is the set of indices $j \in \{0, 1, ..., n\}$ representing the contracts that offer the highest utility to workers of type i. We can then define worker i's *labor supply correspondence* for market j as

$$S_{ij}(\mathbf{w}) := \begin{cases} p_i L & \text{if } \{j\} = B_i(\mathbf{w}) \\ 0 & \text{if } \{j\} \not\subseteq B_i(\mathbf{w}) \\ [0, p_i L] & \text{if } \{j\} \subset B_i(\mathbf{w}). \end{cases} \tag{8}$$

An equilibrium is characterized by a set of vectors of wages and quantities $(\mathbf{w}^*, \mathbf{x}_1^*, ...,$

$\mathbf{x}_n^*$), where $\mathbf{x}_i = (x_{i0}, x_{i1}..., x_{in})$ are non-negative values. Quantity x_{i0} is the mass of workers of type i employed in the inflexible sector, and $(x_{i1}..., x_{in})$ are the masses of workers of type i employed in each flexible market $j = 1, ..., n$.

Definition 1. A *competitive equilibrium* is defined by the following supply and demand conditions:

(i) The mass of workers employed in each market must belong to the demand correspondence for that market:

$$\sum_{i=1}^{n} x_{i0}^* \in D_0(w_0^*) \cup D_0(\mathbf{w}^*),$$

and

$$\sum_{i=1}^{n} x_{ij}^* \in D_j(\mathbf{w}^*) \text{ for all } j \in \{1, ..., n\}.$$

(ii) The mass of workers of type i employed in each market must belong to i's supply correspondence for that market:

$$x_{ij}^* \in S_{ij}(\mathbf{w}^*) \text{ for all } i \in \{1, ..., n\} \text{ and } j \in \{0, ..., n\}.$$

(iii) Workers need to work in one of the two sectors (i.e., labor supply is inelastic):

$$\sum_{j=0}^{n} x_{ij}^* = p_i L, \text{ for all } i \in \{1, ..., n\}.$$

(iv) Total employment must not be greater than aggregate labor demand:

$$\sum_{j=1}^{n} \sum_{i=1}^{n} x_{ij}^* \leq F_1 \text{ and } \sum_{i=1}^{n} x_{i0}^* + \sum_{j=1}^{n} \sum_{i=1}^{n} x_{ij}^* \leq F_0 + F_1.$$

Before discussing the characteristics of the equilibrium, we introduce some concepts and notation. In equilibrium, only workers of type j may accept contract $j \geq 1$ (we can always define contract j so that this is true). To simplify notation, we denote the equilibrium employment in market j by x_j^*. If $x_j^* > 0$, we say that market j is *active* in equilibrium.

Also, define α_k such that $s_0 = h(\alpha_k)$. That is, k is the type for which the inflexible job quality level s_0 is optimal. Without loss of generality, we assume that $p_k \in (0, \epsilon)$, i.e., there is a positive but arbitrarily small ($\epsilon \to 0$) mass of workers of type k.

The next lemma is a consequence of profit equalization in competitive markets:

Lemma 1 (Profit Equalization). *In equilibrium, firms in the inflexible sector have zero profit (i.e., $\pi(s_0, w_0^*) = 0$) and firms in the flexible sector have strictly positive profit, $\pi(s_j^*, w_j^*) = \pi^* > 0$ for all $j \in \{1, ..., n\}$ such that $x_j^* > 0$.*

Lemma 1 implies that profits are the same across all active markets in the flexible sector. That is, in the cross-section of flexible firms, there is no relation between profit and the s-attribute. Lemma 1 also implies that $w_0^* = y - c(s_0)$.

Note that the profit potential is $v(\alpha_i) = \pi\left(s_i^*, \frac{\alpha_i s_0 + (1-\alpha_i) w_0^* - \alpha_i s_i^*}{1-\alpha_i}\right)$. Proposition 1 implies that $v(\alpha_i)$ is strictly U-shaped. Because $v'(\alpha_i) = (1-\alpha_i)^{-2}(s_i^* - s_0)$ (see equation (A.4) in the Appendix), the profit potential reaches its minimum value at α_k. Thus, for each $j \in \{1, ..., k\}$, there exists at most one $j' > k$ such that $v(\alpha_j) = v(\alpha_{j'})$. For expositional simplicity, we make the following assumption:

Assumption 2. *There is no pair $(j, j') \in \{1, ..., n\}^2$ for which $v(\alpha_j) = v(\alpha_{j'})$.*

This assumption allows us to rule out measure-zero cases in which the equilibrium may not be unique, but otherwise, it is not important for the results.[9] We prove the existence of a unique equilibrium in the next proposition.

Proposition 2 (Existence and Uniqueness). *A competitive equilibrium exists. Under Assumptions 1 and 2, there exists a unique type $z \in \{1, ..., n\}$ such that the equilibrium quantities are*

[9] Note that Assumption 2 is violated if the distribution of worker types is continuous. However, in that case, the equilibrium is always unique and Assumption 2 is irrelevant.

$x_0^* = L - \sum_{j=1}^{n} x_j^*$ and

$$x_j^* = \begin{cases} p_j L & \text{if } v(\alpha_j) > v(\alpha_z) \\ 0 & \text{if } v(\alpha_j) < v(\alpha_z) \\ F_1 - \sum_{j \in \{j \neq z : x_j^* > 0\}} p_j L & \text{if } j = z \end{cases} \tag{9}$$

for $j \in \{1, ..., n\}$. The equilibrium wages are $w_0^* = y - c(s_0)$ and

$$w_j^* = \begin{cases} y - c(s_j^*) - v(\alpha_z) & \text{if } x_j^* > 0 \\ w \in \left[y - c(s_j^*) - v(\alpha_z), \frac{\alpha_j s_0 + (1-\alpha_j) w_0^* - \alpha_j s_j^*}{1-\alpha_j} \right] & \text{if } x_j^* = 0 \end{cases} \tag{10}$$

for $j \in \{1, ..., n\}$.

Uniqueness here means unique quantities in each market. Wages are also unique in all active markets (i.e., where $x_j^* > 0$).[10] Proposition 2 shows that the Walrasian auctioneer chooses a wage vector that (i) equalizes profits in all active flexible markets and (ii) maximizes the *profit potential* of flexible firms. Because the profit potential is U-shaped (see Proposition 1), requirement (ii) implies our main result:

Corollary 1 (Polarization). *The equilibrium is polarized: flexible firms cater to the most extreme preferences. Formally, if $j \leq k$, $x_j^* > 0$ implies $x_{j'}^* = p_{j'} L$ for all $j' < j$. If $j \geq k$, $x_j^* > 0$ implies $x_{j'}^* = p_{j'} L$ for all $j' > j$.*

This corollary implies that there is no equilibrium where employees with moderate preferences (i.e., a convex subset of $\{1, ..., n\}$ that includes the "central" type k) are employed in the flexible sector. Because flexible firms cater to those with extreme preferences, in equilibrium, flexible firms are polarized. That is, flexible firms are more extreme than the underlying worker preferences for the s-attribute. This result is important because

[10] To simplify the exposition, we ignore the knife-edge case in which $F_1 = \sum_{j \in \{j : x_j^* > 0\}} p_j L$. In this case, w_z^* may be anywhere in $\left[\frac{\alpha_j s_0 + (1-\alpha_j) w_0^* - \alpha_j s_j^*}{1-\alpha_j}, y - c(s_j^*) - \widehat{v} \right]$, where $\widehat{v} = \max_{j \in \{j : v(\alpha_j) < v(\alpha_z)\}} v(\alpha_j)$.

Lemma 1 implies that flexible firms are also the most profitable (and thus more valuable) firms. Thus, empirically, Corollary 1 implies that firms in the most valuable and profitable sectors will display more polarization in s-quality levels.

The next result confirms that wages fall with the s-attribute:

Corollary 2 (Compensating Differentials). *The equilibrium displays compensating differentials: for $j' > j$, if $x_j^* > 0$ and $x_{j'}^* > 0$, then $s_j^* < s_{j'}^*$ and $w_j^* > w_{j'}^*$.*

That is, in the cross-section, firms with higher levels of the s-attribute offer lower wages to their employees.

Let u^{i*} denote a type-i worker's utility in equilibrium and u_0^i denote her utility if she chose instead to work in the inflexible sector. Let $U_i^* := u^{i*} - u_0^i$ denote the equilibrium surplus enjoyed by a type-i worker. The next corollary summarizes the equilibrium welfare implications for workers:

Corollary 3 (Workers' Surplus Inequality). *In the flexible sector, workers with extreme preferences have higher surpluses: There exists $\hat{\alpha}$ such that if $\alpha_i < \hat{\alpha}$, $U_{i-1}^* \geq U_i^*$, and if $\alpha_i > \hat{\alpha}$, $U_{i+1}^* \geq U_i^*$.*

That is, in equilibrium, workers with extreme preferences benefit more from working in the flexible sector than workers with more moderate preferences toward the s-attribute. Workers in jobs with more surplus have a higher willingness to pay to keep their jobs. Thus, Corollary 3 implies that employee satisfaction is higher in firms with extreme levels of the s-attribute.

Proposition 2 implies the existence of two groups of flexible firms in equilibrium: high-s firms (firms in which $s_j^* > s_0$) and low-s firms (firms in which $s_j^* < s_0$). We define the *degree of firm polarization* as $\rho = s^h - s^l$, where s^h is the minimum s among high-s firms and s^l is the maximum s among low-s firms. The degree of firm polarization is a potentially observable equilibrium outcome. Thus, we use it as one of the outcome variables in our comparative statics exercises.

It is often more convenient to perform comparative statics in the limiting case in which there is a continuum of types distributed according to $P(.)$, with density $p(.)$. In this case, there are no differences between types and indices, thus, we denote a type by $\alpha \in (0,1)$. The equilibrium is then defined by equating supply and demand:

Corollary 4 (Equilibrium under Continuous Types). *If the distribution of types, $P(.)$, is continuous, then the equilibrium is given by a unique type $z \in (k,1)$ such that*

$$F_1 = L\left(\int_0^{\phi(z)} p(\alpha)d\alpha + \int_z^1 p(\alpha)d\alpha\right) \tag{11}$$

where $\phi(\alpha) : (k,1) \to [0,k]$ is defined as

$$\phi(\alpha) := \alpha' \text{ such that } \max_{\alpha' \in [0,k]} v(\alpha') \leq v(\alpha). \tag{12}$$

Because the equilibrium is such that only the extreme types work in the flexible sector, there are two thresholds: $z \in (k,1)$ and $\phi(z) \in [0,k]$. In an interior equilibrium, we have $v(z) = v(\phi(z)) = \pi(z)$, where $\pi(z)$ is the equilibrium profit of the firms in the flexible sector. All types $\alpha \leq \phi(z)$ and $\alpha \geq z$ are employed in the flexible sector. The equilibrium degree of polarization is $\rho = s^*_z - s^*_{\phi(z)}$.

Figure 1 illustrates the equilibrium in the continuous case. For the flexible sector, the wage vector becomes a *wage function* $w(s)$. In equilibrium, all flexible firms must obtain the same profit π^* in all active markets. The wage function is not uniquely determined in inactive markets (see (10)). For simplicity, and without loss of generality, we assume that all flexible markets (active or inactive) are equally profitable. Thus, the wage function is $w(s) = y - \pi^* - c(s)$, which is the isoprofit for profit level π^*. Figure 1 depicts the wage function on the (s,w) plane. Note that $c''(s) > 0$ implies that the wage function is concave. For a given wage function, flexible firms decide where to locate themselves. Since profits are the same everywhere, firms are indifferent as to where they are located.

The wage function $w(s)$ is a menu of choices available to workers. A type-α worker

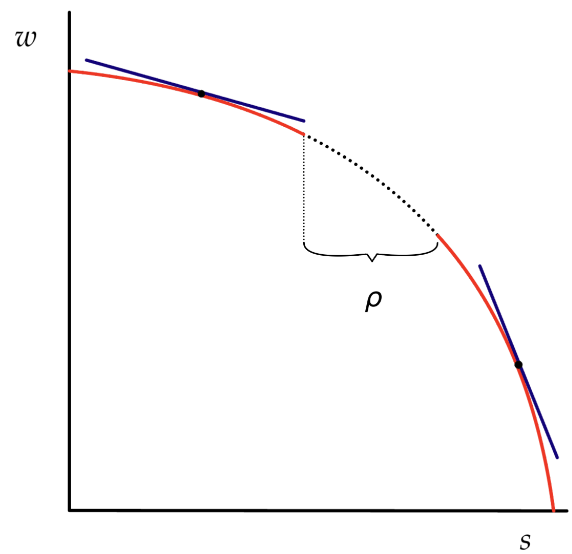

Figure 1: Equilibrium Wage Function and Polarization

solves the problem

$$\max_s \alpha s + (1-\alpha)w(s) \text{ s.t. } \alpha s + (1-\alpha)w(s) \geq \alpha s_0 + (1-\alpha)w_0. \tag{13}$$

As shown in Figure 1, the worker will choose the highest indifference curve given the wage function, and will thus choose $s_\alpha^* = h(\alpha)$, where the (absolute value of the) slope of her indifference curve, $\alpha/(1-\alpha)$, equals the slope of the wage function, $c'(s)$. Because there are fewer flexible firms than workers, there must be empty regions where workers and firms are not located. Because workers with extreme preferences enjoy greater surplus, that region must be an interval: $[s^l, s^h]$. The degree of polarization, ρ, is the length of this interval, as shown in the figure.

It is possible that $s^l = 0$. To see this, use $w_0 = y - c(s_0)$ to write the profit potential as $v(\alpha) = c(s_0) - c(h(\alpha)) + \frac{\alpha}{1-\alpha}(h(\alpha) - s_0)$. The profit potential's intercept is $v(0) = c(s_0)$, which is positive and strictly increasing in s_0. Thus, for s_0 sufficiently low, we will have $\phi(z) = 0$ (i.e., a corner solution). In that case, the degree of polarization becomes $\rho = s^h$.

While ρ measures polarization within the flexible sector, we can also measure the (average) degree of polarization between sectors by

$$\rho_b = \int_0^{\phi(z)} h(\alpha)p(\alpha)d\alpha + \int_z^1 h(\alpha)p(\alpha)d\alpha - s_0. \tag{14}$$

Both degrees of polarization (within-sector and between-sector) are maximal at $s_0 = 0$. Intuitively, as s_0 falls, the flexible sector becomes less valuable for low-α workers and more valuable for high-α workers. Thus, in equilibrium, some low-α workers are replaced by high-α workers. For s_0 sufficiently low, no low-α worker works in the flexible sector: The flexible sector becomes "the high-s sector" and the inflexible sector "the low-s sector."

The distribution of preferences over the s-attribute may change over time. For example, some workers may become more concerned about the environmental impact of their firms. If s measures the extent to which firms use green technologies, such workers would now have higher α. At the same time, it is possible that some workers become *less* concerned about the environment, for example, if they think that environmental concerns have been overblown and politicized. Such workers would then have a lower α.

What would happen to firm polarization, profits and wages if workers became more polarized in their tastes for the s-attribute? To answer this question, we consider changes in $P(.)$ that shift density away from moderate preferences. Mas-Colell et al. (1995, p.198) define an elementary increase in risk as follows: "*$G(.)$ constitutes an elementary increase in risk from $F(.)$ if $G(.)$ is generated from $F(.)$ by taking all the mass that $F(.)$ assigns to an interval $[x', x'']$ and transferring it to the end-points x' and x'' in such a manner that the mean is preserved.*" We generalize the notion of increase in risk and say that $\widehat{P}(.)$ is a *generalized increase in risk* from $P(.)$ if $\widehat{P}(.)$ is generated from $P(.)$ by taking some of the mass that $P(.)$ assigns to an interval $[x', x'']$ and transferring it to points smaller than x' and greater than x'' in such a manner that the mean is preserved. Formally, $\widehat{P}(.)$ is a generalized increase in risk from $P(.)$ if (i) $\int_{x'}^{x''} p(\alpha)d\alpha > \int_{x'}^{x''} \hat{p}(\alpha)d\alpha$ and (ii) $\int_0^1 \alpha p(\alpha)d\alpha = \int_0^1 \alpha\hat{p}(\alpha)d\alpha$. It is immediate that a generalized increase in risk is a mean-preserving spread (and thus $P(.)$

second-order stochastically dominates $\widehat{P}(.)$). Then, we have the following result:

Proposition 3 (Generalized Increase in Risk). *If $\widehat{P}(.)$ is a generalized increase in risk from $P(.)$ for $x' = \phi(z)$ and $x'' = z$, then the equilibrium under $\widehat{P}(.)$:*

i) is more polarized than that under $P(.)$, that is, $\widehat{\rho} > \rho$,

ii) has higher profits than under $P(.)$, that is, $\pi(\widehat{z}) > \pi(z)$,

iii) has lower wages than under $P(.)$, that is, $\widehat{w}^*(\alpha) < w^*(\alpha)$.

An increase in risk implies that more workers have extreme preferences for the s-attribute (either high or low α). Since the flexible technology is more valuable to workers with extreme preferences, flexible firms would cater to those workers, thus becoming more polarized in their provision of the desirable attribute. Profits increase and wages fall because workers with extreme preferences—which are the most valuable to firms—are now less scarce.

3 Polarization, Profit, Wages, and the Labor Share

This section builds upon the core results of the previous section by establishing further empirical predictions. We begin by presenting a parametric version of the model, which we use to illustrate the main equilibrium properties. We then derive additional results linking the degree of firm polarization to profits, wages, and the labor share of a sector's income, and consider how these equilibrium outcomes vary with changes in preferences, technology, and competition.

Proposition 2 and Corollary 4 show how to find the equilibrium for any distribution $P(.)$ and cost function $c(.)$. Here, we consider a parametric version of the model that allows for an analytical solution in closed form. Because it is analytically more convenient to work with the transformed type $a := \frac{\alpha}{1-\alpha}$ (the marginal rate of substitution between s and w), from now on, we refer to a as the worker's type. We assume that a is uniformly distributed on $[a' - \Delta, a' + \Delta]$, for an arbitrary $a' \geq \Delta > 0$.[11] Parameter Δ measures the dispersion of preferences for s around the mean a'. We also assume that the cost function is quadratic: $c_\iota(s) = \frac{\sigma_\iota s^2}{2}$, for $\iota \in \{0, 1\}$.[12] We call this set of assumptions the *quadratic-uniform case*, for short.

We now use our previous results to characterize the equilibrium. Zero profit in the inflexible sector (Lemma 1) implies $w_0^* = y - \frac{\sigma_0 s_0^2}{2}$. The optimal level of the s-attribute in the flexible sector is $h(\alpha) = \frac{a}{\sigma_1}$. The profit potential as a function of a is $v(a) = y - w_0^* - as_0 + \frac{a^2}{2\sigma_1} = \frac{\sigma_0 s_0^2}{2} - as_0 + \frac{a^2}{2\sigma_1}$, which is strictly U-shaped in a (consistent with Proposition 1). The type that minimizes $v(a)$ is $a_k = \sigma_1 s_0$. Let $a_z \in (a' - \Delta, a' + \Delta)$ denote the equilibrium threshold (assuming an interior equilibrium). From Corollary 4, the equilibrium conditions are $v(a_z) = v(\phi(a_z))$ and $\frac{1}{2\Delta}(2\Delta - a_z + \phi(a_z)) = \tau_1$, where $\tau_1 := \frac{F_1}{L}$ measures the tightness of the labor market. Solving these conditions proves the next result.

Proposition 4 (Equilibrium in the Quadratic-Uniform Case). *In an interior equilibrium of the quadratic-uniform case, types $a \in (\sigma_1 s_0 - \Delta(1 - \tau_1), \sigma_1 s_0 + \Delta(1 - \tau_1))$ work in the inflexible sector and are paid wage $w_0^* = y - \frac{\sigma_0 s_0^2}{2}$, and types $a \leq \sigma_1 s_0 - \Delta(1 - \tau_1)$ and $a \geq \sigma_1 s_0 + \Delta(1 - \tau_1)$ work in the flexible sector and are paid wage $w(a) = w_0^* + \frac{\sigma_1 s_0^2}{2} - \frac{\Delta^2}{2\sigma_1}(1 - \tau_1)^2 - \frac{a^2}{2\sigma_1}$.*

Wages decrease with a (consistent with Corollary 2). Note that the solution is interior if $a' \in (a_k - \Delta\tau_1, a_k + \Delta\tau_1)$. To perform comparative statics while keeping the solution interior, from now on we make the simplifying assumption that $a' = a_k$.

Consistent with Corollary 1, flexible firms are polarized. The equilibrium degree of polarization is

$$\rho = \frac{2\Delta(1 - \tau_1)}{\sigma_1}. \tag{15}$$

Equation (15) shows that the degree of polarization is a product of three exogenous parameters: 2Δ (the dispersion in s-preferences), $1 - \tau_1$ (flexible firms' labor market power),

[11]Equivalently, α is distributed according to c.d.f. $P(\alpha) = \frac{\alpha}{1-\alpha}$ on $[\frac{a'-\Delta}{1+a'-\Delta}, \frac{a'+\Delta}{1+a'+\Delta}]$.

[12]Note that we now allow the cost function to differ across sectors. This variation has no implications for the previous analysis but allows for more interesting comparative statics.

and $1/\sigma_1$ (the cost-efficiency in s production). That firm polarization increases with preference dispersion is unsurprising. Less obvious is that firm polarization is also affected by labor market concentration and technological change. In a less tight labor market, flexible firms can focus on catering to those workers with the largest surpluses, which implies more polarization. Similarly, as flexible firms become more efficient in producing s, they will offer more s-quality to all, but disproportionally more to those who value it the most, ultimately increasing the differences between low-s and high-s firms

The equilibrium profit in the flexible sector is

$$\pi^* = \frac{(\sigma_0 - \sigma_1)s_0^2}{2} + \frac{\sigma_1 \rho^2}{8}. \tag{16}$$

The profit is increasing and convex in polarization. Averaging $w(a)$ over all types employed in the flexible sector defines the average wage in that sector:

$$\overline{w} := w_0 + \frac{\sigma_1 s_0^2}{2} - \frac{\Delta^2(1-\tau_1)^2}{2\sigma_1} - M, \tag{17}$$

where M is the average monetary cost of producing s:

$$M := \frac{\int_{a_k-\Delta}^{a_k-\Delta(1-\tau_1)} a^2 da + \int_{a_k+\Delta(1-\tau_1)}^{a_k+\Delta} a^2 da}{4\sigma_1 \tau_1 \Delta} = \frac{3\sigma_1^2 s_0^2 + \Delta^2(3(1-\tau_1)+\tau^2)}{6\sigma_1}. \tag{18}$$

We first consider the impact of Δ on polarization, profits, and wages:

Prediction 1. In sectors with more dispersion in worker preferences for s-quality, firms are more polarized, the profit is higher, and the average wage is lower.

This result is closely related to Proposition 3. An increase in Δ is an increase in risk: it removes mass from intermediate values of a and reallocates this mass to the tails without changing the mean. The average wage decreases for two reasons. First, because the profit increases, there is less surplus left to the workers. Second, because of increased firm polarization, the average cost of producing s increases due to the convexity of the cost function.

Next, we consider the impact of τ_1 on polarization, profits, and wages.

Prediction 2. In more concentrated sectors, firms are more polarized, the profit is higher, and the average wage is lower.

In more concentrated sectors, i.e., sectors with fewer firms, there is less competition for those workers qualified to work in the sector. Thus, firms are more profitable in such sectors. Because firms first target workers with extreme preferences, polarization in s-quality is more pronounced when there are fewer firms.

Finally, we consider the impact of σ_1 on polarization, profits, and wages:

Prediction 3. If flexible firms can offer s-quality at a lower cost (i.e., σ_1 is lower), firms are more polarized, the profit is higher, and the average wage is lower.

When σ_1 falls, firms produce more s, both because they offer higher s-quality to a given type a and because a_k increases, which then increases both a_z and $\phi(a_z)$. Because the distance between a_z and $\phi(a_z)$ remains the same, polarization $\rho = (a_z - \phi(a_z))/\sigma_1$ increases when σ_1 falls. Intuitively, polarization increases because a fall in σ_1 has a larger impact on the marginal cost of producing s for larger values of s.[13]

The flexible sector's profit is higher when σ_1 is lower for two reasons. First, producing s becomes less costly. Second, the flexible sector becomes relatively more cost-efficient than the inflexible sector, which makes the inflexible sector less competitive. The effect of σ_1 on the average wage also has two parts. First, as s-quality becomes cheaper, firms substitute s-quality for wages. Second, as the flexible sector becomes more competitive, flexible firms capture a larger share of the surplus.

An extensive empirical literature documents a decline in the labor share of value added. There are two leading explanations for the decline in the labor share: technological improvements that make superstar firms more efficient (Autor et al. (2020)) and barriers

[13]This is a consequence of the convexity of $c(s)$; the quadratic-cost assumption is not needed.

to entry that reduce competition (Covarrubias, Gutiérrez, and Philippon (2019)). In the cross-section, Barkai (2020) shows that more concentrated industries have higher "pure" profits and lower labor and capital shares. Here, we consider the relationship between the flexible sector's labor share and firm polarization in job quality (and thus, implicitly, the flexible sector's profit). Formally, the flexible sector's labor share is defined (in the general model) as

$$\text{Labor share} := \frac{L\int_0^{\phi(\alpha_z)} w(\alpha)dP(\alpha) + L\int_{\alpha_z}^1 w(\alpha)dP(\alpha)}{F_1\pi^* + L\int_0^{\phi(\alpha_z)} w(\alpha)dP(\alpha) + L\int_{\alpha_z}^1 w(\alpha)dP(\alpha)}, \tag{19}$$

where the numerator is the sector's aggregate wage bill and the denominator is the sector's (financial) value added. In the quadratic-uniform case, we can rewrite the labor share as

$$\text{Labor share} = \frac{w_0 + \frac{\sigma_1 s_0^2}{2} - \frac{\Delta^2(1-\tau_1)^2}{2\sigma_1} - M}{y - M}, \tag{20}$$

which is the average wage over the average value added. The next proposition shows that firm polarization is negatively related to the labor share.

Proposition 5 (**Polarization and the Labor Share**). *In the quadratic-uniform case, the labor share is smaller in more polarized sectors.*

Polarization increases if the flexible sector becomes more concentrated (lower τ_1), if firms become more efficient at producing s (lower σ_1), or if the workers' preferences for s become more dispersed (higher Δ). In all three cases, increased polarization is associated with smaller labor shares. Although all three shocks—reduced competition, improved efficiency, and greater preference dispersion—reduce the labor share, the welfare implications are quite different. A smaller labor share resulting from more industry concentration reduces workers' welfare. In contrast, a lower cost of producing s increases profits and has an ambiguous impact on workers' welfare. More efficient workplace technologies allow firms to offer jobs with higher s-quality at a lower cost. Firms will thus offer contracts with lower wages and higher s-quality. Although the measured labor share falls, some workers are better off because they can choose from an improved menu of wages and s-quality levels. In particular, workers of types $a \geq a_z$ (the before-shock threshold type) benefit. Similarly, some workers with types $a \in (a_k, a_z)$ (defined before the shock) will now be offered jobs in the flexible sector. These workers are also better off. In contrast, some workers with weaker preferences for s-quality will be made worse off. In particular, the worker of type $\phi(a_z)$ (defined before the shock) will no longer be employed in the flexible sector. Intuitively, the welfare impact is heterogeneous because a decrease in σ_1 is a biased technological change that benefits workers with stronger preferences for s-quality. Finally, changing Δ is a preference shock, thus its welfare consequences are not well defined.

4 Outside investors

In this section, we introduce a new type of agent: outside investors. For simplicity, we assume that the outside investors' identities do not overlap with those of other agents (workers and entrepreneurs). In the Internet Appendix, we consider the possibility of such an overlap. Outside investors are atomistic and in large supply. They can buy shares of both flexible and inflexible firms. For simplicity, we normalize the number of shares in each firm to one. To introduce a trading stage, we assume that entrepreneurs first set up their firms and then sell shares to outside investors. Outside investors hold the shares until the end of the period, when firms are liquidated and profits are paid out as dividends. We assume no time discounting and no uncertainty.[14]

We assume that operating costs, $w + c(s)$, are paid out of current cash flows, y, whenever possible. If $y < w + c(s)$, the firm uses its working capital to plug the difference. To invest in working capital, a firm needs to raise funds from outside investors. Let $e_1(s, w) + e_2(s, w)$ denote the total amount that outside investors pay in exchange for one

[14]The lack of risk in our model can be alternatively interpreted as perfect risk sharing. Suppose that each firm produces $y + \epsilon$, with ϵ idiosyncratic. By holding shares in a mass of firms, one can perfectly diversify away all risks.

share of a company that offers contract (s, w), where $e_1(s, w)$ is the amount raised in a primary offering (i.e., the funds stay in the firm) and $e_2(s, w)$ is the secondary offering amount (i.e., the proceeds go to the entrepreneur). Let $d(s, w)$ denote the dividend paid at the end of the period. Limited liability implies that dividends must be non-negative. If $\pi(s, w) \geq 0$, all costs can be funded internally, thus $e_1(s, w) = 0$ and $d(s, w) = \pi(s, w)$. If $\pi(s, w) < 0$, then $e_1(s, w) = -\pi(s, w)$ and $d(s, w) = 0$.

Let $\varepsilon - e_2(s, w) + \beta s + (1 - \beta)\pi(s, w)$ denote the utility of a shareholder who starts with endowment ε and buys one share of a company that offers contract (s, w) by paying $e_2(s, w)$ to the entrepreneur (plus, if needed, $e_1(s, w)$ in a primary offering) and later collects dividend $d(s, w) = \pi(s, w)$ (or $d(s, w) = 0$ if $\pi(s, w) < 0$) .[15] Similar to the workers' preferences, here $\beta \in [0, 1]$ denotes the shareholder's preference for the s-attribute.[16] To simplify the analysis while conveying the main message, we assume that there are two types of outside investors: one with $\beta = 0$ and one with $\beta > 0$. We call investors of the first type "profit-driven investors" (or *π-investors*) and the second "socially responsible investors" (or *s-investors*). We interpret s as an environmental or social attribute that is viewed positively by both workers and investors. Profit-driven investors care about the environmental or social attributes of their investments only because of the financial value they might create. Socially responsible investors care directly about such attributes in addition to financial value.[17] Using Stark's (2023) terminology, π-investors care about financial *value*, while s-investors also care about *values*. We assume that both investor types are in large supply. This assumption implies that, unlike much (but not all) of the

[15]Alternatively, we can write this utility as $\varepsilon - e_2(s, w) + \beta s + (1 - \beta)\,[d(s, w) - e_1(s, w)]$.

[16]We interpret the investor as an ultimate owner. Many (but certainly not all) investors invest through intermediaries (e.g., asset managers) who have a fiduciary duty to maximize stock returns subject to their funds' mandates (i.e., constraints). The intermediary's utility may differ from the ultimate investor's utility, creating agency costs. Because of the benchmark nature of our model, we assume away intermediation. How intermediation affects the sorting of investors to firms is an interesting avenue for further research.

[17]This preference is of a "warm-glow" type. Investors may also care about the aggregate value of s in the economy, regardless of their shareholdings (in Oehmke and Opp's (2022) language, they could have a "broad mandate"). However, because investors are atomistic, such preferences would have no impact on firm outcomes. Pástor et al. (2021) reach a similar conclusion in an asset pricing model with atomistic investors; Dangl et al. (2023) also make a similar point.

literature, the introduction of socially responsible investors expands the set of financing choices, thus increasing the options available to all flexible entrepreneurs.

To characterize the equilibrium, we note first that the efficient s level for a firm owned by an s-investor depends on β. As in the previous section, here we work with the transformed type, $a = \frac{\alpha}{1-\alpha}$. Similarly, define $b := \frac{\beta}{1-\beta}$. Suppose an s-investor matches with a worker of type a. Using the same reasoning as before, we can show that $s^*_{ab} = h(a+b)$. That is, the s-investor increases the efficient s level. Because contracts must be efficient in a competitive equilibrium, the s-investors affect the s levels of the firms in which they invest.

Do socially responsible investors affect s levels through "impact" (i.e., voice) or "divestment" (i.e., exit)? Because the model has no frictions, either channel delivers the same result. To see this, suppose that the entrepreneur cannot commit to a contract; i.e., any contract between a worker of type a and an entrepreneur can be renegotiated after the firm is sold to an s-investor, and either party can unilaterally exit. In this case, the s-investor and the worker will always renegotiate the contract and agree to the efficient s level, s^*_{ab}. Under this interpretation, s investors are "impact investors."

Suppose instead an entrepreneur first commits to a contract (s, w). To maximize the price of the share, the entrepreneur should choose the contract (s^*_{ab}, w^*_{ab}), because it maximizes the surplus for an s-investor subject to the participation of a type-a worker. That is, the most profitable way of attracting investors is to choose the efficient s level. In other words, the s-investors would not invest at the desirable price unless the entrepreneur commits to (s^*_{ab}, w^*_{ab}).

For simplicity, we proceed with the quadratic cost function (none of the results in this section depends on the type distribution $P(.)$). We then have $s^*_{ab} = \frac{a+b}{\sigma_1}$. The next proposition describes the optimal contract in the inflexible sector.

Proposition 6 (Inflexible Sector Equilibrium). *In an equilibrium with two types of shareholders and $c_\iota(s) = \frac{\sigma_\iota s^2}{2}$, only s-investors buy shares of inflexible firms. The equilibrium wage in the inflexible sector is $w^*_0 = bs_0 + y - \frac{\sigma_0 s_0^2}{2}$ and firm profit is $\pi(s_0, w^*_0) = -bs_0$.*

We now consider the equilibrium in the flexible sector. Let $v(a,b)$ denote the profit potential when an s-investor matches with a type-a worker. As in Proposition 1, it is easy to verify that $v(a,b)$ is U-shaped in a. We use $v(a,0)$ to denote the profit potential under a π-investor. We have the following result:

Proposition 7 (Profit Potential and Investor Type). *Let $c_\iota(s) = \frac{\sigma_\iota s^2}{2}$. We have $v(a,b) \geq v(a,0)$ if and only if $a \in [a^-, a^+]$, where*[18]

$$\left\{a^-, a^+\right\} := 1 + \sigma_1 s_0 \pm \sqrt{(1+2\sigma_1 s_0)(1+b) + \sigma_1 s_0^2(\sigma_1 - \sigma_0)}.$$

This proposition implies that s-investors create more value if matched with workers with intermediate preferences, while π-investors create more value if matched with workers with extreme preferences. This result holds because the profit potential function is U-shaped; workers with intermediate preferences should be matched with socially responsible investors because such investors care less about profits. Figure 2 illustrates $v(a,0)$ (solid line) and $v(a,b)$ (dashed line). Under a continuum of worker types, the unique equilibrium is given by the same conditions as in Corollary 4, once we define $v(a) := \max\left\{v(a,0), v(a,b)\right\}$.[19] That is, $v(a)$ is the upper envelope (in red) in Figure 2.

Let a_z denote the equilibrium marginal worker type. Firm $(s^*_{a_z}, w^*_{a_z})$ will be sold for $e_2(s^*_{a_z}, w^*_{a_z}) = v(a_z)$, which will also be the price for all other flexible firms (all flexible entrepreneurs must make the same profit from selling their shares). Because $v(a) \geq v(a,0)$, the entrepreneurs' are (weakly) better off when s-investors are available.

If $a_z \geq a^+$, then s-investors do not invest in the flexible sector. If $a_z < a^+$, s-investors buy shares in firms that hire workers of types $a \in [\min\{a^-, \phi(a_z)\}, a^+]$, while π-investors buy shares in firms that hire workers of types $a \leq \min\{a^-, \phi(a_z)\}$ and $a \geq a^+$. In either case, the equilibrium displays *perfect segmentation*: π-investors buy shares in firms where workers have extreme preferences for s and s-investors buy shares in firms matched with

[18]Equivalently, we have $\alpha \in [\alpha^-, \alpha^+]$, where $\alpha^- := \max\{\frac{a^-}{1+a^-}, 0\}$ and $\alpha^+ := \frac{a^+}{1+a^+}$.

[19]The analysis can be easily generalized to any number m of different types of investors, $\{b_1, ..., b_m\}$, by defining $v(a) = \max\left\{v(a,b_1), ..., v(a,b_m)\right\}$.

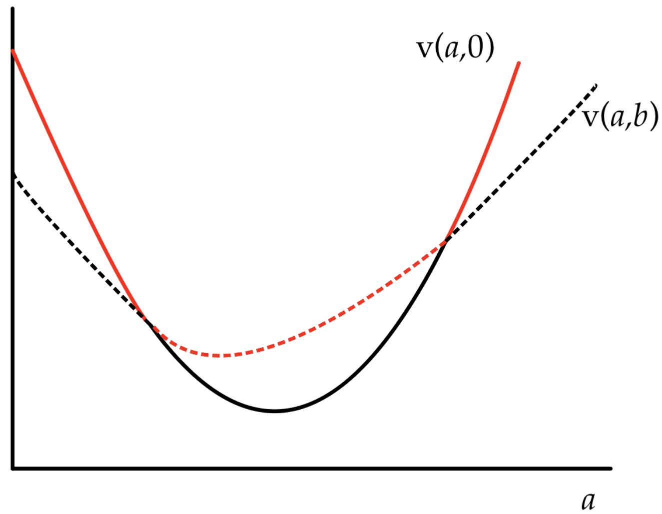

Figure 2: Profit Potential with Socially-Responsible Investors

workers with intermediate preferences.[20] Figure 3 illustrates this result for the case in which $a_z < a^+$ and $\phi(a_z) < a^-$. At first glance, the equilibrium in Figure 3 may seem counterintuitive. Why wouldn't socially responsible investors be more likely to buy shares in high-a firms? Aren't they willing to pay more for firms with high s levels? Our model reveals that the equilibrium effects are subtler than this intuition. Firms that hire workers with very strong preferences for s create large surpluses (see Proposition 1). Thus, profit-driven investors will target such firms because of the potential to extract large profits. Although competition among profit-driven investors will drive their returns to zero,[21] profit-driven investors have a comparative advantage over socially responsible investors in companies where the profit potential is high. Similarly, socially responsible investors

[20]Perfect segmentation is a consequence of the assumption of no uncertainty (or, equivalently, perfect risk-sharing). If we instead assume that risk exists and the number of firms is finite, then diversification would give investors incentives to hold shares of all firms. In that case, s-investors would "tilt" their portfolios towards stocks in which $a \in [a^-, a^+]$, while π-investors would tilt their portfolio away from such stocks.

[21]Note there is no risk or time discounting in our environment, thus zero return is the fair compensation for their investments.

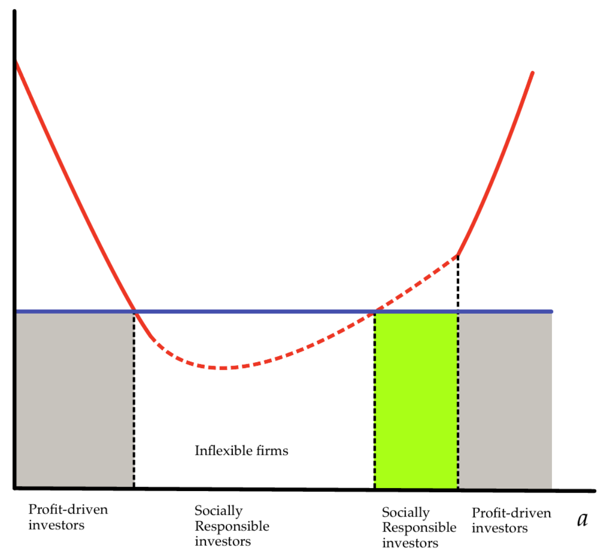

Figure 3: Perfect Segmentation in Equilibrium

have a comparative advantage in the market for low-profit firms.[22]

An increase in b—the intensity of socially responsible investors' preferences for the s-attribute—decreases a^- and increases a^+, thus widening the range of worker types for which s-investors have an advantage relative to π-investors. A larger b also indicates more extreme shareholder preferences with respect to the s attribute. Thus, all else constant, an increase in risk in shareholder preferences increases the number of entrepreneurs willing to sell shares to s-investors and the flexible firms' market values. Conversely, a generalized increase in risk in worker preferences would reduce the number of entrepreneurs who sell to socially responsible investors but also increases market values.

The next proposition compares market valuations and stock returns between flexible and inflexible firms.

Proposition 8 (**Flexibility, Firm Value, and Stock Returns**). *Relative to inflexible firms, flex-*

[22]In the Internet Appendix, we show that when workers are also investors, they typically do not invest in firms of the same type of the firms they work for.

ible firms have higher market valuations and higher expected stock returns.

While it is not always clear which sectors or industries have flexible technologies, such sectors can be empirically identified by their within-sector s-attribute polarization (i.e., how polarized they are in their s choices), which can be measured by ESG metrics or other similar variables. The model then predicts high firm valuations in sectors with high polarization in ESG scores. Similarly, expected stock returns should be higher in sectors where firms are more polarized in their ESG choices (or other similar variables that are viewed positively by both workers and investors).

If $c(s_0)$ is sufficiently low, in equilibrium we have $s^l = 0$, implying that the flexible sector has only high s-quality firms. Thus, if the inflexible sector has very low s-quality or if the cost of producing s falls sufficiently, we have a segmented equilibrium that is also monotonic: all firms with $a < a_z$ are held by socially responsible investors and those with $a > a_z$ are held by traditional investors (in Figure 3, the first region disappears). In that case, expected returns are (weakly) increasing in s and *predictable*: even if s is not observed by investors, wages are.

The model also predicts a link between employee satisfaction and expected stock returns. In particular, firms with the highest stock returns are flexible firms sold to profit-driven investors. These firms also have the highest levels of employee satisfaction (measured by U_i^*, which is the willingness to pay for a job). Because employee satisfaction is also U-shaped in equilibrium, the firms with the lowest employee satisfaction scores are inflexible firms. Such firms also have the lowest stock returns. While the relationship between firm-level employee satisfaction and stock returns does not need to be monotonic, the model predicts that firms at the upper end of employee satisfaction will have higher returns than firms at the low end of employee satisfaction.

5 Firm Creation

For polarization to occur in equilibrium, flexible firms need to be in short supply. In other words, developing or acquiring a flexible technology must be costly. Because flexible firms have positive profits, if the flexible technology were free, entrepreneurs would enter the flexible sector until both profit and polarization were zero. In this section, we model the entrepreneurs' decision to create a firm and enter into one of the sectors. That is, we drop the assumption that F_0 and F_1 are exogenous. We will show that the crucial assumption is that flexible technologies are costly to develop or acquire.

Suppose there is a large number of identical atomistic entrepreneurs. At the ex-ante stage, these entrepreneurs pay cost K_ι to develop technology $\iota \in \{0,1\}$ and create a firm. For simplicity, we set $K_0 = 0$ and $K_1 > 0$. That is, flexible technologies, being more valuable, are also more expensive to develop or acquire. We work with the continuum case. Let $z(F_1)$ denote the equilibrium marginal worker type when the mass of flexible firms is F_1. Note that $z(F_1)$ is continuous and strictly decreasing in F_1 (recall that $z(F_1)$ is defined as the marginal type to the right of α_k). Thus, the equilibrium profit $v(z(F_1))$ for a given F_1 is continuous and strictly decreasing in F_1. This is intuitive: Profit is lower when there are more flexible firms competing for the same workers.

If an entrepreneur needs to pay cost $K_1 > 0$ to create a flexible firm, entrepreneurs would enter the sector if $v(z(F_1)) > K_1$, and not enter if $v(z(F_1)) < K_1$. If $v(z(L)) < K_1$, there exists a unique $F_1^* < L$ such that $v(z(F_1^*)) = K_1$. Thus, F_1^* is the unique equilibrium mass of flexible firms. If $v(z(L)) \geq K_1$, then the only equilibrium requires $F_1^* = L$. In either case, the flexible firms' *ex-post* profit is $\pi^* = K_1$.

When entry is endogenous, entrepreneurs in the flexible sector make zero *ex-ante* profit: $\pi^* - K_1 = 0$. Similarly, entrepreneurs will enter the inflexible sector until their ex-post profits are zero. Outside investors (both s-investors and π-investors) are also in excess supply and thus earn their respective outside utilities. Only workers end up with positive surpluses in equilibrium. This makes sense: Labor is the only scarce resource in this economy. A consequence of this analysis is that polarization is more pronounced when a flexible technology is more expensive to develop.

6 Related Literature

While the empirical literature on compensating differentials is vast, there are few works on the theory of compensating differentials. Our model is inspired by Rosen (1986), who models firms that compete by offering bundles of wages and nonwage attributes (see Lavetti (2023) for a recent review of the Rosen framework). By imposing further structure to Rosen's general framework, we are able to solve for the equilibrium fully and derive testable predictions. Importantly, unlike Rosen (1986), we assume the existence of two sectors with different degrees of technological flexibility. This difference in flexibility is essential for our main result (firm polarization in equilibrium).

An important contribution to the theory of compensating differentials in competitive markets is the work of Berk, Stanton, and Zechner (2010), who present a model in which risk-averse workers accept lower wages in exchange for job stability. They show that firms that commit to job stability choose lower debt levels. Because workers are heterogeneous, firms will cater to different types of workers by offering different bundles of wages and debt levels. Our work differs in several ways, but particularly by focusing on job design by firms with different degrees of technological flexibility and its consequences for equilibrium sorting and polarization. Ferreira and Nikolowa (2024) provide another example of a compensating differentials model à la Rosen. In a dynamic model of careers inside firms, firms compete for workers with preferences over money and prestige. In the Internet Appendix, we extend our model in this paper to allow for careers, following Ferreira and Nikolowa (2024). We then derive additional predictions relating technological parameters to worker turnover and within-firm inequality.

Our model is related to models of product differentiation and spatial competition. In particular, our model resembles Hotelling's (1929) in that firms choose a location along a straight line. In strategic models of spatial competition, such as Hotelling (1929) and Salop (1979), firms have incentives to “maximally differentiate” themselves by locating as far apart from one another in order to gain local market power. Such incentives are absent in our model because we have no strategic interactions. Thus, the model is closer to Rosen’s (1974) model of product differentiation under pure competition. Our firms are price-takers and, thus, most firms choose to locate near or at the same point as others. Firm polarization nevertheless arises in equilibrium because workers (or in the case of product differentiation, consumers) do not enter the market in intermediate locations.

Our paper is also related to a small theoretical literature on the impact of organization and job design on labor market sorting. Aghion and Tirole (1997) show that delegation of decision rights benefits firms through the agent’s participation decision because agents who value autonomy are willing to work for lower compensation. Van den Steen (2005) shows that firms may wish to appoint CEOs with a particular “vision” to attract employees who share such a vision. A shared vision is modeled as shared beliefs in a world of multiple priors. Firms benefit from committing to a vision in multiple ways, such as improved motivation, coordination, and lower compensation costs. Van den Steen (2010) extends the analysis and broadens the interpretation of shared vision to include shared values (i.e., similar preferences). More closely related to our model is Henderson and Van den Steen’s (2015) analysis of purposeful firms. In their model, firms commit to a pro-social purpose to attract employees who wish to develop a reputation for being pro-socially minded. In related work, Song, Thakor, and Quinn (2023) develop a model in which firms and workers are heterogeneous in their preferences for firm purpose. In a search model, they show that firms that offer a higher purpose can save on wage costs by matching with workers with strong purpose preferences. In these models, as in our model, firms that adopt a purpose can be more profitable because employees accept to work for lower wages. In a more recent contribution to this literature, Geelen, Hajda, and Starmans (2022) develop a delegation model of an organization in which controlling and non-controlling stakeholders can have pro-social preferences.

Our model features agents with preferences over nonpecuniary firm attributes in a frictionless competitive environment. A similar approach is found in a strand of the literature on responsible investment, which modifies standard asset pricing models to allow some investors to have social preferences (Heinkel, Kraus, and Zechner (2001); Pástor, Stambaugh, and Taylor (2021); Berk and van Binsbergen (2022)). Despite the absence of frictions, these models deliver many insights. In a sense, our model is the labor market counterpart of these asset market models. While in the asset pricing literature the key scarce resource is capital, in our model the scarce resource is labor. Other related competitive models with few frictions (but no labor markets) include those of Pedersen, Fitzgibbons, and Pomorski (2021), who develop a mean-variance analysis of responsible investing when some investors are unaware of the informational content of ESG scores, Goldstein et al. (2022), who consider a rational expectations equilibrium model of stock prices when information about cash flow and ESG risk is dispersed among atomistic investors (who can be either green or traditional investors), and Landier and Lovo (2023), who present a general equilibrium model of responsible investing in which the matching between entrepreneurs and capital is subject to frictions.

More generally, our paper is related to the theoretical literature on socially responsible investing. A vast literature has developed since the pioneering work of Heinkel, Kraus, and Zechner (2001); for brevity, we review only the papers that share some of our modeling choices and applications. Most papers in this literature assume that some firms have technological flexibility. For example, Chowdry, Davies, and Waters (2019) consider a model of impact investing in which a manager allocates a scarce resource (e.g., attention) between a for-profit technology and a social technology. Oehmke and Opp (2022) present a corporate-finance model of socially responsible investing in which an entrepreneur chooses between two productive technologies—clean and dirty—and then raises funds from investors that can be either purely financially motivated or socially responsible. Edmans, Levit, and Schneemeier (2023) present a model in which a firm can take costly corrective actions to reduce externalities. They compare different forms of divestment strategies by responsible investors (blanket exclusion versus tilting). Dangl et al. (2023) analyze the equilibrium impact of different kinds of social preferences on corporate investment. They explicitly model firms' choices between green and brown technologies.

Broccardo, Hart, and Zingales (2022) consider a model in which some agents care about the welfare of those affected by a decision. They show how investor voice (i.e., voting) can have an impact when investors are socially responsible. Piatti, Shapiro, and Wang (2023) consider a model in which some investors care about public good provision. Those investors invest more in firms delivering the public good (green firms) and may also invest more in brown firms for hedging reasons.

In addition, many models of sustainable investing consider the interactions between financial markets and corporate insiders, such as employees and managers. Davies and Van Wesep (2018) show that divestment campaigns can backfire because executive compensation typically rewards stock *returns*, not prices. Bond and Levit (2022) develop a model of imperfect competition in labor markets where an ESG policy is a commitment to pay workers above the market wage. In a similar vein, Stoughton, Wong, and Yi (2020) and Xiong and Yang (2023) model CSR as a commitment device, for firms with market power, to consider consumer or employee interests. In Albuquerque, Koskinen, and Zhang (2019), firms that adopt a CSR technology directly impact the consumers' demand by decreasing the elasticity of substitution. Thus, the adoption of a CSR technology decreases profit sensitivity to aggregate productivity shocks. Bisceglia, Piccolo, and Schneemeier (2022) present a model in which ex-ante identical firms can choose between two different technologies—brown and green—and a fraction of their customers and investors may have socially responsible preferences. Bucourt and Inostroza (2023) consider a setup where a manager exerts costly effort to increase the firm's ES quality, and heterogeneous investors trade shares based on their beliefs about the firm's ES quality. The authors show investor heterogeneity reduces the firm's ES investments.

7 Conclusion

When workers have preferences for purposeful or socially responsible jobs, profit-maximizing firms will cater to such preferences. By designing jobs with these positive attributes, firms can lower their wage bills. Conversely, firms can also benefit from making a job *less* socially responsible or sustainable because it may cost less to produce using a "dirty" technology. When facing workers with heterogeneous preferences for CSR/ES, firms that have flexible technologies will cater to workers with the most extreme preferences. That is, such firms will appear more polarized in their CSR/ES choices than the preferences of the underlying population.

Firm polarization in CSR/ES investments has several normative and positive implications. In the cross-section, firm value and stock returns are U-shaped in ES qualities. Sectors with more polarization in CSR/ES metrics should have higher expected stock returns. These predictions are still untested. Our model also predicts that both high and low CSR/ES firms are harmed by regulations that constrain their freedom to cater to workers, such as the imposition of minimum environmental standards or working benefits and conditions.[23] Thus, in the absence of other forces, both types of firms are equally likely to oppose policies such as maximum emissions or diversity quotas. In addition, because all firms benefit when worker preferences become more polarized, firms would welcome the spread of conflicting information that is likely to polarize opinions and entrench extreme views.

Our model has the surprising result that socially responsible investors may be less likely to invest in very high ES firms than are purely financial investors. This result is explained by the higher profit potential of high-ES firms. This potential for profit attracts profit-driven investors, pushing share prices up. Socially responsible investors prefer firms with intermediate CSR/ES levels. Such investors have a direct impact on their firms' CSR/ES levels, which are higher than they would have been if sold to purely finan-

[23]We present a formal analysis of the introduction of a minimum standard in the Internet Appendix.

cial investors. These firms have lower valuations, and also lower stock returns because the marginal investor in these firms is willing to sacrifice some basis points in exchange for additional investments in CSR/ES.

Our model is relevant to the discussion on corporate greenwashing and sustainability disclosures by companies. Concerned about firms engaging in "climate cheap talk," the SEC has adopted rules to standardize climate-related disclosures.[24] However, firms have few credible signals of green credentials at their disposal. Because workers are better informed about firms' green investments, wage concessions can serve as a signal of such investments.

Appendix

Proof of Proposition 1. We first characterize the efficient contract set. The Lagrangian for the problem in (1) is:

$$\max_{s,w} \omega \left(f(u^i(s,w)) - f(\underline{u}) \right) + (1-\omega)(\pi(w,s) - \underline{\pi}) - \lambda(\underline{u} - u^i(s,w)) - \mu(\underline{\pi} - \pi(s,w)). \tag{A.1}$$

The first-order conditions are:

$$\begin{aligned} &\omega\alpha_i f'(u^i(s,w)) - (1-\omega)c'(s) + \lambda\alpha_i - \mu c'(s) = 0 \\ &\omega(1-\alpha_i) f'(u^i(s,w)) - (1-\omega) + \lambda(1-\alpha_i) - \mu = 0. \end{aligned} \tag{A.2}$$

Only one of the two participation constraints can bind, so there are three cases: $\lambda = \mu = 0$, $\lambda > 0$ and $\mu = 0$, or $\lambda = 0$ and $\mu > 0$. In each of these three cases, from (A.2) we find that $\frac{\alpha_i}{1-\alpha_i} = c'(s_i)$, and therefore $s_i^* = h(\alpha_i) = c^{'-1}\left(\frac{\alpha_i}{1-\alpha_i}\right)$.

For $\omega = 0$, the worker's participation constraint binds, i.e., $u^{i*} := u^i(s_i^*, w_i^*) = u^i(s_0, w_0) = w_0 + \alpha_i(s_0 - w_0)$. Thus:

$$v'(\alpha_i) = \lambda(s_i^* - w_i^* - s_0 + w_0) \tag{A.3}$$

[24] https://www.sec.gov/news/press-release/2024-31

Since $w_i^* = w_0 + \frac{\alpha}{1-\alpha}(s_0 - s_i^*)$, we can simplify equation (A.3) as follows

$$v'(\alpha_i) = \lambda(s_i^* - w_i^* - s_0 + w_0) = \frac{s_i^* - s_0}{(1-\alpha_i)^2}. \tag{A.4}$$

Define α_k such that $s_0 = h(\alpha_k)$. For $\alpha_i < \alpha_k$, $v'(\alpha_i) < 0$, and for $\alpha_i > \alpha_k$, $v'(\alpha_i) > 0$, that is $v(\alpha_i)$ is strictly U-shaped and reaches its minimum value at α_k. □

Proof of Lemma 1. To show that $\pi(s_0, w_0^*) = 0$, we need to consider two possible cases. First, suppose that $\sum_{i=1}^n x_{i0}^* = F_0$. Since $L < F_0 + F_1$, it follows that some flexible firms do not employ anyone, which means that all flexible firms have zero profit in equilibrium, in every market they can possibly operate, including market $j = 0$, which implies $\pi(s_0, w_0) = 0$. Second, suppose that $\sum_{i=1}^n x_{i0}^* < F_0$. Now some inflexible firms do not operate. The labor demand correspondence says that if $\pi(s_0, w_0) > 0$, $D_0(w_0) = F_0$, in which case the labor demand at market 0 is greater than the labor supply, which cannot happen in equilibrium. Trivially, we cannot have $\pi(s_0, w_0) < 0$ because no inflexible firm would operate. Therefore, in equilibrium, we must have $\pi(s_0, w_0) = 0$.

To show that $\pi(s_j^*, w_j^*) = \pi^* > 0$ for all $j \in \{1, ..., n\}$ such that $x_j^* > 0$, note that the labor demand correspondence implies that all flexible firms must have the same profit in equilibrium, i.e., $\pi(s_j^*, w_j^*) = \pi^* \geq 0$ for all $j \in \{1, ..., n\}$ such that $x_j^* > 0$. Suppose that $\pi(s_j^*, w_j^*) = 0$ for $x_j^* > 0$ and $j \neq k$. Note that the profit potential at α_k is $v(\alpha_k) = 0$. Because $v(.)$ is U-shaped and reaches its minimum at α_k, for $j \neq k$ we have $v(\alpha_j) > \pi(s_j^*, w_j^*) = 0$. Therefore, $u^j(s_j^*, w_j^*) - u^j(s_0, w_0) > 0$ for all types $j \neq k$. Such types will offer to work in the flexible sector, implying that labor supply to the flexible sector is arbitrarily close to L (because p_k is arbitrarily small), which is not possible in equilibrium because $L > F_1$. Thus, we must have $\pi^* > 0$. □

Proof of Proposition 2. Assumption 2 implies that $v(\alpha_j)$ can be strictly ranked for all $j \in \{1, ..., n\}$. Let m index types according to $v(\alpha_j)$, that is, $m + 1 > m$ implies $v(\alpha_{m+1}) > v(\alpha_m)$ for all $m \in M = \{1, ..., n\}$. That is, we reorder all n types, from $m = 1$ to $m = n$, so that lower indices mean a lower profit potential. Note that the type that leads to the minimum profit potential is α_k. Define z as the largest element in M such that $\sum_{m=z}^{n} p_m L \geq F_1$. Note that z always exists (because $L > F_1$) and is uniquely defined. Note that the subset $\{z, ..., n\} \subset M$ includes all types with extreme preferences because $v(\alpha_j)$ is strictly U-shaped.

We first show that $x_m^* = 0$ if and only if $m < z$. To show the sufficiency part, suppose that $m < z$ and $x_m^* > 0$. For this quantity to be feasible, there must exist at least one $m' \geq z$ such that $x_{m'} < p_{m'} L$. That is, there is at least one individual of type m' that is employed in the inflexible sector. Without loss of generality, assume then that $m' = z$. Because not all workers of type z are employed in the flexible sector, workers of that type must be indifferent between working in the flexible or the inflexible sector. Thus, the flexible firms' profit in this market must be $v(\alpha_z)$. Because the profit in market m is at most $v(\alpha_m)$, which is lower than $v(\alpha_z)$ by the definition of z, the demand for workers in market m must be zero. Thus, if $m < z$, then $x_m^* = 0$.

To show necessity, suppose $x_m^* = 0$. Then, it must be that all workers of type m work in the inflexible sector. That is, $w_m^* + \alpha_m(s_m^* - w_m^*) \leq w_0^* + \alpha_m(s_0 - w_0^*)$. Thus, $\pi(s_m^*, w_m^*) \geq v(\alpha_m)$. If $m \geq z$, then there must exist $m' < z$ such that $x_{m'}^* > 0$; otherwise we have $\sum_{m=1}^{n} x_m^* < F_1$, which implies that not all flexible firms are active and thus must have zero profit, contradicting Lemma 1. Because m' is an active market, by Lemma 1 it offers profit $\pi^* > 0$. Then, we have $\pi^* \leq v(\alpha_{m'})$ (because $v(.)$ gives the maximum profit for that market), $v(\alpha_{m'}) < v(\alpha_m)$ (because $m' < m$), and $v(\alpha_m) \leq \pi(s_m^*, w_m^*)$ (as argued above). Thus, $\pi(s_m^*, w_m^*) > \pi^*$, which is a contradiction. Thus, if $x_m^* = 0$, then $m < z$.

We now show that for all $m > z$, we have $x_m^* = p_m L$. Because z is the lowest index $j \in M$ such that $x_j^* > 0$, then $\pi^* \leq v(\alpha_z)$. If $x_m^* < p_m L$, the profit in that market is $v(\alpha_m) > v(\alpha_z) \geq \pi^*$, which violates Lemma 1. Thus, for all $m > z$, we have $x_m^* = p_m L$. If $m = z$, $x_z^* = F_1 - \sum_{j \in \{j \neq j : x_z > 0\}} p_j L$, which follows from the aggregate constraints.

The equilibrium wages for $x_j^* > 0$ follow immediately from the fact that all flexible firms have the same profit $v(z)$. If $x_j^* = 0$, wages can be anywhere between $y - c(s_j^*) -$

$v(\alpha_z)$ and $\frac{C_j(s_0,w_0^*)-\alpha_j s_j^*}{1-\alpha_j}$ since both supply and demand can be zero for such (s, w) pairs. □

Proof of Corollary 1. If $j > 1$, $j \leq k$, and $x_j^* > 0$, then $v(\alpha_{j-1}) > v(\alpha_j) \geq v(\alpha_z)$, thus (9) implies $x_{j-1}^* = p_{j-1}L$. The argument is symmetric for the other case. □

Proof of Corollary 2. Since profits are the same across all active markets, the equilibrium wages in markets j and j' are such that:

$$w_j^* = w_{j'}^* + c(s_{j'}^*) - c(s_j^*), \tag{A.5}$$

where $s_{j'}^* = h(\alpha_{j'}) > h\left(\alpha_j\right) = s_{j'}^*$, and $c(s_{j'}^*) > c(s_j^*)$. It follows that $w_j^* > w_{j'}^*$. □

Proof of Corollary 3. Define the *utility surplus potential* as

$$\vartheta(\alpha_i) := \max_{(s,w)} U_i(s, w) \text{ subject to } y - w - c(s) = \pi^*. \tag{A.6}$$

In an equilibrium with profit π^*, the surplus of a type-i worker employed in the flexible sector is $\vartheta(\alpha_i)$. By the Envelope Theorem, $\vartheta'(\alpha_i) = s_i^* - s_0 - w(s) + w_0$, where $w(s) = y - \pi^* - c(s)$. We have $\vartheta''(\alpha_i) = s_i^{*\prime} + c'(s)s_i^{*\prime} > 0$, thus the utility surplus potential is strictly convex in α_i.

Suppose first that the equilibrium is such that type $i = 1$ is hired by a flexible firm. At $i = 1$, we have $\vartheta'(\alpha_1) = s_1^* - s_0 - w(s_1^*) + w_0$. We have $s_1^* < s_0$ because $s_0 = s_k$. It must then be that $w(s_i^*) > w_0$, otherwise $U_1(s_1^*, w(s_1^*)) = \alpha_1(s_i^* - s_0) + (1 - \alpha_i)(w(s_1^*) - w_0) < 0$, implying that market 1 cannot simultaneously support profit π^* and a non-negative worker surplus. Thus, $\vartheta'(\alpha_1) < 0$. Because $\lim_{\alpha\to\infty} \vartheta'(\alpha) = \infty$, $\vartheta(\alpha_i)$ is strictly U-shaped, and the result follows.

If, instead, type $i = 1$ is not hired by a flexible firm, the equilibrium threshold type z is such that $s_z^* > s_0$. Because $v(\alpha_k) = 0$, then $\vartheta(\alpha_k) < 0$, implying $w(s_0) < w_0$. Thus, $0 < \vartheta'(\alpha_k) < \vartheta'(\alpha_z)$ (the latter inequality follows from the strict convexity of $\vartheta(\alpha_i)$), and the result follows. □

Proof of Corollary 4. A density $p(.)$ can be approximated by a discrete probability function $\hat{p}(.)$ for n equidistant types. Proposition 2 implies the existence of z which characterizes the equilibrium under $\hat{p}(.)$. As $n \to \infty$, if $\hat{p}(.) \to p(.)$, then $z \to z^*$. We then define $z = z^*$. □

Proof of Proposition 3. First, note that $v(\alpha)$ does not depend on the distribution and, thus, it is not affected by a generalized increase in risk. From the definition of a generalized increase in risk, we have $\int_z^{\phi(z)} p(\alpha)d\alpha > \int_z^{\phi(z)} \hat{p}(\alpha)d\alpha$, and therefore

$$F_1 < L\left(\int_0^{\phi(z)} \widehat{p}(\alpha)d\alpha + \int_z^1 \widehat{p}(\alpha)d\alpha\right). \tag{A.7}$$

Since the right-hand side of equation (A.7) is continuous and strictly decreasing in z, it follows that $\widehat{z} > z$, where $\widehat{z}$ is given by: $F_1 = L\left(\int_0^{\phi(\widehat{z})} \widehat{p}(\alpha)d\alpha + \int_{\widehat{z}}^1 \widehat{p}(\alpha)d\alpha\right)$. Parts ii) and iii) of the proposition follow directly from $\widehat{z} > z$. □

Proof of Predictions 1-3. Polarization is $\rho = \frac{2\Delta(1-\tau_1)}{\sigma_1}$. Thus it follows that $\frac{\partial \rho}{\partial \sigma_1} < 0$, $\frac{\partial \rho}{\partial \Delta} > 0$, and $\frac{\partial \rho}{\partial \tau_1} < 0$. The equilibrium profit is $\pi^\star = \frac{\sigma_0 s_0^2}{2} - \frac{\sigma_1 s_0^2}{2} + \frac{\Delta^2(1-\tau_1)^2}{2\sigma_1}$. Therefore, $\frac{\partial \pi^\star}{\partial \sigma_1} < 0$, $\frac{\partial \pi^\star}{\partial \Delta} > 0$, and $\frac{\partial \pi^\star}{\partial \tau_1} < 0$. The average wage is $\overline{w} = y - \frac{\sigma_1 s_0^2}{2} - \frac{\Delta^2(1-\tau_1)^2}{2\sigma_1} - \frac{\Delta^2(1-\tau_1)}{2\sigma_1} - \frac{\Delta^2\tau_1^2}{6\sigma_1}$. It follows that: $\frac{\partial \overline{w}}{\partial \sigma_1} > 0$, $\frac{\partial \overline{w}}{\partial \Delta} < 0$, $\frac{\partial \overline{w}}{\partial \tau_1} = \frac{2\Delta^2(1-\tau_1)}{2\sigma_1} + \frac{\Delta^2}{2\sigma_1} - \frac{2\Delta^2\tau_1}{6\sigma_1} = \frac{\Delta^2}{2\sigma_1}(3 - \frac{8\tau_1}{3}) > 0$. □

Proof of Proposition 5. The expression for the Labor share can be rewritten as follows:

$$\text{Labor share} = \frac{w_0 - \frac{\Delta^2(1-\tau_1)^2}{2\sigma_1} - \frac{\Delta^2(1-\tau_1)}{2\sigma_1} - \frac{\Delta^2\tau_1^2}{6\sigma_1}}{y - \frac{\sigma_1 s_0^2}{2} - \frac{\Delta^2(1-\tau_1)}{2\sigma_1} - \frac{\Delta^2\tau_1^2}{6\sigma_1}} \tag{A.8}$$

We now derive the effect of σ_1, Δ, and τ_1 on the labor share (the intermediate steps are fully presented in the Internet Appendix).

$$\frac{\partial \text{Labor share}}{\partial \sigma_1} = \frac{\frac{\Delta^2(1-\tau_1)^2}{2\sigma_1^2}(y-M) + \left(\frac{\Delta^2(1-\tau_1)}{2\sigma_1^2} + \frac{\Delta^2\tau_1^2}{6\sigma_1^2}\right)\pi^* + \frac{s_0^2}{2}\overline{w}}{\left(y - \frac{\sigma_1 s_0^2}{2} - \frac{\Delta^2(1-\tau_1)}{2\sigma_1} - \frac{\Delta^2\tau_1^2}{6\sigma_1}\right)^2} > 0 \tag{A.9}$$

$$\frac{\partial \text{Labor share}}{\partial \Delta} = \frac{-\frac{\Delta(1-\tau_1)^2}{\sigma_1}(y-M) - \left(\frac{\Delta(1-\tau_1)}{\sigma_1} + \frac{\Delta\tau_1^2}{3\sigma_1}\right)\pi^*}{\left(y - \frac{\sigma_1 s_0^2}{2} - \frac{\Delta^2(1-\tau_1)}{2\sigma_1} - \frac{\Delta^2\tau_1^2}{6\sigma_1}\right)^2} < 0 \tag{A.10}$$

$$\frac{\partial \text{Labor share}}{\partial \tau_1} = \frac{\frac{\Delta^2(1-\tau_1)}{\sigma_1}(y-M) + \frac{\Delta^2}{2\sigma_1}\left(1 - \frac{2\tau_1}{3}\right)\pi^*}{\left(y - \frac{\sigma_1 s_0^2}{2} - \frac{\Delta^2(1-\tau_1)}{2\sigma_1} - \frac{\Delta^2\tau_1^2}{6\sigma_1}\right)^2} > 0 \tag{A.11}$$

□

Proof of Proposition 6. Suppose that $\pi(s_0, w_0^*) = 0$. While π-investors would pay zero for an inflexible firm, s-investors would be willing to pay up to $\beta s_0 > 0$. Thus, only s-investors buy shares in inflexible firms in equilibrium and $\pi(s_0, w_0^*) < 0$. These investors are in excess supply and will thus pay to the entrepreneur $e_2(s_0, w_0^*) = \beta s_0 + (1-\beta)\pi(s_0, w_0^*)$ for each share. Competition among inflexible entrepreneurs should drive their profits from selling shares to zero: $e_2(s_0, w_0^*) = 0$, implying $\pi(s_0, w_0^*) = -\frac{\beta s_0}{1-\beta}$ and $w_0^* = \frac{\beta s_0}{1-\beta} + y - \frac{\sigma_0 s_0^2}{2}$. □

Proof of Proposition 7. Simple algebra shows that

$$v(a,0) = y - w_0 - a s_0 + \frac{a^2}{2\sigma_1}, \tag{A.12}$$

$$v(a,b) = v(a,0) - \beta\left(\frac{a^2}{2\sigma_1} - \frac{(1+\sigma_1 s_0)}{\sigma_1}a + y - w_0 - \frac{b}{2\sigma_1}\right). \tag{A.13}$$

Equation (A.13) shows that the difference $v(a,b) - v(a,0)$ is a quadratic and concave function of a, with roots

$$\{a^-, a^+\} \equiv (1+\sigma_1 s_0) \pm \sqrt{(1+\sigma_1 s_0)^2 + b - 2\sigma_1(y - w_0)}. \tag{A.14}$$

Replacing $w_0^* = b s_0 + y - \frac{\sigma_0 s_0^2}{2}$ (From Proposition 6) in (A.14) proves the result. □

Proof of Proposition 8. After investment $e_1(s, w)$ is made, all flexible firms can be sold for $e_2(s, w) = v(a_k) > 0$, while inflexible firms are sold for $e_2(s, w) = 0$. Thus, flexible firms have higher market valuations than inflexible firms. To prove that flexible firms have higher expected stock returns, note first that inflexible firms cost bs_0 and return $-bs_0$ in profit (see Proposition 6). Thus, investors in such firms obtain a -100% return, i.e., they lose all their (financial) investment. For flexible firms, we have both π-investors and s-investors. π-investors always get zero return (which is the fair risk-adjusted return), otherwise, they do not invest. s-investors earn negative returns, which can be no lower than -100%. □

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Internet Appendix for "Polarization, Purpose and Profit"

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1 Excess supply of workers

In this section, we extend the analysis to the case where workers, rather than firms, are in excess supply. That is, we assume that $L > F_0 + F_1$. This assumption implies that some workers will remain unemployed in equilibrium. The contract of an unemployed worker is normalized to $(0, 0)$.

To illustrate the differences between the baseline model and the model with workers in excess supply, we derive the results in the quadratic-uniform case (see Section 3 of the main article). The equilibrium conditions that determine a_z are unchanged, implying $a_z = \sigma_1 s_0 + \Delta(1 - \tau_1)$. The wage function is thus $w(a) = w_0^* + \frac{\sigma_1 s_0^2}{2} - \frac{\Delta^2}{2\sigma_1}(1 - \tau_1)^2 - \frac{a^2}{2\sigma_1}$. To solve for the equilibrium, we need to determine w_0^*. Because all inflexible firms will now hire workers, we also need to determine which workers are hired. Firms in the inflexible sector prefer to hire workers with higher a. Thus, for a given equilibrium a_z, there exists a threshold type $a_0 = a_z - 2\Delta\tau_0 = \sigma_1 s_0 + \Delta(1 - \tau_1 - 2\tau_0)$ such that inflexible firms hire all types in (a_0, a_z). Because we normalize the utility of unemployed workers to zero, we

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find that $w_0^* = -s_0 a_0 = -s_0(\sigma_1 s_0 + \Delta(1 - \tau_1 - 2\tau_0))$.

This case has the same qualitative properties as when workers are in short supply. In addition, wages and profits in the flexible sector now depend on the inflexible sector market tightness, τ_0, and the profit in the inflexible sector is given by $\pi_0^* = y + s_0^2(\sigma_1 - \frac{\sigma_0}{2}) + s_0\Delta(1 - \tau_1 - 2\tau_0)$. While most of the previous comparative statics apply, this version of the model allows for new comparative statics with respect to τ_0 and the effect of parameters on wages and profits in the inflexible sector. In particular, because w_0^* decreases with τ_1, and the value added per firm $(y - \frac{\sigma_0 s^2}{2})$ is independent of τ_1, we have the following result:

Prediction IA.1. If flexible firms become more efficient at producing s-quality (i.e., σ_1 is lower), the inflexible sector's labor share increases.

Proof. The labor share in the inflexible sector is:

$$\text{Labor share} = \frac{w_0^*}{\pi_0^* + w_0^*} = \frac{-s_0(\sigma_1 s_0 + \Delta(1 - \tau_1 - 2\tau_0))}{y - \frac{\sigma_0 s_0^2}{2}} \tag{IA.1}$$

It then follows that $\frac{\partial \text{Labor share}}{\partial \sigma_1} < 0$. □

Intuitively, as the flexible sector becomes more efficient at producing the s-attribute, its firms hire workers with stronger preferences for s. As a_z decreases, inflexible firms have to hire workers with weaker preferences for the s-attribute. Such workers demand higher wages, increasing the labor share of the inflexible sector. While the effect of σ_1 on the flexible sector's labor share is non-monotonic, lowering τ_1 will eventually decrease the flexible sector's labor share. Thus, as the flexible sector becomes more efficient at creating high-quality jobs, its labor share eventually decreases while the inflexible sector's labor share increases. While the labor share has been decreasing in most sectors in the US, in the financial sector, the labor share has been increasing (see Autor et al. (2020)). These patterns are compatible with most sectors becoming more efficient at improving job quality, forcing the (likely inflexible) financial sector to increase wages to attract workers.

2 General Utility Function

Here we show that our main results hold for a large family of utility functions. By "main results" we mean the existence of a unique equilibrium as described in Proposition 1 (characterized in an analogous way) and Corollaries 1 to 4.

For utility $u^i(s,w)$, define (as in the proof of Corollary 3) the *utility surplus potential* as

$$\vartheta(\alpha_i) := \max_{(s,w)} u^i(s,w) - u^i(s_0,w_0) \text{ subject to } y - w - c(s) = \pi^*. \tag{IA.2}$$

In an equilibrium with profit π^*, the surplus of a type-i worker employed in the flexible sector is $\vartheta(\alpha_i)$. A sufficient (but not necessary) condition for our results to hold is a strictly convex $\vartheta(\alpha_i)$. To see this, notice that strict convexity implies that $\vartheta(\alpha_i)$ is increasing, decreasing, or U-shaped. If it is U-shaped, it is immediate that the profit potential is also U-shaped, and our results follow. If it is increasing or decreasing, only high-α workers or low-α workers will be employed in the flexible sector. In either case, polarization occurs, and the other corollaries hold as well. Thus, here we focus on establishing sufficient conditions for $\vartheta(\alpha_i)$ to be strictly convex.

We consider a utility function $u^i(s,w)$ with the following properties.

Condition IA.1. *Utility $u^i(s,w)$ is strictly increasing in (s,w) and quasi-concave.*

Condition IA.1 simply says that both s and w are goods and indifference curves are convex.

Condition IA.2. *We can write $u^i(s,w) = f\big(g_1(\alpha_i)h_1(s,w) + g_2(1-\alpha_i)h_2(s,w)\big)$, where $f(.)$, $g_1(.)$ and $g_2(.)$ are strictly increasing, and*

$$\begin{aligned} g_1'(\alpha_i)\tfrac{\partial h_1(s,w)}{\partial s} - g_2'(1-\alpha_i)\tfrac{\partial h_2(s,w)}{\partial s} > 0 \\ g_1'(\alpha_i)\tfrac{\partial h_1(s,w)}{\partial w} - g_2'(1-\alpha_i)\tfrac{\partial h_2(s,w)}{\partial w} < 0 \end{aligned} \tag{IA.3}$$

for all $s > 0$, w, and $\alpha \in (0,1)$.

Note that (IA.3) is merely definitional: it defines α as a parameter that increases the marginal utility of s and decreases the marginal utility of w. Without this condition, we would not be able to interpret α.

There are several families of utility functions that satisfy Conditions 1 and 2, including all the standard functions commonly used in consumer theory. Note that the linear utility used in the main text is a special case where $f(x) = x$, $g_1(x) = g_2(x) = x$, $h_1(s, w) = s$ and $h_2(s, w) = w$. Note that all Cobb-Douglas functions such as $u(s, w) = Ks^{g_1(\alpha)}w^{g_2(1-\alpha)}$ also satisfy these conditions, because they can be equivalently written as $g_1(\alpha) \ln s + g_2(1 - \alpha) \ln w$. Similarly, CES functions of the type $\left(g_1(\alpha)s^r + g_2(1-\alpha)w^r\right)^{\frac{v}{r}}$ also satisfy these properties (for $r < 1$). An example of a nonstandard function that also satisfies Conditions 1 and 2 is $u(s, w) = g_1(\alpha)h(s + kw) + g_2(1-\alpha)h(ks + w)$ where $h(.)$ is concave and strictly increasing and $k \in (0, 1)$.

Not all functions satisfying Conditions 1 and 2 imply a convex utility surplus potential. As we will show, we need to impose further conditions on the second derivative of $g_1(.)$ and $g_2(.)$. First, we begin with a simple example.

Example: Cobb-Douglas. Suppose $u(s, w) = s^{\alpha}w^{1-\alpha}$. For this example, we need to restrict the domain to $s > 0$ and $w > 0$. As we will see below, we can impose this condition indirectly by choosing α_1 and α_n (i.e., the min and the max types) suitably. Working with the log transformation, the first-order condition for (IA.2) is

$$\frac{\alpha}{s} - \frac{(1-\alpha)c'(s)}{y - \pi^* - c(s)} = 0 \tag{IA.4}$$

and the second-order condition is

$$-\frac{\alpha}{s^2} - \frac{(1-\alpha)c''(s)(w + c'(s)^2)}{w^2} < 0. \tag{IA.5}$$

Note that (IA.4) defines function $s(\alpha)$ and that $s'(\alpha) > 0$. Replacing $s(\alpha)$ in the constraint yields $w(\alpha) = y - \pi^* - c(s(\alpha))$, implying $w'(\alpha) < 0$. By the Envelope Theorem, we have

$$\vartheta'(\alpha) = \ln s - \ln w - \ln s_0 + \ln w_0 \tag{IA.6}$$

and

$$\vartheta''(\alpha) = \frac{1}{s}s'(\alpha) - \frac{1}{w}w'(\alpha) > 0. \tag{IA.7}$$

Thus, $\vartheta(\alpha)$ is strictly convex. In this case, we can also show that $\vartheta(\alpha)$ is strictly U-shaped. As $\alpha \to 0$, $s \to 0$ and $w \to y - \pi^* > 0$. Thus, $\lim_{\alpha \to 0} \vartheta'(\alpha) = -\infty$, implying that $\vartheta(\alpha)$ is initially decreasing. As $\alpha \to 1$, the lower bound $w = 0$ eventually binds for some $\hat{\alpha}$, implying that as $\lim_{\alpha \to \hat{\alpha}} \vartheta'(\alpha) = \infty$. Thus, as long as α_1 is close to zero and α_n is close to $\hat{\alpha}$, $\vartheta(\alpha)$ is strictly U-shaped in its relevant domain. We also note that the strict convexity of $c(s)$ is not necessary here; the problem is also well-behaved if $c(s)$ is, e.g., linear everywhere. Thus, the strict convexity of the utility surplus potential (and the profit potential) does not hinge on $c(s)$ being convex. In general, if the utility function is *strictly* quasi-concave, strict cost convexity is not necessary for a unique solution.

We now consider more general functions. We first show that, if $g_1(.)$ and $g_2(.)$ are linear, Conditions 1 and 2 are sufficient to guarantee that $\vartheta(\alpha)$ is strictly convex. Write the utility as $u(s, w) = \alpha h_1(s, w) + (1 - \alpha)h_2(s, w)$. To simplify notation, we write condition (IA.3) as $h_{1s}(s, w) - h_{2s}(s, w) > 0$ and $h_{1w}(s, w) - h_{2w}(s, w) < 0$. The first-order condition is

$$\alpha\big(h_{1s}(s, w) - h_{1w}(s, w)c'(s)\big) + (1 - \alpha)\big(h_{2s}(s, w) - h_{2w}(s, w)c'(s)\big) = 0. \tag{IA.8}$$

Quasi-concavity plus strict convexity of $c(s)$ imply that the problem is globally (strictly) concave, thus there is a unique solution, which we denote by $s(\alpha)$. Differentiating IA.8 with respect to α yields

$$h_{1s}(s, w) - h_{2s}(s, w) + \big(h_{2w}(s, w) - h_{1w}(s, w)\big)c'(s) > 0 \tag{IA.9}$$

which is positive because of (IA.3). Thus, we have that $s'(\alpha) > 0$ and $w'(\alpha) < 0$. By the Envelope Theorem, we have

$$\vartheta'(\alpha) = h_1(s, w) - h_2(s, w) - h_1(s_0, w_0) + h_2(s_0, w_0) \tag{IA.10}$$

and

$$\vartheta''(\alpha) = \big(h_{1s}(s,w) - h_{2s}(s,w)\big)s'(\alpha) + \big(h_{1w}(s,w) - h_{2w}(s,w)\big)w'(\alpha) > 0. \tag{IA.11}$$

Thus, $\vartheta(\alpha)$ is strictly convex.

The case for general $g_1(.)$ and $g(.)$ is solved similarly. Quasi-concavity and $c'' > 0$ imply a unique solution for each α and (IA.3) implies $s'(\alpha) > 0$ and $w'(\alpha)$. By the Envelope Theorem, we have

$$\vartheta'(\alpha) = g_1'(\alpha)h_1(s,w) - g_2'(1-\alpha)h_2(s,w) - g_1'(\alpha)h_1(s_0,w_0) + g_2'(1-\alpha)h_2(s_0,w_0) \tag{IA.12}$$

and

$$\vartheta''(\alpha) = \big(g_1'(\alpha)h_{1s}(s,w) - g_2'(1-\alpha)h_{2s}(s,w)\big)s'(\alpha) + \tag{IA.13}$$

$$\big(g_1'(\alpha)h_{1w}(s,w) - g_2'(1-\alpha)h_{2w}(s,w)\big)w'(\alpha) + \tag{IA.14}$$

$$g_1''(\alpha)\big(h_1(s,w) - h_1(s_0,w_0)\big) + g_2''(1-\alpha)\big(h_2(s,w) - h_2(s_0,w_0)\big). \tag{IA.15}$$

We have that (IA.13) and (IA.14) are positive for all values. Thus, the utility surplus potential is convex at all points where (IA.15) is not "too negative." In particular, if $g_1(.)$ and $g_2(.)$ are linear, then (IA.15) is zero everywhere and $\vartheta(\alpha)$ is convex, as we showed before. Thus, if the absolute value of g_1'' and g_2'' is small, $\vartheta(\alpha)$ is convex.

Alternatively, we can verify Proposition 1 directly. Without loss of generality, set $f(x) = x$. Conditions 1 and 2 apply. The maximum profit a flexible firm could extract from a worker of type α_i whose outside option is to work for an inflexible firm is:

$$v(\alpha_i) := \max_{s,w} \pi(s,w) \quad \text{s.t. } u^i(s,w) \geq u^i(w_0,s_0). \tag{IA.16}$$

The Lagrangian for the problem is:

$$L = \pi(s,w) - \lambda(u^i(s_0,w_0) - u^i(s,w)). \tag{IA.17}$$

The first-order conditions are:

$$\begin{aligned} \frac{\partial L}{\partial w} &= -1 + \lambda u^i_w(s,w) = 0 \\ \frac{\partial L}{\partial s} &= -c'(s) + \lambda u^i_s(s,w) = 0. \end{aligned} \tag{IA.18}$$

From the two first-order conditions we have $c'(s) = \frac{u^i_s(s,w)}{u^i_w(s,w)}$. From $\lambda = \frac{1}{u^i_w(s,w)} > 0$, it follows that the participation constraint holds with equality. Therefore

$$u^i(s,w) = u^i(s_0,w_0) \Leftrightarrow h_2(s,w) - h_2(s_0,w_0) = -\frac{g_1(\alpha_i)}{g_2(1-\alpha_i)}(h_1(s,w) - h_1(s_0,w_0)) \tag{IA.19}$$

and

$$\begin{aligned} v'(\alpha_i) &= \lambda\big(\tfrac{\partial u^i(s,w)}{\partial \alpha_i} - \tfrac{\partial u^i(s_0,w_0)}{\partial \alpha_i}\big) \\ &= \tfrac{1}{u^i_w(s,w)}\big(g'_1(\alpha_i)h_1(s,w) - g'_2(1-\alpha_i)h_2(s,w) - g'_1(\alpha_i)h_1(s_0,w_0) + g'_2(1-\alpha_i)h_2(s_0,w_0)\big) \\ &= \tfrac{1}{u^i_w(s,w)}\big(g'_1(\alpha_i) + g'_2(1-\alpha_i)\tfrac{g_1(\alpha_i)}{g_2(1-\alpha_i)}\big)(h_1(s,w) - h_1(s_0,w_0)) \end{aligned} \tag{IA.20}$$

We now derive sufficient conditions under which there exists a unique α^*_i for which $v'(\alpha^*_i) = 0$. For this we need: $\frac{\partial h_1(s,w)}{\partial \alpha} > 0$ and for $\alpha = 0$, $h_1(s,w) < h_1(s_0,w_0)$ and

$$\frac{\partial}{\partial \alpha}\left(\frac{1}{u^i_w(s,w)}\big(g'_1(\alpha_i) + g'_2(1-\alpha_i)\frac{g_1(\alpha_i)}{g_2(1-\alpha_i)}\big)\right) > 0.$$

that is

$$\begin{aligned} &\tfrac{-u^i_{w\alpha}(s,w)}{(u^i_w(s,w))^2}\big(g'_1(\alpha_i) + g'_2(1-\alpha_i)\tfrac{g_1(\alpha_i)}{g_2(1-\alpha_i)}\big) \\ &+\tfrac{1}{u^i_w(s,w)}\big(g''_1(\alpha_i) + \tfrac{g'_1(\alpha_i)g'_2(1-\alpha_i)+g'_2(1-\alpha_i)^2}{g_2(1-\alpha_i)} - g''_2(1-\alpha_i)\tfrac{g_1(\alpha_i)}{g_2(1-\alpha_i)}\big) > 0. \end{aligned} \tag{IA.21}$$

As before, note that if $g''(.) = g''(.) = 0$ (i.e., linearity in α), the expression above is positive. A sufficient condition for (IA.21) to hold is

$$\frac{g''_1(\alpha_i)}{g''_2(1-\alpha)} \geq \frac{g_1(\alpha_i)}{g_2(1-\alpha_i)}. \tag{IA.22}$$

3 Worker-Investors

While we interpret investors as being different from workers, conceptually it makes no difference if investors are also workers. To see this, consider the case in which all workers are born with an endowment ε, which they can use to buy shares. Without loss of generality, we assume that an agent invests in only one firm (because there is no risk, there is no reason to diversify investment). To retain the assumption that investors of all types are in large supply, we assume that ε is large. Now, worker-investors (from now on, *agents*) derive utility from working for a firm with contract (s, w) and from investing in a firm with contract (s', w'). That is, agents can work for one firm and invest in another if they wish.

An agent's utility is thus

$$u^i(s, w, s', w') = \alpha_i s + (1 - \alpha_i)w + \varepsilon - e_2(s', w') + \beta_i s' + (1 - \beta_i)\pi(s', w'). \qquad \text{(IA.23)}$$

Because all agents are atomistic, their investment and working decisions are independent, and the equilibrium is the same regardless of whether agents have a dual role or not.

Note that this conclusion is independent of the correlation between α_i and β_i. It is natural to assume a positive correlation: workers who care about s in their own firms may also prefer to invest in firms with high s'.[1] Thus, suppose that $\alpha_i = \beta_i$. We can rewrite the utility function as

$$u^i(s, w, s', w') = \alpha_i(s + s') + (1 - \alpha_i)(w + \pi(s', w')) + \varepsilon - e_2(s', w'). \qquad \text{(IA.24)}$$

A natural question now is: Will workers invest in firms similar to their own firms? Consider a simple example with three types, where $a_1 = 0$, $a_2 = s_0$, and $a_3 > s_0$. We assume a quadratic cost function, with $\sigma_0 = \sigma_1 = 1$ to simplify the algebra. Agents make their labor supply and investment decisions independently, thus the equilibrium is just as de-

[1] There are other realistic cases. For example, "effective altruism" is the idea that one should make money first and then invest it in projects with social benefits. We can model an effective altruist as an agent with $\alpha_i = 0$ and large $\beta_i > 0$.

scribed in the article. Because the worker type that minimizes $v(a,b)$ is $s_0 - b$, we have that both $v(a,a_2)$ and $v(a,a_3)$ are strictly increasing in a, while $v(a,a_1)$ is U-shaped. We also have that $v(0,a_2) < v(0,a_3)$ and $v'(a,a_2) > v'(a,a_3)$, thus there exists a unique $\hat{a}$ such that $v(\hat{a},a_2) = v(\hat{a},a_3)$. Algebra show that

$$\hat{a} = 1 + s_0 + \sqrt{(1+s_0)^2 - 2(y - w_0^*) - A} \tag{IA.25}$$

where $A := \left(\alpha_3 a_3 - \alpha_2 a_2\right) / \left(\alpha_3 - \alpha_2\right)^{-1}$. Thus, labor market allocations are as follows. First, type $a_1 = 0$ is hired by investors of type a_1. Thus, any worker of type a_1 matches with investors of the same type. Second, if F is large enough so that some workers of type a_2 are hired by flexible firms, these workers match with investors of type a_3. Intuitively, type a_3 investors are those who care less about money, thus they should match with workers with low profit potential (in this case, type a_2). Third, type a_3 workers are hired by either type a_1 or type a_2. Finally, we note that, as before, firms in the inflexible sector are owned by type a_3 investors.

To conclude, workers typically do not invest in the same type of firms they work for. Intuitively, workers with low α prefer to work for firms with low s, but would like to invest in firms with either very low or very high s, because these firms have the highest profit potential. Similarly, workers with high α prefer to work for firms with high s, but, as investors, their comparative advantage is to invest in firms with intermediate levels of s.

4 Career Paths

Most high-skilled workers join firms in entry-level jobs that may eventually lead to promotion. In this section, we extend our model to the case in which firms compete for workers by offering career paths where both wages and other job attributes may change with career progression. This extension is non-trivial because now agents may care not only about the attributes of the entry-level job but also about the attributes of higher-level jobs, as well as the probability of promotion. We use the model to study the effects of flexible technologies on worker turnover rates and within-firm inequality.

At each period $t = 1, 2, ...$, a mass $\frac{p_j L}{2}$ of workers of type $j \in \{1, ..., n\}$ join the labor force. Workers live for two periods. As before, there are F_1 flexible firms and F_0 inflexible firms.[2] Firms live forever. As before, each firm can hire one worker to fill a vacancy, which we now interpret as a high-level position, called *job h*. In addition, a firm can now create m vacancies in a different job, which we call *job l*.[3] The utility of a job that offers consumption C is $\tau u(C)$, where $\tau \in \{l, h\}$. We normalize $h = 1$ and assume $l = \theta < 1$, that is, job l is a lower quality job. Job $\tau \in \{l, h\}$ generates revenue y_τ, with $y_l \leq y_h$. To simplify the exposition, we assume that $u(C) = C^\gamma$, $\gamma < 1$, and $c(s) = \frac{\sigma s^2}{2}$, with $\sigma > 0$.

Suppose a firm matches with a worker of type α_j who is then assigned to job τ. What is the efficient level of the s-attribute for this worker in this job? Using the same reasoning as in the previous sections, cost minimization implies that $s^*_{j\tau} = \frac{\alpha_j}{\sigma(1-\alpha_j)} =: s^*_j$ (for simplicity, we consider only profit-driven investors). Note that the optimal level of the s-attribute is independent of τ.

We first consider *spot contracts*, which are one-period contracts of the form (s_j, w_j). We assume that a worker would never accept a position in job l under a spot contract:

Assumption IA.1. *Job l is individually undesirable:*

$$\theta \left(\alpha_j s^*_j + (1-\alpha_j) \left(y_l - \frac{\sigma s^{*2}_j}{2} \right) \right)^\gamma < \left(\alpha_j s_0 + (1-\alpha_j) w_0 \right)^\gamma \text{ for all } j \in \{1, ..., n\}. \quad \text{(IA.26)}$$

The left-hand side of (IA.26) is the utility of a type-j worker in job l with wage $w_j = (y_l - \frac{\sigma s^{*2}_j}{2})$ (the wage that equates the profit of a flexible firm to zero). The right-hand side of (IA.26) is the utility of the same type of worker in job h under contract (s_0, w_0) (the inflexible contract). Assumption IA.1 says that if a flexible firm offers job l under a spot contract, no worker will accept it; the worker prefers to work in the inflexible sector.

[2]We can endogenize F_ι by introducing an entry stage at $t = 0$, when entrepreneurs (who discount future profits) decide whether to invest K_ι to enter Sector ι.

[3]Formally, a unit mass of firms can create mass m of positions in job l.

Despite job l being undesirable as a standalone job, job l can be used in conjunction with job h in a long-term contract in which workers are offered a career path inside the firm. Suppose a firm in Sector ι chooses to hire young workers of type j on a career path contract. This contract specifies the s level and the wage for an entry position in job l, $(s^{\iota}_{jl}, w^{\iota}_{jl})$, the s level and the wage for a top-level position in job h, $(s^{\iota}_{jh}, w^{\iota}_{jh})$, and the number of workers m^{ι}_{j} who will compete for the top-level position after working in job l for one period. Suppose a young type-j worker has an outside lifetime (i.e., two-period) utility of U_j. The firm must choose a long-term contract $(m^{\iota}_{j}, s^{\iota}_{jl}, w^{\iota}_{jl}, s^{\iota}_{jh}, w^{\iota}_{jh})$ that maximizes profit subject to providing lifetime utility U_j to the worker. Under such contracts, a young worker (of type α_j) first joins the firm in job l and then is promoted to job h (with probability $\frac{1}{m^{\iota}_{j}}$) or fired (with probability $1 - \frac{1}{m^{\iota}_{j}}$) .[4] In what follows, to simplify the exposition, we characterize only interior solutions in which $m^{\iota}_{j} > 1$ in all active markets. We also assume that not all young workers will be employed by flexible firms in equilibrium.

In the following proposition, we show that career path contracts are never adopted by firms in the inflexible sector.

Proposition IA.1 (Optimal contract in the inflexible sector). *Inflexible firms do not employ workers in job l. To hire workers for job h, inflexible firms offer spot contracts* (s_0, w_0)*, with* $w_0 = y_h - c(s_0)$*, to both young and old workers.*

Proof. Because workers in the inflexible sector are in short supply, if inflexible firms offer career path contracts, it must be that $m^0_j = 1$. That is, inflexible firms must offer either safe career paths or spot contracts. An inflexible firm with a vacancy can approach a young worker in a career path contract in another inflexible firm with the offer of two consecutive spot contracts with wage $w_0 = y_h - c(s_0) - \epsilon$ with ϵ arbitrarily small. In that case, we have

$$\begin{aligned} 2u(C_j(s_0, y_h - c(s_0))) &> u(C_j(s_0, x)) + u(C_j(s_0, y_h + y_l - x - 2c(s_0))) \\ &> \theta u(C_j(s_0, x)) + u(C_j(s_0, y_h + y_l - x - 2c(s_0))). \end{aligned}$$

[4] Ferreira and Nikolowa (2024) prove the optimality of such up-or-out contracts in a similar setup.

Thus, the worker would accept this contract. Therefore, only spot contracts that yield zero profits will be offered by inflexible firms. □

Because in the inflexible sector workers are in short supply, inflexible firms will only offer spot contracts that yield zero profit. In what follows, for simplicity and without loss of generality, we assume that $s_0 = y_h - c(s_0) = w_0$. This assumption implies that $C_{j0} = C_{j'0} = C_0$ for all $j, j' \in \{1, ..., n\}$ and that the lifetime utility of an employee with type j must be such that $U_j \geq 2C_0^\gamma$.

We now consider the problem of a flexible firm. We assume that the firm offers long-term contracts.[5] A flexible firm that operates in market j chooses a contract to maximize its profit, taking the lifetime utility of type-j workers, U_j, as given. The representative flexible firm's maximization problem (per period) in market j is:[6]

$$V(\alpha_j, U_j) := \max_{(m_j, s_{jl}, w_{jl}, s_{jh}, w_{jh})} m_j \left(y_l - \frac{\sigma s_{jl}^2}{2} - w_{jl} \right) + y_h - \frac{\sigma s_{jh}^2}{2} - w_{jh} \tag{IA.27}$$

subject to

$$\theta(\alpha_j s_{jl} + (1-\alpha_j) w_{jl})^\gamma + \frac{1}{m_j}(\alpha_j s_{jh} + (1-\alpha_j) w_{jh})^\gamma + \frac{m_j - 1}{m_j} C_0^\gamma \geq U_j. \tag{IA.28}$$

The profit (IA.27) has two parts: the (per period) profit per worker in job l times the mass of such workers (m_j) plus the (per period) profit per worker in job h times the mass of such workers (which is one). Equation (IA.28) is the participation constraint of a young worker of type j. This worker works in the entry-level job in the first period, where she earns w_{jl} and enjoys s_{jl}, and then is promoted with probability $\frac{1}{m_j}$ to the high-level job, where she earns w_{jh} and enjoys s_{jh}. With probability $\frac{m_j - 1}{m_j}$, the worker is not promoted, leaves the firm when old and finds a job in an inflexible firm, where she enjoys consumption C_0. The

[5] It can be shown that long-term contracts dominate spot contracts for sufficiently high h (see Ferreira and Nikolowa (2024))

[6] Because only flexible firms will offer career path contracts, we drop the superscript ι from the contract variables.

young worker's participation constraint (IA.28) must be binding in an optimal solution. It is easy to see that an old worker who is promoted strictly prefers to stay in the firm instead of leaving for an inflexible firm.

In the following proposition, we characterize the optimal career path contract for a given (α_j, U_j).

Proposition IA.2 (Optimal career path contract). *In a career path equilibrium,* $s_j^* = \frac{\alpha_j}{\sigma(1-\alpha_j)}$, $w_{jh}^* = \frac{C_{jh}^* - \alpha_j s_j^*}{1-\alpha_j} > \frac{C_{jl}^* - \alpha_j s_j^*}{1-\alpha_j} = w_{jl}^*$*, and* $C_{jh}^* = \theta^{\frac{1}{\gamma-1}} C_{jl}^*$ *where* C_{jl}^* *is implicitly determined by*

$$C_{jl} = \frac{C_{jl}^{1-\gamma}}{\theta(1-\gamma)}(U_j - C_0^{\gamma}) - \frac{\gamma}{1-\gamma}\overline{C}_{jl}, \tag{IA.29}$$

where $\overline{C}_{jl} := (1-\alpha_j)y_l + \frac{\alpha_j^2}{2\sigma(1-\alpha_j)}$.

Proof. The firm maximizes its per-period profit subject to the the workers lifetime participation constraint.

$$\begin{aligned} \max_{(m_j, s_{jl}, w_{jl}, s_{jh}, w_{jh})} \quad & m_j(y_l - \tfrac{\sigma s_{jl}^2}{2} - w_{jl}) + y_h - \tfrac{\sigma s_{jh}^2}{2} - w_{jh} \\ & -\lambda(U_j - \theta(\alpha_j s_{jl} + (1-\alpha_j)w_{jl})^{\gamma} - \tfrac{1}{m_j}(\alpha_j s_{jh} + (1-\alpha_j)w_{jh})^{\gamma} - \tfrac{m_j - 1}{m_j} C_0^{\gamma})) \end{aligned}$$

The first-order conditions are:

$$-m_j + \lambda\theta(1-\alpha_j)\gamma C_{jl}^{(\gamma-1)} = 0, \tag{IA.30}$$

$$-1 + \frac{\lambda}{m_j}(1-\alpha_j)\gamma C_{jh}^{(\gamma-1)} = 0, \tag{IA.31}$$

$$-m_j\sigma s_{jl} + \lambda\theta\alpha_j\gamma C_{jl}^{(\gamma-1)} = 0, \tag{IA.32}$$

$$-\sigma s_{jh} + \frac{\lambda}{m_j}\alpha_j\gamma C_{jh}^{(\gamma-1)} = 0, \tag{IA.33}$$

$$y_l - \frac{\sigma s_{jl}^2}{2} - w_{jl} - \frac{\lambda}{m_j^2}\left(C_{jh}^{\gamma} - C_0^{\gamma}\right) = 0, \tag{IA.34}$$

$$\frac{1}{m_j} - \frac{U_j - C_0^{\gamma} - \theta C_{jl}^{\gamma}}{C_{jh}^{\gamma} - C_0^{\gamma}} = 0. \tag{IA.35}$$

From (IA.30) and (IA.32) we have: $\sigma s_{jl} = \frac{\alpha_j}{1-\alpha_j}$. From (IA.31) and (IA.33) we have: $\sigma s_{jh} = \frac{\alpha_j}{1-\alpha_j}$. Thus, $s_{jl}^* = s_{jh}^* = \frac{\alpha_j}{\sigma(1-\alpha_j)}$. From (IA.30) and (IA.31), we have $\theta C_{jl}^{(\gamma-1)} = C_{jh}^{(\gamma-1)} \Rightarrow C_{jh}^* = \frac{1}{\theta^{\frac{1}{1-\gamma}}} C_{jl}^*$. Since $\theta < 1$ and $\gamma < 1$, this implies that $C_{jh}^* > C_{jl}^*$. It then follows that $w_{jh}^* = \frac{C_{jh}^*}{1-\alpha_j} - \frac{\alpha_j^2}{\sigma(1-\alpha_j)^2} > \frac{C_{jl}^*}{1-\alpha_j} - \frac{\alpha_j^2}{\sigma(1-\alpha_j)^2} = w_{jl}^*$. To find C_{jl}^*, we simplify (IA.34) as follows:

$$y_l + \frac{\alpha_j^2}{2\sigma(1-\alpha_j)^2} - \frac{C_{jl}}{1-\alpha_j} - \frac{1}{m_j\theta(1-\alpha_j)\gamma C_{jl}^{(\gamma-1)}}\left(C_{jh}^{\gamma} - C_0^{\gamma}\right) = 0 \tag{IA.36}$$

$$y_l + \frac{\alpha_j^2}{2\sigma(1-\alpha_j)^2} - \frac{C_{jl}}{1-\alpha_j} - \frac{1}{m_j\theta(1-\alpha_j)\gamma C_{jl}^{(\gamma-1)}}\left(C_{jh}^{\gamma} - C_0^{\gamma}\right) = 0 \tag{IA.37}$$

$$\Leftrightarrow (1-\alpha_j)y_l + \frac{\alpha_j^2}{2\sigma(1-\alpha_j)} - C_{jl} = \frac{1}{\theta\gamma C_{jl}^{(\gamma-1)}}(U_j - C_0^{\gamma} - \theta C_{jl}^{\gamma}) \tag{IA.38}$$

$$C_{jl} = \frac{C_{jl}^{(1-\gamma)}}{\theta(1-\gamma)}(U_j - C_0^{\gamma}) - \frac{\gamma}{1-\gamma}\overline{C}_{jl}. \tag{IA.39}$$

To show existence, define

$$f(C_{jl}) = \frac{C_{jl}^{(1-\gamma)}}{\theta(1-\gamma)}(U_j - C_0^{\gamma}) - \frac{\gamma}{1-\gamma}\overline{C}_{jl}. \tag{IA.40}$$

Note that $f'(C_{jl}) > 0$. $f(0) < 0$ and

$$\begin{aligned} f(\overline{C}_j) - \overline{C}_j &= \tfrac{\overline{C}_j^{(1-\gamma)}(U_j - C_0^{\gamma})}{\theta(1-\gamma)} - \tfrac{\gamma\overline{C}_j}{1-\gamma} - \overline{C}_j \\ &\geq \tfrac{\overline{C}_j^{(1-\gamma)}}{1-\gamma}\left(\tfrac{C_0^{\gamma}}{\theta} - \overline{C}_j^{\gamma}\right) > 0. \end{aligned}$$

because $U_j \geq 2C_0^{\gamma}$ and individual undesirability implies $C_0^{\gamma} > \theta\overline{C}^{\gamma}$. Thus, there exists a unique $C_{jl} = C_{jl}^{*} \in (0, \overline{C}_{\gamma})$. □

This proposition shows two important properties of career path contracts. First, the optimal level of the s-attribute is constant across jobs in the same firm. Second, wages increase with career progression. Thus, in an optimal contract, within-firm inequality is driven by differences in wages rather than differences in nonwage attributes. This is consistent with Ouimet and Tate's (2022) findings of little within-firm variation in nonwage benefits but significant within-firm variation in wages.

Proposition IA.2 characterizes the optimal contracts conditional on a given vector of lifetime utilities $\{U_1, ..., U_n\}$. The equilibrium vector of lifetime utilities is, in turn, determined by supply and demand, as described in Section 3. Let $z \in \{1, ..., n\}$ denote the marginal type (as defined in Proposition 2), i.e., the worker type with the lowest surplus among those who are hired by flexible firms in equilibrium. Set this worker's equilibrium lifetime utility to $U_z^{*} = 2C_0^{\gamma}$. The next proposition shows that the profit potential is U-shaped:

Proposition IA.3 (Profit potential in career path equilibrium). *The profit potential $V(\alpha_j, U_z^{*})$ is U-shaped in α_j.*

Proof. A direct way to determine the effect of α_j on the profit potential is to use the Envelope Theorem:

$$\begin{aligned}\frac{\partial V(\alpha_j, 2C_0^\gamma)}{\partial \alpha_j} &= \lambda(s_j^* - w_{jl}^*)\gamma\theta C_{jl}^{*(\gamma-1)} + \lambda\frac{1}{m_j^*}(s_j^* - w_{jh}^*)\gamma C_{jh}^{*(\gamma-1)} \\ &= \frac{m_j^*}{1-\alpha_j}(s_j^* - w_{jl}^*) + \frac{1}{1-\alpha_j}(s_j^* - w_{jh}^*) \\ &= \frac{1}{(1-\alpha_j)}\left[s_j^*(m_j^*+1) - m_j^* w_{jl}^* - w_{jh}^*\right]\end{aligned} \tag{IA.41}$$

Since $w_{jl} = \frac{C_{jl}}{1-\alpha_j} - \frac{\alpha_j s_j^*}{1-\alpha_j}$, $w_{jh} = \frac{C_{jh}}{1-\alpha_j} - \frac{\alpha_j s_j^*}{1-\alpha_j}$, and $C_{jh} = \frac{1}{\theta^{\frac{1}{1-\gamma}}} C_{jl}$ it follows that:

$$\begin{aligned}\frac{1}{1-\alpha_j}(s^* - \frac{w_1+pw_2}{1+p}) &= \frac{1}{(1-\alpha_j)^2}\left(s_j^* - \frac{m_j^* C_{jl}^* + C_{jh}^*}{m_j^*+1}\right) \\ &= \frac{1}{(1-\alpha_j)^2}\left(s_j^* - \frac{C_{jl} + C_{jh}\frac{C_0^\gamma - \theta C_{jl}^\gamma}{C_{jh}^\gamma - C_0^\gamma}}{1+\frac{C_0^\gamma - \theta C_{jl}^\gamma}{C_{jh}^\gamma - C_0^\gamma}}\right) \\ &= \frac{1}{(1-\alpha_j)^2}\left(s_j^* - \frac{C_{jl}^{1-\gamma} C_0^\gamma}{\theta}\right),\end{aligned} \tag{IA.42}$$

where C_{jl} is given by: $C_{jl} = \frac{C_{jl}^{(1-\gamma)}}{\theta(1-\gamma)} C_0^\gamma - \frac{\gamma}{1-\gamma}\overline{C}_{jl}$. We now show that there exists a unique α_j for which

$$\frac{1}{(1-\alpha_j)^2}\left(s_j^* - \frac{C_{jl}^{1-\gamma} C_0^\gamma}{\theta}\right) = 0 \tag{IA.43}$$

The derivative of the left-hand side of (IA.43) is:

$$\frac{1}{(1-\alpha_j)^2}\left(\frac{\partial s_j^*}{\partial \alpha_j} - \frac{C_0^\gamma(1-\gamma)}{\theta C_{jl}^\gamma}\frac{\partial C_{jl}}{\partial \alpha_j}\right) + \frac{2}{(1-\alpha_j)^3}\left(s_j^* - \frac{C_{jl}^{1-\gamma} C_0^\gamma}{\theta}\right)$$

where

$$\frac{\partial C_{jl}}{\partial \alpha_j}\left(\frac{C_0^\gamma}{\theta C_{jl}^\gamma} - 1\right) = \frac{\gamma}{1-\gamma}\left(-y_l + \frac{\alpha_j(2-\alpha_j)}{2\sigma(1-\alpha_j)^2}\right),$$

$$\frac{\partial s_j^*}{\partial \alpha_j} = \frac{1}{\sigma(1-\alpha_j)^2}.$$

$$\begin{aligned}
\tfrac{1}{(1-\alpha_j)^2}\left(\tfrac{\partial s_j^*}{\partial \alpha_j} - \tfrac{C_0^\gamma(1-\gamma)}{\theta C_{jl}^\gamma}\tfrac{\partial C_{jl}}{\partial \alpha_j}\right) &= \tfrac{1}{(1-\alpha_j)^2}\left(\tfrac{\gamma C_0^\gamma y}{C_0^\gamma - \theta C_{jl}^\gamma} + \tfrac{1}{\sigma(1-\alpha)^2}\left(1 - \tfrac{\gamma C_0^\gamma}{C_0^\gamma - \theta C_{jl}^\gamma}\tfrac{\alpha(2-\alpha)}{2}\right)\right) \\
&= \tfrac{1}{(1-\alpha_j)^2}\left(\tfrac{\gamma C_0^\gamma y}{C_0^\gamma - \theta C_{jl}^\gamma} + \tfrac{1}{\sigma(1-\alpha)^2}\tfrac{2(C_0^\gamma(1-\alpha_j\gamma) - \theta C_{jl}^\gamma) + \alpha^2\gamma C_0^\gamma}{2(C_0^\gamma - \theta C_{jl}^\gamma)}\right) \\
&> 0.
\end{aligned}$$

We now show that there exists a unique $\alpha_{j'}$ such that $s_{j'}^* = \frac{C_{j'l}^{1-\gamma}C_0^\gamma}{\theta}$. First, we note that $s_j \mid_{\alpha_j=0} = 0$ and $s_j \mid_{\alpha_j=1} \to \infty$. $\frac{C_{jl}^{1-\gamma}C_0^\gamma}{\theta}$ first decreases and then increases in α_j, with $C_{jl} \in (0, \overline{C}_j)$. It follows that for $\alpha_j = 0$, $\frac{C_{jl}^{1-\gamma}C_0^\gamma}{\theta} > s_j^*$ and for $\alpha_j = 1$, $s_j^* > \frac{C_{jl}^{1-\gamma}C_0^\gamma}{\theta}$. Therefore $\alpha_{j'}$ such that $s_{j'}^* = \frac{C_{j'l}^{1-\gamma}C_0^\gamma}{\theta}$ exists. For $\alpha_j \leq \alpha_{j'}$, $\frac{C_{jl}^{1-\gamma}C_0^\gamma}{\theta} \geq s_j^*$; for $\alpha_j \geq \alpha_{j'}$, $\frac{C_{jl}^{1-\gamma}C_0^\gamma}{\theta} \leq s_j^*$. This and the fact that $\frac{1}{(1-\alpha_j)^2}\left(\frac{\partial s_j^*}{\partial \alpha_j} - \frac{C_0^\gamma(1-\gamma)}{\theta C_{jl}^\gamma}\frac{\partial C_{jl}}{\partial \alpha_j}\right) > 0$ imply that the left-hand side of (IA.43) is either always increasing or first decreasing and then increasing. For $\alpha_j = 0$ the left-hand side of (IA.43) is negative, while for $\alpha_j = 0$, it goes to ∞. It therefore follows that $\alpha_{j'}$ exists and is unique.

□

Proposition IA.3 is the equivalent of Proposition 1 for the case where firms offer career path contracts. It shows that the per-period profit potential for a fixed level of a worker's lifetime utility is also U-shaped. Since the shape of the profit potential function drives the main properties of the equilibrium, such properties continue to hold in the case of career path contracts. The proof of equilibrium existence now follows the same steps as in Proposition 2, which we omit for brevity.

The model in this section allows us to study the effect of the flexible technology on within-firm inequality and turnover rates.

Proposition IA.4 (**Technology, Inequality, and Turnover**). *A lower σ increases within-firm inequality and turnover rates.*

Proof. First, we show that $\frac{\partial C_{jl}}{\partial \sigma} < 0$. From (IA.29) we have:

$$\frac{\partial C_{jl}}{\partial \sigma}\left(\frac{U_j - C_0^{\gamma}}{\theta C_{jl}^{\gamma}} - 1\right) = \frac{\gamma}{1-\gamma}\frac{\partial \overline{C}_{jl}}{\partial \sigma} \tag{IA.44}$$

$$\Leftrightarrow \frac{\partial C_{jl}}{\partial \sigma} = -\frac{\gamma}{1-\gamma}\frac{\alpha_j^2}{2\sigma^2(1-\alpha_j)}\frac{1}{\left(\frac{U_j - C_0^{\gamma}}{\theta C_{jl}^{\gamma}} - 1\right)} < 0 \tag{IA.45}$$

Since $w_{jh}^* - w_{jl}^* = \frac{1}{1-\alpha_j}(C_{jh} - C_{jl})$,

$$\frac{\partial (w_{jh}^* - w_{jl}^*)}{\partial \sigma} = \frac{1}{1-\alpha_j}\left(\frac{1}{\theta^{\frac{1}{1-\gamma}}} - 1\right)\frac{\partial C_{jl}}{\partial \sigma} < 0. \tag{IA.46}$$

The probability of promotion is $p_j^* = \frac{U_j - C_0^{\gamma} - \theta C_{jl}^{\gamma}}{C_{jh}^{\gamma} - C_0^{\gamma}}$, where $C_{jh}^{\gamma} = \frac{1}{\theta^{\frac{\gamma}{1-\gamma}}} C_{jl}^{\gamma}$.

$$\frac{\partial p_j^*}{\partial \sigma} = -\frac{\partial C_{jl}}{\partial \sigma}\frac{\gamma C_{jl}^{\gamma-1}}{(C_{jh}^{\gamma} - C_0^{\gamma})^2}\left(\frac{U_j - C_0^{\gamma}}{\theta^{\frac{\gamma}{1-\gamma}}} - \theta C_0^{\gamma}\right) > 0 \tag{IA.47}$$

□

A change in the flexible technology that makes the firm more efficient in producing the s-attribute (i.e., σ is lower) translates into higher equilibrium s levels in both jobs. All else constant, such jobs become more attractive to workers. The firm benefits from improved labor supply by hiring more entry-level workers, which reduces the probability of promotion and increases turnover rates. Given the higher number of entry-level employees, the firm prefers to pay lower wages initially in exchange for higher wages for those who are promoted, thus increasing the wage gap between young and old workers.

5 Minimum Standards Regulation

Here we consider the effect of a simple regulatory proposal, such as a minimum requirement for s. For example, regulators can impose a minimum environmental standard, require a minimum provision of workplace amenities, or impose a minimum quota on workforce diversity. Let $\tilde{s}$ be the minimum s-quality requirement. We assume that the requirement is binding only for low-s firms.

Proposition IA.5 (Minimum standards). *Let z denote an unconstrained equilibrium. If a minimum standard $\tilde{s} \in \left(s^*_{\phi(z)}, s_0\right)$ is introduced, then the new equilibrium, $\tilde{z}$, is such that $\tilde{z} < z$, $\pi(\tilde{z}) < \pi(z)$, and $\tilde{w}^*(\alpha) > w^*(\alpha)$ for $\alpha > \tilde{z}$.*

Proof. For any $\alpha \leq \tilde{\alpha}$, where $h(\tilde{\alpha}) = \tilde{s}$, the firms are constrained to offer a sustainability level $\tilde{s}$. The maximum profit under the minimum standard is as follows: For $\alpha \leq \tilde{\alpha}$, $\tilde{v}(\alpha) = y - \tilde{w}(\alpha) - c(\tilde{s})$, where $\tilde{w}(\alpha) = w_0 + \frac{\alpha}{1-\alpha}(s_0 - \tilde{s})$; For $\alpha \geq \tilde{\alpha}$, $\tilde{v}(\alpha) = v(\alpha)$ (i.e., the minimum standard does not bind). It follows that $\tilde{v}(\alpha)$ is decreasing in α for $\alpha < k$ and increasing in α for $\alpha > k$. Thus the new equilibrium is determined by conditions in Corollary 4 for the function $\tilde{v}(\alpha)$.

Because $\tilde{s} > s^*_{\phi(z)}$ the minimum standard constraint binds at point $\phi(z)$ and therefore $\tilde{v}(\phi(z)) < v(z)$. This implies that $\tilde{\phi}(z) < \phi(z)$ and therefore

$$F > L\left(\int_0^{\tilde{\phi}(z)} p(\alpha)d\alpha + \int_z^1 p(\alpha)d\alpha\right), \tag{IA.48}$$

so the equilibrium $\tilde{z}$, must be such that $\tilde{z} < z$. This implies $\pi(\tilde{z}) < \pi(z)$, and $\tilde{w}(\alpha) > w(\alpha)$ for $\alpha > \tilde{z}$. □

The introduction of a binding minimum standard implies that low-s firms can no longer offer the efficient levels of the s-attribute to workers with low-s preferences. This constraint leads to a decrease in the equilibrium profits of all flexible firms. High-s workers benefit from the introduction of $\tilde{s}$ because they now earn higher wages and consume more. The next corollary describes the effect of the introduction of a minimum standard on the average s level in the flexible sector.

Corollary IA.1 (Minimum Standards and Average S-Quality). *The minimum standard increases the average s in the flexible sector by*

$$\int_0^{\tilde{\phi}(z)} (\tilde{s} - s_\alpha) p(\alpha) d\alpha + \int_{\tilde{z}}^{z} s_\alpha p(\alpha) d\alpha - \int_{\phi(z)}^{\tilde{\phi}(\tilde{z})} s_\alpha p(\alpha) d\alpha. \tag{IA.49}$$

Proof. The difference in the average s level with and without the minimum standard $\tilde{s}$ is:

$$\int_0^{\tilde{\phi}(\tilde{z})} \tilde{s} p(\alpha) d\alpha + \int_{\tilde{z}}^{1} s_\alpha p(\alpha) d\alpha - \int_0^{\phi(z)} s_\alpha p(\alpha) d\alpha - \int_z^1 s_\alpha p(\alpha) d\alpha. \tag{IA.50}$$

Since $\int_{\tilde{z}}^{z} p(\alpha) d\alpha = \int_{\tilde{\phi}(\tilde{z})}^{\phi(z)} p(\alpha) d\alpha$, equation (IA.50) becomes:

$$\int_0^{\phi(z)} \tilde{s} p(\alpha) d\alpha + \int_{\tilde{z}}^{z} s_\alpha p(\alpha) d\alpha - \int_0^{\phi(z)} s_\alpha p(\alpha) d\alpha - \int_{\tilde{z}}^{z} \tilde{s} p(\alpha) d\alpha \tag{IA.51}$$

The increase in the average s in the flexible sector is:
$\int_0^{\phi(z)} (\tilde{s} - s_\alpha) p(\alpha) d\alpha + \int_{\tilde{z}}^{z} (s_\alpha - \tilde{s}) p(\alpha) d\alpha$. □

As expected, the introduction of a binding minimum standard leads to an increase in the average s level in the flexible sector. However, the low-s firms' reaction to introducing a minimum standard is heterogeneous. Some firms adjust on the intensive margin by increasing their s levels to meet the minimum standard (i.e., $\tilde{s}$). This effect is measured by $\int_0^{\tilde{\phi}(z)} (\tilde{s} - s_\alpha) p(\alpha) d\alpha$. Other firms adjust on the extensive margin by becoming high-s firms. This effect is measured by $\int_{\tilde{z}}^{z} s_\alpha p(\alpha) d\alpha - \int_{\phi(z)}^{\tilde{\phi}(\tilde{z})} s_\alpha p(\alpha) d\alpha$. As more firms now choose to locate at the high-s end, high-s workers benefit from an increase in the demand for their types.

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