Liability for Non-Disclosure in Equity Financing

Law Working Paper N° 678/2023
February 2023

Albert H. Choi
University of Michigan and ECGI

Kathryn E. Spier
Harvard University and NBER

ECGI Working Paper Series in Law

Liability for Non-Disclosure in Equity Financing

Working Paper N° 678/2023
February 2023

Albert H. Choi
Kathryn E. Spier

We would like to thank Stephen Choi, Ofer Eldar, Ann Lipton, Paul Mahoney, Nadya Malenko, Teddy Mekonnen, Harry Pei, Adam Pritchard, and James Spindler; seminar participants at University of Michigan Law School, University of Michigan Economics Department, 2022 NBER Organizational Economics Workshop, 2022 NBER Summer Institute Law and Economics Workshop, Hong Kong University of Science and Technology (HKUST) Economics Department, Corporate Law Academic Webinar Series (CLAWS), and University of Michigan Finance Department; and conference participants at 2021 Theoretical Law and Economics Conference at Northwestern University Law School, 2022 Society for Institutional & Organizational Economics (SIOE) Conference at University of Toronto, 2022 Tulane Corporate and Securities Law Conference, and 2022 Corporate and Securities Litigation Workshop for many helpful comments and suggestions.

Abstract

This paper analyzes the effects of holding firms liable for non-disclosure of material information when raising capital. A privately-informed entrepreneur may choose to withhold material information from prospective investors. After cash flows are realized, investors may sue the firm ex post for the entrepreneur's (alleged) non-disclosure. Any damages award received by investors is partially offset by the reduced value of their equity stake. Absent liability, entrepreneurs have an excessive incentive to withhold bad news and pursue socially-wasteful projects. Liability for non-disclosure deters the entrepreneur and prevents misallocation of capital. The socially-optimal damage award may be supra-compensatory, exceeding the overcharge paid by the investors. Depending on the likelihood of court error and litigation cost (including the rent captured by lawyers), socially optimal liability may be either zero or the minimum necessary for full deterrence. After presenting the basic results, liability waivers and empirical implications are also discussed.

Keywords: Equity Financing, IPO Liability, Non-Disclosure, Securities Litigation, Liability Waiver, Class Action Waiver

JEL Classifications:

Albert H. Choi*
Paul G. Kauper Professor of Law
University of Michigan
3230 Jeffries Hall
Ann Arbor, MI 48109-3091, USA
e-mail: alchoi@umich.edu

Kathryn E. Spier
Domenico De Sole Professor of Law
Harvard University
Hauser 302 1563 Massachusetts Avenue
Cambridge, MA 02138, USA
e-mail: kspier@law.harvard.edu

*Corresponding Author

Liability for Non-Disclosure in Equity Financing*

Albert H. Choi
University of Michigan Law School

Kathryn E. Spier
Harvard Law School

January 16, 2023

Abstract

This paper analyzes the effects of holding firms liable for non-disclosure of material information when raising capital. A privately-informed entrepreneur may choose to withhold material information from prospective investors. After cash flows are realized, investors may sue the firm ex post for the entrepreneur's (alleged) non-disclosure. Any damages award received by investors is partially offset by the reduced value of their equity stake. Absent liability, entrepreneurs have an excessive incentive to withhold bad news and pursue socially-wasteful projects. Liability for non-disclosure deters the entrepreneur and prevents misallocation of capital. The socially-optimal damage award may be supra-compensatory, exceeding the overcharge paid by the investors. Depending on the likelihood of court error and litigation cost (including the rent captured by lawyers), socially optimal liability may be either zero or the minimum necessary for full deterrence. After presenting the basic results, liability waivers and empirical implications are also discussed.

*We would like to thank Stephen Choi, Ofer Eldar, Ann Lipton, Paul Mahoney, Nadya Malenko, Teddy Mekonnen, Harry Pei, Adam Pritchard, and James Spindler; seminar participants at University of Michigan Law School, University of Michigan Economics Department, 2022 NBER Organizational Economics Workshop, 2022 NBER Summer Institute Law and Economics Workshop, Hong Kong University of Science and Technology (HKUST) Economics Department, Corporate Law Academic Webinar Series (CLAWS), and University of Michigan Finance Department; and conference participants at 2021 Theoretical Law and Economics Conference at Northwestern University Law School, 2022 Society for Institutional & Organizational Economics (SIOE) Conference at University of Toronto, 2022 Tulane Corporate and Securities Law Conference, and 2022 Corporate and Securities Litigation Workshop for many helpful comments and suggestions. Comments are welcome to alchoi@umich.edu and kspier@law.harvard.edu.

1 Introduction

On May 17, 2012, Facebook went public by selling more than 421 million shares of common stock at $38 per share to public investors on the Nasdaq and raised about $16 billion from the investors. Unlike many other initial public offerings that experience an initial price surge, Facebook's stock price declined shortly after the initial public offering (IPO), hitting a low of $18. It took more than a year for the stock price to rebound to the IPO price of $38. Many public investors, who bought Facebook's shares at the IPO or shortly after, were quite unhappy and brought class action suits against the company under the US Securities Act, claiming that Facebook failed to disclose the fact that more users were using their mobile phones to access Facebook's websites instead of their computers, and the company's advertising revenues were lower than as described in the IPO documents.[1] After more than five years of pre-trial procedures, immediately before the case was to go to trial, the litigants agreed to settle the case for $35 million in 2018.[2]

As the Facebook story demonstrates, when a company raises capital by selling securities to outside investors, the US securities laws require the company to disclose relevant material information it possesses to the prospective investors. In case the company fails to do so, the investors can bring suit against the company to recover compensatory damages. Presumably, such a liability regime ensures that the outside investors will receive necessary information from the company to make an informed decision as to whether to purchase the offered security. At the same time, though, critics have argued that the private enforcement regime, especially the class action system, is too costly and encourages indiscriminate lawsuits against even innocent companies.[3] To what extent does such a liability regime induce the company to disclose relevant material information to the investors? Does liability lead to a more efficient allocation of capital? Should the investors be allowed to bring class actions or be required to bring suit on an individual basis, as some advocates have argued? The objective of this paper is to answer some of these questions with the help of game theoretic modeling.

The paper presents a simple, stylized model of a resource-constrained entrepreneur

[1]See Atkins (2018) and Graf (2018).

[2]Id. Out of $35 million settlement, plaintiff's attorneys received almost $14 million as fees and costs. See Graf (2018). Although the size of settlement in the Facebook case was exceptionally small compared to the size of the IPO and the potential recovery, according to Lowry and Shu (2002), the average size of settlement in their sample was about 11% of the total proceeds raised in IPOs.

[3]See Scott (2017 and 2019). Scott has argued that most of the securities class actions are without merit and the companies should be allowed to bar securities class actions through a mandatory non-class arbitration provision in their charters or bylaws.

who seeks outside funding to pursue a risky business venture.[4] The capital market is competitive and investors are sophisticated. Before raising the necessary capital, the entrepreneur may observe a private signal revealing the project's future cash flow (high or low). High-value projects are socially worthwhile while low-value projects are socially wasteful. Both the arrival of the signal and its contents are privately observed by the entrepreneur. If the entrepreneur learns that the value is high, the entrepreneur discloses the good news to the investors and lowers the cost of capital. If the entrepreneur learns that the value is low, on the other hand, the entrepreneur is tempted to withhold the bad news and masquerade as having an "average" project to secure funding. Since low-value projects are socially wasteful, withholding bad news and raising outside funding leads to a misallocation of capital.

Firm liability for non-disclosure can deter the entrepreneur and prevent the misallocation of capital. After the cash flows are realized and evidence mounts that the entrepreneur hid material information from investors, the investors may sue the firm for damages. Litigation is costly (with lawyers capturing some rent) and the court may find innocent entrepreneurs liable (false positives). Furthermore, any damages award received by the investors is offset in part by the reduced value of their equity stake in the firm. Notwithstanding the complexities, the analysis shows that when the damages award is below a threshold, the entrepreneur is either partially deterred (randomizes between pursuing and abandoning the low-type projects) or not deterred at all. Deterrence is stronger when the entrepreneur has a larger personal stake in the venture and when the likelihood that the entrepreneur is privately informed is larger.[5] When the damage award is above a threshold, on the other hand, full deterrence is obtained. Full deterrence may require supra-compensatory damages, where the damage award collected by investors exceeds the overcharge that they paid for the equity stake.[6]

Our analysis delivers both normative and positive insights. First, if the social cost of litigation (lawyers' opportunity cost) outweighs the deterrence benefits of firm liability, then it is socially desirable to eliminate firm liability altogether. Interestingly, the entrepreneur's private incentive to allow investor lawsuits can be either excessive or insufficient in equilibrium. On the one hand, the option to litigate serves as a type

[4]If the entrepreneur has sufficiently deep pockets and can commit her own resources to the venture, then liability for non-disclosure is unnecessary. The entrepreneur has a private incentive to abandon socially wasteful projects. If the entrepreneur is sufficiently resource constrained, liability is necessary to deter the entrepreneur from withholding bad news from investors.

[5]If it is common knowledge that the entrepreneur has the private information, full unraveling occurs as in Grossman (1981) and Milgrom (1981) and we get full deterrence.

[6]This result is driven by the fact that the damage award received by investors is paid by the firm, and the firm's equity is held by both the entrepreneur and the investors.

of "warranty" and signals to the investors that the entrepreneur is not withholding any material information. Since the entrepreneur who possesses bad news mimics the uninformed entrepreneur, there may be too many lawsuits in equilibrium. On the other hand, because lawsuit leads to lawyers' capturing rent and adjudication error, the entrepreneur may opt out of liability even though litigation is beneficial for society. In short, we cannot rely solely upon market self-regulation to choose the optimal liability regime.

Our analysis also delivers various empirical predictions. First, holding the liability system fixed, as the fraction of outside equity ownership rises, the number of private securities lawsuits will fall. Moreover, when lawsuits are brought, they will tend to be initiated by small retail investors rather than large institutional investors. Large institutional investors, concerned about the drop in their equity holdings, would be much more hesitant to bring suit. Second, when the level of deterrence is weaker, the frequency of over-pricing at the IPO stage rises while the magnitude of the over-pricing shrinks.[7] Estimating the frequency and size of over-pricing may offer an indirect way of measuring the level of deterrence. Finally, as the court becomes more prone to err and the lawyers capture a larger rent from the litigation, firms would become more likely to waive liability (or advocate for liability waiver).

This paper extends the literature on the disclosure of information prior to the sale of an asset. Grossman (1981) and Milgrom (1981) introduced the famous unraveling result when sellers are privately informed about asset quality. Sellers of high quality assets have a clear incentive to disclose this information (to obtain a better selling price) and, as a consequence, sophisticated buyers draw adverse inferences when sellers *do not* disclose. Grossman and Hart (1981) explored the implications of full unraveling for disclosure laws in corporate takeovers.[8] Dye (1985), Farrell (1986), and Shavell (1994) show that complete unraveling does not occur when buyers are uncertain as to whether the sellers actually have private information. In their analyses, sellers with low quality assets have an incentive to withhold the information from the market and pool with the uninformed types.[9] These papers all assume that disclosure, if

[7] On the flip side, the frequency of under-pricing rises and the magnitude of the under-pricing grows.

[8] Grossman and Hart conclude that "the commonly held view that firms withhold information (which is free to release) in order to mislead traders into giving them better terms is false." (p. 333)

[9] Shavell (1994) focuses on the incentive of sellers to acquire information prior to a sale. Although mandatory disclosure may be socially desirable conditional on the seller acquiring the information, mandatory disclosure may chill the collection of socially valuable information. Polinsky and Shavell (2012) show that when sellers are strictly liable for consumer harms stemming from defective products, and can take precautions to reduce product risks, then mandatory and voluntary disclosure are equivalent.

mandated, is perfectly enforced and that the seller does not retain an equity stake in the asset.

Dye (2017) explores a model where mandatory disclosure is imperfectly enforced. If the seller fails to disclose material information, the court awards damages that are proportional to the over-payment by the buyer (relative to what they would have paid had the seller disclosed the information). Our analysis differs from Dye's core model in several important respects. First, in Dye (2017), the asset sale is assumed to always takes place (and efficient) and the seller simply chooses the disclosure strategy to maximize the selling price. Liability for non-disclosure affects the seller's equilibrium disclosure strategy but does not affect the asset allocation.[10] In our analysis, whether or not the entrepreneur raises capital (sells equity) is a *choice variable*. Liability for non-disclosure is socially valuable because it deters the entrepreneur from selling equity in socially-wasteful projects and avoids the misallocation of capital. Second, in Dye (2017), the seller is personally liable for the damage payment. In our analysis, *the firm itself* is liable for the damage payment. The seller's accountability for non-disclosure is limited to their (endogenous) financial stake in the firm.[11] In our model, the damage award received by the buyers is paid, in part, by the buyers themselves: the buyers are in effect taking money out of one pocket and putting it into the other. Third, our analysis considers the possibility of false positives and the cost of litigation and the impact they have on disclosure and deterrence. This also allows us to examine the entrepreneur's incentive of managing the ex post verification process through liability and class action waivers.

Several scholars have also examined the impact on liability system on the securities markets, especially on the IPO market. Hughes and Thakor (1992), for instance, examine the idea of an underwriter deliberately under-pricing its stock at the IPO so as to avoid potential lawsuit ex post. In their analysis, over or under-pricing at IPO can happen because the underwriter can be either "myopic" or "nonmyopic" in making its pricing decision.[12] Lowry and Shu (2002) empirically examines the litigation risk on IPO under-pricing and show that firms with higher legal exposure

[10]Dye (2017) Section 7 allows for inefficient delay and fractional ownership. Social welfare falls when the seller retains a larger fraction of the asset. In our model, increasing the seller's stake is socially efficient insofar as it deters sellers with low-value assets from participating. There are other differences. Dye (2017) does not allow for false positives (see Section 3).

[11]Caskey (2014) develops a securities pricing model to explore the price effects of pending litigation, focusing on the role of litigation insurance. The damages received by investors is offset by a dilution of their equity stake. Caskey (2014) also does not consider the misallocation of capital. Spindler (2007) examines the put option feature of the liability regime and examines the entrepreneur's behavior post IPO.

[12]In an earlier paper, Hughes (1986) allows for a privately informed firm to disclose information to an underwriter who, in turn, verifies the disclosure. Alexander (1993) takes issue with Hughes

tend to under-price their offerings more and also that under-pricing decreases the expected litigation costs.[13] Focusing more on the class action securities lawsuits, Scott and Silverman (2013) have argued that the class action system has many deficiencies and we should allow firms to adopt mandatory individual arbitration when they go public.[14] This paper attempts to examine the issues of disclosure more closely and to shed some light on the optimal liability system, including whether allowing class action waivers can be beneficial.

The paper is organized as follows. Section 2 presents the baseline model without any court error or litigation cost. Without any potential deadweight loss from litigation cost and court error, firm liability improves disclosure incentives and leads to a more efficient allocation of capital. Section 3 presents a general model that allows for both court error and litigation cost (lawyer cost). While the equilibrium is qualitatively similar to that in the baseline model, the general model highlights the divergence of social and private welfare. Building on this, the section examines whether, and the conditions under which, the firm's incentive to opt out of liability (through a liability waiver) may be excessive or insufficient. Section 4 offers various empirical predictions that the analysis generates, and the last section concludes with thoughts for future research.

2 The Baseline Model

Suppose that an entrepreneur E owns a firm that needs capital of $c > 0$. When E raises c and incurs a personal cost of $e > 0$, $c + e$ is invested and the cash-flow stream of $x > 0$ is realized, where $x \in \{x_h, x_l\}$ and $prob(x = x_l) = q \in (0, 1)$.[15] Let $k \equiv c + e$, where k stands for the total investment necessary for the project, and $\overline{x} \equiv q \cdot x_l + (1 - q) \cdot x_h$. We assume that $max\{e, c\} < x_l < k < x_h$ and $\overline{x} > k$ so that financing is efficient either when $x = x_h$ or when E is uninformed, but not when

and Thakor (1992) and argues that when we take into consideration the more complex legal issues, it is unlikely that the legal liability will lead to under-pricing of IPO shares.

[13] Prior studies showed little or no difference in returns in firms that were sued versus those who were not or little difference in characteristics among the cases that were filed. See Tinic (1988), Drake and Vetsuypens (1993), and Bohn and Choi (1996). See Ritter and Welch (2002) and Ritter (2011) for a more extensive review of the literature.

[14] See also Scott (2017, 2019). Webber (2015), on the other hand, argues that elimination of the class action system can lead to cross-subsidization by small, individual shareholders to large, institutional ones.

[15] We can think of E's personal cost of e as either E's costly effort or the amount of personal financial capital E has to pledge to get financing.

$x = x_l$.[16]

We assume that E raises capital from a competitive capital market by having the firm sell fraction $\alpha \in [0, 1]$ of the firm's equity to outside investors, whose reservation value is normalized to zero. For instance, with complete information and when $x = x_h$, with competitive capital markets, the outside investors would pay c for a fraction $\alpha = c/x_h$ of the equity of the firm. The outside investors break even in this complete information scenario, since $\alpha \cdot x_h - c = (c/x_h) \cdot x_h - c = 0$.

There are five periods in the game with no discounting, $t \in \{0, 1, 2, 3, 4\}$, and the timing of the game is as follows. At $t = 0$, Nature chooses $x = x_l$ with probability q and $x = x_h$ with probability $1-q$. E learns the realized x with probability $\pi \in (0, 1)$. E who learns x is "informed" while E who does not learn x is "uninformed." Among the informed E, we denote E (and the firm) who knows that $x = x_h$ as the "h-type" and that $x = x_l$, the "l-type." We denote the uninformed E as the "u-type." Hence, there are three possible types of E (or the firm): u-type, h-type, and l-type.

At $t = 1$, E decides whether to participate and raise capital. If E chooses not to participate, then the game ends and both the investors and E get a return of zero. If E chooses to participate, then the game continues.[17]

At $t = 2$, the informed E chooses whether to disclose x or not disclose ("withhold") x. Following the literature, disclosures are accurate, i.e., E cannot present false evidence, and that the uninformed E cannot pretend to know x. The outside investors observe the entrepreneur's disclosure decision and the disclosed information, if any, and update their belief about x.

At $t = 3$, the firm attempts to raise capital c by issuing equity stake α to outside investors.[18] Outside investors are rational and forward looking and the capital market is competitive. The equity stake α allows investors to break even given their (endogenous) beliefs about the value of the firm x and their returns from future litigation. If

[16] The assumption that $e < x_l$ rules out the uninteresting case where the l-type would never participate, even if $\alpha = 0$. Also, the assumption that $c < x_l$ is made for largely technical convenience. Taken together, our assumptions imply that $q < \overline{q} = \frac{x_h - k}{x_h - x_l} \in (0, 1)$.

[17] We are imagining that E's choice to participate or not participate is a commitment, and cannot change her mind later. This will get rid of the equilibria where investors make positive profits and simplifies the equilibrium characterization.

[18] Although the firm can raise capital by issuing debt and the liability system is not confined to equity securities, because debt tends to be (much) less "information sensitive" and most of the legal issues arise from stock sales, we focus on equity financing.

the firm fails to raise capital, then the game ends.[19]

At $t = 4$, investments c and e are sunk and all returns are realized. Investors (now, shareholders of the firm) learn (1) value $x \in \{x_h, x_l\}$ and (2) whether E withheld information at $t = 2$, and can decide to bring a suit against the firm.[20] If E withheld information, the court awards damages $d \in [0, x_l]$.[21] Note that we are assuming that the limited liability principle applies and the firm cannot be responsible for more than its cash-flow (x_l). Also, while it is natural to assume that the damages are compensatory and equal to the overcharge paid by investors,[22] we also allow for no damages ($d = 0$) and punitive damages.

Our equilibrium concept is perfect Bayesian Nash equilibrium (PBNE). Several observations will simplify the analysis of this game. First, if the l-type chooses to participate, it will not disclose x_l to the market. To see why, suppose that the l-type did disclose x_l to the market. Investors would demand equity stake $\alpha_l = c/x_l$ in return for investing c in the business venture. The l-type's net return if it participates and discloses x_l is $(1 - \alpha_l)x_l - e = x_l - c - e < 0$: the l-type is losing money. So, if the l-type participates at $t = 1$, it will not disclose x_l to the capital market. (Depending on the parameter values, the l-type may or may not participate in equilibrium.)

Second, in any equilibrium where the l-type participates and raises capital with a positive probability, the u-type participates, too. To see why, let α^* be the equilibrium equity stake demanded by investors when there is no disclosure. The l-type's return, $(1 - \alpha^*)x_l - e$ (minus any damages owed to outside investors), is strictly smaller than the u-type's return, $(1 - \alpha^*)\overline{x} - e$. So, if the l-type's financial return is non-negative, then the u-type's return is strictly positive.

Finally, in any equilibrium, the h-type will participate in the market, disclose its information (of x_h), and succeed in raising capital. Since the capital market draws

[19] Initially, we assume that the future litigation system is dictated by the legal system and the litigation parameters (such as the damages and the cost of litigation) are commonly known. Later, we will relax this assumption to allow the firm to choose a different liability system, for instance, through a liability waiver or a class action waiver.

[20] We are assuming here than the entrepreneur is not directly liable for non-disclosure. Technically, the security is being sold by and the representations are being made by the firm. Hence, imposing liability on the firm would seem natural. If we assume that the entrepreneur does not have sufficient assets to pay for monetary damages, such an assumption may also be realistic. The case of holding the entrepreneur directly liable for non-disclosure is considered in the Online Appendix.

[21] For now, we are assuming that there are no "false positives," the u-type cannot be found liable for non-disclosure after x_l has been realized. These assumptions are relaxed in Section 3.

[22] Instead of fixed damages d, we can allow the investors to recover $max\{0, min\{\theta(\alpha x_l - c), x_l\}\}$ where $\theta \in [0, \infty)$. In that case, the analysis on the liability system will examine changes in θ instead of d. We are adopting d for its analytical simplicity.

adverse inferences from non-disclosure, the h-type firm has an incentive to disclose x_h to secure a better deal with investors. These observations are summarized in the following Lemma.[23]

Lemma 1. *In any PBNE where the u-type participates, the h-type participates, discloses x_h, and issues equity stake $\alpha_h = c/x_h$; the u-type issues equity stake $\alpha^* \geq c/\overline{x} > \alpha_h$; the l-type participates with probability $\beta^* \in [0, 1]$, does not disclose x_l, and pools with the u-type.*

In the analysis that follows, we will construct the PBNE where the h-type participates and discloses, the u-type participates and does not disclose, and the l-type partially pools with the u-type. In particular, we characterize values (α^*, β^*) where α^* is the associated equity stake demanded by investors (conditional on non-disclosure) and β^* is the l-type's participation probability.

2.1 Full-Information Benchmark

Since $x_l < k < \overline{x} < x_h$, it is socially efficient for the entrepreneur to raise capital and pursue the venture *unless* the project is known to have low value (x_l). If a social planner possessed the same information as the entrepreneur, the social planner would fund the project if the value was known to be high ($x = x_h$) or if the project had unknown value, but not if $x = x_l$. As the following proposition demonstrates, this outcome would be obtained in a competitive market if the investors have the same information as the entrepreneur.

Proposition 1. *Suppose that the capital market has the same information as the entrepreneur.*

1. *If the entrepreneur and capital market learn that $x = x_h$, then the investors pay c for equity stake $\alpha_h = c/x_h$. E's return is $x_h - c - e > 0$.*

2. *If the entrepreneur and capital market learn that $x = x_l$, then no capital is raised. E's return is zero.*

3. *If the entrepreneur and capital market do not learn x, then investors pay c for equity stake $\overline{\alpha} = c/\overline{x}$. E's return is $\overline{x} - c - e > 0$.*

[23] The formal proof is in the Appendix. Later, we will prove that an equilibrium with u-type participation always exists. For some parameter values, there will also exist trivial PBNE where the u-type does not participate, supported by the market's belief that if there is no disclosure then the firm is the l-type for sure. The additional assumption that $e < (1 - c/x_l)\overline{x} - e$ would rule out such equilibria.

The entrepreneur raises capital and pursues the business venture *unless* the project is commonly known to be of low value (i.e., unless the market knows $x = x_l$).[24] Finally, note that when x is not observed then the equity stake demanded by investors reflects the average value ($\overline{x}$).[25] E's equity stake $1 - \overline{\alpha}$ is just large enough to allow the investors to break even on average.

2.2 Equilibrium Characterization

We now characterize the PBNE where the outside investors demand an equity stake $\alpha^* \in [0,1]$ and the l-types participate and raise capital with probability β^*. Let's begin with the l-type's decision to participate and raise capital. If the l-type participates, the outside investors will bring suit against the firm for damages d. After the payment of damages to the investors, the residual firm value of $x_l - d \geq 0$ is divided between the outside investors and the entrepreneur in proportions α^* and $(1 - \alpha^*)$. The l-type raises capital if the gross return for the entrepreneur, $(1 - \alpha^*)(x_l - d)$, exceeds the personal investment e. We have the following characterization of β^*:

$$\begin{cases} \beta^* = 0 & \text{if} \quad (1-\alpha^*)(x_l - d) - e < 0 \\ \beta^* \in [0,1] & \text{if} \quad (1-\alpha^*)(x_l - d) - e = 0 \\ \beta^* = 1 & \text{if} \quad (1-\alpha^*)(x_l - d) - e > 0 \end{cases} \tag{1}$$

Depending on the level of damages d, the equilibrium may involve full deterrence of the l-type ($\beta^* = 0$), partial deterrence ($\beta^* \in (0,1)$), or no deterrence ($\beta^* = 1$).

Next, consider the equity stake demanded by outside investors. If the entrepreneur is the uninformed u-type, by assumption, there is no lawsuit and the investors get, in expectation, a net return of $\alpha^*\overline{x} - c$. If the entrepreneur is the informed l-type, the outside investors bring suit and collect damages of d from the firm. Because the damages are paid by the firm, the value of the outside investors' equity stake falls to $\alpha^*(x_l - d)$.[26] Thus, the damages awarded to the investors are paid, in part, by the investors themselves. The investors' break-even condition (from the *ex ante*

[24] If $x_l > k$ then raising capital when $x = x_l$ is socially efficient. Investors would be willing to pay c for equity stake $\alpha_l = c/x_l$.

[25] As $q \to 0$ then $\overline{x} \to x_h$ and $\overline{\alpha} \to \alpha_h$. As $q \to \overline{q} = \frac{x_h - k}{x_h - x_l}$ then $\overline{x} \to c + e$ and $\overline{\alpha} \to \frac{c}{c+e}$.

[26] We are assuming here that the investors have not sold their shares when they bring suit. We can relax this assumption later. When the parameters of the lawsuit are common knowledge and the financial market is sufficiently forward-looking, when the investors sell the stock, the stock price would reflect the returns from prospective lawsuits. When litigation is costly, however, the credibility constraint will differ depending on whether the investors have sold their stock. See Assumption 2 in Section 3.

perspective) is:[27]

$$(1-\pi)(\alpha^*\overline{x}-c)+\pi q\beta^*(\alpha^* x_l - c + (1-\alpha^*)d) = 0. \tag{2}$$

As the investors' break-even condition shows, the equilibrium α^* will depend on the equilibrium probability of participation by the l-type ($\beta^* \in [0,1]$).

If the damage award d is above a threshold, l-type entrepreneur will be fully deterred from raising capital: $\beta^* = 0$. We now characterize this upper threshold, $\overline{d}$. First, consider the investors' willingness to supply capital. Setting $\beta^* = 0$ in the investors' break-even condition (2), we have $\alpha^*\overline{x}-c = 0$. Thus, if the outside investors believe that the l-type entrepreneur is fully deterred, they will demand equity stake of

$$\alpha^* = \overline{\alpha} = \frac{c}{\overline{x}} < 1. \tag{3}$$

The equity stake when no information is disclosed reflects the average value of the uninformed u-type only. Now, consider the l-type's decision to participate and raise capital. From (1), the l-type is fully deterred when $(1-\overline{\alpha})(x_l - d) - e < 0$. Since the left-hand side is a decreasing function of d, the threshold damage award $\overline{d}$ satisfies this expression with equality. The following Lemma characterizes this threshold.

Lemma 2. *(Full Deterrence.) There exists a threshold $\overline{d} \geq 0$ such that the l-type is fully deterred, $\beta^*(d,\pi) = 0$, if $d > \overline{d}$. If $e \geq (1-c/\overline{x})x_l$ then $\overline{d} = 0$.[28] If $e < (1-c/\overline{x})x_l$ then*

$$\overline{d} = x_l - \frac{e}{1-c/\overline{x}} > 0. \tag{4}$$

The full-deterrence threshold $\overline{d}$ in (4) has several notable properties. First, $\overline{d}$ is independent of π. With the complete deterrence of the l-type, the outside investors need not worry about the "degree" of adverse selection, represented by π. Second, $\overline{d}$ is a strictly decreasing function of e. Deterrence is easier to achieve when the entrepreneur has more at stake in the venture. Third, as e approaches 0, $\overline{d}$ approaches x_l. When the entrepreneur personally invests very little in the venture, full-deterrence requires damages that effectively liquidate the firm's assets with nothing remaining for the entrepreneur. Interestingly, full deterrence may require supra-compensatory damages in the sense that the investors collect more in damages than the overcharge

[27] Conditional on non-disclosure by the entrepreneur, the investors' break-even constraint is:

$$\frac{1-\pi}{(1-\pi)+\pi q}(\alpha^*\overline{x}-c)+\frac{\pi q}{(1-\pi)+\pi q}\beta^*(\alpha^* x_l - c + (1-\alpha^*)d) = 0.$$

This is equivalent to (2). The investors' equilibrium ex ante return from the h-type is $\alpha_h x_h - c = 0$.

[28] Since $k = c + e$, we can rewrite $(1-c/\overline{x})x_l = \frac{\overline{x}-k}{\overline{x}-x_l}x_l \equiv \widehat{e}$. Then, $\overline{d} < (>) 0$ when $e > (<) \widehat{e}$.

for the assets.[29] Fourth, when e approaches $(1 - c/\overline{x})x_l$, the full-deterrence threshold $\overline{d}$ approaches zero. If $e \geq (1 - \overline{\alpha})x_l$ then the l-type is fully deterred for all $d \geq \overline{d} = 0$, so liability is unnecessary for deterrence.[30] To streamline the analysis we make the following assumption.

Assumption 1: $e < (1 - \overline{\alpha})x_l$.

Assumption 1 implies that in the absence of liability, $d = 0$, the l-type is less-than-fully deterred.

At the opposite end of the spectrum, if the damage award d is below a threshold, the l-type entrepreneur will be completely undeterred, $\beta^* = 1$. Unlike the full deterrence case, with full participation by the l-type, now the degree of adverse selection (π) matters. The next Lemma characterizes the lower threshold, $\underline{d}(\pi)$. As the Lemma (along with its proof) demonstrates, full participation by the l-type (no deterrence) becomes more likely as d, π, or both get smaller.

Lemma 3. *(No Deterrence.) There exist thresholds $\underline{d}(\pi) \in [0, \overline{d}]$ and $\widehat{\pi}_0 \in (0, 1)$ such that the l-type is undeterred, $\beta^*(d, \pi) = 1$, when $d < \underline{d}(\pi)$. If $\pi > \widehat{\pi}_0$ then $\underline{d}(\pi) = 0$. If $\pi \leq \widehat{\pi}_0$ then*

$$\underline{d}(\pi) = x_l - \frac{e}{1 - c/\overline{x} - r(\pi)} \tag{5}$$

where

$$r(\pi) = \frac{\pi q}{1 - \pi} \cdot \frac{k - x_l}{\overline{x}}. \tag{6}$$

$\underline{d}(\pi)$ is strictly decreasing in π with $\underline{d}(0) = \overline{d}$, $\underline{d}(\widehat{\pi}_0) = 0$, and $r(\widehat{\pi}_0) = 1 - c/\overline{x} - e/x_l$.

While the formal details are in the proof of Lemma 3, the result may be understood intuitively. The function $r(\pi)$ defined in (6) is the incremental equity stake demanded by the outside investors when the l-type is (just) undeterred, $\beta^* = 1$. To just break even in expectation, the outside investors require equity stake $c/\overline{x} + r(\pi)$. Intuitively, if the l-types are undeterred, the outside investors will demand a "premium" to compensate for the increased risk that they face. The risk premium demanded by outside investors, $r(\pi)$ in equation (6), is higher when the fraction of informed types, π, is larger.

[29]This is true given that $\overline{d}$ is decreasing in e and $\overline{d} = x_l > c - \alpha x_l \ \forall \alpha \in (0, 1]$ when $e = 0$.

[30]In an initial public offering, the insiders (including the founders, officers, and venture capitalists) are often contractually prohibited from selling their stock for a certain period (the "lock-up" agreement). The entrepreneur making a personal investment of e can be thought of as being similar to such an arrangement, since without a lock-up agreement, the entrepreneur may be able to divest her investment quickly after raising capital from the outside investors before any of the informational issues are uncovered.

Note also that the threshold $\underline{d}(\pi)$ in Lemma 3 is a decreasing function of π. When π rises, full participation of the l-types is unsustainable in equilibrium. It is not hard to see why this is true. When π rises and the outside investors demand a larger equity stake compensate them for the increased risk, the entrepreneur's cost of capital becomes higher, too. The higher cost of capital discourages the l-types from participating in the market. When π is above a threshold $\widehat{\pi}_0$, therefore, the l-type is partially deterred ($\beta^* < 1$) even when there is no liability ($d = 0$). At the same time, without any liability ($d = 0$), full deterrence is no longer feasible.

When the damage award is in an intermediate range ($\underline{d} < d < \overline{d}$) the l-type randomizes between participating and not: $\beta^* \in (0, 1)$. From (1), the l-type entrepreneur is indifferent between participating and not if:

$$(1 - \alpha^*)(x_l - d) - e = 0. \tag{7}$$

Notice that if e or d rise, the outside ownership stake α^* must fall. When the entrepreneur must make a larger personal investment in the venture, or faces greater liability for non-disclosure, it is necessary for the entrepreneur's ownership stake $(1-\alpha^*)$ must be larger (to encourage participation).

Using (7) and allows us to rewrite the investors' break-even condition as:[31]

$$(1 - \pi)(\alpha^* \overline{x} - c) - \pi q \beta^* (c + e - x_l) = 0. \tag{8}$$

The expression, $\alpha^* \overline{x} - c > 0$, is the investors' net return from the uninformed u-type, and the expression, $c + e - x_l = k - x_l > 0$, is the investors' loss associated with the l-type. Notice that the expression $k - x_l > 0$ represents the social loss from the l-type's participation. Since the l-type is indifferent between participating and not, i.e., the l-type's expected return is equal to zero, the outside investors bear the entire social loss in expectation. Solving equations (7) and (8) gives unique closed-form solutions for α^* and β^*. The following Proposition formalizes the results.

Proposition 2. *(Equilibrium Characterization.) Consider liability thresholds $\overline{d}$ and $\underline{d}(\pi)$ defined in (4) and (5), respectively.*

1. *Full Deterrence. If $d > \overline{d}$, then investors demand equity stake $\alpha^*(d, \pi) = \overline{\alpha} = c/\overline{x}$ and the l-type is fully deterred, $\beta^*(d, \pi) = 0$.*

2. *Partial Deterrence. If $\underline{d}(\pi) \leq d \leq \overline{d}$, then investors demand equity stake*

$$\alpha^*(d, \pi) = 1 - \frac{e}{x_l - d}, \tag{9}$$

[31] Expression (7) implies $\alpha^*(x_l - d) x_l - d - e$. Substituting this into equation (1) gives the result.

where $\alpha^*(d, \pi)$ is decreasing in d and does not depend on π. The l-type is partially deterred,

$$\beta^*(d, \pi) = \frac{1-\pi}{\pi q} \cdot \frac{\left(1 - \frac{e}{x_l - d}\right)\overline{x} - c}{c + e - x_l}, \tag{10}$$

where $\beta^*(d, \pi)$ is decreasing in d and π with $\lim_{d \to \overline{d}} \beta^*(d, \pi) = 0$ and, if $\pi \leq \widehat{\pi}_0$, $\lim_{d \to \underline{d}(\pi)} \beta^*(d, \pi) = 1$.

3. *No Deterrence. If $d < \underline{d}(\pi)$, then investors demand equity stake*

$$\alpha^*(d, \pi) = \frac{(1-\pi)c + \pi q(c-d)}{(1-\pi)\overline{x} + \pi q(x_l - d)}, \tag{11}$$

where $\alpha^*(d, \pi)$ is decreasing in d and increasing in π. The l-type is undeterred, $\beta^*(d, \pi) = 1$.

The full deterrence and no deterrence equilibria were explained in the earlier Lemmas. Consider the second case of partial deterrence where $\underline{d}(\pi) < d < \overline{d}$. The equilibrium l-type participation rate $\beta^*(d, \pi)$ characterized (10) has some notable and intuitive properties. First, the l-type participation rate β^* is smaller when the damage award d is larger. In other words, the l-type is deterred by legal liability. This is intuitive. When the liability system is stronger then it becomes more costly for the l-type to not disclose the information, and therefore easier to partially deter the l-type from participating in the market. Second, the l-type's participation rate β^* is decreasing in π, the fraction of informed entrepreneurs.[32] When π is larger, the adverse selection problem is worse. To maintain investor indifference, fewer l-types participate. In the limit, when $\pi \to 1$, $\beta^* \to 0$: when the entrepreneur is perfectly informed, the full unraveling result (Grossman (1981) and Milgrom (1981)) obtains. For comparison, we know from Lemma 3 that when $\pi \to 0$, $\beta^* = 1$ $\forall d < \overline{d}$, and welfare loss again goes to zero. The equilibrium is illustrated in Figure 1.

Finally, it is worth emphasizing the importance of our assumption that the l-type's investment is socially wasteful: $x_l - c - e < 0$. According to Proposition 2, liability for non-disclosure deters the l-type entrepreneur from participating in the capital market and this leads to a more efficient allocation of capital. If instead the l-type investment was socially worthwhile, $x_l - c - e > 0$, it is easy to show that the l-type will always participate and raise capital *regardless of the liability rule.*[33] In

[32] Recall that $\overline{x} = qx_l + (1-q)x_h$ and does not depend on π.

[33] Consider a PBNE with full-disclosure by the l-type. If the l-type discloses, $\alpha_l = c/x_l$ and the l-type's payoff is $(1 - \alpha_l)x_l - e = x_l - c - e > 0$. If the l-type were to mimic the u-type and

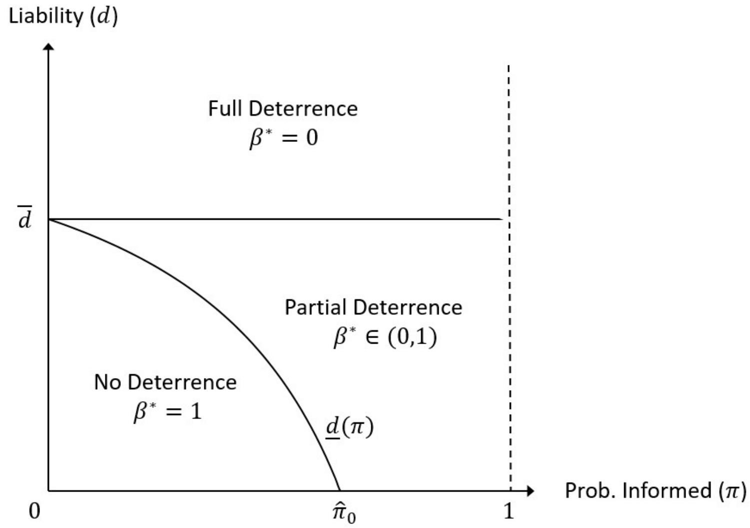

Figure 1: Equilibrium Characterization

that setting, when d is sufficiently large, the l-type will simply disclose the bad news, $x = x_l$, and sell equity stake of $\alpha_l = c/x_l$ to the outside investors. Thus, if the l-type investment was socially worthwhile, holding the entrepreneur liable for non-disclosure of material information would have no effect on the allocation of capital or on social welfare.

2.3 Welfare Implications

It is straightforward to evaluate the implications for social and private welfare. In the PBNE, the h-type and u-type participate with certainty while the l-type participates with probability $\beta^*(d, \pi) \in [0, 1]$. From an ex ante perspective, the expected social welfare is $SW(d, \pi)$ is given by:

$$SW(d, \pi) = (1 - \pi)(\overline{x} - k) + \pi(1 - q)(x_h - k) + \pi q \beta^*(d, \pi)(x_l - k). \tag{12}$$

The first and second terms, which are positive, reflect the value created by the u-types and h-types, respectively. The third term, which is negative, is the welfare loss associated with the l-type's participation: πq is the probability that the entrepreneur

withhold information, investors demand $\overline{\alpha} = c/\overline{x}$ and the l-type's payoff becomes $(1-c/\overline{x})(x_l-d)-e$. Setting these expressions equal gives the threshold $d = c(\overline{x} - x_l)/(\overline{x} - c) > 0$. When d is above the threshold then the equilibrium involves full disclosure by the l-type; when d is below the threshold the equilibrium involves no disclosure by the l-type. In both cases the l-type project is pursued, which is socially efficient.

is the l-type and β^* is the probability of l-type's non-disclosure and participation. As assumed earlier, it is inefficient for the l-type entrepreneurs to participate in the market because the gross return from the venture, x_l, is smaller than the cost of the venture, $k = c + e$.

Equation (12) reveals important comparative statics. First, social welfare depends on d through its affect on the the probability of l-type participation $\beta^*(d, \pi)$ characterized in Proposition 2. If $d > \overline{d}$ so the l-type is fully deterred from participating, then there is no additional social benefit of increasing d. Similarly, when $d < \underline{d}(\pi)$ then the l-type is undeterred and there is no social benefit from raising d incrementally. But when $d \in [\underline{d}(\pi), \overline{d})$ and there is partial deterrence, then social welfare is a strictly increasing function of d. This stands to reason, as the l-type is less likely to participate when d is higher.

Analysis of equation (12) also reveals that social welfare is (weakly) increasing in π, the proportion of informed entrepreneurs. Suppose d is in the high range so the l-type is fully deterred. Setting $\beta^*(d, \pi) = 0$ in equation (12) one can see that as π increases, the proportion of h-types increases relative to u-types and this increases social welfare since $x_h > \overline{x}$. Social welfare also increases with respect to π when d is in the intermediate range. This is true because the probability of l-type participation $\beta^*(d, \pi) \in (0, 1)$ is a decreasing function of π (see Case 2 of from Proposition 2). In the extreme, if the entrepreneur is informed with certainty ($\pi = 1$), there would be full unraveling and the first-best outcome would be obtained. More generally, the society is better off as the entrepreneur has more information.[34]

Proposition 3. *(Socially-Optimal Liability Rules.) Social welfare is maximized with liability rule* $d \geq d_s^* = \overline{d}$.

Now consider the effect of liability on private welfare. First, as d rises, the u-type is (weakly) better off. To see why, recall that the u-type's return is $(1-\alpha^*(d, \pi))\overline{x} - e$. Note that the level of liability d affects the u-type's payoff only through its impact on the equity stake demanded by investors, $\alpha^*(d, \pi)$. According to Proposition 2, $\alpha^*(d, \pi)$ is a decreasing function of d. To see why, suppose for example that $d < \underline{d}(\pi)$ so the l-type is undeterred, $\beta^*(d, \pi) = 1$. As d rises, investors expect to receive a larger "rebate" when they sue the l-types for non-disclosure. As a consequence, the equity stake $\alpha^*(d, \pi)$ demanded by outside investors characterized in equation (11)

[34]When firm liability is in the low range then the l-type is undeterred ($\beta^*(d, \pi) = 1$), and social welfare does not depend on π. Since the l-type participates with certainty, the average welfare from an informed entrepreneur, some of whom are h-types and disclose x_h and others who are l-types and do not disclose their types, is simply $\overline{x} - k$.

falls and the u-type is better off.[35] Second, and not surprisingly, as the level of liability d rises the l-type's return falls.[36]

Proposition 4. *(Privately-Optimal Liability Rule.) The u-type's return is maximized with liability rule $d \geq d_u^* = \overline{d}$. The l-type's return is maximized with liability rule $d = d_l^* = 0$.*

While a more complete discussion is reserved until after the general model has been presented, from the baseline model, we can already see the divergence between social and private optimality. Because the u-type's profit is (weakly) increasing with respect to d, the u-type does not have an incentive to waive liability in equilibrium. The u-type will embrace liability even when the liability regime does not lead to any deterrence, i.e., when $d < \underline{d}(\pi)$. One reason for such divergence is that, any rent captured by the l-type represents a loss from the u-type's perspective (through a higher equilibrium α^*), while for social welfare, such rent is a simple transfer without any welfare consequences. These issues will play an important role in the general model (with litigation cost and false positives), as well.

3 The General Model

The previous section presented the baseline model. While the model delivered important insights, it also relied on several simplifying assumptions, such as no litigation costs and no court error. In this section, we relax these assumptions and consider both (1) the possibility that the court can find u-type entrepreneur liable (false positives); and (2) positive litigation cost. After presenting the results from the richer model, we will also consider important policy question of whether to allow the firm to opt out of the liability regime.

In the baseline model, we assumed that the l-type will be found liable when the investors observe x_l, while the u-type, when it generates the revenue of x_l, is not found liable. This is tantamount to assuming that when x_l is observed, the investors (and the court) can distinguish whether the it is the l-type or the u-type, neither of which provided any information to the investors at the time of equity sale, that has generated the cash-flow of x_l. We also assumed that litigation was costless for both

[35] $\alpha^*(d, \pi)$ is also a decreasing function of d in the intermediate range, $\underline{d}(\pi) < d < \underline{d}(\pi)$, and does not depend on d if $d > \overline{d}$.

[36] The proof is straightforward. Social welfare defined in (12) is the sum of the returns of the investors, the h-types, the u-types, and the l-types. The investors break even on average, the h-type's return does not depend on d, and the u-type's return is increasing in d. Since social welfare is weakly increasing in d, the l-type's return must be weakly decreasing in d.

the plaintiff-shareholders and the defendant-firm. In reality, lawsuits are costly and the court (and the investors) may have difficulty distinguishing between the u-type and the l-type after observing x_l.

Suppose, after x_l has been observed, the investors do not know for certain whether they are facing the l-type or the u-type, and can bring lawsuit against either one. More precisely, when x_l is observed, though the l-type will be sued for certain, the u-type gets sued with probability $\lambda \in [0, 1]$ by the investors.[37] Litigation is also costly. We assume that, conditional on d, the plaintiff-shareholders bear the litigation cost of $\gamma \cdot d$ where $\gamma \in [0, 1)$.[38] Note that, as the size of damages (d) grows, the litigation cost rises as well. We think of $\gamma \cdot d$ as (foremost) the cost of legal representation for the plaintiff-shareholders: the larger the potential recovery, more lawyer hours are spent in litigation. We let $\gamma_0 \leq \gamma$ represent the lawyer's outside reservation value (opportunity cost). When $\gamma > \gamma_0$ and there is a lawsuit, the lawyer captures $(\gamma - \gamma_0)d$ as rent.

3.1 Equilibrium Characterization

We now characterize the PBNE where the outside investors demand an equity stake $\alpha^* \in [0, 1]$ and the l-type participates and raises capital with probability β^*. Let's begin with the l-type's participation decision. If the l-type participates, the outside investors will bring suit against the firm for damages d. The characterization of β^* is the same as in the baseline model and given by (1).

Now, consider the investors. With both false positives and plaintiff-shareholder litigation costs, the investors' ex ante expected break-even condition, conditional on non-disclosure, is:

$$(1 - \pi)[\alpha\overline{x} - c + q\lambda(1 - \gamma - \alpha)d] + \pi q\beta[\alpha x_l - c + (1 - \gamma - \alpha)d] = 0. \tag{13}$$

This expression extends the break-even condition (2) in the baseline model to include legal error (λ) and litigation costs ($\gamma \cdot d$). Suppose the entrepreneur is the u-type. In this case, the investors bring a lawsuit and are (erroneously) awarded damages of d with probability $q\lambda$. The gross award d is offset by a reduction in firm value and payments to the lawyers, and investors get an expected (net) award $q\lambda(1 - \gamma - \alpha)d$.

[37] We can motivate this setup by assuming that, in addition to the cash-flow (x_l), the investors (and the court) also observe some informative signal (not explicitly modeled) that conveys information on the likelihood of prevailing in litigation.

[38] Although the setup assumes that only the plaintiff-shareholders bear the cost of litigation, we do this for the sake of simplicity. The model can be easily generalized to include litigation costs for the defendant-firm without changing the qualitative results. See Online Appendix B for details.

This is shown in the first part of equation (13). If the entrepreneur is the l-type, then investors get an expected net award of $(1-\gamma-\alpha)d$ as shown in the second part of equation (13).

With positive litigation cost and the fact that, in equilibrium, the investors own a fraction of the firm that pays damages, litigation credibility can become an issue. To ensure that litigation is credible against both the u-type and the l-type, we need to make sure that the equilibrium α and γ are sufficiently small, so that $1-\alpha-\gamma \geq 0$. We make the following technical assumption:

Assumption 2: $\max\left\{\frac{x_l - e}{x_l}, \frac{(1-\pi)c + \pi q c}{(1-\pi)\overline{x} + \pi q x_l}\right\} \leq 1-\gamma.$

This assumption will ensure that the litigation will be (at least weakly) credible in equilibrium. More precisely, if the equilibrium fraction sold to the investors is given by α^*, the assumption will ensure that $1-\alpha^*-\gamma \geq 0$, since the left hand side of the inequality represents the highest α that will be attained in equilibrium.

As in the baseline case, when liability exceeds a threshold, $d \geq \overline{d}$, the l-type is fully deterred from raising capital. We now construct the equilibrium equity stake $\alpha^*(d,\pi)$ that would be demanded by the investors. Setting $\beta^*(d,\pi)=0$ in the investors' break-even constraint in (13) gives:

$$\alpha^*(\overline{x} - q\lambda d) - c + q\lambda(1-\gamma)d = 0.$$

Solving for α^* gives the equilibrium equity stake $\alpha^* = \overline{\alpha}(d)$ where

$$\overline{\alpha}(d) = \frac{c - q(1-\gamma)\lambda d}{\overline{x} - q\lambda d}. \tag{14}$$

Notice that if the court error rate is zero, $\lambda = 0$, then $\alpha^* = \overline{\alpha}(d) = c/\overline{x}$, just as in equation (3) in the baseline model. We can easily show that $\overline{\alpha}(d)$ is a decreasing function of d.[39] In other words, similar to the baseline case, as d rises, because the investors' net recovery from litigation rises, they demand, in equilibrium, a smaller fraction of the firm.

Lemma 4. *(Full Deterrence.) There exists a threshold $\overline{d} \geq 0$ such that the l-type is fully deterred, $\beta^*(d,\pi) = 0$, when $d > \overline{d}$. If $e \geq (1-c/\overline{x})x_l$, then $\overline{d} = 0$.*[40] *If*

[39] Rewrite (14) as $\overline{\alpha}(\overline{x} - q\lambda d) - c + q(1-\gamma)\lambda d = 0$. Totally differentiating with respect to $\overline{\alpha}$ and d gives $(\overline{x} - q\lambda d)\Delta\overline{\alpha} + q\lambda(1-\overline{\alpha}-\gamma)\Delta d = 0$. Both coefficients are positive, proving that the derivative is negative.

[40] Since $k = c + e$, we can rewrite $(1-c/\overline{x})x_l = \frac{\overline{x}-k}{\overline{x}-x_l}x_l \equiv \widehat{e}$. Then, $\overline{d} < (>) 0$ when $e > (<) \widehat{e}$.

$e < (1 - c/\overline{x})x_l$, then $\overline{d}$ is implicitly defined by:

$$\overline{d} = x_l - \frac{e}{1 - \overline{\alpha}(\overline{d})} > 0, \tag{15}$$

where $\overline{\alpha}(d)$ is defined in (14). As λ increases or γ decreases, $\overline{d}$ increases: $\frac{\partial \overline{d}}{\partial \lambda} \geq 0$ and $\frac{\partial \overline{d}}{\partial \gamma} \leq 0$. As d increases, $\overline{\alpha}$ decreases: $\frac{\partial \overline{\alpha}}{\partial d} \geq 0$.

As in the baseline case, when the damages are sufficiently large ($d \geq \overline{d}$), notwithstanding the possibility of false positives and litigation cost, the l-type will be completely deterred and only the u-type will participate in financing, conditional on non-disclosure. Due partly to the fact that the cost of litigation depends on the size of damages, the threshold $\overline{d}$ is only implicitly defined.

Lemma 4 also reveals that changes in λ and γ have opposite effects on $\overline{d}$. This is fairly intuitive. As the rate of false positives grows (λ increases), the investors recover more from the u-type. This leads to a lower equilibrium α which, in turn, makes it more attractive for the l-type to participate in financing. To restore full deterrence, $\overline{d}$ has to increase. On the other hand, when the cost of litigation rises (γ increases), this lowers the investors' net recovery from litigation, thereby leading them to demand a higher α to satisfy their break-even condition. With a larger fraction of the firm sold to the investors, it becomes less attractive for the l-type to participate in financing, thereby lowering $\overline{d}$. At the opposite end of the spectrum, when the damages award d is below a threshold, the l-type entrepreneur will be completely undeterred, $\beta^* = 1$.

As in the baseline model, when the damages award d is below a threshold, the l-type entrepreneur will be completely undeterred, $\beta^* = 1$.

Lemma 5. *(No Deterrence.) There exist thresholds $\underline{d}(\pi) \in [0, \overline{d}]$ and $\widehat{\pi}_0 \in (0, 1)$ such that the l-type is undeterred, $\beta^*(d, \pi) = 1$, if $d < \underline{d}(\pi)$. If $\pi > \widehat{\pi}_0$, then $\underline{d}(\pi) = 0$. If $\pi \leq \widehat{\pi}_0$, then $\underline{d}$ is implicitly defined by:*

$$\underline{d} = x_l - \frac{e}{1 - \overline{\alpha}(\underline{d}) - r(\underline{d}, \pi)} \tag{16}$$

where $\overline{\alpha}(\underline{d})$ is defined in (14) and

$$r(\underline{d}, \pi) = \frac{\pi q}{1 - \pi} \cdot \frac{k + \gamma \underline{d} - x_l}{\overline{x} - q\lambda \underline{d}}. \tag{17}$$

$\underline{d}(\pi)$ is strictly decreasing in π, $\frac{\partial \underline{d}}{\partial \pi} < 0$, with $\underline{d}(0) = \overline{d}$ and $\underline{d}(\widehat{\pi}_0) = 0$.

While the formal details are in the proof of Lemma 5, the result may be understood intuitively. The function, $r(\underline{d}, \pi)$, defined in (17) is the incremental equity stake demanded by the outside investors when the l-type is (just) undeterred, $\beta^* = 1$. The risk premium $r(\underline{d}, \pi)$ is an increasing function of $\underline{d}$ and π. That is, conditional on no deterrence, the investors will demand a higher fraction of the firm as their litigation recovery ($\underline{d}$) decreases or as the degree of adverse selection (π) increases. This will establish an inverse relationship between $\underline{d}$ and π.

The thresholds $\overline{d}$ and $\underline{d}(\pi)$, defined in Lemmas 4 and 5, allow us to characterize the full equilibrium in Proposition 5. As in the baseline case, the equilibrium divided into three regions: a region where the l-type is fully deterred ($\beta^* = 0$); where the l-type is not deterred ($\beta^* = 1$), and where l-type is only partially deterred ($\beta^* \in (0, 1)$).

Proposition 5. *(Equilibrium Characterization.) Consider liability thresholds $\overline{d}$ and $\underline{d}(\pi)$ defined in Lemma 4 and Lemma 5, respectively.*

1. *Full Deterrence. If $d > \overline{d}$, then investors demand equity stake $\alpha^*(d, \pi) = \overline{\alpha}(d)$, defined in (14), where $\alpha^*(d, \pi)$ is increasing in d and does not depend on π. The l-type is fully deterred: $\beta^*(d, \pi) = 1$.*

2. *Partial Deterrence. If $\underline{d}(\pi) \leq d \leq \overline{d}$, then investors demand equity stake*

$$\alpha^*(d, \pi) = 1 - \frac{e}{x_l - d}, \tag{18}$$

where $\alpha^*(d, \pi)$ is decreasing in d and does not depend on π. The l-type is partially deterred,

$$\beta^*(d, \pi) = \frac{1 - \pi}{\pi q} \cdot \frac{\left(1 - \frac{e}{x_l - d}\right)(\overline{x} - q\lambda d) - (c - q(1 - \gamma)\lambda d)}{k + \gamma d - x_l}, \tag{19}$$

where $\beta^*(d, \pi)$ is decreasing in d and π with $\lim_{d \to \overline{d}} \beta^*(d, \pi) = 0$ and, if $\pi \leq \widehat{\pi}_0$, $\lim_{d \to \underline{d}(\pi)} \beta^*(d, \pi) = 1$.

3. *No Deterrence. If $d < \underline{d}(\pi)$, then investors demand equity stake*

$$\alpha^*(d, \pi) = \frac{(1 - \pi)(c - q(1 - \gamma)\lambda d) + \pi q(c - (1 - \gamma)d)}{(1 - \pi)(\overline{x} - q\lambda d) + \pi q(x_l - d)}. \tag{20}$$

The l-type is undeterred: $\beta^*(d, \pi) = 1$. As d increases, α^* decreases: $\frac{\partial \alpha^*}{\partial d} < 0$.

Although the proposition's statements are a bit involved, the results are fairly straightforward. The equilibrium α^* also allows us to more concretely tie to the credibility assumption made earlier. When the l-type is only partially deterred and the investors receive $\alpha^*(d,\pi) = 1 - \frac{e}{x_l - d}$, note that $\alpha^*(d,\pi)$ is decreasing with respect to d and when $d = 0$, we get $\alpha^* = \frac{x_l - e}{x_l}$. Similarly, when the l-type is undeterred and $d = 0$, the equilibrium is given by: $\alpha^* = \frac{(1-\pi)c + \pi q c}{(1-\pi)\overline{x} + \pi q x_l}$. Note that the assumption $\max\left\{\frac{x_l - e}{x_l}, \frac{(1-\pi)c+\pi q c}{(1-\pi)\overline{x}+\pi q x_l}\right\} \leq 1-\gamma$ ensures that the equilibrium α^* is small enough to satisfy credibility.

3.2 Welfare Implications

While the equilibrium with both false positives and litigation cost is qualitatively similar to that in the baseline case, there are some important differences when social welfare and, more importantly, the divergence between private and social welfare are concerned. This subsection explores these issues. We first characterize the socially and privately-optimal liability rules and then examine the circumstances under which the private optimum diverges from the social optimum.

3.2.1 The Socially-Optimal Liability Rule

It is straightforward to evaluate the effect of firm liability on social welfare. In the PBNE in Proposition 5, the h-type and the u-type participate with certainty and the l-type participates with probability $\beta^*(d,\pi)$. From an ex ante perspective, the expected social welfare is:

$$SW(d,\pi) = (1-\pi)(\overline{x} - k - q\lambda\gamma_0 d) + \pi(1-q)(x_h - k) + \pi q \beta^*(d,\pi)(x_l - k - \gamma_0 d). \quad (21)$$

Note the differences from the social welfare function in equation (12) of the baseline model. The third term is the social loss associated with a participating l-type: $x_l - k - \gamma_0 d < 0$. This is lower than in the baseline model, as there is an opportunity cost $\gamma_0 \cdot d$ of the lawyers' time. Similarly, the first term in (21), the social value associated with the u-type, includes the expected opportunity cost of the lawyers, $q\lambda\gamma_0 d$. The social welfare function in (21), together with the characterization of $\beta^*(d,\pi)$ in Proposition 5, delivers the following comparative statics.

Lemma 6. *(Social Welfare Comparative Statics.)*

1. *Full Deterrence. If $d \geq \overline{d}$, then $\beta^*(d,\pi) = 0$. Social welfare is decreasing in d.*

2. *Partial Deterrence. If $\underline{d}(\pi) \leq d \leq \overline{d}$, then $\beta^*(d,\pi) \in [0,1)$. Social welfare is increasing in d.*

3. *No Deterrence. If $d < \underline{d}(\pi)$, then $\beta^*(d, \pi) = 1$. Social welfare is decreasing in d.*

With the results from Lemma 6, we can now characterize the socially-optimal liability rule. When d rises, social welfare is falling in the no-deterrence region ($d \geq \overline{d}$), rising in the partial deterrence region ($\underline{d}(\pi) \leq d \leq \overline{d}$), and falling in the full deterrence region ($d < \underline{d}(\pi)$). The result straightforwardly implies that the socially-optimal damage award is $d_s^* \in \{0, \overline{d}\}$. Furthermore, the social planner prefers $d = 0$ to $d = \overline{d}$ if and only if $SW(0, \pi) \geq SW(\overline{d}, \pi)$. This leads to two cases that we need to consider.

According to Lemma 5, if the proportion of informed entrepreneurs is above a threshold, $\pi \geq \widehat{\pi}_0$, then $\underline{d}(\pi) = 0$. This in turn implies that $d \geq \underline{d}(\pi)\ \forall d \geq 0$. Hence, we are in cases 1 or 2 of Lemma 6 and social welfare is maximized by $d = \overline{d}$. Suppose, on the other hand, $\pi < \widehat{\pi}_0$. Lemma 5 implies that $\underline{d}(\pi) > 0$. If $d = 0$, then we are in the no-deterrence region and $\beta^*(d, \pi) = 1$. Using (21), one can show that the social planner prefers no liability $d = 0$ to full deterrence with $d = \overline{d}$ if and only if:[41]

$$(1 - \pi) q \lambda \gamma_0 \overline{d} > \pi q (k - x_l). \tag{22}$$

This expression is intuitive. The left-hand side is the efficiency loss (from the ex ante perspective) when $d = \overline{d}$ and there is full deterrence. Although the l-type is fully deterred, there is a loss of legal resources of $\gamma_0 \overline{d}$ when the u-type is sued, which happens with probability $(1 - \pi) q \lambda$. The right-hand side is the expected efficiency loss when there is no liability ($d = 0$) as the l-type participates with probability one. We have the following result.

Proposition 6. *(Socially-Optimal Liability Rule.) Social welfare is maximized with liability rule $d = d_s^* \in \{0, \overline{d}\}$ where $\overline{d}$ is defined in (15). Let threshold $\widehat{\pi}_s \in (0, \widehat{\pi}_0)$ satisfy*

$$\frac{1 - \widehat{\pi}_s}{\widehat{\pi}_s q} = \frac{k - x_l}{q \gamma_0 \lambda \overline{d}}. \tag{23}$$

If $\pi < \widehat{\pi}_s$, the socially-optimal liability rule is $d_s^* = 0$. If $\pi \geq \widehat{\pi}_s$, the socially-optimal liability rule is $d_s^* = \overline{d}$.

The proposition tells us that the socially-optimal liability rule is either the minimal damages required for full deterrence, $d_s^* = \overline{d}$, or no liability at all, $d_s^* = 0$. If the proportion of informed types π is below a threshold $\widehat{\pi}_s$, then no liability is the socially-optimal rule. This makes intuitive sense. Suppose $\pi \to 0$. In the limit, the welfare loss from l-type's participation becomes vanishingly small, so the social benefit of liability is negligible. The social cost of liability is significant, however, since court

[41] See the proof of Proposition 6 in the appendix for details.

error implies lawsuits will be brought against the u-type. Lawsuits against the u-type involve wasted economic resources, i.e., the opportunity cost of the lawyers' time and effort. Thus, when π is small, no liability is optimal: $d_s^* = 0$. When π is large, on the other hand, the optimal damage award is exactly $d_s^* = \overline{d}$ for the opposite reasons.[42] While deterring the l-type from participating is significant, wary of the welfare loss that stems from lawyer's opportunity cost, maximizing social welfare requires setting $d_s^* = \overline{d}$.

Note also that the equilibrium threshold $\widehat{\pi}_s$ defined in (23) is (1) positively related to the lawyers' opportunity cost (γ_0) and the probability of legal error (λ) but (2) negatively related to the deadweight loss from the l-type's participation ($k - x_l$). Holding $k - x_l$ fixed, if either γ_0 or λ approaches zero, the social cost of lawsuits brought against the innocent u-type (measured by $\gamma_0 \lambda \overline{d}$) gets very small and the socially-optimal liability rule becomes $d_s^* = \overline{d}$.[43] What matters for socially-optimal policy is how much resources (lawyers' opportunity cost) are being "wasted" by lawsuits that are brought against the u-type. As the amount of resources spent against the u-type gets smaller, it becomes socially more desirable to provide full deterrence against the l-type. Similarly, as the deadweight loss from the l-type's participation gets larger ($k - x_l$ gets bigger), preventing the l-type from participating becomes more important and $\widehat{\pi}_s$ becomes smaller.

3.2.2 The Privately-Optimal Liability Rule

The last subsection considered the effect of firm liability on social welfare. Social welfare was defined as the sum of the ex ante expected returns for all of the stakeholders. We now consider the effect of firm liability on the returns of the entrepreneur. This will allow us to understand why entrepreneurs may (or may not) seek to include liability waivers and evaluate the divergence between their private incentives and the social incentives (see Section 3.3). The following Lemma characterizes the effect of firm liability d on the private returns of the u-type and l-type.

Lemma 7. *(Private Welfare Comparative Statics.)*

1. *Full Deterrence. If $d > \overline{d}$, then $\beta^*(d, \pi) = 0$. The u-type's expected return is decreasing in d and the l-type's return is independent of d.*

2. *Partial Deterrence. If $\underline{d}(\pi) \leq d \leq \overline{d}$, then $\beta^*(d, \pi) \in [0, 1)$. The u-type's expected return is increasing in d and the l-type's return is independent of d.*

[42] If the damages award were to exceed this threshold, the social cost of litigation $\gamma_0 \cdot d$ would be unnecessarily excessive and raising d will only reduce welfare.

[43] From Lemma 4, $\overline{d}$ is an increasing function of λ and does not depend on γ_0.

3. *No Deterrence. If $d < \underline{d}(\pi)$, then $\beta^*(d, \pi) = 1$. The u-type's expected return may be increasing or decreasing in d and is a convex function of d, and the l-type's return is decreasing in d. As $\lambda \to 0$, the u-type's return is strictly increasing with respect to d.*

With the results from Lemma 7, we can characterize the privately-optimal liability rules. Before we start, we make a few observations based on the Lemma. First, and most obviously, l-type (weakly) prefers lower level of liability, as it may free-ride on the u-type, participate in the market, and earn rents. We also know from earlier that when the l-type is identified by the investors, the l-type cannot profitably engage in financing. Hence, unless there is full deterrence ($\beta^* = 0$), whichever liability regime is chosen by the u-type, the l-type will follow. This implies that in analyzing the privately-optimal liability rule, we need to focus on the u-type's incentive.

More interestingly, Lemma 7 implies that the u-type's privately-optimal liability rule is either no liability ($d = 0$) or liability just high enough to deter the l-types ($d = \overline{d}$). To see why, suppose that the proportion of informed types π is sufficiently small so that $\underline{d}(\pi) > 0$.[44] If $d < \underline{d}(\pi)$, then we are in Case 3 of Lemma 7. In this region, the u-type's return is a convex function of d, and hence cannot obtain an interior maximum. Thus, in Case 3 of Lemma 7, the u-type's return is maximized at a corner solution, $d = 0$ or $d = \underline{d}(\pi)$. Note further that if $d > \underline{d}(\pi)$ then we are in Cases 1 and 2 of Lemma 7. Since the u-type's expected return is increasing in d in Case 2 and their expected return is decreasing in d in Case 1, we know the u-type's privately-optimal liability rule is either $d_u^* = 0$ or $d_u^* = \overline{d}$.

With these observations in hand, we may now characterize the u-type's privately-optimal liability rule. Suppose that $\pi < \widehat{\pi}_0$, so the l-type is completely undeterred when $d = 0$. As shown in the appendix, the u-type prefers no liability $d = 0$ and no deterrence to $d = \overline{d}$ and full deterrence if and only if

$$(1 - \pi)q\gamma\lambda\overline{d} > \pi q\big[(k - x_l) + (1 - \alpha^*(0, \pi))x_l - e\big] \tag{24}$$

where, using (20),

$$\alpha^*(0, \pi) = \frac{(1 - \pi)c + \pi qc}{(1 - \pi)\overline{x} + \pi q x_l}. \tag{25}$$

The left-hand side of (24) is the u-type's expected loss (from the ex ante perspective) when $d = \overline{d}$. If $d = \overline{d}$, the l-type is fully deterred but the u-type must bear (indirectly) the payments to the lawyers. The expression, $(1 - \pi)q\lambda$, represents the (ex ante) probability that the u-type gets sued under full deterrence. The right-hand side is

[44] According to Lemma 5, if $\pi < \widehat{\pi}_0$, then $\underline{d}(\pi) > 0$.

the u-type's loss if there is no liability, $d = 0$. This includes the social loss from the participation of the l-type $(k - x_l)$ plus the information rents that accrue to the l-type $((1 - \alpha^*(0, \pi))x_l - e)$. These losses are multiplied by the ex ante probability of the l-type, πq. Substituting (25) into (24), and rearranging gives the following result.

Proposition 7. *(The Privately-Optimal Liability Rule.) The u-type's return is maximized with liability rule $d = d_u^* \in \{0, \overline{d}\}$. Let $\widehat{\pi}_u \in (0, \widehat{\pi}_0)$ satisfy*

$$\frac{1 - \widehat{\pi}_u}{\widehat{\pi}_u q} = \frac{(1 - x_l/\overline{x})c - q\gamma\lambda\overline{d}x_l/\overline{x}}{q\gamma\lambda\overline{d}}. \tag{26}$$

If $\pi < \widehat{\pi}_u$, the privately-optimal liability rule is $d_u^* = 0$. If $\pi \geq \widehat{\pi}_u$, the privately-optimal liability rule is $d_u^* = \overline{d}$. The l-type's return is maximized with liability rule $d_l^* = 0$.

The u-type's privately-optimal liability rule is either the minimal damages required for full deterrence, $d_s^* = \overline{d}$, or no liability at all, $d_s^* = 0$. If the proportion of informed type π is below a threshold $\widehat{\pi}_u$, then no liability is the u-type's preferred rule. Suppose that $\pi \to 0$. In the limit, there is no l-type to deter so the private benefit of deterrence is negligible. The private cost of liability is significant, however, since court error implies lawsuits will be brought against the u-type. Lawsuits against the u-type involve expected legal cost of $q\gamma\lambda\overline{d}$. Thus, when π is small, the u-type prefers no liability: $d_u^* = 0$. When π is large, however, the u-type's private benefit of deterring the l-types outweighs the costs and $d_u^* = \overline{d}$.

3.2.3 The Divergence between Private and Social Incentives

Comparing Proposition 6 and Proposition 7 reveals that the u-type entrepreneur's ideal liability rule diverges from the social planner's in a systematic way. The divergence stems from the fact that the u-type does not care about any rent captured by the l-type and the lawyers, while the social planner does. That is, the returns of the l-type and the lawyers are included in the social planner's calculus, but not in the calculus of the self-interested u-type. As a consequence, the u-type's private incentive to impose liability may be either stronger or weaker than the social planner's, $\widehat{\pi}_s > \widehat{\pi}_u$ or $\widehat{\pi}_s < \widehat{\pi}_u$.

Comparing conditions (22) and (24), the u-type's private incentive to impose liability $d = \overline{d}$ defined in (15) is weaker than the social planner's if and only if

$$(1 - \pi)q(\gamma - \gamma_0)\lambda\overline{d} > \pi q[(1 - \alpha^*(0, \pi))x_l - e] \tag{27}$$

where $\overline{d}$ is implicitly defined by (14) and (15) and $\alpha^*(0, \pi)$ is defined in (25).[45]

This expression is intuitive. The left-hand side of (27) represents the lawyers' rent, from the ex ante perspective, when $d = \overline{d}$ and the l-type is fully deterred. Importantly, the rents captured by the lawyers do not generate any direct loss of social welfare, as the well-being of the lawyers is included in the social welfare function. However, these rents do have a negative impact on the u-type's return, as investors will demand a larger equity stake to compensate for higher legal expenses (and to break even). With the l-type fully deterred, any increase in the equity stake (α) directly affects the u-type's profit. The right-hand side (27) represents the l-type's rent, from the ex ante perspective, when $d = 0$ and the l-type is completely undeterred ($\beta^* = 1$). These rents are neutral from a social-welfare perspective, but adversely affect the l-type's return. So (27) tells us that the u-type's private incentive to impose liability is socially insufficient if and only if the rent captured by the lawyers when $d = \overline{d}$ exceeds the rent captured by the l-types when $d = 0$.

Corollary 1. *(Divergence Between Private and Social Incentives.) If the market for legal services is competitive, $\gamma = \gamma_0$, then the u-type's private incentive to impose liability socially excessive, $\widehat{\pi}_u < \widehat{\pi}_s$. If the market for legal services is not competitive, $\gamma > \gamma_0$, there exists a unique court-error rate $\widehat{\lambda} \in (0, 1]$ where the u-type's private incentive to impose liability is socially insufficient, $\widehat{\pi}_u > \widehat{\pi}_s$, if and only if $\lambda > \widehat{\lambda}$.*

Corollary 1 suggests that when courts are more prone to error $\lambda > 0$ and the lawyers earn rents (i.e., the market for legal services is less-than-fully competitive), then the u-type's private incentive to impose liability may be weaker than the social incentive. To understand why, recall that when $d = \overline{d}$ and there is full deterrence, the social cost of litigation reflects the opportunity cost of the lawyers' time, $(1-\pi)q\gamma_0\lambda\overline{d}$, while the u-type's private cost reflects the legal fees, $(1 - \pi)q\gamma\lambda\overline{d}$. Thus, the private cost of litigation exceeds the social cost of litigation by $(1 - \pi)q(\gamma - \gamma_0)\lambda\overline{d} > 0$. The left-hand side, which represents the lawyers' rents, is positive when $\lambda > 0$ and $\gamma > \gamma_0$, is an increasing function of λ, and is a decreasing function of γ_0.[46]

3.3 Liability Waiver

The previous section examined both privately and socially-optimal liability rules and demonstrated that the u-type's ideal liability rule diverges from the social planner's ideal rule in a systematic way. In this section, we extend the results from the previous

[45] $\overline{d}$ depends on γ and λ and $\alpha^*(0, \pi)$ is independent of γ and λ.

[46] See proof of Corollary 1 in the appendix. The right-hand side of (27) is independent of $\lambda, \gamma, \gamma_0$, and π.

section to explore the divergence between the private and social incentives to waive liability. We first start with the straightforward case of choosing between full liability of $d = \overline{d}$ and no liability ($d = 0$). We then extend the analysis and discuss the social and private incentive to opt out of any liability ($d > 0$).

To begin, consider choosing (either by the social planner or the u-type) between a legal regime where the damages are set at the full-deterrence threshold, $d = \overline{d}$, defined in (15), and no liability ($d = 0$). If (27) holds, then we have $\widehat{\pi}_s < \widehat{\pi}_u$. In that case, if $\pi \in (\widehat{\pi}_s, \widehat{\pi}_u)$, the u-type has a private incentive to choose $d = 0$ over $d = \overline{d}$, while the society is better off with $d = \overline{d}$. The private incentive to waive liability is socially excessive. Conversely, if (27) does not hold, then we have $\widehat{\pi}_u < \widehat{\pi}_s$. In this case, if $\pi \in (\widehat{\pi}_u, \widehat{\pi}_s)$, the u-type will embrace liability ($d = \overline{d}$), while the society is better off if liability is waived ($d = 0$). The private incentive to waive liability is socially insufficient. On the other hand, when $\pi \leq min\{\widehat{\pi}_s, \widehat{\pi}_u\}$, both the social planner and the u-type will prefer no liability ($d = 0$) over full liability ($d = \overline{d}$), whereas when $\pi \geq max\{\widehat{\pi}_s, \widehat{\pi}_u\}$, both will choose full liability over no liability. In these outside regions, private and social incentives are aligned when choosing between full liability and no liability.

What if the choice is between *any* liability ($d > 0$) and no liability ($d = 0$)? Building on the results from the previous section, we can make general statements about liability waivers when the damages award is not set exactly at the full-deterrence threshold ($d \neq \overline{d}$). That is, conditional on any $d > 0$, we ask whether the u-type will have an incentive to waive liability (choose $d = 0$ over $d > 0$) and whether such an incentive is consistent with maximizing social welfare. We start with a formal statement in the following proposition.

Proposition 8. *(General Incentive to Waive Liability.) Given any liability $d > 0$, the social planner's and the u-type's incentive to waive liability (i.e., choose $d = 0$) is given by the following.*

1. *Social Planner's Incentive. If $\pi < \widehat{\pi}_s$, the social planner will always waive liability. If $\pi \geq \widehat{\pi}_s$, $\exists$ $\underline{\delta}_s(\pi) \in [\underline{d}(\pi), \overline{d}]$ and $\overline{\delta}_s(\pi) \geq \overline{d}$, such that the social planner will not waive liability if $d \in [\underline{\delta}_s(\pi), \overline{\delta}_s(\pi)]$ and waive liability otherwise. When $\pi > \widehat{\pi}_0$, $\underline{\delta}_s(\pi) = 0$ and $\overline{\delta}_s(\pi) = \frac{(1-e/x_l)\overline{x}-c}{q\lambda\gamma_0}$; and $\underline{\delta}_s(\widehat{\pi}_s) = \overline{\delta}_s(\widehat{\pi}_s) = \overline{d}$.*

2. *The u-type's Incentive. If $\pi < \widehat{\pi}_u$, the u-type will always waive liability. If $\pi \geq \widehat{\pi}_u$, $\exists$ $\underline{\delta}_u(\pi) \leq \overline{d}$ and $\overline{\delta}_u(\pi) \geq \overline{d}$, such that the u-type will not waive liability if $d \in [\underline{\delta}_u(\pi), \overline{\delta}_u(\pi)]$ and waive liability otherwise. When $\pi > \widehat{\pi}_0$, $\underline{\delta}_u(\pi) = 0$ and $\overline{\delta}_u(\pi) = \frac{(1-e/x_l)\overline{x}-c}{q\lambda\gamma}$; and $\underline{\delta}_u(\widehat{\pi}_u) = \overline{\delta}_u(\widehat{\pi}_u) = \overline{d}$.*

While the statements in the proposition are a bit involved, the main idea is fairly straightforward. For both the social planner and the u-type, the incentive to waive liability (and choose $d = 0$, instead) depends foremost on whether π is bigger or smaller than $\widehat{\pi}_s$ or $\widehat{\pi}_u$, respectively. Foremost, for the social planner, we know from Proposition 6 that, when $\pi < \widehat{\pi}_s$ the social welfare is decreasing with respect to d when $d > 0$. Hence, whenever $d > 0$, the social welfare increases by opting out of liability (i.e., by choosing $d = 0$). The story is comparable for the u-type: from Proposition 7, we know that the u-type's expected profit is maximized with $d = 0$ when $\pi < \widehat{\pi}_u$. In short, the social planner will always waive liability when $\pi < \widehat{\pi}_s$ and, similarly, the u-type will always waive liability when $\pi < \widehat{\pi}_u$.

On the other hand, when $\pi \geq \widehat{\pi}_s$ for the social planner (or $\pi \geq \widehat{\pi}_u$ for the u-type), whether the social planner (or the u-type) will waive liability depends on how far d deviates from $\overline{d}$. We know that in these respective regions, for both the social planner and the u-type, the optimal liability is given by $d = \overline{d}$: the social welfare and the u-type's expected profit are maximized with $d = \overline{d}$. Intuitively, then, when the size of liability (d) doesn't deviate "too much" from $\overline{d}$, the social planner and the u-type will not waive liability. Conversely, when d is substantially different from $\overline{d}$, they will waive liability and choose $d = 0$, instead. In Proposition 8, these are expressed using the respective sets of thresholds: $(\underline{\delta}_s(\pi), \overline{\delta}_s(\pi))$ for the social planner and $(\underline{\delta}_u(\pi), \overline{\delta}_u(\pi))$ for the u-type, where $\underline{\delta}_s(\pi) \leq \overline{d} \leq \overline{\delta}_s(\pi)$ and $\underline{\delta}_u(\pi) \leq \overline{d} \leq \overline{\delta}_u(\pi)$. See the dashed lines in Figure 2. For instance, if $d \in [\underline{\delta}_s(\pi), \overline{\delta}_s(\pi)]$ (or $d \in [\underline{\delta}_u(\pi), \overline{\delta}_u(\pi)]$), the social planner (or the u-type) will not waive liability, whereas if $d > \overline{\delta}_s(\pi)$ or $d < \underline{\delta}_s(\pi)$ (or $d > \overline{\delta}_u(\pi)$ or $d < \underline{\delta}_u(\pi)$), the social planner (or the u-type) will waive liability. With these results, we can also see how the u-type's incentive to waive liability differs from the social planner's. The formal statement is presented in the following corollary.

Corollary 2. *If $\pi > \widehat{\pi}_0$, then $\overline{\delta}_u(\pi) < \overline{\delta}_s(\pi)$. When $\pi > \widehat{\pi}_0$ and $d \in [\overline{\delta}_u(\pi), \overline{\delta}_s(\pi)]$, the u-type will waive liability while the social planner will not: u-type's incentive to waive liability $d > 0$ is socially excessive. When $\widehat{\pi}_s < \widehat{\pi}_u$, if $\pi \in [\widehat{\pi}_s, \widehat{\pi}_u)$ and $d \in [\underline{\delta}_s(\pi), \overline{\delta}_s(\pi)]$, the social planner will waive liability while the u-type will not: u-type's incentive to waive liability $d > 0$ is socially insufficient. When $\widehat{\pi}_s > \widehat{\pi}_u$, if $\pi \in [\widehat{\pi}_u, \widehat{\pi}_s)$ and $d \in [\underline{\delta}_u(\pi), \overline{\delta}_u(\pi)]$, the u-type will waive liability while the social planner will not: u-type's incentive to waive liability $d > 0$ is socially excessive.*

The results of the corollary follow directly from Proposition 8. The corollary demonstrates that the social planner's and the u-type's incentives to waive liability do not necessarily align. Foremost, when $\pi > \widehat{\pi}_0$, while both the social planner and the u-type will waive liability only when d is sufficiently larger than $\overline{d}$ ($d > \overline{d}_s > \overline{d}$ for the social planner and $d > \overline{d}_u > \overline{d}$ for the u-type), the u-type will generally have too

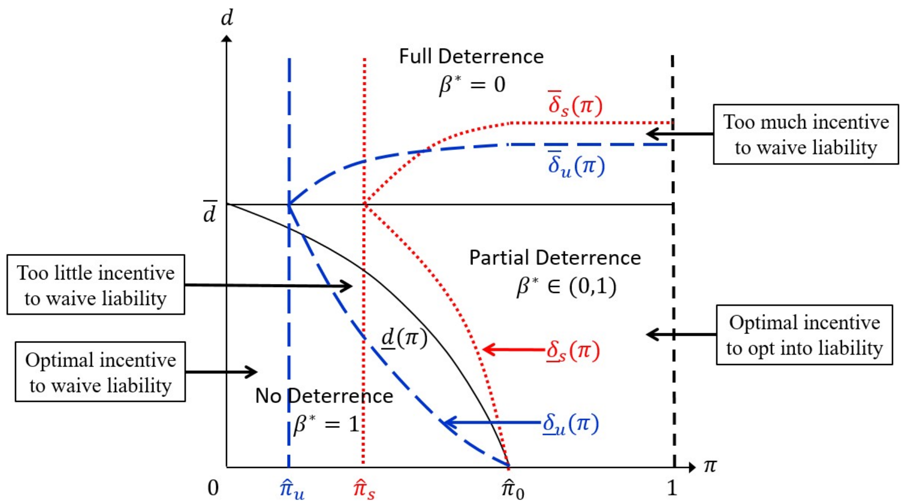

Figure 2: Divergence Between Private and Social Incentive to Waive Liability

much incentive to waive liability. See Figure 2. When $\pi \leq \widehat{\pi}_0$, the incentive alignment will depend (among others) on whether $\widehat{\pi}_s$ is smaller or bigger than $\widehat{\pi}_u$, which, in turn, depends on λ and $\gamma - \gamma_0$, as was shown in Corollary 1. From Corollary 1, we know that when $\gamma - \gamma_0 > 0$ and as λ gets larger, we are more likely to have $\widehat{\pi}_s < \widehat{\pi}_u$. In that case, when $\pi \in [\widehat{\pi}_s, \widehat{\pi}_u)$ and $d \in [\underline{\delta}_s(\pi), \overline{\delta}_s(\pi)]$, even though it is socially optimal to not waive liability, the u-type, finding litigation too privately costly, will waive liability. The opposite will occur with small λ: when $\pi \in [\widehat{\pi}_u, \widehat{\pi}_s)$ and $d \in [\underline{\delta}_u(\pi), \overline{\delta}_u(\pi)]$, the u-type will opt into liability even though litigation is socially wasteful and liability waiver is socially optimal. Figure 2 presents the case of $\widehat{\pi}_u < \widehat{\pi}_s$.

4 Empirical Predictions

In addition to the normative implications regarding liability waiver, the analysis also renders a number of empirical predictions. The first prediction is with respect to shareholders' incentive to pursue private securities litigation. Griffith and Lund (2020), for instance, found that even though shareholder litigation offers potential benefits for large mutual funds, such as Vanguard and BlackRock, most class actions are brought by small institutional or retail investors. With buy-and-hold strategies, large mutual funds have essentially "forfeited" their right to bring litigation.[47] From our model, when a shareholder owns a large fraction of a firm's outstanding equity

[47] See also Webber (2015). Platt (2020) documents an "enforcement shortfall" by the big three institutional investors (Vanguard, BlackRock, and Statestreet). Bebchuk and Hirst (2019) similarly show that mutual funds do not serve as lead plaintiffs in shareholder class actions.

(when α gets large), holding constant the recovery (d), the shareholder would be less inclined (or disinclined) to file or actively participate in a lawsuit since the net return from litigation would be smaller or even non-existent. Formally, as α gets larger, it would become more difficult to satisfy the credibility condition (Assumption 2).[48] Furthermore, when class action lawsuits are filed by other investors, such as retail or small institutional investors with little or no ownership stake in the firm, the institutional investors with a large ownership interest may be more inclined to have the lawsuits dismissed or be settled relatively quickly.[49]

The second set of implications is with respect to initial public offering and a possible way of estimating the size of the deterrence effect. As the analysis shows, foremost, that the relationship between the level of deterrence (the size of d) and IPO under or over-pricing depends on the equilibrium we are in. For instance, when we are in the partial deterrence equilibrium (with $\beta^* \in (0,1)$), as the level of deterrence gets weaker (d gets smaller), because the probability with which the l-type participates in financing (β^*) gets (weakly) higher, over-pricing at the initial offerings becomes more frequent (while the fraction IPOs that is under-priced gets lower).[50] At the same time, while the frequency of over-pricing goes up with lower deterrence, as the rational investors begin to discount the stock more (i.e., demand a larger α in the model), the (average) size of over-pricing should become smaller (and the (average) size of under-pricing becomes larger).

On the other hand, when we are in a region where the l-type's participation is insensitive to the changes in d, either due to full deterrence or no deterrence, the frequency of under-pricing will remain (relatively) constant in the respective region. With full deterrence ($\beta^* = 0$), under and over-pricing frequency will be determined by the realized revenue of the uniformed type (x_h or x_l), whereas with no deterrence

[48] The credibility constraint did not play a major role in our analysis. We have shown, in a separate analysis, that when Assumption 2 is not satisfied, it imposes a credibility threshold, $d_c(\pi) \geq 0$, such that investors bring suit only when $d \geq d_c(\pi)$.

[49] Choi and Spier (2018), for instance, discuss scenarios where shareholders with a long position on the firm would be more willing to accept relatively low settlement offers.

[50] Using a sample of IPOs from 1965 through 2005, Lowry, Officer, and Schwert (2010) show that while on average, IPOs are under-priced, about one-third of IPOs are over-priced (had a negative return, measured over twenty trading days after the IPO). Our model is constructed using the fraction of the firm (α) sold to the investors, but we can easily translate the result using per-share price. Suppose both the l-type and the u-type firms have n total number of shares and sell m shares to the outside investors, so that $m/n = \alpha$. Conditional on non-disclosure, if the investors' expected valuation of the firm is v, we get $p = v/n$, where p stands for the IPO price. We also have $v = \gamma x_l + (1-\gamma)\overline{x}$, where γ stands for the conditional probability of facing the l-type (which depends on β). As γ rises (when β rises), the fraction of IPOs that are over-priced increases but because v falls, the size of over-pricing (measured by $(v - x_l)/n$) decreases.

($\beta^* = 1$), the frequency of over-pricing gets higher due to the presence of the l-type. At the same time, however, the size of over and under-pricing will still be affected by the degree of liability (d). As liability gets stronger, because the size of litigation deadweight loss increases, the size of under-pricing will get bigger (and over-pricing smaller) in both regions as investors would demand a lower IPO price as a compensation for the future litigation cost. In sum, incorporating the liability issue into the initial public offering scenario offers more nuanced understanding of the IPO process. The analysis also offers a way of (indirectly) measuring the level of deterrence, i.e., by estimating the frequency and size of over-pricing (or under-pricing) at IPOs.

The third set of empirical implications deals with when we may be able to observe better or worse deterrence. Foremost, the model shows that when the size of litigation recovery (d) is small and/or the degree of adverse selection (π) is small, the firms (and the entrepreneurs) are much less likely to be deterred (they participate with a high β^*). As the Facebook example in the introduction shows, most of securities class actions are settled and many are settled for relatively small amount (compared to the potential recovery).[51] If we think that the investors expect to receive relatively little from future securities litigation or they believe that the problems of adverse selection isn't severe (π is small), they expect that the damages will produce little or no deterrence and will simply take strategic non-disclosure as given. On the other hand, even when expected recovery may be moderate, when the investors believe that the problems of strategic non-disclosure are substantial, we would observe more frequent litigation and also better deterrence.

The final set of implications deals with conditions under which firms' preference (demand) for a private ordering regime (under which they can tailor their own liability system) can be consistent with social welfare. As we saw earlier, whether the private incentive to waive liability is consistent with social welfare depends on several factors, such as the amount of rent captured by the lawyers in the litigation system ($\gamma - \gamma_0$), size of damages (d), the likelihood of court error (λ), and the probability that the firm raising capital through equity sale has private information (π). As the size of lawyers' rent or the likelihood of false positives grow, for instance, firms become more likely to prefer a liability waiver, whereas with less lawyers' rent and a more accurate adjudication system, firms may be too willing to opt into a liability system. We would assume that these factors are not the same across all firms, and the model tells us how firm characteristics correlate with their incentive to either waive or stay in the

[51] According to Bohn and Choi (1996), over 90% of securities class actions end in settlement. See also Choi, Choi, and Pritchard (2022). Lowry and Shu (2002) shows that the average settlement size, which did not include litigation costs, is about 11% of the total proceeds raised at IPOs.

liability system.

5 Conclusion

Under the current legal system, when a firm sells its stock (or other securities) to outside investors to raise capital, the firm can be held liable to the investors for non-disclosure of material information. Although the general objective of the liability system is to discourage firms from withholding material information and to promote better functioning capital markets, including the IPO market, the system has also attracted criticism for being inefficient and too costly. Is firm liability an effective deterrent? Do the benefits of firm liability outweigh the costs? Instead of having a one-size-fits-all mandatory system, should the firm be able to design their own liability system?

The paper has analyzed how the liability system affects a firm's incentive to disclose material information using a simple game-theoretic model. In the model, the firm can sell stock to the outside investors to raise capital while deciding whether to withhold material information from the prospective investors, and the investors can bring a lawsuit against the firm to recover damages. Investors are rational and forward looking, and anticipate being plaintiffs in future litigation in case non-disclosure is uncovered. The paper has shown that the equilibrium (non-disclosure and financing) depends on a number of factors, including the amount of personal capital that the entrepreneur needs to invest, the size of damages that the investors can recover, the frequency with which the entrepreneur is privately informed (the degree of adverse selection), and the cost of litigation, including both the lawyers' fees and court error.

Building on the analysis, the paper examined several policy proposals, in particular, whether to allow firms to choose their own liability system with a liability waiver. We showed that the firms' choice of liability system may or may not align with the socially-optimal choice. The reason for the divergence comes from the fact that while the firms care about maximizing their own profits (and not about disclosure per se), social welfare depends on deterring strategic non-disclosure and minimizing the dead-weight loss from litigation. The analysis showed that the firms may have a too much or too little incentive to opt out of liability to reduce their cost of capital.[52] The divergence depends, among others, on the amount of rent captured by the lawyers and also on the likelihood of court error. For instance, as the lawyers capture more rent and the court is more likely to find innocent non-disclosing firms liable, the firms

[52]Intuitively, if investors are fully rational and expect the future returns from litigation, they would be willing to finance a firm's investment at a lower cost.

may have too much incentive to waive liability. In such cases, disallowing liability waiver can actually increase social welfare.

Our simple model abstracted from several relevant factors, including externalities on third parties and the bounded rationality of investors. If the returns from litigation are captured in part by non-investing third parties, the link between ex post liability and ex ante financing cost becomes more tenuous. Other factors, such as investor myopia and settlement (that benefits third parties at the expense of investors), can also come into play. When these factors are taken into account, the private incentives to opt out of costly litigation (with liability waivers or class action waivers) may be socially excessive rather than socially insufficient. We intend to analyze these factors in future research.

References

[1] Aggarwal, Dhruv, Albert H. Choi, and Ofer Eldar. "Federal Forum Provisions and the Internal Affairs Doctrine." *Harvard Business Law Review*, Vol. 10 (2020), pp. 383–434.

[2] Alexander, Janet. "The Lawsuit Avoidance Theory of Why Initial Public Offerings are Underpriced." *UCLA Law Review*, Vol. 41 (1993), pp. 17–73.

[3] Atkins, Dorothy. "Facebook's $35M Deal to End IPO Suit Gets Initial OK." *Law360*, 26 February 2018.

[4] Bebchuk, Lucian and Scott Hirst. "Index Funds and the Future of Corporate Governance: Theory, Evidence, and Policy." *Columbia Law Review*, Vol. 119 (2019), pp. 2029–2145.

[5] Bohn, James and Stephen Choi. "Fraud in the New-Issues Market: Empirical Evidence on Securities Class Actions." *University of Pennsylvania Law Review*, Vol. 144 (1996), pp. 903–982.

[6] Caskey, Judson. "The Pricing Effects of Securities Class Acton Lawsuits and Litigation Insurance." *The Journal of Law, Economics, & Organization*, Vol. 30 (2014), pp. 493–532.

[7] Choi, Albert H., Stephen Choi, and Adam Pritchard. "Just Say No? Shareholder Voting on Securities Class Actions." *forthcoming* in *University of Chicago Business Law Review* (2022).

[8] Choi, Albert H. and Kathryn Spier. "Class Actions and Private Antitrust Litigation." *American Economic Journal: Microeconomics*, Vol. 14 (2022), pp. 131–163.

[9] Choi, Albert H. and Kathryn Spier. "The Economics of Class Action Waivers." *Yale Journal on Regulation*, Vol. 38 (2021), pp. 543–565.

[10] Choi, Albert H. and Kathryn Spier. "Taking a Financial Position in Your Opponent in Litigation." *American Economic Review*, Vol. 108 (2019), pp. 3636–3650.

[11] Drake, Philip and Michael Vetsuypens. "IPO Underpricing and Insurance against Legal Liability." *Financial Management*, Vol. 22 (1993), pp. 64–73.

[12] Dye, Ronald. "Disclosure of Nonproprietary Information." *Journal of Accounting Research*, Vol. 23 (1985), pp. 123–145.

[13] Dye, Ronald. "Optimal Disclosure Decisions When There Are Penalties for Nondisclosure." *The RAND Journal of Economics*, Vol. 48 (2017), pp. 704–732.

[14] Farrell, Joseph. "Voluntary Disclosure: Robustness of the Unraveling Result." In R. Grieson, ed. *Antitrust and Regulation*. New York: Lexington Books, 1986.

[15] Graf, Rachel. "Attys in $35M Facebook Deal Seeks $14M in Fees, Costs." *Law360*, 2 August 2018.

[16] Griffith, Sean and Dorothy Lund "A Mission Statement for Mutual Funds in Shareholder Litigation." *University of Chicago Law Review*, Vol. 87 (2020), pp. 1149–1240.

[17] Grossman, Sanford J. "An Introduction to the Theory of Rational Expectations under Asymmetric Information." *Review of Economic Studies*, Vol. 48 (1981), pp. 541–559.

[18] Grossman, Sanford J. and Oliver Hart. "Disclosure Laws and Takeover Bids." *The Journal of Finance*, Vol. 35 (1981), pp. 323–334.

[19] Hughes, Patricia. "Signalling by Direct Disclosure under Asymmetric Information," *Journal of Accounting and Economics*, Vol. 8 (1986), pp. 119–142.

[20] Hughes, Patricia and Anjan Thakor. "Litigation Risk, Intermediation, and the Underpricing of Initial Public Offerings," *Review of Financial Studies*, Vol. 5 (1992), pp. 709–742.

[21] Kim, Moonchul and Jay Ritter. "Valuing IPOs." *Journal of Financial Economics*, Vol. 53 (1999), pp. 409–437.

[22] Lowry, Michelle, Micah Officer, and William Schwert. "The Variability of IPO Initial Returns." *Journal of Finance*, Vol. 65 (2010), pp. 425–465.

[23] Lowry, Michelle and Susan Shu. "Litigation Risk and IPO Underpricing," *Journal of Financial Economics*, Vol. 65 (2002), pp. 309–335.

[24] Milgrom, Paul. "Good News and Bad News: Representation Theorems and Applications," *The Bell Journal of Economics*, Vol. 12 (1981), pp. 380–391.

[25] Platt, Alexander. "Index Fund Enforcement," *UC Davis Law Review*, Vol. 53 (2020), pp. 1453–1529.

[26] Polinsky, A. Mitchell and Steven Shavell. "Mandatory Versus Voluntary Disclosure of Product Risks." *The Journal of Law, Economics, & Organization*, Vol. 28 (2012), pp. 360–379.

[27] Ritter, Jay. "Equilibrium in the Initial Offerings Market." *Annual Review of Financial Economics*, Vol. 3 (2011), pp. 347–374.

[28] Ritter, Jay and Ivo Welch. "A Review of IPO Activity, Pricing, and Allocations," *Journal of Finance*, Vol. 57 (2002), pp. 1795–1828.

[29] Scott, Hal. "Shareholders Deserve Right to Choose Mandatory Arbitration." *Columbia BlueSky Blog*, 28 July 2017. Available at https://clsbluesky.law.columbia.edu/2017/08/21/shareholders-deserve-right-to-choose-mandatory-arbitration/.

[30] Scott, Hal. "The SEC's Misguided Attack on Shareholder Arbitration." *The Wall Street Journal*, 21 February 2019.

[31] Scott, Hal and Leslie Silverman. "Stockholder Adoption of Mandatory Individual Arbitration for Stockholder Disputes." *Harvard Journal of Law and Public Policy*, Vol. 36 (2013), pp. 1187–1230.

[32] Shavell, Steven. "Acquisition and Disclosure of Information Prior to Sale." *The RAND Journal of Economics*, Vol. 25 (1994), pp. 20–36.

[33] Spindler, James. "IPO Liability and Entrepreneurial Response." *University of Pennsylvania Law Review*, Vol. 155 (2007), pp. 1187–1228.

[34] Tinic, Seha. "Anatomy of Initial Public Offerings of Common Stock." *Journal of Finance*, Vol. 43 (1988), pp. 789–822.

[35] Webber, David. "Shareholder Litigation without Class Actions." *Arizona Law Review*, Vol. 57 (2015), pp. 201–267.

Appendix A: Proofs

Proof of Lemma 1. Consider an equilibrium where the u-type participates with positive probability. Suppose there is no disclosure and let b_h, b_u, b_l be the capital market's conditional beliefs where $b_h + b_u + b_l = 1$ where $b_u \in (0, 1]$. Since outside investors break even, α^* satisfies

$$b_h\alpha^*x_h + b_u\alpha^*\overline{x} + b_l(d + \alpha^*(x_l - d)) - c = 0. \tag{28}$$

We will prove $b_h = 0$ (the h-type does not pool with the u-type) and that $\alpha^* \geq c/\overline{x}$.

First consider a PBNE where the l-type does not participate, $b_l = 0$. Suppose that conditional no disclosure, the market believes $b_h > 0$ and $b_u > 0$. In this case, $\alpha^* \in \left(\frac{c}{x_h}, \frac{c}{\overline{x}}\right)$. This isn't a PBNE since the h-type would disclose and secure $\alpha_h = \frac{c}{x_h}$. Suppose instead that the market believes $b_u = 0$ and $b_h = 1$. In this case, $\alpha^* = \alpha_h = c/x_h$. This isn't a PBBE since the u-type would participate. Therefore in a PBNE with $b_l = 0$ we have $b_h = 0$, $b_u = 1$, and $\alpha^* = c/\overline{x}$.

Next consider a PBNE where the l-type participates with positive probability, $b_l > 0$. The l-type's payoff is $(1-\alpha^*)(x_l-d)-e \geq 0$. Rearranging gives $d+\alpha^*(x_l-d) \leq x_l - e$. Substituting into the investors' break-even condition (28),

$$b_h\alpha^*x_h + b_u\alpha^*\overline{x} + b_l(x_l - e) - c \geq 0. \tag{29}$$

We can prove $\alpha^* \geq c/\overline{x}$ by contraction. Suppose first that that $\alpha^* \leq c/x_h < c/\overline{x}$. Substituting into (29),

$$b_hc + b_u\frac{\overline{x}}{x_h}c + b_l(x_l - e) - c \geq 0. \tag{30}$$

Since $\frac{\overline{x}}{x_h} < 1$, this implies

$$b_hc + b_uc + b_l(x_l - e) - c = b_l(x_l - e - c) > 0. \tag{31}$$

This is a contradiction since $x_l - e - c < 0$. Therefore $\alpha^* > c/x_h$. Next, suppose $\alpha^* \in (c/x_h, c/\overline{x})$. The h-type will disclose and secure $\alpha_h = c/x_h < \alpha^*$ and so $b_h = 0$. Substituting into (29),

$$b_uc + b_l(x_l - e) - c = b_l(x_l - e - c) > 0, \tag{32}$$

a contradiction. Therefore $\alpha^* \geq c/\overline{x}$. This completes the proof that if the u-type participates with positive probability then the h-type discloses and the equity stake conditional on non-disclosure is $\alpha^* \geq c/\overline{x}$. $\square$

Proof of Lemma 2. (Baseline Model Full Deterrence Threshold.) Note that $\beta^* = 0$, $\alpha^* = \overline{\alpha} = c/\overline{x}$ defined in (3), and $d = \overline{d}$ defined in (4) satisfy equilibrium conditions (1) and (2) with equality.

Suppose that $d > \overline{d}$. We will prove that $\beta^* = 0$ by contradiction. Suppose not: $\beta^* > 0$. Condition (1) implies that $(1-\alpha^*)(x_l - d) - e \geq 0$. Since $(1-\overline{\alpha})(x_l - \overline{d}) - e = 0$ and $d > \overline{d}$, we must have $\alpha^* < \overline{\alpha}$. Rearranging condition (1) gives $\alpha^*(x_l - d) \leq x_l - d - e$. Substituting this into (2) gives $(1-\pi)(\alpha^*\overline{x} - c) + \pi q \beta^*(x_l - k) \geq 0$. Solving for α^* and using the definition of $\overline{\alpha}$ in (3) gives

$$\alpha^* \geq \overline{\alpha} + \frac{\pi q \beta^*}{1-\pi} \cdot \frac{k - x_l}{\overline{x}}. \tag{33}$$

If $\beta^* > 0$ then $\alpha^* > \overline{\alpha}$, a contradiction. Thus, if $d > \overline{d}$ then $\beta^* = 0$. □

Proof of Lemma 3. (Baseline Model No Deterrence Threshold.) We will prove that if $d < \underline{d}(\pi)$ defined in (5) then $\beta^* = 1$. To do this, we will prove that if $\beta^* < 1$ then $d \geq \underline{d}(\pi)$.

Suppose $\beta^* < 1$. Condition (1) implies $(1-\alpha^*)(x_l - d) - e \leq 0$. Rearranging, $\alpha^*(x_l - d) \geq x_l - d - e$. Substituting into (2) gives $(1-\pi)(\alpha^*\overline{x} - c) + \pi q \beta^*(x_l - k) \leq 0$. Solving for α^* gives:

$$\alpha^* \leq \frac{c}{\overline{x}} + \frac{\pi q \beta^*}{1-\pi} \cdot \frac{k - x_l}{\overline{x}}.$$

Since $\beta^* < 1$ by assumption, and using the definition of $r(\pi)$ in (6),

$$\alpha^* < \frac{c}{\overline{x}} + r(\pi).$$

Next, since condition (1) implies $(1-\alpha^*)(x_l - d) - e \leq 0$ we have

$$d \geq x_l - \frac{e}{1-\alpha^*}.$$

Substituting $\alpha^* < c/\overline{x} + r(\pi)$, gives

$$d > x_l - \frac{e}{1 - c/\overline{x} - r(\pi)}.$$

The right-hand side is $\underline{d}(\pi)$ defined in (5). This establishes that if $\beta^* < 1$ then $d \geq \underline{d}(\pi)$. Therefore if $d < \underline{d}(\pi)$ then $\beta^* = 1$.

We will prove that $\overline{d}(\pi)$ in (5) is decreasing in π and the existence of threshold $\widehat{\pi}_0 \in (0,1)$ where $\overline{d}(\widehat{\pi}_0) = 0$. $r(\pi)$ in (6) is an increasing function of π with $r(0) = 0$

and $\lim_{\pi\to 1} r(\pi) = \infty$. Comparing (5) and (4), and using Assumption 1, confirms $\underline{d}(0) = \overline{d} > 0$. Furthermore, since $r(\pi)$ is increasing in π, $\overline{d}(\pi)$ is decreasing in π for $r(\pi) < 1 - c/\overline{x}$. To find $\widehat{\pi}_0$, note that (5) implies:

$$r(\pi) = 1 - \frac{c}{\overline{x}} - \frac{e}{x_l} = \frac{x_l(\overline{x} - k) - e(\overline{x} - x_l)}{\overline{x} x_l}.$$

Setting this equal to $r(\pi)$ defined in (6) and rearranging terms gives:

$$\frac{1 - \widehat{\pi}_0}{\widehat{\pi}_0 q} \cdot \frac{x_l(\overline{x} - k) - e(\overline{x} - x_l)}{x_l(k - x_l)} = 1. \tag{34}$$

□

Proof of Proposition 2. (Baseline Model Equilibrium Characterization.)

Case 1. The full deterrence result follows from Lemma 2.

Case 2. The partial deterrence result may be found by solving (7) and (8) simultaneously. The comparative statics are immediate. Using the formula for $\widehat{\pi}_0$ in (34) confirms that $\beta^*(d, \pi) \to 1$ as $\pi \to \widehat{\pi}_0$.

Case 3. In the no-deterrence range, the equilibrium $\alpha^*(d, \pi)$ in (11) may be found by setting $\beta^* = 1$ in the investors' break-even condition (2). Notice that $\alpha^*(d, 0) = \overline{\alpha} = c/\overline{x} > 0$ and $\alpha^*(0, \pi) > 0$ for all $\pi \in (0, 1)$.

We will prove that $\alpha^*(d, \pi)$ is decreasing in d and increasing in π. Letting $z = \pi q/(1 - \pi)$, we may rewrite (11) as:

$$\alpha^*(d, z) = \frac{c + z(c - d)}{\overline{x} + z(x_l - d)}. \tag{35}$$

We will now show that $\alpha^*(d, z)$ in (35) is an increasing function of z and, by extension, an increasing function of π. When we differentiate with respect to z, we get:

$$\begin{aligned}
\frac{\partial \alpha^*(d, z)}{\partial z} &= \frac{(\overline{x} + z(x_l - d))(c - d) - (c + z(c - d))(x_l - d)}{(\overline{x} + z(x_l - d))^2} \\
&= \frac{\overline{x}(c - d) - c(x_l - d)}{(\overline{x} + z(x_l - d))^2} \\
&= \frac{c(\overline{x} - x_l) - d(\overline{x} - c)}{(\overline{x} + z(x_l - d))^2}.
\end{aligned}$$

The numerator is a decreasing function of d. To prove that it is positive, it is sufficient to show that this is true when $d = \overline{d}$ defined in (4). If $d = \overline{d}$, the numerator is:

$$c(\overline{x} - x_l) - \left[x_l - \frac{e}{1 - c/\overline{x}}\right](\overline{x} - c)$$

$$= c(\overline{x} - x_l) - x_l(\overline{x} - c) + \frac{e(\overline{x} - c)}{1 - c/\overline{x}}$$

$$= c(\overline{x} - x_l) - x_l(\overline{x} - c) + e\overline{x}$$

$$= \overline{x}(c + e - x_l) = \overline{x}(k - x_l) > 0.$$

Since the numerator is positive when $d = \overline{d}$, it is positive for all $d < \overline{d}$. Thus $\alpha^*(d, z)$ is an increasing function of z and (by extension) $\alpha^*(d, \pi)$ an increasing function of π.

Next we show that $\alpha^*(d, z)$ in (35) decreasing in d. Differentiating with respect to d,

$$\frac{\partial \alpha^*(d, z)}{\partial d} = \frac{(\overline{x} + z(x_l - d))(-z) - (c + z(c - d))(-z)}{(\overline{x} + z(x_l - d))^2}.$$

The denominator is positive. The numerator is negative if

$$-\overline{x} - z(x_l - d) + c + z(c - d) < 0$$

$$-\overline{x} - zx_l + c + zc < 0$$

$$-z(x_l - c) - (\overline{x} - c) < 0.$$

Recall that $z = \pi q/(1 - \pi) > 0$ for all $\pi \in (0, 1)$ and our assumption that $\max\{e, c\} < x_l$ implies that $\overline{x} - c > x_l - c > 0$. Therefore the left-hand side is negative. We have established that $\alpha^*(d, \pi)$ is a decreasing function of d and this completes the proof. $\square$

Proof of Lemma 4. (General Model Full Deterrence Threshold.) Note that $\beta^* = 0$, $\overline{\alpha}(d)$ defined in (14), and $d = \overline{d}$ defined in (15) satisfy equilibrium conditions (1) and (13) with equality.

Suppose that $d > \overline{d}$. We will prove that $\beta^* = 0$ by contradiction. Suppose not: $\beta^* > 0$. Condition (1) implies that $(1 - \alpha^*)(x_l - d) - e \geq 0$. Since $(1 - \overline{\alpha})(x_l - \overline{d}) - e = 0$ and $d > \overline{d}$, we must have $\alpha^* < \overline{\alpha}$. Condition (1) also implies that $\alpha^*(x_l - d) \leq x_l - d - e$. Substituting this into (13) gives

$$(1 - \pi)(\alpha^*(\overline{x} - q\lambda d) - c + q\lambda(1 - \gamma)d) + \pi q\beta^*(x_l - k - \gamma d) \geq 0$$

Solving for α^* gives

$$\alpha^* \geq \frac{c - q\lambda(1-\gamma)d}{\overline{x} - q\lambda d} + \frac{\pi q \beta^*}{1-\pi} \cdot \frac{k + \gamma d - x_l}{\overline{x} - q\lambda d}.$$

Using the definition of $\overline{\alpha}(d)$ in (14),

$$\alpha^* \geq \overline{\alpha}(d) + \frac{\pi q \beta^*}{1-\pi} \cdot \frac{k + \gamma d - x_l}{\overline{x} - q\lambda d},$$

so $\alpha^* > \overline{\alpha}(d)$, a contradiction. Thus, if $d > \overline{d}$ defined in (15) then $\beta^* = 0$.

We now consider comparative statics. We will prove that $\overline{d}$ is increasing in λ and decreasing in γ. Write (14) as $\overline{\alpha} = f(\overline{d}; \lambda, \gamma)$ and (15) as $\overline{\alpha} = f(\overline{d}; \lambda, \gamma)$. The equilibrium is $\overline{\alpha}(\lambda, \gamma)$ and $\overline{d}(\lambda, \gamma)$ and

$$f(\overline{d}(\lambda, \gamma); \lambda, \gamma) - g(\overline{d}(\lambda, \gamma); \lambda, \gamma) = 0.$$

Using the implicit function theorem,

$$\frac{\partial \overline{d}(\lambda, \gamma)}{\partial \lambda} = -\frac{\partial f(\overline{d}; \lambda, \gamma)/\partial \lambda - \partial g(\overline{d}; \lambda, \gamma)/\partial \lambda}{\partial f(\overline{d}; \lambda, \gamma)/\partial \overline{d} - \partial g(\overline{d}; \lambda, \gamma)\partial \overline{d}} \tag{36}$$

and

$$\frac{\partial \overline{d}(\lambda, \gamma)}{\partial \gamma} = -\frac{\partial f(\overline{d}; \lambda, \gamma)/\partial \gamma - \partial g(\overline{d}; \lambda, \gamma)/\partial \gamma}{\partial f(\overline{d}; \lambda, \gamma)/\partial \overline{d} - \partial g(\overline{d}; \lambda, \gamma)\partial \overline{d}}. \tag{37}$$

We will now find the partial derivatives on the right-hand sides of (36) and (37). The functions $\overline{\alpha} = f(\overline{d}; \lambda, \gamma)$ from equation (14) and $\overline{\alpha} = g(\overline{d}; \lambda, \gamma)$ equation (15) can be written as:[53]

$$\overline{\alpha}(\overline{x} - q\lambda\overline{d}) - c + q\lambda(1-\gamma)\overline{d} = 0 \tag{38}$$

and

$$(1 - \overline{\alpha})(x_l - \overline{d}) - e = 0. \tag{39}$$

To begin, we will prove that the denominator of (36) is positive. (Since the denominator of 37) is identical to the denominator of (36), this will prove that the denominator of 37) is positive, too.) Totally differentiating (38) and (39) with respect to $\overline{\alpha}$ and $\overline{d}$,

$$(\overline{x} - q\lambda\overline{d})(\Delta\overline{\alpha}) + q\lambda(1 - \overline{\alpha} - \gamma)(\Delta\overline{d}) = 0, \tag{40}$$

$$-(x_l - \overline{d})(\Delta\overline{\alpha}) - (1 - \overline{\alpha})(\Delta\overline{d}) = 0. \tag{41}$$

[53] $\overline{\alpha}$ and $\overline{d}$ solve the system of equations.

Solving each for $\Delta\overline{\alpha}/\Delta\overline{d}$ gives:

$$\frac{\partial f(\overline{d};\lambda,\gamma)}{\partial\overline{d}} = -\frac{q\lambda(1-\overline{\alpha}-\gamma)}{\overline{x}-q\lambda\overline{d}} < 0 \quad \text{and} \quad \frac{\partial g(\overline{d};\lambda,\gamma)}{\partial\overline{d}} = -\frac{1-\overline{\alpha}}{x_l-\overline{d}} < 0.$$

Substituting these expressions into the denominator of (36),

$$\frac{\partial f(\overline{d};\lambda,\gamma)}{\partial\overline{d}} - \frac{\partial g(\overline{d};\lambda,\gamma)}{\partial\overline{d}} = -\frac{q\lambda(1-\overline{\alpha}-\gamma)}{\overline{x}-q\lambda\overline{d}} + \frac{1-\overline{\alpha}}{x_l-\overline{d}}$$

$$= \frac{-q\lambda(1-\overline{\alpha}-\gamma)(x_l-\overline{d}) + (1-\overline{\alpha})(\overline{x}-q\lambda\overline{d})}{(\overline{x}-q\lambda\overline{d})(x_l-\overline{d})}$$

$$= \frac{(1-\overline{\alpha})(\overline{x}-q\lambda x_l) + q\lambda\gamma(x_l-\overline{d})}{(\overline{x}-q\lambda\overline{d})(x_l-\overline{d})} > 0.$$

Therefore the denominator of (36) is positive. Since the denominator of (37) is identical, it is positive, too.

Now consider the numerator of (36). Totally differentiating (38) and (39) with respect to $\overline{\alpha}$ and λ,

$$(\overline{x}-q\lambda\overline{d})(\Delta\overline{\alpha}) + q(1-\overline{\alpha}-\gamma)\overline{d}(\Delta\lambda) = 0, \tag{42}$$

$$-(x_l-\overline{d})(\Delta\overline{\alpha}) + (0)(\Delta\lambda) = 0. \tag{43}$$

Solving each for $\Delta\overline{\alpha}/\Delta\lambda$ gives:

$$\frac{\partial f(\overline{d};\lambda,\gamma)}{\partial\lambda} = -\frac{q(1-\overline{\alpha}-\gamma)\overline{d}}{\overline{x}-q\lambda\overline{d}} < 0 \quad \text{and} \quad \frac{\partial g(\overline{d};\lambda,\gamma)}{\partial\lambda} = 0.$$

The numerator of (36) is therefore negative:

$$\frac{\partial f(\overline{d};\lambda,\gamma)}{\partial\lambda} - \frac{\partial g(\overline{d};\lambda,\gamma)}{\partial\lambda} = -\frac{q(1-\overline{\alpha}-\gamma)\overline{d}}{\overline{x}-q\lambda\overline{d}} < 0.$$

Since the numerator of the fraction in (36) is negative and the denominator is positive, and since the fraction is preceded by a negative sign, we have proven $\partial\overline{d}(\lambda,\gamma)/\partial\lambda > 0$.

Now consider the numerator of (37). Totally differentiating (38) and (39) with respect to $\overline{\alpha}$ and γ,

$$(\overline{x}-q\lambda\overline{d})(\Delta\overline{\alpha}) - q\lambda\overline{d}(\Delta\gamma) = 0, \tag{44}$$

$$-(x_l-\overline{d})(\Delta\overline{\alpha}) + (0)(\Delta\gamma) = 0. \tag{45}$$

Solving each for $\Delta\overline{\alpha}/\Delta\gamma$ gives:

$$\frac{\partial f(\overline{d};\lambda,\gamma)}{\partial\gamma}=\frac{q\lambda\overline{d}}{\overline{x}-q\lambda\overline{d}}>0\quad\text{and}\quad\frac{\partial g(\overline{d};\lambda,\gamma)}{\partial\gamma}=0.$$

The numerator of (36) is therefore negative:

$$\frac{\partial f(\overline{d};\lambda,\gamma)}{\partial\gamma}-\frac{\partial g(\overline{d};\lambda,\gamma)}{\partial\gamma}=\frac{q\lambda\overline{d}}{\overline{x}-q\lambda\overline{d}}>0.$$

Since the numerator and denominator of (37) are positive, and since the fraction is preceded by a negative sign, we have proven $\partial\overline{d}(\lambda,\gamma)/\partial\gamma<0$. □

Proof of Lemma 5. (General Model No-Deterrence Threshold.) When $d=\underline{d}$, the l-type is just indifferent about participating and not. Setting $\beta^*=1$ in the investors' break-even condition in (13),

$$(1-\pi)(\alpha^*\overline{x}-c+q(1-\gamma-\alpha^*)\lambda\underline{d})+\pi q[\alpha^* x_l-c+(1-\gamma-\alpha^*)\underline{d}]=0.$$

Rearranging,

$$(1-\pi)[\alpha^*(\overline{x}-q\lambda\underline{d})-c+q(1-\gamma)\lambda\underline{d}]+\pi q[\alpha^*(x_l-\underline{d})-c+(1-\gamma)\underline{d}]=0. \quad (46)$$

Suppose that the l-type is just indifferent between participating and not participating. From (1) we have $(1-\alpha^*)(x_l-\underline{d})-e=0$ so $\alpha^*(x_l-\underline{d})=x_l-\underline{d}-e$. Substituting this into (46), we get:

$$(1-\pi)[\alpha^*(\overline{x}-q\lambda\underline{d})-c+q(1-\gamma)\lambda\underline{d}]+\pi q[x_l-\underline{d}-e-c+(1-\gamma)\underline{d}]=0.$$

$$\iff(1-\pi)(\alpha^*(\overline{x}-q\lambda\underline{d})-c+q(1-\gamma)\lambda\underline{d})+\pi q[x_l-k-\gamma\underline{d}]=0.$$

$$\iff\alpha^*(\overline{x}-q\lambda\underline{d})=c-q(1-\gamma)\lambda\underline{d}-\frac{\pi q}{1-\pi}\cdot[x_l-k-\gamma\underline{d}]$$

$$\iff\alpha^*=\frac{c-q(1-\gamma)\lambda\underline{d}}{\overline{x}-q\lambda\underline{d}}+\frac{\pi q}{1-\pi}\cdot\frac{k+\gamma\underline{d}-x_l}{\overline{x}-q\lambda\underline{d}}.$$

This is the equity share demanded by investors when the l-type is just indifferent and participates for sure, $\beta^*=1$. This may be rewritten as:

$$\alpha^*(\underline{d},\pi)=\overline{\alpha}(\underline{d})+r(\underline{d},\pi)$$

where $\overline{\alpha}(\underline{d})$ is defined in (14) and $r(\underline{d},\pi)$ is the risk premium defined in (17). Substituting this into (1) gives the formula for $\underline{d}$ in equation (16). Substituting $\pi=\widehat{\pi}_0$

defined in (34) into (16) gives $\underline{d}(\widehat{\pi}_0) = 0$ and substituting $\pi = 0$ into (16) gives $\underline{d}(0) = \overline{d}$, where $\overline{d}$ is defined in (15).

We will now show that $\underline{d}(\pi)$ is a decreasing function of π. Rewriting (16),

$$1 - \overline{\alpha}(\underline{d}) - r(\underline{d}, \pi) - \frac{e}{x_l - \underline{d}} = 0.$$

Substituting $\overline{\alpha}(\underline{d})$ from (14) and $r(\underline{d}, \pi)$ from (17),

$$1 - \frac{c - q(1-\gamma)\lambda\underline{d}}{\overline{x} - q\lambda\underline{d}} - \frac{\pi q}{1-\pi} \cdot \frac{k + \gamma\underline{d} - x_l}{\overline{x} - q\lambda\underline{d}} - \frac{e}{x_l - \underline{d}} = 0.$$

Multiplying by $\overline{x} - q\lambda\underline{d}$,

$$(\overline{x} - q\lambda\underline{d}) - (c - q(1-\gamma)\lambda\underline{d}) - \frac{\pi q}{1-\pi} \cdot (k + \gamma\underline{d} - x_l) - \frac{e(\overline{x} - q\lambda\underline{d})}{x_l - \underline{d}} = 0.$$

$$(\overline{x} - c - q\gamma\lambda\underline{d}) - \frac{\pi q}{1-\pi} \cdot (k + \gamma\underline{d} - x_l) - \frac{e(\overline{x} - q\lambda\underline{d})}{x_l - \underline{d}} = 0.$$

Totally differentiating with respect to $\underline{d}$ and π,

$$\left[-q\gamma\lambda - \frac{\pi q}{1-\pi} \cdot \gamma - \frac{e(\overline{x} - q\lambda x_l)}{(x_l - \underline{d})^2}\right] \Delta\underline{d} - \frac{q}{(1-\pi)^2} \cdot (k + \gamma\underline{d} - x_l)\Delta\pi = 0.$$

Since the coefficients for $\Delta\underline{d}$ and $\Delta\pi$ are both negative, $\Delta\underline{d}/\Delta\pi < 0$.

□

Proof of Proposition 5. (General Model Equilibrium Characterization.)

Case 1. The full deterrence result follows immediately from Lemma 4.

Case 2. The partial deterrence equilibrium $\alpha^*(d, \pi)$ and $\beta^*(d, \pi)$ in (18) and (19) may be found by solving the l-type's indifference condition (1) and the investors' break-even condition (13) simultaneously.

We now show that $\beta^*(d, \pi)$ defined in (19) is a decreasing function of d. Notice that denominator of (19), $k + \gamma d - x_l$, is increasing in d. If we prove the numerator is decreasing in d we are done. Using the formula for $\alpha^*(d, \pi)$ in (18), the numerator of (19) may be written:

$$(1 - \alpha^*)(\overline{x} - q\lambda d) - c + q\lambda d(1-\gamma)$$

The slope of the numerator of (19) is

$$\frac{\partial\alpha^*(d,\pi)}{\partial d}(\overline{x}-q\lambda d)+q\lambda(1-\alpha^*-\gamma).$$

Using (18) we have $1-\alpha^*=\frac{e}{x_l-d}$ and $\frac{\partial\alpha^*}{\partial d}=\frac{-e}{(x_l-d)^2}$. Substituting these expressions, the slope of the numerator is:

$$\frac{-e}{(x_l-d)^2}(\overline{x}-q\lambda d)+q\lambda\left(\frac{e}{x_l-d}-\gamma\right)$$

$$=\frac{1}{(x_l-d)^2}\left(-e(\overline{x}-q\lambda x_l)-q\lambda(x_l-d)^2\right)<0$$

This proves $\beta^*(d,\pi)$ is a decreasing function of d. Finally, since $\beta^*(d,\pi)$ characterized in (19) is proportional to $(1-\pi)/\pi q$, β^* is decreasing in π.

Case 3. In the no-deterrence range, the equilibrium $\alpha^*(d,\pi)$ in (20) may be found by setting $\beta^*=1$ in the investors' break-even condition (13) and solving for $\alpha^*(d,\pi)$. We will show that $\alpha^*(d,\pi)$ is decreasing in d. Setting $\beta^*=1$ and totally differentiating the investors' break-even condition (13) with respect to α and d,

$$[(1-\pi)(\overline{x}-q\lambda d)+\pi q(x_l-d)]\Delta\alpha^*+[(1-\pi)q\lambda(1-\alpha^*-\gamma)+\pi q(1-\alpha^*)]\Delta d=0.$$

Since the coefficients of $\Delta\alpha^*$ and Δd are both positive, the slope $\Delta\alpha^*/\Delta d$ is negative. Rearranging,

$$\frac{\Delta\alpha^*}{\Delta d}=-\frac{(1-\pi)q\lambda(1-\alpha^*-\gamma)+\pi q(1-\alpha^*)}{(1-\pi)(\overline{x}-q\lambda d)+\pi q(x_l-d)}<0. \tag{47}$$

□

Proof of Lemma 6. (Social Welfare Comparative Statics.) In *Case 1* and *Case 3* of Proposition 5 β^* fixed, so social welfare in (21) is a decreasing function of d. Consider *Case 2*. Some parts of the social welfare function in (21) do not depend on d. The relevant part to examine:

$$-(1-\pi)q\lambda\gamma_0 d-\pi q\beta^*(k+\gamma_0 d-x_l).$$

Substituting β^* from (19) above, this becomes

$$-(1-\pi)q\lambda\gamma_0 d-(1-\pi)\frac{k+\gamma_0 d-x_l}{k+\gamma d-x_l}\left(\alpha^*(\overline{x}-q\lambda d)-c+q(1-\gamma)\lambda d\right)$$

Dropping the $1-\pi$ and substituting $\alpha^* = 1 - \frac{e}{x_l - d}$,

$$-q\lambda\gamma_0 d - \frac{k+\gamma_0 d - x_l}{k+\gamma d - x_l}\left((\overline{x} - q\lambda d) - \frac{e}{x_l - d}(\overline{x} - q\lambda d) - c + q(1-\gamma)\lambda d\right)$$

$$= -q\lambda\gamma_0 d - \frac{k+\gamma_0 d - x_l}{k+\gamma d - x_l}\left(\overline{x} - c - q\gamma\lambda d - \frac{e(\overline{x} - q\lambda d)}{x_l - d}\right)$$

$$= \left(\frac{-q\gamma_0\lambda d(k-x_l) + q\gamma\lambda d(k-x_l)}{k+\gamma d - x_l}\right) - \frac{k+\gamma_0 d - x_l}{k+\gamma d - x_l}\left(\overline{x} - c - \frac{e(\overline{x} - q\lambda d)}{x_l - d}\right).$$

Combining terms in the first bracket, and get

$$= \left\{\frac{q\lambda d(k-x_l)(\gamma-\gamma_0)}{k+\gamma d - x_l}\right\} - \left[\frac{k+\gamma_0 d - x_l}{k+\gamma d - x_l}\right]\left(\overline{x} - c - \frac{e(\overline{x} - q\lambda d)}{x_l - d}\right)$$

Now we can examine each piece. The first term in curly brackets is increasing in d. Since $\gamma_0 < \gamma$, the second term in square brackets is decreasing in d. Finally, the third term in brackets is decreasing in d. This establishes that social welfare is an increasing function of d in the partial deterrence region. This completes the proof. □

Proof of Proposition 6. (Socially-Optimal Liability Rule.) Suppose $\pi \geq \widehat{\pi}_0$. From Lemma 5 we have $\underline{d}(\pi) = 0$. Lemma 6 establishes that social welfare is increasing in d when $d \in [0, \overline{d}]$ (partial deterrence) and decreasing in d when $d > \overline{d}$ (full deterrence). Therefore the socially-optimal damage award $d_s^* = \overline{d}$.

Suppose $\pi < \widehat{\pi}_0$. From Lemma 5 we have $\underline{d}(\pi) > 0$. Lemma 6 establishes that social welfare is decreasing in d when $d \in [0, \underline{d})$ (no deterrence), increasing in d when $d \in [\underline{d}, \overline{d}]$ (partial deterrence), and decreasing in d when $d > \overline{d}$ (full deterrence). Therefore the socially-optimal damage award $d_s^* \in \{0, \overline{d}\}$.

Using the social welfare function in (21), society prefers $d = 0$ and no deterrence ($\beta^* = 1$) to $d = \overline{d}$ and full deterrence ($\beta^* = 0$) if:

$$(1-\pi)(\overline{x}-k) + \pi(1-q)(x_h - k) + \pi q(x_l - k) > (1-\pi)(\overline{x} - k - q\lambda\gamma_0\overline{d}) + \pi(1-q)(x_h - k).$$

Terms cancel and we get

$$\pi q(k - x_l) < (1-\pi)q\lambda\gamma_0\overline{d}. \tag{48}$$

The left-hand side is the social loss if $d = 0$ and there is no deterrence. In this case, the l-types are undeterred so the social loss is $k - x_l$. The right-hand side is the social loss if there is full deterrence. In this case, the social loss reflects the opportunity cost of the lawyer's time. Rearranging (48), the social planner prefers $d = 0$ to $d = \overline{d}$ if and only if $\pi < \widehat{\pi}_s$ where:

$$\frac{1 - \widehat{\pi}_s}{\widehat{\pi}_s q} = \frac{k - x_l}{q\gamma_0\lambda\overline{d}}. \tag{49}$$

This expression is the same as (23) in the text. Since the right-hand side is positive we have $\widehat{\pi}_s > 0$. Since social welfare is strictly increasing in d when $\pi = \widehat{\pi}_0$ and $d \in [0, \overline{d})$, we have $\widehat{\pi}_s < \widehat{\pi}_0$ (by continuity). □

Proof of Lemma 7. (Private Welfare Comparative Statics.) The expected returns of the u-type, l-type, and lawyers, respectively, are:

$$\Pi_u(d, \pi) = (1 - \pi)[(1 - \alpha^*(d, \pi))(\overline{x} - q\lambda d) - e], \tag{50}$$

$$\Pi_l(d, \pi) = \pi q[(1 - \alpha^*(d, \pi))(x_l - d) - e], \tag{51}$$

$$\Pi_a(d, \pi) = (1 - \pi)(\gamma - \gamma_0)q\lambda d + \pi q\beta^*(d, \pi)(\gamma - \gamma_0)d. \tag{52}$$

Case 1. Suppose the l-type is fully deterred, $\beta^*(d, \pi) = 0$. Since the l-type does not participate, the l-type's return is zero and independent of d. From (52) we see that the attorney's return is increasing in d. Since social welfare is decreasing in d (see Lemma 6) we know that the u-type's return is decreasing in d.

Case 2. Suppose the l-type is partially deterred, $\beta^*(d, \pi) \in (0, 1)$. Since the l-type is randomizing between participating and not participating, their expected return is zero for all d. We will now show that the u-type's expected return is increasing in d. From (18) in Proposition 5 we have $1 - \alpha^* = \frac{e}{x_l - d}$. Substituting this into the u-type's return in (50) gives:

$$\frac{e(\overline{x} - q\lambda d)}{x_l - d} - e = \frac{(\overline{x} - q\lambda d) - (x_l - d)}{x_l - d}e = \frac{(\overline{x} - x_l) + (1 - q\lambda)d}{x_l - d}e$$

Since the denominator is decreasing in d and the numerator is increasing in d, we have that the u-type's payoff is increasing in d.

Case 3. Suppose the l-type is undeterred, $\beta^*(d, \pi) = 1$. First, the attorneys' expected return in (52) is increasing in d. Now consider the l-type's expected return. Using expression (1), when $\beta^* = 1$, the l-type's return is:

$$\pi q[(1 - \alpha^*)(x_l - d) - e].$$

Differentiating with respect to d, we get:

$$\pi q\left[-\frac{\partial\alpha^*}{\partial d}(x_l-d)-(1-\alpha^*)\right].$$

Since $\partial\alpha^*/\partial d$ is negative, the first term is positive. The second term is negative. Using the formula for $\Delta\alpha^*/\Delta d$ from (47) above, the expression, after some algebra, becomes

$$\pi q\left[\frac{q(1-\gamma-\alpha^*)[(1-\pi)\lambda+\pi]}{(1-\pi)(\overline{x}-q\lambda d)+\pi q(x_l-d)}(x_l-d)-(1-\alpha^*)\right]$$

$$=\pi q\left[\frac{q(x_l-d)[(1-\pi)\lambda+\pi]-(1-\pi)(\overline{x}-q\lambda x_l)-q(x_l-d)[(1-\pi)\lambda+\pi]}{(1-\pi)(\overline{x}-q\lambda x_l)+q(x_l-d)[(1-\pi)\lambda+\pi]}(1-\gamma-\alpha^*)-\gamma\right]$$

Hence, the slope of the l-type's aggregate return with respect to d is negative:

$$\pi q\left[\frac{-(1-\pi)(\overline{x}-q\lambda x_l)(1-\gamma-\alpha^*)}{(1-\pi)(\overline{x}-q\lambda x_l)+q(x_l-d)[(1-\pi)\lambda+\pi]}-\gamma\right]<0.$$

This completes the proof that the l-type's return is decreasing in d.

Now, consider the u-type's expected return. From (50), the aggregate u-type return is

$$(1-\pi)[(1-\alpha^*)(\overline{x}-q\lambda d)-e].$$

Differentiating with respect to d, we get:

$$(1-\pi)\left[-\frac{\Delta\alpha^*}{\Delta d}(\overline{x}-q\lambda d)-q\lambda(1-\alpha^*)\right].$$

When we substitute the expression for $\frac{\Delta\alpha^*}{\Delta d}$ in (47) from above, the expression becomes:

$$(1-\pi)\left[\frac{q(1-\gamma-\alpha^*)[(1-\pi)\lambda+\pi]}{(1-\pi)(\overline{x}-q\lambda d)+\pi q(x_l-d)}(\overline{x}-q\lambda d)-q\lambda(1-\alpha^*)\right]$$

When we (i) pull the q out, (i) rearrange the numerator of first term, and (iii) add and subtract $\lambda\gamma$ in the square brackets, we get:

$$q(1-\pi)\left[\frac{[(1-\pi)\lambda+\pi](\overline{x}-q\lambda d)}{(1-\pi)(\overline{x}-q\lambda d)+\pi q(x_l-d)}(1-\gamma-\alpha^*)-\lambda(1-\gamma-\alpha^*)-\lambda\gamma\right]$$

Expanding the numerator and combining the two terms that include $1-\gamma-\alpha^*$,

$$q(1-\pi)\left[\frac{(1-\pi)\lambda(\overline{x}-q\lambda d)+\pi(\overline{x}-q\lambda d)-\lambda(1-\pi)(\overline{x}-q\lambda d)-\lambda\pi q(x_l-d)}{(1-\pi)(\overline{x}-q\lambda d)+\pi q(x_l-d)}(1-\gamma-\alpha^*)-\lambda\gamma\right]$$

The first and third terms in the numerator cancel:

$$q(1-\pi)\left[\frac{\pi(\overline{x}-q\lambda d)-\lambda\pi q(x_l-d)}{(1-\pi)(\overline{x}-q\lambda d)+\pi q(x_l-d)}(1-\gamma-\alpha^*)-\lambda\gamma\right]$$

The numerator simplifies further. The slope of the u-type's payoff return function is:

$$=q(1-\pi)\left[\frac{\pi(\overline{x}-\lambda q x_l)}{(1-\pi)(\overline{x}-q\lambda d)+\pi q(x_l-d)}(1-\gamma-\alpha^*)-\lambda\gamma\right]$$

This slope is an increasing function of d. To see why, note that *Case 3* or Proposition 5 establishes that $\partial\alpha^*/\partial d<0$, so $(1-\gamma-\alpha^*)$ is an increasing function of d. The denominator of the fraction is obviously a decreasing function of d. Since the slope of the u-type's return is an increasing function of d, the u-type's return is convex. Therefore the u-type's expected return cannot obtain a local maximum in the no-deterrence region. □

Proof of Proposition 7. (Privately-Optimal Liability Rule.) According to Lemma 7, the u-type's return is decreasing in d in *Case 1*, so we know that $d_u^*\le\overline{d}$. Since the u-type's return is increasing in d in *Case 2* we know that $d_u^*\notin(\underline{d}(\pi),\overline{d})$. Finally, since the u-type's return is convex in d in *Case 3* we know that their return $d_u^*\notin(0,\underline{d}(\pi))$. Therefore $d_u^*\in\{0,\overline{d}\}$.

If $\pi\ge\widehat{\pi}_0$ then $\underline{d}(\pi)=0$ and so the u-type's private optimum is $d_u^*=\overline{d}$. Suppose $\pi<\widehat{\pi}_0$ so $\underline{d}(\pi)>0$. We now consider this case.

Suppose $d=0$ then were are in *Case 3* of Proposition 5 where there is no deterrence, $\beta^*(0,\pi)=1$. Substituting $d=0$ into the u-type's return in (50) and letting $\alpha^*=\alpha^*(0,\pi)$ defined in (25) gives:

$$\begin{aligned}\Pi_u(0,\pi)=(1-\pi)[(1-\alpha^*)\overline{x}-e]&=(1-\pi)[\overline{x}-e-\alpha^*\overline{x}]\\&=(1-\pi)[\overline{x}-k-(\alpha^*\overline{x}-c)].\end{aligned}\tag{53}$$

Substituting $\beta^*=1$ and $d=0$ into the investors' break-even constraint in (13) gives

$$(1-\pi)(\alpha^*\overline{x}-c)+\pi q(\alpha^* x_l-c)=0.$$

$$\alpha^*\overline{x} - c = -\frac{\pi q}{1-\pi}(\alpha^* x_l - c). \tag{54}$$

Substituting (54) into (53) we rewrite the u-type's return:

$$\begin{aligned}\Pi_u(0,\pi) &= (1-\pi)(\overline{x}-k) + \pi q(\alpha^* x_l - c)\\ &= (1-\pi)(\overline{x}-k) + \pi q(\alpha^* x_l - k + e - x_l + x_l)\\ &= (1-\pi)(\overline{x}-k) + \pi q[-(k-x_l) - ((1-\alpha^*)x_l - e)]\end{aligned}$$

$$\Pi_u(0,\pi) = (1-\pi)(\overline{x}-k) - \pi q(k-x_l) - \pi q[(1-\alpha^*)x_l - e]. \tag{55}$$

This expression is intuitive. The first term is the u-type's aggregate return in a perfect world with no asymmetric information. The u-type does not operate in this perfect world. The second term is the social loss when the l-types participate. The third term are the l-types' rents from participation.

Next, suppose $d = \overline{d}$ defined in (15) so there is full deterrence, $\beta^*(\overline{d},\pi) = 0$. Using (21), social welfare is:

$$SW(\overline{d},\pi) = (1-\pi)(\overline{x} - k - q\lambda\gamma_0\overline{d}) + \pi(1-q)(x_h - k).$$

$$= (1-\pi)(\overline{x} - k - q\lambda\gamma\overline{d}) + (1-\pi)(\gamma-\gamma_0)q\lambda\overline{d} + \pi(1-q)(x_h - k).$$

Recall that social welfare is the sum of the u-type, l-type, h-type, and attorney expected returns. Since there is full deterrence, the l-types get zero. From (52), the attorneys' expected return is $\Pi_a(\overline{d},\pi) = (1-\pi)(\gamma-\gamma_0)q\lambda\overline{d}$. The h-type's expected return is $\Pi_h(\overline{d},\pi) = \pi(1-q)(x_h-k)$. Therefore we have

$$SW(\overline{d},\pi) = (1-\pi)(\overline{x} - k - q\lambda\gamma\overline{d}) + \Pi_a(\overline{d},\pi) + \Pi_h(\overline{d},\pi)$$

and so

$$\Pi_u(\overline{d},\pi) = (1-\pi)(\overline{x}-k) - (1-\pi)q\gamma\lambda\overline{d}. \tag{56}$$

This expression is intuitive. The first term is the u-type's aggregate return in a perfect world with no litigation. The second term is the expected litigation cost.

We now characterize the threshold $\widehat{\pi}_u \in (0, \widehat{\pi}_0)$. The u-type will prefer $d = 0$ to $d = \overline{d}$ when (55) is larger than (56) or:

$$(1-\pi)(\overline{x}-k) - \pi q(k-x_l) - \pi q[(1-\alpha^*)x_l - e] > (1-\pi)(\overline{x}-k) - (1-\pi)q\gamma\lambda\overline{d}$$

$$\begin{aligned} -\pi q(k-x_l) - \pi q[(1-\alpha^*)x_l - e] &> -(1-\pi)q\gamma\lambda\overline{d}\\ (1-\pi)q\gamma\lambda\overline{d} &> \pi q(k-x_l) + \pi q[(1-\alpha^*)x_l - e] \end{aligned} \tag{57}$$

where $\alpha^* = \alpha^*(0, \pi)$ is defined in (25). This expression is intuitive. The left-hand side is the u-type's loss if $d = \overline{d}$. The loss is the expected payment to the lawyers. The right-hand side is the u-type's loss from $d = 0$. This includes the social loss from the participation of the l-types, $k - x_l$, plus the information rents that accrue to the l-types.

Rearranging (24),

$$\frac{1-\pi}{\pi q} q\gamma\lambda\overline{d} > c - \alpha^*(0,\pi)x_l.$$

Substituting $\alpha^*(0, \pi)$ from (25),

$$\frac{1-\pi}{\pi q} q\gamma\lambda\overline{d} > c - \frac{(1-\pi)cx_l + \pi q c x_l}{(1-\pi)\overline{x} + \pi q x_l}$$

$$\iff \frac{1-\pi}{\pi q} q\gamma\lambda\overline{d} > \frac{(1-\pi)c\overline{x} + \pi q c x_l - (1-\pi)cx_l - \pi q c x_l}{(1-\pi)\overline{x} + \pi q x_l}$$

$$\iff \frac{1-\pi}{\pi q} q\gamma\lambda\overline{d} > \frac{(1-\pi)(\overline{x} - x_l)c}{(1-\pi)\overline{x} + \pi q x_l}$$

Rearranging, the u-type prefers $d = 0$ to $d = \overline{d}$ when

$$q\gamma\lambda\overline{d} > \frac{\pi q(\overline{x} - x_l)c}{(1-\pi)\overline{x} + \pi q x_l}. \tag{58}$$

Using (58) we may easily characterize the threshold $\widehat{\pi}_u \in (0, \widehat{\pi}_0)$ where if $\pi < \widehat{\pi}_u$ the u-type wants $d = 0$ and if $\pi > \widehat{\pi}_u$ the u-type wants $d = \overline{d}$. From (58), $\widehat{\pi}_u$ is implicitly defined by

$$q\gamma\lambda\overline{d} = \frac{\widehat{\pi}_u q(\overline{x} - x_l)c}{(1-\widehat{\pi}_u)\overline{x} + \widehat{\pi}_u q x_l}.$$

Since $\overline{d} > 0$ defined in (15) does not depend on π, we know that $\widehat{\pi}_u > 0$. Rearranging, the u-type prefers $d = 0$ to $d = \overline{d}$ if and only if $\pi < \widehat{\pi}_u$ where:

$$\frac{1-\widehat{\pi}_u}{\widehat{\pi}_u q} = \frac{(1 - x_l/\overline{x})c - q\gamma\lambda\overline{d}x_l/\overline{x}}{q\gamma\lambda\overline{d}}. \tag{59}$$

This is equation (26) in the text. Recall that we proved above that $\widehat{\pi}_u > 0$, so the numerator of (26) is positive. Since the u-type strictly prefers $d = d_u^* = \overline{d}$ to $d = 0$ when $\pi = \widehat{\pi}_0$, we know $\widehat{\pi}_u < \widehat{\pi}_0$.

□

Proof of Corollary 1. (Divergence Between Private and Social Incentives.) The right-hand side of (27) is independent of $\lambda, \gamma, \gamma_0$, and π because $\alpha^*(0, \pi)$ defined in (20) doesn't depend on these parameters. If $\gamma = \gamma_0$, the left-hand side of (27) is equal to zero so (27) is violated. Therefore the private incentive of the u-type to impose liability is socially excessive, $\widehat{\pi}_u < \widehat{\pi}_s$. If $\gamma > \gamma_0$, the left-hand side of (27) is positive. Lemma 4 implies $\overline{d}$ is weakly increasing in λ. Therefore $\lambda\overline{d}$ on the left-hand side of (27) is also an increasing function of λ. So, as λ increases, the private incentive to impose liability gets weaker relative to the social incentive. When $\lambda = 0$ then (27) is not satisfied. So there exists $\widehat{\lambda} \in (0, 1]$ so that the private incentive to impose liability is stronger than the social incentive for $\lambda \leq \widehat{\lambda}$. □

Proof of Proposition 8. (General Liability Waiver.) We will divide the proof into three cases: (1) when $\pi < \widehat{\pi}_s$ for the social planner or $\pi < \widehat{\pi}_u$ for the u-type, respectively; (2) when $\pi > \widehat{\pi}_0$; and (3) when $\pi \in [\widehat{\pi}_s, \widehat{\pi}_0]$ for the social planner or $\pi \in [\widehat{\pi}_u, \widehat{\pi}_0]$ for the u-type, respectively.

Case 1. Suppose $\pi < \widehat{\pi}_s$ and consider the social planner. From Proposition 6, we know that $SW(d = 0) > SW(d = \overline{d})$. We also know from Lemma 6, that $\frac{\partial SW}{\partial d} < 0$, whenever $d > 0$. Therefore, social welfare is maximized with $d = 0$ ($SW(d = 0) > SW(d) \forall d > 0$) and the social planner will always waive liability (and choose $d = 0$) when $d > 0$.

Now, consider the u-type and suppose $\pi < \widehat{\pi}_u$. Similar to the case for the social planner, we know that $\Pi_u(d = 0, \pi) > \Pi_u(d = \overline{d}, \pi)$. We also know, from Lemma 7, that when $d \geq \overline{d}$, $\frac{\partial \Pi_u(d,\pi)}{\partial d} < 0$. Hence, $\forall d \geq \overline{d}$, $\Pi_u(d, \pi) < \Pi_u(d = 0, \pi)$. When $d \in (0, \overline{d})$, (1) the convexity of $\Pi_u(d, \pi)$ with respect to d and (2) $\Pi_u(d = 0, \pi) > \Pi_u(d = \overline{d}, \pi)$ imply that $\Pi_u(d = 0, \pi) > \Pi_u(d, \pi)$ when $d \in (0, \overline{d})$. Therefore, similar to the social planner, the u-type will waive liability when $d > 0$.

Case 2. Suppose $\pi > \widehat{\pi}_0$. We know, from Propositions 6 and 7, that the optimal liability is $d = \overline{d}$ for both the social planner and the u-type. Furthermore, when $d < \overline{d}$, $\frac{\partial SW}{\partial d} > 0$ and $\frac{\partial \Pi_u(d,\pi)}{\partial d} > 0$. When $d = 0$, although $\beta^* \geq 0$, since the l-type's expected profit is zero ($\Pi_l(d = 0, \pi) = 0$), both the expected welfare loss and the expected loss for the u-type are given by $\pi q \beta^*(0, \pi)(k - x_l)$. From Proposition (5), we know that $\beta^*(0, \pi) = \frac{1-\pi}{\pi q} \cdot \frac{(1-e/x_l)\overline{x}-c}{k-x_l}$. With this expression, the expected welfare and the u-type's profit loss (when $d = 0$) becomes:

$$\pi q \beta^*(0, \pi)(k - x_l) = \pi q \cdot \frac{1 - \pi}{\pi q} \cdot \frac{(1 - e/x_l)\overline{x} - c}{k - x_l} \cdot (k - x_l) = (1 - \pi)((1 - e/x_l)\overline{x} - c).$$

When $d \geq \overline{d}$, with $\beta^*(d \geq \overline{d}, \pi) = 0$, the expected welfare loss and the u-type's expected loss are given by $(1 - \pi)q\lambda\gamma_0 d$ and $(1 - \pi)q\lambda\gamma d$, respectively. Note that both expressions are strictly increasing with respect to d (or, equivalently, $\frac{\partial SW}{\partial d} < 0$ and $\frac{\partial \Pi_u(d,\pi)}{\partial d} < 0$). Hence, $\exists \overline{\delta}_s > \overline{d}$ and $\overline{\delta}_u > \overline{d}$, such that, $(1-\pi)q\lambda\gamma_0\overline{\delta}_s = (1-\pi)q\lambda\gamma\overline{\delta}_u = (1-\pi)((1-e/x_l)\overline{x} - c)$. When we rearrange and solve for $\overline{\delta}_s$ and $\overline{\delta}_u$, we get:

$$\overline{\delta}_u = \frac{(1-e/x_l)\overline{x} - c}{q\lambda\gamma} \leq \frac{(1-e/x_l)\overline{x} - c}{q\lambda\gamma_0} = \overline{\delta}_s,$$

where the inequality is strict whenever $\gamma > \gamma_0$. Note that the expressions are independent of π. When $\pi > \widehat{\pi}_0$, therefore, the social planner will waive liability if and only if $d > \overline{\delta}_s = \frac{(1-e/x_l)\overline{x}-c}{q\lambda\gamma_0}$, and the u-type will waive liability if and only if $d > \overline{\delta}_u = \frac{(1-e/x_l)\overline{x}-c}{q\lambda\gamma}$. This also implies that $\underline{\delta}_s = \underline{\delta}_u = 0$.

Case 3. Suppose $\pi \in [\widehat{\pi}_s, \widehat{\pi}_0]$ and consider, first, the social planner. When $d = 0$, the expected welfare loss is given by $WL(d = 0) = \pi q(k - x_l)$, and when $d \geq \overline{d}$, the expected welfare loss is $WL(d \geq \overline{d}) = (1-\pi)q\lambda\gamma_0 d$. We also know, from Proposition 6, that $\pi q(k - x_l) \geq (1-\pi)q\lambda\gamma_0\overline{d}$. Given that $WL(d \geq \overline{d}) = (1-\pi)q\lambda\gamma_0 d$ is strictly increasing with respect to d, this implies that $\exists \overline{\delta}_s \geq \overline{d}$ such that $\pi q(k - x_l) = (1-\pi)q\lambda\gamma_0\overline{\delta}_s$. When we solve for $\overline{\delta}_s(\pi)$, we get:

$$\overline{\delta}_s(\pi) = \frac{\pi}{1-\pi} \cdot \frac{k - x_l}{\lambda\gamma_0}.$$

Note that, as π gets larger, $\overline{\delta}_s$ gets larger. Also, when $\pi = \widehat{\pi}_0$, we know that $\widehat{\pi}_0 q(k - x_l) = (1-\widehat{\pi}_0)q\lambda\gamma_0\overline{d}$. Hence, $\overline{\delta}_s(\widehat{\pi}_0) = \frac{(1-e/x_l)\overline{x}-c}{q\lambda\gamma_0}$.

To establish $\underline{\delta}_s(\pi)$, first, when $d < \underline{d}(\pi)$, we know that $\frac{\partial SW}{\partial d} < 0$ and when $d \geq \underline{d}(\pi)$, $\frac{\partial SW}{\partial d} \geq 0$. Therefore, when $d \in (0, \underline{d}(\pi)]$, we have $WL(d \in (0, \underline{d}(\pi)]) > WL(d = 0)$. Given that $WL(\overline{d}) \leq WL(d = 0)$, this implies that, for any given $\pi \in [\widehat{\pi}_s, \widehat{\pi}_0]$, $\exists \underline{\delta}_s(\pi) \in (\underline{d}(\pi), \overline{d}]$ such that:

$$WL(\underline{\delta}_s) = (1-\pi)q\lambda\gamma_0\underline{\delta}_s + \pi q\beta^*(\underline{\delta}_s, \pi)(k - x_l + \gamma_0\underline{\delta}_s) = \pi q(k - x_l) = WL(d = 0)$$

This implicitly defines the function $\underline{\delta}_s(\pi)$. When $\pi = \widehat{\pi}_0$, from Lemma 6, we know that $\underline{d}(\widehat{\pi}_0) = 0$, and $\frac{\partial SW}{\partial d} > 0 \ \forall d > 0$. Hence, $\underline{\delta}_s(\widehat{\pi}_0) = 0$. On the other hand, when $\pi = \widehat{\pi}_s$, given that $WL(d = 0) = WL(d = \overline{d})$, we have $\underline{\delta}_s(\widehat{\pi}_0) = \overline{d}$.

Now, consider the u-type and suppose $\pi \in [\widehat{\pi}_u, \widehat{\pi}_0]$. The analysis is fairly similar to that for the social planner. Foremost, when $d = \overline{d}$, the u-type's expected loss is given by $(1-\pi)q\lambda\gamma\overline{d}$, and when $d = 0$, the u-type's expected loss is $\pi q((k - x_l) + (1 - \alpha^*(d = 0, \pi))x_l - e)$. We also know, from Lemma 7, that $(1-\pi)q\lambda\gamma\overline{d} \leq \pi q((k - x_l) + (1 - \alpha^*(d = 0, \pi))x_l - e)$. First, suppose that $d \geq \overline{d}$. Given that the u-type's expected loss $((1-\pi)q\lambda\gamma d)$ is strictly increasing with respect to d and that $(1-\pi)q\lambda\gamma\overline{d} \leq \pi q((k-x_l)+(1-\alpha^*(d=0,\pi))x_l-e)$, $\exists\overline{\delta}_u \geq \overline{d}$ such that $(1-\pi)q\lambda\gamma\overline{\delta}_s = \pi q((k-x_l)+(1-\alpha^*(d=0,\pi))x_l-e)$. When we rewrite the expression, we get:

$$\overline{\delta}_u(\pi) = \frac{\pi q}{1-\pi} \cdot \frac{(k-x_l)+(1-\alpha^*(d=0,\pi))x_l-e}{\lambda\gamma}.$$

When we compare this with $\overline{\delta}_s(\pi) = \frac{\pi q}{1-\pi} \cdot \frac{k-x_l}{\lambda\gamma_0}$, since $(1-\alpha^*(d=0,\pi))x_l-e \geq 0$ and $\gamma \geq \gamma_0$, $\overline{\delta}_u(\pi) \lesseqgtr \overline{\delta}_s(\pi)$ when $\pi \in [\widehat{\pi}_u, \widehat{\pi}_0]$. On the other hand, as $\pi \to \widehat{\pi}_0$, we know that $(1-\alpha^*(d=0,\pi))x_l-e \to 0$ and $\overline{\delta}_u(\widehat{\pi}_0) \to \frac{\pi q}{1-\pi} \cdot \frac{k-x_l}{\lambda\gamma} < \frac{\pi q}{1-\pi} \cdot \frac{k-x_l}{\lambda\gamma_0} = \overline{\delta}_s(\widehat{\pi}_0)$.

Finally, suppose $d < \overline{d}$. When $d \in (0,\overline{d})$, $\Pi_u(d) = (1-\pi)((1-\alpha^*(d,\pi))(\overline{x} - q\lambda d)-e)$, and the u-type's expected loss is given by $(1-\pi)q\lambda\gamma d + \pi q((k-x_l)+(1-\alpha^*(d,\pi))(x_l-d)-e)$. First, $\Pi_u(d=\overline{d}) > \Pi_u(d=0)$ implies that $\exists\tilde{d} \in [0,\overline{d}]$ such that $\frac{\partial \Pi_u}{\partial d}|_{\tilde{d}>0}$. This, in turn, implies that $\exists\underline{\delta}_u \in [0,\overline{d}]$ such that $\Pi_u(\underline{\delta}_u) = \Pi(d=0)$. Second, the fact that $\Pi_u(d)$ is convex with respect to d implies that: (1) $\frac{\partial \Pi_u}{\partial d}|_{\underline{\delta}_u>0}$, since otherwise we cannot have $\underline{\delta}_u$, and (2) $\Pi_u(d > \underline{\delta}_u) > \Pi_u(\underline{\delta}_u) \geq \Pi_u(d \leq \underline{\delta}_u)$. In other words, the u-type will waive liability when $d > \underline{\delta}_u$, but not waive liability when $d \leq \underline{\delta}_u$. $\square$

Online Appendix B

Two-Sided Litigation Costs: A Transformation. In the main text, we simplified the analysis by assuming that only the investor-plaintiffs bear litigation costs. Suppose that the firm-defendant must bear litigation costs, too. Let d be the court-awarded damages, $\gamma_d d$ the firm-defendant's litigation costs; $\gamma_p d$ the investor-plaintiffs' litigation costs; $\gamma_p d$ and $\gamma_d d$ the lawyers' opportunity costs. Let's define new variables:

$$\widehat{d} = d(1+\gamma_d), \quad \widehat{\gamma} = \frac{\gamma_p + \gamma_d}{1+\gamma_d}, \quad \textit{and} \quad \widehat{\gamma}_0 = \frac{\gamma_{0p} + \gamma_{0d}}{1+\gamma_d} \tag{60}$$

Our model with litigation costs for the plaintiff and firm is isomorphic to a model where the firm's litigation cost is 0, the plaintiffs' litigation cost is $\widehat{\gamma}\widehat{d}$, and the lawyers' opportunity cost is $\widehat{\gamma}_0\widehat{d}$. The following analysis demonstrates the equivalence.[54]

With litigation costs, the l-type's payoff is:

$$(1-\alpha)(x_l - (1+\gamma_d)d) - e.$$

Using the formula for $\widehat{d}$ from (60), the l-type's payoff is:

$$(1-\alpha)(x_l - \widehat{d}) - e.$$

This is aligned with equation (1) in the main text. Now consider the u-type's payoff is:

$$(1-\alpha)(\overline{x} - q\lambda(1+\gamma_d)d) - e$$

Using the formula for $\widehat{d}$ from (60), the u-type's payoff is is:

$$(1-\alpha)(\overline{x} - q\lambda\widehat{d}) - e$$

This is the same as in the main text. Notice that with the transformation, it appears "as if" the firm has no litigation costs. Finally, consider the investors. Suppose $x = x_l$ and there is a lawsuit. The investors' payoff with the old notation:

$$\alpha x_l - c + (1 - \gamma_p - \alpha(1+\gamma_d))d.$$

Rearranging this expression, with some algebra:

$$= \alpha x_l - c + \left(1 - \frac{\gamma_p + \gamma_d}{1+\gamma_d} - \alpha\right) d(1+\gamma_d).$$

[54]This is ignoring the credibility constraint for the plaintiffs.

Using the formulas for $\widehat{d}$ and $\widehat{\gamma}$ from above, the investors' payoff if $x = x_l$ and there is a lawsuit is:

$$\alpha x_l - c + (1 - \widehat{\gamma} - \alpha)\,\widehat{d}.$$

Now consider social welfare. Using the old notation, social welfare is:

$$(1-\pi)(\overline{x} - k - q\lambda(\gamma_{0p} + \gamma_{0d})d) + \pi(1-q)(x_h - k) + \pi q\beta(x_l - k - (\gamma_{0p} + \gamma_{0d})d)$$

Using the formulas for $\widehat{d}$ and $\widehat{\gamma}_0$ above,

$$(\gamma_{0p} + \gamma_{0d})d = \left(\frac{\gamma_{0p} + \gamma_{0d}}{1 + \gamma_d}\right)(1 + \gamma_d)d = \widehat{\gamma}_0\widehat{d}.$$

The social welfare function is:

$$(1-\pi)(\overline{x} - k - q\lambda\widehat{\gamma}_0\widehat{d}) + \pi(1-q)(x_h - k) + \pi q\beta(x_l - k - \widehat{\gamma}_0\widehat{d})$$

This is aligned with (21) in the main text. This proves that a model with proportional litigation costs on both sides is isomorphic to a model with proportional litigation costs for the plaintiff only. □

about ECGI

The European Corporate Governance Institute has been established to improve *corporate governance through fostering independent scientific research and related activities.*

The ECGI will produce and disseminate high quality research while remaining close to the concerns and interests of corporate, financial and public policy makers. It will draw on the expertise of scholars from numerous countries and bring together a critical mass of expertise and interest to bear on this important subject.

The views expressed in this working paper are those of the authors, not those of the ECGI or its members.

Electronic Access to the Working Paper Series

The full set of ECGI working papers can be accessed through the Institute's Web-site (https://ecgi.global/content/working-papers) or SSRN:

Finance Paper Series	http://www.ssrn.com/link/ECGI-Fin.html
Law Paper Series	http://www.ssrn.com/link/ECGI-Law.html