

# The Voting Premium

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The Voting Premium

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We are grateful to Rui Albuquerque, Archishman Chakraborty, Ingolf Dittmann, Alex Edmans, Axel Kind, Kristian Rydqvist, conference participants at the 2021 Midwest Finance Association, 8th Annual Conference on Financial Market Regulation, and seminar participants at McGill University, University of Rochester, and University of Washington for helpful comments and discussions.

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#### **Abstract**

This paper develops a unified theory of blockholder governance and the voting premium. It explains how and why a voting premium emerges in the absence of takeovers and controlling shareholders. The model features a minority blockholder and dispersed shareholders who trade shares in a competitive market. Those who own shares after trading vote on a proposal at a shareholder meeting. A voting premium can emerge in equilibrium from the blockholderís desire to influence who exercises control, rather than from exercising control himself. We show that the voting premium is unrelated to measures of voting power and that empirical measures of the voting premium generally do not reflect the economic value of voting rights. Consistent with recent empirical studies, the model can generate a negligible voting premium even when the allocation of voting rights is important. The model can also explain a negative voting premium, which has been documented in several studies. It arises because of free-riding by dispersed shareholders on the blockholder's trades. Finally, the model has novel implications for the liquidity of voting vs. non-voting shares, the relationship between the voting premium and the price of a vote, competition for control among blockholders, the block premium, and corporate influence more generally.

Keywords: Voting, trading, voting premium, blockholders, ownership structure, shareholder rights, corporate governance

JEL Classifications: D74, D82, D83, G34, K22

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# The Voting Premium\*

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#### Abstract

This paper develops a unified theory of blockholder governance and the voting premium. It explains how and why a voting premium emerges in the absence of takeovers and controlling shareholders. The model features a minority blockholder and dispersed shareholders who trade shares in a competitive market. Those who own shares after trading vote on a proposal at a shareholder meeting. A voting premium can emerge in equilibrium from the blockholder's desire to influence who exercises control, rather than from exercising control himself. We show that the voting premium is unrelated to measures of voting power and that empirical measures of the voting premium generally do not reflect the economic value of voting rights. Consistent with recent empirical studies, the model can generate a negligible voting premium even when the allocation of voting rights is important. The model can also explain a negative voting premium, which has been documented in several studies. It arises because of free-riding by dispersed shareholders on the blockholder's trades. Finally, the model has novel implications for the liquidity of voting vs. non-voting shares, the relationship between the voting premium and the price of a vote, competition for control among blockholders, the block premium, and corporate influence more generally.

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### 1 Introduction

Voting is a central mechanism of corporate governance. It empowers shareholders of publicly traded companies to elect directors, approve major corporate transactions, and decide on governance, social, and environmental policies. Most corporations have blockholders who are large enough to influence voting outcomes (La Porta et al. (1999); McCahery, Sautner, and Starks (2016); Edmans and Holderness (2017); Dasgupta, Fos, and Sautner (2020)). Blockholders' desire to accumulate voting power and exert corporate influence can affect stock prices and give rise to a voting premium.

The asset pricing implications of control rights have been studied extensively. The theoretical literature followed Grossman and Hart (1988) and Harris and Raviv (1988), and attributed the voting premium almost exclusively to control contests and takeovers. This is puzzling in light of a large empirical literature in this area. First, the most common measure of the voting premium is arguably the dual-class premium, which appears to be largest in economies in which firms are well-protected against takeovers and control contests are rare. Second, while all studies find the voting premium to be positive on average, many studies document negative voting premiums for some firms, which is difficult to explain in a model with bidding contests. Third, studies that construct non-voting shares synthetically to estimate the voting premium, find that it is largest around shareholder meetings compared to other periods of the year (e.g., Kalay, Karakas, and Pant (2014)), which highlights the importance of voting on proposals for the existence of a voting premium. Last, a more recent empirical literature estimates the voting premium from fees in equity lending markets or price changes around record dates. These studies create a new puzzle, because they usually find negligible values for voting rights, which appears to be in conflict with the earlier literature.

Overall, these gaps and conflicting conclusions suggest that the theoretical underpinnings of the voting premium are still incomplete. To address these challenges, this paper develops a unified theory of blockholder governance and the voting premium. We study how and why a voting premium emerges in the absence of takeovers or controlling shareholders, which is

<sup>&</sup>lt;sup>1</sup>We discuss the theoretical literature in more detail in Section 2.

<sup>&</sup>lt;sup>2</sup>See our extensive discussion of the empirical literature in Section 8 and in the Appendix, which shows large dual class premiums for France, Israel, and Italy, among others, whereas the lowest dual-class premiums are found in the US.

arguably the empirically most relevant setting.

We analyze a model with a continuum of atomistic dispersed shareholders and one minority blockholder. The baseline model features one-share-one-vote. Shareholders first trade with each other in a competitive stock market. Those who own shares after trading then vote on a proposal at a shareholder meeting. Shareholders observe a public signal about the quality of the proposal before they cast a vote, and the proposal is approved if the majority of votes are in favor. Shareholders differ in their preferences for the proposal: While some shareholders need a lot of favorable evidence to be convinced to vote in favor, others are more disposed toward the proposal and generally support it. Such heterogeneity may arise due to differences in investment horizons; tax status; ownership of other firms; attitudes toward risk, corporate governance philosophies, and social and political ideologies.<sup>3</sup>

In our framework, the voting outcomes, the composition of the shareholder base, and asset prices are all endogenous. In equilibrium, the proposal is approved if and only if the public signal about its quality exceeds a certain cutoff. Shareholders are heterogeneous, so the proposal is accepted too often from the point of view of some, and rejected too often from the perspective of others. We call the shareholder who fully agrees with the decision rule implied by the cutoff the "median voter"; the median voter's identity completely characterizes the expected voting outcome. Importantly, the median voter can be either a dispersed shareholder or a blockholder, and his identity is determined by the composition of the shareholder base after trading. Hereafter, the term "median voter" is used interchangeably with the expected voting outcome.<sup>4</sup>

The blockholder and dispersed shareholders trade in anticipation of the expected voting outcome and its impact on their valuations; shareholders' valuations could differ because of heterogeneous preferences. Trading reallocates cash flow and voting rights across shareholders,

<sup>&</sup>lt;sup>3</sup>See the following literature on each of these issues: Investor time horizons: Bushee (1998) and Gaspar, Massa, and Matos (2005); tax status: Desai and Jin (2011); conflicts of interest and common ownership: Cvijanovic, Dasgupta, and Zachariadis (2016) and He, Huang, and Zhao (2019); attitudes to corporate governance and social and political ideologies: Bolton et al. (2020) and Bubb and Catan (2019). Hayden and Bodie (2008) provide a comprehensive overview of different sources of shareholder heterogeneity.

<sup>&</sup>lt;sup>4</sup>We adapt this label from the political science literature and note two caveats. First, the voter who fully agrees with the decision is the median voter only with a simple majority rule, whereas our model features a general majority rule. Second, in the political science literature, the median voter is often characterized as the voter who is indifferent between accepting and rejecting the proposal. In our model, the median voter is only indifferent if the public signal is just equal to the cutoff. In all other cases, if the signal is above or below the cutoff, the median voter has a clear preference in favor or against.

since shares are bundles of both. Price-taking dispersed shareholders trade only for cash flow reasons, i.e., if the share price differs from their private valuations. By contrast, the blockholder can be pivotal for the voting outcome, so he may also purchase shares to increase his ability to sway the voting outcome, that is, push the median voter in his preferred direction.

The equilibrium share price can be decomposed into two terms. The first term captures the market clearing price in the counterfactual scenario where all shareholders anticipate exactly the same decision rule regarding the proposal, but take it as exogenously given. This price would emerge if trading of shares did not reallocate voting rights across shareholders, e.g., if trade happened after the record date or if shares did not contain voting rights. The second term, which we define as the voting premium, is the extra component in the stock price that arises exactly because trading of shares affects the decision rule by reallocating the voting rights across shareholders. We show that this term reflects the blockholder's equilibrium net marginal value from owning an additional voting right.

Our theoretical definition of the voting premium has two appealing empirical counterparts. First, our definition of the voting premium captures the dual-class premium: In an extension to a dual-class setting, the price differential between voting and non-voting shares reflects the blockholder's marginal net value from an additional voting right. Second, the voting premium can be considered as the difference between the pre-record date and the post-record date share price. The expected voting outcome is the same at both moments in time, but trading no longer reallocates voting rights across shareholders after the record date.<sup>5</sup>

We show that a positive voting premium can arise in equilibrium even though there are no takeovers in our model and the blockholder cannot obtain a controlling stake in the firm. Intuitively, as the blockholder buys more shares, he moves the median voter, who becomes more similar to the blockholder. If the blockholder accumulates enough shares, he even becomes the median voter himself, in which case the voting outcome is fully aligned with his preferences. However, since voting rights are not traded separately from cash flow rights, this accumulation of voting power requires more dispersed shareholders who like the expected voting outcome to sell their shares, which results in price impact. If this price impact is significant, the blockholder optimally limits his accumulation of shares and hence his voting power, which implies that his

<sup>&</sup>lt;sup>5</sup>Although our analogy to trades around the record date focuses on high frequency, our model is static in nature, and thus, this analogy abstracts from dynamic aspects of trade that could potentially affect the share price.

marginal benefit from an additional vote remains positive. Thus, the voting premium is also positive; it captures the blockholder's marginal value from an additional vote in equilibrium.

At the same time, if the price impact from accumulating voting power is moderate, the blockholder will trade to become the median voter. In this case, any additional purchase of voting shares leaves the voting outcome unchanged and the voting premium is zero. Therefore, our model can explain recent empirical studies that document a negligible voting premium.

The case of a zero voting premium illustrates the general principle that the voting premium does not reflect the economic value of voting rights, because it captures only the blockholder's marginal value from an additional vote, evaluated at his optimal ownership level. However, the blockholder's total value from voting rights reflects his average propensity to buy votes, which includes all the infra-marginal shares he trades from his initial endowment to his equilibrium ownership. In this respect, the voting premium underestimates the importance of voting rights. This observation is important for interpreting empirical findings, since some proxies for the voting premium measure the marginal value of a vote (dual-class share premium; price drop on record days), whereas others are more related to the average value of voting rights (block premium).

For the same reasons, the voting premium is not a good measure of voting power. Voting power is related to the blockholder's likelihood to be pivotal and swing the voting outcome. Since an increase in the blockholder's voting power decreases his distance from the median voter, the magnitude of the voting premium is generally unrelated to the voting power. The relationship between voting power and the voting premium can even be negative when the blockholder becomes the median voter himself: Then his voting power is large and the voting premium is zero. This discussion underscores that the voting premium does not emerge from exercising control, but rather from influencing who exercises control.

Our model can also rationalize a negative voting premium, which has been documented in several studies. A negative voting premium implies that the blockholder limits his purchases of voting shares in order to prevent the median voter from moving too close to himself. Put differently, the blockholder commits to a reduced influence on the voting outcome. Such a strategy may appear puzzling, because the value of the blockholder's endowment always increases if he moves the median voter towards himself. However, some blockholders have small endowments and their main focus is on their trading profits. Consider a scenario in which

the blockholder favors acceptance of a certain proposal, e.g., adoption of an environmentally friendly production technology, but dispersed shareholders are on average more environmentalist and value this voting outcome even more than the blockholder. Then additional purchases of shares by the blockholder would increase the stock price by more than his own valuation and result in negative trading profits because of the free riding by dispersed shareholders. Then the blockholder limits his purchases, and the value of voting control becomes negative.

The discussion above reflects a more general insight that the voting rights embedded in the shares can either amplify or attenuate the price impact of trades. If the blockholder's trades move the median voter in the direction preferred by dispersed shareholders, they increase the price at which they supply their shares to the blockholder. Then his price impact is amplified compared to the scenario without voting considerations, e.g., if shares did not contain voting rights. However, if the blockholder is in conflict with dispersed shareholders, then his trades push the median voter away from their desired point and thus reduce the price at which they are willing to sell, attenuating the price impact. Overall, this argument implies that liquidity, which is commonly measured by price impact, is endogenous in our setting and generally differs between voting and non-voting shares.

We extend the model in a number of ways to explore additional questions. First, we consider a setting in which voting rights are traded separately, e.g., through share lending, and show that the price of a separately traded vote is conceptually different from the premium for a share in which voting and cash flow rights are bundled. Second, we consider multiple blockholders to analyze how the competition among them and their heterogeneous preferences affect the voting premium. Third, we study the block premium by introducing a first stage in which a new investor can acquire the block from an incumbent. Last, we consider a setting in which decisions are made not by voting, but by managers who consider the preferences of the entire shareholder base. We show that the blockholder's trades can then give rise to an "influence premium" on the share price, which is different from the voting premium and can even be larger.

Overall, our paper makes three contributions. First, it examines the trading between small and large shareholders and the ownership structure of the firm in a context in which blockholders affect voting outcomes without majority control. Second, it contributes to our understanding of asset prices by showing how and when a voting premium emerges when blockholders

can acquire voting control only through securities in which cash flow rights are bundled with voting rights. Third, it provides guidance to the empirical literature by showing how different proxies for the voting premium are related and why they may be different from each other.

#### 2 Discussion of the literature

We contribute a new theory of the value of voting rights. The primary approach in the literature, pioneered by Grossman and Hart (1988) and Harris and Raviv (1988), considers settings with control contests of firms with dual class shares. In this approach, rival bidders and incumbent managers differ in their ability to generate cash flows that are shared by all shareholders, and in their valuation of private benefits from controlling the firm. Bidders compete for control and pay a premium to the holders of the voting shares.<sup>6</sup> Studies in this literature have explored a range of alternative settings, including different types of admissible bids (conditional or unconditional; restricted or unrestricted); variation in the ability to extract private benefits; settings without a free-rider problem; and frictions from asymmetric information. Moreover, some studies have considered deviations from the one-share one-vote principle through trading in derivatives rather than in non-voting shares (e.g., Blair, Golbe, and Gerard (1989); Kalay and Pant (2010); Burkart and Lee (2010); Dekel and Wolinsky (2012)). Independently of the details, a wide range of settings give rise to a voting premium in a bidding contest. Yet, the voting premium appears to be largest in those economies in which firms are well-protected against takeovers and control contests hardly ever take place.<sup>8</sup> Hence, our theory contributes by showing how a voting premium emerges without takeovers and control contests. In our setting, the voting premium arises because the blockholder's trades affect the anticipated voting outcome and, as a result, change both the blockholder's and small shareholders' valuations. We also contribute the new insight that because the blockholder's and small shareholders'

<sup>&</sup>lt;sup>6</sup>Burkart and Lee (2008) survey theoretical work on the role of the security-voting structure and the control premium in the context of takeovers.

<sup>&</sup>lt;sup>7</sup>Types of admissible bids: Vinaimont and Sercu (2003); Dekel and Wolinsky (2012); variation in whether one party has private benefits: none: Bergström and Rydqvist (1992); one party is the main case in Grossman and Hart (1988); both parties: Vinaimont and Sercu (2003); Burkart, Gromb, and Panunzi (1998); there is no free-rider problem in Bergström and Rydqvist (1992); asymmetric information: Burkart and Lee (2010). Some empirical contributions also include further modeling efforts to motivate specific empirical analyses, e.g., Zingales (1995); Rydqvist (1996).

<sup>&</sup>lt;sup>8</sup>We discuss the respective literature and the arguments related to takeovers in more detail in Section 8.

valuations can be affected differently, the voting premium can be negative.

A complementary literature analyzes a market in which votes trade separately from shares.<sup>9</sup> While these papers differ significantly regarding their chosen settings and normative conclusions, they all conclude that the value of separately traded votes is negligible, either because dispersed shareholders value votes in proportion to their probability of being pivotal (Neeman and Orosel (2006); Brav and Mathews (2011); Speit and Voss (2020)) or because uninformed shareholders would like their votes to be picked up and cast by informed shareholders (Esö, Hansen, and White (2014)).<sup>10</sup> As our analysis emphasizes, the price of a vote traded separately is very different from the price of a vote that is traded in conjunction with cash flow rights. In particular, in our extension to a separate market for votes, small shareholders are willing to supply votes for an arbitrarily small price. In contrast, in our extension to a dual class share structure, the price of the voting stock generally exceeds that of the non-voting stock, even though small shareholders still do not value the voting rights per se, since they are never pivotal (see Section 8.1 for more details).

The only approach that has derived a significant voting premium without control contests is Rydqvist (1987), who builds on Milnor and Shapley (1978) and introduces the notion of an oceanic Shapley value to the analysis of dual-class shares. The critical step here is that the ocean of atomistic shareholders can *collectively* become pivotal and thus value their voting power. However, this leaves open how these atomistic shareholders resolve their collective action problem. In our setting, each dispersed shareholder maximizes only his individual payoff.

Our paper also contributes to the literature on the equilibrium ownership structure of firms and the analysis of blockholders. A large strand of this literature is on direct intervention by blockholders ("voice").<sup>12</sup> Another strand of this literature analyzes how trading by blockholders

<sup>&</sup>lt;sup>9</sup>It is largely motivated by concerns about the incentives created by decoupling votes from cash flow rights ("empty voting"), triggered by a sequence of papers by Hu and Black, e.g., Hu and Black (2007); Hu and Black (2015).

<sup>&</sup>lt;sup>10</sup>These papers focus on vote trading in corporations. A related literature in political science examines how vote trading allows agents with a higher intensity of preferences to buy votes from those who care about the decision less. See, e.g., Casella, Llorente-Saguer, and Palfrey (2012) and the literature surveyed in that paper.

<sup>&</sup>lt;sup>11</sup>Rydqvist (1987) develops an empirical measure of relative voting power that is widely used (e.g., Zingales (1994); Chung and Kim (1999); Nenova (2003)).

<sup>&</sup>lt;sup>12</sup>See Admati, Pfleiderer, and Zechner (1994), Bolton and von Thadden (1998), Kahn and Winton (1998), and Maug (1998) for earlier contributions to this literature. See the surveys of Edmans (2014), Edmans and Holderness (2017), and Dasgupta, Fos, and Sautner (2020) for more recent work and further details.

affects governance through its impact on stock prices and managers' incentives ("exit").<sup>13</sup> By contrast, in our setting, the blockholder exercises influence by affecting the identity of the median voter. This is empirically important because many blockholders, notably financial institutions, rely on voting to influence firms' policies.<sup>14</sup> Dhillon and Rossetto (2015), Bar-Isaac and Shapiro (2019), and Meirowitz and Pi (2020) also consider blockholder models with voting, but differently from our paper, they do not study the voting premium and focus on the effects of blockholders on, respectively, the risk taking of the firm and information aggregation.

More broadly, our paper is related to an earlier literature on the existence of equilibrium and the objectives of the firm in a context with incomplete markets and shareholders with heterogeneous preferences.<sup>15</sup> In particular, Drèze (1985) and DeMarzo (1993) develop models with the board of directors as a group of controlling blockholders. To this literature, we contribute by analyzing the voting premium and a richer characterization of the interplay between small shareholders and blockholders. This also distinguishes our paper from Levit, Malenko, and Maug (2020), who analyze trading and voting by atomistic shareholders – a setting in which the voting premium does not arise.

### 3 Model

Consider a publicly traded firm, which is initially owned by a continuum of measure one of dispersed shareholders and one large blockholder. The blockholder is endowed with  $\alpha \in [0, 1)$  shares, and each dispersed shareholder is endowed with  $e = 1 - \alpha$  shares, so the total number of outstanding shares is 1. In the baseline setting, each share has one vote. There is a proposal on which shareholders vote. The proposal could relate to director elections, M&As, executive compensation, corporate governance, or social and environmental policies. The proposal can either be approved (d = 1) or rejected (d = 0).

Preferences. Shareholders' preferences over the proposal depend on two components,

<sup>&</sup>lt;sup>13</sup>See Admati and Pfleiderer (2009), Edmans (2009), Edmans and Manso (2011), as well as the surveys cited in the previous footnote.

<sup>&</sup>lt;sup>14</sup>Thus, our paper contributes to the broader literature on corporate voting (e.g., Maug and Rydqvist, 2009; Levit and Malenko, 2011; Van Wesep, 2014; Malenko and Malenko, 2019; and Cvijanovic, Groen-Xu, and Zachariadis, 2020).

<sup>&</sup>lt;sup>15</sup>See Gevers (1974), Drèze (1985), DeMarzo (1993), and Kelsey and Milne (1996).

which reflect a common value and private values. The common value component depends on an unknown state  $\theta \in \{-1,1\}$ : if  $\theta = -1$  ( $\theta = 1$ ), accepting the proposal is value-decreasing (increasing). In other words, the common value is maximized if the policy matches the state (d = 1 if  $\theta = 1$ ), as common in the strategic voting literature, e.g., Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1996).

Shareholders also have private values from the proposal, which reflect the heterogeneity in their preferences. For simplicity, we refer to these private values as biases and denote them by b. A shareholder with bias b > 0 (b < 0) receives additional (dis)utility if the proposal is accepted. The distribution of biases b among the initial dispersed shareholders is given by a publicly known differentiable cdf G, which has full support with positive density g on  $\left[-\bar{b}, \bar{b}\right]$ , where  $\bar{b} \in (0,1)$ . Differences in shareholders' preferences can stem from time horizons, private benefits, social or political views, common ownership, risk aversion, or tax considerations. As noted in the introduction, the evidence for preference heterogeneity is pervasive. <sup>16</sup>

The value of a share from the perspective of a dispersed shareholder with bias b is

$$v(d, \theta, b) = v_0 + (\theta + b) d, \tag{1}$$

where  $v_0 \geq 0$  ensures that shareholder value is always non-negative. Notice that because of heterogeneous preferences, shareholders apply different hurdle rates for accepting the proposal: a shareholder with bias b would like the proposal to be accepted if and only if his expectation of  $\theta + b$  is positive. We will refer to shareholders with a higher b as being "more activist".

The blockholder has the same preference structure as dispersed shareholders, and his bias is  $\beta$ . Hence, the value of a share from the perspective of a blockholder is  $v(d, \theta, \beta)$ .

**Timeline**. All shareholders are initially uninformed about the state  $\theta$  and have the same prior about its distribution, which we specify below. Shareholders first trade and then vote on the proposal. This timing allows us to focus on how trading affects the composition of the voter base, which is crucial for the analysis of the voting premium. At the trading stage, each dispersed shareholder can buy any number of shares  $x \geq -e$ , where x < 0 corresponds to the shareholder selling shares and  $x \geq -e$  implies that short sales are not allowed. A dispersed

<sup>&</sup>lt;sup>16</sup>With minor modifications, our modeling approach can also capture differences of opinions (priors), when investors agree to disagree.

shareholder's utility from buying x shares is

$$u(d, \theta, b, x; \gamma, e) = (e + x) v(d, \theta, b) - \frac{\gamma}{2} x^2,$$
(2)

where  $\gamma > 0$  can be motivated by risk aversion or as capturing trading frictions (e.g., illiquidity, transaction costs, wealth constraints), which limit shareholders' ability to build large positions in the firm. Similarly, the blockholder can buy any number of shares  $y \geq -\alpha$ , and his utility from buying y shares is  $u(d, \theta, \beta, y; \eta, \alpha)$ , where  $u(\cdot)$  is given by (2) and  $\eta > 0$  captures the blockholder's risk aversion or trading costs. We assume that neither dispersed shareholders nor the blockholder find it in their best interest to short sell, i.e., to sell e or more, and  $\alpha$  or more shares, respectively.<sup>17</sup> For simplicity, we assume that the blockholder submits his order y first, and dispersed shareholders observe y and submit their orders next.

We denote the market clearing share price by p. After the market clears, but before voting takes place, all shareholders observe a public signal about the state  $\theta$ , which may stem from disclosures by management, proxy advisors, or analysts. Let  $q = \mathbb{E}[\theta|\text{public signal}]$  be shareholders' posterior expectation of the state following the signal. For simplicity, we assume that the public signal is q itself, and that q is distributed according to a differentiable cdf F with mean zero and full support with positive density f on  $[-\Delta, \Delta]$ , where  $\Delta \in (\bar{b}, 1)$ . Thus, the ex-ante expectation of  $\theta$  is zero. In what follows, we always refer to  $H(q^*) \equiv \Pr[q > q^*]$  rather than to the cdf. The symmetry of the support of q around zero is not necessary for any of the main results.

After observing the public signal q, each shareholder votes the shares he owns after the trading stage. Hence, we assume that the record date, which determines who is eligible to participate in the vote, is after the trading stage. This timeline applies well to important votes, such as the votes on M&As, proxy fights, and high-profile shareholder proposals, which are typically known well ahead of the record date. The proposal is accepted if at least fraction  $\tau \in (0,1)$  of all shares are cast in favor; otherwise, the proposal is rejected. We assume that the blockholder's initial stake and ability to buy shares are not large enough to grant him the power to accept the proposal unilaterally, as well as to veto the proposal solely with his own

<sup>&</sup>lt;sup>17</sup>The exact conditions that guarantee this are formulated in Lemma 3 in the Online Appendix. If  $\alpha = 0$ , which we analyze as a special case and separately from Proposition 2, the no-short-selling constraint can bind for the blockholder, but it does not change our main results.

votes. In particular, Lemma 3 in the Online Appendix shows that if  $\alpha < \min \{\tau, 1 - \tau\}$  and  $\eta$  is sufficiently large, then  $\alpha + y < \min \{\tau, 1 - \tau\}$  in any equilibrium.

We analyze subgame perfect Nash equilibria in undominated strategies of the voting game. The restriction to undominated strategies is common in voting games, which usually impose the equivalent restriction that dispersed shareholders vote as-if-pivotal. This implies that an investor with bias b, whether he is a dispersed shareholder or the blockholder, votes in favor of the proposal if and only if

$$b + q > 0. (3)$$

# 4 Analysis

We begin by showing that for any trading outcome, proposal approval at the voting stage takes the form of a cutoff decision rule:

**Lemma 1.** In any equilibrium, there exists  $q^*$  such that the proposal is approved by shareholders if and only if  $q > q^*$ .

Intuitively, this is because all shareholders value the proposal more if it is more likely to be value-increasing, i.e., if  $\theta = 1$  is more likely.

We proceed in several steps. First, for any possible blockholder's trade y, Sections 4.1 and 4.2 characterize the trading of dispersed shareholders and the voting stage as a function of y. In Section 4.3, we solve for the optimal trading strategy of the blockholder,  $y^*$ , and for the equilibrium share price.

# 4.1 Trading of dispersed shareholders

Motivated by Lemma 1, suppose that dispersed shareholders expect the proposal to be accepted if and only if  $q > q_e^*$  for some cutoff  $q_e^*$  (we later derive the equilibrium cutoff such that shareholders' expectations are rational). Let  $v(b, q_e^*)$  denote the valuation of a shareholder

<sup>&</sup>lt;sup>18</sup>See, e.g., Baron and Ferejohn (1989) and Austen-Smith and Banks (1996). This restriction helps rule out trivial equilibria, in which shareholders are indifferent between voting for and against because they are never pivotal.

with bias b prior to the realization of q, as a function of the cutoff  $q_e^*$ . Then

$$v\left(b, q_e^*\right) = \mathbb{E}\left[v\left(\mathbf{1}_{q>q_e^*}, \theta, b\right)\right],\tag{4}$$

where the indicator function  $\mathbf{1}_{q>q_e^*}$  equals one if  $q>q_e^*$  and zero otherwise, and  $v\left(d,\theta,b\right)$  is defined by (1). Then (4) can be rewritten as

$$v(b, q_e^*) = v_0 + (b + \mathbb{E}[\theta|q > q_e^*]) H(q_e^*), \tag{5}$$

which increases in b. Dispersed shareholders are price takers, so for any expected share price p, each dispersed shareholder solves

$$\max_{x} \left\{ \left( e + x \right) v \left( b, q_e^* \right) - xp - \frac{\gamma}{2} x^2 \right\} \tag{6}$$

and optimally chooses

$$x(b, q_e^*, p) = \frac{v(b, q_e^*) - p}{\gamma}.$$
 (7)

Thus, shareholder b buys shares if his valuation exceeds the market price,  $v(b, q_e^*) > p$ , sells shares if  $v(b, q_e^*) < p$ , and does not trade otherwise. Given the blockholder's order y, the market clears if and only if

$$\int_{\bar{b}}^{\bar{b}} x(b, q_e^*, p) g(b) db + y = 0$$
(8)

$$\Leftrightarrow p^*(y, q_e^*) = \gamma y + v\left(\mathbb{E}\left[b\right], q_e^*\right). \tag{9}$$

It follows that the equilibrium share price increases in y, and the price impact of the block-holder's trade is larger if  $\gamma$  is larger. Therefore, we can interpret  $\gamma$  as measuring the illiquidity of the market, i.e., the inverse of  $\gamma$  reflects market depth. Equation (9) shows that the price equals the sum of the valuation of the average dispersed shareholder,  $v (\mathbb{E}[b], q_e^*)$ , and the price impact of the blockholder,  $\gamma y$ . From (5), (7), and (9), dispersed shareholders' demand as a function of the blockholder's trade can be written as

$$x(b, y, q_e^*) = \frac{1}{\gamma} (b - \mathbb{E}[b]) H(q_e^*) - y.$$
 (10)

The post-trade ownership structure. Next, we characterize the post-trade ownership structure. After the trading stage, the blockholder owns  $\alpha + y$  shares, a dispersed shareholder with bias b owns  $1 - \alpha + x$  ( $b, y, q_e^*$ ) shares, and all dispersed shareholders collectively own  $1-\alpha-y>0$  shares. Thus, the proportion of shares owned post-trade by dispersed shareholders with bias b, conditional on the expected decision rule  $q_e^*$  and blockholder's trade y, is given by

$$r(b; y, q_e^*) \equiv g(b) \frac{1 - \alpha + x(b, y, q_e^*)}{1 - \alpha - y}.$$
 (11)

Note that  $r(b; y, q_e^*)$  is a density function, i.e.,  $\int_{-\bar{b}}^{\bar{b}} r(b; y, q_e^*) db = 1$ . Thus, the post-trade dispersed shareholder base is characterized by the cdf  $R(b; y, q_e^*)$  given by

$$R(b'; y, q_e^*) = \int_{-\overline{b}}^{b'} r(b; y, q_e^*) db$$

$$= G(b') \left( 1 - \frac{\mathbb{E}[b] - \mathbb{E}[b|b < b']}{\gamma} \frac{H(q_e^*)}{1 - \alpha - y} \right), \tag{12}$$

where the second equality follows from (10) and (11). The cdf R characterizes the post-trade dispersed shareholder base, whereas G characterizes the pre-trade dispersed shareholder base. Note that R(b) < G(b) for any b, i.e., R dominates G in the sense of first-order stochastic dominance. Hence, trading shifts the shareholder base in such a way that more activist shareholders own a larger proportion of the firm after trading. Moreover,  $R(b'; y, q_e^*)$  increases in  $q_e^*$ ; hence, a more activist decision rule (lower  $q_e^*$ ) makes the post-trade shareholder base more activist. Intuitively, shareholders' heterogeneous attitudes towards the proposal create gains from trade, so the shareholder base moves in the direction of the expected outcome.

## 4.2 Voting

The composition of the post-trade shareholder base determines the voting outcome. To derive the conditions under which the proposal is approved, we characterize the identity of the *median voter*, who is defined as the shareholder whose individual vote always coincides with the collective decision on the proposal. In other words, whenever the median voter votes in favor (against), the proposal is accepted (rejected).

To characterize the median voter, we first analyze the votes of dispersed shareholders.

Denote by  $s_{y,q_e^*}(q)$  the number of votes cast by dispersed shareholders in favor of the proposal if signal q is realized, the blockholder traded y shares, and the expected decision rule is  $q_e^*$ . Then,

$$s_{y,q_e^*}(q) \equiv (1 - \alpha - y) (1 - R(-q; y, q_e^*)),$$
 (13)

which is the number of shares held by dispersed shareholders,  $1 - \alpha - y$ , multiplied by the proportion of dispersed shareholders for whom b > -q. Consistent with intuition,  $s_{y,q_e^*}(q)$  is increasing in q.

If the realization of q is either sufficiently high or sufficiently low, then the votes of dispersed shareholders fully determine the vote outcome, and the blockholder's vote does not matter. Specifically, if  $s_{y,q_e^*}(q) \geq \tau$ , which is equivalent to  $q \geq s_{y,q_e^*}^{-1}(\tau)$ , then the value of the proposal is expected to be sufficiently high, so there is enough support among dispersed shareholders to approve the proposal even if the blockholder were to vote against. Similarly, if  $s_{y,q_e^*}(q) + \alpha + y \leq \tau$ , which is equivalent to  $q \leq s_{y,q_e^*}^{-1}(\tau - \alpha - y)$ , then there is enough opposition among dispersed shareholders to reject the proposal even if the blockholder were to vote in favor. Thus, the blockholder's vote is pivotal for the outcome if and only if

$$s_{y,q_e^*}(q) < \tau < s_{y,q_e^*}(q) + \alpha + y,$$
 (14)

in which case the proposal is accepted if and only if the blockholder supports it. Condition (14) is illustrated in the left panel of Figure 1, which plots the number of votes in favor of the proposal as a function of signal q and the post-trade position of the blockholder,  $\alpha + y$ .

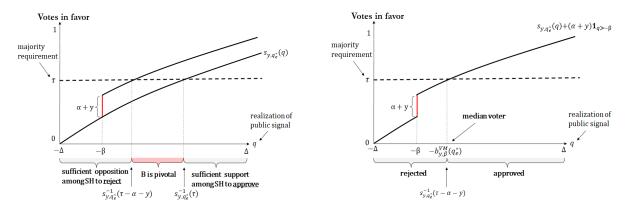


Figure 1 - The pivotal voter and the median voter

We next use condition (14) to characterize the median voter. Let  $b_{y,\beta}^{MV}(q_e^*)$  denote the bias of the median voter if the expected decision rule is  $q_e^*$ , the blockholder traded y shares, and his bias is  $\beta$ .

Importantly, if the blockholder's bias is extreme, he is not the median voter even though he is often pivotal. To see this, recall that the blockholder supports the proposal whenever  $\beta > -q$ . Suppose that  $\beta$  is sufficiently large so that the blockholder always supports for the proposal whenever he is pivotal,  $\beta \geq -s_{y,q_e}^{-1} (\tau - \alpha - y)$ . Then the proposal is accepted if and only if  $s_{y,q_e^*}(q) + \alpha + y \geq \tau$  or, equivalently,  $q \geq s_{y,q_e^*}^{-1} (\tau - \alpha - y)$ , i.e., if and only if a dispersed shareholder with bias  $-s_{y,q_e^*}^{-1} (\tau - \alpha - y)$  votes in favor. By definition, this implies that this dispersed shareholder is the median voter:  $b_{y,\beta}^{MV}(q_e^*) = -s_{y,q_e^*}^{-1} (\tau - \alpha - y)$ . This scenario is illustrated in the right panel of Figure 1. It shows that the median voter is determined by the intersection point between the majority requirement  $\tau$  and the positive votes curve, and this intersection point lies to the right of  $-\beta$ . Similarly, suppose the blockholder's bias  $\beta$  is "small enough,"  $\beta \leq -s_{y,q_e^*}^{-1}(\tau)$ , such that he always votes against the proposal whenever he is pivotal. Then, the proposal is accepted if and only if  $s_{y,q_e^*}(q) \geq \tau$ , i.e., the median voter in this case is a dispersed shareholder with bias  $-s_{y,q_e^*}^{-1}(\tau)$ . The blockholder's vote depends on the realization of q when he is pivotal only in the remaining cases, when his bias is moderate. Then he is the median voter himself, i.e.,  $b_{y,\beta}^{MV}(q_e^*) = \beta$ .

The distinction between the pivotal voter and the median voter is an important implication of this argument. A voter is pivotal if his vote can sway the decision on the proposal. Since dispersed shareholders are atomistic, only the blockholder can be pivotal in our setting. In contrast, the median voter is the shareholder whose vote always coincides with the decision on the proposal. Dispersed shareholders can be the median voter, even though they are atomistic, as in the first two cases described above. This distinction is important for the value of voting rights (see Section 5).

We conclude that if shareholders anticipate decision rule  $q_e^*$  when trading, then the decision rule at the voting stage,  $-b_{y,\beta}^{MV}(q_e^*)$ , is characterized by the three cases above. In equilibrium, shareholders' expectations  $q_e^*$  must be consistent with the actual decision rule. Hence, an equilibrium can be found as a fixed point of  $q_e^*$  such that  $-b_{y,\beta}^{MV}(q_e^*) = q_e^*$ . Using this logic, the equilibrium at the voting stage is characterized as follows.

**Proposition 1 (Voting stage).** If the blockholder trades y shares, then the proposal is approved if and only if  $q > q^*(y)$ , where  $q^*(y)$  solves

$$-b_{y,\beta}^{MV}(q^*) = q^*. (15)$$

There exists  $\overline{\gamma} < \infty$  such that if  $\gamma > \overline{\gamma}$ , then the solution of (15) is unique. In this case, there exists  $\overline{y}$  such that if  $y > \overline{y}$ , the median voter is the blockholder  $(-q^*(y) = \beta)$ , whereas if  $y < \overline{y}$ , the median voter is a dispersed shareholder with bias  $-q^*(y) \neq \beta$ , and  $|q^*(y) + \beta|$  is decreasing in y.a

In general, there can be multiple solutions to (15), and hence multiple equilibria at the voting stage. This is because for small  $\gamma$ , the shifts in the shareholder base are sensitive to the expected decision rule  $q_e^*$ , which can give rise to self-fulfilling expectations.<sup>19</sup> However, if  $\gamma$  is large enough, then dispersed shareholders trade less aggressively, the distribution of the post-trade shareholder base is less sensitive to  $q_e^*$ , and the equilibrium is unique. From this point on, we focus on parameterizations for which the equilibrium at the voting stage is unique.

Importantly, Proposition 1 shows that the blockholder can change the identity of the median voter,  $-q^*(y)$ , and thus the vote outcome, with his trades y. By buying more shares, the blockholder exerts more influence on the voting outcome, which pushes the bias of the median voter closer to  $\beta$ , as captured by the result that  $|q^*(y) + \beta|$  decreases. This can be seen in the left panel of Figure 2, which shows that a larger y pushes  $-q^*(y)$  to the left, closer to  $-\beta$ . Once the blockholder buys enough shares  $(y > \overline{y})$ , the vote outcome exactly coincides with the blockholder's own voting rule (which is to support the proposal whenever  $q > -\beta$ ), so the blockholder becomes the median voter. In this case, while the accumulation of additional voting power would increase the probability of the blockholder being pivotal, it would not change the expected vote outcome, that is, the identity of the median voter. This can be seen

<sup>&</sup>lt;sup>19</sup>In particular, the cdf of the post-trade shareholder base, given by (12), increases in  $q_e^*$ , and hence a more activist *expected* decision rule (lower  $q_e^*$ ) makes the post-trade shareholder base more activist. A more activist shareholder base, in turn, is more likely to approve the proposal for any given signal, leading to a lower *realized* cutoff for approving the proposal, confirming the ex-ante expectations.

in the right panel of Figure 2.

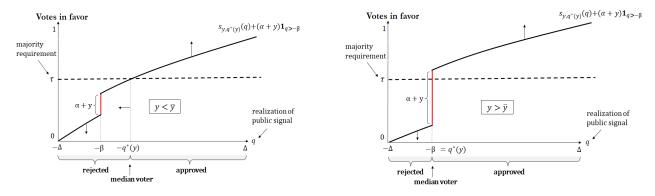


Figure 2 - The effect of the blockholder's trade y on the equilibrium median voter  $q^*(y)$ 

#### 4.3 Blockholder trading

Given the blockholder's trade y, all shareholders correctly anticipate that the decision rule at the voting stage will be  $q^*(y)$ , as given by (15), and that the market clearing price will be

$$p^{*}(y) = \gamma y + v\left(\mathbb{E}\left[b\right], q^{*}(y)\right) \tag{16}$$

from (9). In equilibrium, the blockholder chooses y to maximize

$$\Pi(y) \equiv (\alpha + y) v(\beta, q^*(y)) - yp^*(y) - \frac{\eta}{2} y^2.$$

$$\tag{17}$$

The marginal effect of buying additional shares on the blockholder's expected payoff is

$$\frac{\partial \Pi(y)}{\partial y} = \underbrace{(\beta - \mathbb{E}[b]) H(q^*(y)) - (2\gamma + \eta) y}_{\text{marginal propensity to buy cash flow rights, } MPC(y)}$$
(18)

$$+\frac{\partial\left(-q^{*}\left(y\right)\right)}{\partial y}\times\left[\alpha\left(q^{*}\left(y\right)+\beta\right)+y\left(\beta-\mathbb{E}\left[b\right]\right)\right]f\left(q^{*}\left(y\right)\right),\tag{19}$$

marginal propensity to buy voting rights, MPV(y)

and can be rewritten as

$$\frac{\partial \Pi(y)}{\partial y} = MPC(y) + MPV(y). \tag{20}$$

The term MPC(y) is the marginal payoff from trading the cash flow rights: if  $\beta > \mathbb{E}[b]$  ( $\beta < \mathbb{E}[b]$ ), the blockholder values shares more (less) than dispersed shareholders, which creates gains from trade. The term MPV(y) is the marginal payoff from trading the voting rights; it captures the additional incentives to trade in order to shift the median voter  $-q^*(y)$ . We discuss the intuition and determinants of the MPV in Section 5.

The next proposition characterizes the equilibrium of the game, including the blockholder's optimal trading strategy. As before, we focus on the case when the equilibrium is unique and, accordingly, assume that  $\gamma$  is large enough.

**Proposition 2 (Equilibrium).** Suppose the blockholder has an endowment  $\alpha > 0$ . There exist  $\overline{\gamma} < \infty$  and  $\overline{\eta} < \infty$  such that if  $\gamma > \overline{\gamma}$  and  $\eta > \overline{\eta}$ , the equilibrium exists and is unique. In this equilibrium:

(i) The blockholder's trade satisfies

$$y^* = \frac{1}{2\gamma + \eta} (\beta - \mathbb{E}[b]) H(q^*(y^*)) + \frac{1}{2\gamma + \eta} MPV(y^*), \qquad (21)$$

and the trade of a dispersed shareholder with bias b is given by

$$x^{*}(b) = \frac{1}{\gamma} (b - b_{MT}) H(q^{*}(y^{*})) - \frac{1}{2\gamma + \eta} MPV(y^{*}), \qquad (22)$$

where

$$b_{MT} = \mathbb{E}[b] + \frac{\gamma}{2\gamma + \eta} (\beta - \mathbb{E}[b]). \tag{23}$$

(ii) The bias of the median voter is

$$-q^*(y^*) = \begin{cases} \beta_L(y^*) > \beta & \text{if } \beta < G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}) \\ \beta & \text{if } \beta \in \left(G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}), G^{-1}(\frac{1-\tau}{1-\alpha})\right) \\ \beta_H(y^*) < \beta & \text{if } \beta > G^{-1}(\frac{1-\tau}{1-\alpha}), \end{cases}$$
(24)

where  $\beta_L\left(\cdot\right)$  and  $\beta_H\left(\cdot\right)$  solve (41) and (42) in the appendix, respectively.

(iii) The share price is given by

$$p^* = v(b_{MT}, q^*(y^*)) + \frac{\gamma}{2\gamma + \eta} MPV(y^*),$$
 (25)

where 
$$MPV\left(y^{*}\right)=0$$
 if  $\beta\in\left(G^{-1}\left(\frac{1-\alpha-\tau}{1-\alpha}\right),G^{-1}\left(\frac{1-\tau}{1-\alpha}\right)\right)$  and  $MPV\left(y^{*}\right)>0$  otherwise.

The blockholder's optimal trade  $y^*$  consists of two terms. The first term reflects the incentives to trade for cash flow considerations, and it is positive if and only if  $\beta > \mathbb{E}[b]$ . That is, the blockholder has incentives to buy (sell) shares whenever his intrinsic valuation of the proposal is higher (lower) than the average dispersed shareholder's valuation. The second term reflects the blockholder's additional incentives to buy shares in order to utilize the embedded voting rights, and it is proportional to  $MPV(y^*)$ . Under the assumptions of Proposition 2, the equilibrium  $MPV(y^*)$  is always (weakly) positive (we relax these assumptions in Section 5). Hence, the blockholder's incentives to buy (sell) shares are weakly stronger (weaker) if voting rights are bundled with cash flow rights.

To explain the intuition behind Proposition 2, the bold black curve in Figure 3 plots the equilibrium bias of the median voter as a function of the blockholder's bias. There are two distinct scenarios. First, if  $\beta < G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})$  ( $\beta > G^{-1}(\frac{1-\alpha}{1-\alpha})$ ), then the blockholder's preferences are extreme relative to dispersed shareholders' preferences. As a result, the median voter has a larger (smaller) bias toward the proposal than the blockholder: the black curve is above (below) the 45-degree line. In this region, the blockholder's trades move the median voter and hence the expected voting outcome, and a strictly positive  $MPV(y^*)$  reflects the blockholder's marginal benefit from doing so. Second, if  $\beta \in (G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}), G^{-1}(\frac{1-\tau}{1-\alpha}))$ , then the blockholder's preferences regarding the proposal are moderate and roughly aligned with those of dispersed shareholders, so the black curve coincides with the 45-degree line. In this region,  $MPV(y^*)$  is zero because acquiring additional voting rights would not change the voting outcome.

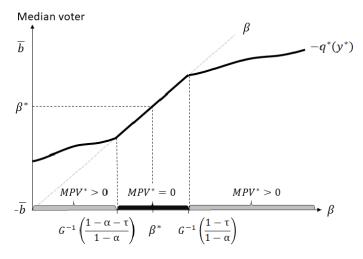


Figure 3 - Equilibrium median voter,  $-q^{*}(y^{*})$ .

## 5 Understanding the voting premium

To understand the intuition behind the MPV and to relate it to the voting premium, consider a hypothetical scenario in which the decision rule is set exogenously at the level  $q^*(y^*)$ . For example, this would be the case if the decision on the proposal is made by a manager with bias  $-q^*(y^*)$ , rather than by a shareholder vote. In this scenario, the decision rule is not affected by the blockholder's trades, so (18)–(19) imply that the blockholder's marginal payoff from buying additional shares is simply MPC, whereas MPV is zero. Then, (25) implies that the stock price in this hypothetical scenario is exactly  $v(b_{MT}, q^*(y^*))$ . We can think of  $v(b_{MT}, q^*(y^*))$  as the valuation of the marginal trader, who has bias  $b_{MT}$  and is indifferent between buying and selling (see (22)).

It follows from (25) that  $MPV(y^*)$  is proportional to the difference between the share price if the proposal is decided by shareholder voting, and the hypothetical share price if the proposal were decided exogenously by the same decision rule. In other words,  $MPV(y^*)$  not only measures the blockholder's marginal willingness to pay for additional votes, but also translates into a price premium, namely, the *voting premium*. In Section 6 below, we introduce the second class of shares with no voting rights and show that the analog of MPV in that setting is proportional to the dual-class premium. For this reason, we next focus on understanding the

determinants of  $MPV(y^*)$ . To this end, we decompose MPV(y) as follows:

$$MPV\left(y\right) = \underbrace{\frac{\partial \left(-q^{*}\left(y\right)\right)}{\partial y}}_{\text{ability to move MV}} \times \underbrace{\left[\underbrace{\left(\beta+q^{*}\left(y\right)\right)\left(\alpha+y\right)}_{\text{stake incentive to move MV}} - \underbrace{\left(\mathbb{E}\left[b\right]+q^{*}\left(y\right)\right)y}_{\text{price incentive to move MV}}\right] \times f\left(q^{*}\left(y\right)\right), (26)$$

where MV stands for the "median voter." Thus, MPV(y) can be decomposed into the block-holder's *ability* to influence the median voter,  $\frac{\partial (-q^*(y))}{\partial y}$ , and his *incentive* to influence the median voter's identity, which is the remainder of the expression in (26). We decompose the expression for incentives further below.

Ability and the zero voting premium. Consider first the blockholder's ability to influence the median voter. From Proposition 1 and the discussion of Figure 3,  $\frac{\partial(-q^*(y))}{\partial y} = 0$  whenever the blockholder is the median voter regardless of marginal changes in his trades. Hence, he cannot influence the voting outcome by buying additional shares in this region. Then his marginal propensity to buy voting rights is zero, and so is the voting premium. This observation further highlights the difference between the pivotal voter and the median voter discussed in Section 4.2: It is the latter and not the former that affects the blockholder's payoff, and hence, the voting premium.

A zero voting premium does not imply that the blockholder does not value voting rights, or that he would not benefit from further influencing the voting outcome. In particular, the incentives from moving the median voter in (26) will generally differ from zero. A zero voting premium implies only that the blockholder cannot influence the position of the median voter through additional trades of voting shares, an issue we follow up on in the discussion of a separate market for votes in Section C.1 of the Online Appendix.

Note also that the blockholder's overall benefits from accumulating voting rights can be positive even if the marginal benefits are zero, because these marginal benefits are evaluated at the blockholder's equilibrium ownership  $y^*$ . By contrast, the overall benefits from owning voting rights also come from the blockholder's inframarginal trades. In addition, even if the equilibrium voting premium is zero, the voting outcome is affected by the blockholder's accumulation of voting rights,  $q^*(y^*) \neq q^*(0)$ . Hence, the voting premium is likely to underestimate

the overall value of voting rights.

Incentives and the positive voting premium. Next, we consider the blockholder's *incentives* to change the identity of the median voter. From the expression in square brackets in (26), these incentives can be broken down into two effects from moving the median voter. The first effect captures the marginal effect from moving the median voter on the blockholder's post-trade ownership of the firm,  $\alpha + y$ , and we refer to it as the *stake incentive*. From (5),

stake incentive = 
$$(\alpha + y) \frac{\partial v(\beta, q^*)}{\partial (-q^*)} = (\alpha + y) (\beta + q^*) f(q^*).$$
 (27)

By buying additional shares, the blockholder moves the median voter closer to his own bias  $\beta$  (see Proposition 1), which increases the value of his stake. In particular, if the blockholder is more activist than the median voter ( $\beta > -q^*$ ), then additional purchases by the blockholder make the median voter more activist, and if the blockholder is less activist ( $\beta < -q^*$ ), his purchases have the opposite impact. In both cases, he moves the median voter closer to himself and benefits from a higher valuation of his shares. Hence, the blockholder always values a marginal vote for its impact on his stake. For ease of exposition, the subsequent discussion is based on the case in which the blockholder is more activist and his purchases make the median voter more activist.<sup>20</sup>

The second part in square brackets in (26) is the *price incentive* to move the median voter, holding the size of the trade y itself constant. We evaluate how an incremental change in the median voter  $-q^*$  affects the stock price from (9):

price incentive 
$$= y \frac{\partial p^*}{\partial (-q^*)} = y \frac{\partial v \left( \mathbb{E} \left[ b \right], q^* \right)}{\partial (-q^*)} = y \left( \mathbb{E} \left[ b \right] + q^* \right) f \left( q^* \right).$$
 (28)

The sign of the price incentive is ambiguous. For the case considered here, when the block-holder's trades make the median voter  $-q^*$  more activist, small shareholders benefit if they are on average more activist than the median voter. Then, the median voter moves towards them, and they increase the price at which they supply their shares to the blockholder. By contrast, if small shareholders are less activist than the median voter, then an even more activist median

<sup>&</sup>lt;sup>20</sup>The opposite case, in which the blockholder is less activist, can be obtained from inverting the signs of the expressions for the stake incentive, the price incentive, and the direction of the change of the median voter as the blockholder trades.

voter hurts them, and they supply their shares for a lower price.

Based on expressions (27) and (28), we can interpret the incentive part of the MPV in (26) as a combination of the stake incentive and the price incentive to move the median voter. The blockholder benefits from moving the median voter whenever this increases the value of his stake by more than it increases the overall price he pays on his trades. A positive voting premium emerges whenever the blockholder benefits from moving the median voter and he also has the ability to do so. This happens if the net benefit from moving the median voter is positive, and if the blockholder is not the median voter himself. Hence, at the margin, the blockholder does not pay a voting premium for being in control of the voting outcome, but for his influence on the composition of the shareholder base, i.e., his influence on the identity of the dispersed shareholder who becomes the median voter.

**Price impact and the voting premium.** The voting rights embedded in the shares could have quite different, and potentially opposite, effects on the *price* of the shares and the *price impact* of trades. In particular, the voting rights could lead to a positive voting premium but a lower price impact, and vice versa.

From (9), the blockholder's trade y has a direct impact on prices because of trading costs, and an additional indirect effect through its influence on the voting outcome. Specifically:

$$\frac{\partial p^*}{\partial y} = \gamma + f\left(q^*\left(y\right)\right) \left(\mathbb{E}\left[b\right] + q^*\left(y\right)\right) \frac{\partial \left(-q^*\left(y\right)\right)}{\partial y}.$$
 (29)

The first term,  $\gamma$ , reflects dispersed shareholders' trading costs and would be present even absent voting considerations, e.g., for non-voting shares. The second term reflects the indirect effect of the blockholder's trades on the median voter,  $-q^*(y)$ , and is directly related to the price incentive to move the median voter shown in equation (28).

Whether this indirect effect is positive depends on whether the resulting change in the median voter benefits or hurts dispersed shareholders, i.e., it depends on whether the interests of the blockholder and those of dispersed shareholders are aligned or in conflict.

If the blockholder and dispersed shareholders are in conflict in that the median voter is between them,  $\mathbb{E}[b] < -q^*(y^*) < \beta$ , then the blockholder's accumulation of shares moves the median voter away from dispersed shareholders. This reduces the reservation price at which

they are willing to supply additional shares, makes the price function flatter, and attenuates the price impact of trades. However, the MPV and the voting premium are positive in this case, because a more activist median voter not only increases the value of the blockholder's stake, but also reduces the price he has to pay for a marginal share. Thus, the embedded voting rights increase the price of the shares but decrease the price impact of trades.

By contrast, suppose that the blockholder's and dispersed shareholders' interests are aligned,  $-q^*(y^*) < \min\{\beta, \mathbb{E}[b]\}$ , such that the median voter moves in the direction preferred by both. Then dispersed shareholders increase the reservation price at which they are willing to supply additional shares. This makes the small shareholders' supply function steeper and amplifies the price impact of the blockholder's trades. At the same time, it reduces the voting premium due to the price incentive component, which makes acquiring an additional voting share more expensive.

Two observations result from this discussion, both of which are relevant in relation to the empirical literature.

Liquidity. Price impact is a common measure of liquidity. The discussion above shows that liquidity measured in this way is endogenous in our setting and is affected by the voting rights bundled with the cash flow rights. It also implies that the liquidity of voting shares generally differs from that of non-voting shares. More specifically, if the blockholder's trades move the median voter towards dispersed shareholders, then price impact is larger, and thus liquidity of a voting share is smaller compared to a scenario without voting considerations, e.g., for a non-voting share with otherwise identical trading costs. (We discuss dual-class share structures in greater detail in Section 6 below.)

Free-riding and a negative voting premium. The discussion of (29) shows that dispersed shareholders may free-ride on the blockholder's trades if their incentives are aligned with those of the blockholder in the following sense: As the blockholder's trades move the median voter in the dispersed shareholders' preferred direction, they increase the price at which they are willing to sell. Generally, this does not prevent a positive voting premium. For example, under the assumptions of Proposition 2, i.e., when the blockholder has a positive endowment  $\alpha$  and  $\gamma$  is large, the voting premium is always non-negative. This is so because the stake

incentive is sufficiently strong, as the blockholder's ability to move the median voter in his preferred direction increases the value of his entire post-trade stake,  $\alpha + y$ , whereas the price incentive effect only applies to his trade y. However, if the blockholder has no endowment, then a negative voting premium may arise:

**Proposition 3.** Suppose  $\alpha = 0$ . There exist  $\overline{\gamma} < \infty$  and  $\overline{\eta} < \infty$  such that if  $\gamma > \overline{\gamma}$  and  $\eta > \overline{\eta}$ , then the equilibrium exists and is unique. In equilibrium,  $MPV(y^*) < 0$  if and only if  $\mathbb{E}[b] < \beta < G^{-1}(1-\tau)$ . In this case, the blockholder buys shares  $(y^* > 0)$  and the share price exhibits a negative voting premium:  $p^* < v(b_{MT}, q^*(y^*))$ .

Intuitively, if the blockholder is less activist than the median voter, then the median voter becomes less activist as the blockholder buys more shares. Moreover, note that the expression for the MPV simplifies if  $\alpha = 0$ :

$$MPV(y)_{\alpha=0} = y \left(\beta - \mathbb{E}[b]\right) f\left(q^*(y)\right) \frac{\partial \left(-q^*(y)\right)}{\partial y}, \tag{30}$$

which is simply the relative sensitivity of the blockholder's and small shareholders' valuations to a change in the median voter. We may refer to (30) as the net trading profits from moving the median voter, since the expression reflects the difference between the valuations of the blockholder and the share price. These net trading profits are negative if the average dispersed shareholder is even less activist than the blockholder ( $\mathbb{E}[b] < \beta$ ): Then a less activist median voter increases the valuation of the average dispersed shareholder, and thereby the stock price, even more than the valuation of the blockholder.<sup>21</sup>

The scenario in Proposition 3 obtains because the interests of the blockholder and those of dispersed shareholders are aligned, but the dispersed shareholders are more extreme and benefit more from the resulting change in the voting outcome. For example, the blockholder may be reluctant to support a certain management proposal, but dispersed shareholders may be more strongly biased against this proposal than the blockholder himself. Then the value of voting control becomes negative for the blockholder, who then buys fewer shares than if he could buy cash flow rights separately and they were not bundled with voting rights. Hence,

<sup>&</sup>lt;sup>21</sup>While Propositions 2 and 3 imply that for large  $\gamma$ , a negative voting premium arises only when the block-holder has no initial endowment ( $\alpha = 0$ ), the existence of a negative voting premium is more general: If  $\gamma$  is not too large, a negative MPV can also arise for small but strictly positive values of  $\alpha$ .

free-riding by dispersed shareholders results in a negative voting premium if the blockholder has no endowment and cares only about his trading profits.

#### 6 Dual-class shares

In this section, we investigate how the MPV is related to measures of the voting premium estimated in the empirical literature (see Section 8). To do so, we extend our model to a setting with two classes of shares with different voting rights. Specifically, suppose that in addition to the traded voting shares, investors can also trade non-voting shares. This setting can capture companies with a dual-class share structure (e.g., Zingales (1995); Nenova (2003)).

We assume that the blockholder and each dispersed shareholder are endowed with  $\hat{\alpha} \in [0, 1]$  and  $\hat{e} \in [0, 1 - \hat{\alpha}]$  non-voting shares, respectively, so that the total number of outstanding non-voting shares lies in the interval [0, 1]. Notice that we allow for the supply of non-voting shares to be zero (i.e.,  $\hat{\alpha} = \hat{e} = 0$ ), which could capture the creation of non-voting securities in derivatives markets. Denote by  $\hat{x}$  and  $\hat{y}$  the trades of dispersed shareholders and the blockholder in the non-voting shares, respectively. The utility of dispersed shareholders is given by

$$\hat{u}(d,\theta,b,x,\hat{x};\gamma,e,\hat{e}) = (e+x)v(d,\theta,b) - \frac{\gamma}{2}x^2 + (\hat{e}+\hat{x})v(d,\theta,b) - \frac{\gamma}{2}\hat{x}^2,$$
(31)

which means that in this extension,  $\gamma$  is best interpreted as capturing trading costs, rather than as risk aversion. We assume that the trading costs are the same for these two securities, to make sure that the price differential between voting and non-voting shares does not stem from differences in the microstructure of these markets. Similarly, the blockholder's utility is given by  $u(d, \theta, \beta, y, \hat{y}; \eta, \alpha, \hat{\alpha})$ . Notice that in principle, shorting of non-voting shares is feasible. However, we assume that the trading costs  $\gamma$  and  $\eta$  are large enough, so that  $e + x + \hat{e} + \hat{x} > 0$  and  $\alpha + y + \hat{\alpha} + \hat{y} > 0$ , i.e., the net cash flow exposure of each investor is always non-negative.

Consider first the case in which the decision rule  $q^*$  is exogenous, e.g., if decisions are taken by management with bias  $-q^*$ . With an exogenous decision rule  $q^*$ , the trading strategies of all investors in each market are given by the expressions in Proposition 2, assuming that  $\frac{\partial(-q^*(y))}{\partial y} = 0$ , and hence, MPV = 0. Indeed, if the decision rule is not affected by trading, then the existence of the market for non-voting shares does not affect trading in the market

for voting shares, and vice versa. This is because investors have no budget constraints and the trading costs apply to each market separately. Moreover, although the endowments of non-voting shares could be different from the endowments of voting shares, the trading quantities are the same as in our model, since they are invariant to the levels of the endowment.

This observation implies that with an exogenous cutoff  $q^*$ , the prices of voting and non-voting shares must be identical. Indeed, given  $(y, \hat{y})$  and  $q^*$ , the difference in prices is

$$p(y, q^*) - p(\hat{y}, q^*) = \gamma y + v(\mathbb{E}[b], q^*) - (\gamma \hat{y} + v(\mathbb{E}[b], q^*)) = \gamma(y - \hat{y}),$$

and since  $y = \hat{y}$ , the two prices are the same.

Next, consider the model with voting. By assumption, the net positions of dispersed share-holders and the blockholder are always non-negative. Therefore, a dispersed shareholder with bias b votes for the proposal if and only if q + b > 0, and the blockholder votes for the proposal if and only if  $q + \beta > 0$ . Note also that for a given  $q^*$  and  $(y, \hat{y})$ , the trading strategies of dispersed shareholders in the voting and non-voting shares are the same as in the baseline model. Thus, the identity of the median voter as a function of the blockholder's trade, namely  $q^*(y)$ , is determined as in the baseline model (see Proposition 1). This implies that given y, the median voter is unaffected by  $\hat{y}$ , i.e., the trades that take place in the market for non-voting shares. However, the presence of non-voting shares changes the blockholder's trades of voting shares, because he internalizes the effect of the voting outcome on the value of his non-voting shares. The objective of the blockholder becomes:

$$\max_{y,\hat{y}} \Pi(y,\hat{y}) = (\alpha + y) v(\beta, q^*(y)) - yp^*(y) - \frac{\eta}{2} y^2$$
(32)

+ 
$$(\hat{\alpha} + \hat{y}) v(\beta, q^*(y)) - \hat{y}\hat{p}^*(\hat{y}) - \frac{\eta}{2}\hat{y}^2$$
. (33)

We obtain the following result:

**Proposition 4 (Dual-class shares)** If the blockholder and dispersed shareholders can trade in voting and non-voting shares, the voting premium is:

$$p_{voting}^* - p_{non-voting}^* = \gamma \left( y^* - \hat{y}^* \right) = \frac{\gamma}{2\gamma + \eta} MPV \left( y^*, \hat{y}^* \right), \tag{34}$$

where

$$MPV\left(y,\hat{y}\right) = \frac{\partial\left(-q^{*}\left(y\right)\right)}{\partial y} f\left(q^{*}\left(y\right)\right) \left[\left(\alpha+\hat{\alpha}\right)\left(q^{*}\left(y\right)+\beta\right)+\left(y+\hat{y}\right)\left(\beta-\mathbb{E}\left[b\right]\right)\right]. \tag{35}$$

Proposition 4 shows that the dual-class voting premium is proportional to the MPV. Thus, the blockholder's increased willingness to buy additional shares to affect the voting outcome translates into an actual price difference between voting and non-voting shares. Note also that  $MPV\left(y^*, \hat{y}^*\right)$  depends on  $\hat{y}^*$ , which means that the volume of trades in the market for non-voting shares affects the blockholder's incentives to buy voting shares, and hence the voting premium. Intuitively, the blockholder's position in non-voting shares gives him additional incentives to change the median voter for the same reasons as his position in voting shares – the endowment benefits  $(\hat{\alpha}\left(q^*\left(y\right) + \beta\right))$ , and the net trading benefits  $(\hat{y}\left(\beta - \mathbb{E}\left[b\right]))$ .

#### 7 Extensions

In this section, we briefly discuss several other implications of the baseline model and its extensions. The complete analysis and explanation of these results is in the Online Appendix.

Exit and a positive voting premium. In the context of our baseline model, we show that although the blockholder has the power to gain influence over the voting outcome by buying additional shares, he may nevertheless choose to do the opposite: sell shares to dispersed shareholders and thereby give up his influence over the voting outcome, while demanding a premium from the dispersed shareholders. Thus, the tension between exit and voice (e.g., Hirschman (1970)) also exists in our model, which demonstrates that the incentives to exit can prevail even when the voting premium is positive. Hence, a positive voting premium does not necessarily indicate a more concentrated ownership structure. This analysis is presented in Section B.1 of the Online Appendix.

Vote trading. Our baseline model focuses on the case where one security has voting rights bundled with cash flow rights, which may limit the blockholder's ability to accumulate votes and implement his agenda. In Section C.1 of the Online Appendix, we study whether allowing for separate trading of voting rights, e.g., through share lending, can help the blockholder

achieve his objectives through voting. We show that while a moderately biased blockholder can use the market for votes to successfully pursue his agenda, a blockholder with a strong bias cannot. Thus, blockholders with strong views about corporate policies could potentially benefit from combining voting with other channels of corporate influence, such as engaging with management behind the scenes, lobbying regulators and proxy advisors, and running media campaigns. This result is consistent with the evidence in McCahery, Sautner, and Starks (2016) that institutional investors use a broad range of governance mechanisms to influence their portfolio companies. In addition, this extension emphasizes that the price of a vote traded separately is different from the voting premium for a share that combines cash flow and voting rights.

Influence premium. In practice, blockholders can exert influence even without having formal control rights if they can influence the company's management. In Section C.2 of the Online Appendix, we analyze a version of the model in which decisions are taken by management rather than by voting, and management gives some weight to the value of its current shareholder base, represented by the preferences of the post-trade average shareholder. Then the blockholder's trades influence decisions by changing these preferences, and the blockholder values this influence, which may give rise to an "influence premium" on the share price. The influence premium is different from the voting premium and can even be larger. In particular, accumulating more shares always increases the blockholder's influence on management, but it does not always increase the blockholder's impact if decisions are taken by a shareholder vote.

Multiple blockholders. Section C.3 of the Online Appendix generalizes our model to the case with multiple blockholders. We show that if they share the same preferences toward the proposal, then the voting premium declines as the number of blockholders increases. In contrast, if the blockholders are sufficiently heterogeneous, their trades pull the median voter in opposite directions. Then, as the blockholders' biases become more extreme, each blockholder tries harder to gain influence over the voting outcome, which results in a higher voting premium.

**Block trading.** In another extension, we endogenize the blockholder by introducing a first stage in which new investors can acquire the block from an incumbent, as in Burkart, Gromb,

and Panunzi (2000), or through a private placement, in which the firm auctions off a block and places it with the highest bidder (e.g., Hertzel and Smith (1993)). We show that block trading may increase the difference in preferences between blockholders and dispersed shareholders, and characterize when the block premium is negative or positive. The analysis is presented in Section C.4 of the Online Appendix.

# 8 Empirical implications and measures of the voting premium

There is a large empirical literature that provides measures of the voting premium and analyses of the cross-sectional and time-series variation of the voting premium. The purpose of this section is to locate the model developed above in the context of the existing empirical evidence and, conversely, shed some light on the empirical discussion by exploring the implications of our model. Specifically, it is not the purpose of this section to offer a comprehensive survey of empirical studies and methodologies and their potential strengths and shortcomings.<sup>22</sup>

Broadly, there are five major strategies that have been developed in the literature to measure the voting premium and the economic value of voting power. We survey 40 studies in more detail in Table 1 in the Appendix and provide a summary in the table below. Of these studies, 15 use data on the US, 4 on Germany, 3 on Italy, 3 are cross-country studies, and the rest provide evidence on 11 other countries.

Methodology	Avg. (%)	Median (%)	Number of studies
Dual-class shares	23.59	14.53	23
Block-trade premium	41.50	29.55	9
Option replication	0.20	0.16	5
Equity lending	0.01		2
Record-day trading	0.09	0.12	1

<sup>&</sup>lt;sup>22</sup>Some papers already contain surveys of different strands of this literature. Rydqvist (1992) provides an early survey of studies on dual-class shares and Dittmann (2004), Adams and Ferreira (2008), and Kind and Poltera (2013) provide more recent updates.

The most salient feature of these studies is that they report very divergent estimates of the voting premium. Below, we first discuss why estimates of the voting premium may vary across methodologies (Section 8.1) and then the cross-sectional variation of voting premiums within methodologies (Section 8.2) and relate them to our model.

#### 8.1 Differences across methodologies

Marginal values vs. block values. Most methods to estimate the voting premium measure the value of a marginal vote. This applies to all methods that rely on stock market prices, i.e., all methods except for the block-trade premium. By contrast, block trades reveal the average valuation of a voting right for the entire block. The table above shows that block trades are associated with significantly larger premiums (average: 41.50%; median: 29.55%) than found in studies of dual-class share premiums (average: 23.59%; median: 14.53%) or those using the three other methods. Based on our model, we would expect the blockholder's willingness to pay for an entire block of shares to be larger than his willingness to pay for an additional voting share. In particular, the equilibrium MPV in our model may equal zero if the blockholder is the median voter at his equilibrium trading amount  $y^*$  (Proposition 2), resulting in a zero dual-class share premium (Proposition 4, equation (34)). However, his average, per-share willingness to pay for a block of votes of size  $y^*$  in addition to his endowment  $\alpha$  equals  $\int_0^{y^*} MPV(y) dy$  and may be much larger.

Voting yields and capitalized voting premiums. In addition, it is salient from the table that studies relying on dual-class shares and block-trades obtain much larger estimates than the other three methods. We attribute this to the fact that the former two methods capitalize the value of the voting right over longer time horizons, which span potentially infinitely many future shareholder meetings. In contrast, the three other studies estimate the voting yield, which captures a period of one year or less. In the Online Appendix, we calibrate a simple valuation model and show that once the difference in the time horizon is accounted for, the estimates from these two sets of methods are in fact consistent with each other.

Separate vs. joint trading of cash flow and voting rights. Another important difference between the methodologies is whether they estimate the price of the vote that is traded

separately (as in the equity lending market) or the price of the vote that is traded in conjunction with cash flow rights (as in the comparison of two classes of stock with differential voting rights). Our analysis emphasizes that the two types of methodologies could give very different estimates of the price of the vote. Indeed, in the extension to a separate market for votes in Section C.1 of the Online Appendix, we show that dispersed shareholders are willing to sell their votes for an arbitrarily small price since they are never pivotal. In contrast, in the extension of Section 6, there are two separate markets for voting and non-voting shares. In this case, a voting premium on the voting stock (relative to the non-voting stock) emerges, since blockholders need to buy cash flow rights to accumulate voting power. Importantly, in the dual-class share setting, dispersed shareholders still do not value their voting rights per se, since they are atomistic and hence never pivotal. Rather, they value the shares depending on the identity of the median voter, i.e., their anticipation of the voting outcome. Hence, their reservation price for selling their shares to the blockholder changes depending on whether the blockholder's trades move the median voter in their preferred direction, and this, in turn, affects the voting premium.

#### 8.2 The cross-sectional variation in the voting premium

This section offers observations on the cross-sectional variation of the voting premium and discusses them in the context of our model.

Negative values of the voting premium. One implication from our analysis is that the voting premium can sometimes be negative, which emanates from the free-rider effect (see Proposition 3 and the related discussion). Interestingly, while the estimates of the mean and median of the voting premium in the studies surveyed in Table 1 are always positive, many studies report that the voting premium is negative for some companies.<sup>23</sup> These findings are consistent with our model, but are difficult to interpret in the context of extant theories. Empirical studies often explain them by pointing out that voting shares may suffer from a liquidity discount relative to non-voting shares.<sup>24</sup>

<sup>&</sup>lt;sup>23</sup>E.g., see Rydqvist (1996), Nenova (2003), and Caprio and Croci (2008) for the dual-class share premium and Albuquerque and Schroth (2010) and Albuquerque and Schroth (2015) for the block trading premium.

<sup>&</sup>lt;sup>24</sup>Odegaard (2007) separates liquidity effects from control effects in Norway, which used to have three classes of shares that differed in their voting rights and the possibility of foreign ownership.

Voting premiums, takeovers, and shareholder meetings. One of the standard explanations for how the blockholder's willingness to pay a premium for voting control is translated into higher prices for voting shares is the takeover mechanism, and several empirical studies find support for this explanation.<sup>25</sup> However, this theory has some limitations. First, since the 1990s, many countries have enacted coattail provisions, which mandate equal treatment of all classes of shares in control changes (Maynes (1996); Nenova (2003)). Second, Dittmann (2004) surveys 12 studies of companies with dual-class share structures and shows that if investors would correctly anticipate the ex-post frequencies of takeovers and takeover premiums paid, then the premium on voting shares in dual-class firms should be smaller by about one order of magnitude compared to the observed premium in most countries. Hence, the takeover explanation is probably only a partial explanation of premiums on voting shares.

Differently from this argument, our analysis shows how the voting premium can arise without contests for majority control, and solely as a result of blockholders' desire to influence the voting outcomes at shareholder meetings. This prediction is consistent with the findings of the more recent literature, which analyzes the time-series variation in the voting premium and finds that the voting premium is largest around shareholder meetings compared to other periods of the year (see Kind and Poltera (2013); Kalay, Karakas, and Pant (2014); Kind and Poltera (2017); Fos and Holderness (2020)).

Voting premiums and ownership structure. Studies on the relationship between the voting premium and ownership concentration show that it is often non-monotonic: the value of voting rights is small both if ownership is very dispersed and if it is very concentrated with one blockholder who has majority control (Kind and Poltera (2013)). Therefore, one common methodology uses the probability of being pivotal inferred from oceanic Shapley values instead of ownership concentration to predict the voting premium.<sup>26</sup> Our analysis in Section C.3 of the Online Appendix suggests a new empirical direction by showing that it is not only the concentration of ownership and the probability of being pivotal that matter, but also

<sup>&</sup>lt;sup>25</sup>For models, see Grossman and Hart (1988); Harris and Raviv (1988); Bergström and Rydqvist (1992). For empirical evidence see Bergström and Rydqvist (1992); Zingales (1995); Rydqvist (1996); Smith and Amoako-Adu (1995).

<sup>&</sup>lt;sup>26</sup>The method was pioneered by Rydqvist (1987) and is based on the theory of oceanic Shapley values of Milnor and Shapley (1978). For applications, see Zingales (1994), Zingales (1995), Chung and Kim (1999), Caprio and Croci (2008), and Nenova (2003).

the preferences of blockholders. Specifically, if blockholders have similar preferences, then ownership concentration is positively correlated with the voting premium, and if blockholders disagree with each other, the voting premium increases the more they disagree.

#### 9 Conclusion

We develop a theory of voting and trading in which a blockholder and dispersed shareholders trade with each other and then vote on a proposal. We analyze the trading decisions of blockholders, when they would be willing to pay a higher price in order to accumulate voting power, and how their trades translate into a premium for voting shares. The model generates a number of insights about the voting premium and the equilibrium ownership structure of the firm.

We find that the voting premium does not reflect the economic value of voting rights to the blockholder, and that it is also unrelated to the voting power of the blockholder. Moreover, common measures of the voting premium may often underestimate the true value of voting rights to their owners. Our analysis also shows that a negative voting premium can arise when dispersed shareholders free-ride on the blockholder's trades, and that liquidity of voting shares can be different from that of non-voting shares. We extend the model to explore the role of the market for votes, the interaction between multiple blockholders, and the pricing of block trades. Overall, our analysis emphasizes how asset prices are affected by blockholders' desire to move the voting outcome in their preferred direction when voting and cash flow rights are bundled in shares.

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### Appendix - Proofs

**Proof of Lemma 1.** Given the realization of q, a shareholder indexed by b votes his shares for the proposal if and only if q > -b. Denote the fraction of post-trade shares voted to approve the proposal by  $\Lambda(q)$ . Note that  $\Lambda(q)$  is weakly increasing. If we have  $\Lambda(\Delta) < \tau$  for the highest possible  $q = \Delta$ , then  $q^*$  in the statement of the lemma is equal to  $\Delta$ . Similarly, if we have  $\Lambda(-\Delta) > \tau$  for the lowest possible  $q = -\Delta$ , then  $q^*$  in the statement of the lemma is equal to  $-\Delta$ . Finally, if  $\Lambda(-\Delta) \leq \tau < \Lambda(\Delta)$ , there exists  $q^* \in [-\Delta, \Delta)$  such that the fraction of votes voted in favor of the proposal is greater than  $\tau$  if and only if  $q > q^*$ . Hence, the proposal is approved if and only if  $q > q^*$ .

**Proof of Proposition 1.** Recall that  $s_{y,q_e^*}(q)$ , given by (13), is increasing in q. Denote

$$\beta_l(y, q_e^*) \equiv -s_{y, q_e^*}^{-1}(\tau), \qquad (36)$$

$$\beta_h(y, q_e^*) \equiv -s_{y, q_e^*}^{-1}(\tau - \alpha - y),$$
 (37)

As follows from the arguments prior to the proposition, the proposal is approved if and only if  $q > -b_{y,\beta}^{MV}(q_e^*)$ , where

$$b_{y,\beta}^{MV}(q_e^*) = \begin{cases} \beta_l(y, q_e^*) & \text{if } \beta \le \beta_l(y, q_e^*) \\ \beta & \text{if } \beta_l(y, q_e^*) < \beta \le \beta_h(y, q_e^*) \\ \beta_h(y, q_e^*) & \text{if } \beta_h(y, q_e^*) < \beta \end{cases}$$
(38)

is the identity of the median voter.

Since shareholders' expectations  $q_e^*$  at the trading stage have to be consistent with the actual decision rule at the voting stage, the equilibrium at the voting stage can be characterized as follows: the proposal is approved if and only if  $q > q^*(y)$ , where

$$-q^{*}(y) = \begin{cases} \beta_{L}(y) & \text{if } \beta < \beta_{L}(y) \\ \beta & \text{if } \beta_{L}(y) < \beta < \beta_{H}(y) \end{cases}$$

$$\beta_{H}(y) & \text{if } \beta_{H}(y) < \beta$$

$$(39)$$

$$= \begin{cases} \max \left\{ \beta, \beta_L(y) \right\} & \text{if } \beta < \beta^* \\ \min \left\{ \beta, \beta_H(y) \right\} & \text{if } \beta > \beta^*, \end{cases}$$

$$\tag{40}$$

is the identity of the median voter,  $\beta_{L}\left(y\right)$  and  $\beta_{H}\left(y\right)$  are the solutions of

$$s_{y,-\beta_L}(-\beta_L) = \tau, \tag{41}$$

$$s_{y,-\beta_L}(-\beta_L) = \tau,$$

$$s_{y,-\beta_H}(-\beta_H) = \tau - \alpha - y,$$

$$(41)$$

and  $\beta^* \equiv \beta_H(-\alpha) = \beta_L(-\alpha)$ .

Condition (41) can be rewritten from (13) as

$$\beta_L = \beta_l(y, -\beta_L) \Leftrightarrow R(\beta_L; y, -\beta_L) = 1 - \frac{\tau}{1 - \alpha - y}. \tag{43}$$

Similarly, condition (42) can be rewritten as

$$\beta_H = \beta_h (y, -\beta_H) \Leftrightarrow R(\beta_H; y, -\beta_H) = 1 - \frac{\tau - \alpha - y}{1 - \alpha - y}. \tag{44}$$

From (12),  $R(b'; y, q^*)$  is a cdf and lies in the unit interval. Moreover,

$$\lim_{\beta \to -\bar{b}} R(\beta; y, -\beta) = 0 \text{ and } \lim_{\beta \to \bar{b}} R(\beta; y, -\beta) = 1.$$
(45)

Hence, solutions to (43) and (44), and, therefore, of (41) and (42), must exist. For  $y = -\alpha$ , the right hand sides of (43) and (44) are identical and  $\beta^*$  is defined from  $R(\beta^*; y, -\beta^*) = 1 - \tau$ . The derivative of  $R(\beta; y, -\beta)$  with respect to  $\beta$  is:

$$\frac{\partial R(\beta; y, -\beta)}{\partial \beta} = g(\beta) \left( 1 + \frac{\beta - \mathbb{E}[b]}{\gamma} \frac{H(-\beta)}{1 - \alpha - y} \right) -G(\beta) \frac{f(-\beta)}{1 - \alpha - y} \frac{\mathbb{E}[b] - \mathbb{E}[b|b < \beta]}{\gamma}.$$
(46)

The first line of (46) equals  $r(\beta; y, -\beta) > 0$ . Since,  $\mathbb{E}[b] > \mathbb{E}[b|b < \beta]$ , the second line is negative. Hence,  $R(\beta; y, -\beta)$  and  $s_{y,-\beta}(-\beta)$  may be non-monotonic in  $\beta$ . From (46),  $\frac{\partial R(\beta; y, -\beta)}{\partial \beta} > 0$  if and only if

$$\frac{\frac{G(\beta)}{g(\beta)}f\left(-\beta\right)\left(\mathbb{E}\left[b\right]-\mathbb{E}\left[b|b<\beta\right]\right)+H\left(-\beta\right)\left(\mathbb{E}\left[b\right]-\beta\right)}{1-\alpha-y}<\gamma,$$

and thus, there exists  $\overline{\gamma} < \infty$  such that if  $\gamma > \overline{\gamma}$ , then  $\frac{\partial R(\beta;y,-\beta)}{\partial \beta}$  for every  $y \geq -\alpha$ . In this case, (45) implies that the solutions to (41)-(42) exist and are unique. Lemma 2 derives the properties of  $\beta_L(y)$ ,  $\beta_H(y)$  in this case, and in particular shows that  $\beta_L(y)$  is decreasing and  $\beta_H(y)$  is increasing in y.

Combining the properties of  $\beta_L(y)$ ,  $\beta_H(y)$  from Lemma 2 with the arguments above, it follows that there exists  $\overline{y}$  such that if  $y > \overline{y}$ , the median voter is the blockholder  $(-q^*(y) = \beta)$ , whereas if  $y < \overline{y}$ , the median voter is a dispersed shareholder with bias  $-q^*(y) \neq \beta$ , where  $|q^*(y) + \beta|$  is decreasing in y.

Figure A1 plots the median voter (vertical axis) as a function of the blockholder's trade y (horizontal axis). The figure shows that function  $\beta_H(y)$  ( $\beta_L(y)$ ) is upward (downward) sloping, starting at  $\beta^*$  for  $y = -\alpha$  and reaching a maximum of  $\bar{b}$  (minimum of  $-\bar{b}$ ) as y approaches  $\tau - \alpha$  (1 –  $\alpha - \tau$ ); this property is shown in the proof of Lemma 2.

The left panel of the figure considers the case in which the blockholder's bias is  $\beta > \beta^*$ . For any trade  $y < y_H \equiv \beta_H^{-1}(\beta)$ , the median voter is a dispersed shareholder whose bias is strictly increasing in y, whereas for any  $y \geq y_H$ , the median voter is the blockholder himself. Hence, the median voter is given by min  $\{\beta, \beta_H(y)\}$ , shown as the bold line in the figure. It follows that by choosing the appropriate trade in the interval  $[-\alpha, y_H]$ , the blockholder can change the identity of the median voter to be any point in the interval  $[\beta^*, \beta]$ . In particular, by buying more (or selling fewer) shares, the blockholder can push the bias of the median voter closer to  $\beta$ . However, the blockholder cannot choose a median voter outside of the interval  $[\beta^*, \beta]$ . Intuitively, the median voter cannot be larger than  $\beta$  because the blockholder's optimal voting strategy (to support the proposal whenever  $q > -\beta$ ) prevails when his stake becomes sufficiently large. Likewise, the median voter cannot be lower than  $\beta^*$  because the collective preferences of dispersed shareholders prevail when the blockholder exits his position.

The right panel of the figure considers the case in which the blockholder's bias is  $\beta < \beta^*$ , which is a mirror image of the left panel. In particular, the blockholder can influence the identity of the median voter to be any point in the interval  $[\beta, \beta^*]$  by choosing the appropriate

trade in the interval  $[-\alpha, y_L]$ .

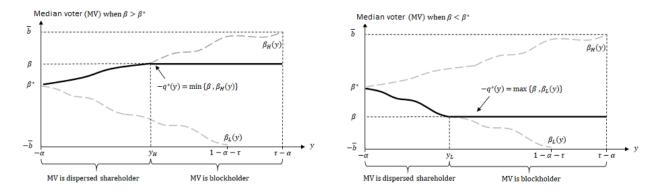


Figure A1 - median voter as a function of the blockholder's trade

Lemma 2 (Properties of the median voter) Suppose that the solutions  $\beta_L(y)$  and  $\beta_H(y)$ of (41) and (42) are unique for all y. Then:

$$\frac{\partial R(\beta_L; y, -\beta_L)}{\partial \beta_L} > 0 \text{ and}$$

$$\frac{\partial \beta_L(y)}{\partial y} = -\frac{\frac{1 - G(\beta_L)}{1 - \alpha - y}}{\frac{\partial R(\beta_L; y, -\beta_L)}{\partial \beta_L}} < 0.$$
(47)

Moreover,  $\lim_{y\nearrow 1-\tau-\alpha}\beta_L\left(y\right)=-\underline{b}$  and  $\lim_{y\nearrow 1-\tau-\alpha}\frac{\partial\beta_L}{\partial y}=-\left[\tau\frac{\partial R(\beta_L;y,-\beta_L)}{\partial\beta_T}\right]^{-1}$ .

(ii) 
$$\frac{\partial R(\beta_H; y, -\beta_H)}{\partial \beta_H} > 0$$
 and 
$$\frac{\partial \beta_H(y)}{\partial y} = \frac{\frac{G(\beta_H)}{1 - \alpha - y}}{\frac{\partial R(\beta_H; y, -\beta_H)}{\partial \beta_H}} > 0. \tag{48}$$

Moreover,  $\lim_{y \nearrow \tau - \alpha} \beta_H(y) = \underline{b}$  and  $\lim_{y \nearrow \tau - \alpha} \frac{\partial \beta_H(y)}{\partial y} = \left[ (1 - \tau) \frac{\partial R(\beta_H; y, -\beta_H)}{\partial \beta_H} \right]^{-1}$ .

- (iii) If the blockholder sells all her shares  $(y = -\alpha)$ , then  $\beta_L(-\alpha) = \beta_H(-\alpha) \equiv \beta^*$ .
- (iv) As  $\gamma$  becomes large, we have:

$$\lim_{\gamma \to \infty} \beta_L(y) = G^{-1} \left( 1 - \frac{\tau}{1 - \alpha - y} \right), \ \lim_{\gamma \to \infty} \beta_H(y) = G^{-1} \left( \frac{1 - \tau}{1 - \alpha - y} \right), \tag{49}$$

and

$$\lim_{\gamma \to \infty} \frac{\partial \beta_L(y)}{\partial y} = -\frac{1 - G(\lim_{\gamma \to \infty} \beta_L(y))}{g(\lim_{\gamma \to \infty} \beta_L(y))(1 - \alpha - y)},$$

$$\lim_{\gamma \to \infty} \frac{\partial \beta_H(y)}{\partial y} = \frac{G(\lim_{\gamma \to \infty} \beta_H(y))}{g(\lim_{\gamma \to \infty} \beta_H(y))(1 - \alpha - y)},$$
(51)

$$\lim_{\gamma \to \infty} \frac{\partial \beta_H(y)}{\partial y} = \frac{G(\lim_{\gamma \to \infty} \beta_H(y))}{g(\lim_{\gamma \to \infty} \beta_H(y))(1 - \alpha - y)},$$
(51)

$$\lim_{\gamma \to \infty} \beta^* = G^{-1} (1 - \tau). \tag{52}$$

**Proof of Lemma 2.** To simplify the expressions, define

$$X(b'; y, q^*) = \frac{\left(\mathbb{E}\left[b\right] - \mathbb{E}\left[b\left|b \le b'\right]\right) H(q^*)}{\gamma \left(1 - \alpha - y\right)}.$$
 (53)

With this definition, we have

$$R(b'; y, q^*) = G(b')(1 - X(b'; y, q^*)) \Leftrightarrow -G(b')X(b'; y, q^*) = R(b'; y, q^*) - G(b')$$
(54)

and

$$\frac{\partial R\left(b';y,q^{*}\right)}{\partial y} = -\frac{X\left(b';y,q^{*}\right)G\left(b'\right)}{1-\alpha-y}.$$
(55)

(i) If  $\beta_L(y)$  is the unique solution to (41), then  $\frac{\partial R(\beta_L;y,-\beta_L)}{\partial \beta_L} > 0$ . We apply the implicit function theorem to condition (41), which requires

$$\left[ (1 - \alpha - y) \frac{\partial R(\beta_L; y, -\beta_L)}{\partial y} + 1 - R(\beta_L; y, -\beta_L) \right] dy 
+ (1 - \alpha - y) \frac{\partial R(\beta_L; y, -\beta_L)}{\partial \beta_L} d\beta_L = 0,$$
(56)

where  $\frac{\partial R(\beta_L;y,-\beta_L)}{\partial y}$  is given by the same expression as above and  $R(\beta_L;y,-\beta_L)$  is again given from (43) so that

$$1 - R\left(\beta_L; y, -\beta_L\right) = \frac{\tau}{1 - \alpha - y}.\tag{57}$$

Substituting for  $1 - R(\beta_L; y, -\beta_L)$  and dividing by  $1 - \alpha - y$  allows us to rewrite (56) as

$$\left[\frac{\partial R\left(\beta_{L}; y, -\beta_{L}\right)}{\partial y} + \frac{\tau}{\left(1 - \alpha - y\right)^{2}}\right] dy + \frac{\partial R\left(\beta_{L}; y, -\beta_{L}\right)}{\partial \beta_{L}} d\beta_{L} = 0.$$
 (58)

Hence,

$$\frac{\partial \beta_L}{\partial y} = -\frac{\frac{\partial R(\beta_L; y, -\beta_L)}{\partial y} + \frac{\tau}{(1 - \alpha - y)^2}}{\frac{\partial R(\beta_L; y, -\beta_L)}{\partial \beta_L}}.$$
 (59)

We next use (55) and (57) to rewrite the numerator of (59) as

$$\begin{split} \frac{\partial R\left(\beta_{L};y,-\beta_{L}\right)}{\partial y} + \frac{\tau}{\left(1-\alpha-y\right)^{2}} &= \frac{1}{1-\alpha-y}\left(-G\left(\beta_{L}\right)X\left(\beta_{L},y,-\beta_{L}\right) + 1 - R\left(\beta_{L};y,-\beta_{L}\right)\right) \\ &= \frac{1-G\left(\beta_{L}\right)}{1-\alpha-y} > 0, \end{split}$$

where the second transformation uses (54). Hence,  $\frac{\partial \beta_L}{\partial y} < 0$  if  $\frac{\partial R(\beta_L; y, -\beta_L)}{\partial \beta_L} > 0$ . For  $y \nearrow 1 - \tau - \alpha$ , almost all dispersed shareholders are required to pass the proposal

without the blockholder. Then  $R \to 0$  and  $\beta_L \to -\bar{b}$ ,  $G(\beta_L) \to 0$ , and  $\frac{1-G(\beta_L)}{1-\alpha-y} \to \frac{1}{\tau}$ . Then

$$\frac{\partial \beta_L}{\partial y} \to -\frac{1}{\frac{\partial R(\beta_L; y, -\beta_L)}{\partial \beta_L} \tau} < 0. \tag{60}$$

(ii) If  $\beta_H(y)$  is the unique solution to (42), then  $\frac{\partial R(\beta_H, y, -\beta_H)}{\partial \beta_H} > 0$ . We apply the implicit function theorem to condition (44), which requires

$$\left[ (1 - \alpha - y) \frac{\partial R(\beta_H; y, -\beta_H)}{\partial y} - R(\beta_H, y, -\beta_H) \right] dy + (1 - \alpha - y) \frac{\partial R(\beta_H; y, -\beta_H)}{\partial \beta_H} d\beta_H = 0.$$
(61)

Substituting for  $R(\beta_H, y, -\beta_H)$  from (44)and dividing by  $1 - \alpha - y$  gives

$$\[ \frac{\partial R(H, y, -\beta_H)}{\partial \beta_H} - \frac{1 - \tau}{(1 - \alpha - y)^2} \] dy + \frac{\partial R(\beta_H, y, -\beta_H)}{\partial \beta_H} d\beta_H = 0.$$
 (62)

Hence,

$$\frac{\partial \beta_H}{\partial y} = -\frac{\frac{\partial R(\beta_H, y, -\beta_H)}{\partial y} - \frac{1 - \tau}{(1 - \alpha - y)^2}}{\frac{\partial R(\beta_H, y, -\beta_H)}{\partial \beta_H}}.$$
(63)

We use (55) and (44) to rewrite the numerator of (63) as

$$\frac{\partial R\left(\beta_{H}, y, -\beta_{H}\right)}{\partial y} - \frac{1-\tau}{\left(1-\alpha-y\right)^{2}} = \frac{1}{1-\alpha-y} \left(-G\left(\beta_{H}\right) X\left(\beta_{H}, y, -\beta_{H}\right) - R\left(\beta_{H}, y, -\beta_{H}\right)\right) \\
= -\frac{G\left(\beta_{H}\right)}{1-\alpha-y} < 0,$$

where the second line uses (54). Hence,  $\frac{\partial \beta_H}{\partial y} > 0$  in any equilibrium in which  $\frac{\partial R(\beta_H, y, -\beta_H)}{\partial \beta_H} > 0$ . For  $y \nearrow \tau - \alpha$ , the number of dispersed shareholders needed to pass the proposal becomes negligible. Then (44) implies  $R \to 1$  and (45) implies  $\beta_H \to \bar{b}$  and  $\mathbb{E}\left[b \mid b \leq \beta_h\right] \to \mathbb{E}\left[b\right]$ . Then  $\frac{-G(\beta_h)}{1-\alpha-y} \to \frac{-1}{1-\tau}$  and (63) simplifies to

$$\frac{\partial \beta_H}{\partial y} \to \frac{1}{\frac{\partial R(\beta_H, y, -\beta_H)}{\partial \beta_H} (1 - \tau)} > 0. \tag{64}$$

(iii) If  $y = -\alpha$ ,  $\beta_h(y, q_e^*) = \beta_l(y, q_e^*)$  from (36) and (37), hence  $\beta_L(y) = \beta_H(y)$ , which are both assumed to be unique. Hence,  $\beta^*$  can be obtained as the unique solution to (41).

(iv) From (12),  $\lim_{\gamma \to \infty} R(-q^*; y, q^*) = G(-q^*)$ . Substituting into (43) and (44) gives (49). From (46),  $\lim_{\gamma \to \infty} \frac{\partial R(-q^*; y, q^*)}{\partial (-q^*)} = g(-q^*)$ . Substituting into (47) and (48) gives (50).

**Proof of Proposition 2.** We start by noting that given (4) and (9), we can rewrite

$$\Pi(y) = (\alpha + y) v (\beta, q^*(y)) - yp^*(y) - \frac{\eta}{2} y^2$$

$$= \alpha v (\beta, q^*(y)) + y (\beta - \mathbb{E}[b]) H (q^*) - (\gamma + \eta/2) y^2$$

$$= \alpha v_0 + \alpha \mathbb{E}[\theta | q > q^*(y)] H (q^*) + ((\alpha + y) \beta - yE[b]) H (q^*) - (\gamma + \eta/2) y^2,$$

which explains the derivation of

$$\frac{\partial\Pi\left(y\right)}{\partial y}=\left(\beta-\mathbb{E}\left[b\right]\right)H\left(q^{*}\right)-\left(2\gamma+\eta\right)y+\frac{\partial\left(-q^{*}\left(y\right)\right)}{\partial y}\left[\alpha\left(q^{*}\left(y\right)+\beta\right)+y\left(\beta-\mathbb{E}\left[b\right]\right)\right]f\left(q^{*}\left(y\right)\right),$$

as claimed in the main text. Note that  $\frac{\partial (-q^*(y))}{\partial y}$  and  $\frac{\partial \Pi(y)}{\partial y}$  do not exist when  $\beta = \beta_L(y)$  or  $\beta = \beta_H(y)$ , which correspond to values  $y_L$  and  $y_H$  in Figure A1. In those cases, we interpret  $\frac{\partial (-q^*(y))}{\partial y}$  as the right derivative of  $-q^*(y)$ , which is zero, and  $\frac{\partial \Pi(y)}{\partial y}$  as the right derivative of  $\Pi(y)$ , which is MPC(y).

Recall that by assumption, the blockholder's trade in equilibrium is in the interval  $(-\alpha, 1-\alpha)$  So hereafter we assume  $y \in (-\alpha, 1-\alpha)$ . We start by giving sufficient conditions under which  $\Pi(y)$  is "well-behaved," namely, continuous, concave, and has a unique maximizer. From Proposition 1, there exists a  $\overline{\gamma}_1$  such that, if  $\gamma > \overline{\gamma}_1$ , then  $\beta_L(y)$  and  $\beta_H(y)$  are uniquely determined and both are continuous functions of y. If so,  $\Pi(y)$  is a continuous function of y as well. In addition, in the online appendix, we show that there exists  $\overline{\eta}_1 < \infty$  such that if  $\eta > \overline{\eta}_1$ , then  $\Pi(y)$  is a concave function. Combined, if  $\gamma > \overline{\gamma}_1$  and  $\eta > \overline{\eta}_1$ , then  $\Pi(y)$  is a continuous and concave function, and hence, it has a unique maximizer. We denote the unique maximizer by  $y^*$ .

Next, we define  $y^{**}$ . If  $y^{*}$  is such that  $\beta_{L}(y^{*}) < \beta < \beta_{H}(y^{*})$ , then it must be  $q_{a}^{*}(y^{*}) = -\beta$  and  $\frac{\partial(-q_{a}^{*}(y))}{\partial y} = 0$ . Therefore, using (18)-(19),  $y^{*}$  must solve

$$(\beta - \mathbb{E}[b]) \Pr[q > -\beta] - (2\gamma + \eta)y^* = 0 \Leftrightarrow$$

$$y^* = y^{**} \equiv \frac{1}{2\gamma + \eta} (\beta - \mathbb{E}[b]) (1 - F(-\beta)).$$

$$(65)$$

Notice that

$$\lim_{\gamma \to \infty} y^{**} = 0.$$

Second, recall that  $\beta_L(-\alpha) = \beta_H(-\alpha) = \beta^*$  from Proposition 1, and note that

$$\lim_{\gamma \to \infty} \beta^* = G^{-1} \left( 1 - \tau \right) \in \left( -\bar{b}, \bar{b} \right).$$

Next, we consider two cases:

1. Suppose  $\beta \in [-\bar{b}, \beta^*)$ . We argue that there exists a unique  $\underline{y} \in (-\alpha, 1 - \alpha - \tau)$  such that: (i)  $\beta = \beta_L(\underline{y})$ , (ii) if  $y \in (-\alpha, \underline{y})$  then  $\beta \in [-\bar{b}, \beta_L(\underline{y})]$ , and (iii) if  $y > \underline{y}$  then  $\beta \in (\beta_L(\underline{y}), \beta_H(\underline{y}))$ . To see why, recall that: (1)  $\beta_H(\underline{y})$  is an increasing function of y, (2)  $\beta_L(\underline{y})$  is a decreasing function of y, (3)  $\beta < \beta^* = \beta_L(-\alpha) = \beta_H(-\alpha)$ , and (4)  $\lim_{\underline{y} \nearrow 1 - \alpha - \tau} \beta_L(\underline{y}) = -\bar{b}$ . Combined, these four facts prove the arguments above.

Moreover, these arguments imply that the median voter is given by

$$-q_{a}^{*}\left(y\right) = \begin{cases} \beta_{L}\left(y\right) & \text{if } -\alpha < y < \underline{y} \\ \beta & \text{if } \underline{y} < y < 1 - \alpha, \end{cases}$$

Notice that by the definition of  $\beta_L(\cdot)$ , y is given by the solution of

$$R(\beta, \underline{y}, -\beta) = 1 - \frac{\tau}{1 - \alpha - \underline{y}} \Leftrightarrow$$

$$\underline{y} = 1 - \alpha - \frac{\tau}{1 - G(\beta)} + \frac{1}{\gamma} \frac{G(\beta)}{1 - G(\beta)} \left( \mathbb{E}[b] - \mathbb{E}[b|b < \beta] \right) \left( 1 - F(-\beta) \right),$$

where  $\lim_{\gamma \to \infty} \underline{y} = 1 - \alpha - \frac{\tau}{1 - G(\beta)}$  and  $\lim_{\gamma \to \infty} \underline{y} < 0 \Leftrightarrow \beta > G^{-1}(\frac{1 - \alpha - \tau}{1 - \alpha})$ . Also notice that

$$\Pi'\left(y\right) = \left(\beta - \mathbb{E}\left[b\right]\right) \Pr\left[q > q_a^*\left(y\right)\right] - \left(2\gamma + \eta\right)y + \begin{cases} MPV\left(y\right) & \text{if } -\alpha < y < \underline{y} \\ 0 & \text{if } y > \underline{y}, \end{cases}$$

and it is not defined for y = y. For  $-\alpha < y < y$ , we have

$$MPV(y) = \frac{\partial \beta_L(y)}{\partial y} f(-\beta_L(y)) [\alpha(\beta - \beta_L(y)) + y(\beta - \mathbb{E}[b])]$$

and

$$\lim_{\gamma \to \infty} MPV(y) = -\frac{1 - G(\lim_{\gamma \to \infty} \beta_L(y))}{g(\lim_{\gamma \to \infty} \beta_L(y))} \times \frac{f(-\lim_{\gamma \to \infty} \beta_L(y))}{1 - \alpha - y} \left[\alpha(\beta - \lim_{\gamma \to \infty} \beta_L(y)) + y(\beta - \mathbb{E}[b])\right],$$
(66)

where  $\lim_{\gamma \to \infty} \beta_L(y) = G^{-1}(1 - \frac{\tau}{1-\alpha-y})$ . Thus,  $\lim_{\gamma \to \infty} MPV(y)$  is bounded (recall  $y < 1 - \alpha$ ). We consider two subcases.

- (a) First, suppose  $\beta > G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})$ . Then for a large  $\gamma$  we have  $\underline{y} < 0 \approx y^{**}$ . Fix any  $\varepsilon \in (0, -\lim_{\gamma \to \infty} \underline{y})$ , and notice that for any  $y \in (-\alpha, \lim_{\gamma \to \infty} \underline{y} + \varepsilon)/\{\lim_{\gamma \to \infty} \underline{y}\}$  we have  $\lim_{\gamma \to \infty} \overline{\Pi'}(y) = \infty$ . Thus, there exists  $\overline{\gamma}_3 < \infty$  such that if  $\gamma > \overline{\gamma}_3$  then  $\Pi'(y) > 0$  for any  $y \in (-\alpha, \underline{y} + \varepsilon)/\{\underline{y}\}$ . Therefore, the maximizer of  $\Pi(y)$  is greater than  $\underline{y}$ , that is,  $y^* > \underline{y}$ . Recall that if  $y > \underline{y}$  then  $\beta \in (\beta_L(y), \beta_H(y))$ , which implies MPV(y) = 0. Therefore, it must be  $MPV(y^*) = 0$ ,  $-q_a^*(y^*) = \beta$ , and  $y^* = y^{**}$ .
- (b) Second, suppose  $\beta < G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})$ . Then, for a large  $\gamma$  we have  $1-\alpha>\underline{y}>0\approx y^{**}$ . Fix any  $\varepsilon\in(0,\lim_{\gamma\to\infty}\underline{y})$ , and notice that for any  $y\in(\lim_{\gamma\to\infty}\underline{y}-\varepsilon,1-\alpha)/\{\lim_{\gamma\to\infty}\underline{y}\}$  we have  $\lim_{\gamma\to\infty}\Pi'(y)=-\infty$ . Thus, there exists  $\overline{\gamma}_4<\infty$  such that if  $\gamma>\overline{\gamma}_4$  then  $\Pi'(y)<0$  for any  $y\in(\underline{y}-\varepsilon,1-\alpha)/\{\underline{y}\}$ . Therefore, the maximizer of  $\Pi(y)$  is smaller than  $\underline{y}$ , that is,  $y^*<\underline{y}$ . In particular, since  $\Pi(y)$  is continuous and concave, and since  $\Pi'(y)|_{y=-\alpha}>0$  for a large  $\gamma$ , there is a unique  $\hat{y}\in(-\alpha,\underline{y})$  such that  $\Pi'(y)|_{y=\hat{y}}=0$ . Therefore, the optimizer of  $\Pi(y)$  is  $y^*=\hat{y}$  and the median voter is a dispersed shareholder with a bias  $\beta_L(\hat{y})>\beta$ .

Since  $\lim_{\gamma \to 0} \hat{y} = 0$  and  $\lim_{\gamma \to \infty} \beta_L(y) = G^{-1}(1 - \frac{\tau}{1 - \alpha - y})$ , (66) implies

$$\lim_{\gamma \to \infty} MPV\left(\hat{y}\right) = \frac{\tau}{1 - \alpha} \frac{f\left(-G^{-1}\left(\frac{1 - \alpha - \tau}{1 - \alpha}\right)\right)}{g\left(G^{-1}\left(\frac{1 - \alpha - \tau}{1 - \alpha}\right)\right)} \frac{\alpha}{1 - \alpha} \left(G^{-1}\left(\frac{1 - \alpha - \tau}{1 - \alpha}\right) - \beta\right) > 0. \quad (67)$$

In addition,

$$\lim_{\gamma \to \infty} MPC\left(\hat{y}\right) = \left(\beta - \mathbb{E}\left[b\right]\right) H\left(-G^{-1}\left(\frac{1-\alpha-\tau}{1-\alpha}\right)\right) - 2\lim_{\gamma \to \infty}\left[\gamma\hat{y}\right].$$

Since  $\Pi'(y)|_{y=\hat{y}} = 0$  in this case, it must be  $\lim_{\gamma \to \infty} MPC(\hat{y}) + \lim_{\gamma \to \infty} MPV(\hat{y}) = 0$ , that is

$$\lim_{\gamma \to \infty} \left[ 2\gamma \hat{y} \right] = \lim_{\gamma \to \infty} MPV(\hat{y}) + (\beta - \mathbb{E}[b]) H\left( -G^{-1}(\frac{1 - \alpha - \tau}{1 - \alpha}) \right).$$

This implies that  $\hat{y}$  and  $MPV\left(\hat{y}\right)$  could have different signs. For example, if  $\alpha$  is small,  $\beta < \mathbb{E}\left[b\right]$  (which is likely given  $\beta < G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})$ ), then  $\lim_{\gamma \to \infty} MPV\left(\hat{y}\right) > 0$  but is close to zero, whereas  $(\beta - \mathbb{E}\left[b\right]) H\left(-G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})\right) < 0$  and is bounded from zero, so  $\lim_{\gamma \to \infty} \left[2\gamma\hat{y}\right] < 0$ , which implies that  $\hat{y}$  converges to 0 from below, i.e.,  $\hat{y} < 0$  while  $MPV\left(\hat{y}\right) > 0$ .

2. Suppose  $\beta \in (\beta^*, \overline{b}]$ . We argue that there exists a unique  $\overline{y} \in (-\alpha, \tau - \alpha)$  such that: (i)  $\beta = \beta_H(\overline{y})$ , (ii) if  $y \in (-\alpha, \overline{y})$  then  $\beta \in (\beta_H(y), \overline{b}]$ , and (iii) if  $y > \overline{y}$  then  $\beta \in (\beta_L(y), \beta_H(y))$ . To see why, recall that: (1)  $\beta_H(y)$  is an increasing function of y, (2)  $\beta_L(y)$  is a decreasing function of y, (3)  $\beta > \beta^* = \beta_L(-\alpha) = \beta_H(-\alpha)$ , and (4)  $\lim_{y \nearrow \tau - \alpha} \beta_H(y) = \overline{b}$ . Combined, these four facts prove the arguments above. Moreover, these arguments imply that the median voter is given by

$$-q_a^*(y) = \begin{cases} \beta_H(y) & \text{if } -\alpha < y < \overline{y} \\ \beta & \text{if } \overline{y} < y < 1 - \alpha, \end{cases}$$

Notice that by the definition of  $\beta_H(\cdot)$ ,  $\overline{y}$  is given by the solution of

$$R(\beta, \overline{y}, -\beta) = 1 - \frac{\tau - \alpha - \overline{y}}{1 - \alpha - \overline{y}} \Leftrightarrow$$

$$\overline{y} = 1 - \alpha - \frac{1 - \tau}{G(\beta)} - \frac{1}{\gamma} \left( \mathbb{E}[b] - \mathbb{E}[b|b < \beta] \right) \left( 1 - F(-\beta) \right),$$

where  $\lim_{\gamma \to \infty} \overline{y} = 1 - \alpha - \frac{1-\tau}{G(\beta)}$  and  $\lim_{\gamma \to \infty} \overline{y} < 0 \Leftrightarrow \beta < G^{-1}(\frac{1-\tau}{1-\alpha})$ . Also notice that

$$\Pi'(y) = (\beta - \mathbb{E}[b]) \Pr[q > q_a^*(y)] - (2\gamma + \eta)y + \begin{cases} MPV(y) & \text{if } -\alpha < y < \overline{y} \\ 0 & \text{if } y > \overline{y}, \end{cases}$$

and it is not defined for  $y = \overline{y}$ . For  $-\alpha < y < \overline{y}$ , we have

$$MPV(y) = \frac{\partial \beta_H(y)}{\partial y} f(-\beta_H(y)) [\alpha(\beta - \beta_H(y)) + y(\beta - \mathbb{E}[b])]$$

and

$$\lim_{\gamma \to \infty} MPV(y) = \frac{G(\lim_{\gamma \to \infty} \beta_H(y))}{g(\lim_{\gamma \to \infty} \beta_H(y))} \times \frac{f(-\lim_{\gamma \to \infty} \beta_H(y))}{1 - \alpha - y} \left[ \alpha(\beta - \lim_{\gamma \to \infty} \beta_H(y)) + y(\beta - \mathbb{E}[b]) \right],$$
(68)

where  $\lim_{\gamma \to \infty} \beta_H(y) = G^{-1}(\frac{1-\tau}{1-\alpha-y})$ . Thus  $\lim_{\gamma \to \infty} MPV(y)$  is bounded (recall  $y < 1-\alpha$ ). We consider two subcases.

- (a) First, suppose  $\beta < G^{-1}(\frac{1-\tau}{1-\alpha})$ . Then for a large  $\gamma$  we have  $\overline{y} < 0 \approx y^{**}$ . Fix any  $\varepsilon \in (0, -\lim_{\gamma \to \infty} \overline{y})$ , and notice that for any  $y \in (-\alpha, \lim_{\gamma \to \infty} \overline{y} + \varepsilon)/\{\lim_{\gamma \to \infty} \overline{y}\}$  we have  $\lim_{\gamma \to \infty} \Pi'(y) = \infty$ . Thus, there exists  $\overline{\gamma}_5 < \infty$  such that if  $\gamma > \overline{\gamma}_5$  then  $\Pi'(y) > 0$  for any  $y \in (-\alpha, \overline{y} + \varepsilon)/\{\overline{y}\}$ . Therefore, the maximizer of  $\Pi(y)$  is greater than  $\overline{y}$ , that is,  $y^* > \overline{y}$ . Recall that if  $y > \overline{y}$  then  $\beta \in (\beta_L(y), \beta_H(y))$ , which implies MPV(y) = 0. Therefore, it must be  $MPV(y^*) = 0$ ,  $-q_a^*(y^*) = \beta$ , and  $y^* = y^{**}$ . Moreover, since  $y > \overline{y} \Rightarrow MPV(y) = 0$ , it must be  $-q_a^*(y^*) = \beta$ ,  $MPV(y^*) = 0$ , and  $y^* = y^{**}$ .
- (b) Second, suppose  $\beta > G^{-1}(\frac{1-\tau}{1-\alpha})$ . Then, for a large  $\gamma$  we have  $1-\alpha > \overline{y} > 0 \approx y^{**}$ . Fix any  $\varepsilon \in (0, \lim_{\gamma \to \infty} \overline{y})$ , and notice that for any  $y \in (\lim_{\gamma \to \infty} \overline{y} \varepsilon, 1-\alpha)/\{\lim_{\gamma \to \infty} \overline{y}\}$  we have  $\lim_{\gamma \to \infty} \Pi'(y) = -\infty$ . Thus, there exists  $\overline{\gamma}_6 < \infty$  such that if  $\gamma > \overline{\gamma}_6$  then  $\Pi'(y) < 0$  for any  $y \in (\overline{y} \varepsilon, 1 \alpha)/\{\overline{y}\}$ . Therefore, the maximizer of  $\Pi(y)$  is smaller than  $\overline{y}$ , that is,  $y^* < \overline{y}$ . In particular, since  $\Pi(y)$  is continuous and concave, and since  $\Pi'(y)|_{y=-\alpha} > 0$  for a large  $\gamma$ , there is a unique  $\hat{y} \in (-\alpha, \overline{y})$  such that  $\Pi'(y)|_{y=\hat{y}} = 0$ . Therefore, the optimizer of  $\Pi(y)$  is  $y^* = \hat{y}$  and the median voter is a dispersed shareholder with a bias  $\beta_H(\hat{y}) < \beta$ .

Since  $\lim_{\gamma \to 0} \hat{y} = 0$  and  $\lim_{\gamma \to \infty} \beta_H(y) = G^{-1}(\frac{1-\tau}{1-\alpha-y})$ , (68) implies

$$\lim_{\gamma \to \infty} MPV(\hat{y}) = \frac{1 - \tau}{1 - \alpha} \times \frac{f(-G^{-1}(\frac{1 - \tau}{1 - \alpha}))}{g(G^{-1}(\frac{1 - \tau}{1 - \alpha}))} \frac{\alpha}{1 - \alpha} (\beta - G^{-1}(\frac{1 - \tau}{1 - \alpha})) > 0.$$
 (69)

In addition,

$$\lim_{\gamma \to \infty} MPC\left(\hat{y}\right) = \left(\beta - \mathbb{E}\left[b\right]\right) H\left(-G^{-1}\left(\frac{1-\tau}{1-\alpha}\right)\right) - 2\lim_{\gamma \to \infty}\left[\gamma\hat{y}\right].$$

Since  $\Pi'(y)|_{y=\hat{y}} = 0$  in this case, it must be  $\lim_{\gamma \to \infty} MPC(\hat{y}) + \lim_{\gamma \to \infty} MPV(\hat{y}) = 0$ , that is

$$\lim_{\gamma \to \infty} \left[ 2\gamma \hat{y} \right] = \lim_{\gamma \to \infty} MPV\left( \hat{y} \right) + \left( \beta - \mathbb{E}\left[ b \right] \right) H\left( -G^{-1}(\frac{1-\tau}{1-\alpha}) \right).$$

Potentially,  $\hat{y}$  and  $MPV\left(\hat{y}\right)$  could again have different signs. For example, if  $\alpha$  is small,  $\beta < E\left[b\right]$  (which is possible if  $G^{-1}(\frac{1-\tau}{1-\alpha}) < E\left[b\right]$ ), then  $\lim_{\gamma \to \infty} MPV\left(\hat{y}\right) > 0$  but is close to zero, whereas  $(\beta - \mathbb{E}\left[b\right]) H\left(-G^{-1}(\frac{1-\tau}{1-\alpha})\right) < 0$  and is bounded from zero, so  $\lim_{\gamma \to \infty} \left[2\gamma\hat{y}\right] < 0$ , which implies that  $\hat{y}$  converges to 0 from below, i.e.,  $\hat{y} < 0$  while  $MPV\left(\hat{y}\right) > 0$ .

Notice that  $\lim_{\gamma\to\infty}\beta^*=G^{-1}(1-\tau)\in \left(G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}),G^{-1}(\frac{1-\tau}{1-\alpha})\right)$ . According to part 1.a and 2.a, for a large  $\gamma$ , if  $G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})<\beta<\beta'$  or  $\beta^*<\beta< G^{-1}(\frac{1-\tau}{1-\alpha})$ , then  $-q_a^*(y^*)=\beta$ ,  $MPV(y^*)=0$ , and  $y^*=y^{**}$ . This establishes part (i) in the statement. According to part 1.b, if  $\beta<\min\{G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}),\beta^*\}=G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})$ , then the median voter is a dispersed shareholder with bias  $\beta_L(y^*)>\beta$ . According to part 2.b, if  $\beta>\max\{G^{-1}(\frac{1-\tau}{1-\alpha}),\beta^*\}=G^{-1}(\frac{1-\tau}{1-\alpha})$ , then the median voter is a dispersed shareholder with bias  $\beta_H(y^*)<\beta$ . Combined, this establishes part (ii).

To see expression (25) for the share price, we simply plug in  $y^*$  and  $q^*(y^*)$  into (16). Finally, given (67) and (69), there exists  $\overline{\gamma}_2 > 0$  such that if  $\gamma > \overline{\gamma}_2$ , then  $MPV(y^*) > 0$  for  $\beta \notin (G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}), G^{-1}(\frac{1-\tau}{1-\alpha}))$ .

Letting  $\overline{\gamma} = \max \{\overline{\gamma}_1, \overline{\gamma}_2, \overline{\gamma}_3, \overline{\gamma}_4, \overline{\gamma}_5, \overline{\gamma}_6\}$  completes the proof.

Proposition 3 is a special case of the following result.

**Proposition 5.** Suppose  $\alpha = 0$ . There exist  $\overline{\gamma} < \infty$  and  $\overline{\eta} < \infty$  such that if  $\gamma > \overline{\gamma}$  and  $\eta > \overline{\eta}$ , then the equilibrium exists and is unique. In equilibrium, the median voter is a dispersed investor with bias

$$-q^*(y^*) = \begin{cases} \beta_L(y^*) > \beta & \text{if } \beta < G^{-1}(1-\tau) \\ \beta_H(y^*) < \beta & \text{if } \beta > G^{-1}(1-\tau). \end{cases}$$
 (70)

Moreover:

(i) If  $\mathbb{E}[b] < \beta$ , then the blockholder's equilibrium trade satisfies  $y^* > 0$  and

$$y^* = \frac{1}{2\gamma + \eta} (\beta - \mathbb{E}[b]) H(q^*(y^*)) + \frac{1}{2\gamma + \eta} MPV(y^*), \qquad (71)$$

the share price is given by

$$p^* = v(b_{MT}, q^*(y^*)) + \frac{\gamma}{2\gamma + \eta} MPV(y^*),$$
 (72)

where MPV  $(y^*) < 0$  if and only if  $\beta < G^{-1}(1-\tau)$ .

(ii) If  $\beta < \mathbb{E}[b]$ , then the blockholder does not trade in equilibrium (i.e.,  $y^* = 0$ ), the no-short-selling constraint binds, MPV(0) = 0, and

$$p^{*} = v(\mathbb{E}[b], q^{*}(0))$$
  
=  $v(b_{MT}, q^{*}(0)) + \frac{\gamma}{2\gamma + \eta}(\mathbb{E}[b] - \beta) H(q^{*}(0)).$ 

**Proof.** We build on the proof of Proposition 2, and adjust to the special case with  $\alpha = 0$ . Showing the existence of a unique maximizer, which we denote by  $y^*$ , follows the same

arguments and hence is omitted. Recall that  $\beta_L(0) = \beta_H(0) = \beta^*$  from Proposition 1, and note that

$$\lim_{\gamma \to \infty} \beta^* = G^{-1} \left( 1 - \tau \right) \in \left( -\bar{b}, \bar{b} \right).$$

Next, we consider two cases:

1. Suppose  $\beta \in [-\bar{b}, \beta^*)$ . As in the proof of Proposition 2, there exists a unique  $\underline{y} \in (0, 1 - \tau)$  such that: (i)  $\beta = \beta_L(\underline{y})$ , (ii) if  $y \in (0, \underline{y})$  then  $\beta \in [-\bar{b}, \beta_L(y))$ , and (iii) if  $y > \underline{y}$  then  $\beta \in (\beta_L(y), \beta_H(y))$ . Thus, the median voter is given by

$$-q^{*}(y) = \begin{cases} \beta_{L}(y) & \text{if } 0 < y < \underline{y} \\ \beta & \text{if } \underline{y} < y < \overline{1}. \end{cases}$$

Moreover, as in the proof of Proposition 2, and by the definition of  $\beta_L(\cdot)$  we have  $\lim_{\gamma \to \infty} \underline{y} = 1 - \frac{\tau}{1 - G(\beta)}$ , and notice that  $\lim_{\gamma \to \infty} \underline{y} > 0 \Leftrightarrow \beta < G^{-1}(1 - \tau)$ . Also notice that

$$MPV\left(y\right) = \begin{cases} \frac{\partial \beta_{L}\left(y\right)}{\partial y} f(-\beta_{L}\left(y\right)) y \left(\beta - \mathbb{E}\left[b\right]\right) & \text{if } 0 < y < \underline{y} \\ 0 & \text{if } y > \underline{y}, \end{cases}$$

and it is not defined for y = y. Notice that

$$\lim_{\gamma \to \infty} MPV\left(y\right) = -\frac{1 - G(\lim_{\gamma \to \infty} \beta_L\left(y\right))}{g(\lim_{\gamma \to \infty} \beta_L\left(y\right))} \frac{f(-\lim_{\gamma \to \infty} \beta_L\left(y\right))}{1 - y} y \left(\beta - \mathbb{E}\left[b\right]\right),$$

where  $\lim_{\gamma\to\infty} \beta_L(y) = G^{-1}(1-\frac{\tau}{1-y})$ . Thus,  $\lim_{\gamma\to\infty} MPV(y)$  is bounded (recall y<1). Since  $\beta<\beta^*$  and  $\lim_{\gamma\to\infty} \beta^*=G^{-1}(1-\tau)$ , for a large  $\gamma$  we have  $1>\underline{y}>0$ , and for the same reasons as in the proof of Proposition 2, the maximizer of  $\Pi(y)$  is smaller than  $\underline{y}$  and given by the solution of FOC subject to the no-short-selling constraint that we impose below. That is, the optimal trade solves

$$y^* = \frac{1}{2\gamma + \eta} \left(\beta - \mathbb{E}\left[b\right]\right) H(q^*\left(y^*\right)) + \frac{1}{2\gamma + \eta} MPV\left(y^*\right),$$

subject to being non-negative. In particular, notice that  $\lim_{\gamma \to \infty} y^* = 0$ , and hence  $\lim_{\gamma \to \infty} MPV(y^*) = 0$ . Suppose the no-short-selling constraint does not bind in the limit, that is,  $y^*$  converges to zero from above. Then, the FOC implies

$$\lim_{\gamma \to \infty} \left(\beta - \mathbb{E}\left[b\right]\right) H\left(q^* \left(\lim_{\gamma \to \infty} y^*\right)\right) - \lim_{\gamma \to \infty} 2\gamma \hat{y} + \lim_{\gamma \to \infty} MPV\left(y^*\right) = 0 \Leftrightarrow \left(\beta - \mathbb{E}\left[b\right]\right) H\left(q^*\left(0\right)\right) = \lim_{\gamma \to \infty} 2\gamma y^*.$$

Thus,  $y^*$  converges to zero from above if and only if  $\beta > \mathbb{E}[b]$ . If  $\beta < \mathbb{E}[b]$ , then the no-short-selling constraint must bind in the limit, and in that case, the blockholder does not trade (i.e.,  $y^* = 0$ ). If  $\beta > \mathbb{E}[b]$ , then the no-short-selling constraint does not bind in the limit, and having  $y^*$  converging to zero from above implies that  $MPV(y^*)$  converges to zero from below. That is, if  $\beta > \mathbb{E}[b]$ , then for large  $\gamma$  it must be

$$MPV\left(y^*\right) < 0 < y^*.$$

Then, the share price is  $p^* = v\left(b_{MT}, q^*\left(y^*\right)\right) + \frac{\gamma}{2\gamma + \eta} MPV\left(y^*\right)$ , where  $MPV\left(y^*\right) < 0$ .

2. Suppose  $\beta \in (\beta^*, \overline{b},]$ . As in the proof of Proposition 2, there exists a unique  $\overline{y} \in (0, \tau)$  such that: (i)  $\beta = \beta_H(\overline{y})$ , (ii) if  $y \in (0, \overline{y})$  then  $\beta \in (\beta_H(y), \overline{b}]$ , and (iii) if  $y > \overline{y}$  then  $\beta \in (\beta_L(y), \beta_H(y))$ . Thus, the median voter is given by

$$-q^{*}(y) = \begin{cases} \beta_{H}(y) & \text{if } 0 < y < \overline{y} \\ \beta & \text{if } \overline{y} < y < 1, \end{cases}$$

Moreover, as in the proof of Proposition 2, and by the definition of  $\beta_H(\cdot)$  we have  $\lim_{\gamma\to\infty} \overline{y} = 1 - \frac{1-\tau}{G(\beta)}$ , and notice that  $\lim_{\gamma\to\infty} \overline{y} > 0 \Leftrightarrow \beta > G^{-1}(1-\tau)$ . Also notice that

$$MPV\left(y\right) = \begin{cases} \frac{\partial \beta_{H}(y)}{\partial y} f(-\beta_{H}\left(y\right)) y \left(\beta - \mathbb{E}\left[b\right]\right) | & \text{if } 0 < y < \overline{y} \\ 0 & \text{if } y > \overline{y}, \end{cases}$$

and it is not defined for  $y = \overline{y}$ . Notice that

$$\lim_{\gamma \to \infty} MPV\left(y\right) = \frac{G(\lim_{\gamma \to \infty} \beta_{H}\left(y\right))}{g(\lim_{\gamma \to \infty} \beta_{H}\left(y\right))} \frac{f(-\lim_{\gamma \to \infty} \beta_{H}\left(y\right))}{1 - y} y \left(\beta - \mathbb{E}\left[b\right]\right),$$

where  $\lim_{\gamma\to\infty}\beta_H(y)=G^{-1}(\frac{1-\tau}{1-y})$ . Thus,  $\lim_{\gamma\to\infty}MPV(y)$  is bounded (recall y<1). Since  $\beta>\beta^*$  and  $\lim_{\gamma\to\infty}\beta^*=G^{-1}(1-\tau)$ , for a large  $\gamma$  we have  $1>\overline{y}>0$ , and for the same reasons as in the proof of Proposition 2, the maximizer of  $\Pi(y)$  is smaller than  $\overline{y}$  and given by the solution of FOC subject to the no-short-selling constraint that we impose below. That is, the optimal trade solves

$$y^* = \frac{1}{2\gamma + \eta} \left(\beta - \mathbb{E}\left[b\right]\right) H(q^*\left(y^*\right)) + \frac{1}{2\gamma + \eta} MPV\left(y^*\right),$$

subject to being non-negative. In particular, notice that  $\lim_{\gamma \to \infty} y^* = 0$ , and hence  $\lim_{\gamma \to \infty} MPV(y^*) = 0$ . Suppose the no-short-selling constraint does not bind in the limit, that is,  $y^*$  converges to zero from above. Then, the FOC implies

$$\lim_{\gamma \to \infty} (\beta - \mathbb{E}[b]) H\left(q^* \left(\lim_{\gamma \to \infty} y^*\right)\right) - \lim_{\gamma \to \infty} 2\gamma \hat{y} + \lim_{\gamma \to \infty} MPV(y^*) = 0 \Leftrightarrow (\beta - \mathbb{E}[b]) H(q^*(0)) = \lim_{\gamma \to \infty} 2\gamma y^*.$$

Thus,  $y^*$  converges to zero from above if and only if  $\beta > \mathbb{E}[b]$ . If  $\beta < \mathbb{E}[b]$ , the no-short-selling constraint must bind in the limit, and in that case, the blockholder does not trade  $(y^* = 0)$ . If  $\beta > \mathbb{E}[b]$ , the no-short-selling constraint does not bind in the limit, and having  $y^*$  converging to zero from above implies that  $MPV(y^*)$  converges to zero from above. That is, if  $\beta > \mathbb{E}[b]$ , then for large  $\gamma$  it must be  $0 < MPV(y^*)$  and  $0 < y^*$ . Then, the share price is  $p^* = v\left(b_{MT}, q^*(y^*)\right) + \frac{\gamma}{2\gamma + \eta}MPV(y^*)$ , where  $MPV(y^*) > 0$ .

Proof of Proposition 4. The objective  $\Pi(y,\hat{y})$  of the blockholder with dual-class shares can be rewritten as:

$$\max_{y,\hat{y}} \Pi(y,\hat{y}) = (\alpha + y) v (\beta, q_a^*(y)) - yp^*(y) - \frac{\eta}{2} y^2 + (\hat{\alpha} + \hat{y}) v (\beta, q_a^*(y)) - \hat{y}\hat{p}^*(\hat{y}) - \frac{\eta}{2}\hat{y}^2$$

$$= \alpha v (\beta, q_a^*(y)) + y (\beta - \mathbb{E}[b]) \Pr[q > q_a^*(y)] - (\gamma + \eta/2)y^2$$

$$+ \hat{\alpha}v (\beta, q_a^*(y)) + \hat{y} (\beta - \mathbb{E}[b]) \Pr[q > q_a^*(y)] - (\gamma + \eta/2)\hat{y}^2$$

$$= (\alpha + \hat{\alpha}) v (\beta, q_a^*(y)) + (y + \hat{y}) (\beta - \mathbb{E}[b]) \Pr[q > q_a^*(y)] - (\gamma + \eta/2) (y^2 + \hat{y}^2)$$

$$= (\alpha + \hat{\alpha}) v_0 + (\alpha + \hat{\alpha}) \Pr[q > q_a^*(y)] \mathbb{E}[\theta|q > q_a^*(y)]$$

$$+ ((\alpha + \hat{\alpha} + y + \hat{y}) \beta - (y + \hat{y}) \mathbb{E}[b]) \Pr[q > q_a^*(y)] - (\gamma + \eta/2) (y^2 + \hat{y}^2) .$$

We rewrite the first-order condition with respect to y,  $\frac{\partial \Pi(y,\hat{y})}{\partial y} = 0$ , as

$$\begin{bmatrix} (\beta - \mathbb{E}[b]) \Pr[q > q_a^*(y)] - (2\gamma + \eta)y \\ \text{marginal propensity to buy cash flows} \end{bmatrix} = 0 \Leftrightarrow \\ \frac{\partial (-q_a^*(y))}{\partial y} f(q_a^*(y)) [(\alpha + \hat{\alpha}) (q_a^*(y) + \beta) + (y + \hat{y}) (\beta - \mathbb{E}[b])]}{\text{marginal propensity to buy votes } MPV(y,\hat{y})} = 0 \Leftrightarrow \\ (\beta - \mathbb{E}[b]) \Pr[q > q_a^*(y)] - (2\gamma + \eta)y + MPV(y,\hat{y}) = 0 \Leftrightarrow \\ y^* = \frac{1}{2\gamma + \eta} (\beta - \mathbb{E}[b]) \Pr[q > q_a^*(y)] + \frac{1}{2\gamma + \eta} MPV(y,\hat{y}).$$

We rewrite the first-order condition with respect to  $\hat{y}$ ,  $\frac{\partial \Pi(y,\hat{y})}{\partial \hat{y}} = 0$ , as

$$\underbrace{\left(\beta-\mathbb{E}\left[b\right]\right)\Pr\left[q>q_{a}^{*}\left(y\right)\right]-\left(2\gamma+\eta\right)\hat{y}}_{\text{marginal propensity to buy cash flows in non-voting shares}}=0\Leftrightarrow$$

$$\hat{y}^* = \frac{1}{2\gamma + \eta} \left( \beta - \mathbb{E}\left[ b \right] \right) \Pr \left[ q > q_a^* \left( y \right) \right],$$

which implies (34).

### Appendix - Table 1

The following table lists 40 studies that measure the voting premium using five different methodologies.<sup>27</sup> Methods of measurement within a given methodology may differ slightly. The studies on dual-class tender offers by Bradley (1980) and DeAngelo and DeAngelo (1985) are classified as block trades because tender offers are bids for a block of shares, not for individual shares. Christoffersen et al. (2007) is listed as two separate studies.

Study		Sample			Voting premium		
Authors	Year of publication	Country	Period	Size	Dual-class premium	Block- trade premium	Voting yield
Panel A: Dual-class shares							
Lease et al.	1983	USA	1940-1978	26	5.44%		
Levy	1983	Israel	1974-1980	25	45.50%		
Horner	1988	Switzerland	1973-1983	45	22.40%		
Megginson	1990	UK	1955-1982	152	13.30%	32.10%	
Zingales	1994	Italy	1987-1990	96	81.50%		
Smith and Amoako-Adu	1995	Canada	1981-1986	62	10.40%	57.20%	
Zingales	1995	USA	1984-1990	94	10.47%	81.50%	
Rydqvist	1996	Sweden	1983-1990	65	12.00%		
Kunz and Angel	1996	Switzerland	1990-1991	29	18.00%		
Maynes	1996	Canada	1984	46	6.66%		
Muus	1998	France	1986-1996	25	51.35%		
Chung and Kim	1999	Korea	1992-1993	119	9.60%		
Hoffmann-Burchardi	1999	Germany	1988-1997	84	26.30%	21.70%	
Cox and Roden	2002	USA	1984-1999	98	7.70%		
Daske and Ehrhardt	2002	Germany	1956-1998	101	17.23%	20.60%	
Dittmann	2003	Germany	1960-2001	82	12.62%		
Neumann	2003	Denmark	1992-1999	34	12.31%		
Nenova	2003	cross-country	1997	661	13.85%		
Muravyev	2004	Russia	1997-2003	37	64.72%		
Ødegaard	2007	Norway	1988-2005	226	5.60%		
Caprio and Croci	2008	Italy	1974-2003	116	56.51%		
Bigelli and Croci	2013	Italy	1999-2008	74	20.35%		
Broussard and Vaihekoski	2019	Finland	1982-2018	50	27.20%		

<sup>&</sup>lt;sup>27</sup>The studies are: Aggarwal, Saffi, and Sturgess (2015); Albuquerque and Schroth (2010); Albuquerque and Schroth (2015); Barak and Lauterbach (2011); Barclay and Holderness (1989); Bergström and Rydqvist (1992); Bigelli and Croci (2013); Bradley (1980); Broussard and Vaihekoski (2019); Caprio and Croci (2008); Christoffersen et al. (2007); Chung and Kim (1999); Cox and Roden (2002); Daske and Ehrhardt (2002); DeAngelo and DeAngelo (1985); Dittmann (2003); Dyck and Zingales (2004); Fos and Holderness (2020); Franks and Mayer (2001); Gurun and Karakas (2020); Hoffman-Burchardi (1999); Horner (1988); Jang, Kim, and Mohseni (2019); Kalay, Karakas, and Pant (2014); Kind and Poltera (2013); Kind and Poltera (2017); Kunz and Angel (1996); Lease, McConnell, and Mikkelson (1983); Levy (1983); Maynes (1996); Megginson (1990); Muravyev (2004); Muus (1998); Nenova (2003); Neumann (2003); Odegaard (2007); Rydqvist (1996); Smith and Amoako-Adu (1995); Zingales (1994); Zingales (1995).

Panel B: Block trades and dual-class tender offers    Bradley	Voting yield
Bradley 1980 USA 1962-1977 161 13.00% DeAngelo and DeAngelo 1985 USA 1960-1980 144 130.90% Barclay and Holderness 1989 USA 1978-1884 63 20.40% Bergström and Rydqvist 1992 Sweden 1980-1990 40 15.20% 27.00% Franks and Mayer 2001 Germany 1988-1997 57 36.32% Dyck and Zingales 2004 cross-country 1990-2000 393 14.00% Albuquerque and Schroth 2010 USA 1990-2006 120 19.62% Barak and Lauterbach 2011 Israel 1993-2005 54 46.96% Albuquerque and Schroth 2015 USA 1990-2006 114 59.74%  Panel C: Option replications  Kind and Poltera 2013 cross-country 2003-2010 155 Kalay et al. 2014 USA 1996-2007 4768 Kind and Poltera 2017 USA 2002-2013 1359 Jang et al. 2019 USA 1996-2015 1648 Gurun and Karakas 2020 USA 1996-2015 5223	
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Barclay and Holderness       1989       USA       1978-1884       63       20.40%         Bergström and Rydqvist       1992       Sweden       1980-1990       40       15.20%       27.00%         Franks and Mayer       2001       Germany       1988-1997       57       36.32%         Dyck and Zingales       2004       cross-country       1990-2000       393       14.00%         Albuquerque and Schroth       2010       USA       1990-2006       120       19.62%         Barak and Lauterbach       2011       Israel       1993-2005       54       46.96%         Albuquerque and Schroth       2015       USA       1990-2006       114       59.74%         Panel C: Option replications         Kind and Poltera       2013       cross-country       2003-2010       155         Kalay et al.       2014       USA       1996-2007       4768         Kind and Poltera       2017       USA       2002-2013       1359         Jang et al.       2019       USA       1996-2015       1648         Gurun and Karakas       2020       USA       1996-2015       5223	
Bergström and Rydqvist       1992       Sweden       1980-1990       40       15.20%       27.00%         Franks and Mayer       2001       Germany       1988-1997       57       36.32%         Dyck and Zingales       2004       cross-country       1990-2000       393       14.00%         Albuquerque and Schroth       2010       USA       1990-2006       120       19.62%         Barak and Lauterbach       2011       Israel       1993-2005       54       46.96%         Albuquerque and Schroth       2015       USA       1990-2006       114       59.74%         Panel C: Option replications         Kind and Poltera       2013       cross-country       2003-2010       155         Kalay et al.       2014       USA       1996-2007       4768         Kind and Poltera       2017       USA       2002-2013       1359         Jang et al.       2019       USA       1996-2015       1648         Gurun and Karakas       2020       USA       1996-2015       5223	
Franks and Mayer         2001         Germany         1988-1997         57         36.32%           Dyck and Zingales         2004         cross-country         1990-2000         393         14.00%           Albuquerque and Schroth         2010         USA         1990-2006         120         19.62%           Barak and Lauterbach         2011         Israel         1993-2005         54         46.96%           Albuquerque and Schroth         2015         USA         1990-2006         114         59.74%           Panel C: Option replications         Kind and Poltera         2013         cross-country         2003-2010         155           Kalay et al.         2014         USA         1996-2007         4768           Kind and Poltera         2017         USA         2002-2013         1359           Jang et al.         2019         USA         1996-2015         1648           Gurun and Karakas         2020         USA         1996-2015         5223	
Dyck and Zingales         2004         cross-country 1990-2000         393         14.00%           Albuquerque and Schroth         2010         USA         1990-2006         120         19.62%           Barak and Lauterbach         2011         Israel         1993-2005         54         46.96%           Albuquerque and Schroth         2015         USA         1990-2006         114         59.74%           Panel C: Option replications         Kind and Poltera         2013         cross-country         2003-2010         155           Kalay et al.         2014         USA         1996-2007         4768           Kind and Poltera         2017         USA         2002-2013         1359           Jang et al.         2019         USA         1996-2015         1648           Gurun and Karakas         2020         USA         1996-2015         5223	
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Panel C: Option replications         Kind and Poltera       2013       cross-country 2003-2010       155         Kalay et al.       2014       USA       1996-2007       4768         Kind and Poltera       2017       USA       2002-2013       1359         Jang et al.       2019       USA       1996-2015       1648         Gurun and Karakas       2020       USA       1996-2015       5223	
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Jang et al.     2019     USA     1996-2015     1648       Gurun and Karakas     2020     USA     1996-2015     5223	0.16%
Gurun and Karakas 2020 USA 1996-2015 5223	0.28%
	0.12%
	0.09%
Panel D: Equity lending	
Christoffersen et al. 2007 USA 1998-1999 6764	
Christoffersen et al. 2007 UK 2003-2005 822	
Aggarwal et al. 2015 USA 2007-2009 7415	0.02%
Panel E: Record-day price effects	
Fos and Holderness 2019 USA 1996-2016 107530	

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