The Threat of Intervention

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Abstract

We develop a model in which an activist shareholder can discipline management through intervention and through the threat of intervention. A weaker disciplinary role played by the intervention mechanism leads to lower firm value and more frequent ex post interventions. Thus, more frequent ex post interventions are not necessarily a sign of enhanced economic efficiency. In general, we show that the ex ante threat and ex post intervention can act as complements or substitutes. Because we endogenize the activist's choice of toehold, we also show that the effect of liquidity trading on firm value depends on the timing of liquidity trading.

One of the fundamental issues in modern corporate finance is the problem of separation of firm ownership from control. The gap between management and shareholders is potentially wide and the danger is great for agency problems to divert a widely-held firm's resources from their efficient use. Therefore it is important to understand what mechanisms are available for reconciling these interests, to what extent they are used, and to what extent they are effective.

If a shareholder decides he does not like what a firm's management is doing, he has two alternatives: He can *intervene* or he can *exit*—that is, he can work directly on changing the behavior of the firm's management or he can sell his shares. Intervention, sometimes referred to as "voice," includes a variety of possible actions to compel changes in managerial behavior: replacement of boards of directors, support for takeover bids, and proxy initiatives to limit management discretion or to affect management compensation.

However, shareholder activity can also have indirect effects, because the foreknowledge by managers of the possible reactions of dissatisfied shareholders can alter managerial behavior. Thus we are not only interested in exit and intervention as behaviors by the blockholder, we are also interested in how they affect managerial behavior. That is, we are also interested in the ex ante incentive effects on managers of the *threats* of shareholder exit or intervention. Beginning with Admati and Pfleiderer (2009) and Edmans (2009), a series of recent articles has shown that, provided management compensation is tied in the short run to share price, the threat of exit and the resultant reduction of share price can serve as a disciplinary device. On the other hand, despite empirical investigations of intervention, surprisingly little theoretical attention has been paid to the role of *the threat of intervention*.

In this paper we focus on the dual aspect of the intervention mechanism: Intervention can improve the firm ex post (through direct action by the activist) or ex ante (through the threat to management). We ask the following research questions: In equilibrium when does the threat of intervention affect managerial behavior? Under what circumstances does intervention play a stronger disciplinary role? Under what circumstances does intervention play a stronger correction role ex post?

To address these questions, we provide a model in which an activist shareholder can accumulate a toehold of shares. After observing the activist's toehold size, the manager decides whether to consume private benefits at the expense of shareholders. Once the managerial action is taken, the activist decides whether to extend the toehold and intervene, or to sell shares. Thus the process can improve firm value through two channels: the direct intervention itself, and the effect of the threat of intervention on managerial behavior.

In the model, an important role is played by the market's revelation through prices of the activist's response to managerial behavior. If the market fully reveals the activist's private information, the activist has no incentive to accumulate the toehold. For this reason it is important to consider the effect of liquidity trading. The presence of liquidity trades enables the activist to a certain degree to hide his information. In this paper we consider liquidity trading during toehold accumulation period as well as during the intervention decision period. As we will show, liquidity trading in different phases of activist activity will have different impacts on the relation between market liquidity and economic efficiency.

Our research framework is relevant in modern financial markets, since most publicly traded firms can be subject to governance though the threat of intervention. Recent empirical evidence shows that the threat of intervention is likely to play a strong disciplinary role. Fos (2016) shows that when the likelihood of a proxy contest increases, firms take actions to increase firm value and therefore reduce the chances of intervention. Gantchev et al. (2016) show that when an activist hedge funds target a firm, other firms in the target's industry are more likely to take actions with the intention to increase firm value and therefore reduce the chances of intervention.

The model reveals several key results.

The model shows how ex ante threat and ex post intervention interact and how they are related to economic efficiency. For instance, in the model, more frequent ex post interventions are not necessarily a sign of enhanced economic efficiency. A weaker disciplinary role played by the intervention mechanism leads to lower firm value (because the manager is not disciplined ex ante), which can lead to more frequent ex post interventions, which are costly (both to the activist and the manager) but only partially recover the damage made to the firm value. Thus in this case more frequent ex post interventions are a sign of worsening corporate governance.

Therefore it is important to understand the circumstances in which observed levels of intervention actually correlate with improvement in firm value. Because we endogenize both the activist's decision to engage in activism and the manager's decision to take the bad action, we are better able to track the relation between the effectiveness of intervention and apparent empirical measures of that effectiveness. While the interventions themselves improve firm value, to the extent that they are substituting for the more efficient alternative of an ex ante threat disciplining the manager, the observation of interventions should correlate with decreasing firm value.

We provide conditions under which we predict a positive or negative correlation between firm value and observed degree of intervention. The sign of the correlation will depend not only on the source of the variation but also on the degree to which the two channels act as complements or substitutes. For instance if variation is due to differences in the effectiveness of activists in punishing firm management—as would be the case in business cycle downturns, when loss of a job would have more dire consequences—then the two mechanisms act as substitutes. When the variation is due to differences in the activist's ex post liquidity needs—activists flush with cash will be unlikely to need to sell for liquidity purposes—then the two mechanisms will be complementary. In the latter case, we should observe increases in intervention correlating with increases in firm value.

The model reveals that it is important to distinguish between sources of liquidity trading. Previous work has emphasized the dual nature of liquidity trading: that it makes it easier for activists to accumulate holdings, but also makes it harder to commit not to dissipate those holdings. Liquidity trading that does not interact with the activist's actions has a positive effect on market liquidity and on the activist's trading profits. It therefore leads to larger toehold accumulated by the activist and consequently increases chances of an equilibrium in which the activist intervenes. In contrast, liquidity trading that interacts with activist's actions leads to wider bid-ask spreads and weaker disciplinary role played by the intervention. This result has important implications for the literature that studies the role of market liquidity in corporate governance. To the best of our knowledge, this is the first paper to contrast two phases of liquidity trading and to show their differential effects on economic outcomes.

Because we endogenize the activist's choice of toehold we can examine the relation between observed blockholdings and the use of intervention as a threat. One of key implications of Shleifer and Vishny (1986) is that the presence of a large blockholder increases chances of blockholder governance through voice. The intuition is that a large block allows the blockholder to capture a larger portion of value creation and therefore to cover the cost of exercising voice—that is, the presence of a large blockholder provides a partial solution to the free-rider problem (Grossman and Hart, 1980). To the best of our knowledge, our model is the first to show that the presence of a large blockholder may lead to fewer incidents of blockholder governance through ex post intervention, to the extent that the threat of intervention is very effective ex ante. In the extreme, one would not observe any intervention events if the threat of intervention were so powerful as to prevent the manager from taking the bad action in any state of the world.[1] Thus, our model argues that the absence of action by large blockholders can in fact be a sign of well-functioning corporate governance.

We investigate the extent to which increases in the toehold are indicative of improved governance. In particular we consider the following

[1]In this extreme case the situation bears a similarity to the theory of contestible markets, where potential competition, even though unobserved, manages to provide market discipline against temptations toward inefficient behavior (Baumol et al., 1988).

questions: When can we expect that size of toehold to be a better/worse indicator than frequency of intervention of the effectiveness of the intervention mechanism? How do changes in the liquidity of the asset market at various points in the activist's cycle of activity affect activist behavior and managerial response?

Related Literature

This paper is related to several strands of the corporate governance literature that studies the role of blockholders in reducing agency costs.[2]

First, the paper contributes to the strand of literature that studies how shareholder intervention can increase firm value ex post (e.g., Shleifer and Vishny, 1986; Kyle and Vila, 1991; Admati et al., 1994; Maug, 1998; Bolton and von Thadden, 1998; Kahn and Winton, 1998; Noe, 2002; Faure-Grimaud and Gromb, 2004; Brav and Mathews, 2011; Back et al., 2016). For example, in their classic paper Shleifer and Vishny (1986) show that the presence of a large minority shareholder provides a partial solution to the free-rider problem and therefore reduces the agency costs.

In this strand of literature, intervention does not play a disciplinary role. Intervention occurs in the absence of managerial action; more effective monitoring does not change the manager's incentives and therefore is beneficial for shareholders only because it increases firm value *ex post*. Thus, our contribution is to introduce the disciplinary role of intervention.

Second, the paper contributes to the literature that studies how corporate governance can affect management's incentives. Grossman and Hart

[2] Edmans (2013) surveys theoretical and empirical literature on the role of blockholders in corporate governance.

(1980) were the first to argue that managers face trade offs between a high profit action with an associated low chance of being raided and a low profit (but high managerial-utility) action which leads to a successful takeover bid. In their model managers are more reluctant to take self-serving actions that lower firm value and increase the probability of a takeover. Scharfstein (1988) explicitly models the source of contractual inefficiencies which was not studied by Grossman and Hart (1980). He explores the conditions under which the takeover threat plays a genuine role (beyond incentive contracts) in disciplining management.[3]

The literature has also studied the governance role of exit and showed that a large shareholder can alleviate conflicts of interest between managers and shareholders through the credible threat of exit on the basis of private information (e.g., Admati and Pfleiderer, 2009; Edmans, 2009; Dasgupta and Piacentino, 2014). Our paper contributes to the corporate governance/management incentive literature by studying the interaction between ex ante and ex post corrections in the intervention equilibrium. Moreover, because we endogenize the activist's choice of toehold, we can study consequences of liquidity trading during toehold-acquiring phase and the activism phase of the blockholder's activity.

[3]While the above papers show that takeover plays a positive disciplinary role, several other papers have highlighted some negative aspects of the threat of intervention (e.g., Stein, 1988; Zwiebel, 1996; Burkart et al., 1997). For example, Stein (1988) develops a model in which takeover pressure can be damaging because it leads managers to sacrifice long-term interests in order to boost current profit.

1. Setup

In the basic model there are three dates 0, 1, and 2 and three types of agents: the manager, whom we denote by $\mathcal{M}$, an activist shareholder, $\mathcal{A}$, and a continuum of uninformed traders (the "market makers"). Markets for shares in the firm occur at each date.

At date 0, $\mathcal{A}$ acquires the initial holding φ of shares of the firm (the "toehold"). The choice of optimal φ, which is analyzed in section 3, will take into account the impact of φ on $\mathcal{M}$'s incentives (and therefore firm value), the cost of holding φ shares, and trading profits.[4]

Trading profits can be positive because participants in the period 0 market expect that with probability θ_0 the purchase order will come from an uniformed activist and with probability $(1 - \theta_0)$ the purchase order comes from an informed activist shareholder. If $\mathcal{A}$ is uninformed, he cannot observe the action taken by $\mathcal{M}$ and therefore cannot engage in activism. Higher values θ_0 correspond to a more liquid period 0 market.[5]

After date 0 trading occurs, market participants, including $\mathcal{M}$, observe φ—that is, they learn the size of $\mathcal{A}$'s holding. Then $\mathcal{M}$ decides whether or not to take a particular action. An agency problem arises because $\mathcal{M}$ and the shareholders have conflicting preferences with respect to the action.

[4]We will assume that the ability to amass large initial positions is limited by disclosure requirements, modeled on current regulatory requirements. Details are specified in the toehold analysis in section 3.

[5]Various methods of modeling liquidity trading exist in the literature; see for example Kyle (1985) and Glosten and Milgrom (1985) for contrasting approaches. The approach we use here, following Admati and Pfleiderer (2009) allows us to use consistent approaches for period 0 and period 1 liquidity, while enabling us to consider the disciplinary role of exit as in Admati and Pfleiderer (2009) (see section 4).

Specifically, we assume the action is "bad" in the sense that it reduces the value of the firm, but provides a private benefit to $\mathcal{M}$. The benefit has the positive value β, known with certainty by all participants.[6] The cost of the damage to the firm is $\tilde{\delta}$, a random value which $\mathcal{M}$ learns privately immediately before making his decision.[7] Let the decision be denoted a (either zero or one); then the value of the firm in period 2 will be $\upsilon - a\tilde{\delta}$, in the absence of intervention by the activist. The value υ is common knowledge. All agents know that the value $\tilde{\delta}$ is drawn from a continuous distribution $F(.)$ with density $f(.)$ and support $[0, \bar{\delta}]$, where $\bar{\delta}$ is sufficiently large. When illustrating some results we will further assume that the distribution of δ is exponential with $F(\delta) = 1 - e^{-\delta\lambda}$.

$\mathcal{M}$'s strategy can be described by defining the set $\Delta \subseteq [0, \bar{\delta}]$, such that $a = 1$ if and only if δ is in the set Δ. Let $\Phi = Pr\{\delta \in \Delta\}$, the ex ante probability that $\mathcal{M}$ chooses $a = 1$.

$\mathcal{A}$ privately observes the action taken by $\mathcal{M}$. Given $\mathcal{M}$'s strategy, observing $\mathcal{M}$'s actions provides $\mathcal{A}$ with a noisy signal of firm value. Given this private information $\mathcal{A}$ must decide whether to buy, sell, or hold his shares at the date 1 market. If $\mathcal{A}$ buys sufficient shares, he can *intervene,* reducing the benefit to $\mathcal{M}$ of taking the bad action, and reducing the damage of the action to the firm. Specifically, if $\mathcal{A}$ intervenes then the benefit to $\mathcal{M}$

[6]Fos and Jiang (2016) document evidence consistent with a manager's value of private benefits of control being 5%-20% of the stock price when the company is targeted in proxy contest.

[7]In a supplement to this paper we also consider the case where $\mathcal{M}$'s action is "good" in that it increases the firm's value at a private cost to the manager. For the most part that G version of the model (to use the terminology of Admati and Pfleiderer) provides results parallel to the B version adopted here.

is reduced to $\beta\gamma$ and the value of the firm is restored to $\upsilon - a\tilde{\delta}\kappa$, where $0 < (1-\gamma) < 1$ measures the effectiveness of $\mathcal{A}$ in reducing the private benefits of control and $0 < (1-\kappa) < 1$ measures his effectiveness in restoring firm value. We further assume that the intervention involves a cost $\eta\tilde{\delta}$ to $\mathcal{A}$. (Thus, it is more costly for $\mathcal{A}$ to repair a larger damage to firm value.) Let $b \in \{0,1\}$ represent the decision to intervene. Then the ultimate value of the firm is $\upsilon - a\tilde{\delta}(\kappa b + 1 - b)$. We assume this value is publicly revealed before the date 2 market, so that trade in the final market occurs at this price.[8]

As we will see, $\mathcal{A}$'s ability to reduce the benefit to $\mathcal{M}$ of taking the bad action, $(1-\gamma)$, and $\mathcal{A}$'s ability to reduce the damage of the action to the firm, $(1-\kappa)$, will play important and distinct roles in the model. Whereas $\mathcal{A}$'s ability to increase firm value ex post, $(1-\kappa)$, will be one of key parameters to determine the existence of the intervention equilibrium, $(1-\gamma)$ will determine the degree of discipline imposed on $\mathcal{M}$ in that equilibrium. Consistent with $(1-\gamma) > 0$, Fos and Tsoutsoura (2014) show that activist shareholders are able to impose a significant career cost on directors of companies targeted in proxy contests. Directors of companies that experience a proxy contest lose seats not only on boards of targeted companies, but also on boards of other companies. Several pieces of evidence motivate $(1-\kappa) > 0$. For example, Brav et al. (2008) show that firm value increases upon intervention by activist

[8] In our formulation, the power of the blockholder to punish managers or to repair damage to the firm are simply taken as parametric. One formulation which has endogenized the ability of outsiders to punish management is that of Fluck (1999). In her account, the size of outsider holdings affects the likelihood of being able to remove the manager. Since she assumes that the firm is less valuable when managers are removed from control, such a threat is not credible except in an infinitely repeated game. She examines credible threats in that framework.

hedge funds.

We next characterize $\mathcal{A}$'s trading in period 1. Similarly to Maug (1998), we assume $\mathcal{A}$ can intervene if he controls a fraction α of the shares in the firm, where α is fixed and publicly known.[9] As we will see, in any equilibrium in which $\mathcal{A}$ decides to sell shares, it is optimal to sell the entire position φ. Similarly, in any equilibrium in which $\mathcal{A}$ decides to buy shares, it is optimal to buy shares until the block size reaches α. Therefore, we assume that if $\mathcal{A}$ decides to sell shares, he sells the entire position φ. If $\mathcal{A}$ decides to buy shares, he buys $\alpha - \varphi$ shares. Thus all participants in trading at date 1 know α and φ.

We follow Admati and Pfleiderer (2009) and assume that $\mathcal{A}$ in period 1 will with probability θ_1 suffer a liquidity shock which requires him to divest himself of any holdings of firm shares and which prevents him from purchasing any shares of the firm. If he does not suffer a liquidity shock, then his purchases and sales will be based on his information and his strategy for future intervention. Thus $\mathcal{A}$'s trades in period 1 may reveal information both about $\mathcal{M}$'s actions and about $\mathcal{A}$'s own intentions for future actions. If $\mathcal{A}$ decides to purchase shares, it could indicate either that the shares are undamaged by $\mathcal{M}$'s action or that $\mathcal{A}$ intends to intervene to repair $\mathcal{M}$'s action. Sales on the other hand could be due either to a bad choice by $\mathcal{M}$ or to $\mathcal{A}$'s liquidity needs. Other participants in the market are unable to observe the liquidity shock of $\mathcal{A}$, and so the price prevailing will take into

[9] Several factors could affect α. For example, a larger α could correspond to cases when $\mathcal{A}$ needs more voting power to make the intervention effective.

account their expectation of the relative likelihood of the shock.[10]

$\mathcal{M}$'s compensation is assumed to be $\omega_2(\upsilon - a\tilde{\delta})$, where ω_2 is a positive coefficient representing the dependence of the compensation on firm value. In section 4 we allow $\mathcal{M}$'s compensation to depend on period 1 price, P_1. $\mathcal{M}$ chooses whether to take the action or not to maximize his expected utility for every realization of $\tilde{\delta}$. When $\mathcal{A}$ is not present, $\mathcal{M}$'s preferred cutoff point, denoted δ_{BM}, is equal to β/ω_2. That is, $\mathcal{M}$ takes the action when $\tilde{\delta} \leq \delta_{BM} = \beta/\omega_2$. Let p_{BM} denote the expected value of the firm when $\mathcal{A}$ is not present.

Next consider the case when $\mathcal{A}$ is present. If $\mathcal{M}$ does not take the action, then $\mathcal{M}$'s utility is simply his compensation, $\omega_2\upsilon$. If $\mathcal{M}$ takes the action, then his utility depends on $\mathcal{A}$'s intervention in period 2. If intervention does not occur, $\mathcal{M}$'s utility is equal to the sum of his compensation and the private benefit $\omega_2(\upsilon-\tilde{\delta})+\beta$. If intervention occurs, $\mathcal{M}$'s utility is equal to the sum of his compensation and the private benefit $\omega_2(\upsilon-\tilde{\delta})+\beta\gamma$.[11] To summarize, the potential impact of $\mathcal{A}$ on $\mathcal{M}$'s decision to take the action comes about through his impact on private benefits of control. We will describe an equilibrium as *disciplinary* if equilibrium cutoff point is lower than δ_{BM}.

We assume that prices are set by risk-neutral, competitive market makers and therefore reflect all of the information publicly available. This means, as noted before, that P_2 equals $\upsilon - a\delta(\kappa b+1-b)$. The date 1 price, P_1 reflects

[10]We have also considered the case when with probability ζ $\mathcal{A}$ will suffer a liquidity shock which requires him to buy $\alpha - \varphi$ shares. This liquidity shock does affect the intervention equilibrium because $\mathcal{A}$ always buys shares in the absence of the sell-side liquidity shock. The effects on the exit equilibrium are ambiguous. The results are available upon request.

[11]Note that $\mathcal{M}$ does not benefit from value creation induced by $\mathcal{A}$'s action. Alternatively, a more productive $\mathcal{A}$ would effectively create an incentive for $\mathcal{M}$ to take the bad action.

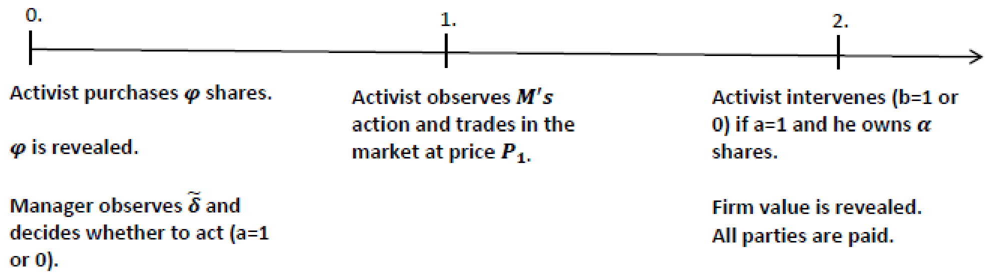

Figure 1: **Time-line.**

the information contained in $\mathcal{A}$'s trading decision.

The timing of events is given in Figure 1. The model is solved backwards. First, we assume $\mathcal{A}$ holds φ shares and characterize equilibrium prices and $\mathcal{A}$'s trading decisions at date 1, and $\mathcal{M}$'s actions. Then we endogenize $\mathcal{A}$'s choice of the initial holding, φ.

2. Date 1 Equilibria and $\mathcal{M}$'s Incentives

Suppose $\mathcal{A}$ holds φ shares. We assume $\mathcal{A}$ is restricted to three actions $T \in \{B, H, S\}$ in the date 1 market: buy enough to get the level to the required amount for intervention (B); sell all holdings (S); or keep holdings unchanged (H for "hold"). As we are going to show later in this section, in any equilibrium in which $\mathcal{A}$ decides to sell shares, it is optimal to sell the entire position. Similarly, in any equilibrium in which $\mathcal{A}$ decides to buy shares, it is optimal to buy shares until the block size reaches α.

It is useful to introduce notation for the prices that would occur if uninformed agents observed $\mathcal{M}$'s action (denote these as p_T^a). If $a = 0$, $p_T^0 = \upsilon$ for any T. If $a = 1$, $p_H^1 = p_S^1 = \upsilon - \Lambda$, where $\Phi \equiv Pr(\delta \in \Delta)$ is the probability of $\mathcal{M}$ taking the action and $\Lambda \equiv \Phi^{-1}E[\mathbb{1}_{\delta\in\Delta}\delta]$ is the expected damage to firm value, conditional on $\mathcal{M}$ taking the action. Note that the price if held is the same as the price if sold, because without having enough of a holding to intervene, $\mathcal{A}$ adds no value to the asset. Finally, $p_B^1 = \upsilon - \kappa\Lambda > p_H^1 = p_S^1$, reflecting the benefit from intervention.[12]

We next consider the value of $\mathcal{A}$'s position π_a^T. The value from holding is $\pi_0^H = \varphi\upsilon$ if $a = 0$ and $\pi_1^H = \varphi(\upsilon - \Lambda)$ if $a = 1$. The value from selling the lot is $\pi_a^S = \varphi p_S$ (note this does not actually depend on a). The value from buying is $\pi_0^B = \alpha\upsilon - (\alpha - \varphi)p_B$ or $\pi_1^B = \alpha(\upsilon - \kappa\Lambda) - \eta\Lambda - (\alpha - \varphi)p_B$, where $\eta\Lambda$ is the expected cost of intervention. Hereafter, we will refer to the value *net* of the undamaged value of the initial holding, $\varphi\upsilon$, as "$\mathcal{A}$'s profits."

A *market equilibrium* for period 1 specifies the probability mixture for $\mathcal{A}$ between buy, hold, and sell $\left(\sigma_a^B, \sigma_a^H, \sigma_a^S\right)$, for $a = 1$ or 0, *conditional on no liquidity shock* and market prices p_B, p_S, such that the probabilities are maximizing choices given prices, and prices are consistent with the probabilities:

$$\sum_{T=B,H,S} \sigma_a^T \pi_a^T \geq \pi_a^{T'} \text{for all } T' \in \{B, H, S\}, \text{ for } a = 0, 1.$$

$$p_T^1 \leq p_T \leq p_T^0, \text{ for } T \in B, S$$

[12]We know that $\Phi > 0$ (and so $\Lambda > 0$) because for any fixed value of ω_2, for δ sufficiently close to zero, $\mathcal{M}$ would prefer to take the action, even if it were publicly observable.

$$
\begin{aligned}
p_S &= \frac{(1-\theta_1)[p_S^1\Phi\sigma_1^S + p_S^0(1-\Phi)\sigma_0^S] + \theta_1[p_S^1\Phi + p_S^0(1-\Phi)]}{(1-\theta_1)[\Phi\sigma_1^S + (1-\Phi)\sigma_0^S] + \theta_1} \\
&= \upsilon - \Lambda\frac{\Phi\sigma_1^S + \bar{\theta}_1\Phi}{\Phi\sigma_1^S + (1-\Phi)\sigma_0^S + \bar{\theta}_1} \\
p_B &= \frac{(1-\theta_1)[p_B^1\Phi\sigma_1^B + p_B^0(1-\Phi)\sigma_0^B]}{(1-\theta_1)[\Phi\sigma_1^B + (1-\Phi)\sigma_0^B]} = \upsilon - \kappa\Lambda\frac{\Phi\sigma_1^B}{\Phi\sigma_1^B + (1-\Phi)\sigma_0^B},
\end{aligned}
$$

where $\bar{\theta}_1 \equiv \theta_1/(1-\theta_1)$ and p_B is defined when the denominator is non-zero. Without loss of generality we can also specify p_B when the denominator is zero: If there is zero probability of buying, we can set $p_B = p_B^0$. To see this, note that if $p_B < p_B^0$ in an equilibrium with no buying, then higher buying prices also yield an equilibrium with the same allocation; moreover p_B cannot exceed p_B^0 in equilibrium.

The complete description of an equilibrium also requires the specification of an intervention program for the activist. The activist can intervene whenever $a = 1$ and his holdings are at least equal to α. However there is a cost of intervention; therefore we must ensure that the activist finds it in his interest to intervene. It can be shown that as long as

$$\eta < \alpha(1-\kappa) \tag{1}$$

an activist with α shares always finds it in his interest to intervene; henceforth we impose this assumption on these parameters.

The following Lemma shows that we can put more structure on equilibrium beliefs.

Table 1: **$\mathcal{A}$'s Profits.** This table describes profits of $\mathcal{A}$, as measured by $\pi - \varphi v$, adjusted to results of Lemma 1. The left column reports profits if $\mathcal{M}$ does not take the action and the right column reports profits if $\mathcal{M}$ takes the action. In the case of each bracketed expression, the second term is to be used in case the denominator of the first term is zero.

	$a = 0$ (no damage)	$a = 1$ (damage)
Buy	$(\alpha - \varphi)\kappa\Lambda\{\frac{\Phi\sigma_1^B}{\Phi\sigma_1^B + (1-\Phi)\sigma_0^B}, 0\}$	$-\alpha\kappa\Lambda - \eta\Lambda + (\alpha - \varphi)\kappa\Lambda\{\frac{\Phi\sigma_1^B}{\Phi\sigma_1^B + (1-\Phi)\sigma_0^B}, 0\}$
Hold	0	$-\varphi\Lambda$
Sell	$-\varphi\Lambda\frac{\Phi(1-\sigma_1^B)+\bar{\theta}_1\Phi}{\Phi(1-\sigma_1^B)+\bar{\theta}_1}$	$-\varphi\Lambda\frac{\Phi(1-\sigma_1^B)+\bar{\theta}_1\Phi}{\Phi(1-\sigma_1^B)+\bar{\theta}_1}$

Lemma 1. *In equilibrium, $\sigma_0^S = 0$ and $\sigma_1^H = 0$.*[13]

In other words, if he observes the bad action was taken, $\mathcal{A}$ will definitely trade at date 1 (although it is possible that he may buy or sell). If he observes that the bad action was not taken, he will not sell voluntarily (although he may be forced to sell for liquidity reasons). $\mathcal{A}$'s profits, as measured by $\pi - \varphi v$, are presented in Table 1.

We next characterize *intervention equilibria*, those in which $\mathcal{A}$ intervenes with certainty (as long as he is not hit by a liquidity shock), i.e., those in which $\sigma_1^B = 1$. Let $\Phi_I, \Lambda_I, \delta_I$ denote the equilibrium values of Φ, Λ, δ.

Proposition 1. *Suppose $\mathcal{A}$ holds φ shares. There exists an intervention equilibrium if and only if*

$$\varphi > \varphi_I \equiv \frac{1}{(1-\kappa)\Phi_I}\{\alpha\kappa(1-\Phi_I) + \eta\}, \tag{2}$$

where $\delta_I = \frac{\beta}{\omega_2}(\theta_1 + (1-\theta_1)\gamma)$, $\Phi_I = F(\delta_I)$, and $\Lambda_I = \Phi_I^{-1}E[\mathbb{1}_{\delta<\delta_I}\delta]$. Equi-

[13]All proofs are in the appendix.

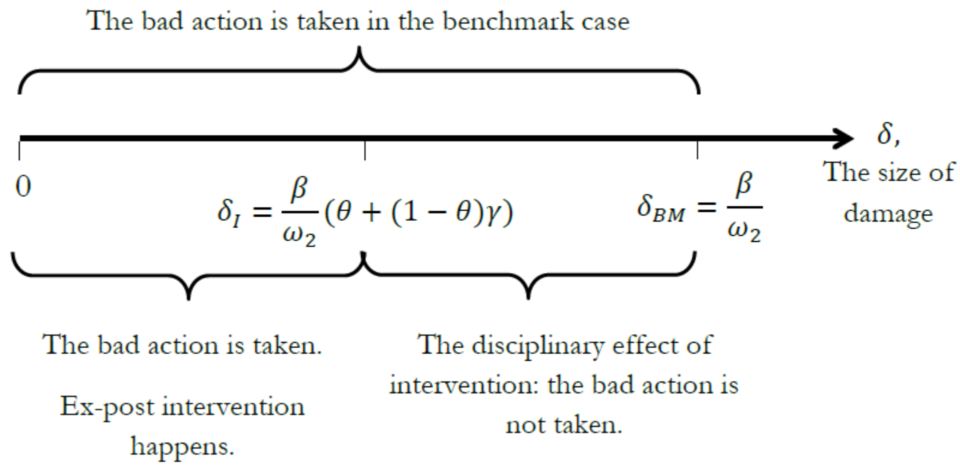

Figure 2: The disciplinary role of Intervention.

librium beliefs are $\left(\sigma_0^B = 1, \sigma_0^H = 0, \sigma_0^S = 0; \sigma_1^B = 1, \sigma_1^H = 0, \sigma_1^S = 0\right)$. *Equilibrium prices are* $p_B = \upsilon - \kappa\Lambda_I\Phi_I$ *and* $p_S = \upsilon - \Lambda_I\Phi_I$. *This equilibrium is always disciplinary,* $\delta_I < \delta_{BM}$.

The impact of the intervention on $\mathcal{M}$'s incentives is illustrated in Figure 2. When $\delta > \delta_{BM}$, $\mathcal{M}$ would not take the bad action even if $\mathcal{A}$ were not present. In this case the damage to firm value is so large that $\mathcal{M}$ prefers to forego the private benefit β. In the intermediate region $\delta_{BM} > \delta > \delta_I$, $\mathcal{A}$'s presence prevents $\mathcal{M}$ from taking the bad action. This is the disciplinary role of the intervention: the mere presence of $\mathcal{A}$ causes $\mathcal{M}$ to not take the bad action. Finally, when $\delta < \delta_I$, $\mathcal{M}$ takes the bad action and ex post intervention takes place. Only in this region will market participants observe incidents of intervention.

The disciplinary role of intervention is increasing in $\mathcal{A}$'s effectiveness in reducing $\mathcal{M}$'s private benefits of control, $(1-\gamma)$. A higher probability of the sell-side liquidity shock, θ_1, shifts δ_I toward δ_{BM} and therefore decreases the impact of intervention on $\mathcal{M}$'s incentives. Thus, a higher θ_1 leads to a less disciplinary equilibria and higher frequency of ex post interventions. It implies that period 1 liquidity generates a tension between ex ante and ex post efficiency of intervention. We discuss this tension in detail in Section 5.4.

The intervention equilibrium exists when $\mathcal{A}$'s initial toehold φ (which we endogenize in Section 3) is larger than φ_I. Equation (2) shows that φ_I depends on several parameters. φ_I is smaller when $(1-\kappa)$ is closer to one ($\mathcal{A}$ is effective in restoring the damage) and as Φ_I increases, that is, when $\mathcal{M}$ is more likely to take the bad action. Among other considerations, this happens when β is large (the agency problem is severe), $(1-\gamma)$ is small ($\mathcal{A}$ is less effective in reducing $\mathcal{M}$'s private benefits of control), and when $\mathcal{A}$ needs a small toehold to intervene, i.e., α is small.

The presence of liquidity trades enables $\mathcal{A}$ to a certain degree to hide his information. The equilibrium sell price, $p_S = \upsilon - \Lambda_I \Phi_I$, reflects equilibrium beliefs that $\mathcal{A}$ sells shares only if he experiences a liquidity shock. Specifically, if $\mathcal{A}$'s sale is caused by the liquidity shock, the probability that $\mathcal{M}$ took the bad action remains equal to its unconditional value Φ_I. In the absence of the liquidity shock, the sell price would be fully revealing and therefore lower $p_S(\theta_1 = 0) = \upsilon - \Lambda_I$. Thus, the presence of the liquidity shock increases the profits from selling the block from $\varphi(\upsilon - \Lambda_I)$ to $\varphi(\upsilon - \Lambda_I \Phi_I)$. The increase in profitability of selling the block due to the possibility of the liquidity shock therefore reduces the chances of the intervention equilibrium. Since $\mathcal{A}$ never voluntarily sells shares in the intervention equilibrium, the presence of liquidity trades is not reflected in his expected profits. Instead, it affects the chances that condition (2) holds.

Finally, note that in the intervention equilibrium $\mathcal{A}$ does not want to deviate from buying $\alpha - \varphi$ shares. If $a = 1$, $\mathcal{A}$ loses money on buying shares because $p_B > \upsilon - \kappa\Lambda_I$. Therefore, he will not buy more than necessary, namely, $(\alpha - \varphi)$. If $a = 0$, an activist $\mathcal{A}$ makes money on buying shares. However, if he buys more shares than $(\alpha - \varphi)$, he reveals that $a = 0$ and therefore drives profits to zero (see section 3 for further details on disclosure requirements).

Next we construct equilibria in which $\mathcal{A}$ does *not* intervene when $\mathcal{M}$ takes the bad action (i.e., $\sigma_1^B = 0$). Again, $\Phi_E, \Lambda_E, \delta_E$ represent values of the endogenous variables in the equilibrium.

Proposition 2. *Suppose $\mathcal{A}$ holds φ shares. There exists a non-disciplinary equilibrium with $\sigma_1^B = 0$ if and only if*

$$\varphi < \varphi_E \equiv (\alpha\kappa + \eta)\frac{\Phi_{BM} + \bar{\theta}_1}{\Phi_{BM} + \Phi_{BM}\bar{\theta}_1}, \tag{3}$$

where $\delta_{BM} = \beta/\omega_2$, $\Phi_{BM} = F(\delta_{BM})$, $\Lambda_{BM} = \Phi_{BM}^{-1}E[\mathbb{1}_{\delta<\delta_{BM}}\delta]$ and equilibrium prices are $p_B = \upsilon$ and $p_S = \upsilon - \Lambda_{BM}\Phi_{BM}\frac{1+\bar{\theta}_1}{\Phi_{BM}+\bar{\theta}_1}$. Equilibrium beliefs are $\left(\sigma_0^B \geq 0, \sigma_0^H \geq 0, \sigma_0^S = 0; \sigma_1^B = 0, \sigma_1^H = 0, \sigma_1^S = 1\right)$.

An equilibrium with no intervention ($\sigma_1^B = 0$) exists when the toehold φ is smaller than the threshold level φ_E. Equation (3) shows that φ_E depends on several parameters. φ_E is larger when $(1 - \kappa)$ is close to zero ($\mathcal{A}$ is not efficient in restoring the damage), when Φ_{BM} decreases, that is, when $\mathcal{M}$ is less likely to take the bad action, and when $\mathcal{A}$ needs a large position to intervene, i.e., α is large. Moreover, the existence of this equilibrium is positively affected by the likelihood of the liquidity shock θ_1 because when θ_1 increases, φ_E increases and condition (3) is more likely to hold. The equilibrium sell price, $p_S = \upsilon - \Lambda_{BM}\Phi_{BM}\frac{1+\bar{\theta}_1}{\Phi_{BM}+\bar{\theta}_1}$, reflects equilibrium beliefs that $\mathcal{A}$ will sell shares if he experiences a liquidity shock. In the absence of the liquidity shock, the sell price would be fully revealing and therefore lower $p_S(\theta_1 = 0) = \upsilon - \Lambda_{BM}$. Thus, the presence of the liquidity shock increases the profits from selling the block and therefore increases the chances of this equilibrium.[14]

We conclude this section by establishing conditions for period 1 equilibria in which $\mathcal{A}$ intervenes with a positive probability $0 < \sigma_1^B < 1$.

Proposition 3. *Suppose $\mathcal{A}$ holds φ shares and $\varphi_E < \varphi_I$. There is a unique mixed strategy disciplinary equilibrium if both conditions (2) and (3) are violated, i.e., $\varphi_E < \varphi < \varphi_I$. Equilibrium beliefs are $\left(\sigma_0^B = 1, \sigma_0^H = 0, \sigma_0^S = 0; \sigma_1^B > 0, \sigma_1^H = 0, \sigma_1^S > 1\right)$. In equilibrium, $p_B = \upsilon - \kappa\Lambda_M\Phi_M\frac{\sigma_1^B}{\Phi_M\sigma_1^B+(1-\Phi_M)}$ and $p_S = \upsilon - \Lambda_M\Phi_M\frac{(1-\sigma_1^B)+\bar{\theta}_1}{\Phi_M(1-\sigma_1^B)+\bar{\theta}_1}$, where $\delta_{BM} < \delta_M = (\beta/\omega_2)\left(1-(1-\theta_1)\sigma_1^B(1-\gamma)\right) < \delta_I$, $\Phi_M = F(\delta_M)$, and $\Lambda_M = \Phi^{-1}E[\mathbb{1}_{\delta<\delta_M}\delta]$.*

Note that when $\varphi_I < \varphi < \varphi_E$, multiple date 1 equilibria are possible (both conditions (2) and (3) are satisfied). As we are going to see in the next section, however, $\mathcal{A}$ will never purchase toehold φ with an intention to be

[14]Proposition 2 shows that an equilibrium with $\sigma_1^B = 0$ is non-disciplinary. Fos and Kahn (2016) analyze a similar model in which they allow $\mathcal{M}$'s compensation to depend on the realized market price of the firm in period 1 and show that in this case there can exist an equilibrium in which $\mathcal{A}$ sells when $\mathcal{M}$ takes the bad action and that this sell plays a disciplinary role.

in a non-disciplinary exit equilibrium. Thus, when (2) holds, we can restrict attention to the intervention equilibria described in Proposition 1, even when $\varphi_I < \varphi < \varphi_E$.

3. Initial toehold

We next analyze the formation of the initial toehold, φ. The initial toehold plays an important role in the model because it determines the type of period 1 equilibrium and therefore $\mathcal{A}$'s and $\mathcal{M}$'s actions. For the reasons noted in the previous paragraph we focus on the case when $\varphi_I < \varphi_E$; analysis of the case where $\varphi_I > \varphi_E$ is available in the Internet Appendix.

First, we describe disclosure requirements. After the initial trade in period 0 occurs, $\mathcal{A}$'s toehold φ becomes common knowledge. This assumption is motivated by the fact that market participants can use Schedule 13F filings to infer changes in stock ownership. If $\mathcal{A}$ purchases a toehold smaller than α, no additional disclosure of the position or trade is necessary until period 0. In this case, $\mathcal{A}$ can potentially benefit from hiding behind liquidity trades, which we introduce in the next paragraph. To capture the role of ownership disclosure requirements that are linked to $\mathcal{A}$'s position size—The Hart–Scott–Rodino Act disclosure requirement (HSR)—we assume that if $\mathcal{A}$ intents to purchase toehold larger than α, market participants become immediately aware of trader's identity and intention and set prices equal to p_1^I. As we're going to see, this assumption implies that $\mathcal{A}$ will not purchase more than α shares in period 0.

Second, we characterize the source of liquidity trading during toehold-accumulation period. Participants in the market expect that with probability

θ_0 the purchase order comes from an uniformed activist, and with probability $(1-\theta_0)$ the purchase order comes from an informed $\mathcal{A}$. Similarly to the role of θ_1 in period 1 markets, higher values θ_0 correspond to a more liquid period 0 market; $\mathcal{A}$'s purchase can be hidden more effectively in the sea of non-monitor purchases.

Next, we describe the relation between $\mathcal{A}$'s toehold size and firm value. If $\mathcal{A}$ is not present, the value of the firm is $p_{BM} = \upsilon - \Lambda_{BM}\Phi_{BM}$. If $\mathcal{A}$ is present and purchases $\varphi < \varphi_I$, Proposition 2 applies. $\mathcal{A}$ always sells shares if the bad action is taken. Since the equilibrium is non-disciplinary and there is no ex post intervention, the value of the firm is still p_{BM}. If $\mathcal{A}$ is present and purchases $\varphi \geq \varphi_I$ which is sufficient to maintain the intervention equilibrium, Proposition 1 applies. The value of the firm is $p_I = (1-\theta_1)p_B + \theta_1 p_S$, where $p_B = \upsilon - \kappa\Lambda_I\Phi_I$ and $p_S = \upsilon - \Lambda_I\Phi_I$.

The actual price on the market in period 0, denoted $p_0(\varphi)$, depends on the size of the purchase order and reflects the market's expectation that $\mathcal{A}$ is participating as well as the activism role played by $\mathcal{A}$ if he is present. If market makers receive an order to purchase $\varphi < \varphi_I$ shares, they set period 0 price to $p_0(\varphi < \varphi_I) = p_{BM}$. If market makers receive an order to purchase $\varphi \geq \alpha$ shares, they set period 0 price to $p_0(\varphi \geq \alpha) = p_I$ (i.e., the HSR disclosure rule is binding). In this two case prices are fully revealing and $\mathcal{A}$ cannot profit from trading. If market makers receive an order to purchase $\varphi_I \leq \varphi < \alpha$ shares, they set period 0 price to $p_0(\varphi_I \leq \varphi < \alpha) = (1-\theta_0)p_I + \theta_0 p_{BM}$. In this case more liquid markets (i.e., higher θ_0) lead to higher trading profits of $\mathcal{A}$.

In period 0 $\mathcal{A}$ takes the market price function $p_0(\varphi)$ as given and decides how many shares φ to buy. Holding φ shares between periods 0 and 1 involves private cost $C(\varphi) = \phi\frac{\varphi^2}{2}$ for $\mathcal{A}$. For example, this cost could correspond to lower diversification of $\mathcal{A}$'s portfolio or binding capital constraints faced by $\mathcal{A}$. This assumption implies that keeping everything else constant, $\mathcal{A}$ prefers to purchase shares later rather than earlier.[15]

$\mathcal{A}$ maximizes expected profits from purchasing φ shares:

$$\max_{\varphi_I \leq \varphi < \alpha} \pi(\varphi; p_I) = \varphi\theta_0(p_I - p_{BM}) - \phi\frac{\varphi^2}{2}. \tag{4}$$

Note that when $\varphi < \varphi_I$ and when $\varphi \geq \alpha$, prices are fully revealing and $\mathcal{A}$'s expected profit is $-\phi\frac{\varphi^2}{2} < 0$. Consequently, $\mathcal{A}$ will prefer $\varphi = 0$ to purchasing fewer than φ_I shares or more than α shares. Note that as long as $\theta_0 = 0$, $\mathcal{A}$ prefers $\varphi = 0$. In other words, if $\mathcal{A}$ needs to purchase φ shares in the open market at a price that reflects $\mathcal{A}$'s impact of firm value, privately-optimal initial stake size will be zero in the absence of liquidity trading. When $\theta_0 > 0$, then, since $p_I > p_{BM}$, $\mathcal{A}$ profits from liquidity trading because prices do not fully reflect $\mathcal{A}$'s impact of firm value. The following propositions characterize $\mathcal{A}$'s optimal φ.

Proposition 4. *Let* $\varphi_A^I = \frac{\theta_0(p_I - p_{BM})}{\phi}$ *be the value of* φ *that maximizes* $\pi(\varphi; p_I)$ *and* φ_0^I *be such that* $\pi(\varphi_0^I; p_I) = 0$.

i. If $\varphi_I \leq \varphi_0^I$, $\mathcal{A}$ *will choose* $\varphi = \max\left(\varphi_A^I, \varphi_I\right)$. *In this case condition (2) holds and the market is in disciplinary intervention equilibrium.*

[15]The analysis can be extended to consider the possibility that higher φ increases the likelihood that $\mathcal{A}$ faces a liquidity shock in period 1.

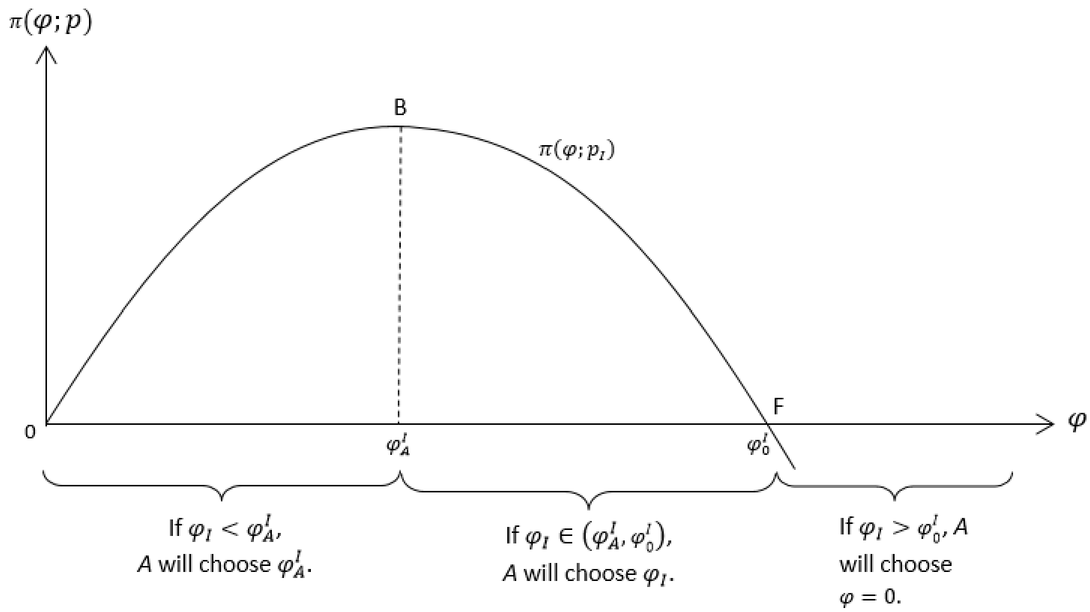

Figure 3: $\mathcal{A}$'s choice of φ.

ii. If $\varphi_I > \varphi_0^I$, $\mathcal{A}$ will choose $\varphi = 0$.

The intuition behind Proposition 4 is presented in Figure 3. When $\varphi_I < \varphi_A^I$, $\mathcal{A}$ finds it optimal to choose $\varphi = \varphi_A^I$. This is because φ_A^I is sufficient to maintain the intervention equilibrium and the corresponding price level, p_I. This case corresponds to the point B on the Figure. When $\varphi_I \in (\varphi_A^I, \varphi_0^I)$, φ_A^I is not large enough to satisfy condition (2) and therefore maintain the intervention equilibrium and the corresponding price level, p_I. In this case, $\mathcal{A}$ finds it optimal to choose $\varphi = \varphi_I$ such that condition (2) holds. This case corresponds to a point on the B-D segment. When $\varphi_I > \varphi_0^I$, $\mathcal{A}$ will choose $\varphi = 0$.

4. Extension

In the base model, exit does not play a disciplinary role, and the intervention equilibrium exists when the initial toehold is large. In other words, when $\mathcal{A}$'s toehold is large, $\mathcal{M}$ is worried about $\mathcal{A}$'s intervention and not about $\mathcal{A}$'s exit. In this section we examine whether including a disciplinary role of exit affects this result.

As in Admati and Pfleiderer (2009), for an exit equilibrium to be disciplinary, it is necessary that a portion of $\mathcal{M}$'s compensation depend on short-term price performance. Specifically, we will assume $\mathcal{M}$'s compensation takes the form $\omega_1 P_1 + \omega_2(\upsilon - a\tilde{\delta})$, where ω_1 and ω_2 are positive coefficients representing the dependence of the compensation on the firm's short-term ("Period 1") and long-term ("Period 2") price performance, respectively.[16] When ω_1 can be positive, the result of proposition 2 are modified as follows. Let $\Phi_E, \Lambda_E, \delta_E$ denote the equilibrium values of Φ, Λ, δ.

Proposition 5. *There exists a disciplinary equilibrium with $\sigma_1^B = 0$ if and only if*

$$\varphi < \varphi_E^* \equiv (\alpha\kappa + \eta)\,\frac{\Phi_E + \bar{\theta}_1}{\Phi_E + \bar{\theta}_1\Phi_E}, \tag{5}$$

where $\delta_E = \frac{\beta}{\omega_2} - (1-\theta_1)\frac{\omega_1}{\omega_2}(p_B - p_S)$, $\Phi_E = F(\delta_E)$, $\Lambda_E = \Phi_E^{-1}E[\mathbb{1}_{\delta<\delta_E}\delta]$ and equilibrium prices are $p_B = \upsilon$ and $p_S = \upsilon - \Lambda_E\Phi_E\frac{1+\bar{\theta}_1}{\Phi_E+\bar{\theta}_1}$. Equilibrium beliefs are $\left(\sigma_0^B \geq 0, \sigma_0^H \geq 0, \sigma_0^S = 0; \sigma_1^B = 0, \sigma_1^H = 0, \sigma_1^S = 1\right)$.

The impact of exit on $\mathcal{M}$'s incentives is summarized in Figure 4. When

[16] $\mathcal{M}$'s sensitivity to short-term prices is taken as exogenous in this paper. It can be motivated, for example, by takeover threats and concern for managerial reputation (Edmans, 2009).

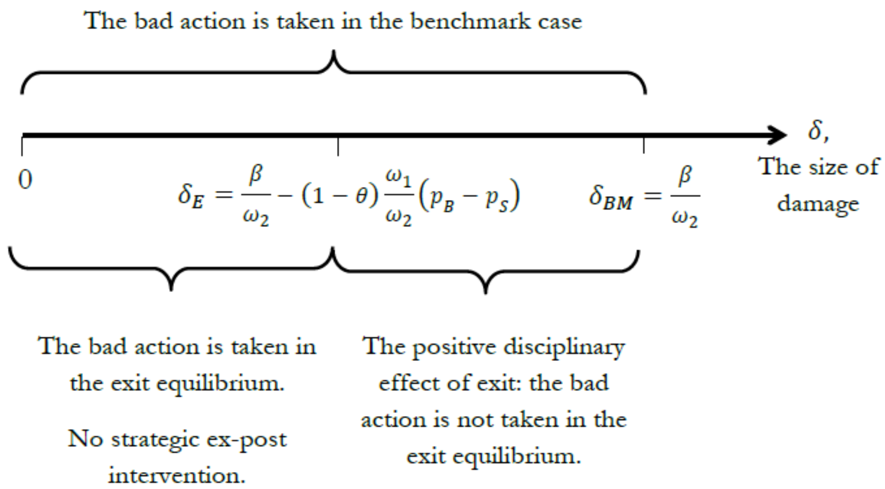

Figure 4: The disciplinary role of Exit.

$\delta > \delta_{BM}$, $\mathcal{M}$ does not take the bad action even when $\mathcal{A}$ is not present. In this case the damage to firm value is so large that $\mathcal{M}$ prefers to forego the private benefit β. In the intermediate region $\delta_{BM} > \delta > \delta_E$, $\mathcal{A}$'s presence prevents $\mathcal{M}$ from taking the bad action. This is the disciplinary role of exit. Finally, when $\delta < \delta_E$, $\mathcal{M}$ takes the bad action and $\mathcal{A}$ sells his stake.

Several parameters affect the disciplinary role of this equilibrium. $\mathcal{M}$ is less likely to take the bad action when ω_1/ω_2 is large ($\mathcal{M}$'s compensation is more dependent on period 1 prices), when β/ω_2 is small (the agency problem is not severe), and when the distribution of δ shifts right. Interestingly, the number of shares owned by $\mathcal{A}$ does not affect the disciplinary role of the exit and intervention equilibria. The chances of a liquidity shock also affect the disciplinary role of the exit equilibrium. Liquidity shocks have a positive impact on the probability that $\mathcal{M}$ takes the bad action because they make prices less informative.

The equilibrium is more likely to exist when $(1-\kappa)$ is close to zero ($\mathcal{A}$ is not efficient in restoring the damage) and when Φ_E decreases, that is, when $\mathcal{M}$ is less likely to take the bad action. The existence of the exit equilibrium is positively affected by ω_1 because when ω_1 increases, $\mathcal{M}$ is less likely to take the bad action. Thus, when $\mathcal{M}$ is more sensitive to the period 1 prices, the exit equilibrium is more likely to exist. Recall that the existence of the intervention equilibrium does not depend on ω_1. Note that the equilibrium is more likely to exist when $\mathcal{A}$ needs a large toehold to intervene, i.e., α is large.

The effect of θ_1 on the existence of the exit equilibrium could be either positive or negative. On one side, the existence of the exit equilibrium is positively affected by θ_1 because when θ_1 increases, condition (5) is more likely to hold for a given level of the probability of bad action, Φ_E. On the other side, the existence of the exit equilibrium is negatively affected by θ because when θ_1 increases, $\mathcal{M}$ is more likely to take the bad action (i.e., Φ_E increases) and therefore condition (5) is less likely to hold.

The equilibrium sell price, $p_S = \upsilon - \Lambda_E \Phi_E \frac{1+\bar{\theta}_1}{\Phi_E+\bar{\theta}_1}$, reflects equilibrium beliefs that $\mathcal{A}$ may sell shares if he experiences a liquidity shock. In the absence of the liquidity shock, the sell price would be fully revealing and therefore lower $p_S(\theta_1 = 0) = \upsilon - \Lambda_I$. Thus, the presence of the liquidity shock increases the profits from selling the block and therefore increases the chances of the exit equilibrium.

To summarize, the analysis in this section shows that when the exit mechanism plays a disciplinary role as in Admati and Pfleiderer (2009), the exit equilibrium exists for a larger range of initial toeholds. The main conclusion—the threat of intervention is more likely to discipline $\mathcal{M}$ when the size of initial toehold is large—still holds.

5. Discussion and Empirical Implications

5.1. Empirical Implications

Our model has important implications for the empirical corporate governance literature that studies consequences of shareholder activism. Suppose we are interested in the relation between shareholder activism and corporate policies (e.g., investments, capital structure, payout). Let Activism_i be an indicator variable for firms that experience activism and y_i be firm i's choice of a corporate policy. For example, consider firm's investment decisions. If $\mathcal{M}$ firm i takes into consideration the threat of intervention when determining the policy, the right empirical model to study this policy is as follows:

$$y_i = \alpha_0 + \beta_0 \cdot \text{Activism}_i + \gamma_0 \cdot \text{Activism}_i^* + \varepsilon_i, \tag{6}$$

where Activism_i^* is the ex ante probability of intervention. Most of the empirical literature on shareholder activism, however, ignores the disciplinary effects of intervention and uses the following regression model:

$$y_i = \alpha_1 + \beta_1 \cdot \text{Activism}_i + u_i. \tag{7}$$

Our theoretical model implies that the OLS estimate of β_1 in regression (7) is biased even if Activism_i is randomly assigned. This is the case because regression residual $u_i = \text{Activism}_i^* + \varepsilon_i$ is correlated with Activism_i.[17] More importantly, the analysis in regression (7) is also biased economically because it ignores the disciplinary role of shareholder activism.

5.2. Toehold Size and Equilibrium Form of Governance

In our model, endogenously determined initial toehold determines the equilibrium form of governance. If the initial toehold is small, $\mathcal{A}$ will exit if $\mathcal{M}$ takes the bad action (Proposition 2). This equilibrium exists for a larger range of initial toeholds when exit plays a disciplinary role (Proposition 5). In contrast, when the initial toehold is large, $\mathcal{A}$ will intervene if $\mathcal{M}$ takes the bad action (Proposition 1).

Importantly, our model implies that when toeholds are large, the equilibrium form of governance is intervention. When $\mathcal{M}$ decides on taking the bad action, he is concerned about the $\mathcal{A}$'s intervention and not about $\mathcal{A}$ walking away. Thus, our model suggests that the threat of intervention disciplines $\mathcal{M}$ when the toehold is large.

5.3. Stock liquidity and economic efficiency

In this section we analyze the relation between the endogenously determined prices and stock liquidity—as measured by bid-ask spread—in the disciplinary intervention equilibrium. We start with period 1 prices and liquidity. In the intervention equilibrium, the period 1 bid-ask spread is $p_B - p_S = (1-\kappa)\Lambda_I\Phi_I$ and the expected price $(1-\theta_1)p_B + \theta_1 p_S = v - (\theta_1 + (1-\theta_1)\kappa)\Lambda_I\Phi_I$.

[17] Heckman (1978) studies econometric models in which the expectation of the treatment affects the outcome.

Corollary 1. *Consider period 1 prices and liquidity in the intervention equilibrium.*

i Higher likelihood of liquidity shocks leads to wider bid-ask spreads and lower firm value, implying a positive correlation between these two endogenously determined values.

ii An increase in $\mathcal{A}$'s effectiveness in restoring the damage leads to wider bid-ask spreads and higher firm value, implying a negative correlation between these two endogenously determined values.

Figure 5 plots equilibrium prices as function of the liquidity shock parameter, θ_1. We see that both bid and ask prices, as well as expected price, decrease when $\mathcal{A}$ is more likely to experience a liquidity shock. This is because θ_1 reduces the disciplinary role of the intervention and therefore increases expected damage to firm value. The ask price, however, is less affected by θ_1 than the bid price because if $\mathcal{A}$ purchases shares of damaged firm, he intervenes and restores part of the damage. Consequently, the bid-ask spread is wider when $\mathcal{A}$ is more likely to experience a liquidity shock. Thus, higher likelihood of liquidity shocks leads to lower measured stock liquidity (wider bid-ask spread) and lower firm value.

The bid-ask spread is positive as long as $\mathcal{A}$ is effective in restoring the damage, $(1-\kappa) > 0$. Only $\mathcal{A}$ knows if there is damage to be restored; thus, $\mathcal{A}$'s activism skill is a source of information asymmetry, even in the absence of liquidity shocks.[18] (As Figure 5 shows, this component of bid-ask spread is positive even when $\theta_1 = 0$.)

[18] See also Back et al. (2016).

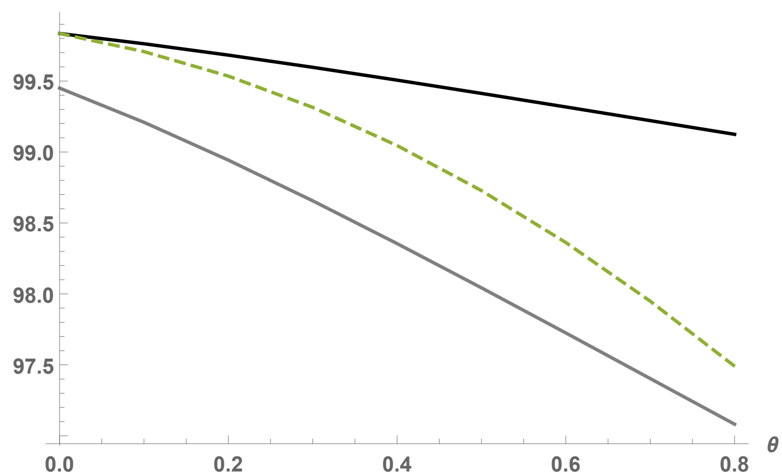

Figure 5: **The effect of θ_1 on prices in the Intervention equilibrium.** The black line plots period 1 buy price, $p_B = \upsilon - \kappa\Lambda_I\Phi_I$. The grey line plots period 1 sell price, $p_S = \upsilon - \Lambda_I\Phi_I$. The dashed line plots the expected period 1 price, $p_1^I = (1-\theta_1)p_B + \theta_1 p_S$. δ_I, Λ_I, and Φ_I are defined in Proposition 1. We assume υ=100, β=25, ω_2=2, γ=0.3; κ=0.3, φ=4, α=5, η=0.1, $f[x] = \lambda\exp(-\lambda x)$, and λ=0.1.

To understand how $\mathcal{A}$'s effectiveness in restoring the damage affects equilibrium outcomes, note that it does not affect the price p_S that $\mathcal{A}$ receives when he sells shares; nor does it affect the disciplinary role of the intervention equilibrium (i.e., δ_I does not depend on κ). It does, however, have a positive impact on the price p_B paid for additional shares, because a more effective activist recovers a larger fraction of the damage. Thus, an increase in $\mathcal{A}$'s effectiveness in restoring the damage leads to lower measured stock liquidity (wider bid-ask spread) and higher firm value.

An important empirical implication of Corollary 1 is that the relation between firm value and stock liquidity depends on the nature of variation in these variables. If changes in liquidity are caused by changes in the likelihood of liquidity shocks, an improvement in stock liquidity will lead to an increase in firm value. In contrast, if changes in liquidity are caused by changes in $\mathcal{A}$'s effectiveness in restoring the damage, an improvement in liquidity will lead to a decrease in firm value. Thus, the causal effect of liquidity on firm value depends on the source of variation in liquidity.

So far, we discussed the role of liquidity parameters that affect period 1 equilibrium. Next, we consider the role of stock liquidity in block formation. In period 0, market participants expect that with probability $(1-\theta_0)$, an order to purchase φ shares will be submitted by $\mathcal{A}$, and with probability θ_0 the order will be submitted by an uninformed agent who will not engage in corporate governance. For example, θ_0 could correspond to the probability that $\mathcal{A}$ experiences a liquidity shock and therefore cannot engage in shareholder activism.

Proposition 7 shows that if period 0 market is completely illiquid ($\theta_0 =$

0), $\mathcal{A}$ will choose $\varphi = 0$ and the disciplinary intervention equilibrium will not exist. Higher θ_0 has a positive impact on period 0 liquidity because it reduces price impact of $\mathcal{A}$'s trade and therefore allows $\mathcal{A}$ to profit from trading. Therefore, higher θ_0 increases chances that $\mathcal{A}$ will accumulate a toehold sufficient to cover costs of holding φ shares as well as expected costs of activism, $\eta\Lambda$. Thus, the model predicts a positive relation between initial period stock liquidity and block formation by activist shareholders.

Corollary 2. *Consider period 0 prices and liquidity in the intervention equilibrium. More liquid period 0 markets lead to a higher initial toehold and therefore increase chances of the disciplinary intervention equilibrium.*

Corollaries 1 and 2 suggest that the timing of a liquidity shock plays an important role. Whereas higher chances of period 0 liquidity shocks make stock markets more liquid and increase chances of the disciplinary intervention equilibrium, higher chances of period 1 liquidity shocks make stock markets less liquid and reduce the disciplinary role of the intervention.

5.4. Ex ante and ex post correction effects

The model emphasizes an important feature of the intervention equilibrium: Intervention has ex ante and ex post correction effects. In this section we discuss how $\mathcal{A}$'s liquidity needs affect $\mathcal{A}$'s and $\mathcal{M}$'s actions and create a tension between ex ante and ex post correction effects.

When $\mathcal{A}$'s liquidity needs increase (measured by θ_1), δ_I decreases and $\mathcal{M}$ is more likely to take the bad action. As Figure 5 indicates, higher θ_1 leads to lower firm value. In the intervention equilibrium, however, $\mathcal{A}$ always intervenes if $\mathcal{M}$ takes the bad action and there is no liquidity shock.

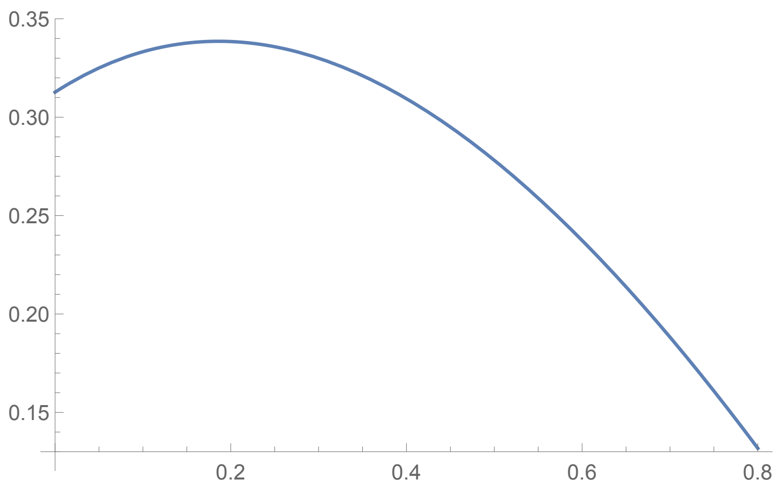

Figure 6: **The effect of θ_1 on the probability of intervention.** The dark line plots the probability of intervention specified in equation (8). δ_I, Λ_I, and Φ_I are defined in Proposition 1. We assume υ=100, β=25, ω_2=2, γ=0.3; κ=0.3, φ=4, α=5, η=0.1, $f[x] = \lambda \exp(-\lambda x)$, and λ=0.1.

Thus, the overall effect of θ_1 on the probability of ex post intervention can be either positive or negative. Specifically, the probability of intervention $Pr(b = 1) = (1 - \theta_1)\Phi_I$ and its derivative with respect to θ_1 is:

$$\frac{\partial Pr(b = 1)}{\partial \theta_1} = -\Phi_I + f_I(1 - \theta_1)(1 - \gamma)\frac{\beta}{\omega_2}. \tag{8}$$

Figure 6 show that the effect of θ_1 of the probability of intervention can be non-monotonic. When θ_1 is small, an increase in θ_1 weakens the ex ante correction effect (lower δ_I) but strengthens the ex post correction effect (more frequent ex post interventions). That is, ex ante and ex post correction effects of intervention are substitutes. In contrast, when θ_1 is large, an increase in θ_1 weakens both the ex ante and ex post correction effects. Thus, ex ante and ex post correction effects of intervention are complements.

When ex ante and ex post correction effects are substitutes, considering the *observable* aspect of the intervention mechanism (i.e. ex post interventions) is not sufficient to reach conclusions about the state of economic efficiency. Starting from a high level of liquidity needs for $\mathcal{A}$, reducing those needs leads to more frequent ex post interventions and enhanced economic efficiency. However, as $\mathcal{A}$'s liquidity needs become sufficiently small, ex post and ex ante correction effects become offsetting, and observed interventions no longer predict increased efficiency.

6. Conclusion

In this paper we have developed a model to study two governance roles played by shareholder interventions: the disciplinary role of intervention and the ex post correction role played by interventions. We have derived predictions as to when one or the other is more likely to be available and more likely to be effective in disciplining the manager.

The model suggests that enhancing the disciplinary role of intervention (i.e., ex ante correction effects) can be more effective than enhancing the ex post correction effects in improving economic efficiency. For instance, the ex ante correction effect reduces the manager's incentive to damage firm value. In contrast, ex post intervention not only recovers only a fraction of the damage, but also requires the activist to bear the cost of intervention.

The model reveals what economic factors can help achieving this goal. For example, increasing personal cost borne by the manager in case of ex post intervention would enhance the disciplinary role of intervention. Similarly, reducing chances of the liquidity shock that forces the activist to sell her toehold would enhance the disciplinary role of intervention.

The model also reveals that ex ante and ex post correction effects can be either complements or substitutes, depending on economic forces that drive variations in these effects. For instance, if the activist is more effective in punishing the manager, these two mechanisms will exhibit substitution: the threat of intervention will be stronger and ex post interventions less frequent. In contrast, if the activist is less likely to experience a liquidity shock that forces him to liquidate her position, ex ante and ex post correction effects can be complements. The ex ante correction effect is stronger because the activist is less likely to be disturbed by the liquidity shock. The ex post correction effect can also stronger because the frequency of ex post intervention can increase following a decrease in the likelihood of liquidity shock.

Finally, because we endogenize the activist's choice of toehold, we can show that the timing of liquidity trading matters. Whereas liquidity trading during the toehold-acquiring phase generally enhances economic efficiency, liquidity shocks experienced during the activism phase of the blockholder's activity generally lead to worsening of economic efficiency.

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Appendix for "The Threat of Intervention"

Appendix A. Proofs

Proof of Lemma 1.

$\mathcal{A}$'s profits, as measured by $\pi - \varphi \upsilon$, are as follows:

	$a = 0$ (no damage)	$a = 1$ (damage)
Buy	$(\alpha-\varphi)\kappa\Lambda\{\frac{\Phi\sigma_1^B}{\Phi\sigma_1^B+(1-\Phi)\sigma_0^B}, 0\}$	$-\alpha\kappa\Lambda - \eta\Lambda + (\alpha-\varphi)\kappa\Lambda\{\frac{\Phi\sigma_1^B}{\Phi\sigma_1^B+(1-\Phi)\sigma_0^B}, 0\}$
Hold	0	$-\varphi\Lambda$
Sell	$-\varphi\Lambda\frac{\Phi\sigma_1^S+\bar{\theta}_1\Phi}{\Phi\sigma_1^S+(1-\Phi)\sigma_0^S+\bar{\theta}_1}$	$-\varphi\Lambda\frac{\Phi\sigma_1^S+\bar{\theta}_1\Phi}{\Phi\sigma_1^S+(1-\Phi)\sigma_0^S+\bar{\theta}_1}$

where in the case of each bracketed expression, the second term is to be used in case the denominator of the first term is zero.

Since $\Phi > 0$, $\pi_0^B - \varphi\upsilon > \pi_0^S - \varphi\upsilon$ implies $\sigma_0^S = 0$. Similarly, $\pi_1^S - \varphi\upsilon > \pi_1^H - \varphi\upsilon$ implies $\sigma_1^H = 0$ if $\Phi < 1$. A sufficient condition for $\Phi < 1$ is that the support of the distribution include sufficiently high values of δ such that the manager is uninterested in taking the action. For example, it is sufficient to assume $F(\beta/\omega_2) < 1$.

Proof of Proposition 1.

In the case when $\sigma_1^B > 0$, $\sigma_0^B = 1$, $\sigma_0^H = 0$, $\sigma_0^S = 0$, and $\sigma_1^H = 0$, $\mathcal{A}$'s profits can be simplified as follows:

	$a = 0$ (no damage)	$a = 1$ (damage)
Buy	$(\alpha-\varphi)\kappa\Lambda\frac{\Phi\sigma_1^B}{\Phi\sigma_1^B+(1-\Phi)}$	$-\alpha\kappa\Lambda - \eta\Lambda + (\alpha-\varphi)\kappa\Lambda\frac{\Phi\sigma_1^B}{\Phi\sigma_1^B+(1-\Phi)}$
Hold	0	$-\varphi\Lambda$
Sell	$-\varphi\Lambda\frac{\Phi\sigma_1^S+\bar{\theta}_1\Phi}{\Phi\sigma_1^S+\bar{\theta}_1}$	$-\varphi\Lambda\frac{\Phi\sigma_1^S+\bar{\theta}_1\Phi}{\Phi\sigma_1^S+\bar{\theta}_1}$

Condition 2 follows from comparing π_1^B and π_1^S when $\sigma_1^B = 1$.

Given the beliefs, $\mathcal{M}$ expects $\mathcal{A}$ to intervene as long as the action is taken and there is no liquidity shock. In equilibrium $\mathcal{M}$ consumes private benefits $\beta(\theta_1+(1-\theta_1)\gamma)$ if $a = 1$. If $\mathcal{M}$ does not take the action, his expected utility is $\omega_2 \upsilon$. If $\mathcal{M}$ takes the action, his expected utility is $\omega_2(\upsilon-\widetilde{\delta})+\beta(\theta_1+(1-\theta_1)\gamma)$. The cutoff point is therefore $\delta_I = \beta/\omega_2\,(\theta_1 + (1-\theta_1)\gamma)$. $\gamma < 1$ implies that the equilibrium is always disciplinary.

Proof of Proposition 2.

If $\sigma_1^B = 0$, $\sigma_1^S = 1$ and $\mathcal{A}$'s profits can be simplified as follows:

	$a = 0$ (no damage)	$a = 1$ (damage)
Buy	0	$-\alpha\kappa\Lambda - \eta\Lambda$
Hold	0	$-\varphi\Lambda$
Sell	$-\varphi\Lambda\frac{\Phi+\bar{\theta}_1\Phi}{\Phi+\bar{\theta}_1}$	$-\varphi\Lambda\frac{\Phi+\bar{\theta}_1\Phi}{\Phi+\bar{\theta}_1}$

Condition 3 follows from comparing π_1^B and π_1^S.

If $\mathcal{M}$ does not take the action, his expected utility is $\omega_2 \upsilon$. If $\mathcal{M}$ takes the action, his expected utility is $\omega_2(\upsilon - \widetilde{\delta}) + \beta$. The cutoff point is therefore $\delta_E = \frac{\beta}{\omega_2} = \delta_{BM}$ and the equilibrium is non-disciplinary. Period 1 prices are $p_B = \upsilon$ and $p_S = \upsilon - \Lambda\Phi\frac{1+\bar{\theta}_1}{\Phi+\bar{\theta}_1}$.

Proof of Proposition 3.

We want to construct an equilibrium with $\sigma_1^B > 0$ and $\sigma_1^S > 0$. When $\sigma_1^B > 0$, $\sigma_0^B = 1$. Consider $G(\sigma_1^B) \equiv \pi_1^B - \pi_1^S$ when $\sigma_1^B \in (0, 1)$:

$$G(\sigma_1^B) = -\alpha\kappa\Lambda - \eta\Lambda + (\alpha-\varphi)\kappa\Phi\Lambda\frac{\sigma_1^B}{\Phi\sigma_1^B + (1-\Phi)} + \varphi\Phi\Lambda\frac{(1-\sigma_1^B)+\bar{\theta}_1}{\Phi(1-\sigma_1^B)+\bar{\theta}_1}.$$

For there to be an equilibrium with $\sigma_1^B \in (0,1)$ for a given Φ, it is necessary and sufficient that $G(\sigma_1^B) = 0$ at some $\sigma_1^B > 0$. By conjecture, $\lim_{\sigma_1^B \to 0} > 0$ and $\lim_{\sigma_1^B \to 1} < 0$. Since the function is quadratic there is exactly one crossing $\in (0,1)$.

The values of p_B and p_S are immediate from their definitions. Given the beliefs, $\mathcal{M}$ expects $P_1(a=0) = (1-\theta_1)p_B + \theta_1 p_S$ and $P_1(a=1) = (1-\theta_1)\left[\sigma_1^B p_B + (1-\sigma_1^B)p_S\right] + \theta_1 p_S$. Moreover, in equilibrium $\mathcal{M}$ is expected to consume private benefits $\beta\left[1-(1-\theta_1)\sigma_1^B(1-\gamma)\right]$. Thus, if $\mathcal{M}$ does not take the action, his expected utility is $\omega_2 \upsilon$. If $\mathcal{M}$ takes the action, his expected utility is $\omega_2(\upsilon - \widetilde{\delta}) + \beta\left[1-(1-\theta_1)\sigma_1^B(1-\gamma)\right]$. The cutoff point is therefore $\delta_M = (\beta/\omega_2)\left(1-(1-\theta_1)\sigma_1^B(1-\gamma)\right)$.

Proof of Proposition 2.

If $\sigma_1^B = 0$, $\sigma_1^S = 1$ and $\mathcal{A}$'s profits can be simplified as follows:

	$a=0$ (no damage)	$a=1$ (damage)
Buy	0	$-\alpha\kappa\Lambda - \eta\Lambda$
Hold	0	$-\varphi\Lambda$
Sell	$-\varphi\Lambda\frac{\Phi+\bar{\theta}_1\Phi}{\Phi+\theta_1}$	$-\varphi\Lambda\frac{\Phi+\bar{\theta}_1\Phi}{\Phi+\theta_1}$

Condition 5 follows from comparing π_1^B and π_1^S.

Given the beliefs, $\mathcal{M}$ expects $P_1(a=0) = (1-\theta_1)p_B + \theta_1 p_S$ and $P_1(a=1) = p_S$. Note that we assumed that stock price is p_B is $\mathcal{M}$ does not take the action and $\mathcal{A}$ holds φ shares. Moreover, in equilibrium $\mathcal{M}$ consumes private benefits β if $a=1$. Thus, if $\mathcal{M}$ does not take the action, his expected utility is $\omega_1\left(p_B - \theta_1(p_B - p_S)\right) + \omega_2\upsilon$. If $\mathcal{M}$ takes the action, his expected utility is $\omega_1 p_S + \omega_2(\upsilon - \widetilde{\delta}) + \beta$. The cutoff point is therefore $\delta_E = \frac{\beta}{\omega_2} - (1-\theta_1)\frac{\omega_1}{\omega_2}(p_B - p_S)$, where $p_B - p_S = \Lambda\Phi\frac{1+\bar{\theta_1}}{\Phi+\bar{\theta_1}}$.

Appendix B. Alternative Equilibria

Multiple equilibria are also possible. One possibility is that both condition (2) and condition (3) are satisfied for their respective values of Φ, in which case we will have one pure strategy equilibrium of each sort. The next proposition provides sufficient conditions for this to occur.

Lemma 2. *If the following conditions are satisfied:*

$$\begin{aligned}\varphi &> \frac{1}{(1-\kappa)\Phi_E}(\alpha\kappa(1-\Phi_E)+\eta)\\ \varphi &< (\alpha\kappa+\eta),\end{aligned}$$

then there exists $\theta_1^* < 1$ such that for all θ_1 in the open interval $(\theta_1^*, 1)$ both an intervention equilibrium and a non-disciplinary equilibrium exist.

Proof. As θ_1 approaches 1, Φ_I approaches Φ_E so φ_I and φ_E in conditions (2) and condition (3) continuously approach the right sides of the two inequalities above. ■

This leads immediately to the following proposition:

Proposition 6. *Provided*

$$\alpha\kappa+\eta > \frac{1}{(1-\kappa)\Phi_E}(\alpha\kappa(1-\Phi_E)+\eta)$$

there is an open interval of values of φ such that for large enough values of θ_1 both an intervention equilibrium and a non-disciplinary equilibrium exist.

It is not difficult to find parameter values for ($\alpha, \kappa, \eta, \beta$ and ω_2) such that this condition holds. For example, as $\frac{\beta}{\omega_2}$ grows Φ_E approaches 1,and the condition simplifies to the maintained assumption:

$$(1-\kappa)\alpha > \eta.$$

Appendix C. Section 3: $\varphi_E < \varphi_I$ Case

In this section we consider the case when $\varphi_E < \varphi_I$ and mixed-strategy equilibrium is possible.

We begin by describing the relation between $\mathcal{A}$'s toehold size and firm value. If $\mathcal{A}$ is not present, the value of the firm is $p_{BM} = \upsilon - \Lambda_{BM}\Phi_{BM}$. If $\mathcal{A}$ is present and purchases $\varphi < \varphi_E$, Proposition 2 applies. $\mathcal{A}$ always sells shares if bad action is taken. Since the equilibrium is non-disciplinary and there is no ex post intervention, the value of the firm is still p_{BM}. If $\mathcal{A}$ is present and purchases $\varphi \in (\varphi_E, \varphi_I)$ which is sufficient to maintain the mixed-strategy equilibrium, Proposition 3 applies. $\mathcal{A}$ intervenes with positive probability if bad action is taken. In equilibrium, firm value is higher than in the benchmark case, $p_M > p_{BM}$. If $\mathcal{A}$ is present and purchases $\varphi \geq \varphi_I$ which is sufficient to maintain the intervention equilibrium, Proposition 1 applies. The value of the firm is $p_I = (1-\theta_1)p_B + \theta_1 p_S$, where $p_B = \upsilon - \kappa\Lambda_I\Phi_I$ and $p_S = \upsilon - \Lambda_I\Phi_I$. Note that it follows from Propositions 1 and 3 that the mixed strategy equilibrium is less disciplinary than the intervention equilibrium. Thus, $p_I > p_M > p_{BM}$, implying that period 0 firm value is weakly increase in $\mathcal{A}$'s toehold size.

Second, we describe disclosure requirements. After the initial trade in period 1 occurs, $\mathcal{A}$'s toehold φ becomes common knowledge. This assumption is motivated by the fact that market participants can use Schedule 13F filings to infer changes in stock ownership. If $\mathcal{A}$ purchases a toehold smaller than α, no additional disclosure of the position or trade is necessary until period 0. In this case, $\mathcal{A}$ can potentially benefit from hiding behind liquidity trades, which we introduce in the next paragraph. To capture the role of ownership disclosure requirements that are linked to $\mathcal{A}$'s position size—The Hart–Scott–Rodino Act disclosure requirement (HSR)—we assume that if $\mathcal{A}$ purchases toehold larger than α, market participants become immediately aware of trader's identity and intention and set prices equal to p_1^I. As we're going to see, this assumption implies that $\mathcal{A}$ will not purchase more than α shares in period 0.

Next, we characterize the source of liquidity trading during toehold-accumulation period. Participants in the market expect that with probability θ_0 the purchase order comes from an uniformed agent (either from a non-activist shareholder who cannot change firm value or from an uninformed activist), and with probability $(1 - \theta_0)$ the purchase order comes from an informed $\mathcal{A}$. Similarly to the role of θ in period 1 markets, higher values θ_0 correspond to a more liquid period 0 market; $\mathcal{A}$'s purchase can be hidden more effectively in the sea of non-monitor purchases.

The actual price on the market in period 0, denoted $p_0(\varphi)$, depends on the size of the purchase order and reflects the market's expectation that $\mathcal{A}$ is participating as well as the activism role played by $\mathcal{A}$ if he is present. If market makers receive an order to purchase $\varphi < \varphi_E$ shares, they set period 0 price to $p_0(\varphi < \varphi_E) = p_{BM}$. If market makers receive an order to purchase $\varphi \geq \alpha$ shares, they set period 0 price to $p_0(\varphi \geq \alpha) = p_I$ (i.e., the HSR disclosure rule is binding). In this two case prices are fully revealing and $\mathcal{A}$ cannot profit from trading.

If market makers receive an order to purchase $\varphi_E \leq \varphi < \varphi_I$ shares, they set period 0 price to $p_0(\varphi_E \leq \varphi < \varphi_I) = (1-\theta_0)p_M + \theta_0 p_{BM}$. If market makers receive an order to purchase $\varphi_I \leq \varphi < \alpha$ shares, they set period 0 price to $p_0(\varphi_I \leq \varphi < \alpha) = (1-\theta_0)p_I + \theta_0 p_{BM}$. In these two cases more liquid markets (i.e., higher θ_0) lead to higher trading profits of $\mathcal{A}$.

In period 0 $\mathcal{A}$ takes the market price function $p_0(\varphi)$ as given and decides how many shares φ to buy. Holding φ shares between periods 0 and 1 involves private cost $C(\varphi) = \phi\frac{\varphi^2}{2}$ for $\mathcal{A}$. For example, this cost could correspond to lower diversification of $\mathcal{A}$'s portfolio or binding capital constraints faced by $\mathcal{A}$. This assumption implies that keeping everything else constant, $\mathcal{A}$ prefers to purchase shares later rather than earlier.[19]

Let $\pi(\varphi; p) = \varphi\theta_0(p - p_{BM}) - \phi\frac{\varphi^2}{2}$ be $\mathcal{A}$'s expected profit given period 0 price p. $\mathcal{A}$ maximizes expected profits from purchasing φ shares:

$$\max_{\varphi_E \leq \varphi < \alpha} \pi(\varphi) = \begin{cases} \pi(\varphi; p_I), \text{ if } \varphi_E \leq \varphi < \varphi_I, \\ \pi(\varphi; p_M), \text{ if } \varphi_I \leq \varphi < \alpha. \end{cases} \tag{C.1}$$

Note that when $\varphi < \varphi_E$ and when $\varphi \geq \alpha$, prices are fully revealing and $\mathcal{A}$'s expected profit is $-\phi\frac{\varphi^2}{2} < 0$. Consequently, $\mathcal{A}$ will prefer $\varphi = 0$ to

[19]The analysis can be extended to consider the possibility that higher φ increases the likelihood that $\mathcal{A}$ faces a liquidity shock in period 1.

purchasing fewer than φ_E shares or more than α shares. Note that as long as $\theta_0 = 0$, $\mathcal{A}$ prefers $\varphi = 0$. In other words, if $\mathcal{A}$ needs to purchase φ shares in the open market at a price that reflects $\mathcal{A}$'s impact of firm value, privately-optimal initial stake size will be zero in the absence of liquidity trading. When $\theta_0 > 0$, then, provided $p_1(\varphi) \neq p_{BM}$, $\mathcal{A}$ profits from liquidity trading because prices do not fully reflect $\mathcal{A}$'s impact of firm value. The following propositions characterize $\mathcal{A}$'s optimal φ.

Proposition 7. *Let $\varphi_A^I = \frac{\theta_0(p_I - p_{BM})}{\phi}$ be the value of φ that maximizes $\pi(\varphi; p_I)$ and $\varphi_A^M = \frac{\theta_0(p_M - p_{BM})}{\phi}$ be the value of φ that maximizes $\pi(\varphi; p_M)$. Further, let φ_0^I be such that $\pi(\varphi_0^I; p_I) = 0$, φ_0^M be such that $\pi(\varphi_0^M; p_M) = 0$, and $\overline{\varphi_I}$ such that $\pi(\overline{\varphi_I}; p_I) = \pi(\varphi_A^M; p_M)$.*

i. If $\varphi_I \leq \overline{\varphi_I}$, $\mathcal{A}$ will choose $\varphi = \max\left(\varphi_A^I, \varphi_I\right)$. In this case condition (2) holds and the market is in disciplinary intervention equilibrium.

ii. If $\varphi_I \in (\overline{\varphi_I}, \varphi_0^I)$, $\mathcal{A}$ will choose either $\varphi = \varphi_I$ or $\varphi = \max\left(\varphi_A^M, \varphi_M\right)$:

ii(a) If $\pi(\varphi_I; p_I) \geq \pi(\max\left(\varphi_A^M, \varphi_M\right); p_M)$, $\mathcal{A}$ will choose $\varphi = \varphi_I$. Condition (2) holds and the market is in disciplinary intervention equilibrium.

ii(b) If $\pi(\varphi_I; p_I) < \pi(\max\left(\varphi_A^M, \varphi_M\right); p_M)$, $\mathcal{A}$ will choose $\varphi = \max\left(\varphi_A^M, \varphi_M\right)$. Conditions (2) and (3) are violated and the market is in disciplinary mixed-strategy equilibrium.

iii. If $\varphi_I > \varphi_0^I$ and $\max\left(\varphi_A^M, \varphi_M\right) \leq \varphi_0^M$, $\mathcal{A}$ will choose $\varphi = \max\left(\varphi_A^M, \varphi_M\right)$. In this case conditions (2) and (3) are violated and the market is in disciplinary mixed-strategy equilibrium.

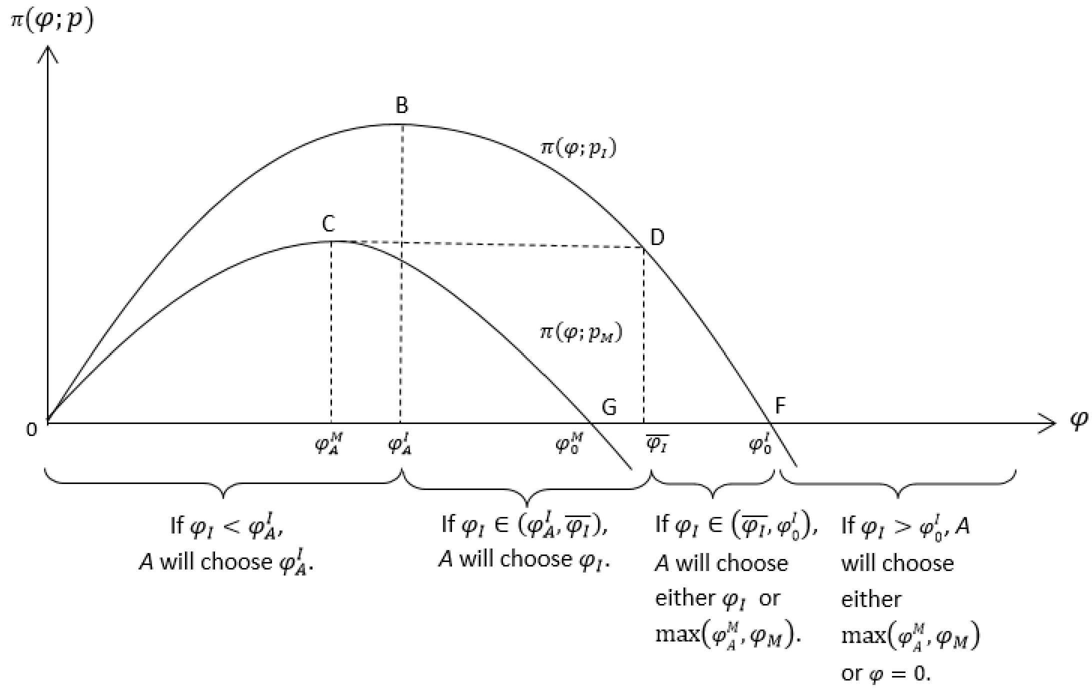

Figure A1: $\mathcal{A}$'s choice of φ.

iv. If $\varphi_I > \varphi_0^I$ and $\varphi_M > \varphi_0^M$, $\mathcal{A}$ will choose $\varphi = 0$.

The intuition behind Proposition 7 is presented in Figure A1. When $\varphi_I < \varphi_A^I$, $\mathcal{A}$ finds it optimal to choose $\varphi = \varphi_A^I$. This case corresponds to the point B on the Figure. When $\varphi_I \in (\varphi_A^I, \overline{\varphi_I})$, φ_A^I is not large enough to satisfy condition (2) and therefore maintain the intervention equilibrium and the corresponding price level, p_I. In this case, $\mathcal{A}$ finds it optimal to choose $\varphi = \varphi_I$ such that condition (2) holds. This case corresponds to a point on the B-D segment.

If $\varphi_I \in (\overline{\varphi_I}, \varphi_0^I)$, $\mathcal{A}$ will choose either $\varphi = \varphi_I$ and will on the D-F segment or $\varphi = \max\left(\varphi_A^M, \varphi_M\right)$ and will be on the C-G segment. If $\varphi_I > \varphi_0^I$

and $\max\left(\varphi_A^M, \varphi_M\right) \leq \varphi_0^M$, $\mathcal{A}$ will choose $\varphi = \max\left(\varphi_A^M, \varphi_M\right)$. This case corresponds to a point on the C-G segment. Finally, when $\varphi_I > \varphi_0^I$ and $\varphi_M > \varphi_0^M$, $\mathcal{A}$ will choose $\varphi = 0$.

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