european corporate governance institute

ecgi

Uncertainty, Disagreement, and Allocation of Control in Firms

Finance Working Paper N° 396/2013

February 2022

David L. Dicks
Baylor University

Paolo Fulghieri
University of North Carolina CEPR and ECGI

ECGI Working Paper Series in Finance

Uncertainty, Disagreement, and Allocation of Control in Firms

Working Paper N° 396/2013
February 2022

David L. Dicks
Paolo Fulghieri

We would like to thank Jonathan Berk, Jonathan Cohn, Diego Garcia, Lorenzo Garlappi, Simon Gervais, Ron Giammarino, Christopher Hennessy, Ali Lazrak, Doron Levit, Adair Morse, Kevin Murphy, Jacob Sagi, Merih Sevilir, Anjan Thakor, Geo§ Tate, and seminar participants at the UNC Brown Bag Seminar, the University of Houston, the UBC Winter Finance Conference, the Finance, Organizations, and Markets Conference, the Miami Behavioral Finance Conference, the North America Winter Meeting of the Econometric Society, and the SFS Finance Cavalcade for very helpful comments. All errors are only our own.

Abstract

We study the impact of (Knightian) uncertainty on the allocation of control in firms. Uncertainty generates disagreement between insiders and outsiders. Optimal governance depends on both firm characteristics and the composition of the outsider's overall portfolio. Strong governance is desirable when assets in place, relative to the growth opportunity, are sufficiently small or is sufficiently large, suggesting a corporate governance life cycle. Diversified outsiders prefer stronger governance, while outsiders with portfolios heavily invested in similar assets as the firm are more willing to tolerate weak governance. Finally, uncertainty links governance and transparency, whereby firms with weak governance should be opaque.

Keywords: Corporate Governance, Ambiguity Aversion, Disagreement

JEL Classifications: G34, D81

David L. Dicks
Assistant Professor
Baylor University, Hankamer School of Business
Forster Business and Innovation
Waco, Texas 76706, United States
phone: +1 (254) 710-4771
e-mail: David_Dicks@baylor.edu

Paolo Fulghieri*
Macon G. Patton Distinguished Professor of Finance
University of North Carolina Kenan-Flagler Business School
300 Kenan Center Drive
Chapel Hill, NC 27599-3490, United States
phone: +1 919-962-3202
e-mail: Paolo_Fulghieri@unc.edu

*Corresponding Author

Uncertainty, Disagreement, and Allocation of Control in Firms*

David L. Dicks
Hankamer School of Business
Baylor University

Paolo Fulghieri
Kenan-Flagler Business School
University of North Carolina
CEPR and ECGI

February 14, 2022

Abstract

We study the impact of (Knightian) uncertainty on the allocation of control in firms. Uncertainty generates disagreement between insiders and outsiders. Optimal governance depends on both firm characteristics and the composition of the outsider's overall portfolio. Strong governance is desirable when assets in place, relative to the growth opportunity, are sufficiently small or is sufficiently large, suggesting a corporate governance life cycle. Diversified outsiders prefer stronger governance, while outsiders with portfolios heavily invested in similar assets as the firm are more willing to tolerate weak governance. Finally, uncertainty links governance and transparency, whereby firms with weak governance should be opaque.

Keywords: Corporate Governance, Ambiguity Aversion, Disagreement

*We would like to thank Jonathan Berk, Jonathan Cohn, Diego Garcia, Lorenzo Garlappi, Simon Gervais, Ron Giammarino, Christopher Hennessy, Ali Lazrak, Doron Levit, Adair Morse, Kevin Murphy, Jacob Sagi, Merih Sevilir, Anjan Thakor, Geoff Tate, and seminar participants at the UNC Brown Bag Seminar, the University of Houston, the UBC Winter Finance Conference, the Finance, Organizations, and Markets Conference, the Miami Behavioral Finance Conference, the North America Winter Meeting of the Econometric Society, and the SFS Finance Cavalcade for very helpful comments. All errors are only our own. We can be reached at David_Dicks@kenan-flagler.unc.edu and Paolo_Fulghieri@kenan-flagler.unc.edu

Differences of opinions among economic agents are an important driver of trades in financial markets, where agents hold different "beliefs" on the future performance of securities, and the design of corporate financial structures, where agents respond to differences in valuation of firms' assets and investment opportunities. While financial economists have long recognized the importance of disagreement, there is a lack of consensus on the source of such disagreement.

Differences of opinion may reflect differences of information among economic agents (e.g., Grossman, 1976). They may also reflect the fact that economic agents hold heterogeneous priors or that they have a different interpretation of the same facts (see Harris and Raviv, 1993, Morris, 1994 and 1995, Kandel and Pearson, 1995, and more recently Van den Steen 2009). A common feature of this second group of papers is that differences of opinion are determined exogenously, by endowing agents either with heterogeneous priors or with different updating rules (i.e., different likelihood functions) used to form posteriors beliefs. As a result, agents hold different "views of the world," which leads them to differences of opinion.

This paper builds on a novel source of disagreement that is grounded in decision theory: uncertainty aversion. Specifically, uncertainty aversion generates endogenous differences of beliefs among agents, where beliefs are broadly understood as de Finetti probabilities (de Finetti, 1974). Our economy is populated by agents who are endowed with the same set of "core beliefs," but are heterogeneous in other dimensions, such as endowments. Agents' heterogeneity generates differences of beliefs as the outcome of their different exposure to risk factors in the economy. If differences in risk exposure cannot be resolved contractually (due, for example, to contractual frictions or incompleteness), agents will be more concerned about different states of the world, generating heterogeneous beliefs. The key benefit of our approach is that uncertainty aversion creates a direct link between differences of beliefs and economic fundamentals, allowing us to study the effect of changes in fundamentals on disagreement among agents.[1]

We study the impact of disagreement on the allocation of control within firms. Diversity of opinions makes the decision-making process in organizations important and creates the

[1]Our paper has also implications for the question of the persistence of disagreement over time in cases where agents observe (infinitely many) signals on uncertain payoff-relevant parameters (see, for example, Cripps et al., 2008, and Acemoglu, Chernozukov, and Yildiz, 2016). Specifically, our model suggests that if agents' heterogeneity persists over time, beliefs will not necessarily converge.

necessity of corporate governance systems.[2] A corporate governance system represents a set of rules for the allocation of control and, thus, of decision making. Allocation of decision making is critical when agents have different beliefs that cannot be reconciled contractually. In our paper, we explicitly ignore the question of optimal contract design and, rather, we focus on the question of the allocation of control, given a set of existing contracts.[3] Since in our model differences of opinion are endogenous, we can determine how disagreement among a firm's stakeholders evolves as the firm's fundamentals change. In this way, we are able to study the forces that contribute to shape a firm's corporate governance system and the allocation of control over a firm's life cycle.

Our paper allows us to take a new look at classic problems in the theory of the firm. Is a strong corporate governance system, from an investors' perspective, always preferable to weak governance? Is greater firm transparency always preferable to less transparency? More generally, is there a relationship between firm transparency and corporate governance? Can deliberate opacity be potentially desirable for shareholders? Our stylized model provides a new rationale for the separation of ownership and control, and is able to explain the optimality of commonly observed features in corporate organizations such as a low level of transparency at weakly governed firms, and why firms do not report more information than required by their regulatory framework.

We consider a firm endowed by a manager (the "insider") and a large shareholder, such as a blockholder (the "outsider"). There are two classes of risky assets in our economy (in addition to the riskless assets) with different exposure to uncertainty. The outsider

[2] Disagreements among shareholders, boards, and management are common events in corporate life. Examples include the ousting of Carly Fiorina from Hewlett and Packard, allegedly "after she and directors disagreed on how to carry out Hewlett's corporate strategy," (see "Hewlett-Packard's Chief Forced Out, Ending Rocky Tenure," New York Times, February 9, 2005). Similarly, Christopher Galvin was ousted from his position of Chairman and Chief Executive of Motorola because of "disagreements over pace, strategy and progress," especially concerning the company's strategy on semiconductors, one of Motorola's traditional strong products (see "For Motorola, Chief's Ouster Seen Bringing Strategy Shift," New York Times, September 22, 2003).

[3] In a setting with costly state falsification, it is possible to show that optimal contracts mitigate, and do not resolve, disagreement, but at the cost of adding unnecessary complexity to the analysis. Dicks and Fulghieri (2022) examine a setting with persistent disagreement under optimal contracts. Alternatively, agent heterogeneity may derive from differences in preferences, such as risk attitudes, or skills (ability). An example of such contractual incompleteness is the limited ability to insure insiders' human capital. Mukerji and Tallon (2001) show that ambiguity aversion can lead to endogenous market incompleteness, and thus the existence of uninsurable risk.

and insider are heterogenous in that they hold a different portfolio of the two risky assets. Such portfolio heterogeneity may reflect, for example, the insider's undiversifiable human capital or the presence of an (optimal) incentive contract (as in Dicks and Fulghieri, 2022). This portfolio heterogeneity leads the insider and outsider to have endogenous differences of opinion and, thus, to a different assessment of the profitability of the firm's investment opportunities.

The firm is endowed with one type of asset plus a growth opportunity. The growth opportunity can either be an expansion of its current assets in place (that is, a "focused" project), or an investment in the other type of asset (that is, a "diversifying" project). The insider is undiversified and holds only the firm's equity, while the outsider holds a better-diversified portfolio. At the outset, the outsider intervenes in the corporate governance system of the firm, which affects its investment policy. The outsider can either exert control of the firm investment decisions ("strong governance") or delegate them to the manager, who thus obtains control ("weak governance"). In addition, the outsider decides whether the firm will be transparent (where the information available to the insider will be revealed to the outsider), or opaque (where the manager's information remains opaque to the outsider).

We show that the strength of the corporate governance system depends on both firm characteristics and the composition of the outside shareholders' overall portfolio. The outsider benefits from delegation of decision-making authority to the insider in two different ways. First, it avoids suboptimal investment, a feature due to the pessimism caused by uncertainty aversion. Second, by delegating decision-making authority to the insider, the outsider reduces exposure to information revelation, which is harmful to uncertainty-averse agents.[4] These effects always make delegation (and corporate opacity) attractive to the outsider.

The disadvantage of delegation of decision-making authority is disagreement with insiders stemming from endogeneity of beliefs (due to uncertainty aversion), and its effect on the desired level of investment. The extent of this disagreement, however, depends on firm characteristics such as asset and ownership structure. These results are in sharp contrast

[4] Dicks and Kim (2021) provide empirical evidence that the representative investor is information avoidant, due to the earnings announcement premium. Similar to Ai and Bansal (2018), Dicks and Kim (2021) attribute this feature to investor uncertainty aversion. There is empirical evidence that investors are willing to pay a premium for opaque assets: see Coval, Jurek, and Stafford (2009), Henderson and Pearson (2011), and Célérier and Vallée (2017), as well as Sato (2014) for a theoretical treatment.

with what one would obtain under risk aversion only, where delegation of decision making is always (weakly) harmful for the outsider.

We find that a strong corporate governance system is optimal when the value of the firm's assets in place, relative to the growth opportunity, is either sufficiently small or sufficiently large. This property suggests a corporate governance life cycle, whereby stronger governance is optimal for young and mature firms, while weaker governance is optimal for firms at the intermediate stage of their development. In addition, we find that a weaker corporate governance system is also optimal (all else equal) for more productive firms, while stronger corporate governance is optimal for less productive ones. These results suggest that private and public companies will be characterized by different governance structures and investment policies, and predict a role for "going private" transactions such as LBOs.

An additional implication of our model is that uncertainty aversion introduces a direct link between the strength of the corporate governance system and firm transparency. In our model, delegation of control to insiders allows the outsider to reduce (in fact, to eliminate) the need to access information on states of nature that affect the fundamental value of the firm.[5] We show that this property of delegation can be desirable because, due to uncertainty aversion, the outsider prefers (all else equal) to have less, rather than more, information on the true state of the firm if he cannot act on that information. This means that the outsider prefers the firm to be less transparent, unless the outsider can benefit from greater transparency by exerting control. Thus, firms with weaker governance should also optimally be more opaque.

Finally, our paper also has implications for the governance structure of private equity and venture capital funds and of their portfolio companies. We find that more diversified outsiders prefer stronger governance, while outsiders with a portfolio more heavily invested in the same asset class as the firm's tolerate weaker governance systems. This happens because more diversified outsiders are likely to be in greater disagreement with the insider on the firm's investment policy and, thus, to prefer stronger governance. If outsiders are institutional investors such as a venture capital or a private equity fund, this property implies that generalist funds should impose relatively stronger governance systems on their portfolio

[5] The link between information and control is quite subtle and it has been examined by several papers in the literature, such as Aghion and Tirole (1997), Dessein (2002), Harris and Raviv (2005), (2008), and (2010), Adams and Ferreira (2007) and Chakraborty and Yilmaz (2017).

companies. In contrast, specialized funds are more willing to tolerate weaker governance systems, where the portfolio companies' insiders have more leeway in determining corporate investment policies, for portfolio companies falling in their specialty, while imposing strong governance on firms outside their specialty.

The persistence of organizational forms characterized by division of ownership and control is a classic puzzle. Both Adam Smith[6] and Berle and Means[7] passionately warned about the negative implications of the separation of ownership and control. In a deliberately provocative paper, Jensen (1989) advocates the "eclipse of the public company" and proposes the "LBO association" (and the avoidance of the separation of ownership and control) as a superior organizational structure. Rappaport (1990), while acknowledging that separation of ownership and control was suboptimal, countered that LBOs may be optimal organization structures temporarily, and defended the "staying power" of public companies.

Corporate finance models typically examine the costs and benefits of alternative governance systems as mechanisms to impose discipline on a firm's insiders, where typically delegation of decision making to insiders is costly (see, for example, Harris and Raviv, 2008 and 2010, and extensive surveys in Shleifer and Vishny, 1997, and Becht, Bolton, and Roell, 2003). Delegation of decision making to insiders is beneficial either when insiders have access to better information or, as in Aghion and Tirole (1997), to provide agents with better incentive to collect valuable information. More recent literature examines the impact of disagreement on incentives and organization design (Van den Steen, 2004, 2009, 2010a and especially 2010b, and Boot and Thakor, 2011).

[6] "The directors of such companies, however, being the managers rather of other people's money than of their own, it cannot well be expected that they should watch over it with the same anxious vigilance with which the partners in a private copartnery frequently watch over their own. Like the stewards of a rich man, they are apt to consider attention to small matters as not for their master's honour, and very easily give themselves a dispensation from having it. Negligence and profusion, therefore, must always prevail, more or less, in the management of the affairs of such a company." Book 5, Chapter 1, Part 3, Article 1, v1.107, Smith (1776).

[7] "In its new aspect the corporation is a means whereby the wealth of innumerable individuals has been concentrated into huge aggregates and whereby control over this wealth has been surrendered to a unified direction. The power attendant upon such concentration has brought forth princes of industry, whose position in the community is yet to be defined. The surrender of control over their wealth by investors has effectively broken the old property relationships and has raised the problem of defining these relationship anew. The direction of industry by persons other than those who have ventured their wealth has raised the question of the motive force back of such direction and the effective distribution of the returns from business enterprise." p. 2, Berle & Means (1933)

This paper is related to several current strands of literature. The first one is the emerging literature in corporate finance focusing on the effect of disagreement on firms' corporate governance and financing strategy. Van den Steen (2004) and (2010b) examine the impact of disagreement on incentives and organization design. Boot, Gopalan, and Thakor (2006) and (2008) examine a firm's choice between private and public ownership as a trade-off between managerial autonomy and liquidity. Boot and Thakor (2011) argue that potential disagreement with new investors causes the initial shareholders to prefer weak corporate governance (that is, "soft claims" that allows managerial discretion). Huang and Thakor (2013) suggest that heterogeneous beliefs affect firms' decision to do share repurchases. Bayar, Chemmanur, and Liu (2011) examine the impact of heterogeneous beliefs on equity carve-outs. Harris and Raviv (1993) examine the impact of disagreement on the volume of trading and the reaction to public announcements.

A second related strand of literature focuses on the role of large shareholders (blockholders) on the governance of modern corporations.[8] The importance of blockholders in firm ownership structures is well documented. For example, Holderness (2009) finds that 96% of US domestic corporations have at least one 5% or greater shareholder. Several papers suggest large shareholder can increase firm value by monitoring and intervention (see Shleifer and Vishny, 1986, Winton, 1993, and Edmans and Manso, 2011, among others). External interventions, however, may have an adverse effect on the insiders incentives, especially in the presence of disagreement between with the large shareholders on firm policy (Van den Steen, 2010a, and Boot and Thakor, 2011). A key feature of our paper is that disagreement between insiders and outsiders emerges endogenously as the outcome of their differential exposure to uncertainty and is affected by firm characteristics. We argue that the beneficial role of large shareholders' intervention depends on the firm life-cycle. In particular, our paper suggest that shareholder intervention may be particularly valuable for either young or more mature firms, while outsiders in firms at intermediate stages of development should confer more decision-making autonomy to insiders.

Finally, our paper contributes to the literature on the determinants of a firm's disclosure policy, that is, its transparency, with and without disagreement. This includes Boot and Thakor (2001), Fishman and Hagerty (2003), Ferreira and Rezende (2007), and Kogan et al.

[8] An extensive survey of the role of blockholders in corporate governance systems can be found in Edmans (2014) and Edmans and Holderness (2017).

(2010) among many others. Closer to our work, Thakor (2013) examines the optimal disclosure policy in the presence of disagreement, and suggests that firms may prefer to disclose less information, and thus to remain more opaque, when information disclosure increases disagreement. In contrast, in our paper, the benefit of opacity is an outcome of information aversion from Knightian uncertainty. In a different setting, Levit and Malenko (2016) also argue that transparency may be harmful and, in fact, weaken corporate governance quality when expression of dissent by board members affect adversely their reputation and, thus, the value of their outside opportunities. In all these papers disagreement is exogenously given and it derives from heterogeneous priors among a firm's stakeholders (that is, it is a "primitive" of the model). Our paper provides the heterogenous priors approach with an explicit decision-theoretic foundation based on uncertainty aversion.

An exception to the literature mentioned above is Garlappi, Giammarino, and Lazrak (2014). This paper considers a firm with a board of directors that are endowed with heterogenous priors and shows that the corporation will behave as if it is uncertainty averse if the board has a supermajority rule in place, such as the unanimity rule. Thus, in this paper disagreement and desire for consensus lead to uncertainty aversion. In contrast, in our paper we show that uncertainty aversion and agents' heterogeneous characteristics lead to disagreement.

Our paper has two main limitations that can provide fruitful avenues for future research. First, we take the insider's lack of diversification as given. In practice, management may be undiversified for a number of reasons, such as the presence of firm-specific human capital, or as the outcome of an incentive contract due to moral hazard (as in Dicks and Fulghieri, 2022). A second limitation is that, in our model, agents are risk neutral. The presence of risk aversion would provide an additional source of disagreement between insiders and outsiders that could be addressed with optimal contracts (see Ross, 1973).[9] Distinct from an uncertainty-aversion framework, however, a weak corporate governance system (i.e., delegation of control) is always dominated by strong governance in a pure risk-aversion framework. Thus, a more general model that explicitly considers uncertainty aversion with either moral hazard or risk aversion will have to incorporate the effects discussed in this paper as drivers of optimal contracts. For example, our model suggests that, because of uncertainty aversion, insiders should work under contracts that optimally generate some exposure to industry-wide

[9] E.g., if the insider is more risk averse, her optimal investment will be lower than the outsider's.

(or even economy-wide) shocks, in order to reduce the extent of the disagreement with the outsiders.[10]

The paper is organized as follows. In Section 1, we describe the basic model. In Section 2, we derive the paper's main results. In Section 3, we present the paper's empirical implications. Section 4 concludes. All proofs are in the appendix.

1 The Model

We consider the allocation of control between a large shareholder (the outsider), denoted by S, and its insider, the manager, denoted by M. We assume that at the outset the firm outsider owns a fixed fraction $1-\beta$ of the firm equity, and the insider retains the residual fraction β for herself.[11]

We study a simple two-period model with three dates, $t \in \{0, 1, 2\}$. At the beginning of the first period, $t = 0$, the outsider chooses the governance structure, δ, and information structure, ψ, of the firm that maximizes the outsider's payoff. For governance, the outsider must decide whether to retain control of the firm's decisions for themselves, denoted by $\delta = r$ ("retention") or to delegate control to the firm's insider, the management, denoted by $\delta = d$ ("delegation"). We interpret retention of control by the outsider as a "strong" corporate governance system, and delegation as a "weak" corporate governance system. Similarly, the outsider can implement a transparent information system, denoted by $\psi = T$ ("transparency"), or an opaque one, denoted by $\psi = O$ ("opacity"). At $t = 1$, the firm receives an investment opportunity, and the party in control (the manager if $\delta = d$, the outsider if $\delta = r$) selects an investment level. The manager always learns the type of project, while the outsider learns the type of project only if the transparency regime, $\psi = T$, is implemented.

We assume the economy is endowed by three (classes of) assets: a riskless asset (our numeraire), and two types of risky assets: type-A and type-B assets. Type-τ assets, with $\tau \in \{A, B\}$, are risky in that they generate at the end of the second period, $t = 2$, a random payoff denominated in terms of the riskless asset. Specifically, a unit of type-τ asset produces

[10] See Gopalan, Milbourn, and Song (2010).

[11] For example, the insider may be an entrepreneur who, after founding the firm, has divested a fraction $1-\beta$ of its equity to outside investors to raise capital in earlier financing rounds. In turn, the outsider could be a private equity investor or a group of dispersed shareholders.

at $t = 2$ a payoff H (success) with probability p_τ, and a payoff L (failure) with probability $1 - p_\tau$. For notational simplicity, we normalize these payoffs to $H = 1$ and $L = 0$.

1.1 Modeling uncertainty

A critical feature of our model is that both outsider and insider are uncertain about the success probabilities, p_τ. We model uncertainty aversion by adopting the minimum expected utility (MEU) approach developed in Gilboa and Schmeidler (1989).[12] In this framework, economic agents do not have a single prior on future events but, rather, they believe that the probability distribution of future events belongs to a given set $\mathcal{M}$, denoted as the investor's "core beliefs set." Thus, uncertainty-averse agents maximize $\mathcal{U}$, where

$$\mathcal{U} = \min_{\mu \in \mathcal{M}} E_\mu \left[u\left(\cdot\right) \right], \tag{1}$$

where μ is a probability distribution over future events, and $u\left(\cdot\right)$ is a von-Neumann Morgenstern (vNM) utility function.[13] When u is a linear (or affine) function, the economic agent will be risk neutral but uncertainty averse.

Formally, we model sophisticated uncertainty-averse economic agents with consistent planning. In this setting, agents are sophisticated in that they correctly anticipate their future uncertainty aversion. Consistent planning accounts for the fact that agents take into account how they will behave in the future.[14] In the context of our model, following the initial contracting phase (when control and information structure are established) at $t = 0$, the investment project is revealed and an investment level is chosen at $t = 1$. All payoffs are determined at $t = 2$. Players at the initial contracting phase, $t = 0$, correctly anticipate behavior at the interim stage, $t = 1$.

A critical feature of uncertainty aversion is that uncertainty-averse agents weakly prefer randomizations over random variables (more precisely, over acts described in Anscombe and

[12] An alternative approach is "smooth ambiguity" developed by Klibanoff, Marinacci, and Mukerji (2005). In their model, agents maximize expected felicity of expected utility, and agents are uncertainty averse if the felicity function is concave. The main results of our paper will hold in this approach (if the felicity function is sufficiently concave), but at the cost of requiring a substantially greater analytical complexity. Similarly, our results also hold under variational preferences of Maccheroni, Marinacci, and Rustichini (2006) if the ambiguity index $c\left(p\right)$ has a positive cross-partial.

[13] In the traditional framework, players have a single prior μ and maximize expected utility $E_\mu \left[u\left(\cdot\right)\right]$.

[14] Siniscalchi (2011) describes this framework as preferences over trees.

Aumann, 1963) rather than each individual variable in isolation.[15] In the context of MEU, this feature may be seen immediately as follows. Given two random variables, y_k, $k \in \{1, 2\}$, with joint distribution $\mu \in \mathcal{M}$, by the property of the minimum operator, we have that for all $q \in [0, 1]$

$$q \min_{\mu \in \mathcal{M}} E_\mu [u(y_1)] + (1-q) \min_{\mu \in \mathcal{M}} E_\mu [u(y_2)] \le \min_{\mu \in \mathcal{M}} \{q E_\mu [u(y_1)] + (1-q) E_\mu [u(y_2)]\}. \tag{2}$$

A second critical feature of uncertainty aversion is that it induces information avoidance. Specifically, information harms an uncertainty-averse agent who does not use it.

$$\min_{\mu \in \mathcal{M}} \ E_s E_{\mu(\cdot;s)} [u(w)] \ge E_s \min_{\mu(\cdot;s) \in \mathcal{M}_s} E_{\mu(\cdot;s)} [u(w)]. \tag{3}$$

Property (3) implies that information harms uncertainty-averse agents unless they use the information in making subsequent decisions.[16] A potential offsetting advantage of learning, however, may come in cases where the agent's utility also depends on a specific action by the agent. In this case, the agent may find it desirable to choose the action only after learning the realization of the signal to condition the choice of the action to the observed signal.[17] This property creates an endogenous cost of disclosure, and provides a benefit of separation of ownership and control in our model.

We model uncertainty aversion by assuming uncertainty on the success probability of the risky assets, p. Following Hansen and Sargent (2001) and (2008), we characterize the core beliefs set $\mathcal{M}$ in (1) by using the notion of relative entropy.[18] For a given pair of

[15] This is the "uncertainty-aversion axiom" of Gilboa and Schmeilder (1989); formally, for any two uncertain acts that the investor is indifferent between, $f \sim g$, the investor (weakly) prefers the mixture: $\alpha f + (1-\alpha) g \succeq f$. Note that for SEU agents, the latter always holds with indifference.

[16] Note that property (3) mirrors the well-known feature that a portfolio of options is worth more than an option on a portfolio and, thus, that writing a portfolio of options is more costly than writing an option on a portfolio. By similar intuition, more information harms an uncertainty-averse agent since the minimization process can be more fine-tuned.

[17] Though the proof of Lemma 1 assumes the minimum expected utility framework, similar intuition applies in the KMM framework (this works by the same intuition that a mean-preserving spread harms a risk-averse economic agent). Indeed, Caskey (2009) shows that an ambiguity-averse agent will optimally ignore ambiguous information. Caskey (2009) interprets the unambiguous information as aggregate information and ambiguous information as firm-specific information.

[18] This specification of uncertainty aversion, which is often referred to as the "constrained preferences" approach, is a particular case of the larger class of "variational preferences." Strzalecki (2011) provides a general characterization of different approaches to modeling uncertainty aversion.

(discrete) probability distributions $(p, \hat{p})$, the *relative entropy* of p with respect to $\hat{p}$ is the Kullback-Leibler divergence of p from $\hat{p}$:

$$R(p|\hat{p}) \equiv \sum_i p^i \log \frac{p^i}{\hat{p}^i}. \tag{4}$$

Thus, the core beliefs set for the uncertainty-averse investors in our economy is

$$\mathcal{M} \equiv \{p : R(p|\hat{p}) \leq \eta\}, \tag{5}$$

where p is the joint distribution of the success probability of the second stage of the two projects, and $\hat{p}$ is an exogenously given "reference" probability distribution of such success probabilities. From (4), it is easy to see that the relative entropy of p with respect to $\hat{p}$ represents the (expected) log-likelihood ratio of the pairs of distributions $(p, \hat{p})$, when the "true" probability distribution is p. Thus, interestingly, the core beliefs set $\mathcal{M}$ can be interpreted as the set of probability distributions, p, with the property that, if true, the investor would expect not to reject the ("null") hypothesis $\hat{p}$ in a likelihood-ratio test. For simplicity, we will assume type A and B assets are independent under $\hat{p}$.

Intuitively, the core belief set $\mathcal{M}$ includes probability distributions that are not "too unlikely" to be the true (joint) probability distribution that characterizes the two technologies, given the reference distribution $\hat{p}$. Note that a small value of η represents situations where agents have more confidence in the probability distribution $\hat{p}$, while a large value of η corresponds to situations where there is greater uncertainty.[19]

An important effect of restricting beliefs to the core beliefs set (5) is to rule out probability distributions that are "too far" from the reference probability $\hat{p}$. In other words, the maximum entropy criterion implied by (5) has the effect of excluding from the core-belief set probability distributions that give too much weight to extreme events. Because uncertainty-averse agents are essentially concerned about "left-tail" events, we interpret this property as "trimming pessimism." The following lemma provides a simple characterization of the core beliefs set $\mathcal{M}$ that will play a critical role in our paper.

Lemma 1 *Let $\eta < \underline{\eta}(\hat{p})$, defined in the appendix. The core beliefs set $\mathcal{M}$ is a strictly convex*

[19] As in Hansen and Sargent (2001) and (2008), and Epstein and Schneider (2010), relative entropy can be interpreted as characterizing the extent of "misspecification error" that affects investors.

set with smooth boundary. If investors have nonnegative positions in both risky assets, the solution to (1) is on the lower left-hand boundary of M. Finally, information harms an uncertainty-averse agent if the agent does not use it.

Lemma 1 is a direct implication of the fact that relative entropy $R(p|\hat{p})$ is a strictly convex function. Uncertainty-averse agents with exposure to both asset classes select probability assessments that lie in the "lower-left" boundary of the core beliefs set. Thus, the relevant part of the core beliefs set $\mathcal{M}$ is a smooth, decreasing, and convex function (see Figure 1).[20] Further, information weakly harms an uncertainty-averse agent, and is strictly harmful if the worst-case scenario is not constant.

Because there is no closed-form solution for the level set of relative entropy for binomial distributions in (5), for ease of exposition, we follow Dicks and Fulghieri (2019) and (2021) and model the relevant portion of the core beliefs set (namely, the decreasing and convex "lower-left" boundary) by using a lower-dimensional parametrization, as follows. We assume that the success probability of risky asset τ depends on the value of an underlying parameter θ_τ, and is denoted by $p(\theta_\tau)$, with $\theta_\tau \in [\theta_L, \theta_H] \subseteq [\theta_m, \theta_M]$. For analytical tractability, we assume that $p(\theta_\tau) = e^{\theta_\tau - \theta_M}$, with $\tau \in \{A, B\}$.

Uncertainty-averse agents treat the vector $\vec{\theta} \equiv (\theta_A, \theta_B)$ as ambiguous and assess that $\vec{\theta} \in C \subset \{(\theta_A, \theta_B) : (\theta_A, \theta_B) \in [\theta_L, \theta_H]^2\}$. We interpret the parameter combination $\vec{\theta}$ as describing the state of the economy at $t = 2$ and we denote C as the set of "core beliefs." In light of Lemma 1 and subsequent discussion, we assume that for $\vec{\theta} \in C$ we have that $(\theta_A + \theta_B)/2 = \theta_T$, where $\theta_T \equiv (\theta_H + \theta_L)/2$. Importantly, given $\vec{\theta}$, the success probabilities of the second-stage of risky assets are independent. We will characterize the extent of uncertainty as $\alpha \equiv \theta_T - \theta_L$.

We will at times benchmark the behavior of uncertainty-averse agents with the behavior of an uncertainty-neutral agent, and we will assume that an uncertainty-neutral agent has $\theta_L = \theta_H$, so that he assesses $\theta_\tau = \theta_T$. This guarantees that the uncertainty-neutral agent has the same probability assessment on the success probability of each asset class as a diversified uncertainty-averse investor (there is no "hard-wired" difference between the two types of

[20] For a general discussion, see Theorem 2.5.3 and 2.7.2 of Cover and Thomas (2006). Our results hold, generally, when the core belief set $\mathcal{M}$ is a strictly convex set with smooth boundaries (note that "rectangular" core-belief sets do not satisfy such conditions).

agents).[21]

At the beginning of the first period, $t = 0$, the firm is endowed with V_A units of type-A assets and V_0 units of the riskless asset.[22] At the interim date, $t = 1$, the firm has access to a new investment opportunity. The type of investment opportunity which becomes available to the firm is random, and is not known at the outset to both the outsider and the insider. The investment opportunity can either be a project in the same type of assets currently owned by the firm, type-A assets, or an investment in type-B assets. We denote these investment opportunities respectively as the *focused* and the *diversified* project.

The firm has a unique opportunity only discoverable at $t = 1$. Specifically, at $t = 1$, the firm discovers the "location" of the profitable project, which is drawn from $l \sim U[-1, 1]$. Investment at the wrong location, $l' \neq l$, will be worthless. Thus, when $l < 0$, investment is only profitable in type-B assets, while investment is profitable in type-A assets if $l > 0$. Define $\omega_B = \{l < 0\}$ and $\omega_A = \{l > 0\}$. Thus, in state ω_τ the firm can acquire I_τ units of type τ assets at the cost of $c(I_\tau)$ units of the riskless asset. We assume that the firm is not cash-constrained in that it has a sufficient amount of riskless assets to be able to implement the desired investment I_τ in the risky asset $\tau \in \{A, B\}$.[23] We also assume that the project is characterized by decreasing returns to scale and positive investment is always optimal: formally, $c(0) = 0$, $0 \leq c'(0) < e^{\theta_L - \theta_M}$, $c'(I_\tau) > 0$ for $I_\tau > 0$, and $c''(I_\tau) > 0$. The cost function $c(\cdot)$ is the same for type-A and type-B projects. For analytical tractability we assume $c(I) = \frac{1}{Z(1+\gamma)} I^{1+\gamma}$ where $\gamma > 0$, though our main results do not depend on this specification. Z affects the cost of acquiring the risky assets and, thus, the value of the investment project; it will be interpreted as characterizing the firm's "productivity." For simplicity, both agents believe that project types are equally likely, i.e., $\Pr\{\omega_\tau\} = 1/2$, and there is no uncertainty on the states ω_τ: both the insider and the outsider have a single common prior on l.[24]

[21] Alternatively, we could assume the core of beliefs satisfy $\left\{ \overrightarrow{\theta} \,|\, \sum |\theta - \theta^*| \leq \eta \right\}$. This would is isomorphic by selecting $\theta_T = \theta^* - \frac{\eta}{2}$ and $\alpha = \frac{\eta}{2}$. The disadvantage of this approach is that it creates a hard-wired difference in optimal investment between uncertainty-neutral and uncertainty-averse agents.

[22] More generally, we could assume that a certain fraction of firms in the economy is endowed with type-A assets, and the remaining firms with type-B assets.

[23] We leave for future research the important question of raising capital under uncertainty aversion.

[24] Our result follow even if there is uncertainty about which type of project is drawn. Uncertainty on project type is easily modeled by letting $\Pr\{w_A\} = q$ and $\Pr\{w_B\} = 1 - q$ where $q \in \left[\frac{1}{2} - \varepsilon, \frac{1}{2} + \varepsilon\right]$ and $\varepsilon > 0$. The effect of adding uncertainty on project type is that the outsider will overweigh the outcome which

Allocation of control is important because the party in control chooses the level of investment I_τ to maximize his/her expected utility. The choice of the investment level, I_τ, is made by the party in control after observation of the realization of l, the location of the investment project available to the firm. We assume that while the realization of the state of the world ω_τ is always observable by the insider, it will be made observable also to the outsider if the outsider chose a transparent information structure, $\psi = T$.

Lemma 2 *If the outsider retains control, $\delta = r$, it is optimal to implement transparency, $\psi = T$. If the outsider does not retain control, $\delta = d$, it is optimal to implement opacity.*

Lemma 2 shows that the outsider always (weakly) prefers to be blind to the realization of the state of the world if management has control. Thus, if the outsider delegates decision making authority, he will also choose an opaque information environment, and will select a transparent framework only if he retains control.

Finally, at the end, $t = 2$, agents consume their holdings. Agents are endowed with vNM utility functions, $u(\cdot)$, which are linear in the riskless asset. Following Epstein and Schneider (2010), this means that they are risk-neutral MEU or SEU agents.

1.2 Endogenous Beliefs

A critical feature of our model is that uncertainty aversion endogenously generates differences of opinion in an economy populated by heterogeneous agents, even when agents have identical core prior beliefs. This happens because, within a MEU framework, an agent's beliefs are determined by the solution to that agent's expected utility minimization problem. Agent heterogeneity (for example, in their endowments) generates different solutions to the minimization problem and, thus, different beliefs. Therefore, differences of beliefs emerge endogenously. As we show in Section 2, these differences are meaningful and impact the firm's investment decision, making the ex-ante allocation of control meaningful.

Consider an agent endowed with a portfolio $\Pi \equiv \{\bar{w}_A, \bar{w}_B, \bar{w}_0\}$, where $\bar{w}_\tau$, $\tau \in \{A, B\}$ represents the overall units of the risky asset type τ owned by the agent, and $\bar{w}_0$ represents

is worse to him. In equilibrium, however, he will be indifferent whether the project is focused or diversifying in all cases except when both the insider and outsider are uncertainty averse. In the latter case, it can be shown that the region that the outsider prefers delegation will shrink.

the units of riskless asset in the agent's portfolio. For a given value of $\overrightarrow{\theta}$, this portfolio provides the agent with an expected utility of

$$\mathbb{E}\left[u\left(\bar{w}_A, \bar{w}_B, \bar{w}_0\right); \overrightarrow{\theta}\right] = e^{\theta_A - \theta_M}\bar{w}_A + e^{\theta_B - \theta_M}\bar{w}_B + \bar{w}_0 \tag{6}$$

An uncertainty-averse agent fears the worst possible outcome of θ:

$$U\left(\bar{w}_A, \bar{w}_B, \bar{w}_0\right) \equiv \min_{\overrightarrow{\theta} \in C} \mathbb{E}\left[u\left(\bar{w}_A, \bar{w}_B, \bar{w}_0\right); \overrightarrow{\theta}\right] \tag{7}$$

Thus, an uncertainty-averse agent's beliefs, denoted $\overrightarrow{\theta}^a$, minimize the agent's expected utility:

$$\overrightarrow{\theta}^a(\Pi) \equiv \arg\min_{\overrightarrow{\theta} \in C} \mathbb{E}\left[u\left(\bar{w}_A, \bar{w}_B, \bar{w}_0\right); \overrightarrow{\theta}\right]. \tag{8}$$

Note the agent's belief θ^a depends on the composition of his overall portfolio Π. The solution to problem (7) is characterized in the following lemma.

Lemma 3 *For $\tau, \tau' \in \{A, B\}$, $\tau \neq \tau'$, let*

$$\tilde{\theta}_\tau^a(\Pi) \equiv \theta_T + \frac{1}{2}\ln\frac{\bar{w}_{\tau'}}{\bar{w}_\tau}. \tag{9}$$

The beliefs held by an uncertainty-averse agent are

$$\theta_\tau^a(\Pi) = \begin{cases} \theta_L & \tilde{\theta}_\tau^a(\Pi) \leq \theta_L \\ \tilde{\theta}_\tau^a(\Pi) & \tilde{\theta}_\tau^a(\Pi) \in (\theta_L, \theta_H) \\ \theta_H & \tilde{\theta}_\tau^a(\Pi) \geq \theta_H \end{cases}. \tag{10}$$

We refer to $\tilde{\theta}^a(\Pi)$ as the "portfolio-distorted" beliefs. We say that the agent has *interior beliefs* when $\theta_\tau^a \in (\theta_L, \theta_H)$. In this case, the agent's beliefs are equal to the portfolio-distorted beliefs, that is $\theta_\tau^a(\Pi) = \tilde{\theta}_\tau^a(\Pi)$. It is important to note that the beliefs of an uncertainty-averse agent depend essentially on the composition of his portfolio Π, that is, on his endowment. In particular, in our specification, the portfolio-distorted beliefs $\tilde{\theta}^a(\Pi)$ differ from the SEU beliefs $\theta_T \equiv \frac{1}{2}(\theta_L + \theta_H)$ by a term that depends on the degree of heterogeneity of the agent's endowment, $\bar{w}_B/\bar{w}_A$. The following corollary can be immediately verified.

Corollary 1 *Holding type τ assets constant, when the agent has a larger position in type τ' assets, the agent is more pessimistic about type τ' assets and more optimistic about type τ assets. Furthermore, the agent has scale-invariant beliefs that depend only on the ratio $\bar{w}_\tau/\bar{w}_{\tau'}$.*

Corollary 1 shows that when an agent has relatively greater endowment of one type of asset, the agent will be relatively more concerned about the priors that are less favorable to that asset. In other words, the agent will be more "pessimistic" about the future value of (or the return on) that asset. Correspondingly, the agent will become more "optimistic" with respect to the other asset. Note also that portfolio-distorted beliefs $\tilde{\theta}^a$ remain the same when the ratio of the endowment in the two types of assets is constant, that is, θ^a is scale invariant (homogeneous of degree zero in $\bar{w}_B$ and $\bar{w}_A$). Note that, from (9), the beliefs held by an uncertainty-neutral agent coincide with the beliefs of a diversified uncertainty-averse agent, for whom $\bar{w}_A = \bar{w}_B$.

An important property of the MEU approach is that even if agents are endowed with vNM utility functions that are linear in wealth, they display decreasing marginal utility in the value of any single asset in their portfolio, when the amount of all other assets remains constant. This happens because of the negative impact on an agent's beliefs that is due to the increase in the endowment of any specific asset, when the endowment of all other assets remains the same.

Lemma 4 *An agent has decreasing marginal utility from one type of asset, $\frac{d^2U}{d(\bar{w}_\tau)^2} \leq 0$, for $\tau \in \{A, B\}$, holding the position in the other asset type constant. For interior beliefs, this inequality is strict.*

This property plays an important role in the investment problem below.

2 Uncertainty and Allocation of Control

We now consider the allocation of control, $\delta \in \{r, d\}$, faced by the outsider at the beginning, $t = 0$. Because the optimal level of investment chosen by an uncertainty-averse agent depends on her portfolio composition, the allocation of control becomes critical.

As a reference point, we start our discussion by considering the benchmark case where there is no separation of ownership and control: the agent making the decision has full ownership of the firm. We assume that, in addition to the full ownership of the firm, the owner is also endowed with other resources (outside the firm) denoted by $\{w_A, w_B, w_0\}$. Thus, in state ω_A, when $l > 0$, the firm has an investment opportunity involving type-A assets. The overall owner's portfolio after the investment is made becomes $\Pi(I_A, 0) \equiv \{w_A + V_A + I_A, w_B, w_0 + V_0 - c(I_A)\}$. Similarly, in state ω_B, when $l < 0$, the firm has an investment opportunity involving type-B assets, so the owner's portfolio becomes $\Pi(0, I_B) \equiv \{w_A + V_A, w_B + I_B, w_0 + V_0 - c(I_B)\}$.

If the firm discovers a focused project, an uncertainty-averse owner chooses at $t = 1$ the optimal investment in a type-A project by maximizing the minimum expected utility function, U_1:

$$U_1(\Pi(I_A, 0)) \equiv \min_{\overrightarrow{\theta} \in C} \mathbb{E}\left[u(\Pi(I_A, 0)); \overrightarrow{\theta}\right], \tag{11}$$

where

$$\mathbb{E}\left[u(\Pi(I_A, 0)); \overrightarrow{\theta}\right] = e^{\theta_A - \theta_M}(w_A + V_A + I_A) + e^{\theta_B - \theta_M} w_B + w_0 + V_0 - c(I_A), \tag{12}$$

and $\overrightarrow{\theta} \in C$ iff $\theta_\tau \in [\theta_T - \alpha, \theta_T + \alpha]$ and $\frac{1}{2}(\theta_A + \theta_B) = \theta_T$. The owner's beliefs, $\theta^a(\Pi(I_A, 0))$, are given by the solution to (11)

$$\theta^a(\Pi(I_A, 0)) = \arg\min_{\overrightarrow{\theta} \in C} \left\{\mathbb{E}\left[u(\Pi(I_A, 0)); \overrightarrow{\theta}\right]\right\}.$$

By Lemma 3, the owner's beliefs, $\theta^a(\Pi(I_A, 0))$, are given by (10), where

$$\tilde{\theta}^a(\Pi(I_A, 0)) = \theta_T + \frac{1}{2}\ln\left[\frac{w_B}{w_A + V_A + I_A}\right]. \tag{13}$$

Thus, $U_1(\Pi^a(I_A, 0)) = \mathbb{E}[u(\Pi^a(I_A, 0)); \theta^a(\Pi^a(I_A, 0))]$, and the corresponding investment is $I_A^a \equiv \arg\max_{I_A} U_1(\Pi^a(I_A, 0))$. Similarly, if the firm discovers a diversifying project, the availability of an investment project involving type-B assets leads the owner-manager to an investment level of I_B^a, where the portfolio-distorted beliefs are now

$$\tilde{\theta}^a(\Pi(0, I_B) = \theta_T + \frac{1}{2}\ln\left[\frac{w_B + I_B}{w_A + V_A}\right]. \tag{14}$$

When deciding I_τ^a, the owner is sophisticated in that he anticipates the impact of his investment choice on his own beliefs, θ^a. This implies that the agent has "no regret" in the sense that the agent will not change beliefs after the investment I_τ^a is made (and, thus, it remains optimal after it is implemented). It also means that the optimal level of investment is determined by two effects. The first effect is the traditional "marginal cost" effect that is due to the convexity of the cost function, $c(I_\tau)$. The second effect is a "pessimism" effect due to uncertainty aversion: by increasing investment in the type-τ asset, from (13) and (14) the owner changes his beliefs in a way that he becomes more pessimistic about that asset. Thus, the owner limits the investment in those assets. These considerations lead to the following theorem.

Theorem 1 *Optimal levels of investment $\{I_A^a, I_B^a\}$ depend on the owner's pre-existing portfolio. An increase of w_τ leads to a decrease of I_τ^a and an increase of $I_{\tau'}^a$, $\tau' \neq \tau$.*

Theorem 1 implies that, with uncertainty aversion, investment depends on the decision-maker's portfolio. Specifically, optimal investment in a project is decreasing in the owner's exposure to the same asset, and increasing in the exposure to the other asset. This property makes investment projects effectively complementary. This spillover effect happens because an increase in the endowment of one type of asset makes an uncertainty-averse agent relatively more pessimistic about that asset and more optimistic about the other asset, resulting in a decrease in I_τ^a and an increase in $I_{\tau'}^a$.

For the remainder of the paper, we consider the case where the outsider owns fraction $(1-\beta)$ of the firm, in addition to an endowment external to the firm, $\{w_A, w_B, w_0\}$.[25] Thus, the outsider's initial portfolio is $\Pi^S \equiv \{w_A + (1-\beta) V_A, w_B, w_0 + (1-\beta) V_0\}$. Further, we discuss the special case in which the outsider has a balanced overall endowment: $w_B = w_A + (1-\beta) V_A \equiv K$, where K characterizes the size of outsider's "external" portfolio. In contrast, the insider is not well diversified: her entire wealth is tied up in the remaining fraction β of the firm. Thus, the insider's initial portfolio is $\Pi^M \equiv \{\beta V_A, 0, \beta V_0\}$.

We believe the relevant case is when all parties are uncertainty averse. To build intuition, we study the four possible scenarios in which the insider and outsider can, in turn, be MEU or SEU maximizers.

[25] Our model can be easily extended to the case where the outsider owns a fraction of equity smaller than $1-\beta$.

2.1 Expected Utility Outsider, Expected Utility Insider

As a starting point, we begin with the simplest (and least interesting) case in which both parties are SEU maximizers and share a common belief $C = \{(\theta_T, \theta_T)\}$, or equivalently, $\beta = 0$. In this case, both parties choose the same investment levels for either a focused or a diversifying project and, thus, the allocation of control is irrelevant. In addition, as we show in Theorem 4 of Section 2.3, the assumption that $\theta_T \equiv \frac{1}{2}(\theta_L + \theta_H)$ and that the outsider's portfolio is better diversified together imply that the investment preferred ex-ante by the uncertainty-averse outsider is equal to the optimal level of investment for a SEU agent.

Theorem 2 *In the absence of uncertainty aversion, the insider and the outsider choose the same investment level $I_\tau^e = I^e$ for both projects, satisfying $c'(I_\tau^e) = e^{\theta_T - \theta_M}$, independent of who has control. Thus, allocation of control is irrelevant.*

When neither party is uncertainty averse, both insider and outsider share the same beliefs, θ_T, and they agree on the optimal level of investment, I_τ^e, for both the focused and the diversified project. Therefore, in the absence of uncertainty, control rights are irrelevant.[26]

2.2 Expected Utility Outsider, Uncertainty Averse Insider

We now consider the case in which the outsider is uncertainty neutral, while the insider is uncertainty averse. Because the outsider is uncertainty neutral, he chooses an investment level equal to I_τ^e if he retains control (Theorem 2). The uncertainty-averse insider, however, behaves differently.

If the firm has a focused project, that is in state ω_A, by investing I_A the insider obtains a portfolio $\Pi^M(I_A, 0) = \{\beta(V_A + I_A), 0, \beta[V_0 - c(I_A)]\}$. Given the portfolio $\Pi^M(I_A, 0)$, at $t = 1$ the insider's minimum expected utility is

$$U_1^M\left(\Pi^M(I_A, 0)\right) \equiv \min_{\theta \in C} \mathbb{E}\left[u\left(\Pi^M(I_A, 0)\right); \theta\right],$$

[26]Note that Theorem 2 assumes a common core of beliefs. Since, under Subjective Expected Utility, the core of beliefs is a singleton, this means that Theorem 2 effectively assumes that agents have common beliefs. With exogenous difference of opinion, the outsider believes that the insider will make an investment decision he perceives as inefficient, and he will always retain control.

where $\mathbb{E}\left[u\left(\Pi^{M}\left(I_{A},0\right)\right);\theta\right]=e^{\theta_{A}-\theta_{M}}\beta\left(V_{A}+I_{A}\right)+\beta\left(V_{0}-c\left(I_{A}\right)\right)$. Thus, under uncertainty aversion the insider's beliefs, $\theta^{M,a}\left(\Pi^{M}\left(I_{A},0\right)\right)$, are determined by minimization of her expected utility, that is

$$\theta^{M,a}\left(\Pi^{M}\left(I_{A},0\right)\right)=\underset{\theta\in C}{\arg\min}\left\{e^{\theta_{A}-\theta_{M}}\beta\left(V_{A}+I_{A}\right)+\beta\left[V_{0}-c\left(I_{A}\right)\right]\right\}.$$

Because the insider holds only type A assets, the beliefs held by the uncertainty-averse insider, $\theta^{M,a}\left(\Pi^{M}\left(I_{A},0\right)\right)$ are the most pessimistic toward type A assets and optimistic toward type B assets

$$\theta_{A}^{M,a}\left(\Pi^{M}\left(I_{A},0\right)\right)=\theta_{L}:\ \theta_{B}^{M,a}\left(\Pi^{M}\left(I_{A},0\right)\right)=\theta_{H} \tag{15}$$

Since the insider's portfolio is not well diversified, the insider's beliefs give maximum weight to the priors that are least favorable to the only risky asset in which have a long position, asset A. Given the insider's beliefs the optimal investment $I_{A}^{M,a}$ is determined by maximizing the insider's minimum expected utility

$$I_{A}^{M,a}=\ \underset{I_{A}}{\arg\max}\ U_{1}^{M}\left(\Pi^{M}\left(I_{A},0\right)\right), \tag{16}$$

where the insider's MEU is $U_{1}^{M}\left(\Pi^{M}\left(I_{A},0\right)\right)=\mathbb{E}\left[u\left(\Pi^{M}\left(I_{A},0\right)\right);\left(\theta_{L},\theta_{H}\right)\right]$. The optimal level of investment, $I_{A}^{M,a}$, is set by the insider under the "worst-case scenario" belief that $\theta_{A}^{M,a}\left(\Pi^{M}\left(I_{A},0\right)\right)=\theta_{L}$. This makes the insider very conservative when making focused investments.

Similarly, if the firm has a diversifying project, that is in state ω_{B}, by investing I_{B} the insider obtains a portfolio $\Pi^{M}\left(0,I_{B}\right)=\left\{\beta V_{A},\beta I_{B},\beta\left(V_{0}-c\left(I_{B}\right)\right)\right\}$. Thus, the insider's minimum expected utility is

$$U_{1}^{M}\left(\Pi^{M}\left(0,I_{B}\right)\right)=\min_{\theta}\mathbb{E}\left[u\left(\Pi^{M}\left(0,I_{B}\right)\right);\overrightarrow{\theta}\right],$$

where $\mathbb{E}\left[u\left(\Pi^{M}\left(0,I_{B}\right)\right);\overrightarrow{\theta}\right]=e^{\theta_{A}-\theta_{M}}\beta V_{A}+e^{\theta_{B}-\theta_{M}}\beta I_{B}+\beta\left[V_{0}-c\left(I_{B}\right)\right]$. Insider's beliefs are determined by minimization of her expected utility:

$$\theta^{M,a}\left(\Pi^{M}\left(0,I_{B}\right)\right)=\ \underset{\theta\in C}{\arg\min}\left\{e^{\theta_{A}-\theta_{M}}\beta V_{A}+e^{\theta_{B}-\theta_{M}}\beta I_{B}+\beta\left[V_{0}-c\left(I_{B}\right)\right]\right\}.$$

By Lemma 3, beliefs held by an uncertainty-averse insider, toward the diversifying project are

$$\theta_B^{M,a}\left(\Pi^M(0,I_B)\right)=\begin{cases}\theta_L & \tilde{\theta}_B^{M,a}\left(\Pi^M(0,I_B)\right)\leq\theta_L\\ \tilde{\theta}_B^{M,a}\left(\Pi^M(0,I_B)\right) & \tilde{\theta}_B^{M,a}\left(\Pi^M(0,I_B)\right)\in(\theta_L,\theta_H)\\ \theta_H & \tilde{\theta}_B^{M,a}\left(\Pi^M(0,I_B)\right)\geq\theta_H\end{cases}, \tag{17}$$

where

$$\tilde{\theta}_B^{M,a}\left(\Pi^M(0,I_B)\right)=\theta_T+\frac{1}{2}\ln\left(\frac{V_A}{I_B}\right). \tag{18}$$

Given the insider's beliefs, $\theta^{M,a}(\Pi^M(0,I_B))$, the optimal investment $I_B^{M,a}$ chosen by the insider is determined by maximizing the insider's minimum expected utility,

$$I_B^{M,a}=\arg\max_{I_B} U_1^M\left(\Pi^M(0,I_B)\right), \tag{19}$$

where insider's MEU is $U_1^M\left(\Pi^M(0,I_B)\right)=\mathbb{E}\left[u\left(\Pi^M(0,I_B)\right);\theta^{M,a}\left(\Pi^M(0,I_B)\right)\right]$. The optimal investment policy of uncertainty-averse insider is characterized in the following.

Theorem 3 *If in control, the uncertainty-averse insider underinvests in focused projects relative to the investment desired by the SEU outsider. Her investment in diversifying projects depends on firm characteristics, and is an increasing function of the value of assets in place, V_A: if assets in place are sufficiently large, $V_A > I^e$, the insider overinvests in diversifying projects; otherwise, if $V_A < I^e$, she underinvests in diversifying projects. Thus, the uncertainty-neutral outsider prefers not delegating control to an uncertainty-averse insider.*

Because the insider holds only type-A assets, a priori, she places a lower value on type-A assets than an SEU investor. Thus, the insider underinvests in focused projects relative to the investment that is optimal for the outsider, I^e. The extent of underinvestment in the focused project becomes more severe when uncertainty $\alpha=\theta_T-\theta_L$ is larger.

Investment in type-B projects depends on the size of the assets in place, V_A, relative to the size of type-B assets that firm will have after the investment is made. When the firm has a sufficiently large endowment of assets in place, that is, if $V_A > I^e$, the insider finds it desirable to invest relatively more in the diversifying project than the outsider, leading to overinvestment. In contrast, if the size of assets is relatively small, $V_A \leq I^e$, the insider prefers to limit exposure to type-B assets, and she underinvests. In either case, the insider's investment policy will differ from the one preferred by the SEU outsider, who always retain control.

Corollary 2 *Outsider loss of welfare from delegation of control to the insider is an inverted U-shaped function of V_A.*

From Theorem 3 we know that it is never optimal to grant control to the insider. This implies that if control is delegated to the insider, it will always have a negative impact on firm value. In addition, the impact is an inverted U-shaped function of V_A. This means that the loss of value due to delegation of decision making is greater at the extreme cases, either for very young firms where investment is considerably larger than assets in place, $V_A < I^e$, or for mature firm, where investment is substantially smaller that assets in place, $V_A > I^e$. The intuition for this is simple: the insider invests in focused projects according to the worst-case scenario, $\theta_A^M = \theta_L$, so the negative impact on firm value from a focused project is independent of V_A. However, the investment in the diversifying project, $I_B^{M,a}$, is increasing in V_A. When $V_A = I^e$, $I_B^{M,a} = I^e$, so the insider chooses the diversifying investment optimally from the outsider's perspective. Any departure from $V_A = I^e$ results in a greater loss from entrenchment.

2.3 Uncertainty Averse Outsider, Expected Utility Insider

We show that the uncertainty-averse outsider finds optimal delegation of authority to an uncertainty-neutral insider. There are two reasons why the uncertainty-averse outsider prefers to grant control to an uncertainty-neutral insider. First, the diversified uncertainty-averse outsider ex-post prefers to underinvest relative to what he would have wanted to invest ex-ante. This effect is due to the impact of the arrival of the new project on the outsider's ex-post beliefs. Second, the uncertainty-averse outsider would prefer not to learn the realization at $t = 1$ of the state of the world ω_τ, that is, to learn the kind of projects that becomes available to the firm in the intermediate date. This effect is due to the harmful effect of the arrival of new information on uncertainty-averse agents described in Lemma 1. These two effects make delegation of decision making to the insider (and firm opacity) ex-ante desirable to the outsider.

Consider first delegation: $\delta = d$. By Lemma 2, the outsider will optimally implement an opaque information structure, $\psi = O$. If the outsider delegates control to an insider who will select investment levels $\{I_A, I_B\}$, the outsider's ex-ante expected utility is determined as follows. While the outsider anticipates that the insider will implement investment of I_τ in state ω_τ, the outsider will not know which state of the world is realized, and thus, which type of project the firm has actually drawn. In addition, since there is no uncertainty on this random variable (by our simplifying assumption)[27] the outsider's expected utility at $t = 1$ is

$$\begin{aligned} \mathbb{E}\left[u\left(\Pi^S\left(I_A 1_{\tau=A}, I_B 1_{\tau=B}\right)\right);\theta\right] &= e^{\theta_A-\theta_M}\left[w_A + (1-\beta)\left(V_A + \frac{1}{2}I_A\right)\right] \qquad (20) \\ +e^{\theta_B-\theta_M}\left[w_B + (1-\beta)\frac{1}{2}I_B\right] &\quad +(1-\beta)\left[V_0 - \frac{1}{2}c\left(I_A\right) - \frac{1}{2}c\left(I_B\right)\right] \end{aligned}$$

where $1_{\tau=A}$ is the indicator variable for the state ω_A (it equals 1 if $l > 0$). Thus, the outsider's minimum expected utility is

$$U_1^S\left(\Pi^S\left(I_A 1_{\tau=A}, I_B 1_{\tau=B}\right)\right) = \min_{\overrightarrow{\theta}\in C}\mathbb{E}\left[u\left(\Pi^S\left(I_A 1_{\tau=A}, I_B 1_{\tau=B}\right)\right);\overrightarrow{\theta}\right].$$

Because the outsider learns nothing at $t = 1$, $U_0^{S,d} = U_1^{S,d}$ with probability 1. Thus, the outsider's payoff under delegation, $\delta = d$, is $U_0^{S,d}(I_A, I_B)$. What investment policy would the outsider prefer the insider implement? The optimal levels of investment, I_τ^{S*}, solve

$$\left\{I_A^{S*}, I_B^{S*}\right\} = \underset{I_A, I_B}{\arg\max}\ U_1^S\left(\Pi^S\left(I_A 1_{\tau=A}, I_B 1_{\tau=B}\right)\right)$$

and are characterized in the following lemma.

Lemma 5 *If the outsider does not retain control, the ex-ante optimal investment levels are* $I_A^{S*} = I_B^{S*} = I^e$.

Lemma 5 shows the outsiders would like to commit to the level of investment chosen in the absence of uncertainty. Because the insider is uncertainty neutral, she will select $I_A = I_B = I^e$ by Theorem 2. This result depends on our assumptions that the outsider has a balanced portfolio, $w_B = w_A + (1-\beta)V_A$, and that the SEU beliefs are θ_T. Therefore, by delegating control to an expected utility insider, the outsider earns a payoff $U_0^{S,d}(I^e, I^e)$.

[27]Because the insider will execute balanced investment, $I_A = I_B = I^e$, the outsider is indifferent between a focused project and a diversifying project. Thus, the results of this section follow even if he treats the randomization over project type as uncertain.

Alternatively, if the outsider retains control, the optimal levels of investment I_τ in state ω_τ is determined in a way similar to Theorem 1. With a focused project, that is, in state ω_A, an investment level of I_A gives the outsider the portfolio $\Pi^S(I_A, 0)$. Thus, the beliefs held by uncertainty-averse outsider, $\theta^{S,a}\left(\Pi^S(I_A,0)\right)$, are (10) where the portfolio-distorted beliefs, $\tilde{\theta}^{S,a}\left(\Pi^S(I_A,0)\right)$, are

$$\tilde{\theta}_A^{S,a}\left(\Pi^S(I_A,0)\right) = \theta_T + \frac{1}{2}\ln\left[\frac{w_B}{w_A + (1-\beta)(V_A + I_A)}\right]. \tag{21}$$

The optimal investment $I_A^{S,a}$ is determined by maximizing the outsider's minimum expected utility

$$I_A^{S,a} = \arg\max_{I_A} U_1^S\left(\Pi^S(I_A,0)\right), \tag{22}$$

where

$$U_1^S\left(\Pi^S(I_A,0)\right) = \mathbb{E}\left[u\left(\Pi^S(I_A,0)\right); \theta^{S,a}\left(\Pi^S(I_A,0)\right)\right], \text{ and} \tag{23}$$

$\mathbb{E}\left[u\left(\Pi^S(I_A,0)\right); \overrightarrow{\theta}\right] = e^{\theta_A - \theta_M}\left[w_A + (1-\beta)[V_A + I_A]\right] + e^{\theta_B - \theta_M} w_B + (1-\beta)\left[V_0 - c(I_A)\right]$. The optimal level of investment for uncertainty-averse outsider is determined by the combination of the "marginal cost" and the "pessimism" effects we discussed above. Note that the impact of "pessimism" depends on the outsider's overall portfolio. Because the outsider is diversified, $w_A + (1-\beta)[V_A + I_A] > w_B$ for $I_A > 0$. This implies $\tilde{\theta}_A^{S,a}\left(\Pi^S(I_A,0)\right) < \theta_T$ and, thus, that uncertainty-averse outsider is pessimistic ex-post on type-A assets relative to an SEU agent. This implies that $I_A^{S,a} < I_A^e$, or equivalently, the outsider prefers to underinvest in focused projects, relative to what he would like to commit to a priori from Lemma 5.

Similarly, when the firm has a diversifying project, that is in state ω_B, an investment level of I_B gives the outsider's the portfolio $\Pi^S(0, I_B)$. The outsider's beliefs, $\theta^{S,a}\left(\Pi^S(0,I_B)\right)$, are (10) where

$$\tilde{\theta}_B^{S,a}\left(\Pi^S(0,I_B)\right) = \theta_T + \frac{1}{2}\ln\left[\frac{w_A + (1-\beta)V_A}{w_B + (1-\beta)I_B}\right]. \tag{24}$$

Thus the optimal investment level, $I_B^{S,a}$, is chosen by the outsider by maximizing minimum expected utility

$$I_B^{S,a} = \arg\max_{I_B} U_1^S\left(\Pi^S(0,I_B)\right), \tag{25}$$

where

$$U_1^S\left(\Pi^S\left(0,I_B\right)\right)=\mathbb{E}\left[u\left(\Pi^S\left(0,I_B\right)\right);\theta^{S,a}\left(\Pi^S\left(0,I_B\right)\right)\right], \tag{26}$$

where

$$\begin{aligned}\mathbb{E}\left[u\left(\Pi^S\left(0,I_B\right)\right);\overrightarrow{\theta}\right] &= e^{\theta_A-\theta_M}\left[w_A+(1-\beta)V_A\right]+e^{\theta_B-\theta_M}\left[w_B+(1-\beta)I_B\right] \\ &\quad +(1-\beta)\left[V_0-c\left(I_B\right)\right].\end{aligned}$$

Because the outsider is diversified a priori, $w_B+(1-\beta)I_B>w_A+(1-\beta)V_A$ for $I_B>0$. This implies $\tilde{\theta}_B^{S,a}\left(\Pi^S\left(0,I_B\right)\right)<\theta_T$ and, thus, that uncertainty-averse outsider is pessimistic ex-post on type-B assets relative to the absence of uncertainty. This implies that $I_B^{S,a}<I_B^e$ and that the outsider prefers to underinvest in the diversifying project as well.

The above discussion implies that diversified uncertainty-averse outsider prefers to underinvest in both focused and diversifying projects, relative to what he would like to commit to ex ante from Lemma 5, leading to the following Lemma.

Lemma 6 *If retaining control, diversified uncertainty-averse outsider prefers the same investment in focused and diversified projects, and to underinvest due to uncertainty* $I_A^{S,a}=I_B^{S,a}<I^e$*. Underinvestment is more severe when the firm is large (relative to his overall portfolio).*

By combining (23) and (26) we obtain that if the outsider retain control, the outsider's ex-ante expected utility is

$$U_0^{S,r}\left(I_A^{S,a},I_B^{S,a}\right)=\frac{1}{2}U_1^S\left(\Pi^S\left(I_A^{S,a},0\right)\right)+\frac{1}{2}U_1^S\left(\Pi^S\left(0,I_B^{S,a}\right)\right) \tag{27}$$

Lemmas 5 and 6 lead us to Theorem 4, the main result for this section.

Theorem 4 *An uncertainty-averse outsider prefers to delegate control to an uncertainty-neutral insider.*

In summary, an uncertainty-averse outsider has two motivations to prefer granting control to an uncertainty-neutral insider. First, the outsider would like to commit ex-ante to a level of investment that is not optimal ex-post if they retain control. Because of the impact of uncertainty aversion on posterior beliefs, the outsider would prefer to underinvest ex-post in both types of projects. Second, by Lemma 2, the outsider would prefer to be blind to the realization of the interim state of the world ω_τ (i.e., the type of project available to the firm), because knowledge of the project type exposes him to additional uncertainty. Granting control to the insider thus allows the outsider to avoid uncertainty.

The desirability to an outsider of delegation depends on firm characteristics:[28]

Corollary 3 *Delegation is more desirable to the outsider when Z is greater, that is, when the growth options are more valuable.*

Corollary 3 follows for two reasons. First, as growth options increase in value, that is for greater values of Z, the outsider's preference for underinvestment worsens. This happens because an increase of the productivity of growth options Z, increases the values of both the investment level of the uncertainty-neutral insider, I^e, and the investment level of the uncertainty-averse outsider, $I_\tau^{S,a}$. However, the positive impact of Z on investment is greater in the case of I^e than $I_\tau^{S,a}$, making the underinvestment problem of retention of control more severe. The second effect is the adverse impact of information revelation on the uncertainty-averse outsider. Firms with more valuable growth options invest more (greater $I_\tau^{S,a}$), and the new investment becomes a larger portion of his portfolio. From (21) and (24) it easy to see that greater investment levels leads to greater dispersion of the posteriors, $\tilde{\theta}^{S,a}\left(\Pi^S\left(I_A, 0\right)\right)$ and $\tilde{\theta}^{S,a}\left(\Pi^S\left(0, I_B\right)\right)$, which in turn exacerbates the outsider's welfare loss due to uncertainty aversion. Together, these properties imply that while an uncertainty-averse outsider always prefers to grant control to an uncertainty-neutral insider, the value creation from delegation is an increasing function of the value of firm's growth options.

We conclude this section by noting that while the SEU insider and diversified MEU outsider agree ex-ante on the optimal level of investment in both projects, I^e, they disagree ex-post when they learn the state of the world ω_τ. In addition, the MEU outsider will be ex-post more "pessimistic" than the SEU insider. The ex-post disagreement between insiders and outsiders derives endogenously from the effect of uncertainty aversion on ex-post beliefs. In this way, this section mirrors results obtained in models with heterogenous priors.[29] However, in our model, the outsider is better off by delegating authority to the SEU insider, even in the face of ex-post disagreement. The value of delegation derives from

[28] In our comparative statics results, we assume $c\left(I\right) = \frac{1}{Z(1+\gamma)} I^{1+\gamma}$ for analytical tractability.

[29] Boot, Gopalan, and Thakor (2006) and (2008) and Boot and Thakor (2011) share these characteristics.

the combination of time-inconsistency of desired investment levels and the welfare loss of information arrival that characterizes MEU agents.

2.4 Uncertainty Averse Outsider, Uncertainty Averse Insider

The more interesting case is when both insider and outsider are uncertainty averse, which provides the core results of our paper. The choice of whether to implement a strong corporate governance system, and thus retain control, or to allow for a weak governance system and to delegate decision making to the firm's insider is based on the trade-off of two distinct effects.

First is the effect of control on investment. If the outsider retains control, he ex-post underinvest in both types of projects with respect to the level of investment that is preferred ex-ante, I^e. From Corollary 3, we know that this effect is more severe when the growth options are more valuable. In contrast, if given control, the insider always underinvests in focused projects, but either overinvests or underinvests in diversifying projects, depending on the relative size of the existing assets and the value of growth options (Theorem 3). In addition, from Corollary 2, the loss of value due to delegation is more severe when assets in place are very small or very large (relative to I^e).

Second is the negative impact of information resolution on uncertainty-averse agents. This effect always makes the outsider to prefer delegating control to the insider, all else equal. This effect is more severe when the outsider's posterior beliefs differ substantially from their prior beliefs. From (21) and (24), it is easy to see that this happens when the level of investment is large relative to relative to the owners' outside endowment, that is when the outsider has a "small" portfolio. We now characterize these trade-offs explicitly, and derive comparative statics.

If the outsider retains control, $\delta = r$, he behaves as described in Lemma 6. Thus, their payoff is equal to $U_0^{S,r}\left(I_A^{S,a}, I_B^{S,a}\right)$, defined in eq. (27). If given control, $\delta = d$, from Theorem 3, the insider chooses a level of investment $\{I_A^{M,a}, I_B^{M,a}\}$ as described in eq. (16) and (19), respectively. In this case, the outsider's expected utility is

$$\begin{aligned}\mathbb{E}\left[u\left(\Pi^S\left(I_A^{M,a}1_A, I_B^{M,a}1_B\right)\right); \overrightarrow{\theta}\right] &= e^{\theta_A-\theta_M}\left[w_A + (1-\beta)\left(V_A + \frac{1}{2}I_A^{M,a}\right)\right] \\ +e^{\theta_B-\theta_M}\left[w_B + (1-\beta)\frac{1}{2}I_B^{M,a}\right] &\quad + (1-\beta)\left[V_0 - \frac{1}{2}c\left(I_A^{M,a}\right) - \frac{1}{2}c\left(I_B^{M,a}\right)\right]. \end{aligned} \quad (28)$$

The outsider's ex-ante minimum expected utility and their payoff under delegation of control, $\delta = d$, is

$$U_0^{S,d}\left(I_A^{M,a}, I_B^{M,a}\right) = \min_{\overrightarrow{\theta} \in C} \mathbb{E}\left[u\left(\Pi^S\left(I_A^{M,a} 1_A, I_B^{M,a} 1_B\right)\right); \overrightarrow{\theta}\right].$$

The optimal allocation of control – that is, the strength of the firm's governance system – is then determined as follows. When the insider and the outsider prefer the same (or similar) levels of investment, the outsider is strictly better off delegating control to the insider than retaining control, i.e., to have a weak rather than a strong governance system. By delegating control to the insider, the outsider remains blind to the realization of the interim uncertainty, which increases their ex-ante payoff. In contrast, a strong governance system (that is, retention of control) is optimal when the insider chooses investment levels that are very inefficient with respect to the investment that the outsider would prefer if retained control. The optimal allocation of control depends on both firm characteristics, V_A, Z, and the outsider's overall portfolio size, K, as follows.

Theorem 5 *There are critical values* $\{\underline{V}_A, \overline{V}_A, \underline{Z}, \bar{Z}, \bar{K}\}$, *with* $\underline{V}_A \leq \overline{V}_A$, $\underline{Z} \leq \bar{Z}$, *and* $\bar{K} > 0$, *such that the outsider:*

1. *prefers to retain control for all* $V_A < \underline{V}_A$ *and for all* $V_A > \overline{V}_A$, *and delegate for all* $V_A \in \left(\underline{V}_A, \overline{V}_A\right)$;
2. *prefers to retain control for all* $Z < \underline{Z}$ *and delegate control for all* $Z > \bar{Z}$, *where* $\bar{Z}\left(V_A\right)$ *is U-shaped;*
3. *prefers to retain control if* $K \geq \bar{K}$.

A strong governance system is optimal when the value of assets in place, V_A, is either sufficiently small, $V_A \leq \underline{V}_A$, or sufficiently large, $V_A \geq \overline{V}_A$. In these cases, the insider and outsider strongly disagree on the optimal levels of investment. Specifically, relative to the outsider's desired investment levels, the insider underinvests in focused projects and in diversifying projects when $V_A \leq \underline{V}_A$, yet the insider overinvests in diversifying projects when $V_A \geq \overline{V}_A$. Thus, in both cases the outsider prefers to retain control in order to select a level of investment better aligned with their ex-ante objectives, even at the cost of being exposed to the adverse effect of information revelation. In the intermediate range, where $V_A \in \left(\underline{V}_A, \overline{V}_A\right)$, insider's and outsider's optimal investment policies are more closely aligned, limiting disagreement. Thus, weak governance, where the insider has more freedom to decide the firm's investment policy, is optimal.

A strong governance system is also optimal for less productive firms (low values of Z) or when the outsider has a sufficiently large portfolio (a large value of K). This happens because in both cases the realization of the project type (location l) has a small impact on the outsider's wealth levels. In this case, the adverse effect of information revelation on the outsider and the efficiency losses due to underinvestment are both small. Thus, the outsider is better off by establishing a strong governance system and retaining control. Conversely, in more productive firms (large Z) or when the firm is sufficiently large component of the outsider's portfolio (small K), the outsider optimally delegates control to the insider by implementing a weak governance system. Finally, note that the value of assets in place has a non-monotonic effect on the threshold $\bar{Z}\left(V_A\right)$. The effect of the corporate governance system on firm investment policy is examined in the following corollary.

Corollary 4 *Under retention, investment in diversified and focused projects are balanced. Under delegation, investment in diversified projects is larger than in focused projects.*

Corollary 4 has the interesting result that if control is retained by the outsider, the firm has balanced investment ($I_A^{S,a} = I_B^{S,a}$), while the insider, granted control, overinvests in diversifying projects ($I_B^{M,a} > I_A^{M,a}$). This means that firms endowed with a strong governance system follow a more balanced investment policy than firms endowed with a weak governance system, which overinvest in diversifying projects. These results hold even though governance is optimally chosen.

For the discussion in the remainder of this section, it is helpful to define the value of delegation as the difference in firm value under delegation and retention: $U_0^{S,d} - U_0^{S,r}$. Note that this difference can also be interpreted as the differential value of firms with weak and strong corporate governance systems, and is characterized in the following.

Corollary 5 *The value of delegation, $U_0^{S,d} - U_0^{S,r}$, is*

1. *decreasing in outside portfolio size K for diversified portfolios if $\gamma \geq \underline{\gamma} \equiv \frac{2\alpha}{\ln 2}$;*
2. *decreasing in outsider's endowment in w_B;*
3. *increasing in the productivity of the growth options, Z, if Z is large enough.*

Point 1 of Corollary 5 follows by a similar intuition to Theorem 5: increasing the size of the outside portfolio diminishes the impact on the outsider of the adverse effect of information revelation, reducing the benefits of delegation. The intuition for Point 2 is as follows. First, the value of delegation decreases in the size of the outside portfolio, as shown in Point 1. In addition, increasing w_B also increases the ex-ante disagreement between the outsider and insider, aggravating the difference of opinion on desired investment. Both of these effects make retention more attractive.[30] Finally, Point 3 derives from the fact that an increase of the productivity, Z, increases the value of the growth options and the uncertainty exposure of the outsider, making delegation more attractive. This happens only when the productivity parameter Z is sufficiently large, because an increase of productivity, Z, has an indeterminate effect on the disagreement between the insider and outsider on investment, which drives the costs of delegation. In the proof of Theorem 5, however, we have shown that when Z is sufficiently large, both insider's and outsider's beliefs converge to the lower end of the core-belief set (i.e., to the "worst-case" scenario), progressively eliminating this disagreement.

[30] Note that these effects work in opposite directions for w_A, so we cannot derive comparative statics for w_A. Numerical simulations, reported below, suggest the comparative statics are nonmonotonic in w_A.

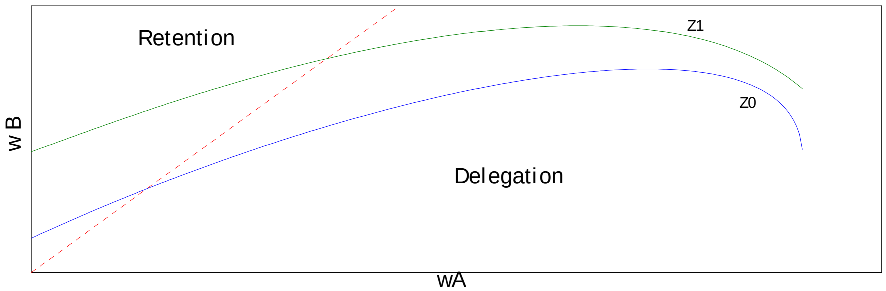

Figure 1: Indifference Curve for Outside Portfolio. The solid lines plot the indifference curve between delegation and retention for different levels of productivity, Z. The dotted line plots $w_B = w_A + (1 - \alpha) V_A$. An increase in w_B causes retention to be more attractive, while an increase in w_A is non-monotonic, favoring first delegation and then retention. When Z is larger, this cutoff increases: $Z1 > Z0$.

We now present several numerical comparative statics results that correspond to Theorem 5 and Corollary 5. Figure 1 shows the indifference curve between retention and delegation as a function of the outsider's endowment, w_A and w_B. First note that, as shown in Point 1 of Corollary 5, when the outsider has a diversified portfolio (that is, along the dotted line) weak governance is optimal when the outsider's portfolio is relatively small, that is, it is closer to the origin, and strong governance is optimal for larger outsider's portfolios, that is for values of w_A and w_B further away from the origin on the dotted line. Second, as shown in Point 2 of Corollary 5, retention is more attractive when w_B is larger. Finally, as anticipated, the effect of w_A is nonmonotonic. Increasing w_A decreases disagreement between the insider and outsider, making delegation more attractive. This happens because insider's and outsider's portfolios become more similar, decreasing disagreement. Increasing w_A also decreases the importance of new investment to the outsider, reducing the importance of uncertainty and, thus, making retention more attractive. This happens because the dispersion of ex post beliefs decreases as w_A increases. The disagreement effect dominates for small values of w_A,

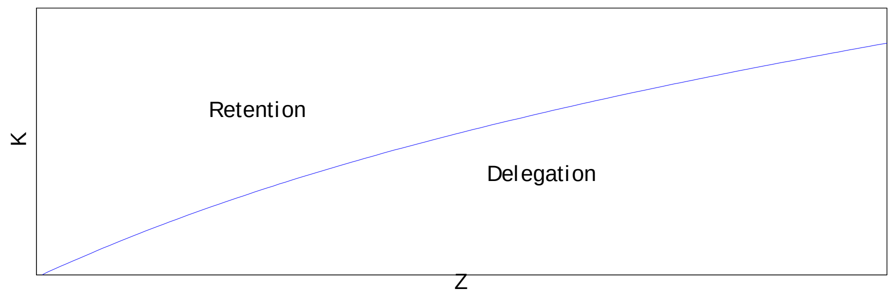

Figure 2: Indifference Curve for Productivity, Z, and Outside Portfolio, K. Increasing the outside portfolio of the outsider, K (where $w_A = K - (1-\alpha) V_A$ and $w_B = K$), makes retention more attractive, but increasing the growth options, Z, makes delegation more attractive.

while the portfolio effect dominates for large values of w_A, resulting in the inverted U-shaped relationship. Finally, as shown in Point 3 of Corollary 5, delegation is more attractive when Z, the value of growth options, is larger.

Figure 2 displays the indifference curve between strong governance (retention) and weak governance (delegation) as a function of the size of the outside portfolio, K, and the value of growth options, Z. The outsider's portfolio is assumed to be diversified, that is, $w_A + (1-\beta) V_A = w_B = K$. As shown in Point 1 of Corollary 5, strong governance (retention) is more attractive as the outside portfolio becomes larger. As shown in Point 3 of Corollary 5, weak governance (delegation) becomes more attractive as the value of growth options increases.

Finally, Figure 3 displays the indifference curve between strong governance (retention) and weak governance (delegation) as a function of productivity of growth options, Z, and the value of assets in place, V_A. First, note that, as suggested in Point 3 of Corollary 5, as productivity Z increases, delegation becomes more attractive. Second, note that, as stated in Point 2 of Theorem 5, the relationship between Z and V_A is a U-shaped function. This

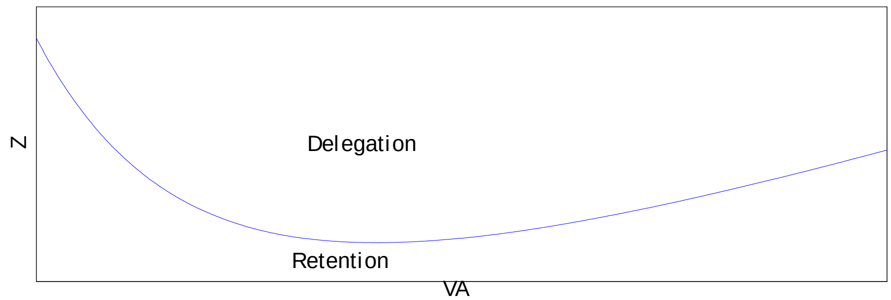

Figure 3: Indifference Curve for Assets in Place, V_A, and Productivity, Z. The line plots the indifference curve between delegation and retention against productivity, Z, and assets in place, V_A. An increase in Z causes delegation to be more attractive, while the relationship with V_A is nonmonotonic.

feature reflects the fact that, as shown in Theorem 5, for a given level of Z, for small values of V_A, retention is optimal, for intermediate values of V_A, delegation is optimal, while for larger values of V_A, retention is once again optimal.

3 Empirical Implications

We determine the optimal governance structure of firms in the presence of disagreement between firm insiders and outsiders. In our model, disagreement emerges endogenously among uncertainty-averse agents with heterogenous portfolios. We show that the strength of a firm's corporate governance system and the allocation of control preferred by the firm's outsiders depend on both firm characteristics and the overall portfolio composition of the firm's outsiders. This link between disagreement and economic fundamentals allows us to formulate the following empirical and policy implications.

1. Corporate governance life-cycle: If the value of a firm's assets in place increases over a firm's life cycle (relative to the value of its growth opportunities), Point 1 of Theorem

5 suggests that firms should follow a governance structure life cycle. In particular, in the earlier stages of development, a young, high-growth firm should have a strong governance system; as the firm ages, it should move to a weak corporate governance system where firm insiders have discretion over investment decisions. Finally, as the firm matures, it should revert back to a strong governance system. Because we expect it is easy to give control to a CEO but difficult to take back control, this suggests a role for LBOs as a mechanism for outside investors to regain control.

2. Diversified outsiders, where the firm represents a smaller fraction of their overall portfolio, prefer a strong governance system. This result follows from Theorem 5 and Corollary 5, which suggest that, all else equal, firms with diversified owners, such as a mutual fund, are more likely to have strong governance. They also suggest that outsiders whose portfolio is focused in sectors different from the firm's core business prefer a strong governance system, while outsiders whose portfolio has the same focus as the firm's core business are more likely to prefer a weak governance system. This means that generalist venture capital or private equity funds should impose strong governance systems on their portfolio companies, while specialized funds are more willing to tolerate weak governance systems, where the management of their portfolio companies have more leeway in determining company corporate policies. In addition, Corollary 5 also implies that diversified outsiders prefer to implement strong governance systems in firms with less productive growth options, but implement a weak governance system in firms with more productive growth options. This result suggests that, all else equal, more valuable firms and firms with more productive growth options (higher Z) should have weak governance. Firms with less productive growth options (smaller Z) should have strong governance.

3. A decline in firm productivity leads to a stronger corporate governance system. Point

3 in Corollary 5 shows that a weak governance system is more valuable to the outsider when the firm is more productive, and that the firm should switch to a strong corporate governance system when productivity decreases. This suggests that a weakening of a firm productivity leads to a strengthening of its corporate governance system. As suggested above, a stronger governance system may be obtained by having the outsider take over the firm through a LBO. This means that weaker firm performance may lead to going-private transactions.

4. *Weak corporate governance systems should also be less transparent.* Firms with weak corporate governance systems should also be more opaque by Lemma 2. If the outsider retains control, the firm implements a transparent information system so that the outsider can make an informed decision. If the outsider delegates control, it optimally implements an opaque information structure. Thus, the model predicts that outsider-controlled firms with a strong governance system will be more transparent, while insider-controlled firms with a weak governance system will be more opaque. Note that this prediction can explain the evidence presented in Coval, Jurek, and Stafford (2009), Henderson and Pearson (2011), and Célérier and Vallée (2017), which document the presence of an opacity premium for certain corporate products.

These observations have implications for the regulation of corporate disclosure. If the government were to implement mandatory disclosure regulation, firms would, in general, be harmed. Mandatory disclosure regulation destroys the benefit of delegation, insulation from uncertainty, so outsiders at all firms would prefer not to delegate. However, firms that would have found it optimal to delegate control to the insider would be harmed. The harmful effect of mandatory disclosure regulation would be worse if control rights have already been delegated to the insider. Dicks and Kim (2021) provide empirical evidence that the representative investor is information avoidant, due to the earnings announcement premium. Further, Dicks and Kim (2021) show that the earnings announcement premium behaves as a measure of firm-specific uncertainty.

Theorem 5 also suggests that private firms should have stronger governance and greater transparency with their shareholders than public firms. This happens because young private firms, that are at the earlier stage of development, have access to valuable growth opportunities and have large inside ownership. Our paper suggests that these firms should have strong governance and, therefore, be transparent with their shareholders. In contrast, more mature public firms have a greater proportion of assets in place relative to their growth opportunities. Our model predicts that these firms should optimally have a weaker and less transparent governance system, where insiders have more control on their firm's decision. Theorem 5 also suggests that private firms should have stronger governance and greater transparency with their shareholders than public firms. This happens because young private firms, that are at the earlier stage of development, have access to valuable growth opportunities and have large inside ownership. Our paper suggests that these firms should have strong governance and, therefore, be transparent with their shareholders. In contrast, more mature public firms have a greater proportion of assets in place relative to their growth opportunities. Our model predicts that these firms should optimally have a weaker and less transparent governance system, where insiders have more control over their firm's decision.

5. Weak-governance firms overinvest in diversifying projects relative to their investment in focused projects. Strong-governance firms implement balanced investment in focused and diversifying projects. This result, which follows directly from Corollary 4, implies that firms with weak corporate governance systems tend to be more diversified than comparable firms with a stronger governance system. In addition, weak-governance firms diversifying projects underperform ex post focused projects, while in strong-governance firms, ex-post performance is similar for focused and diversifying projects. This observation can be seen as follows. A measure of ex-post performance can be obtained by defining $R(I) \equiv I/c(I)$ as the return on investment for a given project. It is easy to verify that $R(I)$ is strictly decreasing in I (from convexity of $c(I)$). From Corollary 4, this implies that firms with weak governance systems underperform in their diversifying investments, $R\left(I_B^{M,a}\right) < R\left(I_A^{M,a}\right)$, while firms with strong governance systems have a more uniform performance across divisions.

4 Conclusions

We study a model where agents' uncertainty aversion generates endogenously differences of opinion between a firm's insider and outsider. We show that the allocation of control and, thus, the strength of the corporate governance system, depends on firm characteristics and the portfolio composition of both the insider and outsider. We predict that less productive firms should have stronger governance, while more productive firms should have weaker governance systems. Firms with weak corporate governance will overinvest in diversifying projects relative to their investment in focused projects, and the diversifying projects underperform the focused projects ex post. In addition, we predict that firms should display a corporate governance life cycle, where both younger and more mature firms should be characterized by a stronger corporate governance system, while firms at their intermediate stage have weaker governance, where the firm insiders have more discretion over corporate investment decisions. Finally, we argue that weaker governance systems are optimally less transparent.

References

[1] Acemoglu, D., Chernozhukov, V., and Yildiz, M. (2016) "Fragility of Asymptotic Agreement under Bayesian Learning," *Theoretical Economics*, **11**: 187-225.

[2] Adams, R. and Ferreira, D. (2007) "A Theory of Friendly Boards," *Journal of Finance*, **62**: 217-250.

[3] Aghion, P. and Tirole, J. (1997) "Formal and Real Authority in Organizations," *Journal of Political Economy*, **105**: 1-29.

[4] Anscombe, F. and Aumann, R. (1963) "A Definition of Subjective Probability," *The Annals of Mathematical Statistics*, **34**: 199-205.

[5] Ai, H. and Bansal, R. (2018) "Risk Preferences and the Macro Announcement Premium," *Econometrica*, **86**: 1383-1430.

[6] Bayar, O., Chemmanur, T., and Liu, M. (2011) "A Theory of Equity Carve-Outs and Negative Stub Values under Heterogeneous Beliefs," *Journal of Financial Economics*, **100**: 616-638.

[7] Becht, M., Bolton, P., and Roell, A. (2003) "Corporate Governance and Control," in: G.M. Constantinides, M. Harris, & R. M. Stulz (ed.), *Handbook of the Economics of Finance*, edition 1, volume 1, chapter 1, pages 1-109, Elsevier.

[8] Berle, A. and Means, G. (1933) *The Modern Corporation and Private Property*. New York: The MacMillan Company.

[9] Boot, A., Gopalan, R. and Thakor, A. (2006) "The Entrepreneur's Choice between Private and Public Ownership," *Journal of Finance*, **61**: 803–836.

[10] Boot, A., Gopalan, R. and Thakor, A. (2008) "Market Liquidity, Investor Participation and Managerial Autonomy: Why do Firms Go Private?" *Journal of Finance*, **63**: 2013-2059.

[11] Boot, A. and Thakor, A. (2001) "The Many Faces of Information Disclosure," *Review of Financial Studies*, **14**: 1021-1057.

[12] Boot, A. and Thakor, A. (2011) "Managerial Autonomy, Allocation of Control Rights, and Optimal Capital Structure," *Review of Financial Studies*, **24**: 3434-3485.

[13] Caskey, J. (2009) "Information in Equity Markets with Ambiguity-Averse Investors," *Review of Financial Studies*, **22**: 3595-3627.

[14] Célérier, C. and Vallée, B. (2017) "Catering to Investors Through Security Design: Headline Rate and Complexity" *Quarterly Journal of Economics* **132**: 1469-1508.

[15] Chakraborty A. and Yilmaz, B. (2017) "Authority, Consensus, and Governance," *Review of Financial Studies*, **30**: 4267-4316.

[16] Coval, J., Jurek, J., and Stafford, E. (2009) "Economic Catastrophe Bonds," *American Economic Review*, **99**: 628-666.

[17] Cover, T. and Thomas, J. (2006) *Elements of Information Theory* (second edition). Hoboken, New Jersey: Wiley-Interscience, John Wiley & Sons, Inc.

[18] Cripps, M., Ely, J., Mailath, G. and Samuelson, L. (2008) "Common Learning," *Econometrica*, **76**: 909-933.

[19] de Finetti, B. (1974) *Theory of Probability: A Critical Introductory Treatment.* John Wiley & Sons Ltd.

[20] Dessein, W. (2002) "Authority and Communication in Organizations," *Review of Economic Studies*, **69**: 811-838.

[21] Dicks, D. and Fulghieri, P. (2019) "Uncertainty Aversion and Systemic Risk," *Journal of Political Economy,* **127**: 1118-1155.

[22] Dicks, D. and Fulghieri, P. (2021) "Uncertainty, Investor Sentiment, and Innovation," *Review of Financial Studies* **34**: 1236-1279.

[23] Dicks, D. and Fulghieri, P. (2022) "Uncertainty, Contracting, and Beliefs in Organizations," SSRN Working Paper 3285458.

[24] Dicks, D. and Kim, H. (2021) "The Earnings Announcement Premium as Uncertainty Aversion: Theory and Evidence," SSRN Working Paper 3424379.

[25] Edmans. A. (2014) "Blockholders and Corporate Governance," *Annual Review of Financial Economics*, **6**: 23-50.

[26] Edmans, A. and C. Holderness (2017) "Blockholders: A Survey of Theory and Evidence," in B. Hermalin and M. Weisbach (eds), *The Handbook of the Economics of Corporate Governance*, Volume 1, Chapter 8, Elsevier, 541-636.

[27] Edmans, A. and Manso, G. (2011) "Governance Through Trading and Intervention: A Theory of Multiple Blockholders," *Review of Financial Studies,* **24**: 2395–2428.

[28] Epstein, L. and Schneider, M. (2010) "Ambiguity and Asset Markets," *Annual Review of Financial Economics*, **2**: 315-346.

[29] Ferreira, D. and Rezende, M. (2007) "Corporate Strategy and Information Disclosure," *Rand Journal of Economics*, **38**: 164-184.

[30] Fishman, M. and Hagerty, K. (2003) "Mandatory Versus Voluntary Disclosure in Markets with Informed and Uninformed Customers," *Journal of Law, Economics, and Organization*, **19**: 45-63.

[31] Garlappi, L., Giammarino, R., and Lazrak, A. (2014) "Ambiguity and the Corporation: Group Decisions, Time Inconsistency, and Underinvestment," SSRN Working Paper 2510235.

[32] Gilboa, I. and Schmeidler, D. (1989) "Maxmin Expected Utility with a Non-Unique Prior," *Journal of Mathematical Economics*, **18**: 141-153.

[33] Gopalan, R., Milbourn, T., and Song, F. (2010) "Strategic Flexibility and the Optimality of Pay for Sector Performance," *Review of Financial Studies*, **23**: 2060-2098.

[34] Grossman, S. (1976) "On the Efficiency of Competitive Stock Markets Where Traders Have Diverse Information," *Journal of Finance* **31**: 573-585.

[35] Hansen, L. and Sargent, T. (2001) "Robust Control and Model Uncertainty," *American Economic Review*, **91**: 60-66.

[36] Hansen, L. and Sargent, T. (2008) *Robustness.* Princeton University Press.

[37] Harris, M. and Raviv, A. (1993) "Differences of Opinion Make a Horse Race," *Review of Financial Studies*, **6**: 473-506.

[38] Harris, M. and Raviv A. (2005) "Allocation of Decision-Making Authority," *Review of Finance*, **9**: 353-383.

[39] Harris, M. and Raviv, A. (2008) "A Theory of Board Control and Size," *Review of Financial Studies*, **21**: 1797-1832.

[40] Harris, M. and Raviv, A. (2010) "Control of Corporate Decisions: Shareholders vs. Management," *Review of Financial Studies*, **23**: 4115-4147.

[41] Henderson, B. and Pearson, N. (2011) "The Dark Side of Financial Innovation: A Case Study of the Pricing of a Retail Financial Product," *Journal of Financial Economics*, **100**: 227-47.

[42] Holderness, C. (2009) "The Myth of Diffuse Ownership in the United States," *Review of Financial Studies*, **22**: 1377-1408.

[43] Huang, S. and Thakor A. (2013) "Investor Heterogeneity, Investor-Management Disagreement and Share Repurchases," *Review of Financial Studies*, forthcoming.

[44] Jensen, M. (1989) "Eclipse of the Public Corporation," *Harvard Business Review*, **67**:5, 61-74.

[45] Kandel, E. and Pearson, N. (1995) "Differential Interpretation of Public Signals and Trade in Speculative Markets," *Journal of Political Economy,* **104**: 831-872.

[46] Klibanoff, P., Marinacci, M. and Mukerji, S. (2005) "A Smooth Model of Decision Making under Ambiguity," *Econometrica*, **73**: 1849–1892.

[47] Kogan, S., Routledge, B., Sagi, J., and Smith, N. (2010) "Information Content of Public Firm Disclosures and the Sarbanes-Oxley Act," SSRN Working Paper 1584763.

[48] Levit, D. and Malenko, N. (2016) "The Labor Market for Directors and Externalities in Corporate Governance," *Journal of Finance,* **71**: 775-808.

[49] Maccheroni, F., Marinacci, M., and Rustichini, A. (2006) "Ambiguity Aversion, Robustness, and the Variational Representation of Preferences," *Econometrica*, **74**: 1447-1498.

[50] Morris, S. (1994) "Trade with Heterogeneous Prior Beliefs and Asymmetric Information," *Econometrica*, **62**: 1327-1347.

[51] Morris, S. (1995) "The Common Prior Assumption in Economic Theory," *Economics and Philosophy*, **11**: 227-253.

[52] Mukerji, S. and Tallon, J. (2001) "Ambiguity Aversion and Incompleteness of Financial Markets," Review of Economic Studies, **68**: 883-904.

[53] Rappaport, A. (1990) "The Staying Power of the Public Corporation," *Harvard Business Review*, **68**:1, 96-104.

[54] Ross, S. (1973) "The Economic Theory of Agency: The Principal's Problem," *American Economic Review*, **63**: 134-139.

[55] Sato, Y. (2014) "Opacity in Financial Markets," *Review of Financial Studies*, 27: 3502-3546.

[56] Shleifer, A. and Vishny, R. (1986) "Large Shareholders and Corporate Control," *Journal of Political Economy* **94**: 461–488.

[57] Shleifer, A. and Vishny R. (1997) "A Survey of Corporate Governance," *Journal of Finance*, **52**: 737-783.

[58] Siniscalchi, M. (2011) "Dynamic Choice under Ambiguity," *Theoretical Economics*, **6**: 379–421.

[59] Smith, A. (1776) *An Inquiry into the Nature and Causes of the Wealth of Nations.* Edwin Cannan, ed. 1904. Library of Economics and Liberty. 20 September 2013. <http://www.econlib.org/library/Smith/smWN.html>.

[60] Strzalecki (2011) "Axiomatic Foundations of Multiplier Preferences." *Econometrica* **79**: 47-73.

[61] Thakor, A. (2013) "Strategic Information Disclosure When There Is Fundamental Disagreement," *Journal of Financial Intermediation*, forthcoming.

[62] Van den Steen, E. (2004) "Rational Overoptimism (and Other Biases)," *American Economic Review*, **94**: 1141-1151.

[63] Van den Steen, E. (2009) "Authority versus Persuasion," *American Economic Review: Papers and Proceedings* **99**: 448–453.

[64] Van den Steen, E. (2010a) "Disagreement and the Allocation of Control." *Journal of Law, Economics & Organization* **26**: 385–426.

[65] Van den Steen, E. (2010b) "Interpersonal Authority in a Theory of the Firm," *American Economic Review*, **100**: 466–490.

[66] Winton, A. (1993) "Limitation of Liability and the Ownership Structure of the Firm," *Journal of Finance* **48:** 487–512.

A Internet Appendix: Proofs

Proof of Lemma 1. Let $x = \{x_A, x_B\}$ be a vector of indicator variables for success of type A and B assets: $x \in \{0,1\}^2$. If the probability of success is $p = \{p_A, p_B\}$ the probability of x is $p_A^{x_A} p_B^{x_B} (1-p_A)^{1-x_A} (1-p_B)^{1-x_B}$. Thus, the relative entropy of p w.r.t. $\hat{p}$ is

$$R(p|\hat{p}) = \sum_{x\in\{0,1\}^2} p_A^{x_A} p_B^{x_B} (1-p_A)^{1-x_A} (1-p_B)^{1-x_B} \ln \frac{p_A^{x_A} p_B^{x_B} (1-p_A)^{1-x_A} (1-p_B)^{1-x_B}}{\hat{p}_A^{x_A} \hat{p}_B^{x_B} (1-\hat{p}_A)^{1-x_A} (1-\hat{p}_B)^{1-x_B}}.$$

Because the log of a product is the sum of the logs, and probabilities sum to one, we can express this as

$$R(p|\hat{p}) = R(p_A|\hat{p}_A) + R(p_B|\hat{p}_B)$$

where $R(p_\tau|\hat{p}_\tau) = p_\tau \ln \frac{p_\tau}{\hat{p}_\tau} + (1-p_\tau) \ln \frac{1-p_\tau}{1-\hat{p}_\tau}$. Because $\frac{\partial^2 R}{\partial p_\tau^2} = \frac{\hat{p}_\tau}{p_\tau} + \frac{1-\hat{p}_\tau}{1-p_\tau}$, $R(p_\tau|\hat{p}_\tau)$ is strictly convex in p_τ. Thus, $R(p|\hat{p})$ is strictly convex in $p = \{p_A, p_B\}$. Also, $\lim_{p_\tau\to 0^+} R(p_\tau|\hat{p}_\tau) = \ln \frac{1}{1-\hat{p}_\tau}$ and $\lim_{p_\tau\to 1^-} R(p_\tau|\hat{p}_\tau) = \ln \frac{1}{\hat{p}_\tau}$. Define $\underline{\eta}(\hat{p}) = \min_{\chi\in Q} \ln \frac{1}{\chi}$, where $Q = \{\hat{p}_A, 1-\hat{p}_A, \hat{p}_B, 1-\hat{p}_B\}$. Therefore, if $\eta < \underline{\eta}(\hat{p})$, $\mathcal{M}$, as the lower level set of a strictly convex function, is strictly convex. Note this generalizes: Theorem 2.5.3 of Cover and Thomas (2006) shows relative entropy is additively separable in independent variables, and Theorem 2.7.2 shows it is strictly convex.

Suppose an agent receives y_A if project A is successful and y_B if project B is successful, both of which are strictly positive. Note R is strictly convex. Because $\frac{\partial R}{\partial p_\tau} = \ln \frac{p_\tau}{\hat{p}_t} - \ln \frac{1-p_\tau}{1-\hat{p}_\tau}$, R achieves a minimum of zero at $p = \hat{p}$, so $\frac{\partial R}{\partial p_\tau} < 0$ for $p_\tau < \hat{p}_\tau$ and $\frac{\partial R}{\partial p_\tau} > 0$ for $p_\tau > \hat{p}_\tau$. The worst-case scenario solves

$$\begin{aligned} \min \quad & \{p_A y_A + p_B y_B\} \\ & R(p|\hat{p}) \leq \eta \end{aligned}$$

Let λ be the multiplier for the constraint, and L be the Lagrangian function. Thus, $L = -(p_A y_A + p_B y_B) - \lambda (R(p|\hat{p}) - \eta)$, so $\frac{dL}{dp_\tau} = -y_\tau - \lambda \frac{\partial R}{\partial p_\tau}$. At the worst-case scenario, $\frac{dL}{dp_\tau} = 0$. Because $y_\tau > 0$, it must be that $\lambda \frac{\partial R}{\partial p_\tau} < 0$. This requires not only that the constraint binds, $\lambda > 0$, but also that p_τ is on the decreasing portion of R, so $p_\tau < \hat{p}_\tau$. If the investor has strictly positive exposure to only one asset type, but not the other, say $y_\tau > 0$ but $y_{\tau'} = 0$, the worst-case scenario involves choosing the worst possible value of p_τ, $R(p_\tau|\hat{p}_\tau) = \eta$ for $p_\tau < \hat{p}_\tau$, and setting $p_{\tau'} = \hat{p}_{\tau'}$. Finally, if $y_A = y_B = 0$, the claim holds WLOG.

Information avoidance holds as a property of the minimum. Suppose we have a set of priors $\mu(w,s) \in \mathcal{M}$, so that each prior gives the joint distribution of wealth, w, and the signal, $s \in S$, and all priors share a common support of s. Define $\mu(\cdot|s)$ as the conditional distribution of wealth given s, and let $\mathcal{M}_s = \{\mu(\cdot|s) | \mu(w,s) \in \mathcal{M}\}$. For all μ and s, $E_\mu[u(w)] = E_s E_{\mu(\cdot|s)}[u(w)]$. Define $\underline{\nu} = \arg\min_{\mu\in\mathcal{M}} E_\mu[u(w)]$: ($\underline{\nu}$ is the worst-case scenario if the agent does not learn s), and $\underline{\nu}(\cdot;s)$ as the conditional distribution of $\underline{\nu}$ given s. Similarly, define $\nu(s) = \arg\min_{\mu(\cdot|s)\in\mathcal{M}_s} E_{\mu(\cdot|s)}[u(w)]$ (the worst-case scenario after the agent has learned s). Because $\underline{\nu}(\cdot|s) \in \mathcal{M}_s$, $E_{\underline{\nu}(\cdot|s)}[u(w)] \geq E_{\nu(s)}[u(w)]$, so $E_s E_{\underline{\nu}(\cdot|s)}[u(w)] \geq E_s E_{\nu(s)}[u(w)]$. Because $E_s E_{\underline{\nu}(\cdot;s)}[u(w)] = \min_{\mu\in\mathcal{M}} E_s E_{\mu(\cdot;s)}[u(w)]$, and $E_s E_{\nu(s)}[u(w)] = E_s \min_{\mu(\cdot;s)\in\mathcal{M}_s} E_{\mu|s}[u(w)]$, the claim is shown. ■

Proof of Lemma 2. If the outsider delegates control, $\delta = d$, opacity is optimal by Lemma 1. Suppose the outsider retains control, $\delta = r$, but attempts to implement opacity, $\psi = O$. Investment is only valuable if it is made at the right location, which the outsider does not know. Thus, the probability of success is 0, and thus the optimal investment is 0. Thus, the payoff from optimal investment under retention and opacity is the payoff from zero investment. Note that the same payoff could be achieved by selecting zero investment in both projects, even if $\psi = T$. Thus, given $\delta = r$, $\psi = T$ is weakly preferred to $\psi = O$. Because we will later show that the firm makes strictly positive investment in the project in equilibrium, this preference will be strict. ■

Proof of Lemma 3. The agent's worst-case scenario solves $U(\Pi) = \min u(\theta; \Pi)$ s.t. $\frac{1}{2}(\theta_A + \theta_B) = \theta_T$, where u is defined in (6). u is strictly convex in θ, so FOCs are sufficient for a minimum. Let L be the Lagrangian and λ be the multiplier: $\frac{\partial L}{\partial \theta_\tau} = -e^{\theta_\tau - \theta_M}\bar{w}_\tau + \frac{\lambda}{2}$, and substituting into $\frac{1}{2}(\theta_A + \theta_B) = \theta_T$, this implies

$$\tilde{\theta}^a_\tau(\Pi) = \theta_T + \frac{1}{2}\ln\frac{\bar{w}_{\tau'}}{\bar{w}_\tau}$$

If $\tilde{\theta}^a_\tau(\Pi) \in [\theta_L, \theta_H]$, $\theta^a_\tau = \check{\theta}^a_\tau$. If $\tilde{\theta}^a_\tau(\Pi) < \theta_L$, $\frac{\partial L}{\partial \theta_\tau} < 0$ for all $\theta_\tau \in [\theta_L, \theta_H]$, so $\theta^a_\tau = \theta_L$. If $\tilde{\theta}^a_\tau(\Pi) > \theta_H$, $\frac{\partial L}{\partial \theta_\tau} > 0$ for all $\theta_\tau \in [\theta_L, \theta_H]$, so $\theta^a_\tau = \theta_H$. Therefore, (10) corresponds to the worst-case scenario for an investor with portfolio Π. ■

Proof of Lemma 4. From (7), $\frac{dU}{d\bar{w}_\tau} = \frac{\partial \mathbb{E}(u)}{\partial \bar{w}_\tau} + \frac{\partial \mathbb{E}(u)}{\partial \theta^a_\tau}\frac{d\theta^a_\tau(\Pi)}{d\bar{w}_\tau} + \frac{\partial \mathbb{E}(u)}{\partial \theta^a_{\tau'}}\frac{d\theta^a_{\tau'}(\Pi)}{d\bar{w}_\tau}$. For corner solutions, $\frac{d\theta^a_\tau(\Pi)}{d\bar{w}_\tau} = \frac{d\theta^a_{\tau'}(\Pi)}{d\bar{w}_\tau} = 0$. For interior solutions, $\frac{\partial \mathbb{E}(u)}{\partial \theta^a_\tau} = \frac{\partial \mathbb{E}(u)}{\partial \theta^a_{\tau'}} = \frac{\lambda}{2}$, so the last two terms sum to $\lambda\frac{\partial}{\partial \bar{w}_\tau}\left\{\frac{1}{2}(\theta^a_A + \theta^a_B)\right\}$, which is zero because $\frac{1}{2}(\theta^a_A + \theta^a_B)$ is constant at θ_T. Therefore, $\frac{dU}{d\bar{w}_\tau} = \frac{\partial \mathbb{E}(u)}{\partial \bar{w}_\tau} = e^{\theta^a_\tau(\Pi) - \theta_M} > 0$, so $\frac{d^2U}{d(\bar{w}_\tau)^2} = e^{\theta^a_\tau(\Pi) - \theta_M}\frac{d\tilde{\theta}^a_\tau(\Pi)}{d\bar{w}_\tau} \leq 0$, because $\frac{d\tilde{\theta}^a(\Pi)}{d\bar{w}_\tau} \leq 0$ (with strict inequality for interior θ^a). ■

Proof of Theorem 1. Consider type τ projects. Applying the envelope theorem, the benefit of increasing investment is $\frac{dU_1}{dI_\tau} = e^{\theta_\tau - \theta_M} - c'(I_\tau)$, where θ_τ is from Lemma 3. Note $\frac{d^2U_1}{dI^2_\tau} = e^{\theta_\tau - \theta_M}\frac{d\theta_\tau}{dI_\tau} - c''(I_B)$, which is strictly negative because $\frac{d\theta_\tau}{dI_\tau} \leq 0$ (Lemma 4) and c is convex. Therefore, FOCs are sufficient for a maximum.

For comparative statics on I_τ, note $\frac{\partial}{\partial w_\tau}\frac{dU_1}{dI_\tau} = e^{\theta_\tau - \theta_M}\frac{\partial \theta_\tau}{\partial w_\tau}$. Because $\frac{\partial \theta_\tau}{\partial w_\tau} \leq 0$, with strict inequality for interior $\overrightarrow{\theta}$, $\frac{\partial}{\partial w_\tau}\frac{dU_1}{dI_\tau} \leq 0$, with strict inequality for interior θ^a. Because $\frac{d^2U_1}{dI^2_\tau} < 0$, $\frac{dI_\tau}{dw_\tau} \leq 0$, with strict inequality for interior $\overrightarrow{\theta}$. Therefore, optimal investment in a type of project is decreasing in the portfolio position of that type of assets. Similarly, for $\tau' \neq \tau$, $\frac{\partial}{\partial w_{\tau'}}\frac{dU_1}{dI_\tau} = e^{\theta_\tau - \theta_M}\frac{\partial \theta_\tau}{\partial w_{\tau'}}$, but $\frac{\partial \theta_\tau}{\partial w_{\tau'}} \geq 0$, so $\frac{dI_\tau}{dw_{\tau'}} \geq 0$ (strict inequality for interior θ^a). ■

Proof of Theorem 2. First, consider a focused project. If the outsider has control, they choose I_A to maximize

$$\mathbb{E}\left[u\left(\Pi^S(I_A, 0)\right); \theta_T\right] = e^{\theta_T - \theta_M}\left[w_A + (1-\beta)\left[V_A + I_A\right]\right] + e^{\theta_T - \theta_M}w_B + (1-\beta)\left[V_0 - c(I_A)\right],$$

so $\frac{d}{dI_A}E\left[u\left(\Pi^S(I_A, 0)\right); \theta^e\right] = (1-\beta)\left[e^{\theta_T - \theta_M} - c'(I_A)\right]$. Thus, the outsider sets $I^{S,e}_A$ so that $c'\left(I^{S,e}_A\right) = e^{\theta_T - \theta_M}$ ($\frac{d^2}{dI^2_A}E\left[u\left(\Pi^S(I_A, 0)\right); \theta_T\right] = -(1-\beta)c''(I_A)$ and c is convex, so SOCs are satisfied). If the insider has control, she chooses I_A to maximize

$$\mathbb{E}\left[u\left(\Pi^M\left(I_A,0\right)\right);\theta_T\right]=e^{\theta_T-\theta_M}\beta\left[V_A+I_A\right]+\beta\left[V_0-c(I_A)\right],$$

so $\frac{d}{dI_A}E\left[u\left(\Pi^M\left(I_A,0\right)\right);\theta_T\right]=\beta\left[e^{\theta_T-\theta_M}-c'\left(I_A\right)\right]$. Thus, the insider chooses $I_A^{M,e}$ so that $c'\left(I_A^{M,e}\right)=e^{\theta_T-\theta_M}$. Therefore, the outsider and the insider will choose the same level of investment for a focused project: $I_A^e\equiv I_A^{S,e}=I_A^{M,e}$.

Second, consider a diversified project. If the outsider has control, they choose I_B to maximize

$$\mathbb{E}\left[u\left(\Pi^S\left(0,I_B\right)\right);\theta_T\right]=e^{\theta_T-\theta_M}\left[w_A+(1-\beta)V_A\right]+e^{\theta_T-\theta_M}\left[w_B+(1-\beta)I_B\right]+(1-\beta)\left[V_0-c(I_B)\right],$$

so $\frac{d}{dI_B}E\left[u\left(\Pi^S\left(0,I_B\right)\right);\theta_T\right]=(1-\beta)\left[e^{\theta_T-\theta_M}-c'\left(I_B\right)\right]$. Thus, the outsider chooses $I_B^{S,e}$ so that $c'\left(I_B^{S,e}\right)=e^{\theta_T-\theta_M}$. If the insider has control, she chooses I_B to maximize

$$\mathbb{E}\left[u\left(\Pi^M\left(0,I_B\right)\right);\theta_T\right]=e^{\theta_T-\theta_M}\beta V_A+e^{\theta_T-\theta_M}\beta I_B+\beta\left[V_0-c(I_B)\right],$$

so $\frac{d}{dI_B}E\left[u\left(\Pi^M\left(0,I_B\right)\right);\theta_T\right]=\beta\left[e^{\theta_T-\theta_M}-c'\left(I_B\right)\right]$. Thus, the insider chooses $I_B^{M,e}$ so that $c'\left(I_B^{M,e}\right)=e^{\theta_T-\theta_M}$. Therefore, the insider and outsider will choose the same level of investment for a diversified project: $I_B^e\equiv I_B^{S,e}=I_B^{M,e}$. Because the same level of investment results independent of who is given control or which project is chosen, $I^e\equiv I_A^e=I_B^e$. Thus, the allocation of control does not matter. ■

Proof of Theorem 3. In this proof, we consider optimal behavior by the insider. Because the outsider is not averse to uncertainty, he will behave as in Theorem 2 if he retain control. Further, he will retain control iff the insider acts suboptimally from their perspective.

For focused projects, the insider's minimum expected utility is

$$U_1^M\left(\Pi^M(I_A,0)\right)=\min_\theta\mathbb{E}\left[u\left(\Pi^M\left(I_A,0\right)\right);\theta\right],$$

where $E\left[u\left(\Pi^M\left(I_A,0\right)\right);\theta\right]=e^{\theta_A-\theta_M}\beta\left(V_A+I_A\right)+\beta\left(V_0-c\left(I_A\right)\right)$. Because she is exposed only to type A assets, her worst-case scenario is $\theta_A^{M,a}\left(\Pi^M\left(I_A,0\right)\right)=\theta_L$ (Lemma 3). Thus, her objective becomes

$$U_1^M\left(\Pi^M\left(I_A,0\right)\right)=e^{\theta_L-\theta_M}\beta\left(V_A+I_A\right)+\beta\left(V_0-c\left(I_A\right)\right),$$

which implies $\frac{d}{dI_A}U_1^M\left(\Pi^M\left(I_A,0\right)\right)=\beta\left[e^{\theta_L-\theta_M}-c'\left(I_A\right)\right]$. Therefore, the insider chooses $I_A^{M,a}$ so that

$$c'\left(I_A^{M,a}\right)=e^{\theta_L-\theta_M}.$$

Because $\theta_L<\theta_T$, $I_A^{M,a}<I^e$, so the insider underinvests in focused projects.

For diversifying projects, the insider's objective is

$$U_1^M\left(\Pi^M\left(0,I_B\right)\right)=\min_\theta\mathbb{E}\left[u\left(\Pi^M\left(0,I_B\right)\right);\theta\right]$$

where $E\left[u\left(\Pi^M(0,I_B)\right);\theta\right]=e^{\theta_A-\theta_M}\beta V_A+e^{\theta_B-\theta_M}\beta I_B+\beta\left[V_0-c\left(I_B\right)\right]$. For a given choice of I_B, she has the portfolio $\Pi^M(0,I_B)=\{\beta V_A,\beta I_B,\beta\left(V_0-c\left(I_B\right)\right)\}$: her beliefs follow from Lemma 3 for a given level of investment I_B. Thus, her endogenous beliefs toward type τ assets are $\theta_\tau^{M,a}$:

$$\theta_\tau^{M,a}\left(\Pi^M(0,I_B)\right)=\begin{cases}\theta_L & \tilde{\theta}_\tau^{M,a}\left(\Pi^M(0,I_B)\right)\le\theta_L\\ \tilde{\theta}_\tau^{M,a}\left(\Pi^M(0,I_B)\right) & \tilde{\theta}_\tau^{M,a}\left(\Pi^M(0,I_B)\right)\in(\theta_L,\theta_H)\\ \theta_H & \tilde{\theta}_\tau^{M,a}\left(\Pi^M(0,I_B)\right)\ge\theta_H\end{cases},$$

where $\tilde{\theta}_A^{M,a}\left(\Pi^M(0,I_B)\right)=\theta_T+\frac{1}{2}\ln\left[\frac{I_B}{V_A}\right]$ and $\tilde{\theta}_B^{M,a}\left(\Pi^M(0,I_B)\right)=\theta_T+\frac{1}{2}\ln\left[\frac{V_A}{I_B}\right]$. Applying the minimax theorem, either $\frac{\partial\mathbb{E}u}{\partial\theta}=0$ or $\frac{d\theta^{M,a}}{dI_B}=0$, so $\frac{d}{dI_B}U_1^M\left(\Pi^M(0,I_B)\right)=\frac{\partial\mathbb{E}u}{\partial I_B}=\beta\left[e^{\theta_B-\theta_M}-c'\left(I_B\right)\right]$. Thus, the insider chooses investment $I_B^{M,a}$ so that

$$c'\left(I_B^{M,a}\right)=e^{\theta_B^{M,a}\left(\Pi^M(0,I_B)\right)-\theta_M}.$$

She may underinvest or overinvest in this situation. Because V_A only enters through beliefs $\theta_B^{M,a}\left(\Pi^M(0,I_B)\right)$, and $\frac{\partial\tilde{\theta}_B^{M,a}}{\partial V_A}=\frac{1}{2V_A}>0$, $\frac{dI_B^{M,a}}{dV_A}>0$.

The optimal investment under expected utility, I^e, satisfies $c'\left(I^e\right)=e^{\theta_T-\theta_M}$ (Theorem 2). If $V_A=I^e$, it follows that $I_B^{M,a}=I^e$, because $\tilde{\theta}_B^{M,a}\left(\Pi^M(0,I^e)\right)=\theta_T$, so $c'\left(I^e\right)=e^{\theta_T-\theta_M}$. Because $\frac{dI_B^{M,a}}{dV_A}>0$, $I_B^{M,a}>I^e$ when $V_A>I^e$ and $I_B^{M,a}<I^e$ when $V_A<I^e$. Therefore, the insider overinvests in diversifying projects if $V_A>I^e$ but underinvests if $V_A<I^e$. Because the insider always underinvests in focused projects, and invests with distortions a.s. in diversifying projects, the SEU outsider refuses to delegate control to her. ■

Proof of Corollary 2. If the outsider delegates control to the insider, his payoff is

$$\begin{aligned}U_0^{S,d} &= e^{\theta_T-\theta_M}\left[w_A+(1-\beta)\left(V_A+\frac{1}{2}I_A^{M,a}\right)\right]+e^{\theta_T-\theta_M}\left[w_B+(1-\beta)\frac{1}{2}I_B^{M,a}\right]\\ &\quad+(1-\beta)\left[V_0-\frac{1}{2}c\left(I_A^{M,a}\right)-\frac{1}{2}c\left(I_B^{M,a}\right)\right].\end{aligned}$$

If he retains control, however, his payoff is

$$\begin{aligned}U_0^{S,r} &= e^{\theta_T-\theta_M}\left[w_A+(1-\beta)\left(V_A+\frac{1}{2}I^e\right)\right]+e^{\theta_T-\theta_M}\left[w_B+(1-\beta)\frac{1}{2}I^e\right]\\ &\quad+(1-\beta)\left[V_0-\frac{1}{2}c\left(I^e\right)-\frac{1}{2}c\left(I^e\right)\right].\end{aligned}$$

If the insider is exogenously granted control, the impact on the outsider's utility is $\Delta=U_0^{S,d}-U_0^{S,r}$, which simplifies to

$$\Delta=\frac{1}{2}(1-\beta)\left[\left(\rho\left(I_A^{M,a}\right)-\rho\left(I^e\right)\right)+\left(\rho\left(I_B^{M,a}\right)-\rho\left(I^e\right)\right)\right]$$

where $\rho(I)=e^{\theta_T-\theta_M}I-c(I)$, the outsider's payoff from investing I in either project. Note $\rho'(I)=e^{\theta_T-\theta_M}-c'(I)$ and $\rho''(I)=-c''(I)<0$. I^e maximizes ρ because $c'\left(I^e\right)=e^{\theta_T-\theta_M}$, so Δ is strictly negative.

Neither I^e nor $I_A^{M,a}$ ($I_A^{M,a}$ satisfies $c'\left(I_A^{M,a}\right) = e^{\theta_L - \theta_M}$) depend on V_A, so neither $\rho\left(I_A^{M,a}\right)$ nor $\rho\left(I^e\right)$ depend on V_A. Theorem 3 showed that $I_B^{M,a}$ is increasing in V_A, and that $I_B^{M,a} = I^e$ when $V_A = I^e$. Thus, an increase in V_A increases Δ when $V_A < I^e$ but decreases Δ when $V_A > I^e$, resulting in the inverted U-shaped relationship. ■

Proof of Lemma 5. Suppose the outsider knows he will not know which type of project the firm draws, but he anticipates that investment of I_A and I_B will be implemented. Thus, his MEU is

$$U_1^S\left(\Pi^S\left(I_A 1_{\tau=A}, I_B 1_{\tau=B}\right)\right) = \min_{\overrightarrow{\theta}\in C} \mathbb{E}\left[u\left(\Pi^S\left(I_A 1_{\tau=A}, I_B 1_{\tau=B}\right)\right); \overrightarrow{\theta}\right],$$

where

$$\begin{aligned}\mathbb{E}\left[u\left(\Pi^S\left(I_A 1_{\tau=A}, I_B 1_{\tau=B}\right)\right); \theta\right] &= e^{\theta_A - \theta_M}\left[w_A + (1-\beta)\left(V_A + \frac{1}{2} I_A\right)\right] \\ &\quad + e^{\theta_B - \theta_M}\left[w_B + (1-\beta)\frac{1}{2} I_B\right] + (1-\beta)\left[V_0 - \frac{1}{2}c\left(I_A\right) - \frac{1}{2}c\left(I_B\right)\right].\end{aligned}$$

Define $\theta^{S,a}\left(\Pi^S\left(I_A 1_{\tau=A}, I_B 1_{\tau=B}\right)\right) = \arg\min_{\overrightarrow{\theta}\in C} E\left[u\left(\Pi^S\left(I_A 1_{\tau=A}, I_B 1_{\tau=B}\right)\right); \overrightarrow{\theta}\right]$. As shown in Lemma 3,

$$\theta_\tau^{S,a}\left(\Pi^S\left(I_A 1_{\tau=A}, I_B 1_{\tau=B}\right)\right) = \begin{cases} \theta_L & \tilde{\theta}^{S,a}\left(\Pi^S\right) \le \theta_L \\ \tilde{\theta}_\tau^{S,a}\left(\Pi^S\left(I_A 1_{\tau=A}, I_B 1_{\tau=B}\right)\right) & \tilde{\theta}^{S,a}\left(\Pi^S\right) \in \left(\theta_L, \theta_H\right) \\ \theta_H & \tilde{\theta}^{S,a}\left(\Pi^S\right) \ge \theta_H \end{cases}$$

where $\tilde{\theta}_A^{S,a}\left(\Pi^S\left(I_A 1_{\tau=A}, I_B 1_{\tau=B}\right)\right) = \theta_T + \frac{1}{2}\ln\left[\frac{w_B + (1-\beta)\frac{1}{2} I_B}{w_A + (1-\beta)\left(V_A + \frac{1}{2} I_A\right)}\right]$ and $\tilde{\theta}_B^{S,a}\left(\Pi^S\left(I_A 1_{\tau=A}, I_B 1_{\tau=B}\right)\right) = 2\theta_T - \tilde{\theta}_A^{S,a}\left(\Pi^S\left(I_A 1_{\tau=A}, I_B 1_{\tau=B}\right)\right)$. Applying the minimax theorem, the FOCs are $\frac{\partial U_1^S}{\partial I_A} = \frac{1}{2}(1-\beta)\left[e^{\theta_A^{S,a} - \theta_M} - c'\left(I_A\right)\right]$ and $\frac{\partial U_1^S}{\partial I_B} = \frac{1}{2}(1-\beta)\left[e^{\theta_B^{S,a} - \theta_M} - c'\left(I_B\right)\right]$, so optimal investment satisfies $c'\left(I_A\right) = e^{\theta_A^{S,a} - \theta_M}$ and $c'\left(I_B\right) = e^{\theta_B^{S,a} - \theta_M}$. Suppose $I_A > I_B$. Because the outsider is diversified ex ante, this implies $\theta_A^{S,a}\left(\Pi^S\left(I_A 1_{\tau=A}, I_B 1_{\tau=B}\right)\right) < \theta_T < \theta_B^{S,a}\left(\Pi^S\left(I_A 1_{\tau=A}, I_B 1_{\tau=B}\right)\right)$, so $e^{\theta_A^{S,a} - \theta_M} < e^{\theta_B^{S,a} - \theta_M}$. This implies, however, that $c'\left(I_A\right) < c'\left(I_B\right)$, which requires $I_A < I_B$. Contradiction. Therefore, $I_A \le I_B$. We can show similarly that $I_B \le I_A$. Thus, $I_A = I_B$, which implies $\theta_\tau^{S,a}\left(\Pi^S\left(I_A 1_{\tau=A}, I_B 1_{\tau=B}\right)\right) = \theta_T$. Therefore, the outsider would like to commit, a priori, to efficient levels of investment: $I_A = I^e$ and $I_B = I^e$. ■

Proof of Lemma 6. With a focused project, an investment level of I_A provides the outsider with utility

$$U_1^S\left(\Pi^S\left(I_A, 0\right)\right) = \min_{\overrightarrow{\theta}\in C} \mathbb{E}\left[u\left(\Pi^S\left(I_A, 0\right)\right); \overrightarrow{\theta}\right],$$

where $E\left[u\left(\Pi^S\left(I_A, 0\right)\right); \overrightarrow{\theta}\right] = e^{\theta_A - \theta_M}\left[w_A + (1-\beta)\left(V_A + I_A\right)\right] + e^{\theta_B - \theta_M} w_B + (1-\beta)\left[V_0 - c\left(I_A\right)\right]$. Beliefs are given by Lemma 3:

$$\theta_{\tau}^{S,a}\left(\Pi^{S}\left(I_{A},0\right)\right)=\begin{cases}\theta_{L} & \tilde{\theta}_{\tau}^{S,a}\left(\Pi^{S}\left(I_{A},0\right)\right)\leq\theta_{L}\\ \tilde{\theta}_{\tau}^{S,a}\left(\Pi^{S}\left(I_{A},0\right)\right) & \tilde{\theta}_{\tau}^{S,a}\left(\Pi^{S}\left(I_{A},0\right)\right)\in\left(\theta_{L},\theta_{H}\right)\\ \theta_{H} & \tilde{\theta}_{\tau}^{S,a}\left(\Pi^{S}\left(I_{A},0\right)\right)\geq\theta_{H}\end{cases}$$

where $\tilde{\theta}_{A}^{S,a}\left(\Pi^{S}\left(I_{A},0\right)\right)=\theta_{T}+\frac{1}{2}\ln\left[\frac{w_{B}}{w_{A}+(1-\beta)(V_{A}+I_{A})}\right]$ and $\tilde{\theta}_{B}^{S,a}\left(\Pi^{S}\left(I_{A},0\right)\right)=2\theta_{T}-\tilde{\theta}_{A}^{S,a}\left(\Pi^{S}\left(I_{A},0\right)\right)$.

$$\frac{d}{dI_{A}}U_{1}^{S}\left(\Pi^{S}\left(I_{A},0\right)\right)=(1-\beta)\left[e^{\theta_{A}^{S,a}\left(\Pi^{S}\left(I_{A},0\right)\right)-\theta_{M}}-c'\left(I_{A}\right)\right].$$

Thus, $I_{A}^{S,a}$ is chosen so that $c'\left(I_{A}^{S,a}\right)=e^{\theta_{A}^{S,a}\left(\Pi^{S}\left(I_{A},0\right)\right)-\theta_{M}}$. Because $w_{A}+(1-\beta)V_{A}=w_{B}$, for all $I_{A}^{S,a}>0$, $\theta_{A}^{S,a}\left(\Pi^{S}\left(I_{A},0\right)\right)<\theta_{T}$, which implies $I_{A}^{S,a}<I^{e}$.

With a diversifying project, an investment level of I_{B} provides the outsider with utility

$$U_{1}^{S}\left(\Pi^{S}\left(0,I_{B}\right)\right)=\min_{\overrightarrow{\theta}\in C}\mathbb{E}\left[u\left(\Pi^{S}\left(0,I_{B}\right);\overrightarrow{\theta}\right]\right],$$

where $E\left[u\left(\Pi^{S}\left(0,I_{B}\right)\right);\overrightarrow{\theta}\right]=e^{\theta_{A}-\theta_{M}}\left[w_{A}+(1-\beta)V_{A}\right]+e^{\theta_{B}-\theta_{M}}\left[w_{B}+(1-\beta)I_{B}\right]+(1-\beta)\left[V_{0}-c\left(I_{B}\right)\right]$. Beliefs are given by Lemma 3:

$$\theta_{\tau}^{S,a}\left(\Pi^{S}\left(0,I_{B}\right)\right)=\begin{cases}\theta_{L} & \tilde{\theta}_{\tau}^{S,a}\left(\Pi^{S}\left(0,I_{B}\right)\right)\leq\theta_{L}\\ \tilde{\theta}_{\tau}^{S,a}\left(\Pi^{S}\left(0,I_{B}\right)\right) & \tilde{\theta}_{\tau}^{S,a}\left(\Pi^{S}\left(0,I_{B}\right)\right)\in\left(\theta_{L},\theta_{H}\right)\\ \theta_{H} & \tilde{\theta}_{\tau}^{S,a}\left(\Pi^{S}\left(0,I_{B}\right)\right)\geq\theta_{H}\end{cases}$$

where $\tilde{\theta}_{B}^{S,a}\left(\Pi^{S}\left(0,I_{B}\right)\right)=\theta^{T}+\frac{1}{2}\ln\left[\frac{w_{A}+(1-\beta)V_{A}}{w_{B}+(1-\beta)I_{B}}\right]$ and $\tilde{\theta}_{A}^{S,a}\left(\Pi^{S}\left(0,I_{B}\right)\right)=2\theta_{T}-\tilde{\theta}_{B}^{S,a}\left(\Pi^{S}\left(0,I_{B}\right)\right)$.

$$\frac{dU_{1}^{S}}{dI_{B}}=(1-\beta)\left[e^{\theta_{B}^{S,a}\left(\Pi^{S}\left(0,I_{B}\right)\right)-\theta_{M}}-c'\left(I_{B}\right)\right].$$

Thus, $I_{B}^{S,a}$ is chosen so that $c'\left(I_{B}\right)=e^{\theta_{B}^{S,a}\left(\Pi^{S}\left(0,I_{B}\right)\right)-\theta_{M}}$. Because $w_{B}=w_{A}+(1-\beta)V_{A}$, for all $I_{B}^{S,a}>0$, $\tilde{\theta}_{B}^{S,a}\left(\Pi^{S}\left(0,I_{B}\right)\right)<\theta_{T}$, so $I_{B}^{S,a}<I^{e}$.

To show that $I_{A}^{S,a}=I_{B}^{S,a}$, note that $\theta_{A}^{S,a}\left(\Pi^{S}\left(I,0\right)\right)=\theta_{B}^{S,a}\left(\Pi^{S}\left(0,I\right)\right)$ for all I because the outsider is diversified a priori, $w_{B}=w_{A}+(1-\beta)V_{A}$. Thus, the pessimism effect is identical for focused and diversifying projects.

Finally, we will show that underinvestment is more severe at large firms (relative to the outsider's portfolio) by showing the equivalent claim – underinvestment is less severe when the outsider's portfolio is larger. Let $K=w_{B}=w_{A}+(1-\beta)V_{A}$. Suppose the uncertainty-averse outsider is faced with a focused project: his portfolio-distorted belief is $\tilde{\theta}_{A}^{S,a}\left(\Pi^{S}\left(I_{A},0\right)\right)=\theta_{T}+\frac{1}{2}\ln\left[\frac{K}{K+(1-\beta)I_{A}}\right]$. Focused investment by the outsider satisfies $c'\left(I_{A}^{S,a}\right)=e^{\theta_{A}^{S,a}\left(\Pi^{S}\left(I_{A},0\right)\right)-\theta_{1}}$. Because $\frac{\partial\theta_{A}^{S,a}\left(\Pi^{S}\left(I_{A},0\right)\right)}{\partial I_{A}^{S,a}}\leq0\leq\frac{\partial\theta_{A}^{S,a}\left(\Pi^{S}\left(I_{A},0\right)\right)}{\partial K}$, this implies that $\frac{dI_A^{S,a}}{dK} \geq 0$. Note that these inequalities will be strict when $\theta_A^{S,a}\left(\Pi^S\left(I_A,0\right)\right) \in (\theta_L,\theta_H)$. Recall $I_A^{S,a} < I^e$ (I^e does not depend on K). Thus, underinvestment is less severe when K is larger, and underinvestment is more severe when K is smaller. This is equivalent to the firm size result, because a large firm will be more important to the portfolio of its owners (the diversifying portfolio will be smaller). Identical results hold for diversifying projects, $\frac{dI_B^{S,a}}{dK} \geq 0$. ■

Proof of Theorem 4. Lemma 1 shows that exposure to information harms an uncertainty-averse outsider. Lemma 6 demonstrates that an uncertainty-averse outsider underinvests, both relative to first best and to what he would like to commit to ex ante by Lemma 5. By Theorem 2, an uncertainty-neutral insider chooses investment optimally, setting $I_A = I_B = I^e$. Thus, the outsider protects himself from uncertainty and achieve efficient investment by delegating control to the insider. ■

Proof of Corollary 3. The outsider's payoff from retention is, from (27),

$$U_0^{S,r}\left(I_A^{S,a}, I_B^{S,a}\right) = \frac{1}{2}U_1^S\left(\Pi^S\left(I_A^{S,a},0\right)\right) + \frac{1}{2}U_1^S\left(\Pi^S\left(0,I_B^{S,a}\right)\right).$$

where $U_1^S\left(\Pi^S\left(I_A^{S,a},0\right)\right)$ and $U_1^S\left(\Pi^S\left(0,I_B^{S,a}\right)\right)$ are defined in (23) and (26). Applying the envelope theorem, the only effect of a change in Z is the direct effect. By direct differentiation, and from $c(I) = \frac{1}{Z(1+\gamma)}I^{1+\gamma}$, we have that

$$\frac{dU_0^{S,r}\left(I_A^{S,a}, I_B^{S,a}\right)}{dZ} = \frac{1-\beta}{2Z}\left[c\left(I_A^{S,a}\right) + c\left(I_B^{S,a}\right)\right].$$

Under delegation, the outsider's payoff is

$$U_0^{S,d}\left(I_A^{M,e}, I_B^{M,e}\right) = \min_{\overrightarrow{\theta}\in C} \mathbb{E}\left[u\left(\Pi^S\left(I_A^{M,e}1_A, I_B^{M,e}1_B\right)\right); \overrightarrow{\theta}\right]$$

where

$$\begin{aligned}\mathbb{E}\left[u\left(\Pi^S\left(I_A^{M,e}1_A, I_B^{M,e}1_B\right)\right); \overrightarrow{\theta}\right] &= e^{\theta_A-\theta_M}\left[w_A + (1-\beta)\left(V_A + \frac{1}{2}I_A^{M,e}\right)\right] \\ +e^{\theta_B-\theta_M}\left[w_B + (1-\beta)\frac{1}{2}I_B^{M,e}\right] &\quad + (1-\beta)\left[V_0 - \frac{1}{2}c\left(I_A^{M,e}\right) - \frac{1}{2}c\left(I_B^{M,e}\right)\right].\end{aligned}$$

By the minimax theorem, $\frac{\partial U_0^{S,d}}{\partial \theta_A}\frac{d\theta_A^S}{dZ} + \frac{\partial U_0^{S,d}}{\partial \theta_B}\frac{d\theta_B^S}{dZ} = 0$. Note that we cannot apply the Envelope Theorem under delegation, because the insider chooses investment optimally for herself. Thus,

$$\frac{dU_0^{S,d}\left(I_A^{M,e}, I_B^{M,e}\right)}{dZ} = \frac{\partial U_0^{S,d}\left(I_A^{M,e}, I_B^{M,e}\right)}{\partial Z} + \frac{\partial U_0^{S,d}\left(I_A^{M,e}, I_B^{M,e}\right)}{\partial I_A}\frac{dI_A^{M,e}}{dZ} + \frac{\partial U_0^{S,d}\left(I_A^{M,e}, I_B^{M,e}\right)}{\partial I_B}\frac{dI_B^{M,e}}{dZ}.$$

$\frac{\partial U_0^{S,d}\left(I_A^{M,e}, I_B^{M,e}\right)}{\partial Z} = \frac{1-\beta}{2Z}[c(I_A^{M,e}) + c(I_B^{M,e})]$, $\frac{\partial U_0^{S,d}\left(I_A^{M,e}, I_B^{M,e}\right)}{\partial I_A} = \frac{1-\beta}{2}[e^{\theta_A^{S,a}(\Pi^S(I_A 1_{\tau=A}, I_B 1_{\tau=B}))-\theta_M} - c'(I_A^{M,e})]$, and $\frac{\partial U_0^{S,d}\left(I_A^{M,e}, I_B^{M,e}\right)}{\partial I_B} = \frac{1-\beta}{2}[e^{\theta_B^{S,a}(\Pi^S(I_A 1_{\tau=A}, I_B 1_{\tau=B}))-\theta_M} - c'(I_B^{M,e})]$. Because the insider is uncertainty-neutral, she sets investment optimally: $c'\left(I_A^{M,e}\right) = c'\left(I_B^{M,e}\right) = e^{\theta_T - \theta_M}$, and investment is balanced, $I_A^{M,e} = I_B^{M,e}$, so $\theta_A^{S,a}\left(\Pi^S\left(I_A 1_{\tau=A}, I_B 1_{\tau=B}\right)\right) = \theta_B^{S,a}\left(\Pi^S\left(I_A 1_{\tau=A}, I_B 1_{\tau=B}\right)\right) = \theta_T$. Thus, $\frac{\partial U_0^{S,d}\left(I_A^{M,e}, I_B^{M,e}\right)}{\partial I_A} = \frac{\partial U_0^{S,d}\left(I_A^{M,e}, I_B^{M,e}\right)}{\partial I_B} = 0$. Therefore, $\frac{dU_0^{S,d}\left(I_A^{M,e}, I_B^{M,e}\right)}{dZ} = \frac{1-\beta}{2Z}[c(I_A^{M,e}) + c(I_B^{M,e})]$ and $\frac{dU_0^{S,r}\left(I_A^{S,a}, I_B^{S,a}\right)}{dZ} = \frac{1-\beta}{2Z}[c(I_A^{S,a}) + c(I_B^{S,a})]$. The insider is uncertainty-neutral, $I_A^{M,e} = I_B^{M,e} = I^e$, but the outsider underinvests ex post (Lemma 6), so $I_A^{S,a} = I_B^{S,a} < I^e$. Therefore, $\frac{dU_0^{S,d}\left(I_A^{M,e}, I_B^{M,e}\right)}{dZ} > \frac{dU_0^{S,r}\left(I_A^{S,a}, I_B^{S,a}\right)}{dZ}$: as growth options improve, delegation becomes more valuable. ■

Proof of Theorem 5. To prove Point (1), we show that the benefit of delegation, $U_0^{S,d} - U_0^{S,r}$, is inverted U-shaped in V_A (holding $w_A + (1-\beta) V_A$ constant).[31] Define $I_B^{M,a}(V_A)$ as the diversifying investment by the insider when the value of the assets in place is V_A. When V_A is small, $V_A \leq V_A^1 \equiv e^{-2\alpha} Z^{\frac{1}{\gamma}} e^{\frac{1}{\gamma}(\theta_L - \theta_M)}$, the insider sets $I_B^{M,a}$ so that $c'\left(I_B^{M,a}\right) = e^{\theta_L - \theta_M}$, or equivalently, $I_B^{M,a} = \left[Z e^{\theta_L - \theta_M}\right]^{\frac{1}{\gamma}}$. When V_A is large, $V_A \geq V_A^2 \equiv e^{2\alpha} Z^{\frac{1}{\gamma}} e^{\frac{1}{\gamma}(\theta_H - \theta_M)}$, the insider sets $I_B^{M,a}$ so that $c'\left(I_B^{M,a}\right) = e^{\theta_H - \theta_M}$, or equivalently, $I_B^{M,a} = \left[Z e^{\theta_H - \theta_M}\right]^{\frac{1}{\gamma}}$. Note $I_B^{M,a}(V_A)$ is constant for $V_A \leq V_A^1$ and for $V_A \geq V_A^2$. For $V_A \in \left(V_A^1, V_A^2\right)$, $I_B^{M,a}$ is chosen to that $c'\left(I_B^{M,a}\right) = e^{\theta_B^{M,a} - \theta_M}$, where $\tilde{\theta}_B^{M,a} = \theta_T + \frac{1}{2}\ln\left[\frac{V_A}{I_B^{M,a}}\right]$, so $I_B^{M,a} = Z^{\frac{2}{2\gamma+1}} e^{\frac{2}{2\gamma+1}(\theta_T - \theta_M)} V_A^{\frac{1}{2\gamma+1}}$. Thus, $I_B^{M,a}(V_A)$ is strictly increasing in V_A for $V_A \in \left(V_A^1, V_A^2\right)$.

For this result, we increase V_A and decrease w_A so that $w_A + (1-\beta) V_A$ remains constant.[32] Thus, define $\tilde{w}_A = w_A - (1-\beta)\varepsilon$ and $\tilde{V}_A = V_A + \varepsilon$. By construction, $\frac{\partial U_0^{S,r}}{\partial \varepsilon} = \frac{\partial U_0^{S,d}}{\partial \varepsilon} = 0$. Similar to the proof of Corollary 3, this implies $\frac{dU_0^{S,r}}{d\varepsilon} = 0$, while

$$\frac{dU_0^{S,d}}{d\varepsilon} = \frac{\partial U_0^{S,d}}{\partial I_A}\frac{dI_A^{M,a}}{d\varepsilon} + \frac{\partial U_0^{S,d}}{\partial I_B}\frac{dI_B^{M,a}}{d\varepsilon}.$$

Also, $\frac{dI_A^{M,a}}{d\varepsilon} = 0$ because $I_A^{M,a} = \left[Z e^{\theta_L - \theta_M}\right]^{\frac{1}{\gamma}}$. As shown above, $\frac{dI_B^{M,a}}{d\varepsilon} = 0$ for $V_A \leq V_A^1$ and for $V_A \geq V_A^2$, but $\frac{dI_B^{M,a}}{d\varepsilon} > 0$ for $V_A \in \left(V_A^1, V_A^2\right)$. Thus, $\frac{dU_0^{S,d}}{d\varepsilon} > \frac{dU_0^{S,r}}{d\varepsilon}$ iff $\frac{\partial U_0^{S,d}}{\partial I_B}\frac{dI_B^{M,a}}{d\varepsilon} > 0$. $\frac{\partial U_0^{S,d}}{\partial I_B} = \frac{1}{2}(1-\beta)\left[e^{\theta_B^S - \theta_M} - e^{\theta_B^{M,a} - \theta^M}\right]$, because $c'\left(I_B^{M,a}\right) = e^{\theta_B^{M,a} - \theta^M}$. For $V_A \in \left(V_A^1, V_A^2\right)$, $I_B^{M,a}(V_A)$ is strictly increasing in V_A. The outsider believes

$$\theta_B^S = \theta_T + \frac{1}{2}\ln\left[\frac{w_A + (1-\beta) V_A + \frac{1}{2}(1-\beta) I_A^{M,a}}{w_B + \frac{1}{2}(1-\beta) I_B^{M,a}(V_A)}\right].$$

Note θ_B^S is decreasing in V_A (because $\frac{d}{d\varepsilon}\left[w_A + (1-\beta) V_A\right] = 0$, $\frac{dI_A^{M,a}}{d\varepsilon} = 0$, and $\frac{dI_B^{M,a}}{d\varepsilon} \geq 0$). The insider

[31] The proof does not require that delegation and retention are both optimal for some values of V_A. For example, if other parameters are such that retention is optimal for all V_A (for example, very large K or very small Z), the result holds by setting $\underline{V_A} = \overline{V_A}$. Alternatively, if other parameters are such that delegation is optimal for all V_A (for example, very large Z), the result holds by setting $\underline{V_A} = 0$ and $\overline{V_A} = \infty$.

[32] We show numerically that the value of delegation is nonmonotonic in w_A. See Figure 1.

believes $\theta_B^{M,a} = \theta_T + \frac{1}{2}\ln\left[\frac{V_A}{I_B^{M,a}}\right]$ where

$$\frac{I_B^{M,a}}{V_A} = Z^{\frac{2}{2\gamma+1}} e^{\frac{2}{2\gamma+1}(\theta_0-\theta^e)} V_A^{-\frac{2\gamma}{2\gamma+1}},$$

so $\theta_B^{M,a}$ is increasing in V_A. Because $\frac{\partial U_0^{S,d}}{\partial I_B} = \frac{1}{2}(1-\beta)\left[e^{\theta_B^S-\theta_M} - e^{\theta_B^{M,a}-\theta^M}\right]$, $\frac{\partial}{\partial V_A}\frac{\partial U_0^{S,d}}{\partial I_B} < 0$. It is easily shown that $\frac{\partial U_0^{S,d}}{\partial I_B} > 0$ for $V_A \leq V_A^1$ and $\frac{\partial U_0^{S,d}}{\partial I_B} < 0$ for $V_A \geq V_A^2$. Thus, there exists a unique $\tilde{V}_A$ such that $\frac{\partial U_0^{S,d}}{\partial I_B} > 0$ for $V_A < \tilde{V}_A$ and $\frac{\partial U_0^{S,d}}{\partial I_B} < 0$ for $V_A > \tilde{V}_A$. Therefore, $\frac{dU_0^{S,d}}{d\varepsilon} > \frac{dU_0^{S,r}}{d\varepsilon}$ for $V_A \in \left(V_A^1, \tilde{V}_A\right)$ and $\frac{dU_0^{S,d}}{d\varepsilon} < \frac{dU_0^{S,r}}{d\varepsilon}$ for $V_A \in \left(\tilde{V}_A, V_A^2\right)$.[33] If $U_0^{S,d}|_{V_A=\tilde{V}_A} > U_0^{S,r}|_{V=\tilde{V}_A}$, then define $\underline{V_A} < \tilde{V} < \overline{V_A}$ such that $U_0^{S,d}|_{V_A=\underline{V_A}} = U_0^{S,r}|_{V_A=\underline{V_A}}$ and $U_0^{S,d}|_{V_A=\overline{V_A}} = U_0^{S,r}|_{V_A=\overline{V_A}}$. If $U_0^{S,d}|_{V_A=\tilde{V}_A} \leq U_0^{S,r}|_{V=\tilde{V}_A}$, define $\underline{V_A} = \overline{V_A} = \tilde{V}_A$ and the claim trivially holds.

Point (2) claims that retention is optimal for $Z < \underline{Z}$, delegation is optimal for $Z > \bar{Z}$, and that $\bar{Z}(V_A)$ is U-Shaped. We will prove these separately. When the project is small (Z small), the pessimism effect disappears, but the insider invests inefficiently, so the outsider retains control. Because $U_0^{S,r}(0,0) = U_0^{S,d}(0,0)$, to show $U_0^{S,r} > U_0^{S,d}$ for all $Z \in (0, \underline{Z})$, it is sufficient to show that $\frac{dU_0^{S,r}}{dZ}|_{Z=\varepsilon} > \frac{dU_0^{S,d}}{dZ}|_{Z=\varepsilon}$ for small positive ε. From the proof of Corollary 3, $\frac{dU_0^{S,r}}{dZ} = \frac{\partial U_0^{S,r}}{\partial Z}$, and $\frac{\partial U_0^{S,r}}{\partial Z} = \frac{1-\beta}{2Z}\left[c\left(I_A^{S,a}\right) + c\left(I_B^{S,a}\right)\right]$. Because $c(I) = \frac{1}{1+\gamma}Z^{\frac{1}{\gamma}}\left[c'(I)\right]^{\frac{1+\gamma}{\gamma}}$ and $c'\left(I_A^{S,a}\right) = e^{\theta_A^S-\theta_M}$ and $c'\left(I_B^{S,a}\right) = e^{\theta_B^S-\theta_M}$, and because $Z \to 0$, $\theta_A^S\left(\Pi^S\left(I_A^{S,a}, 0\right)\right) \to \theta_T$ and $\theta_B^S\left(\Pi^S\left(0, I_B^{S,a}\right)\right) \to \theta_T$, so for small ε,

$$\frac{\partial U_0^{S,r}}{\partial Z}|_{Z=\varepsilon} = \frac{1-\beta}{2(1+\gamma)}\varepsilon^{\frac{1}{\gamma}-1}\left[e^{\frac{1+\gamma}{\gamma}(\theta_T-\theta_M)} + e^{\frac{1+\gamma}{\gamma}(\theta_T-\theta_M)}\right].$$

Similarly, the proof of Corollary 3 shows

$$\frac{dU_0^{S,d}}{dZ} = \frac{\partial U_0^{S,d}}{\partial Z} + \frac{\partial U_0^{S,d}}{\partial I_A}|_{I_A=I_A^{M,a}}\frac{dI_A^{M,a}}{dZ} + \frac{\partial U_0^{S,d}}{\partial I_B}|_{I_B=I_B^{M,a}}\frac{dI_B^{M,a}}{dZ}$$

and $\frac{\partial U_0^{S,d}}{\partial Z} = \frac{1}{2}(1-\beta)\left[\frac{1}{Z}c\left(I_A^{M,a}\right) + \frac{1}{Z}c\left(I_B^{M,a}\right)\right]$. Because the insider is not diversified, $c'\left(I_A^{M,a}\right) = e^{\theta_L-\theta_M}$ and, for sufficiently small Z, $c'\left(I_B^{M,a}\right) = e^{\theta_H-\theta_M}$. Thus, for sufficiently small ε,

$$\frac{\partial U_0^{S,d}}{\partial Z}|_{Z=\varepsilon} = \frac{1-\beta}{2(1+\gamma)}\varepsilon^{\frac{1}{\gamma}-1}\left[e^{\frac{1+\gamma}{\gamma}(\theta_L-\theta_M)} + e^{\frac{1+\gamma}{\gamma}(\theta_H-\theta_M)}\right].$$

For indirect focused-investment effects, $\frac{\partial U_0^{S,d}}{\partial I_A}|_{I_A=I_A^{M,a}} = \frac{1-\beta}{2}\left[e^{\theta_A^S-\theta_M} - c'\left(I_A^{M,a}\right)\right]$. Because $c'\left(I_A^{M,a}\right) = e^{\theta_L-\theta_M}$, $I_A^{M,a} = Z^{\frac{1}{\gamma}}e^{\frac{1}{\gamma}(\theta_L-\theta_M)}$, so $\frac{dI_A^{M,a}}{dZ} = \frac{1}{\gamma}Z^{\frac{1}{\gamma}-1}e^{\frac{1}{\gamma}(\theta_L-\theta_M)}$. As Z gets small (sufficiently small ε), θ^S

[33] For $V_A \leq V_A^1$ and $V_A \geq V_A^2$, $\frac{dU_0^{S,d}}{d\varepsilon} = \frac{dU_0^{S,r}}{d\varepsilon} = 0$.

approaches θ_T, so

$$\frac{\partial U_0^{S,d}}{\partial I_A}|_{I_A=I_A^{M,a}} \frac{dI_A^{M,a}}{dZ}|_{Z=\varepsilon} = \frac{1-\beta}{2\gamma}\varepsilon^{\frac{1}{\gamma}-1}\left[e^{\theta_T-\theta_M} - e^{\theta_L-\theta_M}\right] e^{\frac{1}{\gamma}(\theta_L-\theta_M)}.$$

For indirect diversified-investment effects, $\frac{\partial U_0^{S,d}}{\partial I_B}|_{I_B=I_B^{M,a}} = \frac{1-\beta}{2}\left[e^{\theta_B^S-\theta_M} - c'\left(I_B^{M,a}\right)\right]$. For sufficiently small Z, $\theta_B^M = \theta_H$, so $\frac{dI_B^{M,a}}{dZ} = \frac{1}{\gamma}Z^{\frac{1}{\gamma}-1}e^{\frac{1}{\gamma}(\theta_H-\theta_M)}$. As Z gets small (sufficiently small ε), θ_B^S approaches θ_T, so

$$\frac{\partial U_0^{S,d}}{\partial I_B}|_{I_B=I_B^{M,a}} \frac{dI_B^{M,a}}{dZ}|_{Z=\varepsilon} = \frac{1-\beta}{2\gamma}\varepsilon^{\frac{1}{\gamma}-1}\left[e^{\theta_T-\theta_M} - e^{\theta_H-\theta_M}\right] e^{\frac{1}{\gamma}(\theta_H-\theta_M)}.$$

Define $\phi(\alpha)$ so that

$$\begin{aligned}\phi(\alpha) &= \frac{1}{1+\gamma}\left[e^{\frac{1+\gamma}{\gamma}(\theta_T-\alpha-\theta_M)} + e^{\frac{1+\gamma}{\gamma}(\theta_T+\alpha-\theta_M)}\right] + \frac{1}{\gamma}\left[e^{\theta_T-\theta_M} - e^{\theta_T-\alpha-\theta_M}\right] e^{\frac{1}{\gamma}(\theta_T-\alpha-\theta_M)} \\ &\quad + \frac{1}{\gamma}\left[e^{\theta_T-\theta_M} - e^{\theta_T+\alpha-\theta_M}\right] e^{\frac{1}{\gamma}(\theta_T+\alpha-\theta_M)}.\end{aligned}$$

Note $\frac{dU_0^{S,r}}{dZ}|_{Z=\varepsilon} = \frac{1-\beta}{2}\varepsilon^{\frac{1}{\gamma}-1}\phi(0)$ and $\frac{dU_0^{S,d}}{dZ}|_{Z=\varepsilon} = \frac{1-\beta}{2}\varepsilon^{\frac{1}{\gamma}-1}\phi(\alpha)$. It is sufficient to show ϕ is decreasing with α. With a little rearranging,

$$\phi'(\alpha) = \frac{p(\theta_T)^{\frac{1+\gamma}{\gamma}}}{\gamma^2}\left[e^{\frac{\alpha}{\gamma}} - e^{-\frac{\alpha}{\gamma}} - \left(e^{\alpha\frac{1+\gamma}{\gamma}} - e^{-\alpha\frac{1+\gamma}{\gamma}}\right)\right]$$

Because $\alpha > 0$, $e^{\alpha m} - e^{-\alpha m}$ is increasing in m, so $\phi'(\alpha) < 0$. Therefore, $\frac{dU_0^{S,r}}{dZ}|_{Z=\varepsilon} > \frac{dU_0^{S,d}}{dZ}|_{Z=\varepsilon}$ for sufficiently small positive ε. Thus, $U_0^{S,r} > U_0^{S,d}$ for Z close to zero: equivalently, there exists $\underline{Z}$ such that $U_0^{S,r} > U_0^{S,d}$ for all $Z < \underline{Z}$.

For the second claim of Point (2), we will show that, when growth options are sufficiently large, equilibrium beliefs of the outsider and insider coincide, so equilibrium investment will be the same, which implies that the outsider delegates control to the insider (Lemma 2). Consider the equilibrium beliefs of the outsider who retains control and is faced with a focused project: $\tilde{\theta}_A^S\left(\Pi^S(I_A, 0)\right) = \theta_T + \frac{1}{2}\ln\left[\frac{w_B}{w_A+(1-\beta)(V_A+I_A)}\right]$. Note that $\theta_A^S = \theta_L$ iff $\tilde{\theta}_A^S \leq \theta_L$ iff $I_A \geq \bar{I}_A^S$ where

$$\bar{I}_A^S \triangleq \frac{1}{1-\beta}\left[e^{2\alpha}w_B - w_A - (1-\beta)V_A\right].$$

Thus, if the outsider invests sufficiently, they will agree with the insider (because $\theta_A^{M,a} = \theta_L$ as shown in Theorem 3). This is optimal if $c'\left(\bar{I}_A^S\right) \leq e^{\theta_L-\theta_M}$, or equivalently, if

$$Z \geq \frac{1}{p(\theta_L)}\left\{\frac{1}{1-\beta}\left[e^{2\alpha}w_B - w_A - (1-\beta)V_A\right]\right\}^{\gamma}.$$

Similarly, with a diversifying project, $\tilde{\theta}_B^S\left(\Pi^S(0,I_B)\right) = \theta_T + \frac{1}{2}\ln\left[\frac{w_A+(1-\beta)V_A}{w_B+(1-\beta)I_B}\right]$. Note that $\theta_B^S = \theta_L$ iff $\tilde{\theta}_B^S \leq \theta_L$ iff $I_B \geq \bar{I}_B^S$ where

$$\bar{I}_B^S \triangleq \frac{1}{1-\beta}\left\{e^{2\alpha}\left[w_A + (1-\beta)V_A\right] - w_B\right\}.$$

This is optimal if $c'\left(\bar{I}_B^S\right) \leq e^{\theta_L-\theta_M}$, or equivalently, if

$$Z \geq \frac{1}{p(\theta_L)}\left\{\frac{1}{1-\beta}\left[e^{2\alpha}\left[w_A+(1-\beta)V_A\right]-w_B\right]\right\}^{\gamma}.$$

Because $w_B = w_A + (1-\beta)V_A$, $\bar{I}_A^S = \bar{I}_B^S$ (the cutoffs for Z are symmetric as well). When growth options are sufficiently large, the outsider invests according to the worst-case scenario for the type of project drawn: $\theta_A^S\left(\Pi^S(I_A,0)\right) = \theta_B^S\left(\Pi^S(0,I_B)\right) = \theta_L$ for $I_A \geq \bar{I}_A^S$ and $I_B \geq \bar{I}_B^S$.

The insider always invests in focused projects according to the worst-case scenario: $\theta_A^{M,a}\left(\Pi^M(I_A,0)\right) = \theta_L$. Her portfolio-distorted beliefs for the diversifying project are $\tilde{\theta}_B^{M,a}\left(\Pi^M(0,I_B)\right) = \theta^e + \frac{1}{2}\ln\frac{V_A}{I_B}$. $\theta_B^{M,a} = \theta_L$ iff $I_B \geq e^{2\alpha}V_A$. It is optimal for her to set $I_B \geq e^{2\alpha}V_A$ iff $c'\left(e^{2\alpha}V_A\right) \leq e^{\theta_L-\theta_T}$, which holds iff $Z \geq e^{\theta_L-\theta_T}\left(e^{2\alpha}V_A\right)^{\gamma}$. Therefore, when growth options are sufficiently profitable, the insider will invest the same as the outsider. By Lemma 2, delegation is strictly preferred.

For the third part of Point (2), define $\Delta = U_0^{S,d} - U_0^{S,r}$ as the value of delegation, and define $\bar{Z}(V_A)$ as the value of $\bar{Z}$ for a given V_A, holding everything else constant. Thus, $\Delta > 0$ for all $Z > \bar{Z}(V_A)$ and $\Delta < 0$ for $Z = \bar{Z}(V_A) - \varepsilon$ for small positive ε. This implies that $\frac{\partial\Delta}{\partial Z}|_{Z=\bar{Z}(V_A)} > 0$. By definition of $\bar{Z}(V_A)$, $\Delta\left(\bar{Z}(V_A)\right) = 0$ for all V_A. Totally differentiating Δ with respect to V_A, $\frac{d\Delta}{dV_A} = \frac{\partial\Delta}{\partial V_A} + \frac{\partial\Delta}{\partial Z}\frac{d\bar{Z}(V_A)}{dV_A}$. As shown in the proof of Point (1), $\frac{\partial\Delta}{\partial V_A} = 0$ for $V_A < V_A^1$, $\frac{\partial\Delta}{\partial V_A} > 0$ for $V_A \in \left(V_A^1, \tilde{V}_A\right)$, $\frac{\partial\Delta}{\partial V_A} < 0$ for $V_A \in \left(\tilde{V}_A, V_A^2\right)$, and $\frac{\partial\Delta}{\partial V_A} = 0$ for $V_A > V_A^2$. This implies that $\frac{d\bar{Z}(V_A)}{dV_A} = 0$ for $V_A < V_A^1$, $\frac{d\bar{Z}(V_A)}{dV_A} < 0$ for $V_A \in \left(V_A^1, \tilde{V}_A\right)$, $\frac{d\bar{Z}(V_A)}{dV_A} > 0$ for $V_A \in \left(V_A^2, \tilde{V}_A\right)$, and $\frac{d\bar{Z}(V_A)}{dV_A} = 0$ for $V_A > V_A^2$. Thus, $\bar{Z}(V_A)$ is U-Shaped in V_A.

Finally, for Point (3), when a diversified outsider has a sufficiently large portfolio, he will always want control. Let $w_A = K - (1-\beta)V_A$ and $w_B = K$, so

$$\begin{aligned}
\tilde{\theta}_A^{S,a}\left(\Pi^S(I_A,0)\right) &= \theta_T + \frac{1}{2}\ln\left[\frac{K}{K+(1-\beta)I_A}\right],\\
\tilde{\theta}_B^{S,a}\left(\Pi^S(0,I_B)\right) &= \theta_T + \frac{1}{2}\ln\left[\frac{K}{K+(1-\beta)I_B}\right],\\
\tilde{\theta}^{S,a}\left(\Pi^S(I_A 1_{\tau=A}, I_B 1_{\tau=A})\right) &= \theta_T + \frac{1}{2}\ln\left[\frac{K+\frac{1}{2}(1-\beta)I_B}{K+\frac{1}{2}(1-\beta)I_A}\right].
\end{aligned}$$

As $K \to \infty$, all of these converge to θ_T: that is, the outsider's worst case scenario converges to θ_T for either project. Because the worst-case scenario is not moving around, he does not fear the uncertainty in the limit (Lemma 1 implies strict preference only when the worst-case scenario is not constant). Thus, when the outsider's outside portfolio is sufficiently large, he exerts control. ■

Proof of Corollary 4. Result for the outsider is from Lemma 6, while for insider is from Theorem 3. ■

Proof of Corollary 5. The value of delegation is $U_0^{S,d} - U_0^{S,r}$. To show that Point 1 holds, consider increasing both w_A and w_B by a small amount. Similar to the proof of Corollary 3,

$$\frac{dU_0^{S,d}}{dw_A} = \frac{\partial U_0^{S,d}}{\partial w_A} + \frac{\partial U_0^{S,d}}{\partial I_A}\frac{dI_A^{M,a}}{dw_A} + \frac{\partial U_0^{S,d}}{\partial I_B}\frac{dI_B^{M,a}}{dw_A}.$$

The outside portfolio of the outsider does not affect the investment decisions of the insider, so $\frac{dI_A^{M,a}}{dw_A} = \frac{dI_B^{M,a}}{dw_A} = 0$. Therefore, $\frac{dU_0^{S,d}}{dw_A} = \frac{\partial U_0^{S,d}}{\partial w_A}$. Further, $\frac{\partial U_0^{S,d}}{\partial w_A} = e^{\theta_A^S(\Pi^S(I_A^{M,a}1_A, I_B^{M,a}1_B))-\theta_M}$. $\frac{dU_0^{S,d}}{dw_B} = e^{\theta_B^S(\Pi^S(I_A^{M,a}1_A, I_B^{M,a}1_B))-\theta_M}$ by similar reasoning. Thus, the impact of an increase in outside portfolio on $U_0^{S,d}$ is

$$\frac{dU_0^{S,d}}{dw_A} + \frac{dU_0^{S,d}}{dw_B} = e^{\theta_A^S\left(\Pi^S\left(I_A^{M,a}1_A, I_B^{M,a}1_B\right)\right)-\theta_M} + e^{\theta_B^S\left(\Pi^S\left(I_A^{M,a}1_A, I_B^{M,a}1_B\right)\right)-\theta_M}.$$

Because $\theta_A + \theta_B = \theta_T$ and $I_B^{M,a} \geq I_A^{M,a}$, $\theta_A^S(\Pi^S(I_A^{M,a}1_A, I_B^{M,a}1_B)) = \theta_T + \delta_d$ while $\theta_B^S(\Pi^S(I_A^{M,a}1_A, I_B^{M,a}1_B)) = \theta_T - \delta_d$.

Under retention, because utility is defined recursively,

$$U_0^{S,r} = \frac{1}{2}\left(U_1^{S,a}\left(\Pi^S\left(I_A^{S,a}, 0\right)\right) + U_1^{S,a}\left(\Pi^S\left(0, I_B^{S,a}\right)\right)\right).$$

$\frac{dU_1^{S,a}}{dw_A} = \frac{\partial U_1^{S,a}}{\partial w_A}$, $\frac{\partial U_1^{S,a}(\Pi^S(I_A^{S,a},0))}{\partial w_A} = e^{\theta_A^S(\Pi^S(I_A^{S,a},0))-\theta_M}$, and $\frac{\partial U_1^{S,a}(\Pi^S(0,I_B^{S,a}))}{\partial w_A} = e^{\theta_A^S(\Pi^S(0,I_B^{S,a}))-\theta_M}$, as in Corollary 3, so

$$\frac{dU_0^{S,r}}{dw_A} = \frac{1}{2}\left[e^{\theta_A^S\left(\Pi^S\left(I_A^{S,a},0\right)\right)-\theta_M} + e^{\theta_A^S\left(\Pi^S\left(0,I_B^{S,a}\right)\right)-\theta_M}\right].$$

Similarly,

$$\frac{dU_0^{S,r}}{dw_B} = \frac{1}{2}\left[e^{\theta_B^S\left(\Pi^S\left(I_A^{S,a},0\right)\right)-\theta_M} + e^{\theta_B^S\left(\Pi^S\left(0,I_B^{S,a}\right)\right)-\theta_M}\right].$$

Thus, the impact of increasing both w_A and w_B is

$$\begin{aligned}\frac{dU_0^{S,r}}{dw_A} + \frac{dU_0^{S,r}}{dw_B} &= \frac{1}{2}\left[e^{\theta_A^S\left(\Pi^S\left(I_A^{S,a},0\right)\right)-\theta_M} + e^{\theta_A^S\left(\Pi^S\left(0,I_B^{S,a}\right)\right)-\theta_M}\right] \\ &\quad + \frac{1}{2}\left[e^{\theta_B^S\left(\Pi^S\left(I_A^{S,a},0\right)\right)-\theta_M} + e^{\theta_B^S\left(\Pi^S\left(0,I_B^{S,a}\right)\right)-\theta_M}\right].\end{aligned}$$

By Corollary 4, $I_A^{S,a} = I_B^{S,a}$, so $\theta_A^S(\Pi^S(I_A^{S,a}, 0)) = \theta_B^S(\Pi^S(0, I_B^{S,a})) = \theta_T - \delta_r$ while $\theta_A^S(\Pi^S(0, I_B^{S,a})) = \theta_B^S(\Pi^S(I_A^{S,a}, 0)) = \theta_T + \delta_r$. Therefore,

$$\frac{dU_0^{S,r}}{dw_A} + \frac{dU_0^{S,r}}{dw_B} = \left[e^{\theta_T - \delta_r - \theta_M} + e^{\theta_T + \delta_r - \theta_M}\right].$$

Define $g(\delta) = e^{\theta_T + \delta - \theta_M} + e^{\theta_T - \delta - \theta_M}$. Note $\frac{dU_0^{S,d}}{dw_A} + \frac{dU_0^{S,d}}{dw_B} = g(\delta_d)$ and $\frac{dU_0^{S,r}}{dw_A} + \frac{dU_0^{S,r}}{dw_B} = g(\delta_r)$. Because $g' = e^{\theta_T + \delta - \theta_M} - e^{\theta_T - \delta - \theta_M} > 0$, $\frac{dU_0^{S,d}}{dw_A} + \frac{dU_0^{S,d}}{dw_B} \leq \frac{dU_0^{S,r}}{dw_A} + \frac{dU_0^{S,r}}{dw_B}$ iff $\delta_d \leq \delta_r$, or equivalently, iff $\theta_A^S\left(\Pi^S\left(0, I_B^{S,a}\right)\right) \geq$

$\tilde{\theta}_A^S\left(\Pi^S\left(I_A^{M,a}1_A, I_B^{M,a}1_B\right)\right)$. Because

$$\tilde{\theta}_A^S\left(\Pi^S\left(0, I_B^{S,a}\right)\right) = \theta_T + \frac{1}{2}\ln\left[\frac{w_B + (1-\beta)\, I_B^{S,a}}{w_A + (1-\beta)\, V_A}\right]$$

and

$$\tilde{\theta}^S\left(\Pi^S\left(I_A^{M,a}1_A, I_B^{M,a}1_B\right)\right) = \theta_T + \frac{1}{2}\ln\left[\frac{w_B + (1-\beta)\,\frac{1}{2}I_B^{M,a}}{w_A + (1-\beta)\left(V_A + \frac{1}{2}I_A^{M,a}\right)}\right],$$

so $\frac{dU_0^{S,r}}{dw_A} + \frac{dU_0^{S,r}}{dw_B} \geq \frac{dU_0^{S,d}}{dw_A} + \frac{dU_0^{S,d}}{dw_B}$ iff

$$I_B^{M,a} \leq 2I_B^{S,a} + \frac{w_B + (1-\beta)\, I_B^{S,a}}{w_A + (1-\beta)\, V_A} I_A^{M,a}. \tag{A.1}$$

In numerical simulations, we have never found an occasion when (A.1) failed to hold. Note that (A.1) is satisfied if $I_B^{M,a} \leq 2I_B^{S,a}$, which is guaranteed to hold if $\gamma \geq \underline{\gamma} \equiv \frac{2\alpha}{\ln 2}$, where γ controls the curvature of the cost function: $c(I) = \frac{1}{Z(1+\gamma)} I^{1+\gamma}$. Thus, Point 1 is proven: increasing the outside portfolio size makes retention more attractive. By having a larger outside portfolio, uncertainty is lessened.

For Point 2, the claim is that the value of delegation, $U_0^{S,d} - U_0^{S,r}$, is decreasing in w_B, or equivalently, that $\frac{dU_0^{S,r}}{dw_B} > \frac{dU_0^{S,d}}{dw_B}$. The impact of w_B under retention is

$$\frac{dU_0^{S,r}}{dw_B} = \frac{1}{2}\left[e^{\theta_B^S\left(\Pi^S\left(I_A^{S,a},0\right)\right)-\theta_M} + e^{\theta_B^S\left(\Pi^S\left(0,I_B^{S,a}\right)\right)-\theta_M}\right].$$

Because $e^{\theta-\theta_M}$ is convex in θ, and because $\frac{1}{2}\left[\theta_B^S\left(\Pi^S\left(I_A^{S,a},0\right)\right) + \theta_B^S\left(\Pi^S\left(0, I_B^{S,a}\right)\right)\right] = \theta_T$, $\frac{dU_0^{S,r}}{dw_B} > e^{\theta_T-\theta_M}$. The impact of w_B on the payoff under delegation is $\frac{dU_0^{S,d}}{dw_B} = e^{\theta_B^S\left(\Pi^S\left(I_A^{M,a}1_A, I_B^{M,a}1_B\right)\right)-\theta_M}$. By the Corollary 4, $I_B^{M,a} \geq I_A^{M,a}$, which implies that $\theta_B^S\left(\Pi^S\left(I_A^{M,a}1_A, I_B^{M,a}1_B\right)\right) \leq \theta_T$. Because $e^{\theta-\theta_M}$ is increasing in θ, $\frac{dU_0^{S,d}}{dw_B} \leq e^{\theta_T-\theta_M} < \frac{dU_0^{S,r}}{dw_B}$.

For Point 3, under retention, by the proof of Corollary 3, $\frac{dU_0^{S,r}}{dZ} = \frac{1-\beta}{2Z}\left[c\left(I_A^{S,a}\right) + c\left(I_B^{S,a}\right)\right]$. Under delegation, by the proof of Corollary 3,

$$\frac{dU_0^{S,d}}{dZ} = \frac{\partial U_0^{S,d}}{\partial Z} + \frac{\partial U_0^{S,d}}{\partial I_A}\frac{dI_A^{M,a}}{dZ} + \frac{\partial U_0^{S,d}}{\partial I_B}\frac{dI_B^{M,a}}{dZ}.$$

The insider chooses investment, $c'\left(I_A^{M,a}\right) = e^{\theta_L-\theta_M}$, so $\frac{\partial U_0^{S,d}}{\partial I_A}|_{I_A=I_A^{M,a}} = \frac{1}{2}(1-\beta)\left[e^{\theta_A^S-\theta_M} - e^{\theta_L-\theta_M}\right]$. Because $\theta^S \geq \theta_L$ (usually with strict inequality), $\frac{\partial U_0^{S,d}}{\partial I_A}|_{I_A=I_A^{M,a}} \geq 0$. Further, $\frac{dI_A^{M,a}}{dZ} > 0$, so $\frac{\partial U_0^{S,d}}{\partial I_A}\frac{dI_A^{M,a}}{dZ} > 0$. Similarly, $\frac{\partial U_0^{S,d}}{\partial I_B}|_{I_B=I_B^{M,a}} = \frac{1-\beta}{2}\left[e^{\theta_B^S-\theta_M} - e^{\theta_B^M-\theta_M}\right]$. We cannot sign $\frac{\partial U_0^{S,d}}{\partial I_B}$, but we can say that for Z big enough, it is strictly positive (see below).

It can be quickly verified that $I_B^{M,a}$ is increasing in Z,[34] which implies $\theta_B^{M,a}$ is decreasing in Z. As Z goes to zero, investment goes to zero, so $\theta_B^S\left(\Pi^S\left(I_A^{M,a}1_A, I_B^{M,a}1_B\right)\right)$ approaches θ_T. For small Z, $\theta_B^{M,a}\left(\Pi^M\left(0, I_B^{M,a}\right)\right) = \theta_H$ (type B assets are an insignificant portion of her portfolio). Thus, for small values of Z, $e^{\theta_B^S-\theta_M} < e^{\theta_B^M-\theta_M}$, so $\frac{\partial U_0^{S,d}}{\partial I_B} < 0$. When Z gets large, $Z \geq \frac{e^{\alpha(2\gamma+1)}}{p(\theta_T)}V_A^\gamma$, $I_B^{M,a} \geq e^{2\alpha}V_A$, so $\theta_B^{M,a}\left(\Pi^M\left(0, I_B^{M,a}\right)\right) = \theta_L$, so $I_B^{M,a} = I_A^{M,a}$, which implies $\theta_B^S\left(\Pi^S\left(I_A^{M,a}1_A, I_B^{M,a}1_B\right)\right) = \theta_T$. Thus, $\theta_B^S\left(\Pi^S\left(I_A^{M,a}1_A, I_B^{M,a}1_B\right)\right) = \theta_T$ but $\theta_B^M\left(\Pi^M\left(0, I_B^{M,a}\right)\right) = \theta_L$ for $Z \geq \frac{e^{\alpha(2\gamma+1)}}{p(\theta_T)}V_A^\gamma$, so $\frac{\partial U_0^{S,d}}{\partial I_B} > 0$.[35]

Therefore, $\frac{\partial U_0^{S,d}}{\partial I_A} > 0$ and, for sufficiently large Z, $\frac{\partial U_0^{S,d}}{\partial I_B} > 0$. Further, $\frac{\partial U_0^{S,d}}{\partial Z} = \frac{1-\beta}{2Z}\left[c\left(I_A^{M,a}\right) + c\left(I_B^{M,a}\right)\right]$, so the total impact of an increase in Z on the outsider's utility under delegation is

$$\frac{dU_0^{S,d}}{dZ} = \frac{1-\beta}{2Z}\left[c\left(I_A^{M,a}\right) + c\left(I_B^{M,a}\right)\right] + \frac{\partial U_0^{S,d}}{\partial I_A}\frac{dI_A^{M,a}}{dZ} + \frac{\partial U_0^{S,d}}{\partial I_B}\frac{dI_B^{M,a}}{dZ}.$$

$\frac{\partial U_0^{S,d}}{\partial I_A} > 0$ and $\frac{\partial U_0^{S,d}}{\partial I_B} > 0$ for Z large enough. The total impact of an increase in Z on his utility under retention is

$$\frac{dU_0^{S,r}}{dZ} = \frac{1-\beta}{2Z}\left[c\left(I_A^{S,a}\right) + c\left(I_B^{S,a}\right)\right].$$

$I_j^{k,a}$ is increasing in Z for all $j \in \{A, B\}$ and $k \in \{S, M\}$, yet as Z gets big, any party in control would invest $I_j^{k,a} = I_{\min}$ where $c'\left(I_{\min}\right) = e^{\theta_L-\theta_M}$. Thus, for Z very large, $\frac{dU_0^{S,r}}{dZ}$ equals the first term of $\frac{dU_0^{S,d}}{dZ}$, and the other two terms are strictly positive, so there exists a $\tilde{Z}$ such that $\frac{dU_0^{S,d}}{dZ} > \frac{dU_0^{S,r}}{dZ}$ for all $Z > \tilde{Z}$. ■

[34] $I_B^{M,a}$ satisfies $\phi\left(I_B^{M,a}\right) = 0$ where $\phi\left(I, \theta, Z\right) = e^{\theta_B^{M,a}-\theta_M} - \frac{I^\gamma}{Z}$. The result follows by total differentiation.

[35] For intermediate values of Z, $I_B^{M,a} \geq I_A^{M,a}$, so $\theta_B^S \leq \theta_T$. Depending on the value of Z, $\theta_B^M \in (\theta_L, \theta_H)$, but θ_B^M is strictly decreasing in Z on this range. Thus, there is a $\tilde{Z}$ such that $\frac{\partial U_0^{S,d}}{\partial I_B} > 0$ for all $Z > \tilde{Z}$.

about ECGI

The European Corporate Governance Institute has been established to improve *corporate governance through fostering independent scientific research and related activities.*

The ECGI will produce and disseminate high quality research while remaining close to the concerns and interests of corporate, financial and public policy makers. It will draw on the expertise of scholars from numerous countries and bring together a critical mass of expertise and interest to bear on this important subject.

The views expressed in this working paper are those of the authors, not those of the ECGI or its members.

Electronic Access to the Working Paper Series

The full set of ECGI working papers can be accessed through the Institute's Web-site (www.ecgi.global/content/working-papers) or SSRN:

Finance Paper Series	http://www.ssrn.com/link/ECGI-Fin.html
Law Paper Series	http://www.ssrn.com/link/ECGI-Law.html