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Corporate Governance in the Presence of Active and Passive Delegated Investment

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Abstract

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Keywords: corporate governance, delegated asset management, passive funds, index funds, competition, investment stewardship, engagement, monitoring

JEL Classifications: G11, G23, G34, K22

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Corporate governance in the presence of active and passive delegated investment*

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1 Introduction

Institutional ownership has grown tremendously over the last decades, rising to more than 70% of US public firms. The composition of institutional ownership has also changed, with a remarkable growth in passive fund ownership. The fraction of equity mutual fund assets held by passive funds is now greater than 30%, and the Big Three index fund managers (BlackRock, Vanguard, and State Street) alone cast around 25% of votes in S&P 500 firms (Appel et al., 2016; Bebchuk and Hirst, 2019a). How active and passive asset managers monitor and engage with their portfolio companies has thus become of utmost importance for the governance and performance of public firms. In 2018, the SEC chairman Jay Clayton encouraged the SEC Investor Advisory Committee to examine "how passive funds should approach engagement with companies," and during the 2018 SEC Roundtable on the Proxy Process, Senator Gramm noted that "what desperately needs to be discussed [in the context of index fund growth] ... is corporate governance."[1]

There is considerable debate in the literature about the governance role of asset managers, and passive funds in particular. Empirical studies have produced conflicting results. On the one hand, Appel, Gormley, and Keim (2016, 2019) find that passive ownership is associated with more independent directors, fewer antitakeover defenses, and greater success of activists, and Filali Adib (2019) concludes that it promotes the passage of value-increasing proposals. On the other hand, Schmidt and Fahlenbrach (2017) and Heath et al. (2022) show that passive ownership is associated with less board independence, more CEO power, and worse pay-performance sensitivity.

The debate about the passive funds' role in governance also concerns their incentives to engage.[2] Some scholars argue that passive funds "lack a financial incentive" to stay engaged, both "because passive funds seek only to match the performance of an index—not outperform it" and because "any investment in improving the performance of a company will benefit all funds that track the index equally" (Lund, 2018), whereas other scholars believe that "existing critiques of passive investors are unfounded" (Fisch et al., 2019).

In this paper, we contribute to this debate by providing a theoretical framework to analyze the governance role of active and passive asset managers. We highlight the factors

[1] See the SEC chairman's statement at https://www.sec.gov/news/public-statement/statement-clayton-iac-091318 and the 2018 SEC roundtable transcript at https://www.sec.gov/files/proxy-round-table-transcript-111518.pdf.

[2] See, e.g., Bebchuk and Hirst (2019b) and Lund (2018) on one side of this debate and Fisch et al. (2019) and Kahan and Rock (2020) on the other side.

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that determine funds' incentives to engage and show that the growth of passive funds can improve governance even though their performance indeed simply tracks the performance of the market, and despite the increasingly low fees they have been charging over time. However, such governance improvement is not guaranteed and depends on the sizes of the active and passive fund sectors and competition in the fund industry. Moreover, it may come at the expense of fund investors' well-being. Our analysis has implications for the empirical studies of passive funds' role in governance and, in particular, helps reconcile the conflicting evidence in the existing studies.

In our model, fund investors decide how to allocate their capital by choosing between three options: they can either save privately or invest with either an active or a passive (index) fund manager by incurring a search cost. If an investor decides to invest with a fund manager, they negotiate an asset management fee, which is a certain fraction of the realized value of the fund's assets under management (AUM). Next, trading takes place. Passive funds invest all of their AUM in the value-weighted market portfolio. Active funds invest strategically, exploiting trading opportunities due to liquidity (retail) investors' demand: they buy stocks with low liquidity demand, i.e., those that are "undervalued," and do not invest in "overvalued" stocks with high liquidity demand. After investments are made, fund managers decide how much costly effort to exert to increase the value of their portfolio firms. Effort captures multiple actions that a shareholder can take to add value: interacting with the firm's management, ongoing monitoring activities, submitting shareholder proposals, or nominating directors. Another important example of institutional activism is voting, which requires investing resources to vote informatively, and at a potential cost of alienating the management. For example, proxy contests have become an integral part of the U.S. corporate governance system and, as discussed in Brav et al. (2021) and evidenced by the recent high-profile proxy battle at Exxon, the votes of large asset managers often play a pivotal role in determining contest outcomes.[3] For simplicity, we refer to all of these actions as monitoring, and discuss them in more detail in Section 6.1.

The key determinants of a fund manager's incentives to monitor are the fund's stake in the firm and the fees charged to the fund's investors: the higher the fund's stake, the more its AUM increase in value due to monitoring; and the higher the fees, the more is

[3]In the Exxon battle, "the key to victory, according to two people with knowledge of Engine No. 1's strategy ... was winning over big mutual-fund investors" ("How Exxon Lost a Board Battle With a Small Hedge Fund," *The New York Times*, May 28, 2021).

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captured by the fund manager from this increase in value.[4] (Lewellen and Lewellen (2022) provide empirical estimates of fund managers' incentives to be engaged shareholders based on the analysis of their portfolios and fees.[5]) The equilibrium ownership stakes and fees, in turn, depend on the fund's AUM and the fees of other funds in the market. All of these characteristics are determined endogenously; they are affected by the returns fund managers realize by trading in financial markets and by the competition between funds.

Jointly analyzing these aspects and their combined effect on governance is critical, because focusing only on one aspect (e.g., fund fees) can miss other important effects. For example, it is frequently argued that the growth in passive funds is detrimental to governance due to the low fees they charge investors which, in turn, can lead to lower incentives to stay engaged. However, this argument does not take into account that fees do not change in isolation, and a decrease in fees is accompanied by other changes relevant for governance, such as the reallocation of investor funds from private savings to asset managers and across different types of asset managers, as well as changes in funds' ownership stakes. While our model captures all of these general equilibrium effects, it is very tractable, allowing us to analyze the combined effects on governance, firm valuations, and investors' payoffs.

In particular, one implication of our analysis is that the relation between fund fees and governance is far from obvious: easier access to passive funds (which we model as a reduction in search costs) could simultaneously decrease passive fund fees but increase their monitoring efforts and improve overall governance. Intuitively, when passive funds are more easily available and charge lower fees, their aggregate AUM increase, which increases their ownership stakes and strengthens their incentives to monitor. Moreover, if passive funds primarily crowd out fund investors' private savings, rather than their allocation to active funds, then passive fund growth does not significantly affect active fund fees. Hence, active funds continue to monitor, and the dominant effect of passive fund growth is to replace retail

[4]These properties are consistent with the empirical evidence. For example, Heath et al. (2022) document that index funds with high expense ratios are more likely to vote against management than those with low expense ratios, whereas Iliev and Lowry (2015) and Iliev, Kalodimos, and Lowry (2021) show that funds with higher equity stakes are more likely to conduct governance research and to vote "actively" instead of relying on proxy advisors' recommendations. Relatedly, Lakkis (2021) finds evidence consistent with the hypothesis that an increase in a fund family's combined (across all of its funds) equity stake leads the family to oppose management more often and increases family-level coordination in voting.

[5]For example, Lewellen and Lewellen (2022) estimate that for the top five index fund managers (BlackRock, State Street, Vanguard, Dimensional, and Schwab), a 1% increase in the value of their typical stockholding leads to an extra $133,000 in their annual management fees. This number is comparable to the corresponding estimate of $520,400 for activist investors, i.e., those who file Schedule 13D.

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shareholders (who have neither ability nor incentives to monitor) in firms' ownership structures. As a result, the overall level of investor monitoring increases, so passive fund growth improves aggregate governance and increases the fundamental value of the market portfolio despite the decrease in fund fees.

However, if passive fund growth crowds out investors' allocations to active funds, rather than their private savings, then it can be detrimental to governance. In this case, passive funds primarily replace active funds, rather than retail shareholders, in firms' ownership structures. Since passive funds charge lower fees than active funds, they have lower incentives to stay engaged, so the overall level of investor monitoring can decrease. The accompanying decline in active and passive fund fees further reduces funds' combined incentives to monitor.

An implication of these results is that there can be a trade-off between governance and fund investors' well-being: if passive fund growth substantially increases fund investors' equilibrium returns, then it is detrimental to aggregate governance, and vice versa. Intuitively, passive fund growth is especially beneficial to fund investors if it creates strong competition between funds and substantially decreases active and passive fund fees. But lower fees decrease funds' incentives to stay engaged and hence are detrimental to governance. Put differently, effective fund manager monitoring requires that funds earn sufficient rents from managing investors' assets, which comes at the expense of fund investors.

While the aggregate, market-wide, effect of passive fund growth depends on whether it crowds out investors' private savings or allocations to active funds, there is substantial heterogeneity in its effects across individual firms. As access to passive funds becomes easier, firms that are relatively "overvalued" are likely to see an improvement in governance and an increase in their fundamental value, even if the market-wide effect on governance is negative. In contrast, the fundamental value of the relatively "cheap" stocks weakly decreases, even if the aggregate effect is positive. This heterogeneity reflects differences in ownership structures between these firms and, in particular, which shareholders – active funds or retail investors – are being replaced by passive funds.

Besides passive fund growth, another important development in the U.S. governance system has been the strengthening of shareholder rights. Examples include the move towards annual director elections, proxy access bylaws, and mandatory say-on-pay votes, among others. In the context of our model, such changes can be thought of as reducing funds' monitoring costs, and our analysis shows that their effects are generally subtle. On the one hand, lower monitoring costs induce fund managers to monitor more, which increases

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the value of their portfolio firms. This improvement in governance benefits fund investors on their existing investments through the funds. However, there is also a negative effect: traders in financial markets rationally anticipate the benefits of increased monitoring and bid up the prices, which lowers funds' ability to realize gains from trade and hurts fund investors on their future investments. Moreover, the resulting decline in fund returns also affects the sizes of the active and passive fund sectors. More generally, our analysis suggests that governance regulations have both a direct effect on shareholders' incentives to monitor, and also an indirect effect by changing investors' capital allocation decisions and thereby funds' AUM and ownership stakes.

Our paper has several implications for the empirical studies of passive funds' governance role. First, the debate in the literature often focuses on differences in methodologies as a way to explain the conflicting findings. Our paper suggests another, complementary, way to reconcile the different results, by looking at whether higher passive fund ownership in a given study results from lower retail or lower active fund ownership. As we discuss in Section 5.1, studies that document a positive (negative) governance effect of higher passive fund ownership show no corresponding changes (significant decreases) in active fund ownership, which is exactly consistent with the predictions of our model.

Second, most papers in this literature exploit index reconstitutions, studying how the resulting changes in firms' ownership structures affect governance. Our analysis highlights that the governance effects of endogenous changes in passive ownership (which is the focus of our paper and corresponds to what is observed in the time-series over the last decades) can be quite different from the effects of exogenous changes in the fraction of a firm owned by passive funds (e.g., due to index reconstitutions). This is because the time-series effects reflect not only the changes in firms' ownership structures, but also the simultaneous changes in fund fees and AUM. Both fees and AUM have important aggregate effects on funds' incentives to engage, but stay constant in the index reconstitution setting. Furthermore, the types of investors that passive funds replace in ownership structures in the time-series could differ from those they replace upon index reconstitutions. For these two reasons, it is possible that passive fund growth in the time-series improves governance, while an increase in passive funds' ownership stakes caused by index reconstitutions hurts governance, and vice versa.

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Related literature. Our paper is related to the literature on shareholder activism and the interaction between shareholders' trading and monitoring decisions.[6] Our key contribution to this literature is to study activism by delegated asset managers and examine how the simultaneous presence of active and passive funds affects funds' fees, AUM, and investment decisions, and the effect of these factors on funds' monitoring. Given our interest in these questions, we abstract from more specific details of the activism process, such as the role of the board (Cohn and Rajan, 2013), negotiations with management (Corum, 2021), communication (Levit, 2019), pushing for the sale of the firm (Corum and Levit, 2019; Burkart and Lee, 2021), the role of the activist's reputation (Strobl and Zeng, 2015), and the interaction between shareholders (e.g., Edmans and Manso, 2011; Brav, Dasgupta, and Mathews, 2021).

Our paper is more closely related to studies of the governance role of asset managers (see Dasgupta, Fos, and Sautner (2021) for a comprehensive survey). Dasgupta and Piacentino (2015), Song (2017), Burkart and Dasgupta (2021), and Cvijanovic, Dasgupta, and Zachariadis (2022) focus on asset managers' reputational incentives due to concerns about flows, and examine whether they strengthen or weaken governance via exit and voice. Cocoma and Zhang (2021) analyze how investors' decisions to become active or passive, defined by whether they become informed or remain uninformed, interact with their decisions on activism. Edmans, Levit, and Reilly (2019) and Levit, Malenko, and Maug (2022) study index funds in extensions of their models and focus, respectively, on the interaction between voice and exit, and on index funds' role in voting. Differently from all these papers, our focus is on how fund investors' decisions to delegate their capital affect funds' AUM, fees, and ownership stakes, and how these variables jointly affect funds' incentives to monitor. Two other papers study, like ours, the interaction between active and passive funds in general equilibrium, but focus on different mechanisms. In Baker, Chapman, and Gallmeyer (2020), passive funds do not engage in governance, so a reduction in passive fund fees is detrimental to governance but increases households' diversification opportunities. In contrast, in our paper, both active and passive funds engage in governance, which can make passive fund growth beneficial for governance. Friedman and Mahieux (2021) examine whether passive and active fund monitoring choices are complements or substitutes. In their setting, funds commit to their monitoring levels in advance, so their monitoring efforts do not depend on their fees or AUM. In contrast, our paper focuses on how funds' monitoring incentives are

[6] E.g., Admati, Pfleiderer, and Zechner (1994), Kahn and Winton (1998), and Maug (1998), among many others. Edmans and Holderness (2016) provide an in-depth survey of this literature.

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affected by the equilibrium fees, AUM, and ownership stakes.

Finally, our paper contributes to studies in the delegated asset management literature that analyze the equilibrium levels of active and passive investing and their implications for price efficiency and welfare (e.g., Stambaugh, 2014; Brown and Davies, 2017; Bond and Garcia, 2020; Jin, 2020; Lee, 2020; Malikov, 2020; Garleanu and Pedersen, 2021). Among these papers, the closest is Garleanu and Pedersen (2021), as we build on Garleanu and Pedersen (2018, 2021) in modeling the asset management industry with endogenously determined fees.[7] But differently from all these papers, our focus is on the corporate governance role of delegated asset management. In particular, while the asset payoffs in the above papers are exogenous, the asset payoffs in our paper are determined endogenously by fund managers' monitoring decisions. Buss and Sundaresan (2020), Gervais and Strobl (2021), and Kashyap et al. (2021) also study the effects of delegated asset management on corporate outcomes, but through non-governance channels. Buss and Sundaresan (2020) show that passive ownership reduces firms' cost of capital and induces them to take more risk; Gervais and Strobl (2021) analyze how the presence of asset managers affects the feedback effect of financial markets on real investment when firms learn from prices; and Kashyap et al. (2021) study how firms' investments are affected by benchmarking in fund managers' contracts.

The paper proceeds as follows. Section 2 introduces the setup. Section 3 describes the equilibrium. Section 4 derives the implications for governance, and Section 5 relates them to the empirical evidence. Section 6 discusses the assumptions of the model, Section 7 presents several extensions, and Section 8 concludes.

2 Model setup

There are three types of agents: (1) fund investors, who decide how to allocate their capital; (2) fund managers, who make investment and governance decisions; and (3) liquidity investors (noise traders). All agents are risk-neutral.

The timeline is illustrated in Figure 1. At $t = 1$, fund investors decide whether to pay a (search) cost to invest their capital with a fund manager or to invest it outside the financial market, which we refer to as private savings. At $t = 2$, fund investors negotiate with fund managers over the asset management fees. At $t = 3$, fund managers decide how to invest

[7] Cuoco and Kaniel (2011), Basak and Pavlova (2013), and Buffa, Vayanos, and Woolley (2019) study the asset pricing implications of benchmarking and asset management contracts in general.

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their assets under management, and trading takes place. At $t = 4$, each fund manager decides on the amount of effort to exert for each firm in his portfolio. Finally, at $t = 5$, all firms pay off, and the payoffs are split between fund managers and their investors according to the asset management fees decided upon at $t = 2$. We next describe the three types of agents and each of these stages in more detail.

Figure 1. Timeline of the model.

Fund managers and fund investors

We follow Garleanu and Pedersen (2018, 2021) in modeling investors' search for fund managers and their bargaining over asset management fees. There are two types of risk-neutral fund managers: active and passive (index). In our basic model, there is one fund manager of each type, but the model can be extended to any numbers of active and passive funds, N_A and N_P (see Section 7.3). While the active fund manager optimally chooses his investment portfolio, the passive fund manager is restricted to holding a value-weighted index of stocks.

Assets in financial markets can be accessed by fund investors only through the funds. Each fund manager offers to invest the capital of investors in exchange for an asset management fee. To focus on the effects of the contractual arrangements that are observed in the mutual fund industry, we follow Pastor and Stambaugh (2012) and assume that the fee charged to fund investors is a fraction of the fund's realized value of AUM at date 5 (this assumption is relaxed in Section 7.4). In particular, let f_A and f_P denote the fee as the percentage of AUM charged by the active and passive fund, respectively (we conjecture and later verify that each fund charges the same fee to all its investors). These fees are determined by bargaining between investors and fund managers, as described below. Then, if the realized value of fund manager i's portfolio at date 5 is $\tilde{Y}_i$, he keeps $f_i\tilde{Y}_i$ and distributes $(1 - f_i)\,\tilde{Y}_i$ among fund investors in proportion to their original investments to the fund.

There is a mass of risk-neutral investors, who have combined capital (wealth) W. Each investor has an infinitesimal amount of capital. At $t = 1$, each investor decides whether to invest in the financial market by delegating his capital to one of the fund managers, or whether to invest outside the financial market (private savings). The latter can be interpreted as immediate consumption, savings at a bank, or simply keeping money under the mattress.

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We normalize the (gross) return from private savings to one.

If an investor decides to invest with a fund, he needs to incur a search cost. Specifically, if an investor with wealth ε incurs a cost $\psi_A\varepsilon$ ($\psi_P\varepsilon$), he finds an active (passive) fund manager and can invest with him.[8] These costs can be interpreted as the costs of searching for relevant information, such as the fund's portfolio characteristics, investment process, and fee structure, and spending the time to understand it. For passive fund investors, the key component of these costs is finding out the fund's fee structure; these costs are likely to be larger for less financially sophisticated investors.[9] Consistent with this, Hortaçsu and Syverson (2004) conclude that investors' search frictions contribute to explaining the sizable dispersion in fees across different S&P 500 index funds despite their financial homogeneity, and Choi, Laibson, and Madrian (2010) show, in an experimental setting, that search costs for fees play an important role in decisions to invest across similar S&P 500 index funds. Some sources of growth in index funds over time (e.g., Coates, 2018) have been the move of 401(k) plans into index funds, as well as improved information: increased investor awareness about what index funds do and how their after-fee returns compare to those of active funds; the increased ability to find fund information on the Internet; improved disclosures; and the increased availability of financial advisors. All these trends can be interpreted as a decrease in ψ_P, so we will vary ψ_P as our key parameter to generate passive fund growth.

We assume that $\psi_A \geq \psi_P$. Intuitively, it takes more time and effort to understand the investment strategy and fee structure of an active fund, compared to an index fund. Since active funds in our model exploit trading opportunities and thus outperform passive funds, which simply invest in the market portfolio, fund investors face a trade-off between earning a higher rate of return on their portfolio but at a higher search cost vs. a lower rate of return at a lower cost (if $\psi_A < \psi_P$, then no investor would invest with the passive fund in equilibrium). In a richer model with heterogeneity of skill among active fund managers, ψ_A could be interpreted as the cost of searching for skill.

If an investor incurs the search cost and finds fund manager $i \in \{A, P\}$, the two negotiate fee f_i via Nash bargaining, as in Garleanu and Pedersen (2018, 2021). Fund managers have bargaining power η, and fund investors have bargaining power $1-\eta$. Modeling the fee setting

[8] Alternatively, we could assume that all investors have the same amount of wealth, in which case the proportionality of the search cost to wealth would be a normalization.

[9] See Section III.B in Hortaçsu and Syverson (2004) for a detailed discussion of search frictions in the context of index funds, and Appendix B in Garleanu and Pedersen (2018) for a description of investors' search process and the associated costs.

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via bargaining leads to a very tractable setup, which allows us to derive the equilibrium in closed form. In Section 6.2, we discuss why this assumption helps us abstract from second-order considerations in the fee-setting process, and why the main qualitative effects that arise in our model would also arise in other models of imperfect competition between funds.

We denote by W_A and W_P the AUM of the active and passive fund, respectively, after the investors make their capital allocation decisions.

Assets and trading

There is a continuum of measure one of firms, indexed by $j \in [0,1]$. Each firm's stock is in unit supply. The date-5 payoff of firm j, i.e., its fundamental value, is:

$$R_j = R_0 + \sum_{i=1}^{M_j} e_{ij}, \tag{1}$$

where R_0 is the baseline payoff without shareholder monitoring, M_j is the number of shareholders of firm j, and e_{ij} is the amount of "effort" exerted by shareholder i in firm j at date 4, as described below. To make the key forces of the model more clear, we assume that the effects of shareholders' efforts are additive. A natural extension would be to analyze a setting where efforts are substitutes (e.g., if monitoring by one shareholder makes other shareholders' monitoring redundant) or complements (e.g., if monitoring by a shareholder is more successful if other shareholders also push for similar changes).

The initial owners of each firm are assumed to have low enough valuations to be willing to sell their shares regardless of the price. For example, we can think of these initial owners as venture capitalists, who would like to exit the firm, and normalize their valuations to zero. Thus, the supply of shares in the market is always one. There are three types of traders who initially do not hold any shares and hold the entire supply after trading: the active fund manager, the passive fund manager, and competitive liquidity investors.

The trading model is broadly based on Admati, Pfleiderer, and Zechner (1994), augmented by passive fund managers:[10] (1) the active fund is strategic in that it takes into account the impact of its trading on the price; (2) the passive fund follows the mechanical rule of

[10] We extend Admati, Pfleiderer, and Zechner (1994) to a continuum of firms, multiple shareholders that can take actions (rather than one), and we introduce active and passive delegated asset management. In addition, differently from their paper, in which agents are risk-averse, we assume that all agents are risk-neutral, and trading occurs not due to risk-sharing motives but because of heterogeneous private valuations.

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investing all its AUM in a value-weighted portfolio of all stocks; (3) competitive liquidity investors have rational expectations in their assessment of asset payoffs and trade anticipating the equilibrium effort of fund managers; and (4) the price is set to clear the market (i.e., a Walrasian trading mechanism). It can be microfounded by the following game, which is formalized in the online appendix. First, the active and passive fund each submits a market order, then liquidity investors submit their demand schedules as a function of the price, and the equilibrium price is the price that clears the market. Short sales are not allowed.

More specifically, for each stock, there is a large mass of competitive risk-neutral liquidity investors (noise traders), who can each submit any demand of up to one unit. Liquidity investors value an asset at its common valuation, given by (1), perturbed by an additional private value component. In particular, liquidity investors' valuation of stock j is $R_j - Z_j$, where Z_j captures the amount of liquidity demand driven by hedging needs or investor sentiment. Stocks with large Z_j have relatively low demand from liquidity investors, while stocks with small Z_j have relatively high demand. The role of different realizations of Z_j for different stocks is to create potential gains from active portfolio management.

For simplicity, we assume that Z_j are i.i.d. (across stocks) draws from a binary distribution: $\Pr(Z_j = Z_L) = \Pr(Z_j = Z_H) = \frac{1}{2}$, where $Z_L > Z_H$. We refer to these two types of stocks as L-stocks and H-stocks, i.e., stocks with low and high liquidity demand, respectively. Thus, the L-stocks are relatively more underpriced than the H-stocks. The realizations of Z_j are publicly observed for all j. We assume that $\frac{Z_L+Z_H}{2} > 0$, which automatically also implies $Z_L > 0$ (Z_H could be either positive or negative). In other words, the market portfolio and, even more so, the L-stocks, are undervalued by liquidity investors, which enables fund managers to realize gains from trade by buying these stocks.

In Sections 7.1 and 7.2, we generalize this setup in two directions. First, we allow the misvaluation Z_j of firm j to change with the firm's fundamental value and governance. Second, we allow liquidity investors to have heterogeneous valuations.

Governance stage

Denote by x_{ij} the number of shares held by fund i in firm j. After establishing a position in the firm, each fund manager decides on the amount of effort to exert. If he exerts effort e and is of type $i \in \{A, P\}$, he bears a private cost $c_i(e)$. This cost is not shared with fund investors, capturing what happens in practice (although the equilibrium fees charged to fund investors will be indirectly affected by these costs). Thus, if the fund manager charges fee

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f_i, holds x_{ij} shares, and exerts effort e_{ij}, his payoff from firm j, up to a constant that does not depend on e_{ij}, is:

$$f_i x_{ij} e_{ij} - c_i (e_{ij}) . \tag{2}$$

We impose the standard assumptions that $c_i(0) = 0$, $c_i'(e) > 0$, $c_i''(e) > 0$, $c_i'(0) = 0$, and $c_i'(\infty) = \infty$, which guarantee an interior solution to fund managers' decisions on governance.

As discussed in the introduction, we think of effort as any action that shareholders can take to increase value: communicating with management, submitting shareholder proposals, nominating directors, as well as voting on important corporate decisions, such as proxy contests. All of these tactics are regularly employed by institutional investors, as evidenced by the survey of McCahery, Sautner, and Starks (2016).

While our results hold if active and passive funds have the same costs of monitoring, we also allow for potentially different costs, since different types of funds could be using different strategies given their different comparative advantages. For example, as Fisch et al. (2019) and Kahan and Rock (2020) point out, while active funds' trading in the firm's stock could give them access to firm-specific information and allow them to better identify firm-specific problems, passive funds have the advantage of setting and implementing broad market-wide standards in areas such as governance, sustainability, and risk management. Indeed, the Big Three index fund families perform a large number of private and public engagements, promoting good governance practices across multiple firms in their portfolios (e.g., Gormley et al., 2021). In Section 6.1, we discuss how the relation between active and passive funds' costs of monitoring affects our results.

3 Analysis

We solve the model by backward induction, starting with funds' monitoring decisions.

3.1 Governance stage

Given fund manager i's payoff (2) from firm j, the first-order condition implies that his optimal effort satisfies:

$$e_{ij} = c_i'^{-1} (f_i x_{ij}) . \tag{3}$$

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The fund manager exerts more effort if he owns a higher fraction of the firm (higher x_{ij}) or if he keeps a higher fraction of the payoff rather than distributes it out to fund investors (higher f_i). Note that the level of effort that maximizes the combined payoff of all players is $c_i'^{-1}(1)$. Hence, (3) reflects two layers of the free-rider problem. First, $x_{ij} < 1$ manifests a free-rider problem among shareholders: the fund manager underinvests in effort because other shareholders benefit from his effort but do not bear the cost of it. Second, $f_i < 1$ manifests an agency problem between the fund manager and fund investors: given ownership of x_{ij}, the effort that would maximize their joint payoff if $c_i'^{-1}(x_{ij})$, but the fund manager monitors less because he only captures a fraction of the payoff.

Note also that at this stage, fund investors benefit from the fund manager's monitoring. As we discuss below, however, monitoring does not benefit fund investors from the ex-ante perspective because the price at which the fund buys shares reflects its expected monitoring.

3.2 Trading stage

During the trading stage, all players rationally anticipate that fund managers' effort decisions will be made according to (3).

Liquidity investors. Liquidity investors have rational expectations about the effort that fund managers will exert. Hence, if they expect the active and passive fund to hold x_{Aj} and x_{Pj} shares of stock j, respectively, their assessment of the payoff (1) is:

$$R_j(x_{Aj}, x_{Pj}) = R_0 + c_A'^{-1}(f_A x_{Aj}) + c_P'^{-1}(f_P x_{Pj}). \tag{4}$$

Each liquidity investor finds it optimal to buy stock j if and only if his valuation exceeds the price, i.e., $R_j(x_{Aj}, x_{Pj}) - Z_j \geq P_j$. Recall that the active fund, passive fund, and liquidity investors hold the entire supply of shares after trading. We focus on the parameter range for which liquidity investors hold at least some shares in each type of stock, L and H. This happens when the total funds' AUM, $W_A + W_P$, are not too high, so that funds' combined demand for the stock is lower than the supply, $x_{Aj} + x_{Pj} < 1$. A sufficient condition for this to hold is specified in Proposition 1 below. Then, the price of stock j is given by:

$$P_j = R_j - Z_j. \tag{5}$$

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Equation (5) has intuitive properties. First, the price is lower if liquidity investors' demand is lower (i.e., Z_j is higher), e.g., if there is lower hedging demand or lower investor sentiment. Second, the price is higher if $R_j = R_j(x_{Aj}, x_{Pj})$ is higher, i.e., if either the active or the passive fund holds more shares. This is because all else equal, higher fund ownership implies higher expected monitoring and thus a higher payoff. We assume that $R_0 > Z_L$, which ensures that the price of each stock is always positive.

The fact that market participants incorporate the expected governance improvements into the price implies that the fund cannot make profits on its monitoring efforts. This is similar to the results in Admati, Pfleiderer, and Zechner (1994) and Grossman and Hart (1980), where the benefit of an activist's (raider's) future value improvement is incorporated into the price. Nevertheless, the fund manager in our model exerts effort in equilibrium because once investments are made, exerting effort increases his payoff (see Section 3.1).

Equation (5) also implies that as funds' ownership increases and they monitor more, the return $\frac{R_j}{P_j}$ decreases, so funds realize lower gains from trade. Thus, governance generates decreasing returns to scale from investment.

Passive fund. The passive fund is restricted to investing its AUM W_P into the value-weighted portfolio of stocks. We denote this market portfolio by index M, and note that its price (i.e., the total market capitalization) is $P_M \equiv \int_0^1 P_j dj = \frac{P_L+P_H}{2}$. The passive fund buys x_{Pj} units of stock j, such that the proportion of its AUM invested in this stock, $\frac{x_{Pj}P_j}{W_P}$, equals the weight of this stock in the market portfolio, $\frac{P_j}{P_M}$. It follows that x_{Pj} is the same for all stocks and equals:

$$x_P = \frac{W_P}{P_M}. \tag{6}$$

Active fund. The active fund manager decides which assets to invest in, choosing between stocks of type L and H. We focus on the case where the active fund finds it optimal to only buy L-stocks, and to diversify equally across all L-stocks (a sufficient condition for this to hold is specified in Proposition 1). Intuitively, stocks with higher liquidity demand are "overpriced" relative to stocks with lower liquidity demand, and the active fund only finds it optimal to buy the relatively cheaper stocks. As a result, the active fund holds a less diversified portfolio than the passive fund, consistent with practice. Since the total AUM

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W_A are allocated evenly among mass $\frac{1}{2}$ of L-stocks, the fund's investment in each L-stock is:

$$x_{AL} = \frac{2W_A}{P_L}. \tag{7}$$

Equilibrium at the trading and governance stages. Combining the above arguments, we can characterize the equilibrium payoffs and prices as functions of funds' AUM W_A and W_P and fees f_A and f_P, which are determined at stages 1 and 2. We denote the aggregate liquidity demand for the market portfolio by $Z_M \equiv \frac{Z_L+Z_H}{2}$, and the payoff of the market portfolio by $R_M \equiv \frac{R_L+R_H}{2}$. Since active funds only invest and monitor in L-stocks and passive funds invest and monitor in both L- and H-stocks, the equilibrium prices and payoffs of L-stocks and of the market portfolio are given by the following equations:

$$P_L = R_L - Z_L, \tag{8}$$

$$P_M = R_M - Z_M, \tag{9}$$

$$R_L = R_0 + c_A'^{-1}(f_A x_{AL}) + c_P'^{-1}(f_P x_P), \tag{10}$$

$$R_M = R_0 + \frac{1}{2}c_A'^{-1}(f_A x_{AL}) + c_P'^{-1}(f_P x_P), \tag{11}$$

where x_P and x_{AL} are given by (6) and (7), respectively.

3.3 Capital allocation by investors and fee setting

Infinitesimal investors decide between private savings, which earn a return of one, and investing with an active or passive fund. Consider an investor with wealth ε. The active fund invests the investor's wealth into L-stocks; in particular, it buys $\frac{\varepsilon}{P_L}$ of L-stocks, where the payoff of each stock is R_L. Since the investor incurs a search cost $\psi_A\varepsilon$ to find the active fund and pays fee f_A, the investor's payoff from investing with the active fund is $(1-f_A)\,R_L\frac{\varepsilon}{P_L} - \psi_A\varepsilon$, so his rate of return is $(1-f_A)\frac{R_L}{P_L} - \psi_A$. Similarly, the investor's return from investing with the passive fund is $(1-f_P)\frac{R_M}{P_M} - \psi_P$.

Our baseline analysis focuses on the case where the equilibrium AUM of each fund are positive; a sufficient condition for this to hold is specified in Proposition 1 (we relax this assumption in Section 8.4 of the online appendix). This implies that capital flows into the funds until, in equilibrium, investors earn the same rate of return from investing with the

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active and passive fund, which we denote by λ:

$$\lambda \equiv (1 - f_A) \frac{R_L}{P_L} - \psi_A = (1 - f_P) \frac{R_M}{P_M} - \psi_P. \tag{12}$$

Consider the fee-setting stage. Suppose that an investor with wealth ε has already incurred the cost $\psi_A \varepsilon$ and is now bargaining with the active fund manager over the fee, $\tilde{f}_A$. To determine the Nash bargaining solution, we find each party's payoff upon agreeing and upon negotiations failing. The investor's payoff from agreeing on fee $\tilde{f}_A$ is $(1 - \tilde{f}_A) R_L \frac{\varepsilon}{P_L}$, and his payoff if negotiations fail is $\lambda \varepsilon$ (e.g., he can incur the cost $\psi_P \varepsilon$ and invest with the passive fund for a rate of return λ). Next, note that for the fund manager, the effect of getting additional AUM ε on his utility via a change in effort is second-order by the envelope theorem.[11] Hence, the fund manager's additional payoff from agreeing on fee $\tilde{f}_A$ and getting additional AUM ε is $\tilde{f}_A R_L \frac{\varepsilon}{P_L}$, and his payoff if negotiations fail is zero. Given the fund manager's bargaining power η, fee $\tilde{f}_A$ is determined via the Nash bargaining solution:

$$\max_{\tilde{f}_A} \left((1 - \tilde{f}_A) R_L \frac{\varepsilon}{P_L} - \lambda \varepsilon \right)^{1-\eta} \left(\tilde{f}_A R_L \frac{\varepsilon}{P_L} \right)^{\eta}. \tag{13}$$

Since the total surplus created from bargaining is $R_L \frac{\varepsilon}{P_L} - \lambda \varepsilon$, the fee must be such that the fund manager gets fraction η of this surplus:

$$\tilde{f}_A R_L \frac{\varepsilon}{P_L} = \eta \left(R_L \frac{\varepsilon}{P_L} - \lambda \varepsilon \right). \tag{14}$$

This implies that, as conjectured previously, the active fund fee for all investors is indeed the same, $\tilde{f}_A = f_A$, and is determined by the fixed point equation:

$$f_A = \eta \left(1 - \lambda \frac{P_L}{R_L} \right). \tag{15}$$

Similarly, the passive fund fee is the same for all investors, $\tilde{f}_P = f_P$, and satisfies:

$$f_P = \eta \left(1 - \lambda \frac{P_M}{R_M} \right). \tag{16}$$

[11] See Section 6.2 for a discussion of this property. To see why the effect of ε via a change in effort is second-order, note that the active fund manager's payoff is $\max_e \{ \frac{1}{2} [f_A x_{AL} \left(R_0 + e + c_P'^{-1}(f_P x_P) \right) + \tilde{f}_A \frac{2\varepsilon}{P_L} \left(R_0 + e + c_P'^{-1}(f_P x_P) \right) - c_A(e)] \}$, and by the envelope theorem, the derivative with respect to ε at $\varepsilon = 0$ is $\tilde{f}_A \frac{1}{P_L} \left(R_0 + c_A'^{-1}(f_A x_{AL}) + c_P'^{-1}(f_P x_P) \right) = \tilde{f}_A \frac{R_L}{P_L}$.

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To solve for the equilibrium fees, return λ, and funds' AUM, we next consider investors' decisions on how to allocate their capital. Since we focus on the case where the AUM of each fund are positive, there are two possible cases, depending on the parameters.

In the first case, investors earn a low equilibrium rate of return and are indifferent between all of the three options: saving privately, investing with the active fund, and investing with the passive fund. In this case, $\lambda = 1$ in (12), so investor indifference conditions imply:

$$(1 - f_A) \frac{R_L}{P_L} - \psi_A = 1, \tag{17}$$

$$(1 - f_P) \frac{R_M}{P_M} - \psi_P = 1. \tag{18}$$

In the second case, investors are indifferent between investing with the active fund and the passive fund, and both options strictly dominate private savings, i.e., $\lambda > 1$. Then, the investor indifference conditions (17) and (18) are replaced by: (a) the indifference condition between investing with the active and passive fund,

$$(1 - f_A) \frac{R_L}{P_L} - \psi_A = (1 - f_P) \frac{R_M}{P_M} - \psi_P, \tag{19}$$

and (b) the condition that the combined funds' AUM are equal to total investor wealth W:

$$W_A + W_P = W. \tag{20}$$

3.4 Equilibrium

From this point on, we assume that fund managers' costs of effort are quadratic, i.e.,

$$c_i(e) = \frac{c_i}{2} e^2.$$

While the assumption of quadratic costs is not necessary to characterize the equilibrium and is not important for many equilibrium properties discussed after Proposition 1 and in Section 4,[12] assuming quadratic costs allows us to formulate in closed form the sufficient conditions for the existence of this equilibrium and simplifies the exposition. In particular,

[12] For example, for general costs of effort, the equilibrium characterized by Proposition 1 takes exactly the same form, except that equation (21) becomes $W = \frac{P_L}{2f_A} c_A' (2(R_L - R_M)) + \frac{P_M}{f_P} c_P' (2R_M - R_L - R_0)$. The proof of Proposition 1 in the appendix is presented for this more general case.

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funds' equilibrium effort levels are then given by $e_P = \frac{f_P x_P}{c_P}$ and $e_{AL} = \frac{f_A x_{AL}}{c_A}$.

Given the arguments above, the equilibrium $(f_A, f_P, x_{AL}, x_P, P_L, P_M, R_L, R_M)$ is the solution to the following system of equations: (i) market clearing and optimal monitoring decisions (8)-(11); (ii) fee negotiation conditions (15)-(16); and (iii) investor capital allocation conditions: (17)-(18) in the case of $\lambda = 1$, and (19)-(20) in the case of $\lambda > 1$. This equilibrium is characterized in Proposition 1.

Proposition 1 (equilibrium). *Suppose $\psi_A \geq \psi_P \frac{c_A}{c_P}$, $z_1 < \frac{Z_M}{Z_L} < z_2$, and $w_1 < W < w_2$, where z_i, w_i are given by (40)-(41) in the appendix. Then the equilibrium is as follows.*

(i) *The asset management fees are $f_A = \frac{\eta\psi_A}{\psi_A + \lambda(1-\eta)}$ and $f_P = \frac{\eta\psi_P}{\psi_P + \lambda(1-\eta)}$, and $f_A \geq f_P$.*

(ii) *The payoffs of the L-stocks and the market portfolio are $R_L = (1 + \frac{1-\eta}{\psi_A + (\lambda-1)(1-\eta)})Z_L$ and $R_M = (1 + \frac{1-\eta}{\psi_P + (\lambda-1)(1-\eta)})Z_M$.*

(iii) *The prices of the L-stocks and the market portfolio are $P_L = \frac{1-\eta}{\psi_A + (\lambda-1)(1-\eta)} Z_L$ and $P_M = \frac{1-\eta}{\psi_P + (\lambda-1)(1-\eta)} Z_M$.*

(iv) *There exists $\bar{W}$, such that if $W \geq \bar{W}$, the investors' rate of return satisfies $\lambda = 1$, whereas if $W < \bar{W}$, λ strictly decreases in W and satisfies the fixed point equation:*

$$W = \frac{c_A}{f_A} \left(R_L - R_M\right) P_L + \frac{c_P}{f_P} \left(2R_M - R_L - R_0\right) P_M. \tag{21}$$

The restrictions on parameters in Proposition 1 ensure that we consider the interesting case, i.e., one in which both funds raise positive AUM, do not together hold the entire supply of shares, and the active fund finds it optimal to invest in L-stocks and not in H-stocks. For the remainder of the paper, we assume that these assumptions hold, with a few exceptions that we explicitly point out. The assumption $\psi_A \geq \psi_P \frac{c_A}{c_P}$ is intuitive: if passive and active funds have the same monitoring technologies ($c_P = c_A$), it automatically follows from our earlier assumption that active funds are harder to search for, $\psi_A \geq \psi_P$.

The properties of the equilibrium are as follows. If aggregate investor wealth is limited, asset managers compete for investor funds and offer relatively low fees, allowing investors to earn a rate of return $\lambda > 1$. If aggregate investor wealth is large, investors' outside options in negotiations are limited, which increases the fees charged by asset managers and decreases investors' rate of return, $\lambda = 1$. The active fund outperforms the passive fund before fees,

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$\frac{R_L}{P_L} \geq \frac{R_M}{P_M}$, due to its ability to invest strategically in the most undervalued stocks. As a consequence, and consistent with practice, the fee charged by the active fund is higher than the fee charged by the passive fund: $f_A \geq f_P$ (recall that $\psi_A \geq \psi_P$).

Because we are interested in the governance effects of passive fund growth, the next result demonstrates how the search cost ψ_P affects the equilibrium. As we discuss in Section 2, a decrease in ψ_P can be thought of as easier access to passive funds over time due to their growing inclusion in 401(k) plans, increased investor awareness about them, and improved disclosures about their fee structures.

Proposition 2. *As access to passive funds becomes easier (ψ_P decreases): (1) funds' fees, f_A and f_P, decrease; (2) funds' combined AUM, $W_A + W_P$, increase; and (3) fund investors' rate of return, λ, increases. In particular, there exists a cutoff $\bar{\psi}_P$, such that $\lambda = 1$ for $\psi_P \geq \bar{\psi}_P$ and $\lambda > 1$ for $\psi_P < \bar{\psi}_P$.*

Figure 2 demonstrates Proposition 2 via a numerical example; we use the same numerical example in the next section to illustrate the implications for governance. The x-axis in all panels captures $1/\psi_P$, so that access to passive funds becomes easier as we move to the right. Easier access to passive funds is beneficial for fund investors: it decreases active and passive fund fees (panels (c) and (d)) and increases investors' return on investment (panel (a)). As a result, as panel (b) shows, investors decrease their private savings and start allocating more capital to funds, so funds' combined AUM grow (all the monotonicity statements in Proposition 2 apply in a weak sense). The cutoff $\bar{\psi}_P$ separates the region $\psi_P > \bar{\psi}_P$, where investors are indifferent between investing through the funds and saving privately ($\lambda = 1$), and the region $\psi_P < \bar{\psi}_P$, where they prefer to invest through the funds ($\lambda > 1$). In the first region, easier access to passive funds brings additional money into the asset management industry ($W_A + W_P$ grows in panel (b)), whereas in the second region, all investor wealth is already invested in the funds ($W_A + W_P = W$ in panel (b)), so easier access to passive funds just reallocates capital from active to passive funds.

Proposition 2 is broadly consistent with empirical evidence if we think of the recent trends in the asset management industry as stemming from easier access to passive funds over time, i.e., a decrease in ψ_P. The assets held by passive funds have increased substantially over the last decades, both in absolute value and as a fraction of all fund assets. For example, the total AUM of passive funds have grown from less than \$1 trillion in the early 2000s to more than \$5 trillion in recent years. This growth has been accompanied by a decline in

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both active and passive funds' expense ratios (captured by f_A and f_P in the model), from around 1% (0.23%) for active (passive) funds in the 2000s, to less than 0.7% (0.15%) in recent years.[13]

The result that lower search costs ψ_P decrease fund fees follows from two effects. The first effect is that easier access to passive funds weakly improves investors' outside option in negotiations with fund managers. To see this, consider the case of high investor returns ($\lambda > 1$). A decrease in ψ_P increases the investor's net (of search costs) return from investing with the passive fund and thereby increases his outside option in bargaining with the active fund, which induces the active fund manager to lower his fees. A reduction in active fund fees, in turn, increases the investor's net return from investing with the active fund and thereby increases his outside option in bargaining with the passive fund, resulting in a lower passive fund fee as well. In other words, easier access to the passive fund strengthens the competition between the active and passive fund, resulting in a reduction of their fees. This effect is reflected through a higher λ in the expressions for f_A and f_P in Proposition 1. It is present when $\lambda > 1$ but is absent when $\lambda = 1$, since a reduction in ψ_P improves investors' outside option in the former case but does not affect it in the latter case.

The second effect is that, holding investors' outside option (net equilibrium return λ) constant, a reduction in ψ_P leads to a decrease in the market return $\frac{R_M}{P_M}$ earned by the passive fund. This is because as ψ_P declines, investors' net (of search costs) return from investing with the passive fund increases. To achieve the same λ, capital starts flowing into the passive fund until its *gross* return, $\frac{R_M}{P_M}$, decreases in a way that investors' *net* return remains the same. A decrease in the passive fund's return, in turn, results in a lower passive fund fee (this can be formally seen from (16)). This effect is reflected through a dependence of f_P on ψ_P directly (not via λ) in the expression for f_P in Proposition 1.

[13] These stylized facts are based on the data on funds' AUM and expense ratios from the CRSP Mutual Fund database. We thank Davidson Heath, Daniele Macciocchi, Roni Michaely, and Matthew Ringgenberg for generously sharing these data with us.

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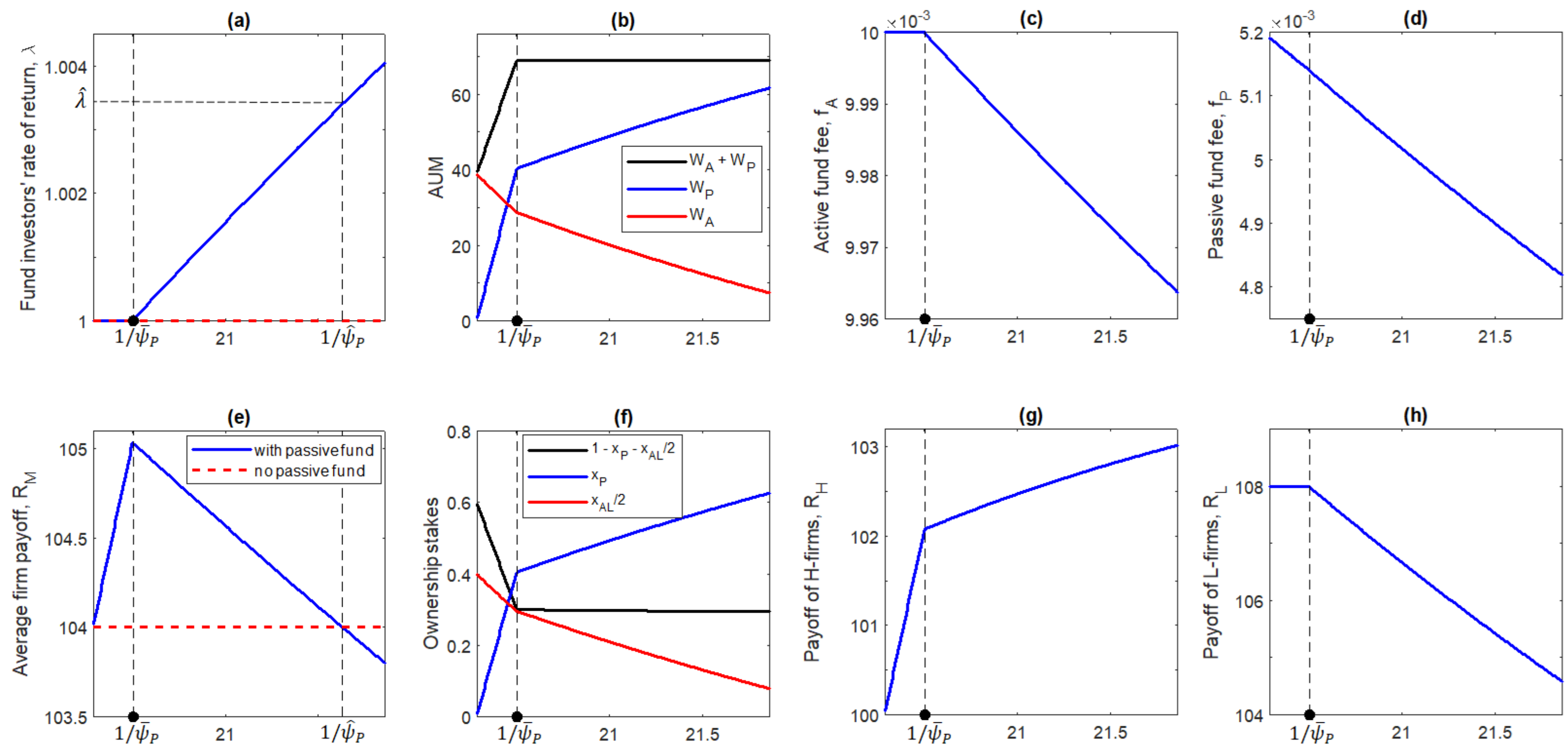

Figure 2. The x-axis in all panels captures $1/\psi_p$, i.e., moving to the right corresponds to easier access to passive funds. The y-axes are: (a) fund investors' rate of return λ; (b) funds' AUM; (c) active fund fee; (d) passive fund fee; (e) market payoff R_M in the baseline parameter specification (solid blue line) and in the benchmark case without a passive fund (dashed red line); (f) average (across all firms) ownership stakes of the passive fund (x_P), active fund ($x_{AL}/2$), and liquidity investors ($1 - x_P - x_{AL}/2$); (g) payoff R_H of H-firms; (h) payoff R_L of L-firms. The parameters are $\eta = 0.1$, $c_A = c_P = 0.001$, $\psi_A = 0.1$, $Z_L = 10.8$, $Z_H = 0$, $R_0 = 100$, and $W = 69$.

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4 Implications for governance

4.1 The governance role of passive funds

It is often argued that passive fund growth is detrimental to governance due to the lower fees that passive fund managers charge and, thereby, their lower incentives to stay engaged. This argument implicitly assumes that as passive funds grow, fund fees decrease, while other factors that affect fund managers' monitoring efforts do not change. However, in reality, fees do not change exogenously and in isolation: changes in fees are likely to be accompanied by other changes, such as changes in funds' AUM, changes in funds' ownership stakes, and the substitution between delegated asset management and private savings. In this section, we use our model to analyze the governance role of passive funds while formally accounting for a combination of these effects. Among other things, we show that passive fund growth can be beneficial for governance even if it results in lower fund fees.

As in Proposition 2, to study the implications of passive fund growth, we consider the comparative statics of parameter ψ_P. To understand its effect on aggregate governance, we examine the payoff of the market portfolio R_M, since R_M reflects the level of investor monitoring in an average firm. Proposition 3 presents our main result.

Proposition 3. *Easier access to passive funds (lower ψ_P) improves aggregate governance R_M if $\psi_P > \bar{\psi}_P$. If, in addition, $c_P \geq c_A$ and $e_{AL} < \frac{Z_L - Z_H}{2}$, then lower ψ_P hurts governance if $\psi_P \leq \bar{\psi}_P$.*

Recall from (3) that funds' incentives to monitor depend on the fees they charge and on their ownership stakes. Thus, Proposition 3 can be understood by analyzing how fund fees and firms' ownership structures change as access to passive funds becomes easier. We illustrate these forces using the example in Figure 2. As ψ_P decreases, both active and passive fund fees weakly decrease (see Proposition 2 and panels (c)-(d) of Figure 2), which, other things equal, weakens both fund managers' incentives to monitor. In addition, firms' ownership changes as well: as ψ_P decreases, capital flows to the passive fund, allowing it to take increasingly large stakes in all firms (higher x_P in panel (f)). This growth in the passive fund's stakes comes at the expense of both the stakes held by liquidity investors and the stakes held by the active fund.[14] The former effect improves governance: liquidity investors

[14] In other words, while for large ψ_P, initial owners primarily sell their shares to the active fund and

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do not monitor (we can think of them as retail shareholders, who have neither the ability nor incentives to be engaged), so if funds replace liquidity investors in firms' ownership, the overall level of investor monitoring increases. However, the latter effect hurts governance. This is because the active fund has a stronger incentive to monitor given its endogenously higher fees ($f_A \geq f_P$), and its ability to monitor is at least as high, as captured by assumption $c_P \geq c_A$ in Proposition 3 (see Section 6.1 for a discussion of this assumption).

These two counteracting effects on ownership can be seen in panels (g) and (h), which depict the value of H- and L-firms, respectively. The active fund does not hold the more expensive H-firms, so as ψ_P decreases, the passive fund only replaces liquidity investors in these firms' ownership structures, which increases shareholder monitoring (R_H increases in panel (g)). In contrast, in L-firms, the passive fund replaces both the active fund and liquidity investors, and the net effect on these firms' value is weakly negative (panel (h)).

The net effect of passive fund growth on the value of the overall market (i.e., the average firm) is thus ambiguous and depends on the interaction of these forces. According to Proposition 3 and panel (e), the key determinant of the aggregate net effect is whether the passive fund primarily competes with investors' private savings ($\psi_P > \bar{\psi}_P$) or with the active fund ($\psi_P < \bar{\psi}_P$) in its competition for investor capital.

Intuitively, when $\psi_P > \bar{\psi}_P$, investors are indifferent between investing in the funds and saving privately. Thus, easier access to passive funds crowds out private savings and brings new capital into the funds ($W_A + W_P$ grows in panel (b)). As a result of this increase in funds' AUM, the reduction in liquidity investors' stake in the average firm is substantial: panel (f), which depicts ownership of the average firm, shows that combined fund ownership significantly increases and liquidity investors' ownership ($1 - x_P - \frac{x_{AL}}{2}$) significantly decreases. Furthermore, the negative effect on incentives through fees is relatively mild: because investors can save privately at the same rate of return as from investing with the funds, active fund fees do not decrease in this region (panel (c)), so the active fund still has strong incentives to monitor on the stakes it continues to own. As a result, as Proposition 3 shows, governance in the average firm improves. The exact reason why the positive effect of replacing liquidity investors always dominates the negative effects (the reduction in x_{AL} and in f_P) is that investors must earn a competitive return from investing with the funds, as we explain in detail in Section 4.1.2.

In contrast, in the region $\psi_P < \bar{\psi}_P$, all investor wealth is invested in the funds, so the

liquidity investors, they sell increasingly more shares to the passive fund as ψ_P decreases.

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growth in passive AUM comes entirely from investors' allocations to the active fund (W_A+W_P is constant in panel (b)). As a result, the passive fund primarily replaces the active fund, and not liquidity investors, in an average firm's ownership structure (panel (f)). This harms governance because, for a given ownership stake, the passive fund monitors less than the active fund given its endogenously lower fees. In addition, since the funds strongly compete with each other in this region, both active and passive fund fees decrease substantially, reducing funds' incentives to monitor on the stakes they own.[15]

Overall, these arguments show that whether passive fund growth crowds out investors' allocations to active funds or brings new investor capital into the fund industry is closely tied to changes in firms' ownership structures, i.e., whether passive funds replace retail investors or active funds as firms' shareholders. This, in turn, has direct implications for governance.

4.1.1 Trade-off between governance and fund investors' well-being

An interesting implication of Proposition 3 is that there can be a trade-off between fund investors' well-being and governance. To see this, note that in the region $\psi_P < \bar{\psi}_P$, as access to passive funds becomes easier, fund investors' equilibrium rate of return increases, whereas aggregate governance worsens (panels (a) and (e) of Figure 2).

The same trade-off arises if we compare the baseline case (in which both the active and passive fund are present) to a benchmark with $\psi_P = \infty$, in which there is no passive fund and investors allocate their wealth between the active fund and private savings. The red dashed line in panel (e) of Figure 2 corresponds to the market payoff R_M in this benchmark.[16] Panels (a) and (e) show that while the introduction of the passive fund always weakly increases λ compared to the benchmark (in which $\lambda = 1$), it only improves governance if it does not decrease ψ_P below $\hat{\psi}_P$ (where $1/\hat{\psi}_P$ is depicted in panel (e)) and, accordingly, does not increase λ too much (above $\hat{\lambda}$ in panel (a)). We summarize these observations as follows:

[15] There are two additional nuanced effects in this case, one negative and one positive. The negative effect is that since the passive fund invests in more expensive stocks than the active fund ($P_H > P_L$), the combined ownership of the two funds declines, while liquidity investors' ownership increases, which further reduces overall investor monitoring. The positive effect is that the reduction in R_L means that the active fund can buy L-stocks at a lower price, and hence the active fund's ownership stakes do not decrease as much. Condition $e_{AL} < \frac{Z_L - Z_H}{2}$ in Proposition 3 ensures that this positive effect is relatively minor. In the proof of Proposition 3, we show that there exists a cutoff $\underline{\psi}_P$ such that this condition is satisfied for $\psi_P < \underline{\psi}_P$.

[16] Lemma 7 in the online appendix presents sufficient conditions for such a "corner" equilibrium to exist.

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Corollary 1. *Easier access to passive funds (lower ψ_P) improves aggregate governance if and only if it does not increase fund investors' returns too much.*

Intuitively, passive fund growth is especially beneficial for fund investors (i.e., increases λ substantially) when it results in strong competition between funds and significantly decreases fund fees. However, this competition implies that the passive fund primarily replaces the active fund, rather than liquidity investors, in firms' ownership structures. Moreover, the substantial reduction in fees implies lower incentives to monitor: to have incentives to stay engaged, fund managers need to earn enough rents from managing investors' portfolios and not leave too much money to fund investors. These effects create a trade-off between governance and fund investor well-being.

This intuition is more general and applies to changes in several other parameters as well. To see this, recall from Proposition 1 that $R_M = (1 + \frac{1-\eta}{\psi_P+(\lambda-1)(1-\eta)})Z_M$. Thus, for any parameter that does not enter this relation (e.g., ψ_A, c_i, or W), a change in this parameter that increases investors' equilibrium return λ, inevitably leads to a decline in aggregate governance R_M, and vice versa. For example, when aggregate investor wealth W is more limited, investors' return is higher because funds compete for investors' capital (see part (iv) of Proposition 1), but governance is worse because lower AUM and ownership stakes of the funds decrease their incentives to monitor in an average firm. A similar intuition applies to search costs for the active fund ψ_A and costs of monitoring c_i; we discuss the comparative statics in c_i in more detail in Section 4.2. Moreover, as we show in Sections 7.1 and 7.4, the trade-off between governance and fund investor well-being is robust to more general assumptions about stock misvaluations, and also arises for general compensation contracts.

4.1.2 Relation between fund fees and governance

The trade-off between governance and fund investor well-being does not arise in the region $\psi_P > \bar{\psi}_P$, where aggregate governance improves even though passive fund fees decline. Thus, the link between asset management fees and funds' incentives to monitor is not immediate:

Corollary 2. *If $\psi_P > \bar{\psi}_P$, then easier access to passive funds (lower ψ_P) improves aggregate governance R_M, even though it decreases fund fees.*

The reason why the relation between fees and governance is positive in this region is that fund fees do not change in isolation: the reduction in fees is accompanied by an increase in the

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passive fund's AUM and ownership stakes, and thereby a replacement of liquidity investors. Why does this positive effect dominate the negative effect of lower passive fees (and the partial crowding out of the active fund) in this region? The intuition is as follows. As ψ_P decreases, capital starts flowing into the passive fund, increasing its AUM and holdings x_P in its portfolio firms, so that in equilibrium, investors remain indifferent between investing with the passive fund and their private savings (e.g., Berk and Green, 2004). In other words, $(1 - f_P) \frac{R_M}{R_M - Z_M} - \psi_P = 1$ (see (18)), and hence, the decrease in ψ_P and f_P must be accompanied by a decrease in $\frac{R_M}{R_M - Z_M}$, i.e., an increase in R_M. This argument emphasizes that fee-related criticisms of passive funds need to take into account that lower fees are frequently accompanied by higher AUM and fund ownership.

4.1.3 Heterogeneous effects of passive fund growth across firms

As alluded to above, passive fund growth can have very different effects on the governance of different firms. For example, the positive effect on the aggregate market in the region $\psi_P > \bar{\psi}_P$ comes entirely from improvements in H-firms, in which the passive fund replaces only liquidity investors. In contrast, the value of L-firms in this region remains unaffected: the passive fund replaces not only liquidity investors but also the active fund in these firms' ownership structures, and the combined effect is neutral.[17] Heterogeneous effects of passive fund growth are also apparent in the region $\psi_P < \bar{\psi}_P$ in Figure 2 (see panels (g) and (h)): the governance of H-firms improves despite the overall negative effect on the aggregate market.

4.2 Who benefits from lower costs of monitoring?

Over the last decades, regulations and corporate charter amendments have empowered shareholders and made it easier for them to promote changes in their portfolio firms. Mandatory say-on-pay votes, the move towards annual director elections, increased use of majority (rather than plurality) voting for directors, and proxy access are only some examples of these changes.[18] In the context of our model, we can think of these changes as reducing both funds' costs of monitoring, c_A and c_P. In addition, individual asset managers have

[17]Formally, because investors are indifferent between investing with the active fund and saving privately, the active fund's after-fee return $(1 - f_A) \frac{R_L}{R_L - Z_L}$ must remain the same (see (17)), which together with (15), implies that both the active fund fee f_A and the return $\frac{R_L}{R_L - Z_L}$ must remain unaffected.

[18]See, e.g., "The Long View: The Role of Shareholder Proposals in Shaping U.S. Corporate Governance (2000-2018)," *Harvard Law School Forum on Corporate Governance*, February 6, 2019.

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been taking steps to decrease their individual costs of monitoring, e.g., by increasing the size of their stewardship teams. In this section, we explore the effects of reductions in funds' monitoring costs on prices, investors' returns, and the sizes of the active and passive sectors.

Proposition 4. *Suppose fund manager i's cost of monitoring c_i decreases and fund manager j's cost of monitoring c_j stays constant or decreases. Then:*

(*i*) *firms' payoffs and prices always weakly increase, and strictly increase if $\psi_P < \bar{\psi}_P$;*

(*ii*) *fund investors' return always weakly decreases, and strictly decreases if $\psi_P < \bar{\psi}_P$;*

(*iii*) *if $\psi_P \geq \bar{\psi}_P$, fund manager i's payoff strictly decreases and fund manager j's payoff weakly decreases.*

Parts (*i*) and (*ii*) show that lower monitoring costs increase fund managers' efforts and thus firms' payoffs, but can make fund investors worse off. The opposite effect of c_i on governance and fund investors' returns is a manifestation of the general trade-off between the two discussed at the end of Section 4.1.1. Intuitively, because investors in financial markets have rational expectations about the effect of c_i on funds' equilibrium monitoring and firms' payoffs, a decrease in c_i translates into higher prices and thereby lower returns. In particular, even though firms' payoffs (R_L and R_M) increase, prices ($P_L = R_L - Z_L$ and $P_M = R_M - Z_M$) increase as well. Higher prices imply that funds now buy a lower number of shares and hence realize lower gains from trade, leading funds' returns to fall. As a result, in the region $\lambda > 1$, investors' equilibrium return declines.[19] Thus, while initial owners of the firm (e.g., venture capitalists) are better off as they can now sell their shares for a higher price, the new owners of the firm (i.e., fund investors) are worse off.

The fact that *all* fund investors are worse off when monitoring becomes cheaper is a property of our static model. In a richer dynamic model, lower monitoring costs would be harmful for some fund investors but beneficial for others. Specifically, suppose that at a given point in time, the fund already has some existing investors and has acquired ownership stakes using their capital. If, at this point, the fund's monitoring cost unexpectedly declines (e.g., due to an unanticipated policy change), this benefits existing investors on the positions that the fund already holds. Indeed, as discussed in Section 3.1, once trade has taken place,

[19] As discussed in Section 3.2, this inability to profit from ex post monitoring is similar to Admati, Pfleiderer, and Zechner (1994) and the free-rider problem in Grossman and Hart (1980).

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fund investors always benefit from more monitoring. However, and for the same reason as in our setting, this decrease in c_i hurts all future investors of the fund, as well as its existing investors on any of their future contributions to the fund.

Whether decreasing c_i is beneficial for fund managers depends on the following trade-off. The positive effect is that for given AUM and fees, and once the fund has established a position in a firm, lower monitoring costs increase the fund manager's equilibrium payoff.[20] However, since fund investors anticipate a lower return on their investments, the fund's AUM may change, and in particular, the fund may attract less capital than before. This is exactly what happens in the region $\psi_P \geq \bar{\psi}_P$, where $\lambda = 1$: as c_i decreases, investors allocate less capital to the fund and increase their private savings, which decreases the fund manager's AUM and thereby the fees he can collect (part (iii) of Proposition 4).[21] Alluding again to the richer dynamic model, the fund manager benefits from stronger shareholder rights and an easier ability to intervene on the investments he has already made. However, he may be worse off in the long run, given his lower ability to attract investor capital in the future.

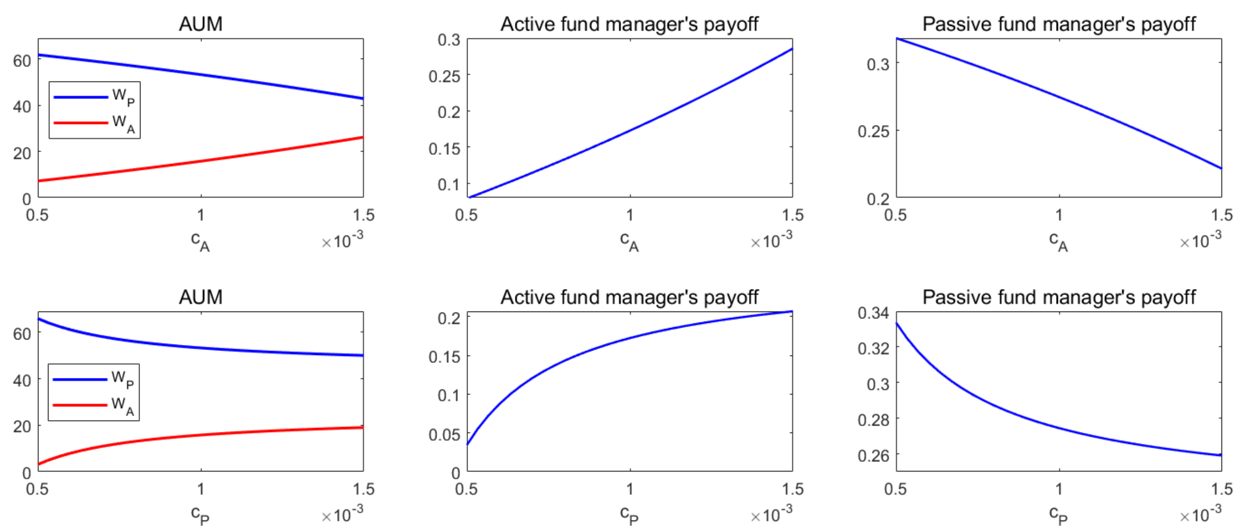

Figure 3. In this figure, we plot funds' AUM and fund managers' payoffs as functions of funds' monitoring costs c_A (top row) and c_P (bottom row) in the region $\psi_P < \bar{\psi}_P$. The parameters are $c_A = 0.001$ (when c_P varies), $c_P = 0.001$ (when c_A varies), $\eta = 0.1$, $\psi_A = 0.1$, $\psi_P = 0.047$, $Z_L = 10.8$, $Z_H = 0$, $R_0 = 100$, and $W = 69$.

[20] In particular, given fee f and stake x in a certain firm, the fund manager's payoff from this firm, up to a constant, is $V(c) = \arg\max_e \{fxe - \frac{c}{2}e^2\}$, and by the envelope theorem, $V'(c) < 0$.

[21] In Section 8.3 of the online appendix, we examine the effect of c_i on the combined welfare of all players—firms' initial owners, fund investors, fund managers, and liquidity investors—and show that decreasing funds' costs of monitoring beyond a certain threshold is detrimental to total welfare.

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When $\psi_P < \bar{\psi}_P$, the dynamics of fund flows is different. In this case, all investor capital is allocated to the funds, so a change in c_i leads to a reallocation of capital from one fund to the other. Numerically we find that as any fund's costs of monitoring decrease, capital flows out of the active fund and into the passive fund. Figure 3 illustrates this dynamic. We consider the same parameters as in Figure 2, but pick the value of ψ_P for which $\lambda > 1$, and vary c_A and c_P. The first column of the figure shows that when either c_A or c_P decreases, W_A decreases and W_P increases. Accordingly, the active fund manager's payoff decreases (second column), and the passive fund manager's payoff increases (third column).

The broad intuition is that under the conditions of Proposition 3, as investor monitoring increases and firm valuations rise, the return of the active fund, $\frac{R_L}{P_L}$, decreases more than the return of the passive fund, $\frac{R_M}{P_M}$, leading investors to reallocate capital from the active fund to the passive fund.[22] Hence, the passive fund can benefit from decreasing its cost of monitoring, which is broadly consistent with the observation that the Big Three index fund families have been increasing the size of their corporate governance teams over the recent years. For example, in his 2018 letter to CEOs, BlackRock's Larry Fink committed "to double the size of the investment stewardship team over the next three years."

Overall, the arguments in this section have two implications. First, stronger shareholder rights and regulations designed to reduce the costs of monitoring not only affect corporate governance, but can also change the sizes of the active and passive fund sectors. Second, the net effect of such regulations is ambiguous: while they improve governance and benefit fund investors and fund managers on the positions that are already established, they may decrease the returns of future fund investors and weaken some funds' ability to attract capital.

5 Empirical implications

5.1 Interpretation of existing empirical studies

The effect of passive funds on corporate governance is a highly debated question in the empirical literature, with several papers exploiting the index assignment setting and finding contradictory results: Appel, Gormley, and Keim (2016, 2019) and Filali Adib (2019) find

[22] To see this, suppose c_A decreases. Then the active fund starts monitoring more, but only in L-stocks (since it does not hold H-stocks), so $\frac{R_L}{P_L}$ decreases more than $\frac{R_M}{P_M}$. Likewise, if c_P decreases, the passive fund starts monitoring more in both types of stocks, but the return of L-stocks again declines more than that of H-stocks because they are cheaper. A formal argument is in the appendix, after the proof of Proposition 4.

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positive effects, whereas Schmidt and Fahlenbrach (2017), Bennett, Stulz, and Wang (2020), and Heath et al. (2022) conclude that there are negative effects. The literature typically alludes to differences in methodologies as a way to explain differences in results.[23] Our paper provides an alternative, unified explanation for these contradictory findings, which is complementary to the methodological explanations. Because of different methodologies, as well as slightly different samples and time periods, these papers differ in whether higher passive ownership results from lower retail or lower active fund ownership, in a manner that can explain the opposite conclusions. In particular, the papers that document a positive governance effect of higher passive ownership find no corresponding changes in active (non-index) fund ownership.[24] In contrast, both Heath et al. (2022) and Bennett, Stulz, and Wang (2020) conclude that the negative effects on governance they document are consistent with the predictions of our model because in both studies, index additions lead to a significant decrease in active fund ownership. Similarly, Schmidt and Fahlenbrach (2017) find no changes in ownership by all institutional investors, consistent with passive funds replacing actively managed institutions.[25]

Our second implication is that policymakers should exercise caution in using the existing studies to understand the governance effects of passive fund growth over the last decades. It is possible that higher passive fund ownership caused by index reconstitutions hurts governance, whereas passive fund growth over time improves governance, and vice versa. This is because to isolate the effects of passive ownership, the literature aims to identify exogenous variation in ownership structures, such as those due to index reconstitutions. In contrast, the endogenous growth in passive ownership over time has coincided with other contemporaneous changes, such as changes in active and passive fund fees, as well as active and passive funds' aggregate AUM, which all remain fixed in an index reconstitution setting. As our market equilibrium model shows, these factors have important implications for aggregate governance and their combined effects are subtle. In addition, and related to the first implication above, the types of shareholders that passive funds replace in the time-series (active funds vs. retail)

[23] For example, Appel, Gormley, and Keim (2020) suggest that the results in Heath et al. (2022) and Schmidt and Fahlenbrach (2017) could be biased because of these papers' failure to control for Russell's proprietary market cap (see p. 27), whereas Schmidt and Fahlenbrach (2017) hypothesize that their differences from Appel, Gormley, and Keim (2016) could be explained by the use of index-switching firms (rather than cross-sectional variation in index membership), as well as by different definitions of index funds (see p. 287).

[24] See Table 2 in Appel, Gormley, and Keim (2016); Table 2 and footnote 17 in Appel, Gormley, and Keim (2019); and p. 11 in Filali Adib (2019).

[25] See Table 3 and pp. 110 and 126 in Heath et al. (2022); Table 3, pp. 4-5, and Section 5.2 in Bennett, Stulz, and Wang (2020); and Figures 2 and 3 in Schmidt and Fahlenbrach (2017).

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could be very different from those they replace in index reconstitution studies, potentially leading to different results. Accordingly, in the next section, we discuss some evidence about the time-series changes in ownership structures.

5.2 Which investors were replaced by passive funds in the past?

To shed light on whether passive funds have been replacing active funds or retail investors over time, we present some simple aggregate statistics. We calculate the average ownership stakes of active (passive) funds by taking the combined AUM of active (passive) funds from the CRSP Mutual Fund database, and dividing them by the overall capitalization of the U.S. stock market.[26] Figure A.1 in the Appendix presents the dynamics of ownership over 2004-2017, defined this way. It shows that the fraction of equity held by passive funds increased from around 4% in 2004 to around 20% in 2017, while the fraction of equity held by active funds increased until 2008 and was then stable until 2011, after which it started dropping. In fact, the combined fraction of equity held by active and passive funds remained relatively stable over 2011-2017. Thus, it appears that between 2011 and 2017, passive funds were primarily replacing active funds from firms' ownership structures, whereas between 2004 and 2011, they were primarily replacing other shareholders.

To understand the patterns of ownership prior to 2004, we rely on French (2008), who covers the 1980-2006 period. In Table I, French (2008) documents that the percent of U.S. equity held by all open-end funds increased from 4.6% in 1980 to 27.6% in 2004, whereas direct holdings decreased from 47.9% in 1980 to 27.1% in 2004. Table II shows that the fraction of U.S. equity open-end fund assets invested in passive funds increased from 1% in 1984 to 12.7% in 2004. Combined, this suggests that prior to 2004, passive funds were replacing investors other than active funds in firms' ownership structures.

Overall, while crude, this evidence suggests that case $\lambda = 1$ of the model was more relevant until 2011, while case $\lambda > 1$ has become more relevant after 2011.

5.3 Implications for hedge fund activism

So far, we have focused on three types of investors: passive funds, active funds, and retail investors. Another category of investors important for governance are activist hedge funds.

[26] This is equivalent to calculating the ownership stakes of active and passive funds within each firm and then taking the market-value-weighted average of those stakes across firms.

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Our model has implications for hedge fund activism if we return to the interpretation of funds' monitoring efforts e_{ij} as voting, in particular, in proxy contests run by activist hedge funds. Proxy contests are typically close votes, and large mutual funds are often pivotal voters (e.g., Fos and Jiang, 2015; Brav et al., 2021). Making an informed voting decision in this situation (i.e., exerting effort) is likely to be costly, both because of the high uncertainty about the value of the dissident vis-à-vis the incumbent management, and because it may require voting against management, risking managerial retaliation. The feature of our model that, other things equal, passive funds are stronger monitors than liquidity (retail) investors, but weaker monitors than active mutual funds, is consistent with the observed voting patterns in proxy contests. Brav et al. (2021) show that while passive funds do frequently dissent, especially when the dissident has a strong case, they are substantially less likely to support dissidents compared to active mutual funds. They also point out that mutual funds are expected to be more diligent and informed voters than retail investors. Relatedly, Appel, Gormley, and Keim (2019) find that an increase in passive fund ownership (instrumented using Russell index assignments) is associated with higher activists' success rates in achieving changes in governance and control, such as reaching a proxy fight settlement. Importantly, in their sample, higher passive ownership corresponds to lower retail ownership, and not to lower ownership by other institutions, such as active mutual funds (see footnote 17 on p. 2759).

Under the interpretation of funds' effort as voting in activist campaigns, one can think of aggregate governance R_M as capturing the success of such campaigns. Our results then suggest that when passive funds primarily crowd out private savings and replace retail investors in firms' ownership structures, activist hedge fund campaigns are more likely to succeed, whereas if passive funds primarily replace active mutual funds, such campaigns are more likely to fail. One might then potentially link the increased flows to hedge fund activists over the last two decades[27] to the increased replacement of retail investors by large asset managers in firms' ownership structures, which has been observed in practice and corresponds to the region $\psi_P > \bar{\psi}_P$ in the model.

6 Discussion of assumptions

In this section, we discuss several assumptions and properties of the model.

[27] See, e.g., "Outlook Remains Bright for Activist Investing," at https://sophisticatedinvestor.com/outlook-remains-bright-for-activist-investing (February 1, 2016).

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6.1 Active and passive funds' monitoring strategies

It is important for our results that both active and passive funds can monitor and increase firm value. While passive funds do not run activist campaigns or take board seats, they have other strategies to influence management that they can and do regularly use. The two channels most commonly used by institutional investors are discussions with management and voting (McCahery, Sautner, and Starks, 2016). Accordingly, Fisch et al. (2019) point out that over the last decades, all institutional investors, but large passive fund managers especially, have become increasingly involved in governance through voting and communications with management. Large passive asset managers have special governance committees that analyze how votes should be cast, and their votes are often pivotal in deciding important issues, such as proxy fights or contentious M&As (e.g., Brav et al., 2021).[28] Passive funds also regularly talk with their portfolio firms about their policies and expectations. For example, in 2017, BlackRock, Vanguard, and State Street had, respectively, over 1600, 950, and 650 conversations with management teams, and also sent hundreds of letters to them.[29] The evidence in Gormley et al. (2021) suggests that governance campaigns by the Big Three passive fund families have a material impact on board composition of their portfolio firms.

While both active and passive funds engage in governance, how different are their costs of doing so? In particular, what is the rationale and the role of the assumption $c_P \geq c_A$ in Proposition 3? This assumption is consistent with the commonly expressed view that "governance interventions are especially costly for passive funds, which do not generate firm-specific information as a byproduct of investing" (Lund, 2018). In addition, Bebchuk and Hirst (2019b) point out that "index fund managers ... have a web of financially significant business ties with corporate managers," which could make them more reluctant to vote against management and increase their costs of monitoring relative to active funds (e.g., Cvijanovic, Dasgupta, and Zachariadis, 2016). Consistent with this idea, Boone et al. (2020), Brav et al. (2021), and Heath et al. (2022) find that active mutual funds are more likely to vote against management than passive funds across multiple proposal types.

[28] Kahan and Rock (2020) discuss that on such consequential issues, when passive fund managers are likely to be pivotal, they tend to invest significant resources in acquiring firm-specific information and deciding the outcome. Consistent with this, BlackRock's Investment Stewardship report writes: "In some cases, we have multiple meetings with both the company and the activist over many months as the situation evolves." (https://www.blackrock.com/corporate/literature/publication/blk-profile-of-blackrock-investment-stewardship-team-work.pdf)

[29] See "At BlackRock, Vanguard and State Street, 'Engagement' Has Different Meanings," *The Wall Street Journal*, January 20, 2018.

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However, this view is not universally held, and some argue that passive funds could be more effective in their monitoring efforts than active funds (e.g., Kahan and Rock, 2020). For example, passive funds' long-term horizon could give credibility to their demands and make it easier for them to influence management, so that they can induce the same changes with lower effort compared to active funds with high turnover. Kahan and Rock (2020) also point out that in addition to issues where firm-specific information is required, there are other issues for which the market-wide expertise of index funds is more valuable. In the context of our model, if these considerations lead passive funds to have lower monitoring costs, $c_P < c_A$, then passive funds replacing active funds in firms' ownership structures could have an ambiguous effect: passive funds would have lower incentives to monitor due to lower fees, but a greater ability to do so. However, all the other effects would remain the same, and hence the trade-offs described in Section 4.1 would arise in this setting as well. In particular, since the numerical example of Figure 2 features $c_A = c_P$, it would remain qualitatively unchanged if c_P is slightly lower than c_A, except that the negative effects of passive fund growth in the region $\psi_P \leq \bar{\psi}_P$ would not be as strongly pronounced.

6.2 Bargaining over fees

Assuming that fees are set via bargaining makes the model tractable and allows us to obtain closed form solutions. This assumption is natural if we think of fund investors as institutional investors, but may be less natural in the context of individual investors. However, the qualitative effects that arise in our model are likely robust to other models of imperfect competition among fund managers. This is because the property of fees that is needed for our effects is that easier access to passive funds, by improving fund investors' outside options, decreases the fees of the active fund, and the extent of this effect depends on whether the active fund primarily competes with the passive fund or with investors' private savings. This property is likely to hold in other models of imperfect competition, e.g., in a model where fund managers set their fees in advance and investors need to incur heterogeneous "transportation" costs to invest with the funds, as in Hotelling (1929) and Salop (1979). The complication that would arise in this alternative setting is that when setting the fees, fund managers would take into account the effect of fees on their future monitoring efforts. This "governance effect" on fees is likely to be second-order in practice. Modeling fee setting via Nash bargaining allows us to abstract from the "governance effect" (see Section 3.3 and

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footnote 11), while capturing the more first-order effects stemming from competition between funds and fund investors' outside options.

6.3 After-fee performance of active and passive funds

In our model, the after-fee return of the active fund is higher than that of the passive fund; otherwise, rational investors would not be willing to incur a higher search cost to invest with the active fund. However, the model could be easily modified to capture the empirically observed after-fee underperformance of active funds (Fama and French, 2010), while delivering the same implications for governance. For example, one possible reason why investors delegate capital to active funds despite their negative after-fee alphas is that they incorrectly overvalue managerial skill, e.g., because they cannot distinguish performance due to skill from performance due to exposures to systematic factors (Song, 2020). Another possible reason is that some fund investors demand a non-market portfolio due to their unique investment needs (e.g., hedging labor income or real estate) and are willing to pay for it via higher fees. Finally, as Pastor and Stambaugh (2012) show, if investors have uncertainty about the extent of decreasing returns to scale, then the equilibrium allocation to active funds would be high despite the historical evidence on their underperformance. Our model could be enriched to incorporate these features. For example, the overvaluation of skill could be captured by assuming that if the equilibrium return of an active fund is r_A, fund investors perceive it to be $r_A + \rho$ for some $\rho > 0$. In such a setting, the active fund manager would charge an excessively high fee, resulting in after-fee underperformance relative to the passive fund. Importantly, our results about governance would remain qualitatively unchanged: a reduction in the search cost ψ_P would reduce fund fees, and its effect on governance would depend on whether the passive fund crowds out the active fund or private savings.

6.4 Heterogeneous outside investment opportunities

All investors in our model get the same payoff from not investing with the funds. As a result of this assumption, a reduction in the search cost ψ_P affects either the combined AUM of active and passive funds (when $\lambda = 1$) or the equilibrium return of fund investors (when $\lambda > 1$), but not both simultaneously. It would be natural to consider a smoother model, in which a reduction in ψ_P changes the combined AUM and investors' equilibrium return simultaneously. This can be done by assuming that the payoff from the outside investment

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opportunity differs across investors, so that for a given return λ, mass $1 - F(\lambda)$ of investors prefer to invest in the outside investment opportunity rather than with the funds, where $F(\cdot)$ is a continuous function over some interval $[\lambda_L, \lambda_H]$. When ψ_P is sufficiently low or sufficiently high, this model would work equivalently to our basic model. When ψ_P is intermediate, the comparative statics in ψ_P will feature both effects from Proposition 3 simultaneously, and the overall effect will depend on the derivative of $F(\cdot)$ at the equilibrium return λ. We conjecture that there are natural restrictions on the distribution $F(\cdot)$, under which the cutoff result from Proposition 3 carries over to this smooth model.

7 Extensions

7.1 Generalization of mispricing

In our basic model, the degree of misvaluation of a firm's stock does not depend on the firm's fundamental value: liquidity investors value stock j at $R_j - Z_j$, where Z_j is independent of R_j. It is plausible that the degree of misvaluation changes with fundamental value, and governance in particular. For example, better governance could be associated with the adoption of better reporting and disclosure practices (e.g., Boone and White, 2015), in which case the degree of misvaluation will decrease with R_j. Alternatively, if misvaluation comes from excessive investor optimism or pessimism about a particular technology the firm is using, and higher R_j leads firm j to increase investment in that technology, then the degree of misvaluation will increase with R_j.

In this section, we extend the model by assuming that if stock j is of type $t_j \in \{L, H\}$, then liquidity investors value it at $R_j - Z_{t_j}(R_j)$, where $Z_{t_j}(R_j) = A_{t_j} + BR_j$ for some constants A_L, A_H, and B, where $A_L > A_H$ and $A_M \equiv \frac{A_L + A_H}{2} > 0$. If $B = 0$, this specification reduces to the one in the basic model. If $B < 0$ ($B > 0$), then the degree of misvaluation is decreasing (increasing) in the fundamental value of the firm, as in the first (second) example.

Notice that the solution of the model is largely unaffected. In particular, the equilibrium fee bargaining equations, (15)-(16), and the investor capital allocation equations, (17)-(20), are unchanged. However, the market-clearing conditions change from (8)-(9) to:

$$P_L = (1 - B) R_L - A_L, \tag{22}$$

$$P_M = (1 - B) R_M - A_M. \tag{23}$$

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As a result, as we show in the appendix, the equilibrium market payoff is now given by:

$$R_M = \frac{A_M}{1 - B - \frac{1-\eta}{\psi_P + \lambda(1-\eta)}}. \tag{24}$$

We focus on $B < 1$ since if $B \geq 1$, the stock price either does not depend on or decreases with the firm' payoff.[30]

To see the effects of this generalization, consider the case in which $\lambda = 1$ in equilibrium. Proposition 5 shows that the result that easier access to passive funds improves governance continues to hold. Moreover, this improvement in governance is higher if B is higher.

Proposition 5. *If $\lambda = 1$ and $B < 1$, easier access to passive funds (lower ψ_P) improves aggregate governance R_M. The change in R_M is higher if B is higher.*

To see the intuition, recall why a reduction in ψ_P improves governance in the basic model when $\lambda = 1$. A reduction in ψ_P induces investors to reallocate capital from private savings to the passive fund until the market return declines to the point where investors again become indifferent between investing with the fund and saving privately. The decline in the market return implies that in equilibrium, the increase in the passive fund's AUM must be sufficiently high, so that the resulting growth in the fund's ownership stakes improves aggregate governance despite lower fees (Section 4.1.2). The same logic holds in this extended model, but parameter B now affects the speed with which the market return decreases as governance improves. If B is higher, then the market return decreases more slowly, since better governance also leads to higher misvaluation of assets by liquidity investors. Thus, if B is higher, a reduction in ψ_P leads to a greater increase in the passive fund's AUM, implying a stronger improvement in governance.

Proposition 5 focuses on the case of $\lambda = 1$. If $\lambda > 1$, then as in the basic model, easier access to passive funds can harm governance because capital is reallocated from the active fund to the passive fund, which has lower incentives to monitor. In addition, (24) implies that as in the basic model, there is a trade-off between governance and investor well-being: for any parameter that does not enter (24), a change in this parameter that improves governance leads to lower investor returns, and vice versa. For example, a decrease in monitoring costs c_i increases investor monitoring and improves governance, but decreases λ, as in Section 4.2.

[30]If $B \geq 1$, the equilibrium has a bang-bang structure: either the passive fund or the active fund has positive AUM, but not both at the same time.

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7.2 Heterogeneous valuations of liquidity investors

The basic model assumes that for a given stock, all liquidity investors have the same valuation. A consequence of this assumption is that the price impact of a mutual fund's trade arises only because of an anticipated change in governance. It is natural to consider the case in which liquidity investors are heterogeneous in their valuations. Then, the price impact will occur not only because of a change in governance but also because of a change in the identity of the marginal liquidity investor.

To analyze this extension, consider the basic model with one change. Suppose that there is a unit mass of liquidity investors for each stock, and that liquidity investor k values stock j at $R_j - Z_{kj}$, where Z_{kj} is a conditionally i.i.d. (across liquidity investors) draw from a uniform distribution over $[Z_j - \Delta, Z_j + \Delta]$, $\Delta \geq 0$ is a constant, and $Z_j \in \{Z_L, Z_H\}$ is, as before, an i.i.d. (across stocks) draw from a binary distribution with $Z_L > Z_H$ and $Z_M = \frac{Z_L + Z_H}{2} > 0$. Thus, as in the basic model, L-stocks are undervalued by liquidity investors compared to H-stocks, in the sense that the distribution of investors' valuations is shifted downwards by a constant. The basic model corresponds to $\Delta = 0$.

This model is solved similarly to the basic model. For a fixed λ, the equilibrium fee bargaining and investor capital allocation conditions, (15)-(20), are unchanged. The only difference is in the market-clearing conditions: conditions (8)-(9) are replaced by:

$$P_L = R_L - Z_L + \Delta \left(2x_P + 2x_{AL} - 1\right), \tag{25}$$

$$P_M = R_M - Z_M + \Delta \left(2x_P + x_{AL} - 1\right), \tag{26}$$

where, as before, x_P and x_{AL} are the ownership stakes of the passive and active fund (see the proof of Proposition 6 for the derivation). The reason why P_L and P_M increase in x_P and x_{AL} is that higher ownership by the funds implies lower ownership by liquidity investors. Since the stock is owned by investors with the highest valuations, higher fund ownership crowds out liquidity investors with the lowest valuations. Hence, the marginal liquidity investor has a higher valuation, so the market-clearing price is higher.

Therefore, the extended model features decreasing returns to scale for two separate reasons. The first, as in the basic model, is due to improvements in governance (higher fund ownership increases the fund's monitoring and the firm's payoff, which decreases the relative amount of mispricing; see Section 3.2); the second is because higher fund ownership increases the valuation of the marginal liquidity investor.

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Does this model lead to similar governance implications? Recall that in the basic model, if $\lambda = 1$, easier access to passive funds always improves governance: the positive effect of higher passive fund's AUM and the replacement of liquidity investors dominates the negative effects of lower passive fund fees and a partial replacement of the active fund. Whether this conclusion holds in the extended model depends on the magnitude of Δ. Intuitively, as ψ_P declines, capital flows into the passive fund until its gross return, $\frac{R_M}{P_M}$, declines to a point where investors again become indifferent between investing with the fund and saving privately (see (18)). As discussed above, the return $\frac{R_M}{P_M}$ declines with AUM for two reasons: an improvement in governance and an increase in the marginal liquidity investor's valuation, and the extent of the second effect is captured by Δ. If Δ is not too high, the second effect is relatively weak, and hence the conclusion that easier access to passive funds improves governance continues to hold:

Proposition 6. *There exists $\bar{\Delta} > 0$ such that for any $\Delta < \bar{\Delta}$, if $\lambda = 1$, easier access to passive funds (lower ψ_P) improves aggregate governance R_M.*

In contrast, if Δ is sufficiently high, an increase in the passive fund's AUM caused by lower ψ_P can be relatively modest, because a rapid increase in the valuations of marginal liquidity investors quickly reduces the fund's return. Then, an increase in the passive fund's AUM does not overcompensate the negative effects, and governance may worsen.

7.3 Multiple active and passive funds

In this section, we extend the basic model to a general number of funds of each type, N_A and N_P. All passive funds hold the market portfolio, and suppose that all active funds find it optimal to diversify across L-stocks and not invest in H-stocks (which can be guaranteed by conditions similar to those imposed in Proposition 1). We restrict attention to symmetric equilibria, in which funds of the same type have the same AUM and asset management fees.

Denote by x_{AL} the combined holdings of all active funds in each L-firm. Then, each active fund manager owns $\frac{x_{AL}}{N_A}$ shares, so his optimal effort is $\frac{f_A x_{AL}}{c_A N_A}$, and all active funds' collective effort in each L-firm is $\frac{f_A x_{AL}}{c_A}$. Similarly, if the combined holdings of all passive funds in each firm are x_P, then the optimal effort of each passive fund manager is $\frac{f_P x_P}{c_P N_P}$, and their collective effort is $\frac{f_P x_P}{c_P}$. Thus, the only thing that matters for governance are the combined holdings of all active fund managers, x_{AL}, and all passive fund managers, x_P, while

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the exact number of funds and their individual ownership stakes do not matter, holding the fees constant. The reason is that under quadratic costs of effort, the following two opposite effects cancel out. First, there is a free-rider effect: with more funds of each type, each fund holds fewer shares, so each fund manager captures a lower fraction of the payoff from his effort. This effect works in the direction of a higher number of funds reducing the total amount of effort. Second, although each fund manager exerts lower effort, there are now more fund managers who exert effort. This effect works in the direction of a higher number of funds increasing the total amount of effort. Under a quadratic cost function, these two effects cancel out, and the total amount of effort depends on the total ownership of each type of funds. If the cost function had more curvature than quadratic (e.g., if $c_i(e) = \frac{c_i}{\alpha} e^{\alpha}$ for $\alpha > 2$), then the second effect would dominate. If the cost function had less curvature than quadratic (e.g., if $c_i(e) = \frac{c_i}{\alpha} e^{\alpha}$ for $\alpha < 2$), then the first effect would dominate.

Since the combined effort of all fund managers is $\frac{f_A x_{AL}}{c_A} + \frac{f_P x_P}{c_P}$, equations (8)-(11) continue to hold. Moreover, for given search costs ψ_A and ψ_P, the fees determined through Nash bargaining are exactly the same as in the basic model. To see this, suppose that investors' equilibrium rate of return is λ. Then investors' and fund managers' payoffs from agreeing and from negotiations failing are given by the same expressions as in the basic model, leading to the same equations for fees, (15)-(16). Investors' capital allocation conditions ((17)-(18) in the case of $\lambda = 1$, and (19)-(20) in the case of $\lambda > 1$) remain the same as well, except that W_A and W_P now stand for the combined AUM of all active and passive funds, respectively. We conclude that Propositions 1, 2, and 3 continue to hold, and hence our predictions about the effects of easier access to passive funds remain unchanged.

In the discussion above, we take N_A and N_P as given, but one could also endogenize funds' entry decisions by introducing costs of entry for each fund type. In such a model, a change in the search cost ψ_P would change the equilibrium number of funds through a change in fund managers' expected payoffs. However, because the equilibrium level of governance R_M does not depend on N_A and N_P as discussed above, this would not change the comparative statics of governance in ψ_P. Alternatively, one could also endogenize the search costs and assume that ψ_A and ψ_P are functions of the equilibrium number of funds, as in Garleanu and Pedersen (2018). In such a model, the search costs and the equilibrium number of funds would be interrelated and determined in equilibrium.

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7.4 General compensation contracts

Our model is also tractable for more general compensation contracts. For example, a hedge fund manager's fee structure typically includes a management fee, a performance fee, as well as high water marks and/or hurdle rates for the performance fee to be paid. In this section, we show how the analysis can be extended for contracts of general shape.

Suppose that fund managers are compensated by contracts from a given ordered set, denoted by $\phi(r, f)$, where $r \in \mathbb{R}$ is the realized gross return of the fund (i.e., $\frac{R_L}{P_L}$ for the active fund and $\frac{R_M}{P_M}$ for the passive fund) and $f \in [f_L, f_H]$ is the index of a contract; $\phi(r, f)$ is the compensation of the fund manager per dollar of investment. After an investor finds a fund manager, they bargain over index $f \in [f_L, f_H]$. As in the basic model, in a symmetric equilibrium, all investors that go to the active (passive) fund sign the same contract with some index f_A (f_P). Function $\phi(r, f)$ can be any function satisfying the following conditions. First, for any $f \in [f_L, f_H]$, $\phi(r, f)$ is increasing in r and locally differentiable at each equilibrium fund return (i.e., at equilibrium levels of $\frac{R_L}{P_L}$ and $\frac{R_M}{P_M}$). Second, for each r, $\phi(r, f)$ is increasing and continuous in $f \in [f_L, f_H]$, and strictly increasing in f at equilibrium values of r. Third, for each r, $\phi(r, f_L) \leq 0$ and $\phi(r, f_H) \geq \max\{r, 0\}$. Intuitively, the first condition ensures that the solution to the governance problem is well-behaved, while the second and third conditions ensure that any surplus division between a fund manager and an infinitesimal investor can be attained with an appropriate contract index f. Note that when $\phi(r, f) = fr$, $f_L = 0$, and $f_H = 1$, this extension is equivalent to the basic model.

We first show how the equilibrium can be derived for such a general compensation contract, and then discuss which features change and which remain the same as in the basic model. If a fund investor invests wealth ε with fund manager $i \in \{A, P\}$ and the fund generates gross return r_i, then the fund manager's compensation is $\phi(r_i, f_i)\varepsilon$, the fund investor's payoff is $r_i\varepsilon - \phi(r_i, f_i)\varepsilon$, and the surplus from bargaining between them is $(r_i - \lambda)\varepsilon$. Thus, using the arguments in Section 3.3, we obtain the following analogs of equation (14):

$$\phi\left(\frac{R_L}{P_L}, f_A\right) = \eta\left(\frac{R_L}{P_L} - \lambda\right) \text{ and } \phi\left(\frac{R_M}{P_M}, f_P\right) = \eta\left(\frac{R_M}{P_M} - \lambda\right), \tag{27}$$

and the analog of the investor's indifference condition (12) is:

$$(1-\eta)\frac{R_L}{P_L} + \eta\lambda - \psi_A = (1-\eta)\frac{R_M}{P_M} + \eta\lambda - \psi_P = \lambda. \tag{28}$$

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It follows that given the equilibrium rate of return λ of fund investors, we can find the equilibrium $R_L, P_L, R_M, P_{M,}, f_A$, and f_P as solutions to the system of equations (27)-(28) and the pricing equations (8)-(9). For R_i and P_i, these solutions (as functions of λ) do not depend on the shape of the compensation contract and are the same as in the basic model. In particular, the equilibrium payoffs and prices are given by (*ii*)-(*iii*) in Proposition 1. However, as shown in the appendix, the equation that determines λ when $\lambda > 1$ is generalized from (21) to:

$$W = \frac{c_A\left(R_L - R_M\right)}{\frac{\partial}{\partial r}\phi\left(\frac{R_L}{P_L}, f_A\right)} P_L + \frac{c_P\left(2R_M - R_L - R_0\right)}{\frac{\partial}{\partial r}\phi\left(\frac{R_M}{P_M}, f_P\right)} P_M. \tag{29}$$

While the equilibrium for a fixed λ is the same, the shape of the compensation contract affects the equilibrium and matters for governance because it affects the equilibrium return λ. For example, suppose that the contract is steeper than in the basic model: instead of $\phi(r, f) = fr$ in the basic model, $\phi(r, f) = f \max\{0, r - w\}$ for some water mark $w > 0$, assumed to be below the equilibrium returns of the funds. Then, to implement the same sharing of surplus between the fund manager and each investor, f_i must be higher than in the basic model. This implies that fund managers will exert higher effort given the same AUM, which in turn will lead to a lower equilibrium λ.[31]

Although the equilibrium changes, the key trade-offs of passive fund growth for governance remain similar. First, suppose that the search cost ψ_P is high enough, so that investors are indifferent between investing with the funds and saving privately, i.e., $\lambda = 1$. Then, Proposition 1 implies that aggregate governance (captured by the payoff of the market portfolio) is given by $R_M = (1 + \frac{1-\eta}{\psi_P})Z_M$. Thus, governance improves when access to passive funds becomes easier, as in the basic model. Second, for a general λ, the fact that $R_M = (1 + \frac{1-\eta}{\psi_P + (\lambda-1)(1-\eta)})Z_M$, again implies a trade-off between governance and investor well-being: a change in any parameter that does not enter this relation (e.g., ψ_A or c_i), improves

[31] Another interesting extension is to allow investors and fund managers to sign any contract, without restricting attention to a specific ordered set of contracts. We conjecture that any equilibrium in the model of Section 7.4 (i.e., an equilibrium that arises for a given ordered set of contracts) is also an equilibrium in this more general extension. Intuitively, when an investor with infinitesimal wealth ε and a fund manager bargain over a contract, the result of the bargaining has no effect on equilibrium in the financial market (since the investor is infinitesimal), and thus both the fund manager and fund investor are indifferent between all contracts that attain the same division of surplus. Since the ordered set of contracts $\{\phi(r, f), f \in [f_L, f_H]\}$ is sufficiently large to cover any division of surplus (given the second and third restrictions on function $\phi(r, f)$ above), the fund manager and investor will not benefit from deviating to a different type of contract.

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governance R_M if and only if it decreases investors' return λ.

8 Conclusion

The governance role of delegated portfolio managers, and passive funds in particular, is the subject of an ongoing debate among academics and policymakers. In this paper, we develop a tractable theoretical framework to study the governance effects of active and passive funds in a general equilibrium setting. Analyzing market equilibrium is critical for understanding the governance implications of passive fund growth because their greater availability changes not only firms' ownership structures, but also the fees and AUM of both active and passive funds, which all affect fund managers' incentives to be engaged shareholders.

We highlight that passive fund growth can improve aggregate governance even if it is accompanied by a reduction in fund fees. However, improvements in governance are not guaranteed and depend on whether passive funds primarily compete with investors' private savings or with active funds; such governance improvements may also come at the expense of fund investors' well-being. Moreover, the effects of higher passive ownership are heterogeneous across firms, depending on whether passive funds primarily replace retail shareholders or active funds in these firms' ownership structures. Our analysis has important implications for the interpretation of empirical studies of passive funds and helps reconcile the conflicting evidence on their governance role.

To focus on the interplay between fund managers' AUM, fees, investment strategies, and ownership stakes, we abstract from several important features of the monitoring process, such as investors' private information about firms, dynamic considerations due to differences in investors' horizons, or potential coordination between shareholders in their monitoring efforts. An in-depth look at these questions and their interaction with the mechanisms we study in the paper provides interesting avenues for future research.

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Appendix

Ownership by active and passive funds over time

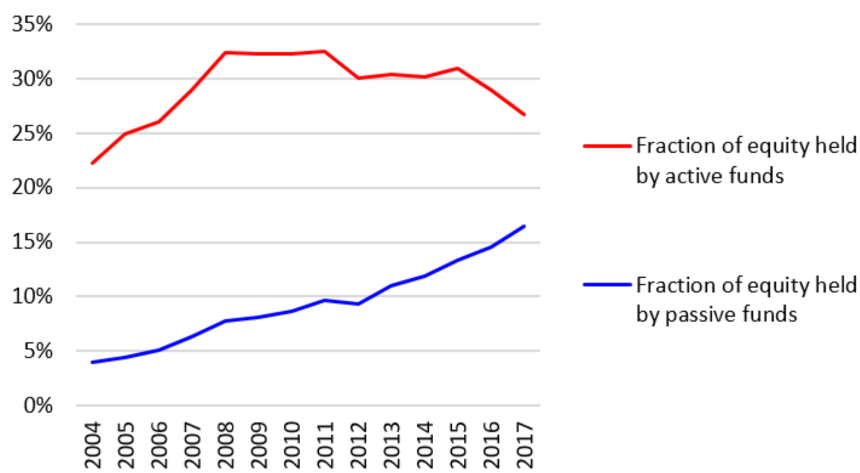

Figure A.1. The fraction of equity held by active (passive) funds is calculated by dividing the combined AUM of active (passive) funds from the CRSP Mutual Fund database by the total market capitalization of U.S. public firms. This is equivalent to calculating the ownership stakes of active and passive funds within each firm and then taking the market-value-weighted average of those stakes across firms.

Proofs

Certain auxiliary results (Lemma 1 through Lemma 8) and derivations (equations (63)–(117)) have been relegated to the online appendix. We refer to these results and equations in some places of the main appendix.

Proof of Proposition 1. There are two possible cases: 1) $\lambda = 1$, and 2) $\lambda > 1$. We consider each case separately.

(1) Equilibrium when $\lambda = 1$.

Consider the three equations for the active fund manager and L-stocks, i.e., (8), (15), and (17), which we can rewrite as:

$$f_A = \eta \frac{Z_L}{R_L} \quad \text{(fee bargaining)} \tag{30}$$

$$(1 - f_A) \frac{R_L}{P_L} = 1 + \psi_A \quad \text{(investor indifference)} \tag{31}$$

$$R_L - P_L = Z_L \quad \text{(market clearing)} \tag{32}$$

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Plugging f_A from (30) and P_L from (32) into (31) gives:

$$\left(1-\frac{\eta Z_L}{R_L}\right)\frac{R_L}{R_L-Z_L}=1+\psi_A \Leftrightarrow (1+\psi_A-\eta)\,Z_L=\psi_A R_L.$$

Hence, $R_L=\left(1+\frac{1-\eta}{\psi_A}\right)Z_L$. Then, (32) implies $P_L=R_L-Z_L=\frac{1-\eta}{\psi_A}Z_L$, and (30) implies

$$f_A=\eta\frac{Z_L}{\frac{1+\psi_A-\eta}{\psi_A}Z_L}=\frac{\eta\psi_A}{1+\psi_A-\eta}.$$

Similarly, we can rewrite the three equations for the passive fund manager and the market portfolio, i.e., (9), (16), and (18), as

$$\begin{aligned} f_P &= \eta\frac{Z_M}{R_M} && \text{(fee bargaining)} \\ (1-f_P)\frac{R_M}{P_M} &= 1+\psi_P && \text{(investor indifference)} \\ R_M-P_M &= Z_M && \text{(market clearing)} \end{aligned}$$

Since this system looks similar to the corresponding system for the active fund and the L-stocks, the solution is: $R_M=\left(1+\frac{1-\eta}{\psi_P}\right)Z_M$, $P_M=\frac{1-\eta}{\psi_P}Z_M$, and $f_P=\frac{\eta\psi_P}{1+\psi_P-\eta}$.

(2) Equilibrium when $\lambda>1$.

We start by deriving (21). Using (6) and (7) and plugging them into (20), we get

$$W=\frac{1}{2}x_{AL}P_L+x_P P_M. \tag{33}$$

Next, using (10) and (11),

$$\begin{aligned} R_L-R_M &= \frac{1}{2}c_A'^{-1}(f_A x_{AL}) \Leftrightarrow c_A'(2\,(R_L-R_M))=f_A x_{AL}, && (34)\\ 2R_M-R_L &= R_0+c_P'^{-1}(f_P x_P) \Leftrightarrow c_P'(2R_M-R_L-R_0)=f_P x_P. && (35) \end{aligned}$$

Plugging these into (33) gives (21). We next characterize the equilibrium as a function of λ, using (8)-(11); (15), (16); and (19), (21).

First, consider L-stocks and the active fund and use (15), (19), and (8):

$$\begin{aligned} f_A\frac{R_L}{P_L} &= \eta\left(\frac{R_L}{P_L}-\lambda\right) && \text{(fee bargaining)} && (36)\\ (1-f_A)\frac{R_L}{P_L} &= \psi_A+\lambda && \text{(investor indifference)} && (37)\\ P_L &= R_L-Z_L && \text{(market clearing)} && (38) \end{aligned}$$

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From (36), $\frac{R_L}{P_L} = \frac{\eta\lambda}{\eta - f_A}$, and plugging this into (37) gives

$$(1 - f_A)\,\frac{\eta\lambda}{\eta - f_A} = \psi_A + \lambda \Leftrightarrow f_A = \frac{\eta\psi_A}{\psi_A + \lambda\,(1-\eta)}.$$

Plugging this into (36) gives

$$\frac{R_L}{P_L}\eta\left(1 - \frac{\psi_A}{\psi_A + \lambda\,(1-\eta)}\right) = \eta\lambda \Leftrightarrow (\psi_A + \lambda\,(1-\eta))\,P_L = (1-\eta)\,R_L,$$

and using (38) gives

$$R_L = \left(1 + \frac{1-\eta}{\psi_A + (\lambda - 1)\,(1-\eta)}\right) Z_L. \tag{39}$$

Finally, using (38) and (39), $P_L = R_L - Z_L = \frac{1-\eta}{\psi_A + (\lambda-1)(1-\eta)} Z_L$.

Second, consider asset M (the market portfolio) and the passive fund manager. Since the system of equations (9), (16), and (19) looks exactly the same as the corresponding system for active fund managers and the L-asset (36)-(38), the solution looks the same as well, which gives the expressions for f_P, R_M, and P_M in the statement of the proposition.

Thus, all equilibrium outcomes – f_A, f_P, R_L, R_M, P_L, P_M – are expressed as a function of λ and the exogenous parameters of the model. The equilibrium λ is then determined from the equilibrium condition that investors invest all of their capital either with the active or with the passive fund manager, i.e., the fixed point solution to (21).

(3) Combining the two cases together.

By Lemma 1 in the online appendix, if $c_P \geq \frac{\psi_P}{\psi_A} c_A$, then λ is decreasing in W. Hence, there exists $\bar{W}$ such that $\lambda > 1$ for $W < \bar{W}$ and $\lambda = 1$ for $W \geq \bar{W}$. As also shown in Lemma 1, λ strictly decreases in W if $W < \bar{W}$ and $c_P \geq \frac{\psi_P}{\psi_A} c_A$. It remains to ensure that in the conjectured equilibrium: (1) the active fund indeed finds it optimal to only invest in L-stocks and to diversify across all L-stocks; (2) both the active and passive fund raise positive AUM; and (3) the active and passive fund combined do not hold all the shares, so that liquidity investors hold at least some shares in each firm. Lemma 2 in the online appendix proves that the active fund will indeed diversify equally across L-stocks. Part (ii) of Lemma 3 in the online appendix imposes conditions that are sufficient for the active fund to not deviate to investing in H-stocks. Lemma 4 in the online appendix imposes sufficient conditions for both funds' AUM to be positive, and Lemma 5 in the online appendix imposes sufficient conditions for the active and passive fund combined to not hold all the shares. Combining these conditions together yields the following two conditions:

$$\max\left\{\frac{\frac{R_0}{Z_L} + \left[1 + \frac{1-\eta}{\psi_A}\right]}{2\left[1 + \frac{1-\eta}{\psi_P}\right]}, \frac{\xi_A\xi_P + \xi_A - \xi_P}{\xi_P^2}\right\} \;<\; \frac{Z_M}{Z_L} < \frac{1 + \frac{1-\eta}{\psi_A}}{1 + \frac{1-\eta}{\psi_P}}, \tag{40}$$

$$\hat{W} \;\leq\; W < \frac{R_0 - Z_L}{2}, \tag{41}$$

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where ξ_A and ξ_P are given by (87)-(88) and $\hat{W} < \bar{W}$ is given by (96) in the online appendix. Finally, we point out that the conditions of the proposition describe a non-empty set of parameters. For example, $\eta = 0.01$, $c_A = 0.001$, $c_P = 0.002$, $\psi_A = 0.1$, $Z_L = 1$, $Z_H = 0.81$, $R_0 = 10.75$, $W = 1.5$, and $\psi_p \in [0.0897; 0.08974]$ satisfy these conditions. ■

Proof of Proposition 2. (1) We start by deriving the expressions for active and passive funds' AUM. Using Proposition 1 and (64),

$$\begin{aligned} W_P = x_P P_M &= \tfrac{c_P e_P}{f_P} \tfrac{R_M}{\frac{\psi_P}{1-\eta}+\lambda} = c_P \left(2R_M - R_L - R_0\right) \tfrac{\psi_P+\lambda(1-\eta)}{\eta\psi_P} \tfrac{R_M(1-\eta)}{\psi_P+\lambda(1-\eta)} \\ &= \tfrac{1-\eta}{\eta} \tfrac{c_P}{\psi_P} R_M \left(2R_M - R_L - R_0\right). \end{aligned} \quad (42)$$

Similarly, using Proposition 1 and (63),

$$\begin{aligned} W_A = \tfrac{1}{2} x_{AL} P_L &= \tfrac{1}{2} \tfrac{c_A e_{AL}}{f_A} \tfrac{R_L}{\frac{\psi_A}{1-\eta}+\lambda} = \tfrac{1}{2} 2c_A \left(R_L - R_M\right) \tfrac{\psi_A+\lambda(1-\eta)}{\eta\psi_A} \tfrac{R_L(1-\eta)}{\psi_A+\lambda(1-\eta)} \\ &= \tfrac{1-\eta}{\eta} \tfrac{c_A}{\psi_A} R_L \left(R_L - R_M\right). \end{aligned} \quad (43)$$

Note, as an auxiliary result, that these expressions imply that when $\lambda = 1$, AUM of fund i are decreasing in ψ_i. Indeed, if $\lambda = 1$, then R_L does not depend on ψ_P, and W_P strictly decreases in ψ_P if and only if

$$-\frac{c_P}{\psi_P^2} R_M \left(2R_M - R_L - R_0\right) + \frac{c_P}{\psi_P} \left(4R_M - R_L - R_0\right) \frac{dR_M}{d\psi_P} < 0,$$

which holds since $2R_M - R_L - R_0 > 0$ and $\frac{dR_M}{d\psi_P} < 0$. Similarly, if $\lambda = 1$, then R_M does not depend on ψ_A, and W_A strictly decreases in ψ_A if and only if

$$-\frac{c_A}{\psi_A^2} R_L \left(R_L - R_M\right) + \frac{c_A}{\psi_A} \left(2R_L - R_M\right) \frac{dR_L}{d\psi_A} < 0,$$

which holds since $R_L - R_M > 0$ and $\frac{dR_L}{d\psi_A} < 0$. Note also that the same arguments hold for the equilibria of Lemma 7 in the online appendix, in which only one fund raises AUM – this is because the above expressions for W_A (W_P) are still valid in the equilibrium where only the active (passive) fund raises AUM.

(2) Next, we show that the combined AUM of active and passive fund managers, $W_A + W_P$, strictly decrease in ψ_P if $\lambda = 1$. This automatically implies that $W_A + W_P$ always weakly decrease in ψ_P (because when $\lambda > 1$, $W_A + W_P = W$). To show that total AUM decrease in ψ_P, note, using (43)-(42), that

$$W_A + W_P = \frac{1-\eta}{\eta} \left(\frac{c_A}{\psi_A} R_L \left(R_L - R_M\right) + \frac{c_P}{\psi_P} R_M \left(2R_M - R_L - R_0\right) \right). \quad (44)$$

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Since R_L does not depend on ψ_P for $\lambda = 1$, $W_A + W_P$ decrease in ψ_P if and only if

$$-\frac{c_A}{\psi_A}R_L\frac{dR_M}{d\psi_P} - \frac{c_P}{\psi_P^2}R_M\left(2R_M - R_L - R_0\right) + \frac{c_P}{\psi_P}\left(4R_M - R_L - R_0\right)\frac{dR_M}{d\psi_P} \;<\; 0 \Leftrightarrow$$
$$\left[-\frac{c_A}{\psi_A}R_L + \frac{c_P}{\psi_P}\left(4R_M - R_L - R_0\right)\right]\frac{dR_M}{d\psi_P} - \frac{c_P}{\psi_P^2}R_M\left(2R_M - R_L - R_0\right) \;<\; 0.$$

Since $2R_M - R_L - R_0 > 0$ and $\frac{\partial R_M}{\partial \psi_P} < 0$, it is sufficient to show that

$$-\frac{c_A}{\psi_A}R_L + \frac{c_P}{\psi_P}\left(4R_M - R_L - R_0\right) \geq 0. \tag{45}$$

Note that $e_P = 2R_M - R_L - R_0 \geq 0$ and hence $2R_M - R_L > 0$, and summing up these two inequalities gives $4R_M - R_L - R_0 > R_L$. This, together with the assumption of Proposition 1 that $\frac{c_P}{\psi_P} \geq \frac{c_A}{\psi_A}$, implies (45), as required. The same result with respect to ψ_P also applies in the equilibrium of Lemma 7 in the online appendix in which only the passive fund raises positive AUM.

The fact that $W_A + W_P$ decrease in ψ_P implies the last statement of the lemma, i.e., that $\lambda = 1$ only when ψ_P is large enough. Indeed, if $\lambda = 1$, fund investors invest their funds both with the fund managers and in private savings, and hence $W_A + W_P < W$, while if $\lambda > 1$, all investor funds are allocated to the fund managers, i.e., $W_A + W_P = W$. Hence, $\lambda = 1$ applies if and only if $W_A + W_P < W$, or if and only if ψ_P is large enough.

(3) Next, we prove that λ decreases in ψ_P under the conditions of Proposition 1. This is weakly satisfied for the region where $\lambda = 1$. To see this for the region where $\lambda > 1$, note that the combined AUM of the two funds, $W_A + W_P$, satisfy (44). In addition, for a fixed λ, R_L does not depend on ψ_P and R_M decreases in ψ_P, so repeating the steps subsequent to (44), implies that for a fixed λ, $W_A + W_P$ decreases in ψ_P. Moreover, if $\lambda > 1$, then $W_A + W_P = W$. On the other hand, as follows from the proof of Lemma 1 in the online appendix, equality (54) holds, where the right-hand side decreases in λ. Combined, we have

$$W_A\left(\lambda, \psi_P\right) + W_P\left(\lambda, \psi_P\right) = W,$$

and hence,

$$\frac{\partial\left(W_A + W_P\right)}{\partial \lambda}\frac{d\lambda}{d\psi_P} + \frac{\partial\left(W_A + W_P\right)}{\partial \psi_P} = 0,$$

where $\frac{\partial(W_A+W_P)}{\partial\lambda} < 0$ and $\frac{\partial(W_A+W_P)}{\partial\psi_P} < 0$. Thus, $\frac{d\lambda}{d\psi_P} < 0$, as required.

(4) Finally, we prove the result for fund fees, i.e., that both f_A and f_P increase in ψ_P. Since $f_A = \frac{\eta\psi_A}{\psi_A + \lambda(1-\eta)}$, it weakly increases in ψ_P (it does not depend on ψ_P if $\lambda = 1$ and strictly increases if $\lambda > 1$ given $\frac{d\lambda}{d\psi_P} < 0$). And, since $f_P = \frac{\eta\psi_P}{\psi_P + \lambda(1-\eta)}$, it always strictly increases in ψ_P: if $\lambda = 1$, this is because $f_P = \frac{\eta\psi_P}{\psi_P + 1 - \eta}$, while if $\lambda > 1$, this is because $\frac{df_P}{d\psi_P} = \frac{\partial f_P}{\partial\lambda}\frac{d\lambda}{d\psi_P} + \frac{\partial f_P}{\partial\psi_P} > 0$, which follows from $\frac{\partial f_P}{\partial\lambda} < 0$, $\frac{d\lambda}{d\psi_P} < 0$, and $\frac{\partial f_P}{\partial\psi_P} > 0$. This completes the proof. ■

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Proof of Proposition 3. Note that $c_P \geq c_A$ and $\psi_P \leq \psi_A$ together imply that $c_P \geq \frac{\psi_P}{\psi_A} c_A$. Recall that by Proposition 2, $\lambda = 1$ if $\psi_P \geq \bar{\psi}_P$ and $\lambda > 1$ if $\psi_P < \bar{\psi}_P$. Therefore, if $\psi_P > \bar{\psi}_P$, Proposition 1 implies that R_M strictly increases as ψ_P decreases.

Second, to establish that the continuity of equilibrium also applies at $\psi_P = \bar{\psi}_P$, we prove that $\lim_{\psi_P \uparrow \bar{\psi}_P} \lambda = 1$, and that $\psi_P = \bar{\psi}_P$ satisfies the fixed point equation (21) with $\lambda = 1$. To see this, note that Propositions 1 and 2 imply that for all $\psi_P < \bar{\psi}_P$, (21) is satisfied for the equilibrium λ. Denote the right hand side of (21) by $RHS(\lambda, \psi_P)$, and recall that by the proof of Proposition 1, $RHS(\lambda, \psi_P)$ represents the total AUM of active and passive funds (that is, $W_A + W_P$). Also note that $RHS(\lambda, \psi_P)$ is continuous w.r.t. λ and ψ_P, is strictly decreasing with ψ_P (by step (3) of the proof of Proposition 2), and is strictly decreasing in λ (by Proposition 1). Therefore, it is sufficient to show that $\psi_P = \bar{\psi}_P$ satisfies (21) with $\lambda = 1$ (since it would also imply that $\lim_{\psi_P \uparrow \bar{\psi}_P} \lambda = 1$). Suppose this is not the case. Then, since $\lambda = 1$ has to hold by Proposition 2, it must be that $W \neq RHS(1, \bar{\psi}_P)$. Since $RHS(\lambda, \psi_P)$ represents the total AUM, it cannot be $W < RHS(1, \bar{\psi}_P)$, and hence it must be $W > RHS(1, \bar{\psi}_P)$. However, then by continuity of $RHS(\lambda, \psi_P)$ in ψ_P, there exists $\delta > 0$ such that $W > RHS(1, \psi_P')$ for any $\psi_P' \in (\bar{\psi}_P - \delta, \bar{\psi}_P)$. Therefore, for any such $\psi_P = \psi_P'$, $\lambda = 1$ should be an equilibrium according to step (1) in the proof of Proposition 1, which yields a contradiction with Proposition 2 since $\psi_P' < \psi_P$.

Third, we prove that if W_A weakly increases as ψ_P decreases and $\psi_P \leq \bar{\psi}_P$, then R_M strictly decreases as ψ_P decreases. Note that as ψ_P decreases, Proposition 2 implies that λ strictly increases, where "strictly" follows step (3) in the proof of Proposition 2. Therefore, Proposition 1 implies that R_L strictly decreases as ψ_P decreases. Therefore, since W_A is given by (43), for W_A to weakly increase it must be that R_M strictly decreases.

Fourth, we re-formulate R_H and R_L. Denote the total capital invested by the passive fund in L-firms and H-firms by W_{PL} and W_{PH}, respectively. Then, using this notation, we can re-formulate R_H and R_L as follows.

(a) Re-formulation of R_H: By (3) and $x_{AH} = 0$, we have $R_H = R_0 + \frac{f_P x_P}{c_P}$. Plugging in $x_P = \frac{W_{PH}}{\frac{1}{2} P_H}$ (since there is $\frac{1}{2}$ measure of H-firms) and $P_H = R_H - Z_H$,

$$\begin{aligned} R_H &= R_0 + \frac{f_P}{c_P} \frac{2W_{PH}}{R_H - Z_H} \Leftrightarrow R_H (R_H - Z_H) = R_0 (R_H - Z_H) + \frac{f_P}{c_P} 2W_{PH} \\ &\Leftrightarrow R_H^2 - (R_0 + Z_H) R_H - \left(\frac{f_P}{c_P} 2W_{PH} - R_0 Z_H \right) = 0. \end{aligned}$$

The discriminant of this quadratic equation is given by $\Delta = (R_0 - Z_H)^2 + 8 \frac{f_P}{c_P} W_{PH}$. Since $\sqrt{\Delta} > R_0 - Z_H$, the smaller root for R_H is smaller then Z_H, contradicting with $P_H = R_H - Z_H > 0$. Therefore, R_H is given by the larger root:

$$R_H = \frac{1}{2} (R_0 + Z_H) + \sqrt{\frac{1}{4} (R_0 - Z_H)^2 + 2 \frac{f_P}{c_P} W_{PH}}. \tag{46}$$

Hence,

$$\frac{dR_H}{d\psi_P} = \frac{2}{2R_H - Z_H - R_0} \left(\frac{f_P}{c_P} \frac{dW_{PH}}{d\psi_P} + \frac{1}{c_P} W_{PH} \frac{df_P}{d\psi_P} \right). \tag{47}$$

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(b) Re-formulation of R_L: By (3), we have $R_L = R_0 + \frac{f_P x_P}{c_P} + \frac{f_P x_{AL}}{c_A}$. Plugging in $x_P = \frac{W_{PL}}{\frac{1}{2}P_L}$ and $x_{AL} = \frac{W_A}{\frac{1}{2}P_L}$ (since $x_{AH} = 0$ and there is $\frac{1}{2}$ measure of H-firms) and using derivations analogous to part (a) yields

$$R_L = \frac{1}{2}(R_0 + Z_L) + \sqrt{\frac{1}{4}(R_0 - Z_L)^2 + \frac{f_P}{c_P}2W_{PL} + \frac{f_A}{c_A}2W_A}. \tag{48}$$

Hence,

$$\frac{dR_L}{d\psi_P} = \frac{2}{2R_L - Z_L - R_0}\left(\frac{f_P}{c_P}\frac{dW_{PL}}{d\psi_P} + \frac{f_A}{c_A}\frac{dW_A}{d\psi_P} + \frac{1}{c_P}W_{PL}\frac{df_P}{d\psi_P} + \frac{1}{c_A}W_A\frac{df_A}{d\psi_P}\right). \tag{49}$$

Fifth, we prove that if W_A strictly decreases as ψ_P decreases, $\psi_P \leq \bar{\psi}_P$, and $Z_L - Z_H > 2e_{AL}$, then $\frac{dR_M}{d\psi_P} > 0$. Note that as noted in the third step above, as ψ_P decreases, λ strictly increases and R_L strictly decreases. Denote the total capital invested by the passive fund in L-firms and H-firms by W_{PL} and W_{PH}, respectively. Then, combining $W_A + W_P = W_A + W_{PL} + W_{PH}$ with $W = W_A + W_P$ (where the latter follows by the arguments in the second step above) yields

$$\frac{dW_A}{d\psi_P} + \frac{dW_{PL}}{d\psi_P} = -\frac{dW_{PH}}{d\psi_P}. \tag{50}$$

(When $\psi_P = \bar{\psi}_P$, we replace all derivatives with left-hand derivatives, i.e., derivatives as $\psi_P \uparrow \bar{\psi}_P$.) Note that $\frac{dW_A}{d\psi_P} > 0$ since we are focusing on the case where W_A strictly decreases as ψ_P decreases. Also note that $\frac{d\lambda}{d\psi_P} < 0$ together with Propositions 1 and 2 imply that $\frac{df_P}{d\psi_P} > 0$ and $\frac{df_A}{d\psi_P} > 0$. There are two scenarios to consider:

(1) Suppose that $\frac{dW_A}{d\psi_P} + \frac{dW_{PL}}{d\psi_P} \leq 0$. Then, (50) implies that $\frac{dW_{PH}}{d\psi_P} \geq 0$. Therefore, $\frac{df_P}{d\psi_P} > 0$ and (47) imply that $\frac{dR_H}{d\psi_P} > 0$, i.e., R_H strictly decreases as ψ_P decreases. Since we have previously established that $\frac{dR_L}{d\psi_P} > 0$, this implies that $\frac{dR_M}{d\psi_P} = \frac{1}{2}\left(\frac{dR_L}{d\psi_P} + \frac{dR_H}{d\psi_P}\right) > 0$.

(2) Suppose that $\frac{dW_A}{d\psi_P} + \frac{dW_{PL}}{d\psi_P} > 0$. Due to (50), this implies that $\frac{dW_{PH}}{d\psi_P} < 0$. Since $\frac{df_P}{d\psi_P} > 0$ and $\frac{df_A}{d\psi_P} > 0$, (47) and (49) imply that to show $\frac{dR_M}{d\psi_P} = \frac{1}{2}\left(\frac{dR_L}{d\psi_P} + \frac{dR_H}{d\psi_P}\right) > 0$, it is sufficient to prove that

$$0 < \frac{1}{2R_H - Z_H - R_0}\frac{f_P}{c_P}\frac{dW_{PH}}{d\psi_P} + \frac{1}{2R_L - Z_L - R_0}\left(\frac{f_P}{c_P}\frac{dW_{PL}}{d\psi_P} + \frac{f_A}{c_A}\frac{dW_A}{d\psi_P}\right). \tag{51}$$

Recall that $\frac{dW_A}{d\psi_P} > 0$. Combining with $c_P \geq c_A$ and $f_P \leq f_A$ (where the latter is by Proposition 1), this implies that to show (51), it is sufficient to show

$$0 < \frac{1}{2R_H - Z_H - R_0}\frac{dW_{PH}}{d\psi_P} + \frac{1}{2R_L - Z_L - R_0}\left(\frac{dW_{PL}}{d\psi_P} + \frac{dW_A}{d\psi_P}\right). \tag{52}$$

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In turn, (50) and $\frac{dW_{PH}}{d\psi_P} < 0$ imply that (52) is equivalent to

$$0 < -\frac{1}{2R_H - Z_H - R_0} + \frac{1}{2R_L - Z_L - R_0} \Leftrightarrow 2R_L - Z_L < 2R_H - Z_H \Leftrightarrow 2e_{AL} < Z_L - Z_H,$$

where the equivalence follows from $R_H = R_0 + e_P$ (since $x_{AH} = 0$) and $R_L = R_0 + e_P + e_{AL}$. Since $Z_L - Z_H > 2e_{AL}$ holds by assumption, this concludes the proof of the proposition.

We now show that there exists a cutoff $\underline{\psi}_P$ such that condition $e_{AL} < \frac{1}{2}(Z_L - Z_H)$ is satisfied if $\psi_P < \underline{\psi}_P$. Since $e_{AL} = 2(R_L - R_M)$, this reduces to $\frac{1}{2}(Z_L - Z_H) > 2(R_L - R_M)$. Plugging in $Z_H = 2Z_M - Z_L$ and R_L and R_M from Proposition 1, this inequality becomes

$$\begin{aligned} Z_L - Z_M &> 2\left(1 + \frac{1-\eta}{\psi_A + (\lambda-1)(1-\eta)}\right) Z_L - 2\left(1 + \frac{1-\eta}{\psi_P + (\lambda-1)(1-\eta)}\right) Z_M \\ \Leftrightarrow \quad & \frac{1 + 2\frac{1-\eta}{\psi_P + (\lambda-1)(1-\eta)}}{1 + 2\frac{1-\eta}{\psi_A + (\lambda-1)(1-\eta)}} > \frac{Z_L}{Z_M}. \end{aligned} \tag{53}$$

Since $\psi_P \leq \psi_A$, the left-hand side decreases in λ. Since $\lambda \leq \lambda_{\max} = \frac{R_0}{R_0 - Z_L} - \psi_A$ by Lemma 6 in the online appendix, it is sufficient to show that (53) holds for $\lambda = \lambda_{\max}$, i.e.,

$$\psi_P < 2\frac{1-\eta}{\frac{Z_L}{Z_M}\left(1 + 2\frac{1-\eta}{\psi_A + (\lambda_{\max}-1)(1-\eta)}\right) - 1} - (\lambda_{\max} - 1)(1-\eta) \Leftrightarrow$$

$$\psi_P < \underline{\psi}_P \equiv 2\frac{1-\eta}{\frac{Z_L}{Z_M}\left(1 + 2\frac{1-\eta}{\psi_A + \left(\frac{R_0}{R_0 - Z_L} - \psi_A - 1\right)(1-\eta)}\right) - 1} - \left(\frac{R_0}{R_0 - Z_L} - \psi_A - 1\right)(1-\eta).$$

■

Proof of Proposition 4.

Note that by Proposition 2, $\lambda = 1$ if $\psi_P \geq \bar{\psi}_P$ and $\lambda > 1$ if $\psi < \bar{\psi}_P$. By Proposition 1, $\lambda = 1$ if $W \geq \bar{W}$ and $\lambda > 1$ if $W < \bar{W}$. Therefore, it must be that if $\psi_P \geq \bar{\psi}_P$, then $W \geq \bar{W}$, and if $\psi_P < \bar{\psi}_P$, then $W < \bar{W}$.

We start by proving (ii). Fund investors' payoff is characterized by their equilibrium rate of return λ. When $W \geq \bar{W}$, their rate of return is $\lambda = 1$ and is unaffected by c_A or c_P. When $W \leq \bar{W}$, λ increases with c_A and c_P. To see this, recall that λ is the solution to

$$W = \frac{c_A}{f_A(\lambda)}(R_L(\lambda) - R_M(\lambda)) P_L(\lambda) + \frac{c_P}{f_P(\lambda)}(2R_M(\lambda) - R_L(\lambda) - R_0) P_M(\lambda), \tag{54}$$

where $f_A(\lambda)$, $f_P(\lambda)$, $R_L(\lambda)$, $R_M(\lambda)$, $P_L(\lambda)$, and $P_M(\lambda)$ are given by Proposition 1. By Lemma 1 in the online appendix, the right-hand side decreases with λ whenever $\psi_A \geq \psi_P$ and $c_A \leq \frac{\psi_A}{\psi_P} c_P$. Since the right-hand side increases in c_A and c_P, it follows that λ increases in c_A and c_P (otherwise, if c_i increased, the right-hand side would increase both through the effect of c_i and through the effect of λ, while the left-hand side would not).

We next prove (i). Consider R_L and R_M. If $W \geq \bar{W}$, they do not depend on c_A or c_P. If $W \leq \bar{W}$, then $R_L = (1 + \frac{1-\eta}{\psi_A + (\lambda-1)(1-\eta)}) Z_L$ and $R_M = (1 + \frac{1-\eta}{\psi_P + (\lambda-1)(1-\eta)}) Z_M$. Since λ

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increases with c_A and c_P as shown above, then both R_L and R_M decrease with c_A and c_P, and thus P_L and P_M decrease with c_A and c_P as well.

Finally, we prove (iii). Let e_P (e_{AL}) denote the passive (active) fund manager's equilibrium effort. Then, the passive fund manager's payoff is given by

$$\begin{gathered} V_P = f_P x_P R_M - \tfrac{c_P}{2} e_P^2 = c_P e_P \left(R_M - \tfrac{1}{2} e_P\right) \\ = c_P \left(2R_M - R_L - R_0\right) \left(R_M - \tfrac{1}{2}\left(2R_M - R_L - R_0\right)\right) = \tfrac{c_P}{2}\left(2R_M - R_L - R_0\right)\left(R_L + R_0\right), \end{gathered} \tag{55}$$

and the active fund manager's payoff is given by

$$\begin{gathered} V_A = \tfrac{1}{2}\left(f_A x_{AL} R_L - \tfrac{c_A}{2} e_{AL}^2\right) = \tfrac{1}{2} c_A e_{AL} \left(R_L - \tfrac{1}{2} e_{AL}\right) \\ = c_A \left(R_L - R_M\right) \left(R_L - \tfrac{1}{2} 2 \left(R_L - R_M\right)\right) = c_A \left(R_L - R_M\right) R_M. \end{gathered} \tag{56}$$

If $W \geq \bar{W}$, then by Proposition 1, R_L and R_M do not change with c_A and c_P. Hence, V_P increases with c_P and V_A increases with c_A. ■

Proof that $\frac{R_L}{P_L}$ decreases more than $\frac{R_M}{P_M}$ upon a decrease in c_i. In this part, we show why the return $\frac{R_L}{P_L}$ declines more than $\frac{R_M}{P_M}$ when either c_A or c_P marginally decreases. Note that under the conditions of Proposition 3, we have $Z_L - Z_H > e_{AL} \Leftrightarrow P_H > P_L$, and hence $P_M > P_L$.

First, consider a marginal decrease in c_A, and suppose that it increases the active fund's effort (in L-stocks) by x. Then the new returns are, respectively, $\frac{R_L+x}{P_L+x}$ and $\frac{R_M+x/2}{P_M+x/2}$. Then

$$\frac{d}{dx}\frac{R_M + x/2}{P_M + x/2} = \frac{1}{2}\frac{-Z_M}{\left(P_M + x/2\right)^2} > \frac{d}{dx}\frac{R_L + x}{P_L + x} = \frac{-Z_L}{\left(P_L + x\right)^2} \Leftrightarrow \frac{2Z_L}{Z_M} > \frac{\left(P_L + x\right)^2}{\left(P_M + x/2\right)^2},$$

which holds because $Z_L > Z_M$ and $P_L + x < P_M + \frac{x}{2}$ for small x because $P_M > P_L$. Thus, the reduction in the return of the passive fund is smaller than the reduction in the return of the active fund.

Second, consider a marginal decrease in c_P, and suppose that it increases the passive fund's effort (in both types of stocks) by x. Then the new returns are, respectively, $\frac{R_M+x}{P_M+x}$ and $\frac{R_L+x}{P_L+x}$. Since $Z_L > Z_M > 0$ and $P_M > P_L$, we have $\frac{d}{dx}\frac{R_M+x}{P_M+x} = \frac{-Z_M}{(P_M+x)^2} > \frac{-Z_L}{(P_L+x)^2} = \frac{d}{dx}\frac{R_L+x}{P_L+x}$, i.e., the reduction in the return of the passive fund is smaller than the reduction in the return of the active fund. ■

Proof of Proposition 5. Using $f_A \frac{R_L}{P_L} = \eta\left(\frac{R_L}{P_L} - \lambda\right)$ and $f_P \frac{R_M}{P_M} = \eta\left(\frac{R_M}{P_M} - \lambda\right)$, the investors' equilibrium indifference condition (12) can be written as

$$\frac{R_L}{P_L} - \frac{\psi_A}{1-\eta} = \frac{R_M}{P_M} - \frac{\psi_P}{1-\eta} = \lambda.$$

Using (23), we obtain

$$\frac{R_M}{\left(1-B\right) R_M - A_M} = \lambda + \frac{\psi_P}{1-\eta}. \tag{57}$$

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In the region where $\lambda = 1$, the left-hand side of (57) increases in ψ_P, and hence a reduction in ψ_P means that the left-hand side must decline. Since it is strictly decreasing in R_M, the equilibrium level of R_M increases if ψ_P decreases, proving the first statement of the proposition. To prove the second statement of the proposition, rewrite (57) as

$$R_M = \frac{A_M \left(\lambda + \frac{\psi_P}{1-\eta}\right)}{\left(\lambda + \frac{\psi_P}{1-\eta}\right)(1-B) - 1}, \tag{58}$$

which is equivalent to (24). The cross-partial derivative of (58) in the region where $\lambda = 1$ is

$$\frac{\partial^2 R_M}{\partial \left(1 + \frac{\psi_P}{1-\eta}\right) \partial B} = -\frac{2A_M \left(1 + \frac{\psi_P}{1-\eta}\right)}{\left(\left(1 + \frac{\psi_P}{1-\eta}\right)(1-B) - 1\right)^3} < 0.$$

Hence, an increase in B increases the effect of a reduction in ψ_P on R_M. Finally, to see that the trade-off between investor well-being and governance extends to this model, note from (58) that $\frac{dR_M}{d\lambda} < 0$. Hence, for any parameter that does not enter (58), a change in this parameter increases λ if and only if it decreases R_M. ■

Proof of Proposition 6. We first derive (25)-(26). Consider an L-stock. If fraction $x_P + x_{AL}$ is owned by the mutual funds, then liquidity investors must own fraction $1 - x_P - x_{AL}$. Since the stock is owned by liquidity investors with the highest valuations (lowest Z_{kj}) and given the uniform distribution of Z_{kj} on $[Z_L - \Delta, Z_L + \Delta]$, this implies that Z_{kj}^* of the marginal liquidity investor satisfies

$$1 - x_P - x_{AL} = \Pr\left(Z_{kj} < Z_{kj}^*\right) = \frac{Z_{kj}^* - (Z_L - \Delta)}{2\Delta},$$

or $Z_{kj}^* = Z_L - \Delta\left(2x_P + 2x_{AL} - 1\right)$. Since $P_L = R_L - Z_{kj}^*$, this gives (25). Similarly, consider an H-stock. If fraction x_P is owned by the mutual funds, then liquidity investors must own fraction $1 - x_P$. This implies that Z_{kj}^* of the marginal liquidity investor satisfies $1 - x_P = \Pr(Z_{kj} < Z_{kj}^*) = \frac{Z_{kj}^* - (Z_H - \Delta)}{2\Delta}$, which gives $Z_{kj}^* = Z_H - \Delta\left(2x_P - 1\right)$. Then,

$$P_H = R_H - Z_H + \Delta\left(2x_P - 1\right). \tag{59}$$

Using $P_M = \frac{P_L + P_H}{2}$ and combining (59) with (25) gives (26).

We now prove the statement of the proposition. Using the market-clearing condition,

$$\frac{R_M}{P_M} = \frac{R_M}{R_M - Z_M + \Delta\left(2x_P + x_{AL} - 1\right)} = 1 + \frac{Z_M - \Delta\left(2x_P + x_{AL} - 1\right)}{R_M - Z_M + \Delta\left(2x_P + x_{AL} - 1\right)}. \tag{60}$$

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On the other hand, (18) combined with (16) for $\lambda = 1$ implies

$$1 - \eta \left(1 - \frac{P_M}{R_M}\right) = (1 + \psi_P)\, \frac{P_M}{R_M} \Leftrightarrow \frac{R_M}{P_M} = 1 + \frac{\psi_P}{1 - \eta}. \tag{61}$$

Equating (60) and (61), we get:

$$\frac{\psi_P}{1 - \eta} = \frac{Z_M - \Delta\,(2x_P + x_{AL} - 1)}{R_M - Z_M + \Delta\,(2x_P + x_{AL} - 1)}$$

A reduction in ψ_P reduces the left-hand side, so the right-hand side must also decline. If Δ is sufficiently small, R_M must increase, implying an improvement in governance. ■

Derivation of eq. (29). Consider the equation linking λ and W.

$$W = \frac{1}{2} x_{AL} P_L + x_P P_M. \tag{62}$$

Next, consider the effort problem. For the active fund manager:

$$\max_e \phi\left(\frac{R_0 + e + e_P}{P_L}, f_A\right) x_{AL} P_L - \frac{c_A}{2} e^2,$$

so the FOC gives:

$$\frac{\partial}{\partial r}\phi\left(\frac{R_0 + e_{AL} + e_P}{P_L}, f_A\right) x_{AL} = c_A e_{AL}.$$

Similarly, the FOC for the passive fund manager is:

$$\frac{\partial}{\partial r}\phi\left(\frac{R_0 + \frac{1}{2} e_{AL} + e_P}{P_M}, f_P\right) x_P = c_P e_P.$$

Next, by (63)-(64) in the online appendix, $e_{AL} = 2\,(R_L - R_M)$ and $e_P = 2R_M - R_L - R_0$. Combining these with FOCs gives

$$\begin{aligned} x_{AL} &= \frac{c_A e_{AL}}{\frac{\partial}{\partial r}\phi\left(\frac{R_L}{P_L}, f_A\right)} = \frac{2c_A\,(R_L - R_M)}{\frac{\partial}{\partial r}\phi\left(\frac{R_L}{P_L}, f_A\right)} \\ x_P &= \frac{c_P e_P}{\frac{\partial}{\partial r}\phi\left(\frac{R_M}{P_M}, f_P\right)} = \frac{c_P\,(2R_M - R_L - R_0)}{\frac{\partial}{\partial r}\phi\left(\frac{R_M}{P_M}, f_P\right)}. \end{aligned}$$

Plugging these into (62) yields (29). ■

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Online appendix for "Corporate governance in the presence of active and passive delegated investment"

8.1 Formalization of the trading stage

The Walrasian equilibrium in the trading stage is defined as follows. First, the active and passive fund simultaneously submit their market orders, x_A and x_P, i.e., the number of stocks each of them is willing to buy. These market orders are subject to the investment mandate that the total value of the portfolio of each fund evaluated at the expected in equilibrium market price equals the total AUM of the fund. Next, liquidity investors observe these two market orders (and thus anticipate funds' future effort levels) and submit limit orders, which specify how many shares they are willing to buy at each price. Each liquidity trader can submit any demand of up to one unit. The market clearing price is the price that equalizes demand and supply.

It is important to specify what happens if the active fund deviates from its equilibrium trading strategy. Since the passive fund submits its market order simultaneously with the active fund, the passive fund does not react to this off-equilibrium deviation. However, liquidity traders observe both market orders and submit their limit orders after that, and because they now anticipate a different level of effort by the active fund, the market clearing price changes. As a result, if the deviation of the active fund changes the price of the market portfolio, the passive fund may pay above or below its AUM off-equilibrium. To ensure that this is consistent with the setup of the model, we assume that the passive fund manager has access to saving and borrowing, which can be utilized off-equilibrium upon a deviation of the active fund manager, but cannot be utilized on-equilibrium path as it will violate the investment mandate of the passive fund. In particular, if the portfolio of the passive fund upon the active fund's deviation ends up costing more (less) than the passive fund's AUM, the passive fund borrows the extra amount (saves the extra amount outside the stock market).

8.2 Auxiliary results for the basic model

Auxiliary Result. Note that

$$\begin{aligned} R_L &= R_0 + e_{AL} + e_P \\ R_M &= R_0 + \frac{e_{AL}}{2} + e_P, \end{aligned}$$

which imply

$$e_{AL} = 2\left(R_L - R_M\right) \tag{63}$$

$$e_P = 2R_M - R_L - R_0. \tag{64}$$

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Using Proposition 1 and since $e_{AL} = \frac{f_A x_{AL}}{c_A}$ and $e_P = \frac{f_P x_P}{c_P}$, we get the following expressions for x_{AL} and x_P as functions of λ and the model parameters:

$$x_{AL} = \frac{2c_A}{f_A(\lambda)} \left[\xi_A(\lambda) Z_L - \xi_P(\lambda) Z_M \right], \tag{65}$$

$$x_P = \frac{c_P}{f_P(\lambda)} \left[2\xi_P(\lambda) Z_M - \xi_A(\lambda) Z_L - R_0 \right], \tag{66}$$

where

$$\xi_A(\lambda) \equiv 1 + \frac{1-\eta}{\psi_A + (\lambda - 1)(1-\eta)} \tag{67}$$

$$\xi_P(\lambda) \equiv 1 + \frac{1-\eta}{\psi_P + (\lambda - 1)(1-\eta)} \tag{68}$$

$$f_A(\lambda) \equiv \frac{\eta\psi_A}{\psi_A + \lambda(1-\eta)}$$

$$f_P(\lambda) \equiv \frac{\eta\psi_P}{\psi_P + \lambda(1-\eta)}.$$

■

Lemma 1 *Consider any equilibrium given by Proposition 1. Then, the rate of return* λ *is decreasing in aggregate wealth* W *if* $|c_P - c_A|$ *is sufficiently small, or if* $\psi_A \geq \psi_P$ *and* $c_A \leq \frac{\psi_A}{\psi_P} c_P$*. Moreover, under either of these conditions,* λ *is strictly decreasing in* W *if* $\lambda > 1$*.*

Proof of Lemma 1. We present the proof for the quadratic cost functions, $c_i(e) = \frac{c_i}{2} e^2$. Note that in any equilibrium where W is strictly larger than the total AUM raised by funds, it has to be $\lambda = 1$, because otherwise $\lambda > 1$ and hence the fund investors that save privately would strictly prefer to deviate and invest in a fund. Therefore, if $\lambda > 1$, then it has to be that W is equal to the total AUM. For this reason, to prove the lemma, it is sufficient to show that the total AUM strictly decreases with λ.

Consider any equilibrium given by Proposition 1. Then, the total AUM raised by funds is

$$\frac{P_L}{2f_A} c_A \left(2\left(R_L - R_M\right)\right) + \frac{P_M}{f_P} c_P \left(2R_M - R_L - R_0\right) \tag{69}$$

$$= c_A \frac{\psi_A + \lambda(1-\eta)}{\eta\psi_A} \begin{pmatrix} \left(1 + \frac{1-\eta}{\psi_A + (\lambda-1)(1-\eta)}\right) Z_L \\ -\left(1 + \frac{1-\eta}{\psi_P + (\lambda-1)(1-\eta)}\right) Z_M \end{pmatrix} \frac{1-\eta}{\psi_A + (\lambda-1)(1-\eta)} Z_L$$

$$+ c_P \frac{\psi_P + \lambda(1-\eta)}{\eta\psi_P} \begin{pmatrix} 2\left(1 + \frac{1-\eta}{\psi_P + (\lambda-1)(1-\eta)}\right) Z_M \\ -\left(1 + \frac{1-\eta}{\psi_A + (\lambda-1)(1-\eta)}\right) Z_L - R_0 \end{pmatrix} \frac{1-\eta}{\psi_P + (\lambda-1)(1-\eta)} Z_M,$$

Note that by the proof of Proposition 1, both funds raise positive AUM, and hence

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$x_{AL} > 0$ and $x_P > 0$. Moreover, λ has a finite upperbound by Lemma 6. Therefore, (3) implies that $e_{AL} = \frac{f_A x_{AL}}{c_A} > 0$ and $e_P = \frac{f_P x_P}{c_A} > 0$, and in turn, (8)-(11) imply that $R_L - R_M = \frac{1}{2} e_{AL} > 0$ and $2R_M - R_L - R_0 = e_P > 0$. Plugging in the expressions for R_L and R_M from Proposition 1 yields

$$0 > -\left(1 + \frac{1-\eta}{\psi_A + (\lambda - 1)(1-\eta)}\right) Z_L + \left(1 + \frac{1-\eta}{\psi_P + (\lambda - 1)(1-\eta)}\right) Z_M \tag{70}$$

and

$$0 > -2\left(1 + \frac{1-\eta}{\psi_P + (\lambda - 1)(1-\eta)}\right) Z_M + R_0 + \left(1 + \frac{1-\eta}{\psi_A + (\lambda - 1)(1-\eta)}\right) Z_L, \tag{71}$$

respectively. Multiplying (69) by $\frac{\eta}{1-\eta}$ and rearranging the terms, we get

$$\begin{aligned} &\frac{c_A}{\psi_A}\left[\begin{array}{c} \left(1 + \frac{1-\eta}{\psi_A+(\lambda-1)(1-\eta)}\right)^2 Z_L \\ -\left(1 + \frac{1-\eta}{\psi_A+(\lambda-1)(1-\eta)}\right)\left(1 + \frac{1-\eta}{\psi_P+(\lambda-1)(1-\eta)}\right) Z_M \end{array}\right] Z_L \\ &+\frac{c_P}{\psi_P}\left[\begin{array}{c} 2\left(1 + \frac{1-\eta}{\psi_P+(\lambda-1)(1-\eta)}\right)^2 Z_M \\ -\left(1 + \frac{1-\eta}{\psi_A+(\lambda-1)(1-\eta)}\right)\left(1 + \frac{1-\eta}{\psi_P+(\lambda-1)(1-\eta)}\right) Z_L \\ -\left(1 + \frac{1-\eta}{\psi_P+(\lambda-1)(1-\eta)}\right) R_0 \end{array}\right] Z_M \end{aligned}$$

Hence, (69) is strictly decreasing in λ if and only if

$$\begin{aligned} 0 \;>\; &\frac{c_A}{\psi_A}\left[\begin{array}{c} -2\left(1 + \frac{1-\eta}{\psi_A+(\lambda-1)(1-\eta)}\right)\frac{(1-\eta)^2}{(\psi_A+(\lambda-1)(1-\eta))^2} Z_L \\ +\frac{(1-\eta)^2}{(\psi_A+(\lambda-1)(1-\eta))^2}\left(1 + \frac{1-\eta}{\psi_P+(\lambda-1)(1-\eta)}\right) Z_M \\ +\left(1 + \frac{1-\eta}{\psi_A+(\lambda-1)(1-\eta)}\right)\frac{(1-\eta)^2}{(\psi_P+(\lambda-1)(1-\eta))^2} Z_M \end{array}\right] Z_L \\ &+\frac{c_P}{\psi_P}\left[\begin{array}{c} -4\left(1 + \frac{1-\eta}{\psi_P+(\lambda-1)(1-\eta)}\right)\frac{(1-\eta)^2}{(\psi_P+(\lambda-1)(1-\eta))^2} Z_M \\ +\frac{(1-\eta)^2}{(\psi_A+(\lambda-1)(1-\eta))^2}\left(1 + \frac{1-\eta}{\psi_P+(\lambda-1)(1-\eta)}\right) Z_L \\ +\left(1 + \frac{1-\eta}{\psi_A+(\lambda-1)(1-\eta)}\right)\frac{(1-\eta)^2}{(\psi_P+(\lambda-1)(1-\eta))^2} Z_L + R_0 \frac{(1-\eta)^2}{(\psi_P+(\lambda-1)(1-\eta))^2} \end{array}\right] Z_M, \end{aligned}$$

or equivalently,

$$\begin{aligned} 0 \;>\; &\frac{c_A}{\psi_A} Z_L \left[\begin{array}{c} \frac{(1-\eta)^2}{(\psi_A+(\lambda-1)(1-\eta))^2}\left(-\left(1 + \frac{1-\eta}{\psi_A+(\lambda-1)(1-\eta)}\right) Z_L + \left(1 + \frac{1-\eta}{\psi_P+(\lambda-1)(1-\eta)}\right) Z_M\right) + \\ \left(1 + \frac{1-\eta}{\psi_A+(\lambda-1)(1-\eta)}\right)\left(-\frac{(1-\eta)^2}{(\psi_A+(\lambda-1)(1-\eta))^2} Z_L + \frac{(1-\eta)^2}{(\psi_P+(\lambda-1)(1-\eta))^2} Z_M\right) \end{array}\right] \\ &+\frac{c_P}{\psi_P} Z_M \left[\begin{array}{c} \frac{(1-\eta)^2}{(\psi_P+(\lambda-1)(1-\eta))^2}\left(-2\left(1 + \frac{1-\eta}{\psi_P+(\lambda-1)(1-\eta)}\right) Z_M + R_0 + \left(1 + \frac{1-\eta}{\psi_A+(\lambda-1)(1-\eta)}\right) Z_L\right) \\ +\left(1 + \frac{1-\eta}{\psi_P+(\lambda-1)(1-\eta)}\right)\left(-2\frac{(1-\eta)^2}{(\psi_P+(\lambda-1)(1-\eta))^2} Z_M + \frac{(1-\eta)^2}{(\psi_A+(\lambda-1)(1-\eta))^2} Z_L\right) \end{array}\right], \end{aligned} \tag{72}$$

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By (70)-(71), the first line in each of the square brackets in (72) is nonpositive. Therefore, to prove that (72) holds, it is sufficient to show that

$$\begin{aligned}
0 &> \frac{c_A}{\psi_A}\left[\begin{array}{c} -\left(1+\frac{1-\eta}{\psi_A+(\lambda-1)(1-\eta)}\right)\frac{1}{(\psi_A+(\lambda-1)(1-\eta))^2}Z_L \\ +\left(1+\frac{1-\eta}{\psi_A+(\lambda-1)(1-\eta)}\right)\frac{1}{(\psi_P+(\lambda-1)(1-\eta))^2}Z_M \end{array}\right]Z_L \\
&+\frac{c_P}{\psi_P}\left[\begin{array}{c} -2\left(1+\frac{1-\eta}{\psi_P+(\lambda-1)(1-\eta)}\right)\frac{1}{(\psi_P+(\lambda-1)(1-\eta))^2}Z_M \\ +\frac{1}{(\psi_A+(\lambda-1)(1-\eta))^2}\left(1+\frac{1-\eta}{\psi_P+(\lambda-1)(1-\eta)}\right)Z_L \end{array}\right]Z_M \\
\Leftrightarrow\ 0 &> \frac{(\psi_A+(\lambda-1)(1-\eta))^2}{(\psi_P+(\lambda-1)(1-\eta))^2}\left[\begin{array}{c} \frac{c_A}{c_P}\frac{\psi_P}{\psi_A}\frac{\psi_A+\lambda(1-\eta)}{\psi_A+(\lambda-1)(1-\eta)}Z_L \\ -2\frac{\psi_P+\lambda(1-\eta)}{\psi_P+(\lambda-1)(1-\eta)}Z_M \end{array}\right]Z_M \\
&+\left[\frac{\psi_P+\lambda(1-\eta)}{\psi_P+(\lambda-1)(1-\eta)}Z_M-\frac{c_A}{c_P}\frac{\psi_P}{\psi_A}\frac{\psi_A+\lambda(1-\eta)}{\psi_A+(\lambda-1)(1-\eta)}Z_L\right]Z_L
\end{aligned}$$

Letting

$$\begin{aligned}
x &\equiv \frac{\psi_A+\lambda(1-\eta)}{\psi_A+(\lambda-1)(1-\eta)}Z_L, \\
y &\equiv \frac{\psi_P+\lambda(1-\eta)}{\psi_P+(\lambda-1)(1-\eta)}Z_M,
\end{aligned}$$

this condition can be expressed as

$$\begin{aligned}
0 &> \frac{(\psi_A+(\lambda-1)(1-\eta))^2}{(\psi_P+(\lambda-1)(1-\eta))^2}\left[\frac{c_A}{c_P}\frac{\psi_P}{\psi_A}x-2y\right]Z_M+\left[y-\frac{c_A}{c_P}\frac{\psi_P}{\psi_A}x\right]Z_L \\
\Leftrightarrow\ 0 &> \frac{(\psi_A+(\lambda-1)(1-\eta))^2}{(\psi_P+(\lambda-1)(1-\eta))^2}[x-2y]\frac{Z_M}{Z_L}+[y-x] \\
&+\left(\frac{c_A}{c_P}\frac{\psi_P}{\psi_A}-1\right)\left[\frac{(\psi_A+(\lambda-1)(1-\eta))(\psi_A+\lambda(1-\eta))}{(\psi_P+(\lambda-1)(1-\eta))(\psi_P+\lambda(1-\eta))}y-x\right].
\end{aligned}$$

Denoting $a\equiv x-y$ and $b\equiv 2y-x$, this condition becomes

$$\frac{(\psi_A+(\lambda-1)(1-\eta))^2}{(\psi_P+(\lambda-1)(1-\eta))^2}\frac{Z_M}{Z_L}b+a > \left(\frac{c_A}{c_P}\frac{\psi_P}{\psi_A}-1\right)\left[\frac{(\psi_A+(\lambda-1)(1-\eta))(\psi_A+\lambda(1-\eta))}{(\psi_P+(\lambda-1)(1-\eta))(\psi_P+\lambda(1-\eta))}y-x\right]. \tag{73}$$

Note that (70) implies that $a\geq 0$ and (71) implies that $b>0$, and hence the left-hand side in (73) is always positive.

Suppose that $|c_P-c_A|$ is sufficiently small. Then, by continuity of λ in c_A and c_P, it is sufficient to show that (73) holds if $c_P=c_A$. Therefore, there are three cases to consider. First, suppose that $\psi_P\geq\psi_A$. Then

$$\frac{(\psi_A+(\lambda-1)(1-\eta))(\psi_A+\lambda(1-\eta))}{(\psi_P+(\lambda-1)(1-\eta))(\psi_P+\lambda(1-\eta))}y-x\leq y-x=-a\leq 0,$$

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and hence the right-hand side in (73) is nonpositive, concluding the argument. Second, suppose that $\psi_P < \psi_A$ and

$$\frac{(\psi_A + (\lambda - 1)(1-\eta))(\psi_A + \lambda(1-\eta))}{(\psi_P + (\lambda - 1)(1-\eta))(\psi_P + \lambda(1-\eta))}y - x \geq 0.$$

Then, the right-hand side in (73) is nonpositive, concluding the argument. Third, suppose that $\psi_P < \psi_A$ and

$$\frac{(\psi_A + (\lambda - 1)(1-\eta))(\psi_A + \lambda(1-\eta))}{(\psi_P + (\lambda - 1)(1-\eta))(\psi_P + \lambda(1-\eta))}y - x < 0.$$

Then,

$$\begin{aligned} a &= x - y > x - \frac{(\psi_A + (\lambda - 1)(1-\eta))(\psi_A + \lambda(1-\eta))}{(\psi_P + (\lambda - 1)(1-\eta))(\psi_P + \lambda(1-\eta))}y \\ &\geq \left(1 - \frac{\psi_P}{\psi_A}\right)\left[x - \frac{(\psi_A + (\lambda - 1)(1-\eta))(\psi_A + \lambda(1-\eta))}{(\psi_P + (\lambda - 1)(1-\eta))(\psi_P + \lambda(1-\eta))}y\right] \\ &= \left(\frac{\psi_P}{\psi_A} - 1\right)\left[\frac{(\psi_A + (\lambda - 1)(1-\eta))(\psi_A + \lambda(1-\eta))}{(\psi_P + (\lambda - 1)(1-\eta))(\psi_P + \lambda(1-\eta))}y - x\right], \end{aligned}$$

which implies that (73) is satisfied since $b > 0$.

Next, suppose that $\psi_A \geq \psi_P$ and $c_A \leq \frac{\psi_A}{\psi_P} c_P$. There are two cases to consider. First, suppose that

$$\frac{(\psi_A + (\lambda - 1)(1-\eta))(\psi_A + \lambda(1-\eta))}{(\psi_P + (\lambda - 1)(1-\eta))(\psi_P + \lambda(1-\eta))}y - x \geq 0.$$

Then, the right-hand side in (73) is nonpositive, concluding the argument. Second, suppose that

$$\frac{(\psi_A + (\lambda - 1)(1-\eta))(\psi_A + \lambda(1-\eta))}{(\psi_P + (\lambda - 1)(1-\eta))(\psi_P + \lambda(1-\eta))}y - x < 0.$$

Then,

$$\begin{aligned} a &= x - y \geq x - \frac{(\psi_A + (\lambda - 1)(1-\eta))(\psi_A + \lambda(1-\eta))}{(\psi_P + (\lambda - 1)(1-\eta))(\psi_P + \lambda(1-\eta))}y \\ &> \left(1 - \frac{c_A}{c_P}\frac{\psi_P}{\psi_A}\right)\left[x - \frac{(\psi_A + (\lambda - 1)(1-\eta))(\psi_A + \lambda(1-\eta))}{(\psi_P + (\lambda - 1)(1-\eta))(\psi_P + \lambda(1-\eta))}y\right] \\ &= \left(\frac{c_A}{c_P}\frac{\psi_P}{\psi_A} - 1\right)\left[\frac{(\psi_A + (\lambda - 1)(1-\eta))(\psi_A + \lambda(1-\eta))}{(\psi_P + (\lambda - 1)(1-\eta))(\psi_P + \lambda(1-\eta))}y - x\right]. \end{aligned}$$

This implies that (73) is satisfied since $b > 0$, concluding the first step of the proof. ■

Lemma 2 (diversification across L-stocks) *If the cost function is quadratic, the active fund finds it optimal to diversify across L-stocks and invest the same amount in each L-stock.*

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Proof of Lemma 2. Consider the problem of the active fund manager subject to investing only in L-firms. What will be the price that an active fund manager needs to pay to acquire x_{Aj} shares of firm j? Since the holdings of the passive fund are fixed by her assets under management and the requirement to hold a value-weighted portfolio, competition among liquidity investors means that the relationship between x_{Aj} and P_j must satisfy:

$$P_j = R_0 + c_A'^{-1}\left(f_A x_{Aj}\right) + c_P'^{-1}\left(f_P x_P\right) - Z_j.$$

Therefore, to acquire x_{Aj} shares, the active fund manager must pay

$$x_{Aj}\left(R_0 + c_A'^{-1}\left(f_A x_{Aj}\right) + c_P'^{-1}\left(f_P x_P\right) - Z_j\right).$$

Her cost of effort for firm j is $c_A\left(e_{Aj}\right) = c_A\left(c_A'^{-1}\left(f_A x_{Aj}\right)\right)$. Thus, the portfolio optimization problem of the active fund manager is:

$$\int \left[f_A x_{Aj}\left(R_0 + c_A'^{-1}\left(f_A x_{Aj}\right) + c_P'^{-1}\left(f_P x_P\right)\right) - c_A\left(c_A'^{-1}\left(f_A x_{Aj}\right)\right)\right] dj$$

subject to

$$\int x_{Aj}\left(R_0 + c_A'^{-1}\left(f_A x_{Aj}\right) + c_P'^{-1}\left(f_P x_P\right) - Z_j\right) dj = W_A.$$

Let

$$F\left(t\right) = \max_e \left\{te - c_A\left(e\right)\right\}. \tag{74}$$

Then, we can re-write this optimization problem as:

$$\begin{array}{rl} & \int \left[f_A x_{Aj}\left(R_0 + c_P'^{-1}\left(f_P x_P\right)\right) + F\left(f_A x_{Aj}\right)\right] dj \\ \text{s.t.} & \int x_{Aj}\left(R_0 + c_A'^{-1}\left(f_A x_{Aj}\right) + c_P'^{-1}\left(f_P x_P\right) - Z_j\right) dj = W_A. \end{array} \tag{75}$$

The proof proceeds in two steps. In the first step, we show that any portfolio in which the fund is not equally invested in all L-stocks cannot be optimal. In the second step, we verify that a portfolio that is equally diversified across all L-stocks is locally optimal, and hence, given the first step, is also globally optimal.

Step 1: We prove that if there are two L-stocks such that the fund's investment in these stocks is not the same, the fund has a profitable deviation.

Suppose there are two L-stocks such that the fund's holdings in them are x_{A1} and x_{A2}, and $x_{A1} < x_{A2}$ (this includes the case of $x_{A1} = 0$). Consider the following deviation: buy $x_{A1} + \delta$ of the first stock and $x_{A2} - \varepsilon$ of the second stock such that the total amount spent remains the same, and hence the budget constraint is still satisfied. We will show that this deviation is profitable for the fund manager for small enough δ. Denote the prices before (after) this deviation by $P_{j,old}$ ($P_{j,new}$) for $j \in \{1,2\}$. Then the budget constraint implies:

$$x_{A1}P_{1,old} + x_{A2}P_{2,old} = \left(x_{A1} + \delta\right)P_{1,new} + \left(x_{A2} - \varepsilon\right)P_{2,new}, \tag{76}$$

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where

$$
\begin{aligned}
P_{1,old} &= (R_0 + e_P - Z_L) + c_A'^{-1}(f_A x_{A1}), && (77)\\
P_{2,old} &= (R_0 + e_P - Z_L) + c_A'^{-1}(f_A x_{A2}), && (78)\\
P_{1,new} &= (R_0 + e_P - Z_L) + c_A'^{-1}(f_A x_{A1} + f_A \delta), && (79)\\
P_{2,new} &= (R_0 + e_P - Z_L) + c_A'^{-1}(f_A x_{A2} - f_A \varepsilon), && (80)
\end{aligned}
$$

where in (79)-(80), we used the fact that e_P stays the same since these two L-stocks have zero mass. Denoting

$$e(x) \equiv c_A'^{-1}(f_A x) \tag{81}$$

and

$$
\begin{aligned}
\Delta_1(\delta) &\equiv P_{1,new} - P_{1,old} = e(x_{A1} + \delta) - e(x_{A1}),\\
\Delta_2(\varepsilon) &\equiv P_{2,old} - P_{2,new} = e(x_{A2}) - e(x_{A2} - \varepsilon),
\end{aligned}
$$

we can rewrite the budget constraint (76) as:

$$
\begin{aligned}
x_{A1}P_{1,old} + x_{A2}P_{2,old} &= (x_{A1} + \delta)(P_{1,old} + \Delta_1(\delta)) + (x_{A2} - \varepsilon)(P_{2,old} - \Delta_2(\varepsilon)) \Leftrightarrow\\
x_{A2}\Delta_2(\varepsilon) + \varepsilon P_{2,old} - \varepsilon \Delta_2(\varepsilon) &= x_{A1}\Delta_1(\delta) + \delta P_{1,old} + \delta \Delta_1(\delta).
\end{aligned}
$$

Differentiating this w.r.to ε and taking the limit $\varepsilon \to 0, \delta \to 0$, we get:

$$x_{A2}e'(x_{A2}) + P_{2,old} = [x_{A1}e'(x_{A1}) + P_{1,old}] \frac{d\delta}{d\varepsilon} \tag{82}$$

Using (75), the fund manager's benefit from this deviation, up to a constant that does not depend on δ and ε, satisfies:

$$
\begin{aligned}
\Pi &= f_A(x_{A1} + \delta)(R_0 + e_P) + F(f_A(x_{A1} + \delta)) && (83)\\
&\quad + f_A(x_{A2} - \varepsilon)(R_0 + e_P) + F(f_A(x_{A2} - \varepsilon)).
\end{aligned}
$$

We next show that such a deviation is profitable, i.e., $\frac{d\Pi}{d\varepsilon} > 0$. Using (74) and applying the envelope theorem, $\frac{dF(f_A(x+\delta))}{d\delta} = f_A e(x + \delta)$. Hence, differentiating (83) w.r.to ε, and taking the limit $\varepsilon \to 0, \delta \to 0$, we have:

$$\frac{d\Pi}{d\varepsilon} = f_A(R_0 + e_P)\frac{d\delta}{d\varepsilon} + f_A e(x_{A1})\frac{d\delta}{d\varepsilon} - f_A(R_0 + e_P) - f_A e(x_{A2}).$$

Using (82),

$$\frac{d\Pi}{d\varepsilon} > 0 \Leftrightarrow (R_0 + e_P + e(x_{A1}))\frac{x_{A2}e'(x_{A2}) + P_{2,old}}{x_{A1}e'(x_{A1}) + P_{1,old}} > R_0 + e_P + e(x_{A2}),$$

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and plugging in (77)-(78), $\frac{d\Pi}{d\varepsilon} > 0 \Leftrightarrow$

$$\begin{aligned} & [R_0 + e_P + e(x_{A1})] \times [x_{A2} e'(x_{A2}) + R_0 + e_P + e(x_{A2}) - Z_L] \\ > \; & [R_0 + e_P + e(x_{A2})] \times [x_{A1} e'(x_{A1}) + R_0 + e_P + e(x_{A1}) - Z_L] \end{aligned}$$

Denoting $R \equiv R_0 + e_P$ and simplifying, $\frac{d\Pi}{d\varepsilon} > 0 \Leftrightarrow$

$$\begin{aligned} Z_L [e(x_{A2}) - e(x_{A1})] \; > \; & [R + e(x_{A2})] [x_{A1} e'(x_{A1}) + e(x_{A1})] + Re(x_{A2}) \\ & - [R + e(x_{A1})] \times [x_{A2} e'(x_{A2}) + e(x_{A2})] - Re(x_{A1}). \end{aligned}$$

Since the left-hand side is strictly positive, a sufficient condition for $\frac{d\Pi}{d\varepsilon} > 0$ is that the right-hand side is weakly negative, i.e.,

$$\begin{aligned} & [R + e(x_{A2})] [x_{A1} e'(x_{A1}) + e(x_{A1})] + Re(x_{A2}) \\ < \; & [R + e(x_{A1})] \times [x_{A2} e'(x_{A2}) + e(x_{A2})] + Re(x_{A1}) \Leftrightarrow \end{aligned}$$

$$\begin{aligned} [R + e(x_{A2})] x_{A1} e'(x_{A1}) \; & < \; [R + e(x_{A1})] x_{A2} e'(x_{A2}) \\ \frac{x_{A2} e'(x_{A2})}{x_{A1} e'(x_{A1})} \; & > \; \frac{R + e(x_{A2})}{R + e(x_{A1})}. \end{aligned}$$

Since $\frac{R+e(x_{A2})}{R+e(x_{A1})}$ decreases in R, a sufficient condition for $\frac{d\Pi}{d\varepsilon} > 0$ is that $\frac{x_{A2} e'(x_{A2})}{x_{A1} e'(x_{A1})} \geq \frac{e(x_{A2})}{e(x_{A1})}$, or equivalently, that function $\frac{xe'(x)}{e(x)}$ is weakly increasing. Since $f_A x = c'_A(e(x))$, we have $f_A = c''_A(e(x)) e'(x)$, and thus $\frac{xe'(x)}{e(x)}$ is weakly increasing if and only if $\frac{c'_A(e)}{ec''_A(e)}$ is a weakly increasing function. In particular, this holds for any power function $c_A(e) = c_A e^{\alpha}, \alpha > 1$ (including quadratic), because $\frac{c'_A(e)}{ec''_A(e)} = \frac{1}{\alpha - 1}$ is a constant. We conclude that $\frac{d\Pi}{d\varepsilon} > 0$, and thus any portfolio in which the fund is not equally invested in all L-stocks cannot be optimal.

Step 2: We prove that a portfolio that is equally diversified across all L-stocks is locally optimal. To prove it, consider problem (75). Let μ_0 denote the Lagrange multiplier of the budget constraint and μ_j denote the Lagrange multiplier of the no short-sale constraint for stock j. Then, the optimal portfolio choice solves $\max_{x_{Aj}, \mu_0, \mu} L$, where

$$\begin{aligned} L \equiv \int \left[f_A x_{Aj} \left(R_0 + c_P'^{-1}(f_P x_P) \right) + F(f_A x_{Aj}) \right] dj \\ + \mu_0 \left(W_A - \int x_{Aj} \left(R_0 + c_A'^{-1}(f_A x_{Aj}) + c_P'^{-1}(f_P x_P) - Z_j \right) dj \right) + \int \mu_j x_{Aj} dj. \end{aligned}$$

The first-order condition with respect to x_{Aj} is (applying the envelope theorem to $F(\cdot)$):

$$\begin{aligned} & f_A \left(R_0 + c_P'^{-1}(f_P x_P) + c_A'^{-1}(f_A x_{Aj}) \right) - \mu_0 \begin{pmatrix} R_0 + c_A'^{-1}(f_A x_{Aj}) + c_P'^{-1}(f_P x_P) \\ -Z_j + x_{Aj} \left[\frac{dc_A'^{-1}(f_A x_{Aj})}{dx_{Aj}} \right] \end{pmatrix} + \mu_j = 0 \\ \Leftrightarrow \; & (f_A - \mu_0) \left(R_0 + c_P'^{-1}(f_P x_P) + c_A'^{-1}(f_A x_{Aj}) \right) - \mu_0 \left(-Z_j + x_{Aj} \frac{f_A}{c''_A\left(c_A'^{-1}(f_A x_{Aj})\right)} \right) + \mu_j = 0. \end{aligned}$$

Suppose that $\mu_j = 0 \; \forall j$. Then, $x_{Aj} = x_{Ak}$ for all j, k. Indeed: we have exactly the same

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equation on all $f_A x_{Aj}$:

$$\begin{aligned}(f_A - \mu_0)\left(R_0 + c_P'^{-1}(f_P x_P) + c_A'^{-1}(f_A x_{Aj})\right) &= \mu_0 \left(-Z_j + \frac{f_A x_{Aj}}{c_A''\left(c_A'^{-1}(f_A x_{Aj})\right)}\right) \Leftrightarrow \\ f_A R_j &= \mu_0 \left(R_j - Z_j + \frac{f_A x_{Aj}}{c_A''\left(c_A'^{-1}(f_A x_{Aj})\right)}\right). \quad (84)\end{aligned}$$

It follows that $\mu_0 > 0$ since both the left-hand-side and the term in brackets are strictly positive.

To check that this is a local maximum, we verify the second-order condition. The second-order derivative of L with respect to x_{Aj} is:

$$(f_A - \mu_0)\frac{f_A}{c_A''(e_{Aj})} - \mu_0 \frac{f_A}{c_A''(e_{Aj})} - \mu_0 x_{Aj} \frac{d^2 e_{Aj}}{dx_{Aj}^2}.$$

Since the Hessian matrix is a diagonal (i.e., the cross-partial derivative w.r.to $x_{Aj}x_{Ak}$ is zero), the second-order condition is simply

$$(f_A - 2\mu_0)\frac{f_A}{c_A''(e_{Aj})} - \mu_0 x_{Aj} \frac{d^2 e_{Aj}}{dx_{Aj}^2} < 0. \quad (85)$$

For a general power function, $e_{Aj} = \left(\frac{f_A x_{Aj}}{\alpha c_A}\right)^{\frac{1}{\alpha-1}}$, and hence, $\frac{d^2 e_{Aj}}{dx_{Aj}^2} \geq 0 \Leftrightarrow \alpha \leq 2$. Thus, for $\alpha \leq 2$, the second term in (85) is non-positive, and hence to prove (85), it is sufficient to prove that $f_A - 2\mu_0 < 0$. We prove it by contradiction. Suppose $f_A \geq 2\mu_0$. Then $f_A R_j \geq 2\mu_0 R_j$, so using (84), we have

$$\begin{aligned}\mu_0\left(R_j - Z_j + \frac{f_A x_{Aj}}{c_A''(e_{Aj})}\right) &\geq 2\mu_0 R_j \Leftrightarrow R_j \leq -Z_j + \frac{f_A x_{Aj}}{c_A''(e_{Aj})} \Leftrightarrow \\ R_0 + c_P'^{-1}(f_P x_P) + Z_j &\leq -e_{Aj} + \frac{f_A x_{Aj}}{c_A''(e_{Aj})}. \quad (86)\end{aligned}$$

Since $R_0 + c_P'^{-1}(f_P x_P) + Z_j > 0$, then to prove the contradiction, it is sufficient to show that $e_{Aj} \geq \frac{f_A x_{Aj}}{c_A''(e_{Aj})}$. Consider $c_A(e) = c_A e^\alpha, \alpha > 1$. Since $e_{Aj} = \left(\frac{f_A x_{Aj}}{\alpha c_A}\right)^{\frac{1}{\alpha-1}}$,

$$e_{Aj} \geq \frac{f_A x_{Aj}}{c_A''(e_{Aj})} \Leftrightarrow e_{Aj} \geq \frac{c_A \alpha e_{Aj}^{\alpha-1}}{c_A \alpha(\alpha-1) e_{Aj}^{\alpha-2}} = \frac{e_{Aj}}{\alpha-1} \Leftrightarrow \alpha \geq 2.$$

Hence, when $\alpha \geq 2$, then (84) does not hold, and hence, by contradiction, $f_A < 2\mu_0$. Combining the arguments, we have proved that when $\alpha = 2$, the second-order condition is satisfied, and hence $x_{Aj} = x_{Ak}$ for all L-stocks is the local maximum. Given Step 1, it is also the global maximum. ■

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Lemma 3 (sufficient conditions for not investing in H-stocks) (i) *For a given set of parameters and the conjectured equilibrium effort levels e_{AL}, e_P, the active fund does not find it optimal to deviate to investing in H-stocks if $Z_L - Z_H > e_{AL}\left(1 + \frac{Z_H}{R_0 + e_P}\right)$.*

(ii) *Suppose $\frac{Z_M}{Z_L} > \frac{\xi_A \xi_P + \xi_A - \xi_P}{\xi_P^2}$, where*

$$\xi_A \equiv \xi_A(\lambda_{\max}) = 1 + \frac{1}{\frac{\psi_A}{1-\eta} + \frac{R_0}{R_0 - Z_L} - \psi_A - 1}, \tag{87}$$

$$\xi_P \equiv \xi_P(\lambda_{\max}) = 1 + \frac{1}{\frac{\psi_P}{1-\eta} + \frac{R_0}{R_0 - Z_L} - \psi_A - 1}. \tag{88}$$

Then, given the equilibrium characterized by Proposition 1, the active fund does not find it optimal to deviate to investing in H-stocks.

(iii) *Suppose*

$$Z_L < \frac{R_0}{\frac{1-\eta}{\psi_A} + \left(1 + \frac{1-\eta}{\psi_A}\right)\frac{Z_H}{R_0}}.$$

Then, given any equilibrium characterized by Lemma 7, the active fund does not find it optimal to deviate to investing in H-stocks.

Proof of Lemma 3.

Proof of part (i)**.** Consider the problem of the active fund manager. Since the holdings of the passive fund are fixed by her assets under management and the requirement to hold a value-weighted portfolio, competition among liquidity investors means that the relationship between x_{Aj} and P_j must satisfy:

$$P_j = R_0 + c_A'^{-1}(f_A x_{Aj}) + c_P'^{-1}(f_P x_P) - Z_j.$$

To acquire x_{Aj} shares, the active fund manager must pay

$$x_{Aj}\left(R_0 + c_A'^{-1}(f_A x_{Aj}) + c_P'^{-1}(f_P x_P) - Z_j\right).$$

Her cost of effort for firm j is $c_A\left(c_A'^{-1}(f_A x_{Aj})\right)$. Thus, the portfolio optimization problem of the active fund manager is:

$$\int \left[f_A x_{Aj}\left(R_0 + c_A'^{-1}(f_A x_{Aj}) + c_P'^{-1}(f_P x_P)\right) - c_A\left(c_A'^{-1}(f_A x_{Aj})\right)\right] dj$$

subject to

$$\int x_{Aj}\left(R_0 + c_A'^{-1}(f_A x_{Aj}) + c_P'^{-1}(f_P x_P) - Z_j\right) dj = W_A.$$

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Let $F(t) = \max_e \{te - c_A(e)\}$. Then, we can re-write this optimization problem as:

$$\begin{array}{c} \int \left[f_A x_{Aj} \left(R_0 + c_P'^{-1}(f_P x_P) \right) + F(f_A x_{Aj}) \right] dj \\ \text{s.t. } \int x_{Aj} \left(R_0 + c_A'^{-1}(f_A x_{Aj}) + c_P'^{-1}(f_P x_P) - Z_j \right) dj = W_A \end{array}$$

Consider the solution in which $x_{Aj} = 0$ for all H-stocks. As shown in Section 2 of this document, for a quadratic cost function, we then have $x_{Aj} = x_{AL} = \frac{2W_A}{P_L}$ for all L-stocks.

We next find sufficient conditions for a small deviation to investing in H-stocks to not be profitable. Consider a deviation to $x_{Aj} = x_{AL} - \delta$ for L-stocks and $x_{Aj} = \varepsilon$ for H-stocks such that the budget constraint is satisfied. Denote the prices before (after) this deviation by $P_{t,old}$ ($P_{t,new}$) for $t \in \{L, H\}$. Then the budget constraint implies:

$$\frac{1}{2}\varepsilon P_{H,new} + \frac{1}{2}(x_{AL} - \delta) P_{L,new} = \frac{1}{2} x_{AL} P_{L,old},$$

where

$$\begin{array}{rcl} P_{L,old} & = & R_0 + c_A'^{-1}(f_A x_{AL}) + c_P'^{-1}(f_P x_P) - Z_L, \\ P_{L,new} & = & R_0 + c_A'^{-1}(f_A x_{AL} - f_A \delta) + c_P'^{-1}(f_P x_P) - Z_L, \\ P_{H,new} & = & R_0 + c_A'^{-1}(f_A \varepsilon) + c_P'^{-1}(f_P x_P) - Z_H, \end{array}$$

where we used the fact that $c_P'^{-1}(f_P x_P)$ stays the same since the active and passive fund submit their market orders simultaneously and thus x_P does not change (see Section 8.1). Since $c_A'^{-1}(y) = \frac{y}{c_A}$, and denoting $e_P = c_P'^{-1}(f_P x_P)$, the budget constraint is equivalent to

$$\varepsilon \left(R_0 + c_A'^{-1}(f_A \varepsilon) + e_P - Z_H \right) = \delta \left(R_0 + c_A'^{-1}(f_A x_{AL} - f_A \delta) + e_P - Z_L \right) + x_{AL} \frac{f_A \delta}{c_A}$$

Differentiating this w.r.to ε and taking the limit $\varepsilon \to 0, \delta \to 0$, we get:

$$\begin{array}{rcl} R_0 + \dfrac{f_A \varepsilon}{c_A} + e_P - Z_H + \varepsilon \left[\dfrac{d}{d\varepsilon} \dfrac{f_A \varepsilon}{c_A} \right] & = & \dfrac{d\delta}{d\varepsilon} \left[\begin{array}{c} R_0 + \frac{f_A x_{AL} - f_A \delta}{c_A} + e_P - Z_L \\ + x_{AL} \frac{f_A}{c_A} + \delta \frac{d}{d\delta} \frac{f_A x_{AL} - f_A \delta}{c_A} \end{array} \right] \\ & \Leftrightarrow & \dfrac{d\delta}{d\varepsilon} = \dfrac{R_0 + e_P - Z_H + 2 \frac{f_A \varepsilon}{c_A}}{R_0 + \frac{2 f_A x_{AL} - 2 f_A \delta}{c_A} + e_P - Z_L}. \end{array}$$

The payoff Π from this deviation satisfies:

$$\begin{array}{rcl} 2\Pi & = & 2 \displaystyle\int \left[f_A x_{Aj} (R_0 + e_P) + F(f_A x_{Aj}) \right] dj \\ & = & f_A (x_{AL} - \delta)(R_0 + e_P) + F(f_A (x_{AL} - \delta)) + f_A \varepsilon (R_0 + e_P) + F(f_A \varepsilon), \end{array} \tag{89}$$

where $F(t) = \max_e \{te - c_A(e)\}$.

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Note that $\frac{dF(f_A(x_{AL}-\delta))}{d\delta} = -\frac{f_A^2(x_{AL}-\delta)}{c_A}$ because by envelope theorem

$$F'_\delta = F'_t \frac{dt}{d\delta} = [t = f_A(x_{AL}-\delta)] = -f_A F'_t = -f_A c_A'^{-1}[f_A(x_{AL}-\delta)] = -\frac{f_A^2(x_{AL}-\delta)}{c_A}.$$

Similarly, $\frac{dF(f_A\varepsilon)}{d\varepsilon} = F'_t \frac{dt}{d\varepsilon} = [t = f_A\varepsilon] = f_A F'_t = f_A c_A'^{-1}[f_A\varepsilon] = \frac{f_A^2\varepsilon}{c_A}$.
Hence, differentiating (89) w.r.to ε, we have:

$$\begin{aligned} 2\frac{d\Pi}{d\varepsilon} &= \frac{d}{d\delta}\left[f_A(x_{AL}-\delta)(R_0+e_P) + F(f_A(x_{AL}-\delta))\right]\frac{d\delta}{d\varepsilon} + \frac{d}{d\varepsilon}\left[f_A\varepsilon(R_0+e_P) + F(f_A\varepsilon)\right] \\ &= \frac{R_0+e_P-Z_H+2\frac{f_A\varepsilon}{c_A}}{R_0+\frac{2f_Ax_{AL}-2f_A\delta}{c_A}+e_P-Z_L}\left[-f_A(R_0+e_P) - \frac{f_A^2(x_{AL}-\delta)}{c_A}\right] + \left[f_A(R_0+e_P) + \frac{f_A^2\varepsilon}{c_A}\right]. \end{aligned}$$

Hence, $\frac{d\Pi}{d\varepsilon} < 0$ if and only if

$$R_0 + e_P + \frac{f_A\varepsilon}{c_A} < \frac{R_0+e_P-Z_H+2\frac{f_A\varepsilon}{c_A}}{R_0+\frac{2f_Ax_{AL}-2f_A\delta}{c_A}+e_P-Z_L}\left[R_0+e_P+\frac{f_A(x_{AL}-\delta)}{c_A}\right]$$

and taking the limit $\varepsilon \to 0, \delta \to 0$,

$$R_0 + e_P < \frac{R_0+e_P-Z_H}{R_0+\frac{2f_Ax_{AL}}{c_A}+e_P-Z_L}\left[R_0+e_P+\frac{f_Ax_{AL}}{c_A}\right]$$

Denoting $r_P \equiv R_0 + e_P$,

$$\begin{aligned} \frac{d\Pi}{d\varepsilon} &< 0 \Leftrightarrow r_P\left[r_P - Z_L + 2\frac{f_Ax_{AL}}{c_A}\right] < (r_P - Z_H)\left[r_P + \frac{f_Ax_{AL}}{c_A}\right] \\ &\Leftrightarrow 0 < r_P\left(Z_L - Z_H - \frac{f_Ax_{AL}}{c_A}\right) - Z_H\frac{f_Ax_{AL}}{c_A}. \\ &\Leftrightarrow 0 < (R_0+e_P)(Z_L - Z_H - e_{AL}) - Z_He_{AL} \\ &\Leftrightarrow Z_L - Z_H > e_{AL}\left(1 + \frac{Z_H}{R_0+e_P}\right), \end{aligned} \tag{90}$$

which proves part (i). This condition is useful for two reasons. First, it serves as the basis to prove part (ii). Second, we use (90) in our numerical examples to verify that under the parameters we use, parts $(i)-(iv)$ of Proposition 20 describe the equilibrium, i.e, the active fund indeed does not want to invest in H-stocks.

Proof of part (ii)**.** To prove this part, we show that the conditions in part (ii) are sufficient for (90) to hold. We reformulate (90) in terms of $Z_M = \frac{Z_H+Z_L}{2}$ and Z_L and use

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(67)-(68):

$$\begin{aligned}
-2Z_M + 2Z_L &> e_{AL}\left(1+\frac{2Z_M - Z_L}{R_0+e_P}\right) = 2\left(R_L - R_M\right)\left(1+\frac{2Z_M - Z_L}{R_0+e_P}\right) \Leftrightarrow \\
-Z_M + Z_L &> \left(\xi_A(\lambda) Z_L - \xi_P(\lambda) Z_M\right)\left(1-\frac{Z_L - 2Z_M}{R_0+e_P}\right).
\end{aligned}$$

Plugging in $e_P = 2\xi_P(\lambda) Z_M - \xi_A(\lambda) Z_L - R_0$, we get

$$\left(2\xi_P(\lambda) Z_M - \xi_A(\lambda) Z_L\right)\left(Z_L - Z_M\right) > \left(\xi_A(\lambda) Z_L - \xi_P(\lambda) Z_M\right)\left(2\xi_P(\lambda) Z_M + 2Z_M - \xi_A(\lambda) Z_L - Z_L\right).$$

Simplifying and rearranging, this is equivalent to

$$\left(\xi_A(\lambda) Z_L - \xi_P(\lambda) Z_M\right)^2 + \xi_P^2(\lambda) Z_M^2 + Z_L Z_M\left(\xi_P(\lambda) - \xi_A(\lambda) - \xi_A(\lambda)\xi_P(\lambda)\right) > 0$$

Since the first term is non-negative, a sufficient condition is that the sum of the second and third term is strictly positive or, equivalently,

$$\frac{Z_M}{Z_L} > \frac{\xi_A(\lambda)\xi_P(\lambda) + \xi_A(\lambda) - \xi_P(\lambda)}{\xi_P^2(\lambda)}. \tag{91}$$

We next show that the right-hand side is increasing in λ. Indeed, denote $L_i \equiv \frac{\psi_i}{1-\eta} + \lambda - 1$, where $L_A \geq L_P$, and notice that

$$\begin{aligned}
\left(\frac{\xi_A(\lambda)\xi_P(\lambda) + \xi_A(\lambda) - \xi_P(\lambda)}{\xi_P^2(\lambda)}\right)' &\geq 0 \Leftrightarrow \\
\xi_A'(\lambda)\xi_P(\lambda)\left(\xi_P(\lambda)+1\right) &\geq \xi_P'(\lambda)\left[\xi_A(\lambda)\xi_P(\lambda) + 2\xi_A(\lambda) - \xi_P(\lambda)\right] \Leftrightarrow \\
\frac{-1}{L_A^2}\left(1+\frac{1}{L_P}\right)\left(2+\frac{1}{L_P}\right) &\geq \frac{-1}{L_P^2}\left[\left(1+\frac{1}{L_A}\right)\left(1+\frac{1}{L_P}\right) + 2 + \frac{2}{L_A} - 1 - \frac{1}{L_P}\right] \Leftrightarrow \\
L_A\left[2L_A L_P + 3L_P + 1\right] &\geq L_P\left(2L_P^2 + 3L_P + 1\right).
\end{aligned}$$

The last inequality automatically follows from the fact that $L_A \geq L_P$. Hence, if (91) is satisfied for the largest possible λ, i.e., $\lambda_{\max}$ from Lemma 6, then it is satisfied for any possible λ. This completes the proof of part (ii).

Proof of part (iii). By Lemma 7, there are two cases to consider: the equilibrium where only the passive funds raises positive AUM, and the equilibrium where only the active fund raises positive AUM. Note that since the arguments made in part (i) apply to these equilibria as well, it is sufficient to show that e_{AL}, e_P satisfy (90). First, suppose that the equilibrium is as described by part (i) of Lemma 7. Then, $x_{AL} = 0$ implies that the active fund exerts no effort, and hence $e_{AL} = 0$, so (90) is satisfied. Second, suppose that the equilibrium is as described by part (ii) of Lemma 7. Then, $\lambda = 1$ and $R_L = \left(1 + \frac{1-\eta}{\psi_A}\right) Z_L$ in equilibrium.

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Combining with $e_P = 0$ (due to $x_P = 0$) and $e_{AL} = R_L - R_0$, (90) is equivalent to

$$Z_L - Z_H > \left(\left(1 + \frac{1-\eta}{\psi_A}\right) Z_L - R_0\right)\left(1 + \frac{Z_H}{R_0}\right) \Leftrightarrow Z_L < \frac{R_0}{\frac{1-\eta}{\psi_A} + \left(1 + \frac{1-\eta}{\psi_A}\right)\frac{Z_H}{R_0}},$$

which completes the proof. ■

Lemma 4 (positive assets under management) *Suppose that*

$$\frac{R_0 + \left[1 + \frac{1-\eta}{\psi_A}\right] Z_L}{2\left[1 + \frac{1-\eta}{\psi_P}\right]} < Z_M < Z_L \left(\frac{\psi_A + 1 - \eta}{\psi_P + 1 - \eta}\right)\frac{\psi_P}{\psi_A} \tag{92}$$

and that $W \geq \hat{W}$ for $\hat{W}$ given by (96). Then $W_A > 0$ and $W_P > 0$.

Proof of Lemma 4. Since $x_{AL} = \frac{2W_A}{P_L}$ and $x_P = \frac{W_P}{P_M}$, then $W_A > 0$ and $W_P > 0$ is equivalent to $x_{AL} > 0$ and $x_P > 0$. Using (65)-(66), this is equivalent to

$$\begin{cases} \left[1 + \frac{1-\eta}{\psi_A + (\lambda-1)(1-\eta)}\right] Z_L > \left[1 + \frac{1-\eta}{\psi_P + (\lambda-1)(1-\eta)}\right] Z_M \\ 2\left[1 + \frac{1-\eta}{\psi_P + (\lambda-1)(1-\eta)}\right] Z_M > R_0 + \left[1 + \frac{1-\eta}{\psi_A + (\lambda-1)(1-\eta)}\right] Z_L \end{cases} \tag{93}$$

Intuitively, Z_L are the trading gains captured by the active fund, and $Z_M < Z_L$ are the trading gains captured by the passive fund. If one fund's trading gains (relative to the costs of searching for that fund) are much larger than for the other, investors will not invest in the second fund.

(1) Let us start with the condition $x_{AL} > 0$, i.e, the first condition in (93). It is equivalent to

$$\begin{gathered} \left(\frac{\psi_A}{1-\eta} + \lambda\right)\left(\frac{\psi_P}{1-\eta} - 1 + \lambda\right) > \frac{Z_M}{Z_L}\left(\frac{\psi_P}{1-\eta} + \lambda\right)\left(\frac{\psi_A}{1-\eta} - 1 + \lambda\right) \Leftrightarrow \\ \Leftrightarrow \lambda^2\left(\frac{Z_L}{Z_M} - 1\right) + \lambda\left(\frac{\psi_A}{1-\eta} + \frac{\psi_P}{1-\eta} - 1\right)\left(\frac{Z_L}{Z_M} - 1\right) + \left[\frac{\psi_A}{1-\eta}\left(\frac{\psi_P}{1-\eta} - 1\right)\frac{Z_L}{Z_M} - \frac{\psi_P}{1-\eta}\left(\frac{\psi_A}{1-\eta} - 1\right)\right] > 0 \\ \Leftrightarrow \lambda^2 + \lambda\left(\frac{\psi_A}{1-\eta} + \frac{\psi_P}{1-\eta} - 1\right) + \frac{\left[\frac{\psi_A}{1-\eta}\left(\frac{\psi_P}{1-\eta} - 1\right)\frac{Z_L}{Z_M} - \frac{\psi_P}{1-\eta}\left(\frac{\psi_A}{1-\eta} - 1\right)\right]}{\frac{Z_L}{Z_M} - 1} > 0 \Leftrightarrow \lambda^2 + B\lambda + C > 0, \end{gathered}$$

where the second to last equivalence follows from $\frac{Z_L}{Z_M} - 1 > 0$, and the last equivalence is simply a new notation. It can be shown that

$$B^2 - 4C \geq 0 \Leftrightarrow \frac{1}{4}\left[\frac{\psi_A}{1-\eta} - \frac{\psi_P}{1-\eta}\right]^2 + \frac{1}{2}\frac{\psi_A}{1-\eta} - \frac{1}{2}\frac{\psi_P}{1-\eta} + \frac{1}{4} \geq -\frac{\frac{\psi_A}{1-\eta} - \frac{\psi_P}{1-\eta}}{\frac{Z_L}{Z_M} - 1},$$

which always holds since $\psi_A > \psi_P$. Since $B^2 - 4C \geq 0$, a sufficient condition for $\lambda^2 + B\lambda + C > 0$ for all $\lambda \geq 1$ is that $\lambda_2 < 1$, where $\lambda_2 = \frac{-B + \sqrt{B^2 - 4C}}{2}$, or equivalently, $\sqrt{B^2 - 4C} < 2 + B$.

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This requires (1) $B+2>0 \Leftrightarrow \frac{\psi_A}{1-\eta}+\frac{\psi_P}{1-\eta}+1>0$, which always holds, and (2) $B^2-4C<B^2+4B+4 \Leftrightarrow B+C+1>0$. Plugging in the expressions for B and C and simplifying, $B+C+1>0$ is equivalent to

$$\left(\frac{\psi_A+1-\eta}{\psi_P+1-\eta}\right)\frac{\psi_P}{\psi_A}\frac{Z_L}{Z_M}>1 \Leftrightarrow \left(\frac{1+\frac{1-\eta}{\psi_A}}{1+\frac{1-\eta}{\psi_P}}\right)\frac{Z_L}{Z_M}>1. \tag{94}$$

(2) Next, consider the condition $x_P>0 \Leftrightarrow$

$$H_P(\lambda)\equiv 2\left[1+\frac{1}{\frac{\psi_P}{1-\eta}-1+\lambda}\right]Z_M-R_0-\left[1+\frac{1}{\frac{\psi_A}{1-\eta}-1+\lambda}\right]Z_L>0.$$

Suppose $H_P(1)>0$, which is equivalent to

$$2\left[1+\frac{1}{\frac{\psi_P}{1-\eta}}\right]Z_M-R_0-\left[1+\frac{1}{\frac{\psi_A}{1-\eta}}\right]Z_L>0. \tag{95}$$

Note that $\lim_{\lambda\to\infty}H_P(\lambda)=Z_H-R_0<Z_L-R_0<0$, which holds by assumption. Then we can define $\hat{\lambda}\equiv\min\{\lambda\geq 1: H_P(\lambda)\leq 0\}$. It follows that sufficient conditions for $x_P>0$ are that 1) first, $H_P(1)>0$, i.e., (95), and 2) second, $\lambda>\hat{\lambda}$. Since λ is decreasing in W by Lemma 1, sufficient conditions are (95) and $W>\hat{W}$, where $\hat{W}$ corresponds to $\hat{\lambda}$, i.e.,

$$\hat{W}=\frac{c_A}{\hat{f}_A}\left(\hat{R}_L-\hat{R}_M\right)\hat{P}_L+\frac{c_P}{\hat{f}_P}\left(2\hat{R}_M-\hat{R}_L-R_0\right)\hat{P}_M, \tag{96}$$

where $\hat{f}_A=\frac{\eta\psi_A}{\psi_A+\hat{\lambda}(1-\eta)}$, $\hat{f}_P=\frac{\eta\psi_P}{\psi_P+\hat{\lambda}(1-\eta)}$, $\hat{R}_L=(1+\frac{1-\eta}{\psi_A+\left(\hat{\lambda}-1\right)(1-\eta)})Z_L$, $\hat{P}_L=\hat{R}_L-Z_L$, $\hat{R}_M=(1+\frac{1-\eta}{\psi_P+\left(\hat{\lambda}-1\right)(1-\eta)})Z_M$, and $\hat{P}_M=\hat{R}_M-Z_M$. Since $H_P(1)>0$, then $\hat{\lambda}>1$, and thus $\hat{W}<\bar{W}$. ■

Lemma 5 *Suppose $W<\frac{R_0-Z_L}{2}$. Then, given the equilibrium characterized by Proposition 1, the active and passive fund combined do not hold all the shares, i.e., liquidity investors hold at least some shares for both stocks of type L and of type H.*

Proof of Lemma 5. Note that $x_{AL}+x_P=\frac{2W_A}{P_L}+\frac{W_P}{P_M}$, where

$$\begin{aligned}P_L &= R_0-Z_L+c_A'^{-1}(f_Ax_{AL})+c_P'^{-1}(f_Px_P)\geq R_0-Z_L,\\ P_M &= R_0-Z_M+\frac{1}{2}c_A'^{-1}(f_Ax_{AL})+c_P'^{-1}(f_Px_P)\geq R_0-Z_M>R_0-Z_L.\end{aligned}$$

Hence,

$$x_{AL}+x_P\leq\frac{2W_A+W_P}{R_0-Z_L}\leq\frac{2(W_A+W_P)}{R_0-Z_L}=\frac{2W}{R_0-Z_L}.$$

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It follows that the condition $W < \frac{R_0 - Z_L}{2}$ ensures that $x_{AL} + x_P < 1$, i.e., the active and passive fund combined do not hold all the shares of L-firms. This, in turn, implies $x_P < 1$, i.e., liquidity investors hold at least some shares of H-firms as well. ■

Lemma 6 (upper bound on λ) *In any equilibrium given by Proposition 1 or Lemma 7, it must be $\lambda \leq \lambda_{\max}$, where*

$$\lambda_{\max} = \begin{cases} \frac{R_0}{R_0 - Z_L} - \psi_A, & \text{if } W_A > 0, \\ \frac{R_0}{R_0 - Z_M} - \psi_P, & \text{otherwise.} \end{cases} \quad (97)$$

Proof of Lemma 6.

First, suppose that $W_A > 0$. Then,

$$\lambda = (1 - f_A) \frac{R_L}{P_L} - \psi_A \leq \frac{R_L}{P_L} - \psi_A,$$

where

$$\frac{R_L}{P_L} = \frac{R_0 + e_{AL} + e_P}{R_0 - Z_L + e_{AL} + e_P},$$

and $\frac{R_0 + e_{AL} + e_P}{R_0 - Z_L + e_{AL} + e_P} \leq \frac{R_0}{R_0 - Z_L}$ since $\frac{R_0 + x}{R_0 - Z_L + x}$ decreases in x. Hence, $\lambda \leq \lambda_{\max}$, as required.

Second, suppose that $W_A = 0$ and $W_P > 0$. Then,

$$\lambda = (1 - f_P) \frac{R_M}{P_M} - \psi_P \leq \frac{R_M}{P_M} - \psi_P,$$

where

$$\frac{R_M}{P_M} = \frac{R_0 + \frac{1}{2} e_{AL} + e_P}{R_0 - Z_M + \frac{1}{2} e_{AL} + e_P},$$

and $\frac{R_0 + \frac{1}{2} e_{AL} + e_P}{R_0 - Z_M + \frac{1}{2} e_{AL} + e_P} \leq \frac{R_0}{R_0 - Z_M}$ since $\frac{R_0 + x}{R_0 - Z_M + x}$ decreases in x. Hence, $\lambda \leq \lambda_{\max}$, as required. ■

8.3 Welfare implications of the reduction in monitoring costs

In this section, we examine the effects of regulations that decrease funds' costs of monitoring on the combined welfare of all the players. To analyze welfare, we interpret Z_i as liquidity investors' private valuations coming from motives such as hedging or liquidity needs, rather than investor sentiment. Whether decreasing funds' costs of monitoring is beneficial for total welfare depends on its combined effect on firms' initial owners, fund investors, fund managers, and liquidity investors. Since liquidity investors trade shares at the price that equals their private valuations, their payoff is zero. Hence, the effect of such policies on total welfare depends on the trade-off between their positive effect on governance and initial owners' payoff on the one hand, and their potential negative effect on fund investors and fund managers on the other hand.

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The next result shows that decreasing funds' costs of monitoring beyond a certain threshold is detrimental to total welfare:

Proposition 7 (welfare effects of decreasing the costs of monitoring). *Define $\bar{c}_i$ as the infimum of c_i for which $\lambda > 1$. If $c_i < \bar{c}_i$, then decreasing c_i harms total welfare.*[32]

The logic is the following. According to Proposition 4, as a fund's cost of monitoring decreases, fund investors' rate of return decreases as well, until it reaches the point (at $c_i = \bar{c}_i$) where investors are indifferent between investing with fund managers and saving privately, i.e., $\lambda = 1$. At this point, a further decrease in the fund's cost of monitoring has no additional marginal benefit because, as follows from Proposition 1, the fund's monitoring levels and hence firm valuations stay constant in c_i when $\lambda = 1$. Therefore, the only welfare effect of further decreasing c_i is the decline in fund managers' profits (condition $\psi_P \geq \bar{\psi}_P$ in part (*iii*) of Proposition 4 corresponds to the case $\lambda = 1$).

The reason why funds' monitoring and thus firm value do not change with c_i when $\lambda = 1$ is as follows. Suppose, for example, that the passive fund's effort increased as c_P decreased (assuming for a moment that the fund's ownership stakes x_P would not change). Higher effort would raise firms' payoffs (R_M) and hence market prices (P_M). Since, as discussed above, the fund does not gain from increased monitoring, the only effect of higher valuations would be the fund's lower ability to realize gains from trade. This would make investing in the fund less attractive to investors relative to saving privately, leading to outflows into private savings and decreasing the fund's AUM. These outflows, in turn, would lead the fund to take smaller positions in the underlying stocks, and these smaller positions would have a counteracting effect of decreasing the fund's incentives to monitor. In equilibrium, the fund's AUM and, accordingly, its ownership stakes x_P decrease in a way that the combined effects of lower c_P and lower x_P on the fund's effort cancel out, so that the equilibrium effort and hence firm valuations remain unchanged.

Overall and more generally, this logic emphasizes that to understand the effects of governance regulations, it is important to consider their potential effects on funds' assets under management, since those effects can potentially counteract the desired effects of regulations.

Note also that as passive funds become easier to access (ψ_P declines), funds' AUM grow and investors are likely to strictly prefer investing with the funds over their private savings (Proposition 2), which makes the counteracting effect described above less likely. Accordingly, as we show in the proof of Proposition 7, the threshold $\bar{c}_i$ increases with ψ_P, which leads to the following implication: Regulations that reduce funds' costs of monitoring are more likely to be welfare improving if (1) passive funds are easier to access, and (2) funds' AUM are sufficiently large.

Suppose that at time 0, there is an unlimited number of active and passive fund managers, who decide whether to pay a cost to enter the market. The entry costs of a fund of type i are k_i, $i \in \{A, P\}$. The equilibrium number of funds is such that for each fund type, the costs of entry are exactly equal to the expected profit of fund managers (ignoring the integer issues). As in Garleanu and Pedersen (2018), suppose that the investor's cost of searching

[32] If this infimum does not exist, i.e., $\lambda = 1$ for all c_i satisfying the conditions of Proposition 1, then decreasing c_i harms total welfare for all c_i satisfying these conditions.

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for a fund manager depends on the total number of fund managers. Specifically, the cost for an investor to find an active (passive) fund manager is $\psi_A(N_A)$ ($\psi_P(N_P)$), where $\psi_A(\cdot)$ and $\psi_P(\cdot)$ are decreasing.

Proof of Proposition 7. Welfare equals the sum of the payoffs of the initial shareholders, the payoffs of liquidity investors, the payoffs of fund managers, and the payoffs of fund investors:

$$Welfare = P_M + 0 + \left[\frac{1}{2}f_A x_{AL} R_L + f_P x_P R_M - \frac{1}{2}\frac{c_A}{2}e_{AL}^2 - \frac{c_P}{2}e_P^2\right] + (\lambda - 1)W \tag{98}$$

The first term is the payoff of the initial owners of the firms, which is $P_M = \frac{P_L + P_H}{2}$ up to a constant that is equal to initial owners' valuations. The second term, which captures the payoffs of liquidity investors, equals zero because liquidity investors trade shares at the price equal to their private valuations. The third term, in the square brackets, captures the combined payoff of the active and passive fund manager, which is their share of the fund's payoff minus their costs of monitoring. The last term captures the payoff of the fund investors: since their initial wealth is W and they earn equilibrium rate of return λ on it, their final payoff is λW. Note that in the expression above, W has a multiplier of $(\lambda - 1)$, rather than just λ. This has an effect on the comparative statics of welfare only with respect to W, and not any other parameters. The rationale behind this choice is that if W increases, the increase in W must be financed from another source in the economy that is not explicitly modeled in our framework. For example, if W increases by ΔW, it must be that ΔW less is invested in the rest of the overall economy, and to capture that, we subtract ΔW from our welfare calculation, resulting in the term $(\lambda - 1)W$.

Using $f_A x_{AL} = c_A e_{AL}$, $f_P x_P = c_P e_P$, $e_{AL} = 2(R_L - R_M) \geq 0$, and $e_P = 2R_M - R_L - R_0 \geq 0$, we can rewrite (98) as

$$\begin{aligned} Welfare &= P_M + \frac{1}{2}c_A e_{AL} R_L + c_P e_P R_M - \frac{1}{2}\frac{c_A}{2}e_{AL}^2 - \frac{c_P}{2}e_P^2 + (\lambda - 1)W \\ &= P_M + \frac{1}{2}c_A e_{AL}\left(R_L - \frac{1}{2}e_{AL}\right) + c_P e_P\left(R_M - \frac{1}{2}e_P\right) + (\lambda - 1)W \\ &= P_M + c_A(R_L - R_M)R_M + \frac{c_P}{2}(2R_M - R_L - R_0)(R_L + R_0) + (\lambda - 1)W. \end{aligned} \tag{99}$$

Below, we show that $\bar{c}_i$ is given by (100)-(101) and prove that $\lambda > 1$ for $c_i > \bar{c}_i$ and $\lambda = 1$ for $c_i \leq \bar{c}_i$. Now, consider any $c_i < \bar{c}_i$, so that $\lambda = 1$. Then, according to Proposition 1, P_M, R_M, and R_L do not change with c_P and c_A. Note that $R_L - R_M = \frac{1}{2}e_{AL} = \frac{1}{2}\frac{f_A x_{AL}}{c_A} > 0$ and $2R_M - R_L - R_0 = e_P = \frac{f_P x_P}{c_P} > 0$, because f_A and f_P are positive by Proposition 1, and both x_{AL} and x_P are positive by the proof of Proposition 1. Hence, (99) implies that welfare strictly increases with c_P and c_A, as required.

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We next show that $\bar{c}_P$ and $\bar{c}_A$ are given by

$$W = \frac{1-\eta}{\eta}\left(\begin{array}{c} \frac{c_A}{\psi_A}\left(1+\frac{1-\eta}{\psi_A}\right)Z_L\left(\left(1+\frac{1-\eta}{\psi_A}\right)Z_L-\left(1+\frac{1-\eta}{\psi_P}\right)Z_M\right) \\ +\frac{\bar{c}_P}{\psi_P}\left(1+\frac{1-\eta}{\psi_P}\right)Z_M\left(2\left(1+\frac{1-\eta}{\psi_P}\right)Z_M-\left(1+\frac{1-\eta}{\psi_A}\right)Z_L-R_0\right) \end{array}\right), \quad (100)$$

$$W = \frac{1-\eta}{\eta}\left(\begin{array}{c} \frac{\bar{c}_A}{\psi_A}\left(1+\frac{1-\eta}{\psi_A}\right)Z_L\left(\left(1+\frac{1-\eta}{\psi_A}\right)Z_L-\left(1+\frac{1-\eta}{\psi_P}\right)Z_M\right) \\ +\frac{c_P}{\psi_P}\left(1+\frac{1-\eta}{\psi_P}\right)Z_M\left(2\left(1+\frac{1-\eta}{\psi_P}\right)Z_M-\left(1+\frac{1-\eta}{\psi_A}\right)Z_L-R_0\right) \end{array}\right), \quad (101)$$

respectively. Indeed, recall that in equilibrium described by Proposition 1, $W_A + W_P$ is given by the right-hand side of (21). Consider any $i \in \{A, P\}$. We show that $\lambda > 1$ for $c_i > \bar{c}_i$ and $\lambda = 1$ for $c_i \leq \bar{c}_i$. First, consider $c_i \leq \bar{c}_i$. Then, it must be $\lambda = 1$. This is because then, (100), (101), and Proposition 1 imply $W \geq W_A + W_P$, which is consistent with $\lambda = 1$. This also implies that it cannot be $\lambda > 1$, because if we had $\lambda > 1$, then (100), (101), Proposition 1, and Lemma 1 in the online appendix would imply that $W > W_A + W_P$, yielding a contradiction since no investor would save privately given $\lambda > 1$. Second, consider $c_i > \bar{c}_i$. Then it must be $\lambda > 1$. Indeed, if we had $\lambda = 1$, then (100), (101), and Proposition 1 would imply $W < W_A + W_P$, yielding a contradiction since then the total investor endowment would not be sufficient for funds to raise total AUM of $W_A + W_P$.

As an auxiliary result, we next also show that $\bar{c}_P$ and $\bar{c}_A$ strictly increase with ψ_P. This follows from (100) and (101), because the right-hand side in both of them is strictly decreasing in ψ_P. To see this, take any $i \in \{A, P\}$, and let $c_i = \bar{c}_i$. If $i = P$, consider (100), and if $i = A$, consider (101). Then $\lambda = 1$, and using the expressions for R_L and R_M from Proposition 1, the partial derivative of the right-hand side w.r.t. ψ_P is negative if and only if

$$0 > \left(-\frac{c_A}{\psi_A}R_L + \frac{c_P}{\psi_P}\left(4R_M - R_L - R_0\right)\right)\frac{\partial R_M}{\partial \psi_P},$$

which always holds since $\frac{\partial R_M}{\partial \psi_P} < 0$, $c_P \geq \frac{\psi_P}{\psi_A}c_A$, and $4R_M - R_L - R_0 > 2R_M > R_L$, where the last set of inequalities follow from $2R_M - R_L - R_0 = e_P > 0$ as argued after expression (99) above.

While this completes the proof, in what follows, we also provide the sufficient conditions that ensure that (1) $c_P > \frac{\psi_P}{\psi_A}\bar{c}_A > 0$ and (2) $\bar{c}_P > \frac{\psi_P}{\psi_A}c_A$. Together, these two inequalities in turn ensure that the set of values of c_i that satisfy both the conditions of Proposition 1 ($c_P \geq \frac{\psi_P}{\psi_A}c_A$) and the condition $c_i < \bar{c}_i$, is non-empty for each $i \in \{A, P\}$. We show that these sufficient conditions are given by $W_L < W < W_H$, where

$$W_L \equiv \frac{1-\eta}{\eta}\max\left\{\begin{array}{c} \frac{c_A}{\psi_A}\left(1+\frac{1-\eta}{\psi_A}\right)Z_L\left(\left(1+\frac{1-\eta}{\psi_A}\right)Z_L-\left(1+\frac{1-\eta}{\psi_P}\right)Z_M\right) \\ +\frac{c_A}{\psi_A}\left(1+\frac{1-\eta}{\psi_P}\right)Z_M\left(2\left(1+\frac{1-\eta}{\psi_P}\right)Z_M-\left(1+\frac{1-\eta}{\psi_A}\right)Z_L-R_0\right), \\ \frac{c_P}{\psi_P}\left(1+\frac{1-\eta}{\psi_P}\right)Z_M\left(2\left(1+\frac{1-\eta}{\psi_P}\right)Z_M-\left(1+\frac{1-\eta}{\psi_A}\right)Z_L-R_0\right) \end{array}\right\}$$

$$W_H \equiv \frac{1-\eta}{\eta}\left(\begin{array}{c} \frac{c_P}{\psi_P}\left(1+\frac{1-\eta}{\psi_A}\right)Z_L\left(\left(1+\frac{1-\eta}{\psi_A}\right)Z_L-\left(1+\frac{1-\eta}{\psi_P}\right)Z_M\right) \\ +\frac{c_P}{\psi_P}\left(1+\frac{1-\eta}{\psi_P}\right)Z_M\left(2\left(1+\frac{1-\eta}{\psi_P}\right)Z_M-\left(1+\frac{1-\eta}{\psi_A}\right)Z_L-R_0\right) \end{array}\right).$$

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Note that $W_L \leq W_H$ is satisfied since $\frac{c_P}{\psi_P} \geq \frac{c_A}{\psi_A}$ as assumed in Proposition 1, and that $W_L < W_H$ whenever $\frac{c_P}{\psi_P} > \frac{c_A}{\psi_A}$. The reason why $W_L < W < W_H$ is a sufficient condition is that from (100)-(101), it follows that $W_L < W$ implies that $\bar{c}_P > \frac{\psi_P}{\psi_A} c_A$ and $c_P > \frac{\psi_P}{\psi_A} \bar{c}_A > 0$, and $W < W_H$ implies $c_P > \frac{\psi_P}{\psi_A} \bar{c}_A$, as required. ■

8.4 The case where only one fund raises positive AUM

Lemma 7 (equilibria with one type of fund) *Suppose*

$$Z_L < \frac{R_0}{\frac{1-\eta}{\psi_A} + \left(1 + \frac{1-\eta}{\psi_A}\right) \frac{Z_H}{R_0}} \tag{102}$$

and

$$\frac{R_0 - Z_L}{2} > W. \tag{103}$$

(*i*) *Suppose*

$$W > W_P(1) \equiv \frac{1-\eta}{\eta} \frac{c_P}{\psi_P} \left(1 + \frac{1-\eta}{\psi_P}\right) Z_M \left(\left(1 + \frac{1-\eta}{\psi_P}\right) Z_M - R_0\right). \tag{104}$$

Then, the equilibrium where $\lambda = 1$ and only the passive fund raises AUM exists if and only if

$$\left(1 + \frac{1-\eta}{\psi_P}\right) Z_M \geq \left(1 + \frac{1-\eta}{\psi_A}\right) Z_L \tag{105}$$

and

$$\left(1 + \frac{1-\eta}{\psi_P}\right) Z_M > R_0. \tag{106}$$

If this equilibrium exists, then $W_P = W_P(1)$, f_P, R_M, and P_M are as described in Proposition 1, $R_L = R_M$, and $P_L = R_L - Z_L$. Moreover, if $\psi_A > \frac{Z_L}{R_0 - Z_L}$, then this equilibrium is unique.

(*ii*) *Suppose*

$$W > W_A(1) \equiv \frac{1}{2} \frac{1-\eta}{\eta} \frac{c_A}{\psi_A} \left(1 + \frac{1-\eta}{\psi_A}\right) Z_L \left(\left(1 + \frac{1-\eta}{\psi_A}\right) Z_L - R_0\right). \tag{107}$$

Then, the equilibrium where $\lambda = 1$ and only the active fund raises AUM exists if and only if

$$\left(1 + \frac{1-\eta}{\psi_A}\right) Z_L + R_0 \geq 2 \left(1 + \frac{1-\eta}{\psi_P}\right) Z_M \tag{108}$$

and

$$\left(1 + \frac{1-\eta}{\psi_A}\right) Z_L > R_0. \tag{109}$$

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If this equilibrium exists, then $W_A = W_A(1)$, f_A, R_L, and P_L are as described in Proposition 1, $R_M = \frac{1}{2}R_0 + \frac{1}{2}R_L$, and $P_M = R_M - Z_M$. Moreover, if $\psi_P > \frac{Z_M}{R_0 - Z_M}$, then this equilibrium is unique.

Proof of Lemma 7. Note that $Z_L < \frac{R_0}{\frac{1-\eta}{\psi_A} + \left(1 + \frac{1-\eta}{\psi_A}\right)\frac{Z_H}{R_0}}$ automatically implies $Z_L < \frac{R_0}{\frac{1-\eta}{\psi_A}}$, and hence condition (102) implies that the conditions of part (iii) of Lemma 3 in the online appendix is satisfied. This, together with Lemma 2 in the online appendix, implies that under the conjectured equilibrium, the active fund does not find it optimal to deviate from its strategy of only investing in L-stocks and equally diversifying across them.

Proof of part (i). Consider the equilibrium in part (i), i.e., where only the passive fund raises positive AUM and the rate of return λ that fund investors earn on their investment satisfies $\lambda = 1$. Then, following the same steps in the proof of Proposition 1 yields the same expressions for f_P, R_M, and P_M as described in that proposition. Note that

$$f_P = \frac{\eta\psi_P}{\psi_P + 1 - \eta} = \eta\frac{Z_M}{R_M}, \tag{110}$$

Since the active fund does not raise any AUM, we have $R_L = R_M$, and $P_L = R_L - Z_L$. Moreover, the AUM of the passive fund are given by

$$\begin{aligned} W_P = x_P P_M = \tfrac{c_P e_P}{f_P} P_M = \tfrac{c_P}{f_P}(2R_M - R_L - R_0)P_M = \tfrac{c_P}{f_P}(R_M - R_0)P_M \\ = c_P \tfrac{\psi_P + 1 - \eta}{\eta\psi_P}\left(\left(1 + \tfrac{1-\eta}{\psi_P}\right)Z_M - R_0\right)\tfrac{1-\eta}{\psi_P}Z_M \equiv W_P(1), \end{aligned} \tag{111}$$

where the second equality follows from (3) and the third equality follows from (10)-(11). Therefore, $W_P = W_P(1)$. Note that $W > W_P(1)$ by assumption, which implies that $W > W_P$, which is consistent with $\lambda = 1$.

Let us now derive the necessary and sufficient conditions for this equilibrium to exist. Note that a fund investor gets a return of $1 + (1-\eta)\left(\frac{R_L}{P_L} - 1\right)$ from his bargaining with the active fund, and therefore the fund investor does not prefer to deviate to search for the active fund if and only if

$$\begin{aligned} 1 \quad &\geq \quad 1 + (1-\eta)\left(\frac{R_L}{P_L} - 1\right) - \psi_A \Leftrightarrow 1 \geq \frac{R_L}{R_L - Z_L} - \frac{\psi_A}{1-\eta} \\ &\Leftrightarrow \quad R_L \geq \left(1 + \frac{1-\eta}{\psi_A}\right)Z_L, \end{aligned}$$

which is equivalent to (105) due to $R_L = R_M$. Positive AUM for the passive fund require $x_P > 0$, i.e., $2R_M - R_L - R_0 > 0$, which is equivalent to (106). Finally, liquidity investors hold at least some shares in this equilibrium, i.e., $x_P < 1$ is satisfied, because

$$x_P = \frac{W_P}{P_M} = \frac{W_P}{R_M - Z_M} < \frac{W}{R_0 - Z_L} < \frac{1}{2},$$

where the last inequality holds by assumption (103).

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Next, we show that if $\psi_A > \frac{Z_L}{R_0 - Z_L}$, then the equilibrium described in part (i) is unique. Proving this result consists of two substeps. First, we show that the investors' return from searching for and investing in the active fund is always strictly smaller than one. This holds because this return is bounded from above by $\frac{R_L}{P_L} - \psi_A$, which satisfies

$$\frac{R_L}{P_L} - \psi_A = \frac{R_L}{R_L - Z_L} - \psi_A < \frac{R_0}{R_0 - Z_L} - \psi_A < 1,$$

where the first inequality follows from $R_L = R_M > R_0$ and the last inequality follows from $\psi_A > \frac{Z_L}{R_0 - Z_L}$. Second, we prove that there is no equilibrium where only the passive fund raises positive AUM and $\lambda > 1$. To see this, consider any equilibrium where $W_A = 0$ and $W_P > 0$, but without restricting λ to be equal to one (that is, allowing for $\lambda > 1$). Then, the derivation of the equilibrium is slightly different than in Proposition 1, because the outside option of the fund investor in his bargaining with the passive fund is not equal to λ, but is equal to one. This is because the only other option of the investor is to save privately, which has a return of one. Therefore, following the same steps as those used in deriving (16), but plugging in ε for the outside option of the investor in the fee bargaining, yields the following fixed point equation:

$$f_P = \eta \left(1 - \frac{P_M}{R_M}\right). \tag{112}$$

Since $R_L = R_M$ and $P_M = R_M - Z_M$ still hold, we have

$$W_P = x_P P_M = \frac{c_P e_P}{f_P} P_M = c_P \frac{R_M}{\eta Z_M}(R_M - R_0)(R_M - Z_M), \tag{113}$$

where the second equality follows from (3) and the third equality utilizes (10)-(11). Note that λ is given by $\lambda = (1 - f_P)\frac{R_M}{P_M} - \psi_P$, and plugging in (112) and $P_M = R_M - Z_M$, λ can be expressed as

$$\lambda = (1 - f_P)\frac{R_M}{P_M} - \psi_P = (1 - \eta)\frac{R_M}{R_M - Z_M} + \eta - \psi_P,$$

which strictly decreases in R_M. Since the right-hand side in (113) strictly increases in R_M, this implies that W_P strictly decreases in λ. Moreover, if $\lambda = 1$, then (113) is equal to (111), since (110) and (112) are equal. Combining this with the continuity of (113) in R_M, as λ converges to 1 from above, (113) converges to (111). Thus, $W_P < W_P(1)$ for all $\lambda > 1$. Since $W_P(1) < W$, this implies that $W_P < W$ for all $\lambda > 1$, and hence it cannot be $\lambda > 1$ in equilibrium, because if it were, then no investor would save privately, resulting in a contradiction.

Proof of part (ii)**.** Consider the equilibrium in part (ii), i.e., where only the active fund raises positive AUM and $\lambda = 1$. Then, following the same steps in the proof of Proposition 1 yields the same expressions for f_A, R_L, and P_L as described in that proposition. Note that

$$f_A = \frac{\eta \psi_A}{\psi_A + 1 - \eta} = \eta \frac{Z_L}{R_L}, \tag{114}$$

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where the last equality follows from (30). Since the passive fund does not raise any AUM, we have $R_M = \frac{1}{2}R_0 + \frac{1}{2}R_L$ and $P_M = R_M - Z_M$. Moreover, the AUM of the active fund are given by

$$\begin{aligned} W_A &= \tfrac{1}{2}x_{AL}P_L = \tfrac{1}{2}\tfrac{c_A e_{AL}}{f_A}P_L = \tfrac{1}{2}\tfrac{c_A}{f_A}2(R_L - R_M)P_L = \tfrac{1}{2}\tfrac{c_A}{f_A}(R_L - R_0)P_L \\ &= \tfrac{1}{2}c_A\tfrac{\psi_A+1-\eta}{\eta\psi_A}\left(\left(1+\tfrac{1-\eta}{\psi_A}\right)Z_L - R_0\right)\tfrac{1-\eta}{\psi_A}Z_L \equiv W_A(1), \end{aligned} \tag{115}$$

where the second equality follows from (3) and the third equality follows from (10)-(11). Therefore, $W_A = W_A(1)$. Note that $W > W_A(1)$ by assumption, which implies that $W > W_A$, which is consistent with $\lambda = 1$.

Let us now derive the necessary and sufficient conditions for this equilibrium to exist. Note that a fund investor gets a return of $1 + (1-\eta)\left(\frac{R_M}{P_M} - 1\right)$ from his bargaining with the passive fund, and therefore the fund investor does not prefer to deviate to search for the passive fund if and only if

$$\begin{aligned} 1 &\geq 1 + (1-\eta)\left(\frac{R_M}{P_M} - 1\right) - \psi_P \Leftrightarrow 1 \geq \frac{R_M}{R_M - Z_M} - \frac{\psi_P}{1-\eta} \\ &\Leftrightarrow R_M \geq \left(1 + \frac{1-\eta}{\psi_P}\right)Z_M, \end{aligned}$$

which is equivalent to (108) due to $R_M = \frac{1}{2}R_L + \frac{1}{2}R_0$. Positive AUM for the active fund require $x_{AL} > 0$, i.e., $R_L - R_0 > 0$, which is equivalent to (109). Finally, liquidity investors hold at least some shares in this equilibrium, i.e., $x_{AL} < 1$ is satisfied, because

$$x_{AL} = \frac{W_A}{\frac{1}{2}P_L} = 2\frac{W_A}{R_L - Z_L} < 2\frac{W}{R_0 - Z_L} < 1,$$

where the last inequality holds by assumption (103).

Next, we show that if $\psi_P > \frac{Z_M}{R_0 - Z_M}$, then the equilibrium described in part (*ii*) is unique. Proving this result consists of two substeps. First, we show that the investors' return from searching for and investing in the passive fund is always strictly smaller than one. This holds because this return is bounded from above by $\frac{R_M}{P_M} - \psi_P$, which satisfies

$$\frac{R_M}{P_M} - \psi_P = \frac{R_M}{R_M - Z_M} - \psi_P < \frac{R_0}{R_0 - Z_M} - \psi_P < 1,$$

where the first inequality follows from $R_M = \frac{1}{2}R_L + \frac{1}{2}R_0 > R_0$ and the last inequality follows from $\psi_P > \frac{Z_M}{R_0 - Z_M}$. Second, we prove that there is no equilibrium where only the active fund raises positive AUM and $\lambda > 1$. To see this, consider any equilibrium where $W_P = 0$ and $W_A > 0$, but without restricting λ to be equal to one (that is, allowing for $\lambda > 1$). Then, the derivation of the equilibrium is again slightly different from that in Proposition 1, because the outside option of the fund investor in his bargaining with the active fund is not equal to λ, but is equal to one. Therefore, following the same steps as those used in deriving (15), but plugging in ε for the outside option of the investor in the fee bargaining, yields the following

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fixed point equation:

$$f_A = \eta \left(1 - \frac{P_L}{R_L}\right). \tag{116}$$

Since $R_M = \frac{1}{2}R_0 + \frac{1}{2}R_L$ and $P_L = R_L - Z_L$ still hold, we have

$$W_A = \frac{1}{2}x_{AL}P_L = \frac{1}{2}\frac{c_A e_{AL}}{f_A}P_L = \frac{1}{2}c_A\frac{R_L}{\eta Z_L}2(R_L - R_M)(R_L - Z_L), \tag{117}$$

where the second equality follows from (3) and the third equality utilizes (10)-(11). Note that λ is still given by (12), and plugging (116) and $P_L = R_L - Z_L$ in (12), λ can be expressed as

$$\lambda = (1 - f_A)\frac{R_L}{P_L} - \psi_A = (1 - \eta)\frac{R_L}{R_L - Z_L} + \eta - \psi_A,$$

which strictly decreases in R_L. Since the right-hand side in (117) strictly increases in R_L, this implies that W_A strictly decreases in λ. Moreover, if $\lambda = 1$, then (117) is equal to (115), since (114) and (116) are equal. Combining this with the continuity of (117) in R_L, as λ converges to 1 from above, (117) converges to (115). Thus, $W_A < W_A(1)$ for all $\lambda > 1$. Since $W_A(1) < W$, this implies that $W_A < W$ for all $\lambda > 1$, and hence it cannot be $\lambda > 1$ in equilibrium, since if it were, then no investor would save privately, resulting in a contradiction. ■

Lemma 8 (decreasing the cost of monitoring when only one fund exists) *Consider the equilibrium of Lemma 7, in which only the passive (active) fund raises positive AUM. Then, the passive (active) fund manager's payoff always strictly decreases if c_P (c_A) decreases.*

Proof of Lemma 8.

The proof immediately follows from the proof of Proposition 4, because this statement has already been proved for the case $\lambda = 1$ in Proposition 4, and the proof applies to equilibria with only one fund as well. ■

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