Equity Issuance Methods and Dilution

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March 2021

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Keywords: Rights offerings, Public offerings, Asymmetric Information, Winner Curse

JEL Classifications: G32

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Equity Issuance Methods and Dilution*

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1 Introduction

Public companies undertake seasoned equity offerings (SEOs) to raise new equity capital from current shareholders and new investors. Broadly speaking, SEOs can be classified into two modes: public offerings and rights offerings.[1] In public offerings, firms announce the issue size, and both current shareholders and new investors can subscribe.[2] In addition, the firm may offer current shareholders some guaranteed allocation of the newly issued shares up to their fractional ownership, which we refer to as dilution protection. In rights offerings, firms announce the issue size and offer short-term in-the-money call options, i.e., rights, to current shareholders on a pro-rata basis. Current shareholders receive the rights for free and decide whether to exercise them and receive new shares. Typically, rights can be sold to other investors who then exercise them. The total issue proceeds are the strike price times the number of rights (or equivalently, shares) issued.

A major friction in capital markets are information asymmetries among the participants, which can lead to mispricing. Such mispricing is a particularly important concern for shareholders and investors at the time when new shares are issued (e.g., Myers and Majluf, 1984). On the one hand, shareholders fear that their holdings get diluted due to underpricing of new shares. On the other hand, prospective investors worry that they may end up purchasing overpriced shares.

As we show, there is, however, a simple solution to the informational friction – a rights offering with a sufficiently low strike price, such that even the most pessimistic shareholders exercise their rights. If all current shareholders exercise their rights, their fractional ownership in the firm remains unchanged and no shares are issued to new investors. Accordingly, any dilution to the existing shares caused by the low strike price is exactly offset by the gains on the new shares. Consequently, all shareholders receive the full information payoff, regardless of any potential informational asym-

[1] A third way to raise equity finance are private placements in which new shares are sold to a small group of qualified investors. We are interested in equity issuance methods where the share price is determined in competitive markets and therefore do not analyse private placements. Though as we briefly discuss in Section 7.5 private placements can be viewed as a special case of public offerings.

[2] In practice, issuing firms are typically assisted by underwriters who provide certification and possibly commitment to purchase all shares not taken up by investors (e.g., Eckbo and Masulis, 1992). As discussed later, we abstract from underwriters.

metries among market participants or among shareholders. In contrast, a public offering always generates some wealth transfer among shareholders and investors because new shares are sold to investors at a premium or discount.

This suggests that rights offerings dominate public offerings in the sense that the former can avoid wealth transfers. In addition, rights offering have lower direct floating costs than public offerings (Smith, 1977; Ecbko, et al., 2007). However, empirical evidence shows that rights offerings are infrequent in the U.S. (e.g., Ecbko, et al., 2007). Outside of the U.S., rights offerings are more common but are often not the predominate issue mode (Massa et al., 2016). This so-called rights puzzle has been explained with adverse selection problems which are mitigated in public offerings through underwriter certification (e.g., Eckbo and Masulis, 1992).[3] Nonetheless, a fundamental question remains: Why do firms bother with underwriter certification in public offerings given that rights offerings can circumvent the information problem?

In this paper, we relax one crucial condition that allows rights offerings to resolve the information problem, namely that all current shareholders have the resources needed to exercise their rights. In other words, we assume that some shareholders cannot, for some exogenous reasons, subscribe to new shares in public offerings or exercise their rights in rights offerings. Henceforth, we refer to these shareholders as cash-poor. As standard in the literature, we also assume throughout the paper that informed capital is scarce. In particular, those current shareholders who know the value of the firm can participate in a offer but cannot purchase all shares or rights. That is, share price and rights price are determined by the uninformed new investors. Otherwise, the offer could be funded by informed shareholders alone and would therefore not be plagued by information problems.

The aim of the paper is to compare public and rights offering in this setting with information asymmetries and some wealth-constrained shareholders. We intentionally do not engage with the security design problem of deriving the optimal selling procedure in this setting.[4] Instead, we

[3] Other explanations for the choice of issue mode are discussed in the related literature.

[4] As in Myers and Majluf (1984), debt would be the optimal security in our setting. Still, the widely documented stock price reactions following equity issuances shows that firms do issue equity despite or also in the presence of asymmetric information problems.

focus on two equity financing methods which are widely used in practice and explore whether they differ and how issue mode and terms affect wealth transfers among current shareholders as well as investment efficiency.

In the main model we assume that there is asymmetric information only about the NPV of the new investment project, while in an extension (Section 6) we consider the case with uncertainty about the assets in place. In the former setting shareholders always benefit if the firm invests. This may not hold in the latter setting which requires to determine which firm types actually want to issue and invest as in Myers and Majluf (1984). We choose the former setting as our main one to focus on the novel feature of our model, namely the strategic participation of informed shareholders. We postpone the analysis of the underinvestment problem to Section 6 where we focus on our the novel trade-off between more investment efficiency and more wealth transfers among shareholders.

The comparison of public and rights offerings in our main setting reveals a surprising results: Cash-poor shareholders fare better in public offerings than in rights offerings, despite the fact that they obtain proceeds from selling their rights, but receive no (extra) compensation in a public offering. Intuitively, rights have a positive value only if the strike price is lower than the equilibrium price in a public offering. Such a lower strike price implies that more new shares must be issued in a rights offerings to fund the investment. However, rights are priced at a discount on average due to the winner's curse problem, similar to Rock (1986). This implies more dilution to the existing holdings, which is not fully compensated by the proceeds from selling the rights. This may explain why firms particularly in the U.S. prefer public offerings to rights offerings.

Next, we analyze how firms choose offer mode and terms when knowing their type (Section 4). The choice of flotation method may therefore serve as a signal to uninformed investors. For this signaling game, we assume that firms maximize the total payoff to all current shareholders, or equivalently, minimize the payoff to new investors. As we show, only two kinds of equilibria can exist: The first kind is a pooling equilibrium in which all firms choose the same dilution protection in a public offer, or alternatively all firms choose the same strike price in a rights offering. In the

second kind of equilibrium, a single rights and a single public offering co-exist where high and low firm types opt for the rights offering, while intermediate firm types select the public offer.

The comparison between the pooling outcomes shows that public offers have smaller discounts, higher announcement returns, and are less underpriced than rights offerings. In the co-existence equilibrium outcomes the discount is also smaller in the public offering. Announcement returns and underpricing in the co-existence outcomes depend on the conditional means of the subset of firms choosing the rights, respectively the public offer, making their ranking dependent on distributional assumptions. Numerical simulations suggest that announcement returns are higher and underpricing is less severe in the public offering than in the rights offering, in line with the predictions based on the comparison of the two pooling outcomes. There is some support in the empirical literature for these predictions as we discuss in Section 5.

In Section 6, we analyse the implications of uncertainty about the assets in place in our setting with strategic shareholder participation. Replacing the project NPV with assets in place as the source of the asymmetric information introduces an underinvestment problem without overturning the earlier analysis. In particular, the above equilibrium outcomes become enriched by the Myers and Majluf (1984) feature that some firm types do not invest in equilibrium. That is, there are single public, respectively single rights, offering equilibria in which all investing firms choose the same mode and terms, but the most highly valued firms do not invest. There may also exist equilibria in which a single public and a single rights offer co-exist and in addition firm types with the highest value assets abstain from investing. Among the investing firms, the high and low firm types choose the rights offering, while intermediate firm types pick the public offer as above. As regards the underinvestment problem, we focus on the possible trade-off between investment efficiency and redistribution among shareholders. In public offers, better dilution protection induces more firm types to invest but also increases redistribution among shareholders. In rights offerings, higher strike prices promote investment but also lead to more wealth transfers among shareholders. Thus, there is indeed a trade-off between investment efficiency and redistribution in either issue mode. It

has to be pointed out that these comparative static results hold for uniformly distributed assets in place.

Finally, we discuss the robustness of our results with respect to shareholder participation in rights offerings (Section 7.1), letting uninformed or cash-poor shareholders participate in the offers (Sections 7.2 and 7.3), or allowing cash-rich shareholders to purchase more shares than those allocated to them on a pro-rata basis (Section 7.4). We also compare public offerings with private placements (Section 7.5).

Literature Review

We focus our discussion on papers that - like ours - consider asymmetric information problems the primary concern when raising equity financing. We only briefly discuss other explanations for the choice of issue method and also abstract from papers that analyse private placements or compare them with either public or rights offerings. The literature recognizes that rights offerings allow current shareholders to avoid – in principle – dilution. If all shareholders participate proportionally in a rights offering they maintain their fractional ownership. Consequently, there are neither adverse selection problems nor wealth transfers (e.g., Myers and Majluf 1984, p. 195 footnote 5; Berk and DeMarzo 2017, p. 856). However, as noted by e.g., Ursel (2006) or Wu et al. (2016), if some shareholders sell their rights to investors, adverse selection problems arise as in the Myers and Majluf (1984) setting.[5] Our analysis shows that the ensuing adverse selection problems are aggravated by the winner's curse problem when some current shareholders strategically decide whether to exercise or sell their rights.

Ecbko and Masulis (1992) argue that underwriter certification and low shareholder take-up can explain why firms prefer public offerings. In their framework, underwritten offers are not direct sales as in Myers and Majluf (1984), but come with a noisy though informative certification of the firms' value. There is no such certification in uninsured rights offerings and the fraction of the

[5]If rights are non-tradeable, wealth transfers between current shareholders and investors are eliminated, though not necessarily transfers among shareholders.

issue taken up by current shareholders is exogenously given. Clearly, undervalued firms experience a wealth loss, which increases as the shareholder take-up becomes smaller. Consequently, the choice of issue mode depends upon the shareholder take-up: If it is high (low), the uninsured rights offering entails less (more) wealth transfers to investors than the underwritten issue. Our framework differs along two important dimensions. First, public offers do not feature an underwriter who plays an informational role or guarantees the offer. Second, shareholder take-up is a strategic decision rather than driven by factors outside the model.[6]

Heinkel and Schwartz (1986) also consider an extended Myers and Majluf (1984) setting to examine the choice between fully underwritten public offers and uninsured rights offerings. In their model, firms differ in the probability distribution of their terminal stock price, and the distribution depends on a parameter which is private information to the firm. All firms want to raise the same amount of equity capital, and if the realized terminal share price is less than the subscription price, the offer fails and the firm incurs a fixed cost per share. Thus, issuing a larger number of shares – as lower quality firm must to raise the financing – makes failure more costly. Failure (costs) are avoided by using an underwriter who guarantees the offer proceeds. The failure cost of the uninsured rights offer enables high quality firms to use the subscription price to credibly reveal their types (expected terminal share price). Low quality firms prefer to sell shares at a pooling price through an uninformed underwriter, because the (expected) failure cost of an uninsured rights offering exceed underwriter fees and possible undervaluation. Heinkel and Schwartz assume that firms choosing an underwritten offer sell their issue to the underwriter at the same price they would announce in a rights offer, if they were to choose that financing method. Therefore, the extent to which shareholders participate in the rights offering plays no role.[7] By contrast, our framework

[6] Eckbo and Norli (2005) add more structure to the framework of Eckbo and Masulis (1992) to prove equilibrium. They also allow for a larger menu of flotation methods. As in Eckbo and Masulis (1992), the exogenous shareholder take-up is the crucial determinant for the issue choice.

[7] In an extension Heinkel and Schwartz introduce standby rights offers as a third issue mode. The underwriter promises to purchases any not taken-up shares in exchange for a fee and also learns the firm type at some cost. In equilibrium, the highest quality firms choose the standby rights offer as they find it less expensive to reimburse the underwriter for becoming informed.

features a meaningful market for rights and current shareholders who strategically decide whether to participate respectively exercise or sell their rights. Furthermore, there is no failure risk because issue respectively rights prices adjust to allow investors to break even in equilibrium.

There are several other explanation for the choice of issuance methods that are not based on informational frictions. Smith (1977) attributes the prevalence of public offerings in the US to agency conflicts among managers and shareholders. Hansen (1988) argues that shareholders face additional flotation cost in form of price concessions in rights offering which are absent in public offerings. Hence, public offerings are more attractive even though the direct flotation costs are larger. Hansen and Pinkerton (1982) propose that differences in ownership structures account for the choice of flotation method. Firms with large blockholders opt for rights offering, whereas dispersedly held firms find public underwritten offer the more cost efficient way to raise new equity financing. Ursel (2006) argues that firms in poor financial condition with low net worth use rights offerings since current shareholders have larger incentives to inject new funds to keep the firm alive than outside investors. Thus, rights issues are a (equity) financing of last resort. Wu et al. (2014) propose that the flotation choice is driven by rent-protection motives of controlling shareholders.[8] In their model, the controlling shareholder can maintain her fractional ownership in a rights offering, but her stake is diluted in a public offer which is (more) costly when private benefits are large. In a cross-country study with a sample of share issues from 41 countries during 1990-2008 McLean et al. (2013) find that the likelihood of public offerings relative to both private placements and rights offerings increase with investor protection. Finally, Holderness (2017) covers in his meta-analysis over 100 studies on equity issues in different countries. He argues that the flotation choice is driven by the presence or absence of mandatory shareholder approval. In countries where shareholders must approve an issue, rights offerings are much more common, whereas public offers are more common in countries that allow management/boards to issue equity without shareholder approval.[9]

[8] Focusing exclusively on rights offerings, Fried and Spamann (2018) show that pre-emptive rights do not protect minority shareholders against expropriation through an equity issue, so-called cheap-stock tunneling.

[9] He reports that shareholder-approved issues are associated with positive and higher announcement returns than managerial issues, and that this holds across and within countries as well as for different issue methods.

2 Model Setup and Benchmark

2.1 Model

Consider an economy that is populated by publicly traded firms with assets in place a and an unfunded investment opportunity which requires an outlay I and generates a payoff $I + b$. For simplicity, we assume that both the value of the assets in place a as well as the investment cost I are the same across all firms and publicly known. By contrast, the net present value (NPV) of the investment $b > 0$ varies across firms and is distributed on $[\underline{b}, \overline{b}]$ according to the density function $f(b)$, respectively its distribution function $F(b)$. As we discuss in Section 6, conditional on investing it is largely inconsequential whether the information asymmetry concerns the assets in place a or the investment b. The number of existing shares is normalized to 1. Since we want to compare equity flotation methods we restrict firms to raise I by issuing new equity through either a public offering (PO) or a rights offering (RO), which we describe later. Current shareholders and competitive new investors are all risk-neutral.

The key frictions in the model are information asymmetry and the scarcity of informed capital. In particular, among current shareholders only a fraction $(1 - \eta)$ know the project's NPV b while the remaining η shareholders merely know its distribution. Like the latter, investors only know the distribution of b. In addition, a fraction π of shareholders have no spare wealth to participate in the equity issuance. Furthermore, these cash-poor shareholders can neither borrow nor sell (part of) their current shares to participate in an offering. We relax this assumption in Subsection 7.3. The remaining $(1 - \pi)$ shareholders have financial slack to purchase additional shares. However, they cannot trade with either cash-poor shareholders or investors. That is, cash-rich shareholders can at most purchase those newly issued shares which are allocated to them on a pro-rata basis. In Subsection 7.4 we ease this restriction and allow cash-rich shareholders to buy more shares or rights and argue that our insights are robust. The key assumption is that some investors participate in the offers, which reflects the way seasoned offerings are conducted in practice. If instead cash-rich

shareholders could purchase all shares or rights, informed capital would not be scarce, and the equity market would be freed from asymmetric information problems.

Our assumption of cash-poor shareholders can be interpreted in different ways. First, some shareholders may have exhausted their buying power, and borrowing on margin account for an extended period of time can be too expensive. Furthermore, we argue in Section 7.3 that borrowing to exercise the rights and immediately selling the underwritten shares to pay back the loan is equivalent to selling the rights directly. Second, inattentive shareholders whose rights are sold by their brokers on their behalf are equivalent to cash-poor shareholders in the model. (See Section 7.1). Finally, managers in public firms typically do not purchase significant amounts of the newly issued shares, since much of their wealth is already tied to the firm.

Throughout the paper, we consider stylized versions of public and rights offerings. In public offerings, the firm issues new shares in the public market, but shareholders may receive some dilution protection. That is, current shareholders are given priority over some fraction $\lambda \in [0, 1]$ of the new shares on a pro-rata basis. Obviously, only the cash-rich shareholders can buy additional shares and possibly benefit from the dilution protection. Investors get to buy all $(1 - \lambda)$ non-dilution protected shares and those dilution protected shares that shareholders do not wish to take up.[10] Shareholders and investors simultaneously decide whether to subscribe. Finally, the investors' break-even condition determines the per share price P_{PO} and the number of newly issued shares such that $N_{PO} = \frac{I}{P_{PO}}$.[11]

The payoff to shareholders in a public offering depends on the offer price P_{PO}, the number of shares issued N_{PO}, and their subscription decision. After issuing new shares and investing, the true firm value is equal to $I + a + b$. Given the number of shares is $N_{PO} + 1$, the true share value must

[10] Dilution protection is very common in the UK. In countries where shares are allocated on a pro-rata basis based on the subscription, the parameter λ can be interpreted as the demand of the shareholders relative to that of the investors (similar to Rock 1986).

[11] Our stylized public offering is a direct share sale and resembles an At-The-Market (ATM) offering, except that an ATM offering may split the total issuance into smaller quantities spread over some time period. In the US ATM offerings have recently become more popular, accounting for 40 percent of Seasoned Equity Offerings in 2015 (Billett et al., 2016).

equal $\frac{1}{N_{PO}+1}(I+a+b)$. If a cash-rich shareholder with β shares subscribes, she receives $\lambda\beta N_{PO}$ new shares in exchange for investing an amount $\lambda\beta N_{PO}P_{PO} = \lambda\beta I$. As a result, her final payoff as a function of the true firm type b is

$$\beta\left[\frac{\lambda N_{PO}+1}{N_{PO}+1}(I+a+b)-\lambda I\right]. \tag{1}$$

If a shareholder chooses not to subscribe or has no cash to do so, her payoff is

$$\beta\frac{1}{N_{PO}+1}(I+a+b). \tag{2}$$

Cash-rich shareholders strategically subscribe to the newly issued shares to receive the higher of (1) and (2).

In rights offerings with a strike price P_S, $N_{RO} = \frac{I}{P_S}$ rights are issued to shareholders on a pro-rata basis at no cost. Each right gives its owner the option to purchase a newly issued share at the strike price P_S. Cash-rich shareholders can choose between exercising the rights or selling them to new investors. Simultaneously, investors decide whether to buy rights which shareholders put up for sale. Since cash-poor shareholders can by assumption neither borrow nor sell their current shares, they have no choice but to sell their rights to new investors. In Subsection 7.3, we discuss the implications of relaxing this assumption. Doing nothing, that is, neither exercising nor selling the rights, is weakly dominated by selling the rights as long as the rights price $P_R \geq 0$. Therefore, we rule out doing nothing as an option here, but discuss it in Section 7.1. The break even constraint of the competitive investors determines P_R, and we exclude negative prices.

Following the rights offering the true firm value is $I+a+b$, and the number of shares is $N_{RO}+1$. Hence, the true share value is $\frac{I+a+b}{N_{RO}+1}$. If a cash-rich shareholder with β shares exercise her rights, she receives βN_{RO} new shares and invests $\beta N_{RO}P_S = \beta I$. As a result, her payoff as a function of the true firm type b is equal to

$$\frac{a+I+b}{N_{RO}+1}(N_{RO}\beta+\beta)-\beta N_{RO}P_S=\beta\left(a+b\right). \tag{3}$$

If a current shareholder sells her rights, her payoff is

$$\frac{a+I+b}{N_{RO}+1}\beta+P_R\beta N_{RO}. \tag{4}$$

Cash-rich shareholders strategically exercise their rights to receive the higher of (3) and (4). In Sections 2.2 and 3, we solve for the Perfect Bayesian Equilibrium outcomes when all firms adopt a given flotation method. That is we derive the cash-rich shareholders' optimal participation decisions for a given dilution protection λ (strike price P_S) and the associated equilibrium price P_{PO} in a public offering (P_R in a rights offering). In Section 4, we let firms choose the offer to maximize the total payoffs to all shareholders.

2.2 Benchmark

Here we consider the public offer and rights offer game when all shareholders have wealth to participate in the offer ($\pi = 0$) but only some of them ($1-\eta$) know b, the NPV of the project. The η uninformed shareholders and the investors merely know the distribution of b. The (outcomes of the) offer games with informational friction but no wealth constraints serves as a benchmark for the subsequent analysis.

As noted in the literature, rights offering can avoid wealth transfers between shareholders and new investors if "stockholders can be compelled to exercise their rights and hold the newly issued shares" (Myers and Majluf, 1984 footnote 5). We extend this intuition by showing that rights offerings can resolve asymmetric information problems among current shareholders, ensuring that each and every shareholder receives the full information payoff $a+b$.

Proposition 1 *Given all current shareholders are cash-rich, they all receive a net payoff of $a+b$ in the unique Perfect Bayesian Equilibrium of a rights offering. Moreover, this equilibrium exists*

only if $P_S \leq a + \underline{b}$, and is implemented by all current shareholders exercising their rights.

When a shareholder exercises the rights allocated to her on a pro-rata basis, her payoff does not depend on the strike price. Indeed, exercising the rights implies that her fractional ownership stake in the firm remains unchanged (see (3)). Therefore, any mispricing of the issue (strike price P_S) is fully offset by a corresponding value change of her "old" shares. However, informed shareholders of firms with low project values b may find it more profitable to sell their rights in the market. A sufficiently low strike price in combination with market beliefs that any rights sold would come from the worst firm type $\underline{b}$ make this an inferior option. As a result, informed as well as uninformed shareholders find it in their interest to exercise their rights. Consequently, they all receive a net payoff equal to $a+b$, as they would under complete information.[12] The proof in the appendix shows that this is the unique Perfect Bayesian Equilibrium. As shown by Myers and Majluf (1984), selling shares to investors in a public offering inevitably leads to wealth transfers. This holds true also in our setting.

Proposition 2 *Any public offering with incomplete dilution protection $\lambda < 1$ leads to wealth transfers among shareholders and investors.*

In public offerings without dilution protection ($\lambda = 0$), new investors purchase all new shares in a successful offering, as in Myers and Majluf (1984). Since they are uninformed, the price P_{PO} must - in equilibrium - be the same for any and all firms, irrespective of the net present value of the investment opportunity. Moreover, investors only purchase shares if the price is such that they break even on average. Consequently, there is mispricing and redistribution across firm types: For firms whose investment project has a low (high) net present value, the new shares are overpriced (underpriced), and investors make a loss (profit). Accordingly, shareholders receive a payoff which is either larger or smaller than $a + b$, their full information payoff.

[12]The $1.3 billion rights offering by Wharf Ltd, a Hong Kong listed property developer, illustrates how rights offerings at low strike prices avoid shareholder dilution and secures full subscription (Wall Street Journal, February 11, 2011).

The asymmetric information problems are complicated by the dilution protection, since it adds a winner's curse problem when investors purchase shares in equilibrium (i.e., when $\lambda < 1$).[13] Informed cash-rich shareholders take up their allocated quota $\lambda(1-\eta)$ only if the issue is underpriced. As a result, investors end up buying more shares when a firm is overpriced. Hence, dilution protection leads to additional redistribution among shareholders and investors. As discussed in Subsection 7.2, uninformed shareholders benefit from taking up their allocated quota $\lambda\eta$.

Comparing Propositions 1 and 2, we establish the following benchmark: Rights offerings dominate public offerings in the sense that the former but not the latter overcomes informational frictions and avoids redistribution both among shareholders and between shareholders and investors. Hence, the widespread use of public offerings cannot be attributed exclusively to asymmetric information problems. There must be at least one other friction. Subsequently, we (re-)introduce wealth constraints of some shareholders (i.e., $\pi > 0$) and show how this may reverse the ranking of the two offer methods.

3 Offer Methods and Wealth Transfer

In this section, we characterize the shareholders' participation decisions in public offerings with any given dilution protection λ (Subsection 3.1) and in rights offerings with any given strike price P_S (Section 3.2). We then compare the wealth transfers among cash-rich and cash-poor shareholders across the different issue methods (Section 3.3).

The model (Subsection 2.1) allows for potentially four types of shareholders: cash-rich informed and uninformed ones and cash-poor informed and uninformed ones. To make the analysis more transparent we simplify matters here and assume that wealth and information are perfectly correlated. That is, there are but two types of shareholders, $(1-\pi)$ informed cash-rich and π uninformed cash-poor shareholders. As we discuss in Subsection 7.2 allowing also for uninformed cash-rich

[13] In the limit ($\lambda = 1$), there cannot be an issue price P_{PO} in equilibrium at which shareholders strictly prefer not to subscribe to the offer, and the public offer becomes de facto a rigths offering (see Subsection 3.3).

and informed cash-poor shareholders does not alter our qualitative results. The sequence of moves in both the public offer and the rights offer game are as before (Subsection 2.1). Given an issue method, shareholders and investors simultaneously decide whether to participate.

3.1 Public Offerings

In a public offering, cash-rich shareholders subscribe to the new shares only if the payoff from subscribing (1) is higher than the payoff from abstaining (2).

Lemma 1 *In a public offering with a given λ, cash-rich shareholders subscribe to the new shares if and only if*

$$b \geq b^*_{PO} \equiv P_{PO} - a. \tag{5}$$

Cash-rich shareholders follow a simple threshold strategy and subscribe to an offer only if the sum of assets in place a and net present value of the investment b (weakly) exceed the price P_{PO}. That is, they subscribe only if the new shares are underpriced, similar to e.g., Rock (1986). Clearly, cash-poor shareholders have no choice but to abstain from the offer.

Since investors do not know the net present value of the investment, their participation in the offer must be unconditional, that is, cannot depend on the firm type b. At the same time they anticipate that cash-poor shareholders never subscribe but that cash-rich shareholders only subscribe if the issue is not overpriced. Accordingly, the investors' collective payoff when subscribing is equal to

$$Pr\left(b < b^*_{PO}\right)\left[\frac{N_{PO}}{N_{PO}+1}(a + I + E[b|b < b^*_{PO}]) - I\right]$$

$$+Pr\left(b \geq b^*_{PO}\right)\left[1 - (1-\pi)\lambda\right]\left[\frac{N_{PO}}{N_{PO}+1}(a + I + E[b|b \geq b^*_{PO}]) - I\right], \tag{6}$$

In the above expression, the first line represents the case when the offer is overvalued and all N_{PO} new shares are purchased by the investors. They receive a fraction $\frac{N_{PO}}{N_{PO}+1}$ of the firm in exchange for contributing I. The second line reflects the case where cash-rich shareholders buy $(1-\pi)\lambda N_{PO}$

shares. The remaining $[1-(1-\pi)\lambda]\,N_{PO}$ shares are purchased by the investors. Rearranging the terms in (6) using the fact that $N_{PO} = \frac{I}{P_{PO}}$ and factoring out $\frac{N_{PO}}{N_{PO}+1}$ yields

$$Pr\left(b \geq b^*_{PO}\right)\left[1-(1-\pi)\lambda\right]\left[a + E[b|b \geq b^*_{PO}] - P_{PO}\right] + Pr\left(b < b^*_{PO}\right)\left[a + E[b|b < b^*_{PO}] - P_{PO}\right],$$

which can be rewritten as

$$a + E(b) - P_{PO} - Pr\left(b \geq b^*_{PO}\right)(1-\pi)\lambda\left[a + E[b|b \geq b^*_{PO}] - P_{PO}\right]. \tag{7}$$

The zero profit condition of the competitive investors, together with condition (5), determines the equilibrium issue price P_{PO}.

Proposition 3 *For any given $\lambda \in [0,1]$, there exists a Perfect Bayesian Equilibrium in which all firms raise I. The equilibrium price P_{PO} is decreasing in λ, and $a + \underline{b} < P_{PO} \leq a + E(b)$.*

Since all firms issue and invest, the price P_{PO} in any equilibrium must exceed the value of firm with the lowest net present investment $(a + \underline{b})$ and can be at most equal to the unconditional mean $(a + E(b))$. Otherwise, investors would on average either earn a profit or not break even. In the limiting case of no dilution protection ($\lambda = 0$), the information advantage of the cash-rich shareholders becomes irrelevant. They never receive any new shares, and there is no winner's curse problem. Hence, investors are willing to purchase the new shares at the unconditional average firm value, that is, $P_{PO} = a + E(b)$. Better dilution protection exacerbates the winner's curse problem, which decreases the equilibrium price P_{PO}. When shareholders enjoy better dilution protection, investors get to buy a smaller fraction $(1-(1-\pi)\lambda)$ of underpriced shares while still buying all overpriced shares. Consequently, they can only break even if the equilibrium price is lower.

We now turn to the wealth transfers between cash-poor and cash-rich shareholders. Since investors break even on average, we can define the ex-ante (prior to knowing b) wealth transfer from cash-poor to cash-rich shareholders as the difference between the expected actual payoff and the

fair expected payoff $(a + E(b))$. Using the payoff of the cash-poor shareholders (2), we can express the wealth transfer as

$$WT_{PO} \equiv a + E\left(b\right) - \frac{1}{1 + \frac{I}{P_{PO}}}\left[I + a + E\left(b\right)\right]. \tag{8}$$

We next rank all public offerings with different dilution protections according to the extent of the ex-ante wealth transfers among current shareholders.

Proposition 4 *The ex-ante wealth transfers from cash-poor to cash-rich shareholders in public offerings with a given dilution protection λ is equal to*

$$WT_{PO} = \frac{I}{I + P_{PO}}\left[a + E(b) - P_{PO}\right] \geq 0, \tag{9}$$

and increases with the dilution protection λ.

Since a better dilution protection grants cash-rich shareholders the option to purchase more new shares, it exacerbates the winner's curse problem. The equilibrium price P_{PO} must decrease to allow investors to break even, which necessitates a larger issue (N_{PO}). As a result, the cash-rich shareholders' ability to subscribe becomes more valuable. Hence, the wealth transfer increases with the dilution protection.

3.2 Rights Offerings

In a rights offering with strike price P_S, $N_{RO} = I/P_S$ rights are issued. Cash-rich shareholders choose between exercising or selling their rights, while investors simultaneously decide whether to purchase the rights put up for sale. Cash-rich shareholders exercise their rights if and only if (3) is weakly greater than (4).

Lemma 2 *In a rights offering, cash-rich current shareholders exercise their rights if and only if:*

$$b \geq b^*_{RO} \equiv P_S + P_R(N_{RO} + 1) - a. \tag{10}$$

As in public offerings, cash-rich shareholders follow a simple threshold strategy. They prefer to sell their rights if the project return falls below the cut-off value b^*_{RO}. The expression of the cutoff value is more complicated than in the public offering because the "fair" value in a rights offering contains the bundle of strike price P_S and rights price P_R. Investors who purchase one right and exercise it have a payoff equal to

$$\frac{a+I+b}{N_{RO}+1} - P_S - P_R.$$

Rational investors also anticipate that cash-poor shareholders always sell their rights, whereas cash-rich shareholders sell them only if the project returns are low ($b < b^*_{RO}$). Consequently, the expected payoff to investors is

$$\begin{aligned} Pr(b \geq b^*_{RO})\pi N_{RO} \left(\tfrac{a+I+E(b|b\geq b^*_{RO})}{N_{RO}+1} - P_S - P_R \right) \\ + Pr(b < b^*_{RO}) N_{RO} \left(\tfrac{a+I+E(b|b<b^*_{RO})}{N_{RO}+1} - P_S - P_R \right) \end{aligned} \tag{11}$$

Investors break even if the sum of strike and rights prices equals the conditional expected firm value (on a per share basis), taking into account when cash-rich shareholders subscribe.

$$P_S + P_R = \frac{1}{N_{RO}+1} \left[a + I + \frac{\pi Pr(b \geq b^*_{RO}) E(b|b \geq b^*_{RO}) + Pr(b < b^*_{RO}) E(b|b < b^*_{RO})}{\pi P(b \geq b^*_{RO}) + Pr(b < b^*_{RO})} \right]. \tag{12}$$

Proposition 5 *For any given $P_S \in (0, a + b^*_{RO}]$, there exists a Perfect Bayesian Equilibrium in which all firms issue rights with the same strike price P_S. The cutoff type b^*_{RO} solves*

$$b^*_{RO} = \frac{(1-\pi) Pr(b < b^*_{RO}) E(b|b < b^*_{RO}) + \pi E(b)}{(1-\pi) Pr(b < b^*_{RO}) + \pi}, \tag{13}$$

lies in $(\underline{b}, E(b))$, and is independent of P_S.

Cash-rich shareholders follow their threshold strategy (10), and the thresholds b^*_{RO} is the solution to (13) in equilibrium. For any solution b^*_{RO}, the equilibrium rights price P_R is then uniquely given by (12). At first glance it may be surprising that the cutoff value b^*_{RO} does not depend on the

strike price P_S. To understand this feature it is perhaps best to consider the sell/exercise decision of cash-rich shareholders in firms with low project returns ($b < b^*_{RO}$). Clearly, exercising is not attractive if the strike price is overvalued, that is, if $P_S > a + b$. Exercising at low strike prices $P_S < a + b$ is attractive, but selling the rights is even more profitable because rights are priced by the investors' beliefs about the conditional average firm value. Since cash-poor investors in all firms sell their rights, the rights price for firms with low investment returns is effectively subsidized. Regardless of the strike price, firms above or below the conditional average belief are the same, and the cutoff value is therefore not affected by the strike price.

The strike price does, however, affect the wealth transfers from cash-poor to cash-rich shareholders. Given a strike price P_S, the payoff to cash-poor shareholders in a type-b firm is equal to

$$\frac{a + I + b}{N_R + 1} + P_R N_R$$

which can be rewritten as

$$a + b + \frac{I}{I + P_S}[b^*_{RO} - b].$$

(See the proof of the subsequent proposition for details.) As in public offerings, we can define the ex-ante wealth transfer among shareholders in rights offerings as the difference between the expected actual payoff and the fair expected payoff ($a + E(b)$):

$$WT_{RO} \equiv \frac{I}{I + P_S}\left[E(b) - b^*_{RO}\right]. \tag{14}$$

Proposition 6 *The ex-ante wealth transfers from cash-poor to cash-rich shareholders in rights offerings with a strike price P_S are equal to*

$$\frac{I}{I + P_S}\left[E(b) - b^*_{RO}\right] > 0 \tag{15}$$

and decrease in P_S.

The intuition is similar to that of Proposition 4. Lower strike prices P_S require more rights N_{RO} to be issued. Since rights are on average underpriced due to the winner's curse problem, the financial ability to exercise the rights becomes more valuable when more rights are issued. For cash-poor shareholders who cannot exercise the rights, a lower strike price implies that a larger fraction of the company is sold at a discount on average. Hence, lower strike prices and larger numbers of rights N_{RO} lead to more wealth transfers from cash-poor to cash-rich shareholders.

Absent information asymmetries, exercising the rights and buying new shares or simply selling the rights yield the same payoff. Gains made from exercising are matched by the proceeds from the rights sale (Farinha et al. 2017). Proposition 6 implies that this does not hold in our setting with asymmetric information. Due to the winner curse problem, the rights price does not fully compensate for the dilution of existing share holdings. Therefore, the pricing of the new shares matters for cash-poor shareholders.

3.3 Comparing Public and Rights Offerings

In rights offerings, non-participating shareholders (can) sell the rights instead of doing nothing in public offerings. In addition, investors are exposed to a larger extent to the winner's curse problem than in public offerings. Despite these two differences, one specific public offering (with full dilution protection) is equivalent to one specific rights offering (with zero rights price).

Proposition 7 *Rights offerings with a strike price P_S such that the equilibrium rights price P_R equals* 0 *are equivalent to public offerings with full dilution protection ($\lambda = 1$).*

Denote by P_{PO} the equilibrium issue price in a public offering with full dilution protection. Intuitively, in a rights offering with a strike price $P_S = P_{PO}$ the value of the rights is zero in equilibrium ($P_R = 0$), because such a right resembles an at-the-money option at expiration. Under this conjecture, the payoffs to the shareholders are the same in both issue modes. First, if cash-rich shareholders participate in the offering, they can maintain their fractional ownership at the same price in either offer. Conversely, if shareholders cannot or choose not to subscribe, their payoffs

must again be the same in either offer. Their holdings are equally diluted since the prices P_{PO} and P_S are equal and hence also the number of newly issued shares. Moreover, selling the rights does not generate any income. Because all shareholders receive the same payoffs in either issue modes, cash-rich shareholders in the same firm types find it profitable to subscribe respectively to abstain, generating the same extent of the winner's curse problem in either offer. The equivalence result allows to rank the flotation methods according to the extent to which they entail wealth transfers between cash-poor and cash-rich shareholders.

Corollary 1 *Any rights offering entails more wealth transfers from cash-poor to cash-rich shareholders than public offerings.*

On the one hand, wealth transfers increase in the dilution protection λ (Proposition 4). Therefore, a public offering with full dilution protection comes with the largest wealth transfers among all public offerings, whereas an offer without any dilution protection features no such wealth transfers. On the other hand, wealth transfers decrease with the strike price P_S in rights offerings (Proposition 6) which in turn is highest when rights have zero (resale) value. Hence, among all rights offerings the one with an equilibrium rights price P_R equal to 0 leads to the least wealth transfer. This least-wealth-transfer rights offering ($P_R = 0$) is equivalent to the aforementioned most-wealth-transfer public offering ($\lambda = 1$). Consequently, any rights offering generates (weakly) more wealth transfer from cash-poor to cash-rich shareholders than any public offerings. This result is depicted in Figure 1.

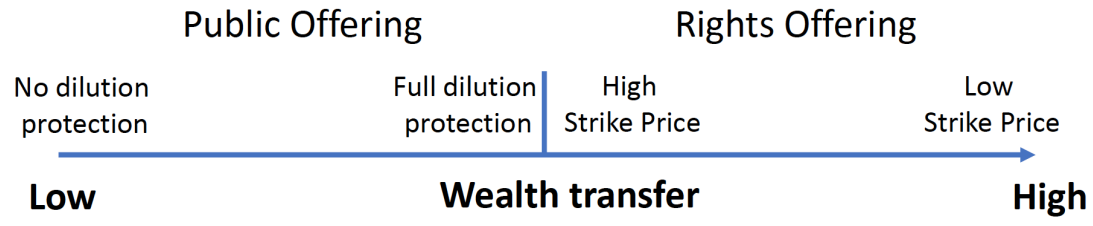

Figure 1: Rank issue modes based on wealth transfers.

The Corollary suggests one consideration that may affect firms' choice of issuance mode — wealth transfers among shareholders. When firms care more about equality among shareholders, they opt for a public offering with no dilution protection to avoid such wealth transfers. Conversely, when firms favor their cash-rich shareholders, they use rights offerings with low strike prices to create wealth transfer from the cash-poor shareholders.

4 Choosing Offer Terms and Modes

We now examine the offer game when firms can strategically choose offer mode and terms, knowing their type b at the time of the issue decision. As in Section 3, there are $(1-\pi)$ informed cash-rich shareholders and π uninformed cash-poor shareholders. After learning its type, the firm decides whether to raise I and, if yes, chooses offer method (public or rights offer) and terms (dilution protection λ or strike price P_S). Thereafter, shareholders and investors simultaneously decide to participate. More specifically, should the firm choose a pubic offer, the cash-rich shareholders then choose whether to buy the λ dilution protected shares on the pro-rata basis $(1-\pi)$, and investors decide whether to buy all remaining shares. Should the firm choose a rights offering, cash-poor shareholders sell their rights, cash-rich shareholders choose whether to exercise or sell their rights, and the investors decide whether buy all rights offered by shareholders and if yes, to exercise them. Furthermore, investors update their beliefs about b based on firms' issue methods and break even on average, as in the baseline model.

When solving for the Perfect Bayesian Equilibria of this signaling game, we assume that firms choose the issue method to maximize current shareholder wealth. This objective function is less evident than it may appear at first sight since in our framework cash-rich and cash-poor shareholders typically disagree over the optimal flotation method, the extent of dilution protection, or the strike price.[14] In view of the diverging preferences among shareholders some arbitrariness in choosing

[14]The French bank Natixis is an example of shareholder disagreement about an equity offering. Natixis opted for a rights issue to strengthen its capital base in 2008 after suffering heavy losses in the US subprime market. Its two main shareholders, Caisse d'Epargne and Banque Populaire, were in favour of the issue presumably because part

the firms' objective function is unavoidable. We opt for firms maximizing the weighted sum of the payoffs to the cash-poor and cash-rich shareholders because it is equivalent to minimizing new investors' payoffs, conditional on the firm's type b. The latter seems to us the least controversial objective function in our framework.

When subscribing to a firm type b public offer with dilution protection λ, price P_{PO}, and number of new shares $N_{PO} = \frac{I}{P_{PO}}$, the investors realize a payoff equal to

$$\Pi_{PO}(b) = \begin{cases} [1 - \lambda(1-\pi)] N_{PO} \left(\frac{a+I+b}{N_{PO}+1} - P_{PO} \right) & \text{if } \frac{a+I+b}{N_{PO}+1} \geq P_{PO} \\ N_{PO} \left(\frac{a+I+b}{N_{PO}+1} - P_{PO} \right) & \text{if } \frac{a+I+b}{N_{PO}+1} < P_{PO}, \end{cases} \tag{16}$$

Similarly, in a rights offering with strike price P_S, number of rights $N_{RO} = \frac{I}{P_S}$, and rights price P_R, the investors' payoff is equal to

$$\Pi_{RO}(b) = \begin{cases} \pi N_{RO} \left(\frac{a+I+b}{N_{RO}+1} - P_S - P_R \right) & \text{if } \frac{a+I+b}{N_{RO}+1} \geq P_S + P_R \\ N_{RO} \left(\frac{a+I+b}{N_{RO}+1} - P_S - P_R \right) & \text{if } \frac{a+I+b}{N_{RO}+1} < P_S + P_R \end{cases}. \tag{17}$$

We begin the analysis of possible equilibrium outcomes by establishing that offers of the same mode (PO or RO) with different terms cannot co-exist in equilibrium.

Lemma 3 *In any equilibrium, all public offerings have the same dilution protection λ, and all rights offerings have a common strike price P_S.*

Intuitively, multiple public or rights offerings cannot co-exist because overvalued firms would invariably deviate to the offer with the highest price. Suppose to the contrary that there are two public offerings. Any low firm type with non participating cash-rich shareholders prefers the offer with the higher issue price, irrespective of whether the dilution protection is greater or smaller than that of the offer with the lower price. Hence, there can only be a single issue price in equilibrium.

of the proceeds were meant to repay their advancements to Natixis. By contrast, US activist investors Greenlight Capital and Royal Capital Management were opposed (Financial Times, September 5, 2008).

This in turn must imply a single dilution protection since cash-rich shareholders in undervalued firms strictly prefer more to less dilution protection.

The argument why multiple rights offerings do not co-exist is similar, though slightly more involved. Suppose that there are two rights offerings with different strike prices. High firm types must prefer the higher strike price, since dilution is more costly for those firms and the lower strike price leads to more dilution of the cash-poor shareholders. Further, for each of the two rights offerings there must be some undervalued and some overvalued firm types. Firms which are undervalued under the offer with the lower strike price prefer to deviate to the offer with the higher strike price and get subsidized by higher-valued types, rather than subsidizing lower-valued types in the rights offer with the low price.

Given Lemma 3, only two kinds of Perfect Bayesian Equilibrium can exist: pooling equilibria where all firms use the same issue method (Subsection 4.1), and a semi-pooling equilibrium where some firms pool on a unique public offering and others pool on a unique rights offering (Subsection 4.2).

4.1 Single Offer Mode Equilibria

When firms can strategically choose offer terms and mode, all firms choosing the same offer remains a Perfect Bayesian Equilibrium. Such pooling outcomes are supported by the beliefs that any deviating firm is perceived to be the lowest firm type $\underline{b}$. We first establish the pooling equilibria for public offerings. Denote by $(P_{PO}(\lambda), N_{PO}(\lambda))$ the public offering equilibrium outcome for any given dilution parameter λ, as characterized by Proposition 3.

Proposition 8 *There exist pooling equilibria in which all firms choose some common dilution protection λ in a public offering. An issue with a given λ is such an equilibrium if and only if*

$$[1-\lambda(1-\pi)]\, N_{PO}(\lambda)\left(\frac{a+I+\overline{b}}{N_{PO}(\lambda)+1}-P_{PO}(\lambda)\right)\leq \pi I\left(\frac{\overline{b}-\underline{b}}{a+I+\underline{b}}\right). \tag{18}$$

Moreover, the condition always holds for any λ sufficiently close to 1.

Proposition 3 guarantees that $P_{PO}(\lambda)$ is indeed the equilibrium issue price associated with dilution protection λ. Condition (18) rules out any deviation to any other public or rights offerings given the investors' belief that any such firm would be the lowest type $\underline{b}$.

Specifically, consider a deviation to another public offering $\hat{\lambda} > \lambda$. A deviating firm would have to sell its new shares at a price $\hat{P}_{PO} = a + \underline{b}$, given the investors' off-equilibrium beliefs. Thus, any firm type (except type $\underline{b}$) deviating to $\hat{\lambda}$ would sell its shares at a discount. Clearly, this is not attractive for all low firm types $b \in [\underline{b}, b^*_{PO}(\lambda))$ which sell overpriced shares to investors in the pooling equilibrium. High firms types $b \in (b^*_{PO}(\lambda), \overline{b}]$ sell underpriced shares in the pooling offer or if they were to deviate, and in either case cash-rich shareholders would participate. By Proposition 3 the pooling price $P_{PO}(\lambda)$ is higher than $\hat{P}_{PO}$ (price effect), but investors can purchase more shares since $\lambda < \hat{\lambda}$ (quantity effect). Clearly, the lower price ($\hat{P}_{PO}$) hurts the shareholders, but the better dilution protection ($\hat{\lambda}$) benefits them. Hence, undervalued firms prefer not to deviate if the price effect dominates, implying lower gains to investors from the pooling offer than from the deviating offer.

The condition for the price effect to prevail is determined by the highest firm type $\overline{b}$ since it suffers most from selling underpriced shares. The left hand side of condition (18) is the investors' equilibrium payoff when subscribing to firm $\overline{b}$. The right hand side is their payoff when firm $\overline{b}$ chooses full dilution protection $\hat{\lambda} = 1$. This is the best deviating public offer since it lets the firm sell as few underpriced shares as possible to investors. Depending on parameters, notably the support of firm types $\left[\underline{b}, \overline{b}\right]$, the condition may not hold for offerings with little dilution protection. In this case, current shareholders in firm $\overline{b}$ prefer to sell fewer shares to new investors at the more deflated price $\hat{P}_{PO}$.

The advantage of selling fewer shares becomes increasingly smaller when the dilution protection λ of the pooling offer increases. In the limit when λ approaches 1, investors buy the same number of shares in the pooling and deviating offer. Once there is only the price effect firms strictly prefer the pooling offer. By continuity, all firm types choose pooling offers with sufficiently good dilution

protection.

Such pooling offers must also dominate deviations to any rights offering $\hat{P}_S$. Given the investors' beliefs, a weakly positive rights price $\hat{P}_R$ must imply a strike price $\hat{P}_S \leq a + \underline{b}$. Hence, buying and exercising the rights would be (weakly) profitable. Therefore, deviating to the rights offering cannot be attractive for low firm types $b \in [\underline{b}, b^*_{PO}(\lambda))$. They prefer to sell overpriced shares to new investors. High firm types $b \in (b^*_{PO}(\lambda), \overline{b}]$ would not want to switch to a rights offering with a zero strike price ($\hat{P}_S = a + \underline{b}$) since it is equivalent to a public offering with full dilution protection (Proposition 7). Furthermore, rights offerings with positive rights prices (but lower strike prices) dilute the stakes of the cash-poor shareholders more at deflated prices. Cash-rich shareholders can maintain their fractional ownership and are therefore indifferent across different combinations of strike and rights prices. Hence, high firm types (in fact, all firms) have no incentive to deviate to a rights offering with a lower strike price ($\hat{P}_S < a + \underline{b}$).

Next we turn our attention to rights offering. For any P_S, denote by $(P_R(P_S), N_{RO}(P_S))$ the rights offering outcome given by Lemma 2 and Proposition 5.

Proposition 9 *There exist pooling equilibria in which all firms choose some common strike price P_S in a rights offering. An issue with a given P_S is such an equilibrium if and only if*

$$\pi N_{RO}(P_S) \left(\frac{a + I + \overline{b}}{N_{RO}(P_S) + 1} - P_S - P_R(P_S) \right) \leq \pi I \left(\frac{\overline{b} - \underline{b}}{a + I + \underline{b}} \right) \tag{19}$$

Moreover, the condition always holds for any $P_S \in [a + \underline{b}, a + b^*_{RO}]$.

Similar to (18) in the pooling public offering equilibrium, condition (19) rules out any deviation to any other public or rights offerings. Consider an initial rights offering with some strike price $P_S \in [a + \underline{b}, a + b^*_{RO}]$. A firm may deviate either to another rights offering or switch to a public offer. In either deviation the financing terms are set by the investors' belief that the firm is of type $\underline{b}$. Hence, if a firm were to choose another rights offering, the strike price would be (weakly) lower ($\hat{P}_S \leq a + \underline{b}$) and the number of new shares (weakly) larger ($I/\hat{P}_S \geq I/P_S$). Clearly, all low firm

types $b \in [\underline{b}, b^*_{RO})$ prefer the initial rights offering since they sell overpriced shares to investors and the stakes of their shareholders get less diluted. High firm types $b \in (b^*_{RO}, \overline{b}]$ sell underpriced shares whether they adhere to the initial rights offering or deviate. In either case cash-rich shareholders subscribe, thereby maintaining their fractional ownership. That is, their payoff is $a + b$ irrespective of the strike price, and they are indifferent. By contrast, cash-poor shareholders in high firm types prefer the initial rights offering as it dilutes their ownership stake (weakly) less. Moreover, any possible difference in rights price $(\hat{P}_R(\hat{P}_S) - P_R(P_S))$ never fully compensates them for being more diluted. The reason is that the sum of $\hat{P}_R$ and $\hat{P}_S$ are such that investors break even for the lowest firm type $\underline{b}$, whereas the sum of P_R and P_S is based on all firm types (taking into account the winner's curse problem).

As in Proposition 8, the highest-value firm type $\overline{b}$ suffers most from issuing underpriced rights/shares. The left hand side of equation (19) is the investors' payoff from purchasing and exercising the rights of firm $\overline{b}$ in the initial offering. If firm $\overline{b}$ were to deviate to another rights offering, it would set $\hat{P}_S = a + \underline{b}$ which in turn implies $\hat{P}_R = 0$, since it dilutes the ownership stakes of its cash-poor shareholders the least. The right hand side of condition (19) is the corresponding payoff to the investors. Such a deviation can only be attractive to firm type $\overline{b}$ if the strike price in the initial rights offering is lower and therefore were to dilute its cash-poor shareholders more. Hence, a pooling equilibrium in which all firms choose the same strike price P_S always exists for $P_S \in [a + \underline{b}, a + b^*_{RO}]$.

As argued in the discussion of Proposition 8, the best deviating public offers for high-value firms is full dilution protection ($\hat{\lambda} = 1$), again because it lets the firm sell as few underpriced shares as possible to investors. Since this public offer is equivalent to the rights offering with $\hat{P}_R = 0$ (Proposition 7), neither the highest-value firm $\overline{b}$ nor any other undervalued type $b \in (b^*_{RO}, \overline{b})$ would want to deviate.

4.2 Coexistence of Rights and Public Offers

While Lemma 3 rules out equilibria with multiple rights offerings, respectively multiple public offerings, the two offer modes may co-exist in equilibrium.

Proposition 10 *Any co-existence equilibrium is characterized by three cutoffs* $\underline{b}^\dagger < b^\dagger < \overline{b}^\dagger$. *Low-value firms* $b \in (\underline{b}, \underline{b}^\dagger)$ *and high-value firms* $b \in (\overline{b}^\dagger, \overline{b})$ *choose rights offering and intermediate types* $b \in (\underline{b}^\dagger, \overline{b}^\dagger)$ *choose public offerings. Furthermore, in all firm types* $b > b^\dagger$, *cash-rich shareholders participate.*

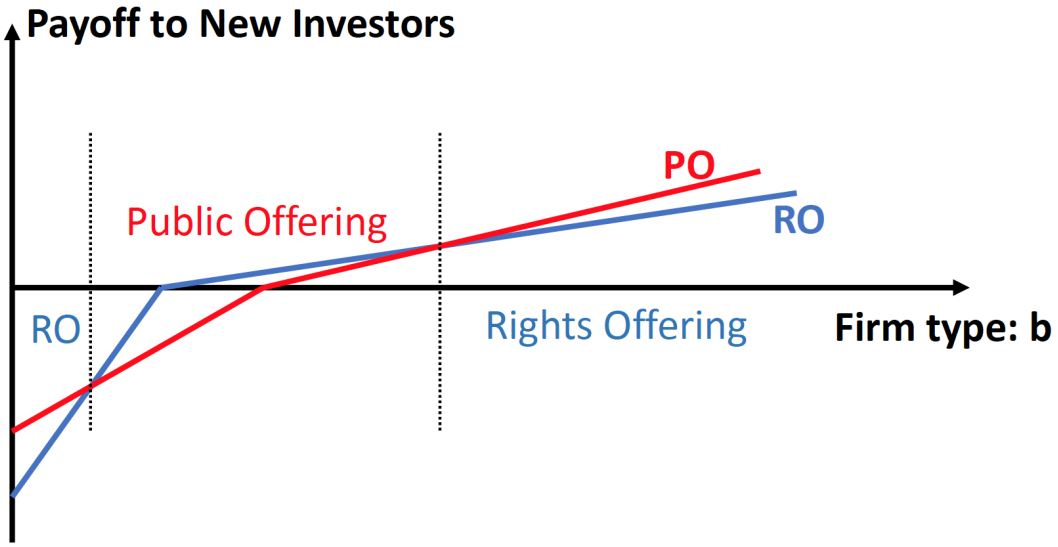

Figure 2: Payoff to new investors in a co-existence equilibrium

Figure 2 plots the investors' payoff as a function of the true firm type b, with the red curve representing the payoff from the public offering and the blue one the payoff from the rights offering. Investors buying shares from overvalued firms ($b < b^*_{PO}$ and $b < b^*_{RO}$) realize a loss, depicted by the parts of the curves below the horizontal axis. Shareholders of these firms do not purchase any new shares, leaving the entire issue to the investors. By contrast, cash-rich shareholders take-up their shares in undervalued offers ($b \geq b^*_{PO}$ or $b \geq b^*_{RO}$), and the investors can buy only a fraction of the new shares. The resulting positive payoffs are the parts of the curves above the horizontal

axis. Since the slope of the curves is equal to the investors' fractional ownership, the curves have a kink at zero.

Since firms maximize the total payoff to shareholders, or equivalently minimizing the payoff to investors, they choose the lower contour of the respective payoff curves in Figure 2. The key feature is that the payoff curve in the negative region ($b < b^*_{RO}$) is steeper for the rights offering than for the public offering. The reason is that the strike price P_S must be smaller than the public offering price P_{PO}, and hence more shares are being issued and sold to investors. As a result, the lowest quality firms choose the rights offering to sell more overvalued shares to investors. In a co-existence equilibrium, some high quality firms must also choose the rights offering. Since the rights offering provides full dilution protection to cash-rich shareholders, investor can buy fewer shares of undervalued firms in the rights offering. Consequently, the payoff curve from the public offer has a steeper slope above the horizontal axis. Being able to sell fewer shares to the investors is more valuable to the highest quality firms which therefore choose rights offerings.[15]

Such co-existence equilibria require that the issue price P_{PO} exceeds the strike price P_S. Intuitively, if P_{PO} would be smaller than P_S, all overvalued firm types whose cash-rich shareholders do not subscribe (to either offer) would prefer the rights offering. It would entail less dilution and (possibly) some revenues from the rights sale. This condition is merely necessary but not sufficient to ensure the existence of these equilibria. To establish existence, we have to resort to an numerical example: The equilibrium payoffs depicted in Figure 3 are based on an investment cost $I = 1$, assets in place $a = 0.2$, a fraction of cash-poor investor $\pi = 0.2$, four equally likely firm types $b \in \{1, 3, 6, 12\}$, dilution protection $\lambda = 0.4$, and a strike price $P_S = 1$. The equilibrium issue price in the public offering $P_{PO} = 4.41$, and the three cutoffs of Proposition 10 are $\underline{b}^\dagger = 2.02$, $b^\dagger = 4.21$, and $\overline{b}^\dagger = 9.61$.[16]

[15] For undervalued firms the benefit of the public offer is the higher price while its cost is the lesser extent to which it protects shareholders from dilution. As the dilution cost increases in the firm type, the higher types among all undervalued firms opt for the rights offering.

[16] This example with four discrete firm types can easily be expanded to one with a continuum of types without affecting the equilibrium outcome and the shape of Figure 3. In fact, any continuous distribution of b with equal (i.e., a $\frac{1}{4}$) probabilities on each of the intervals $[0.5, \underline{b}^\dagger]$, $[\underline{b}^\dagger, b^\dagger]$, $[b^\dagger, \overline{b}^\dagger]$, $[\overline{b}^\dagger, 13]$ and conditional means of 1, 3, 6, 12

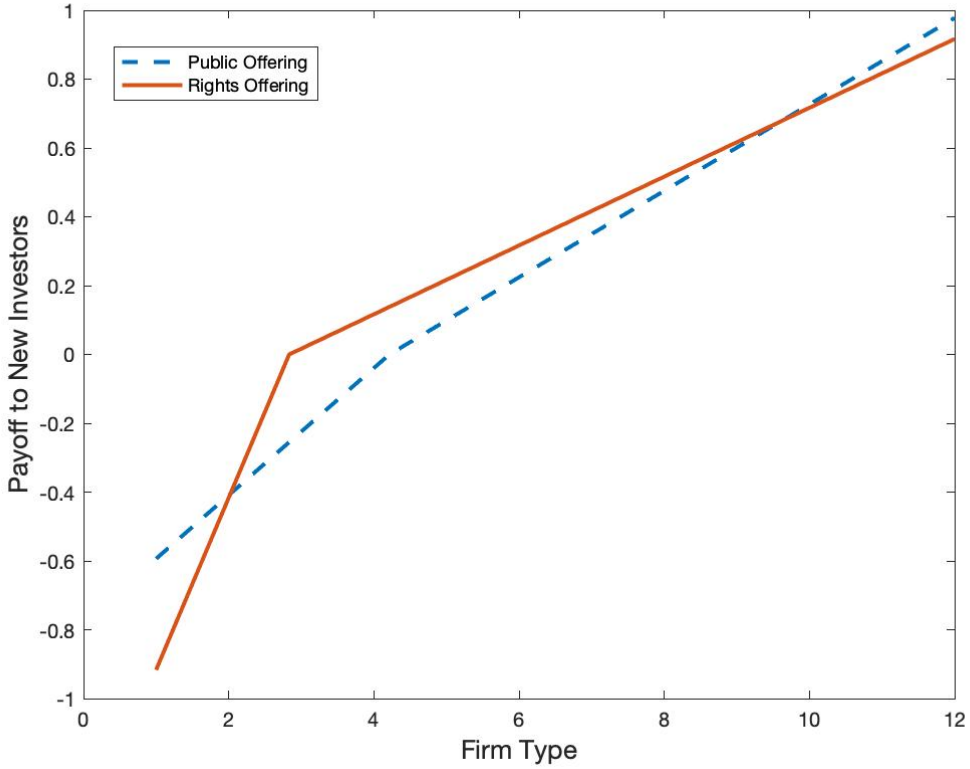

Figure 3: Numerical example for the co-existence equilibrium

For a sample of 85 US rights offerings and 85 matched public offerings Kothare (1997) documents that bid-ask spreads increase after rights issues but decrease after public offerings. Since bid-ask spreads also reflect the extent of information asymmetries, these changes are consistent with the co-existence equilibrium outcome: Firms opting for a right offering are more heterogeneous, and the associated larger adverse selection problem implies larger bid-ask spreads. Relatedly, Ursel (2006) reports that firms with more volatile stock prices tend to use rights offering. To the extent that volatility is also affected by (uncertainty about) the underlying firm values, this finding is in support of more diverse firms choosing rights offerings.

(i.e., the four firm types in the discrete example) respectively generates the same figure and equilibrium outcome.

5 Discounts, Underpricing, and Announcement Returns

In this section, we explore empirical implications of our model. Prior to choosing the flotation method, firm types are indistinguishable to the (uninformed) market participants and therefore trade at a common initial share price P_0. The offer discount in a public offering (rights offering) can be expressed as the difference between initial price and public offering price (strike price), normalized by the initial price:

$$\frac{P_0 - P_{PO}}{P_0} \qquad \text{and} \qquad \frac{P_0 - P_S}{P_0}.$$

We choose to be agnostic about the extent to which market participants anticipate the investment and the offering mode and hence do not pin down the price P_0 at which shares initially trade. Consequently, we cannot make predictions about the size or sign of the discount in either flotation mode but merely rank the discount across the two modes. As discussed in Subsection 3.3, the strike price P_S must be smaller than the issue price P_{PO} in a public offering when all firms choose the same issue mode. Otherwise the rights price would be negative. In any equilibrium where a public offering and a rights offering co-exist (Proposition 10), the strike price must also be smaller than the issue price. Hence, our model implies that discounts are larger in rights offerings than in public offerings.

Following the IPO literature, we define underpricing as the (one-day) return on shares purchased in an issue. That is, underpricing is the difference between the post-issue share price and the public offering price P_{PO} (the strike price P_S), normalized by the public offering price P_{PO} (the strike price P_S). Since all firm types invest, the expected firm value and hence the post-issue price is equal to $E\frac{(I+a+b)}{1+I/P_{PO}}$ following a public offer and equal to $E\frac{(I+a+b)}{1+I/P_S}$ following a rights offering. Formally, the underpricing in a public offering and a rights offering is

$$\frac{\frac{I+a+E(b|PO)}{1+I/P_{PO}} - P_{PO}}{P_{PO}} \qquad \text{and} \qquad \frac{\frac{I+a+E(b|RO)}{1+I/P_S} - P_S}{P_S} \tag{20}$$

which after some manipulation, can be expressed as

$$\frac{a + E\left(b|PO\right) - P_{PO}}{P_{PO} + I} \qquad \text{and} \qquad \frac{a + E\left(b|RO\right) - P_S}{P_S + I}$$

The comparison of underpricing in public and rights offerings is more involved.[17] When comparing underpricing across the pooling equilibria where all firms either opt for the same rights or for the same public offering, the conditional expectations $E\left(b|RO\right)$ and $E\left(b|PO\right)$ reduce to the unconditional mean $E\left(b\right)$. Hence, the comparison is solely driven by the strike and issue prices P_S and P_{PO}. Since, the strike price is (weakly) smaller than the issue price ($P_S \leq P_{PO}$), underpricing is more severe in rights offerings.

In the equilibrium in which a rights and a public offer coexist (Proposition 10), underpricing is determined by the prices P_{PO} and P_S as in the pooling equilibria and, in addition, by the conditional expectations $E\left(b|RO\right)$ and $E\left(b|PO\right)$ which are in general not identical. Unfortunately, we cannot analytically establish generic (qualitative) results. Though, numerical simulations suggest that firms which issue rights are the types with the higher average project NPV b. In the numerical example in Section 4.2, the average project NPV of firms issuing rights is $\frac{1+12}{2} = 6.5$, whereas the average is $\frac{3+6}{2} = 4.5$ for the firms using the public offering. Still, the price effect (P_{PO} vs P_S) clearly dominates the firm quality effect ($E\left(b|RO\right)$ vs $E\left(b|PO\right)$), and underpricing (given by (20)) is 2.85 in the rights offering and 0.0528 in the public offering. Trusting that this example (and our other simulations) are reasonably representative, we argue that there is less underpricing in public offerings.

Finally, the announcement return is the difference between post-issue share price and the initial share price, normalized by the initial price:

$$\frac{E\frac{(I+a+b)}{1+I/P_{PO}} - P_0}{P_0} \qquad \text{and} \qquad \frac{E\frac{(I+a+b)}{1+I/P_S} - P_0}{P_0}$$

Like underpricing, the announcement returns across the two pooling equilibria are purely driven by

[17] The underpricing in rights offering is defined for shareholders rather than investors as the formula does include the rights price P_R.

the price effect since both initial price and post-issue price are the same when all firms either opt for a public or a rights offer. Hence, announcement returns are lower following rights offering since the strike price is weakly smaller than the issue price ($P_S \leq P_{PO}$). In the co-existence equilibria, there are again the opposing price and firm quality effects at work. Our numerical simulations suggest that overall the price effect dominates the firm quality effect. Hence, we are inclined to argue that announcement returns are higher in public offerings.

Our predictions receive some support in the empirical literature. Armitage (2007) studies discounts in rights offers and open offers (similar to public offerings) in the UK. Consistent with our prediction he documents that rights are often issued at a discount of 15% to 20% relative to the market price, whereas open offers are usually discounted by less than 10%.[18] International evidence provides a similar picture. Asem et al. (2016) report that the average discount in US public offerings is around 3%. In contrast, rights offerings have an average discount of 17% in the UK and an average of 19% in Australia.

In terms of announcement return, Slovin et al. (2000) and Barnes and Walker (2006) find that in the UK, abnormal returns are significantly more negative for rights offerings (on average -3.1% announcement return) than for private placements (3.3%). For Hong Kong, Wu et al. (2005) report that rights issues have on average a significant 3-day announcement return of -8.0 percent, while public offers as well as private placements are associated with significantly positive announcement returns. In France where rights and public offerings are both common, Gajewski and Ginglinger (2002) report significant two-day average excess returns of -1.28% for standby rights issues, -2.84% for uninsured rights issues, and an insignificant negative return for public offerings. The proportion of public offerings increases from 4.84% over the 1986-1989 period to 16.84% over the 1990–1996 period.

[18] In the UK, a firm is not permitted to offer shares to the public without initially making an offer to existing qualifying shareholders (Barnes and Walker, 2006). In a rights issues, shareholders who do not wish to take-up their rights can sell them. In an open offer, the new shares are offered pro-rata to the existing shareholders, but the shareholders cannot sell their entitlements. Instead, the placees commit to take the remaining shares. To the extent that the current shareholders do not receive any compensation should they not participate in the issue, an open offer is similar to a public offering in our model.

While we are unaware of any empirical research directly comparing the underpricing in public offerings and rights offerings, we follow Altinkilic and Hansen (2003) and decompose underpricing into the discount and the announcement return.[19] Based on the above empirical evidence, the discount is approximately 10% larger in rights offerings and the announcement return is approximately 4% lower. Hence, we may conclude that rights offerings feature higher underpricing, broadly consistent with our theoretical predictions.

6 Investment Efficiency and Wealth Transfers

In the main model investors and shareholders are asymmetrically informed about the NPV of the project b, whereas the value of the asset in place a is common knowledge. Here we consider the reverse case when shareholders have private information about the assets in place a, while the NPV of the project b is common knowledge. The first observation is that, conditional on issuing shares, the pre-money total firm value $a + b$ is the crucial term incorporating the informational friction. Consequently, it is irrelevant whether the information asymmetry is about the NPV of the project or the value of the assets in place. In fact, the term $a + b$ jointly appears in all payoff expression throughout the analysis. Hence, one merely needs to replace all the project net payoffs b with the asset values a (and vice versa) in the lemmata and propositions, and all the results carry over to the case where the information asymmetry is about the assets in place instead of the project NPV.

The more interesting observation is that asymmetric information about assets in place - though not about project NPV - can lead some firms to abstain from investing as in Myers and Majluf (1984). When firms can choose issue mode as in Section 4 as well as whether to issue at all, the firm types with the most valuable assets in place may prefer to forgo the investment because issuing shares to investors may dilute the stake of the shareholders too much.[20] Our main insight is that

[19] For details see equations (1) and (2) in Altinkilica and Hansen (2003). Follow their notation, denote by p_{-1}, p_0, and p_1 the pre-issue share price, the offering price (i.e., P_S in a rights offering or P_{PO} in a public offering), and the post issue price. Then, underpricing $\equiv \log \frac{p_1}{p_0} = \log \frac{p_1}{p_{-1}} + \log \frac{p_{-1}}{p_0} \equiv$ announcement return + discount.

[20] Asymmetric information about both assets in place and project NPV does not add any qualitative new feature relative to asymmetric information only about the assets in place since the crucial term incorporating the informational

offer methods that lead to more wealth transfers among shareholders also result in more investment.

More specifically, assume that the investment cost I and the NPV of the project b are the same for all firms, while the value of the assets in place is now distributed on $[\underline{a}, \bar{a}]$ with $\underline{a} \geq 0$ according to the density function $f(a)$, respectively its distribution function $F(a)$. As before only the $(1-\pi)$ cash-rich shareholders are informed and know the realization of $a \in [\underline{a}, \bar{a}]$. As in Section 4, we assume that firms maximize the weighted sum of the payoffs to cash-poor and cash-rich shareholders, or equivalently, minimize the investors' payoff. Rather intuitively, the equilibrium outcomes combine features of Myers and Majluf (1984) and of Proposition 8, respectively Propositions 9 and 10. To avoid repetition we relegate the analysis and the formal statement of the results to the appendix and discuss here the equilibrium outcomes informally.

First, there exist semi-pooling equilibria in which all firm types above the cut-off type $\hat{a}_{PO} \in (\underline{a}, \bar{a})$ abstain from investing while all other types $a \in [\underline{a}, \hat{a}_{PO}]$ invest and choose a public offer with some common dilution protection λ. Similar to Proposition 8, cash-rich shareholders in firms which invest only subscribe if the assets in place are (at or) above the threshold value $a^*_{PO} \in (\underline{a}, \hat{a}_{PO})$. Thus, cash-rich shareholders in investing firms only subscribe if the new shares are under-priced $(a \in [a^*_{PO}, \hat{a}_{PO}])$, and firm types with the highest value assets in place $(a \in (\hat{a}_{PO}, \bar{a}])$ do not invest since the discount would exceed the project NPV and shareholders would be worse off.

Second, there exist semi-pooling equilibria in which all firm types above the cut-off type $\hat{a}_{RO} \in (\underline{a}, \bar{a})$ abstain from investing while all other types $a \in [\underline{a}, \hat{a}_{RO}]$ invest and choose a rights offer with some common strike price P_S. The underlying logic is the same as for the semi-pooling public offer equilibria: Firm types with the highest asset values $(a \in (\hat{a}_{RO}, \bar{a}])$ do not issue any rights since the equity stakes of the shareholders would be diluted by more than the project NPV. Among the investing firms $(a \in [\underline{a}, \hat{a}_{RO}])$ cash-rich shareholders exercise their rights only if the right issue is underpriced $(a \in [a^*_{RO}, \hat{a}_{RO}])$.

Finally, there may also exist an equilibrium in which a public offer and a rights offer co-exist

friction is the sum of $a + b$.

and in addition firm types with the highest value assets abstain from investing.

In the remainder of this section, we want to explore whether offer terms, that is, dilution protection λ and strike price P_S, which promote investment efficiency entail more redistribution among shareholders. In general, the investment cut-off types ($\hat{a}_{PO}$ and $\hat{a}_{RO}$) and the subscription cut-off types (a^*_{PO} and a^*_{RO}) do not only depend on the respective offer terms λ and P_S, but also on the distribution function $F(a)$ and its support. As a result, clear-cut comparative static results require to impose distributional assumptions. For simplicity, we assume that the assets in place are uniformly distributed on $[\underline{a}, \bar{a}]$.

As in Section 3 we analyse the redistribution from an ex ante point of view, that is, before the firm type a has realized. Since asymmetric information about the assets in place results in types with the highest assets in place abstaining from investing, the ex-ante wealth redistribution among shareholders is now the product of two terms, the probability of being a type which invests times the redistribution conditional on the firm investing. As for the conditional redistribution equation, (8) respectively (14) still apply except that expectations are taken with respect to assets in place rather than project NPVs. The probability of investing is simply the probability that a firm type is smaller or equal to the investment cut-off type $\hat{a}_{PO}$, respectively $\hat{a}_{RO}$. Thus, the ex-ante wealth transfer in public offers, $\hat{WT}_{PO}$, is equal to

$$\hat{WT}_{PO} \equiv Pr\left(a \leq \hat{a}_{PO}\right) \times \left\{ E\left(a | a \leq \hat{a}_{PO}\right) + b - \frac{1}{1 + \frac{I}{P_{PO}}} \left[I + E\left(a | a \leq \hat{a}_{PO}\right) + b \right] \right\}, \tag{21}$$

and in rights offers the ex-ante wealth transfer,$\hat{WT}_{RO}$, is equal to

$$\hat{WT}_{RO} \equiv Pr\left(a \leq \hat{a}_{RO}\right) \times \left\{ \frac{I}{I + P_S} \left[E(a | a \leq \hat{a}_{RO}) - a^*_{RO} \right] \right\}. \tag{22}$$

Proposition 11 *Given $a \sim U(\underline{a}, \bar{a})$, better dilution protection in public offers increases investment efficiency and wealth transfers among shareholders, and higher strike prices in rights offer increase investment efficiency and wealth transfers among shareholders.*

There is a trade-off between wealth transfer among shareholders and investment efficiency. Better dilution protection λ in public offers results in a higher investment cut-off $\hat{a}_{PO}$: More firm types invest when dilution protection is better since (cash-rich) shareholders can purchase more shares in underpriced issues which in turn increases the asset cut-off value $\hat{a}_{PO}$ at which the entire NPV accrues to the investors. As in the baseline case (Proposition 4), redistribution among shareholders, conditional on investing, increases with better dilution protection since it exacerbates the winner's curse problem. As a result, there are more wealth transfers among shareholders (because investors on average must break-even). Thus, both terms increase in dilution protection, and the ex-ante wealth transfers among shareholders also increase.

Higher strike prices P_S in rights offers lead to more investment (higher cut-off value $\hat{a}_{RO}$) because fewer new shares have to be issued to finance the investment I. That is, the equity stakes of the shareholders become less diluted, and the cut-off asset value $\hat{a}_{RO}$ at which investors extract the entire NPV of the investment increases. The impact of higher strike prices P_S on redistribution among shareholders, conditional on investing, is more intricate than in the baseline case (Proposition 6). On the one hand, higher strike prices lead to less dilution and less wealth transfers. On the other hand, the value of the average asset in place of the investing firms is higher when strike prices are higher. As a result, the winner's curse problem becomes larger, and there is more redistribution among shareholders. As it turns out, these two effects cancel each other in case of uniformly distributed assets in place. Thus, wealth transfers, conditional on investing, remain constant as the strike price increases, and the ex-ante wealth transfer is solely driven by more firms investing.

To conclude, the better dilution protection in public offers increase wealth transfers among shareholders in the main model with uncertainty about the project NPV and in the setting with asymmetric information about the assets in place when investing is not profitable for highest firm types ($a \in (\hat{a}_{PO}, \bar{a}]$). By contrast, higher strike prices in rights offer have opposite effects on wealth transfers in these two settings. Consequently, Corollary 1 does no longer hold with asymmetric information about the assets in place, even though the equivalence result (Proposition 7) remains

true.

7 Extensions and Discussion

In this section we discuss the robustness of our results with respect to the shareholder participation, in particular, allowing cash-poor or uninformed shareholders to participate in the offers as well as cash-rich shareholders to purchase more shares than those allocated to them on a pro-rata basis. Last, we compare public offerings to private placements.

7.1 Shareholder Participation in Rights Offerings

Our model assumes a functioning rights market where shareholders and investors trade without frictions other than the adverse selection problem.[21] Therefore, all non-exercised rights are in equilibrium sold to investors, and rights offerings do not face a subscription risk. This requires that rights are in fact tradable which holds true in most countries (Holderness and Pontiff, 2016). Furthermore, many countries offer protection to shareholders who do not to respond to a rights offering by either having brokers sell the rights on their behalf (e.g. Italy or Sweden) or by having an investment bank sell all non-exercised rights and credit the proceeds to the non-participating shareholders (e.g., Australia). In some countries – most notably – the US, firms can choose whether or not to make rights transferable.[22] In the study of Holderness and Pontiff (2016) about 50 percent of the firms in their US sample opt for transferable rights while in the international sample of Massa et al. (2016) more than 60 percent of the rights offering have tradable rights in countries which do not make transferability mandatory. Thus, shareholders do indeed either exercise or sell their rights (or have them sold on their behalf) in many countries as our model assumes. Still, rights do in practice at times lapse due to inattention, wealth constraints, or restricted transferability.

[21] In their international study Massa et al. (2016) find that rights are typically less liquid than the underlying shares and often undervalued. The latter feature is consistent with a winner's curse problem in the rights market.

[22] In the UK, Singapore, and Hong Kong offers without tradeable rights are called open offers and are separately regulated (Massa et al., 2016).

When valuable rights expire those shareholders who hold these rights lose out even more than the cash-poor shareholders in our analysis.[23]

7.2 Uninformed Current Shareholders

With the exception of the benchmark case in Subsection (2.2), all and only cash-rich shareholders are informed about the true quality of the project b , respectively the assets in place a in Section 6. Given asymmetric information and wealth constraints, the general setting would comprise four types of shareholders; cash-rich informed and uninformed ones and cash-poor informed and uninformed ones. Our (qualitative) results carry over to such a richer setting for two reasons. First, it is immaterial whether cash-poor shareholders are informed or not since they can – by assumption – not act strategically. Either they do nothing in public offerings or mechanically sell their rights. Hence, it is without loss of generality to assume that all cash-poor shareholders are uninformed. Second, uninformed cash-rich shareholders can never purchase more new shares than those allocated to them on a pro-rata basis, that is, $(1-\pi)\eta\lambda N_{PO}$ in a public offer and $(1-\pi)\eta N_{RO}$ in a rights offer. Consequently, they are not directly exposed to the winner's curse problem. Given that the equilibrium prices are set such that investors break even, uninformed cash-rich shareholders make – on average – a gain from participating and therefore always subscribe to new shares, respectively exercise their rights. In equilibrium, these shareholders therefore always take up the same fixed fraction of shares $(1-\pi)\eta\lambda$ in public offerings, respectively $(1-\pi)\eta$ in rights offerings. Hence, one can abstract from these and apply the analysis of the main model to the remaining shares $[1-(1-\pi)\eta\lambda]N_{PO}$, respectively $[1-(1-\pi)\eta]N_{RO}$, generating qualitatively the same results.

[23] In their study of 179 rights offerings in the US, Holderness and Pontiff (2016) find that shareholder non-participation leads to wealth transfers of around seven percent of the raised capital, and that these transfers are typically at the expense of small individual shareholders.

7.3 Margin Borrowing and Trading of Shares

Relaxing the assumption that cash-constrained shareholders can neither borrow nor trade their shares to participate in an offer seems an obvious extension. Here we discuss how the equilibrium outcomes may be affected in such an extended model settings. In a public offering, the strategy of selling existing shares to buy newly issued ones is futile given both shares are traded simultaneously at the same market clearing price. Still, when shareholders can trade their existing shares, investors are confronted with a winner's curse problem in both the primary and secondary market. While this exacerbates the extent of the adverse selection, it does not qualitatively change the nature of the winner's curse problem.

In rights offerings, cash-poor shareholders can sell part of their shares to have the funds to exercise the remaining rights. A complete analysis requires a fully specified trading environment, e.g. whether only rights or also shares can be traded separately. Regardless of the chosen model setup, the cash-poor shareholders always need to sell some shares or rights or both to participate in the rights offering, which again changes the extent but not the qualitative nature of the winner's curse problem.

Instead of selling some of their shares, cash-poor shareholders could borrow to participate in an offer. For example, they could borrow on their margin accounts and exercise undervalued rights. However, if the newly acquired shares are sold immediately to cover the margin loan, the payoff is the same as if the rights were sold instead.[24] Exercising the rights and getting the full information payoff (3) requires shareholders to hold the shares sufficiently long until the true firm value is realized. In practice, this may take a long time, making borrowing on margin accounts prohibitively expensive or even infeasible.

[24] A simple example illustrates this point. Suppose the market price after a rights offering is £10 and the strike price is £4. If a shareholder exercises her right and immediately sell the share at the market price, the payoff is £10 − £4, which is exactly the price the right commands in the market.

7.4 Buying More Shares

Key features of our model are that some shareholders strategically participate in the offer, but that the share, respectively the rights, price is determined by uninformed competitive investors. For simplicity, we assume a fraction of informed cash-rich shareholders who can buy at most those newly issued shares or rights that are allocated to them on a pro-rata basis. Since the cash-poor shareholders cannot purchase any new shares, competitive investors always buy some - or all - shares (rights) in equilibrium, and their break-even constraint determines the price. We are confident that our qualitative results carry over to more general settings which relaxes these assumptions as e.g., introducing cash-rich uninformed shareholders (Section 7.2). Similarly, we could allow for the possibility that cash-rich informed shareholders can buy more rights or shares than those allocated to them on a pro-rata basis, $(1-\pi)$ respectively $\lambda(1-\pi)$.[25] This would reduce the shares or rights that investors can purchase when the issue is underpriced, and therefore exacerbate the winner's curse problem. Since investors must break even on average, the cash-poor shareholders would ultimately bear the cost of the aggravated winner's curse problem. That is, there would be more redistribution among shareholders but the results would not change qualitatively. Obviously, this does not hold if the cash-rich shareholders could purchase all shares or rights since it would fully resolve the adverse selection problem. Hence, our results crucially hinge on the assumption that informed capital is scarce and informed shareholders neither have sufficient available wealth to absorb the entire offering nor can they borrow sufficient amounts to do so. The latter can be motivated by limits of arbitrage arguments (e.g., Barberis and Thaler, 2003). As such limits are orthogonal to our analysis we believe that a limits-to-arbitrage based micro foundation of our purchase limits $(1-\pi)$ respectively $\lambda(1-\pi)$ would neither affect our main results nor generate substantive new insights.

[25] Formally, one merely needs to re-interpret $(1-\pi)$, respectively $\lambda(1-\pi)$, as the proportion of shares that the informed cash-rich shareholders can at most purchase.

7.5 Private Placement

In a private placement, the issuing firm negotiates a share sale to a small group of qualified investors who may be shareholders or investors. Since most - if not all - shareholders do not qualify, private placements can be viewed as being similar to public offerings with zero dilution protection in our model. The key difference is the pricing mechanism. The public offering price P_{PO} is the market clearing price set by competitive investors, whereas the issue price in a private placement is the outcome of the bargaining between firm and qualified investors. In practice, private placements are sold at a discount relative to the current share price (e.g., Eckbo et al., 2007). The discount may be a reflection of the qualified investors' strong bargaining position or it may be compensation for costs of investigating the firm or for valuable monitoring. In either case, there is a wealth transfer between shareholders and qualified investors. Shareholders are treated equally and typically have no available action to take. In this sense, they are similar to the cash-poor shareholders in our model.

8 Conclusion

We analyze seasoned equity offerings where some shareholders are informed and can strategically choose to participate. When all shareholders have wealth to participate in the issue, right offerings achieve the full information outcome and therefore dominate public offerings which necessarily generate wealth transfers. We show that this ranking may be reversed when some of the existing shareholders are wealth-constrained. In rights offerings, investors must purchase the rights to buy the underlying shares, rather than only buying these shares as in a public offering. Hence, a positive right price implies a discount in the strike price relative to the public offering price. Therefore, cash-poor shareholders become more diluted in a rights offering, and lower strike prices increase the wealth transfer from them to informed cash-rich shareholders. More generally, cash-poor and cash-rich shareholders have diverging preference over flotation methods and terms. Moreover, there

is a trade-off between investment efficiency and wealth transfers among shareholders in both rights and public offerings.

When firms choose the flotation mode and terms to maximize the total payoff to all shareholders, there are only two kinds of equilibria. On the one hand, there exist pooling equilibria in which all firms choose the same public offering, or alternatively all firms choose the same rights offering. On the other hand, there exist equilibria with a single rights and a single public offering. In such an equilibrium, high and low quality firms opt for the rights offering, while intermediate firm types choose the public offer. Low quality firms prefer a rights offering to sell a larger fraction of their overvalued firms. High quality firms favor a rights offering because it allows cash-rich shareholders to maintain their fractional ownership, thereby selling fewer undervalued shares to investors.

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A Appendix

A.1 Proofs to Sections 2 to 4

Proof of Proposition 1: In any equilibrium, the informed shareholders can secure a net payoff of at least $a + b$ by exercising the rights. Similarly, the uninformed shareholders must receive at least a net payoff $a + E(b)$. New investors must on average at least break even. Because the total firm value net of investment I is $a+b$, the above payoffs are exactly the equilibrium payoffs for investors, informed and uninformed shareholders.

Next, we show that uninformed shareholders receive exactly $a+b$ net of investment in equilibrium as well. Suppose otherwise, then some uninformed shareholders can earn a net payoff strictly larger than $a + b$. The informed shareholders in the same firm would deviate to this strategy to earn a strictly larger payoff, a contradiction. Thus, all shareholders receive exactly $a + b$, which can be implemented by exercising the rights.

We now prove that no equilibrium exists for strike prices $P_S > a + \underline{b}$. Consider the informed shareholders of a firm type $b \in [\underline{b}, P_S - a)$, which implies $P_S > a + b$. If they choose not to exercise, their payoff is

$$\frac{I + a + b}{1 + \frac{I}{P_S}} > \frac{I + a + b}{1 + \frac{I}{a+b}} = a + b.$$

Thus, these informed shareholders' equilibrium payoff must be strictly higher than $a + b$. A contradiction and therefore, no such equilibrium exists.

To complete the proof, we show that when the strike price $P_S \leq a + \underline{b}$, all shareholders have an incentive to exercise their rights. The equilibrium is supported by the investors' belief that any sold rights come from the worst firm type $\underline{b}$. The price of the rights is therefore

$$P_R = \frac{I + a + \underline{b}}{1 + \frac{I}{P_S}} - P_S.$$

The payoff to shareholders of a type b firm should they choose to sell the rights is

$$\frac{I+a+b}{1+\frac{I}{P_S}}+P_R N_{RO}.$$

Since $P_R \leq \frac{I+a+b}{1+\frac{I}{P_S}} - P_S$ for any $b \geq \underline{b}$, the above payoff is bounded by

$$\frac{I+a+b}{1+\frac{I}{P_S}}+\left(\frac{I+a+b}{1+\frac{I}{P_S}}-P_S\right)\frac{I}{P_S}=a+b,$$

which can be achieved by exercising the rights. Therefore, all shareholders have an incentive to exercise their rights. ■

Proof of Proposition 2: Given $\lambda < 1$, investors must purchase some shares in equilibrium. Since their purchase decision cannot depend on b, their break-even condition implies a unique P_{PO}. The per-share payoff to investors is

$$\frac{I+a+b}{1+N_{PO}}-P_{PO},$$

which is linear in b and has a unique root at $b = P_{PO} - a$. Hence, the investors´ payoff is non-zero for any firm type $b \neq P_{PO} - a$, implying wealth transfers.■

Proof of Lemma 1: Cash-rich shareholders subscribe if and only if

$$(\lambda N_{PO}+1)(I+a+b)-\lambda I(N_{PO}+1) \geq (I+a+b),$$

which is equivalent to

$$\lambda N_{PO}(I+a+b) \geq \lambda I(N_{PO}+1),$$

which is in turn is equivalent to

$$N_{PO}(a+b) \geq I.$$

Together with the fact that $P_{PO}N_{PO} = I$, we have condition (5). ■

Proof of Proposition 3: When $P_{PO} = a + \underline{b}$, condition (5) always holds, and cash-rich shareholders subscribe. In this case, $Pr(b \geq b^*_{PO}) = 1$ and $E(b|b \geq b^*_{PO}) = E(b)$. Therefore, investor payoff (7) becomes

$$[1 - (1 - \pi)\lambda]\,[a + E(b) - P_{PO}]\,,$$

which is strictly positive.

For any $P_{PO} \geq a + E(b)$, it follows by definition that $E(b|b \geq P_{PO} - a) \geq P_{PO} - a$, with strict inequality for $P_{PO} < a + \overline{b}$. Investor payoff (7) for such prices is strictly negative. By continuity, investor payoff (7) as a function of P_{PO} has a root and all roots lie in $(a + \underline{b}, a + E(b))$.

Finally, we show that P_{PO} is decreasing in λ. Suppose $\lambda_1 < \lambda_2$. Denote by $P_{PO,i}$ the corresponding solution to (7) for λ_i $(i = 1, 2)$. Also denote by $Pr_i(b|b \geq b^*_{PO,i})$ and $E_i(b|b \geq b^*_{PO,i})$ the corresponding values for λ_i, where $b^*_{PO,i}$ is given by (5). Since $P_{PO,i} < a + E(b) < a + E_i(b|b \geq b^*_{PO,i})$, we have

$$\begin{aligned} & a + E(b) - P_{PO,1} - Pr_1\left(b|b \geq b^*_{PO,1}\right)(1 - \pi)\lambda_2\left[a + E[b|b \geq b^*_{PO,1}] - P_{PO,1}\right]. \\ < \;& a + E(b) - P_{PO,1} - Pr_1\left(b|b \geq b^*_{PO,1}\right)(1 - \pi)\lambda_1\left[a + E[b|b \geq b^*_{PO,1}] - P_{PO,1}\right] \\ = \;& 0. \end{aligned}$$

Hence, for $\lambda = \lambda_2$, (7) is negative when $P_{PO} = P_{PO,1}$. Since (7) is positive for $P_{PO} = a + \underline{b}$, there must exist a $P_{PO,2} \in (a + \underline{b}, P_{PO,1})$, completing the proof. ■

Proof of Proposition 4: From (8), we have

$$\begin{aligned} WT_{PO} &= [a + E(b)]\left(1 - \frac{P_{PO}}{I + P_{PO}}\right) - \frac{P_{PO}}{I + P_{PO}} I \\ &= [a + E(b)]\frac{I}{I + P_{PO}} - \frac{I}{I + P_{PO}} P_{PO} \\ &= [a + E(b) - P_{PO}]\frac{I}{I + P_{PO}}, \end{aligned}$$

which establishes (9). Since $P_{PO} \leq a + E\,(b)$ from Proposition 3, the wealth transfer $WT_{PO} \geq 0$, which clearly is decreasing in P_{PO}. Furthermore, the issue price P_{PO} is decreasing in λ(Proposition 3). Hence, WT_{PO} is increasing in λ.■

Proof of Lemma 2: Cash-rich shareholders exercise their rights if

$$a + b \geq \frac{I + a + b}{N_{RO} + 1} + P_R N_{RO},$$

which implies

$$N_{RO}(a + b) \geq I + P_R N_{RO}(N_{RO} + 1),$$

which in turn implies

$$a + b \geq \frac{I}{N_{RO}} + P_R(N_{RO} + 1).$$

Using the fact that $\frac{I}{N_{RO}} = P_S$, condition (10) follows immediately. ■

Proof of Proposition 5: Rewriting (10) as

$$P_R = \frac{1}{N_{RO} + 1}\left[I + \frac{(1 - \pi)P(b < b^*_{RO})E(b|b < b^*_{RO}) + \pi E(b)}{(1 - \pi)P(b < b^*_{RO}) + \pi} + a\right] - P_S$$

and using to substitute P_R in (12), we have

$$\begin{aligned} b^*_{RO} &= P_S + \left[\tfrac{1}{(N_{RO}+1)}\left(I + \tfrac{(1-\pi)P(b<b^*_{RO})E(b|b<b^*_{RO})+\pi E(b)}{(1-\pi)P(b<b^*_{RO})+\pi} + a\right) - P_S\right](N_{RO}+1) - a \\ &= P_S + \left(I + \tfrac{(1-\pi)P(b<b^*_{RO})E(b|b<b^*_{RO})+\pi E(b)}{(1-\pi)P(b<b^*_{RO})+\pi} + a\right) - P_S(N_{RO}+1) - a \\ &= P_S + I + \tfrac{(1-\pi)P(b<b^*_{RO})E(b|b<b^*_{RO})+\pi E(b)}{(1-\pi)P(b<b^*_{RO})+\pi} + a - I - P_S - a \\ &= \tfrac{(1-\pi)P(b<b^*_{RO})E(b|b<b^*_{RO})+\pi E(b)}{(1-\pi)P(b<b^*_{RO})+\pi}, \end{aligned}$$

which is the expression in the statement.

Next, we show that $b^*_{RO} \in (\underline{b}, E(b))$. At $b^*_{RO} = \underline{b}$, the right hand side of (13) is $E(b) > \underline{b}$. Since $E(b|b < b^*_{RO}) < E(b)$ whenever $b^*_{RO} < \bar{b}$, the right hand side of (13) is in turn dominated by $E(b)$. Therefore, b^*_{RO} must exist and lie in $(\underline{b}, E(b))$.■

Proof of Proposition 6: The payoff to cash-poor shareholders and to shareholders in firms of type $b < b^*_{RO}$ is

$$\frac{I+a+b}{N_R+1} + P_R N_{RO}$$

Using (10) and the definition of b^*_{RO} (Proposition 5) this payoff can be rewritten as

$$\begin{aligned} & \tfrac{I+a+b}{N_{RO}+1} + \tfrac{N_{RO}}{N_{RO}+1}(I + a + b^*_{RO}) - N_{RO}P_S \\ = \;& \tfrac{a+b+N_{RO}b^*_{RO}+N_{RO}a}{N_{RO}+1} \\ = \;& a + b + \tfrac{N_{RO}}{N_{RO}+1}(b^*_{RO} - b) \\ = \;& a + b + \tfrac{I}{I+P_S}(b^*_{RO} - b) \end{aligned}$$

Since investors break even, the ex ante wealth transfer among shareholders is therefore

$$a + E(b) - \left\{a + E(b) + E\left[\frac{I}{I+P_S}(b^*_{RO} - b)\right]\right\} = \frac{I}{I+P_S}\left[E(b) - b^*_{RO}\right].$$

■

Proof of Proposition 7: For $\lambda = 1$ equation (7) and $b^*_{PO} = P_{PO} - a$ imply the following condition for b^*_{PO}:

$$E(b) - b^*_{PO} - P(b \geq b^*_{PO})(1-\pi)\left[E(b|b \geq b^*_{PO}) - b^*_{PO}\right] = 0.$$

Solving for b^*_{PO}, we have

$$b^*_{PO} = \frac{E(b) - (1-\pi)P(b \geq b^*_{PO})E(b|b \geq b^*_{PO})}{1 - P(b \geq b^*_{PO})(1-\pi)}. \tag{23}$$

Together with the fact that

$$E(b) = P(b < b^*_{PO})E(b|b < b^*_{PO}) + P(b \geq b^*_{PO})E(b|b \geq b^*_{PO}),$$

condition (23) is equivalent to the condition for b^*_{RO} in a rights offering (13). Therefore, $b^*_{PO} = b^*_{RO}$ which implies that $P_{PO} = P_S$ and $N_{PO} = N_{RO}$ as well. In both types of offerings, existing shareholders receive $a+b$ if they participate (subscribe or exercise the rights) and the same payoff $\frac{I+a+b}{N_{RO}+1}$ if they do not participate. Overall, everyone's payoff in a public offering is exactly the same as in a rights offering. ■

Proof of Proposition 8: Consider a public offering equilibrium with issue price $P_{PO}(\lambda)$ and number of new shares $N_{PO}(\lambda)$, as characterized in Proposition 3. Suppose further that any firm which deviates is perceived by the investors as being of type $\underline{b}$. Hence, if a firm deviates to another public offering with $\hat{\lambda} \neq \lambda$, it has to sell its shares at $\hat{P}_{PO} = a + \underline{b}$ with $\hat{N}_{PO} = \frac{I}{\hat{P}_{PO}}$. The payoff (for investors and cash-rich shareholders) from buying these shares is

$$\frac{I + a + b}{\hat{N}_{PO} + 1} - \hat{P}_{PO} = \frac{b - \underline{b}}{\hat{N}_{PO} + 1} \geq 0.$$

As a result, no firm of type $b \leq b^*_{PO}$ wants to deviate to $\hat{\lambda}$ because $\Pi_{PO}(b|\lambda) \leq 0$. Firms of type

$b > b^*_{PO}$ do not deviate either if

$$[1-\lambda(1-\pi)]\, N_{PO}\left(\frac{I+a+b}{N_{PO}+1}-P_{PO}\right) \leq \left[1-\hat{\lambda}(1-\pi)\right]\hat{N}_{PO}\left(\frac{I+a+b}{\hat{N}_{PO}+1}-\hat{P}_{PO}\right) \tag{24}$$

holds. Since both sides are linear in b, it is sufficient to show that the inequality is satisfied at the endpoints b^*_{PO} and $\overline{b}$. By definition of b^*_{PO} (Lemma 2), the left hand side equals 0 for $b = b^*_{PO}$, while the right hand side is

$$\left[1-\hat{\lambda}(1-\pi)\right]\hat{N}_{PO}\left(\frac{b^*_{PO}-\underline{b}}{\hat{N}_{PO}+1}\right) > 0$$

For $b = \overline{b}$, the left-hand side of (24) is the left-hand side of (18). The right-hand side of (24) can be rewritten as

$$\left[1-\hat{\lambda}(1-\pi)\right]\left(\frac{I+a+\overline{b}}{1+\hat{P}_{PO}/I}-I\right) = \left[1-\hat{\lambda}(1-\pi)\right] I\left(\frac{\overline{b}-\underline{b}}{I+a+\underline{b}}\right)$$

which is (weakly) larger than the right-hand side of (18) since $\hat{\lambda} \leq 1$. Hence, condition (18) implies that (24) holds for $b = \overline{b}$.

If a firm deviates to a rights offering with $\hat{P}_S$, the associated rights price $\hat{P}_R$ is given by

$$\hat{P}_R = \frac{I+a+\underline{b}}{\hat{N}_{RO}+1} - \hat{P}_S. \tag{25}$$

For $\hat{P}_R \geq 0$, it must be that $\hat{P}_S \leq a + \underline{b}$. Condition (25) and Lemma 2 imply that

$$b \geq \hat{P}_S + \hat{P}_R(\hat{N}_{RO}+1) - a = \underline{b}.$$

Hence, the payoff from exercising (and buying) rights is (weakly) positive. Consequently, no firm of type $b \leq b^*_{PO}$ wants to deviate to a rights offering since $\Pi_{PO}(b|\lambda) \leq 0$. Firms of type $b > b^*_{PO}$ do

not deviate either if

$$[1-\lambda(1-\pi)]\, N_{PO}\left(\frac{I+a+b}{N_{PO}+1}-P_{PO}\right) \leq \pi \hat{N}_{RO}\left(\frac{I+a+b}{\hat{N}_{RO}+1}-\hat{P}_S-\hat{P}_R\right). \tag{26}$$

As above, both sides are linear in a, and it is sufficient to show that the inequality is satisfied at the endpoints. For $b = b^*_{PO}$, the left hand side equals 0, while the right hand side is

$$\pi \hat{N}_{PO}\left(\frac{b^*_{PO}-\underline{b}}{\hat{N}_{PO}+1}\right) > 0.$$

For $b = \overline{b}$, the right-hand side of (26) is equal to

$$\pi \hat{N}_{RO}\left(\frac{\overline{b}-\underline{b}}{\hat{N}_{PO}+1}\right) = \pi I\left(\frac{\overline{b}-\underline{b}}{I+\hat{P}_S}\right).$$

Since $\hat{P}_S \leq a+\underline{b}$, it must be that

$$\pi I\left(\frac{\overline{b}-\underline{b}}{I+\hat{P}_S}\right) \geq \pi I\left(\frac{\overline{b}-\underline{b}}{I+a+\underline{b}}\right)$$

holds. Hence, condition (18) implies that (26) holds for $a = \overline{a}$.

Finally, as $\lambda \to 1$, the left-hand side of condition (18) becomes

$$\pi N_{PO}(1)\left(\frac{I+a+\overline{b}}{N_{PO}(1)+1}-P_{PO}(1)\right) = \pi\left(\frac{I+a+\overline{b}}{1+\frac{P_{PO}(1)}{I}}-I\right).$$

Since $P_{PO}(1) > a+\underline{b}$ (Proposition 3),

$$\pi\left(\frac{I+a+\overline{b}}{1+\frac{P_{PO}(1)}{I}}-I\right) < \pi\left(\frac{I+a+\overline{b}}{1+\frac{a+\overline{b}}{I}}-I\right) = \pi I\left(\frac{\overline{b}-\underline{b}}{I+a+\underline{b}}\right).$$

Thus, condition (18) is satisfied in the limit and by continuity also holds when λ is sufficiently close to 1. ■

Proof of Proposition 9: Consider a rights offering equilibrium with strike price P_S, number of new shares N_{RO}, and associated rights price P_R, as characterized in Proposition 5. Furthermore, any firm which deviates from this equilibrium rights offering is perceived by the investors as being of type $\underline{b}$. Parallel to the proof of Proposition 8, it suffices to establishes that

$$\pi N_{RO}(P_S)\left(\frac{I+a+\overline{b}}{N_{RO}(P_S)+1}-P_S-P_R(P_S)\right)\leq \pi\hat{N}_{RO}\left(\frac{I+a+b}{\hat{N}_{RO}+1}-\hat{P}_S-\hat{P}_R\right) \tag{27}$$

for any deviating rights offering with $\hat{P}_S$ and that

$$\pi N_{RO}(P_S)\left(\frac{I+a+\overline{b}}{N_{RO}(P_S)+1}-P_S-P_R(P_S)\right)\leq \left[1-\hat{\lambda}(1-\pi)\right]\hat{N}_{PO}\left(\frac{I+a+b}{\hat{N}_{PO}+1}-\hat{P}_{PO}\right) \tag{28}$$

for any deviating public offering with $\hat{\lambda}$. The right-hand sides of (27) and (28) are the same as those in (26) and (24), which are both weakly positive. Since by Lemma 2

$$\pi N_{RO}(P_S)\left(\frac{I+a+\overline{b}}{N_{RO}(P_S)+1}-P_S-P_R(P_S)\right)\leq 0,$$

for any firm type $b\leq b^*_{RO}$, conditions (27) and (28) hold for these types. As for the firm types $b>b^*_{RO}$, it suffices to establishes that (27) and (28) hold for $b=\overline{b}$ due to the linearity in b. For $b=\overline{b}$ the left-hand sides of (27) and (28) are the left-hand side of (19). As shown in the proof of Proposition 8, the right-hand sides of (27) and (28) are weekly larger than $\pi I\left(\frac{\overline{b}-\underline{b}}{I+a+\underline{b}}\right)$, the right-hand side of (19). Hence, condition (19) implies that (27) and (28) hold for $b=\overline{b}$.

Given $P_R(P_S)\geq 0$, the left-hand side of (19) is bounded by

$$\pi N_{RO}(P_S)\left(\frac{I+a+\overline{b}}{N_{RO}(P_S)+1}-P_S\right)=\pi\left(\frac{I+a+\overline{b}}{1+\frac{P_S}{I}}-I\right).$$

Since $P_S \geq a + \underline{b}$,

$$\pi \left(\frac{I + a + \overline{b}}{1 + \frac{P_S}{I}} - I \right) \leq \pi \left(\frac{I + a + \overline{b}}{1 + \frac{a + \underline{b}}{I}} - I \right) = \pi I \left(\frac{\overline{a} - \underline{a}}{I + a + \underline{b}} \right).$$

Thus, condition (19) holds for any $P_S \in [a + \underline{b}, a + b^*_{RO}]$. ■

Proof of Lemma 3: Suppose there were two public offerings with corresponding $\lambda_2 > \lambda_1$ both adopted by some firms. Denote by $P_{PO,i}$ and $b^*_{PO,i}$ $(i = 1, 2)$ the corresponding issue price and cutoff type in each offering. Lemma 1 and Proposition 3 imply that some firms in each offering must weakly lie below the respective cutoff type $b^*_{PO,i}$. Let $b_i \leq b^*_{PO,i}$ $(i = 1, 2)$ be two such firms. For these firms all N_i new shares are issued to the investors. From (16), each firm's optimal choice of issue terms implies

$$N_i \left[\frac{I + a + b_i}{N_i + 1} - P_{PO,i} \right] \leq N_{-i} \left[\frac{I + a + b_i}{N_{-i} + 1} - P_{PO,-i} \right].$$

Since $N_i P_{PO,i} = I$, we have

$$N_i \frac{I + a + b_i}{N_i + 1} \leq N_{-i} \frac{I + a + b_i}{N_{-i} + 1}.$$

Hence, $\frac{N_i}{N_i + 1} \leq \frac{N_{-i}}{N_{-i} + 1}$ for $i = 1, 2$, which implies $N_1 = N_2$ and as a result, $P_{PO,1} = P_{PO,2}$ and $b^*_{PO,1} = b^*_{PO,2}$. Finally, consider a different pair of firms in each issue mode, with their firm types above the respective cutoff types: $b_i > b^*_{PO,i}$. Using the fact that $b^*_{PO,i} = P_{PO,i} - a$, we have

$$\frac{I + a + b_i}{N_i + 1} > \frac{I + a + b^*_{PO,i}}{\frac{I}{P_{PO,i}} + 1} = \frac{I + P_{PO,i}}{\frac{I}{P_{PO,i}} + 1} = P_{PO,i}.$$

In equilibrium, cash-rich shareholders subscribe, and new investors receive $[1 - \lambda_i(1 - \pi)] N_i$ new shares. The optimal choice of issue terms implies

$$[1 - \lambda_i(1 - \pi)] N_i \left[\frac{I + a + b_i}{N_i + 1} - P_{PO,i} \right] \leq [1 - \lambda_{-i}(1 - \pi)] N_{-i} \left[\frac{I + a + b_i}{N_{-i} + 1} - P_{PO,-i} \right].$$

Using the fact that N_i and $P_{PO,i}$ are the same across $i = 1, 2$, the above expression simplifies to

$$1 - \lambda_i(1 - \pi) \leq 1 - \lambda_{-i}(1 - \pi),$$

for $i = 1, 2$. Hence, $\lambda_1 = \lambda_2$. There is at most one public offering in equilibrium.

Suppose there were two rights offerings with strike prices $P_{S,2} > P_{S,1}$, both adopted by some firms. Denote the corresponding rights price by $P_{R,i}$ $(i = 1, 2)$. By Proposition 5 some cash-rich shareholders must choose to (or not to) exercise the rights in each offer. Denote by $b_{i,e}$ $(b_{i,ne})$ $i = 1, 2$ the type of firms that issue rights with strike price $P_{S,i}$, and the cash-rich shareholders exercise (do not exercise) their rights. The optimal choice of offer terms states

$$\pi N_i \left[\frac{I + a + b_{i,e}}{N_i + 1} - P_{S,i} - P_{R,i} \right] \leq \pi N_{-i} \left[\frac{I + a + b_{i,e}}{N_{-i} + 1} - P_{S,-i} - P_{R,-i} \right],$$

which, combined with the fact that $P_{S,i} N_i = I$, implies

$$N_i \left[\frac{I + a + b_{i,e}}{N_i + 1} - P_{R,i} \right] \leq N_{-i} \left[\frac{I + a + b_{i,e}}{N_{-i} + 1} - P_{R,-i} \right].$$

Since $N_1 = \frac{I}{P_{S,1}} > \frac{I}{P_{S,2}} = N_2$, the above condition implies

$$b_{1,e} \leq b^{cf} \equiv \frac{P_{R,1} N_1 - P_{R,2} N_2}{\frac{N_1}{N_1+1} - \frac{N_2}{N_2+1}} - (I + a),$$

and

$$b_{2,e} \geq b^{cf} \geq b_{1,e}.$$

On the other hand, the optimal offer term choice for the $b_{i,ne}$ firm implies

$$N_i \left[\frac{I + a + b_{i,ne}}{N_i + 1} - P_{R,i} \right] \leq N_{-i} \left[\frac{I + a + b_{i,ne}}{N_{-i} + 1} - P_{R,-i} \right],$$

which following the same logic, implies $b_{1,ne} \leq b_{2,ne}$, $b_{1,ne} \leq b^{cf}$, and $b_{2,ne} \geq b^{cf}$. Hence, combined

with Lemma 2, we have

$$b_{1,ne} < b_{1,e} \leq b^{cf} \leq b_{2,ne} < b_{2,e}.$$

However, this relation cannot hold in equilibrium, because $a_{1,e}$ firm has an incentive to deviate to the rights offering with strike price $P_{S,2}$. With $P_{S,1}$, investors in the $b_{1,e}$ firm collectively receive

$$\pi N_1 \left[\frac{I + a + b_{1,e}}{N_i + 1} - P_{S,1} - P_{R,1} \right],$$

which is weakly positive because of Lemma 2 and the fact that the cash-rich shareholders exercise their rights, i.e. $b_{1,e} \geq b^*_{RO,1}$. On the other hand, with $P_{S,2}$, investors in the $a_{1,e}$ firm collectively receive

$$N_2 \left[\frac{I + a + b_{1,e}}{N_i + 1} - P_{S,2} - P_{R,2} \right] < N_2 \left[\frac{I + a + b_{2,ne}}{N_i + 1} - P_{S,2} - P_{R,2} \right],$$

which is in turn weakly negative because the cash-rich shareholders in $b_{2,ne}$ firms choose to sell the rights, i.e. $b_{2,ne} \leq b^*_{RO,2}$. Consequently,

$$\pi N_1 \left[\frac{I + a + b_{1,e}}{N_i + 1} - P_{S,1} - P_{R,1} \right] \geq 0 > N_2 \left[\frac{I + a + b_{1,e}}{N_i + 1} - P_{S,2} - P_{R,2} \right].$$

Hence, the $a_{1,e}$ firm has an incentive to deviate to using a rights offering with strike price $P_{S,2}$. Contradiction! Concluding the proof. ■

Proof of Proposition 10: We begin with the following lemma that establishes the existence of $b^\dagger$.

Lemma 4 *Suppose b_1 (b_2) is any firm type where the cash-rich shareholders (do not) subscribe to the new shares. Then $b_1 > b_2$.*

Proof of Lemma 4: Suppose instead that $b_1 < b_2$. By Lemma 1 and 2 relatively better firms see shareholders participate. Therefore, b_1 and b_2 firms must have different offering modes. Without loss of generality assume b_1 adopts a public offering with dilution protection λ and b_2 adopts a rights

offering with strike price P_S and rights price P_R. Because current shareholders participate, Lemma 1 states that

$$b_1 > P_{PO} - a.$$

Hence, the investors' payoff in firm b_1 is

$$\left[1 - \lambda(1-\pi)\right] N_{PO} \left[\frac{I + a + b_1}{N_{PO} + 1} - P_{PO}\right] > 0.$$

Similarly, Lemma 2 implies the investors' payoff in firm b_2 is

$$N_{RO} \left[\frac{I + a + b_2}{N_{RO} + 1} - P_S - P_R\right] < 0.$$

However, b_1 firm then has an incentive to deviate to the rights offerings because

$$N_{RO} \left[\frac{I + a + b_1}{N_{RO} + 1} - P_S - P_R\right] < N_{RO} \left[\frac{I + a + b_2}{N_{RO} + 1} - P_S - P_R\right] < 0.$$

The contradiction establishes the lemma. ■

As a consequence of Lemma 4, there is a cutoff type $b^\dagger$ below which current shareholders do not participate in the offering. The next lemma establishes the existence of $\underline{b}^\dagger$.

Lemma 5 *There exist $\underline{b}^\dagger < b^\dagger$ such that all firms with $b \in (\underline{b}^\dagger, b^\dagger)$ (resp. $b < b^\dagger$) choose public (resp. rights) offerings.*

Proof of Lemma 5: By Lemma 4, the set of firms in $(\underline{b}, b^\dagger)$ can be partitioned into two subsets B_{RO} and B_{PO}, conducting rights offerings and public offerings respectively. Lemma 1 and Lemma 4 imply that both sets are non-empty. For any firm type $b_1 \in B_{PO}$, the IC condition suggests

$$N_{RO} \left[\frac{I + a + b_1}{N_{RO} + 1} - P_S - P_R\right] \geq N_{PO} \left[\frac{I + a + b_1}{N_{PO} + 1} - P_{PO}\right]. \tag{29}$$

Proposition 7 states that a rights offering with $P_R = 0$ is equivalent to a public offering with $\lambda = 1$,

which in turn implies two public offerings cannot coexist. Thus, it must be that $P_R > 0$. Using the fact $N_{RO}P_S = N_{PO}P_{PO} = I$ and $P_R > 0$, condition (29) implies

$$N_{RO}\frac{I+a+b_1}{N_{RO}+1} > N_{RO}\left[\frac{I+a+b_1}{N_{RO}+1} - P_R\right] \geq N_{PO}\frac{I+a+b_1}{N_{PO}+1}.$$

Hence, $N_{RO} > N_{PO}$. This condition in turn implies

$$b_1 \geq \frac{N_{RO}P_R}{\frac{N_{RO}}{N_{RO}+1} - \frac{N_{PO}}{N_{PO}+1}} - a - I \equiv \underline{b}^{\dagger}.$$

Completing the proof of the lemma. ■

Finally, we are ready to establish the existence of $\overline{b}^{\dagger}$. From Lemma 5, (16), (17), and the issue mode being chosen optimally, we have

$$\begin{cases} \Pi_{PO}(b) > \Pi_{RO}(b) & \text{for } b < \underline{b}^{\dagger} \\ \Pi_{PO}(b) < \Pi_{RO}(b) & \text{for } \underline{b}^{\dagger} < b < b^{\dagger} \end{cases}.$$

Clearly, both Π_{PO} and Π_{RO} are increasing, concave, and piece-wise linear functions. The only kink is the unique root of each respective function. Therefore, the graph of Π_{PO} and Π_{RO} must intersect exactly once at $\underline{b}^{\dagger}$ when $\Pi < 0$ and once when $\Pi > 0$. Denote the intersection by $\overline{b}^{\dagger}$. If such an $\overline{b}^{\dagger}$ does not exist, only one issue mode exists for $b > b^{\dagger}$, when shareholders subscribe, a contradiction.

Since Π_{RO} is steeper than Π_{PO} when $\Pi < 0$ ($N_{RO} > N_{PO}$), the reverse must be true when $\Pi > 0$ to generate an intersection. Hence, we must have

$$\begin{cases} \Pi_{PO}(b) > \Pi_{RO}(b) & \text{for } b > \overline{b}^{\dagger} \\ \Pi_{PO}(b) < \Pi_{RO}(b) & \text{for } b^{\dagger} < b < \overline{b}^{\dagger} \end{cases}.$$

This establishes the proposition. ■

A.2 Section 6: Single Offer Mode Equilibrium Outcomes

Proposition 12 *When the support of* $[\underline{a}, \overline{a}]$ *is large enough and*

$$[1-\lambda(1-\pi)]\, N_{PO}\left(\frac{\hat{a}_{PO}+I+b}{N_{PO}+1}-P_{PO}\right) \leq \pi I\left(\frac{\hat{a}_{PO}-\underline{a}}{\underline{a}+I+b}\right). \tag{30}$$

holds, there exist semi-pooling equilibria in which firm types $a \in (\hat{a}_{PO}, \overline{a}]$ *do not issue shares, firm types* $a \in [\underline{a}, \hat{a}_{PO}]$ *all choose a public offer with a common* λ*, and only cash-rich shareholders in firms* $a \in [a^*_{PO}, \hat{a}_{PO}]$ *subscribe to the offer. The two cut-off types* a^*_{PO} *and* $\hat{a}_{PO}$ *solve:*

$$a^*_{PO} = \frac{(1-\lambda(1-\pi))Pr(a^*_{PO} \leq a < \hat{a}_{PO})E(a|a^*_{PO} \leq a < \hat{a}_{PO}) + Pr(a < a^*_{PO})E(a|a < a^*_{PO})}{(1-\lambda(1-\pi))Pr(a^*_{PO} \leq a < \hat{a}_{PO}) + Pr(a < a^*_{PO})} \tag{31}$$

and

$$(1-\lambda(1-\pi))\frac{N_{PO}}{1+N_{PO}}(I+\hat{a}_{PO}+b) - (1-\lambda(1-\pi))I = b. \tag{32}$$

Moreover, condition (30) holds for $a \sim U(\underline{a}, \overline{a})$ *and either* λ *or* π *sufficiently close to 1.*

Proof: Given cash-rich shareholders subscribe for $a \geq a^*_{PO} \equiv P_{PO} - b$, investors break even when

$$\begin{aligned} &Pr(a < a^*_{PO})\left(\frac{N_{PO}}{N_{PO}+1}(b+I+E(a|a<a^*_{PO})) - I\right) \\ &\quad + Pr(a^*_{PO} \leq a < \hat{a}_{PO}) \times (1-\lambda(1-\pi)) \times \left(\frac{N_{PO}}{N_{PO}+1}(b+I+E(a|a^*_{PO} \leq a < \hat{a}_{PO})) - I\right) = 0 \end{aligned}$$

Solving the break-even condition for P_{PO} yields

$$P_{PO} = \frac{(1-\lambda(1-\pi))Pr(a^*_{PO} \leq a < \hat{a}_{PO})(b+E(a|a^*_{PO} \leq a < \hat{a}_{PO})) + Pr(a < a^*_{PO})(b+E(a|a<a^*_{PO}))}{(1-\lambda(1-\pi))Pr(a^*_{PO} \leq a < \hat{a}_{PO}) + Pr(a < a^*_{PO})}$$

and allows us to express $a^*_{PO} = P_{PO} - b$ as

$$a^*_{PO} = \frac{(1-\lambda(1-\pi))Pr(a^*_{PO} \leq a < \hat{a}_{PO})E(a|a^*_{PO} \leq a < \hat{a}_{PO}) + Pr(a < a^*_{PO})E(a|a < a^*_{PO})}{(1-\lambda(1-\pi))Pr(a^*_{PO} \leq a < \hat{a}_{PO}) + Pr(a < a^*_{PO})}$$

For the cut-off type $\hat{a}_{PO}$ shareholders are indifferent between investing or not and therefore

$$(1-\lambda(1-\pi))\frac{N_{PO}}{1+N_{PO}}(I + \hat{a}_{PO} + b) - (1-\lambda(1-\pi))I = b.$$

Solving this equality for $\hat{a}_{PO}$ gives

$$\begin{aligned}
\hat{a}_{PO} &= (1+N_{PO})\left[\frac{b}{(1-\lambda(1-\pi))N_{PO}} + P_{PO}\right] - I - b \\
&= \left(1 + \frac{I}{a^*_{PO}+b}\right)\left[\frac{b(a^*_{PO}+b)}{(1-\lambda(1-\pi))I} + a^*_{PO} + b\right] - I - b \\
&= \frac{b(a^*_{PO}+b)}{(1-\lambda(1-\pi))I} + \frac{b}{(1-\lambda(1-\pi))} + a^*_{PO} + b + I - I - b \\
&= a^*_{PO} + \frac{b(a^*_{PO}+b+I)}{(1-\lambda(1-\pi))I}. \qquad (33)
\end{aligned}$$

To establish the existence of a^*_{PO} , we use (33) to substitute $\hat{a}_{PO}$ in (31) and define

$$f(x) \equiv x - \frac{(1-\lambda(1-\pi))Pr(x \leq a < x + \frac{b(x+b+I)}{(1-\lambda(1-\pi))I})E(a|x \leq a < x + \frac{b(x+b+I)}{(1-\lambda(1-\pi))I}) + Pr(a < x)E(a|a < x)}{(1-\lambda(1-\pi))Pr(x \leq a < x + \frac{b(x+b+I)}{(1-\lambda(1-\pi))I}) + Pr(a < x)}.$$

Existence of a^*_{PO} is equivalent to a root of $f(x)$. At $x = \underline{a}$

$$f(\underline{a}) = \underline{a} - E\left[a|\underline{a} \leq a < \underline{a} + \frac{b(\underline{a}+b+I)}{(1-\lambda(1-\pi))I}\right] < 0$$

and at $x = \bar{a}$

$$f(\bar{a}) = \bar{a} - E(a|a < \bar{a}) > 0$$

By continuity, a root of $f(x)$ exists in $[\underline{a}, \bar{a}]$ and we define it as a^*_{PO}. The definition of f is independent of $\bar{a}$, and so is a^*_{PO}. Therefore, as long as $\bar{a}$ is large enough, $\hat{a}_{PO} = a^*_{PO} + \frac{b(a^*_{PO}+b+I)}{(1-\lambda(1-\pi))I} > a^*_{PO}$ must also be strictly less than $\bar{a}$, which implies that $\hat{a}_{PO}$ exists in $(a^*_{PO}, \bar{a}]$.

Condition (30) ensures that no type has an incentive to deviate. Conditional on investing $a \in [\underline{a}, \hat{a}_{PO}]$, condition (30) is equivalent to (18). Proposition 8 therefore implies that no issuing types has an incentive to deviate to another issue method. Since type $\hat{a}_{PO}$ is indifferent between investing and not, all types $a < \hat{a}_{PO}$ are better off investing. Conversely, all types $a > \hat{a}_{PO}$ prefer doing nothing to the equilibrium issue mode, which in turn dominates any other issue method for these types.

Finally, the equilibrium condition (32) implies that the left-hand side of condition (30) is equal to b and hence simplifies to

$$b \leq \pi I \left(\frac{\hat{a}_{PO} - \underline{a}}{\underline{a} + I + b} \right).$$

With $a \sim U(\underline{a}, \bar{a})$ we can analytically solve for

$$a^*_{PO} = \underline{a} + \frac{(b + I + \underline{a})b}{I\sqrt{1 - \lambda(1-\pi)} - b},$$

and

$$\hat{a}_{PO} = a^*_{PO} + \frac{b(a^*_{PO} + b + I)}{(1 - \lambda(1-\pi))I}.$$

As $\lambda \to 1$ a^*_{PO} approach $a^{\lim}_{\lambda} \equiv \underline{a} + \frac{(b+I+\underline{a})b}{I\sqrt{\pi}-b} > \underline{a}$, and for or $\pi \to 1$ a^*_{PO} approaches $a^{\lim}_{\pi} \equiv \underline{a} + \frac{(b+I+\underline{a})b}{I-b} > \underline{a}$.

$$\begin{aligned} \pi I \left(\tfrac{\hat{a}_{PO} - \underline{a}}{\underline{a}+I+b} \right) &> \pi I \left(\frac{\frac{b(a^*_{PO}+b+I)}{(1-\lambda(1-\pi))I}}{\underline{a}+I+b} \right) \\ &= b \left(\tfrac{a^*_{PO}+b+I}{\underline{a}+I+b} \right) \left(\tfrac{\pi}{(1-\lambda(1-\pi))} \right) \\ &\to b \tfrac{a^{\lim}_{\lambda}+b+I}{\underline{a}+I+b} > b, \end{aligned}$$

respectively $b\frac{a^{\lim}_{\pi}+b+I}{\underline{a}+I+b} > b$ in the final step for $\pi \to 1$. Hence, condition (30) holds for either λ or π sufficiently close to 1.■

Proposition 13 *When the support of* $[\underline{a}, \bar{a}]$ *is large enough and*

$$\pi N_{RO}\left(\frac{\hat{a}_{RO}+I+b}{N_{RO}+1}-P_S-P_R\right)\leq \pi I\left(\frac{\hat{a}_{RO}-\underline{a}}{\underline{a}+I+b}\right) \tag{34}$$

holds, there exist semi-pooling equilibria in which firm types $a \in (\hat{a}_{RO}, \bar{a}]$ *do not issue any rights, firm types* $a \in [\underline{a}, \hat{a}_{RO}]$ *all choose a rights offer with a common* P_S*, and only cash-rich shareholders in firms* $a \in [a^*_{RO}, \hat{a}_{RO}]$ *subscribe to the offer. The two cut-off types* a^*_{RO} *and* $\hat{a}_{RO}$ *solve:*

$$a^*_{RO}=\frac{\pi Pr(a^*_{RO}\leq a<\hat{a}_{RO})E(a|a^*_{RO}\leq a<\hat{a}_{RO})+Pr(a<a^*_{RO})E(a|a<a^*_{RO})}{\pi Pr(a^*_{RO}\leq a<\hat{a}_{RO})+Pr(a<a^*_{RO})} \tag{35}$$

and

$$\pi N_{RO}(\frac{I+\hat{a}_{RO}+b}{1+N_{RO}}-P_R-P_S)=b. \tag{36}$$

Moreover, condition (34) always holds for any $P_S \geq \underline{a}+b$*.*

Proof: Given cash-rich shareholders subscribe for $a \geq a^*_{RO} \equiv P_S + P_R(N_{RO}+1) - b$, investors break even when

$$P_R+P_S=\frac{1}{N_{RO}+1}\left(b+I+\frac{\pi Pr(a^*_{RO}\leq a<\hat{a}_{RO})E(a|a^*_{RO}\leq a<\hat{a}_{RO})+Pr(a<a^*_{RO})E(a|a<a^*_{RO})}{\pi Pr(a^*_{RO}\leq a<\hat{a}_{RO})+Pr(a<a^*_{RO})}\right)$$

Using the break-even condition to substitute P_R in the definition of a^*_{RO} gives

$$a^*_{RO}=\frac{\pi Pr(a^*_{RO}\leq a<\hat{a}_{RO})E(a|a^*_{RO}\leq a<\hat{a}_{RO})+Pr(a<a^*_{RO})E(a|a<a^*_{RO})}{\pi Pr(a^*_{RO}\leq a<\hat{a}_{RO})+Pr(a<a^*_{RO})}$$

For the cut-off type $\hat{a}_{RO}$ shareholders are indifferent between investing or not and therefore

$$\pi N_{RO}\left(\frac{I+a\hat{R}O+b}{1+N_{RO}}-P_R-P_S\right)=b.$$

Using equations (35) and (36), the cut-off type $\hat{a}_{RO}$ can be written as

$$\begin{aligned}\hat{a}_{RO} &= (1+N_{RO})\left(\frac{b}{\pi N_{RO}} + P_R + P_S\right) - I - b \\ &= (1+N_{RO})\left[\frac{b}{\pi N_{RO}} + \frac{1}{N_{RO}+1}(b+I+a^*_{RO})\right] - I - b \\ &= \frac{b(1+N_{RO})}{\pi N_{RO}} + b + I + a^*_{RO} - I - b = a^*_{RO} + \frac{b}{\pi}\left(\frac{P_S}{I}+1\right). \end{aligned} \tag{37}$$

We now establish the existence of a^*_{RO} and $\hat{a}_{RO}$. We use (37) to substitute $\hat{a}_{RO}$ in (35), and define

$$f(x) = x - \frac{\pi Pr(x \le a < x + \frac{b}{\pi}(\frac{P_S}{I}+1))E(a|x \le a < x + \frac{b}{\pi}(\frac{P_S}{I}+1)) + Pr(a<x)E(a|a<x)}{\pi Pr(x \le a < x + \frac{b}{\pi}(\frac{P_S}{I}+1)) + Pr(a<x)}.$$

Existence of a^*_{RO} is equivalent to a root of $f(x)$. At $x = \underline{a}$,

$$f(\underline{a}) = \underline{a} - E\left[a|\underline{a} \le a < \underline{a} + \frac{b}{\pi}\left(\frac{P_S}{I}+1\right)\right] < 0.$$

and at $x = \bar{a}$,

$$f(\bar{a}) = \bar{a} - E(a|a<\bar{a}) > 0.$$

By continuity, a root of $f(x)$ exists in $[\underline{a}, \bar{a}]$ and we define it as a^*_{RO}. The definition of f is independent of $\bar{a}$, and so is a^*_{RO}. Therefore, as long as $\bar{a}$ is large enough, $\hat{a}_{RO} = a^*_{RO} + \frac{b}{\pi}(\frac{P_S}{I}+1)$ must also be strictly less than $\bar{a}$, which implies that $\hat{a}_{RO}$ exists in $(a^*_{RO}, \bar{a}]$.

Condition (34) ensures that no type has an incentive to deviate. Conditional on investing $a \in [\underline{a}, \hat{a}_{RO}]$, condition (34) is equivalent to (19). Proposition 9 therefore implies that no issuing type has an incentive to deviate to another issue method. Since type $\hat{a}_{RO}$ firm is indifferent between investing and not, all types $a < \hat{a}_{RO}$ are better off investing. Conversely, all types $a > \hat{a}_{RO}$ prefer doing nothing to the equilibrium issue method, which in turn dominates any other methods for

these types.

Finally, the equilibrium condition (36) implies that the left-hand side of condition (34) is equal to b and hence the condition simplifies to

$$b \leq \pi I \left(\frac{\hat{a}_{RO} - \underline{a}}{\underline{a} + I + b} \right).$$

Since $\hat{a}_{RO} = a^*_{RO} + \frac{b}{\pi}(\frac{P_S}{I} + 1) \geq \underline{a} + \frac{b}{\pi}(\frac{P_S}{I} + 1)$ and by assumption $P_S \geq \underline{a} + b$,

$$\pi I \left(\frac{\hat{a}_{RO} - \underline{a}}{\underline{a} + I + b} \right) \geq \pi I \left(\frac{\frac{b}{\pi}(\frac{P_S}{I} + 1)}{\underline{a} + I + b} \right) \geq \pi I \left(\frac{\frac{b}{\pi}(\frac{\underline{a}+b}{I} + 1)}{\underline{a} + I + b} \right) = b.$$

■

A.3 Section 6: Coexisting Equilibrium Outcomes

There are the cut-off types $\underline{a}^\dagger$, $a^\dagger$, and $\overline{a}^\dagger$ parallel to those in terms of project NPV in Proposition 10. When the support of a is large enough, there exists an additional cut-off type $\hat{a} \in \left(\overline{a}^\dagger, \overline{a}\right)$ which satisfies

$$\pi N_{RO} \left(\frac{I + \hat{a} + b}{1 + N_{RO}} - P_{RO} - P_S \right) = b. \tag{38}$$

Since $\hat{a} > \overline{a}^\dagger > a^\dagger$, type $\hat{a}$ firm uses rights offering if it issues new equity, and the cash-rich shareholders exercise their rights. The left-hand side of condition (38) is the payoff to the investors from exercise the rights sold by the cash-poor shareholders. Condition (38) implies that when type $\hat{a}$-firm conducts a rights offering, the investors extract the entire NPV b, leaving the shareholders indifferent about investing or not. Hence, all firm types below $\hat{a}$ issue and those above do not.

A.4 Section 6: Proof of Proposition 11

With $a \sim U(\underline{a}, \bar{a})$ the cut-off type a^*_{PO} as given in equation 31 becomes

$$a^*_{PO} = \frac{(1-\lambda(1-\pi))\frac{\left(\hat{a}_{PO}-a^*_{PO}\right)\left(a^*_{PO}+\hat{a}_{PO}\right)}{2(\bar{a}-\underline{a})} + \frac{\left(a^*_{PO}-\underline{a}\right)\left(a^*_{PO}+\underline{a}\right)}{2(\bar{a}-\underline{a})}}{(1-\lambda(1-\pi))\frac{\hat{a}_{PO}-a^*_{PO}}{\bar{a}-\underline{a}} + \frac{a^*_{PO}-\underline{a}}{\bar{a}-\underline{a}}} \tag{39}$$

Using (33) to substitute $\hat{a}_{PO}$ in (39) yields

$$a^*_{PO} = \frac{(b+I)b + I\underline{a}\sqrt{1-\lambda(1-\pi)}}{I\sqrt{1-\lambda(1-\pi)}-b} = \underline{a} + \frac{(b+I+\underline{a})b}{I\sqrt{1-\lambda(1-\pi)}-b}$$

Hence, a^*_{PO} is increasing in λ, and therefore also $\hat{a}_{PO} = a^*_{PO} + \frac{b(a^*_{PO}+b+I)}{(1-\lambda(1-\pi))I}$ and $Pr(a < \hat{a}_{PO})$. With $a \sim U(\underline{a}, \bar{a})$ the wealth transfer conditional on investing

$$E(a|a < \hat{a}_{PO}) + b - \frac{1}{1+\frac{I}{P_{PO}}}[I + E(a|a < \hat{a}_{PO}) + b]$$

becomes

$$\begin{aligned}
&= \frac{I}{I+P_{PO}}[E(a|a<\hat{a}_{PO}) + b - P_{PO}] = \frac{I}{I+b+a^*_{PO}}[E(a|a<\hat{a}_{PO}) - a^*_{PO}] \\
&= \frac{I}{I+b+a^*_{PO}}\left[E(a|a<a^*_{PO} + \frac{b(a^*_{PO}+b+I)}{(1-\lambda(1-\pi))I}) - a^*_{PO}\right] \\
&= \frac{b}{2[1-\lambda(1-\pi)]} + \frac{I(\underline{a}-a^*_{PO})}{2(I+b+a^*_{PO})} \\
&= \frac{b}{2[1-\lambda(1-\pi)]} + \frac{I\left(\underline{a} - \frac{(b+I)b+I\underline{a}\sqrt{1-\lambda(1-\pi)}}{I\sqrt{1-\lambda(1-\pi)}-b}\right)}{2\left(I+b+\frac{(b+I)b+I\underline{a}\sqrt{1-\lambda(1-\pi)}}{I\sqrt{1-\lambda(1-\pi)}-b}\right)} \\
&= \frac{b}{2[1-\lambda(1-\pi)]} - \frac{b}{2\sqrt{1-\lambda(1-\pi)}} = \frac{b}{2\sqrt{1-\lambda(1-\pi)}}\left(\frac{1}{\sqrt{1-\lambda(1-\pi)}} - 1\right),
\end{aligned}$$

and is increasing in λ, like $Pr(a < \hat{a}_{PO})$. Hence, $\hat{WT}_{PO}$ is increasing in λ.

With $a \sim U(\underline{a}, \bar{a})$ the cut-off type a^*_{RO} as given by equation (35) becomes

$$a^*_{RO} = \frac{\pi \frac{\left(\hat{a}_{RO} - a^*_{RO}\right)\left(a^*_{RO} + \hat{a}_{RO}\right)}{2(\bar{a} - \underline{a})} + \frac{\left(a^*_{RO} - \underline{a}\right)\left(a^*_{RO} + \underline{a}\right)}{2(\bar{a} - \underline{a})}}{\pi \frac{\hat{a}_{RO} - a^*_{RO}}{\bar{a} - \underline{a}} + \frac{a^*_{RO} - \underline{a}}{\bar{a} - \underline{a}}} \tag{40}$$

Using (37) to substitute $\hat{a}_{PO}$ in (40) yields

$$a^*_{RO} = \frac{b}{\sqrt{\pi}}(\frac{P_S}{I} + 1) + \underline{a}$$

Hence, a^*_{RO} is increasing in P_S, and therefore also $\hat{a}_{RO} = a^*_{RO} + \frac{b}{\pi}(\frac{P_S}{I} + 1)$ and $Pr(a < \hat{a}_{RO})$. With $a \sim U(\underline{a}, \bar{a})$ the wealth transfer conditional on investing

$$\frac{I}{I + P_S}\left[E(a | a \leq \hat{a}_{RO}) - a^*_{RO}\right]$$

becomes

$$= \frac{I}{P_S + I}\left(\frac{\underline{a} + a^*_{RO} + \frac{b}{\pi}\left(\frac{P_S}{I} + 1\right)}{2} - a^*_{RO}\right) = \frac{b}{2\pi} + \frac{I}{P_S + I}\left(\frac{\underline{a} - a^*_{RO}}{2}\right)$$
$$= \frac{b}{2\pi} + \frac{I}{P_S + I}\left(\frac{\frac{b}{\sqrt{\pi}}(\frac{P_S}{I} + 1)}{2}\right) = \frac{b}{2\pi} + \frac{b}{2\sqrt{\pi}},$$

and is independent of P_S. Since $Pr(a < \hat{a}_{RO})$ is increasing in P_S, so is $\hat{WT}_{RO}$. ■

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