Joint control and redemption rights in venture capital contracts

Finance Working Paper N°. 37/2004
March 2004

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ECGI Working Paper Series in Finance

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* I am very grateful to Andrew Winton for his many suggestions and support on this paper. I am also thankful to Ross Levine, Paul Povel, Rajdeep Singh, Vasiliki Skreta, and the seminar participants at the Carlson School of Management, University of Minnesota for their comments. All errors are mine.

Abstract

In most venture capital financed firms, neither the VC nor the manager has exclusive authority over some of the key corporate decisions. For example: the decision whether the firm should undertake an IPO or be sold to a larger rival usually requires the approval of both the manager and the VC. This contradicts a strong prediction in the theoretical literature that joint control is suboptimal. In this paper, I show that assigning control jointly to both the agents, and specifying a harsh penalty (such as liquidation) if they fail to reach an agreement, is sometimes better than assigning control exclusively to one of the agents. A key factor is the firm's "financial slack" – the difference between its expected cash flows and its required investments and monitoring costs. Joint control is the optimal control allocation when the firm has low financial slack and a reasonable collateral value, and when the VC's liquidity constraints and cost of monitoring the firm are high. Most VC-financed firms fit this description.

Keywords: Joint control, venture capital, redemption rights

JEL Classifications: G24, G30

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Introduction

Financial contracting theory suggests that the allocation of control over future strategic decisions should be a central feature of the contract between the manager of a start-up firm and the venture capitalist (VC) who finances the firm. The theory predicts that control should remain with the manager if the firm is not financially constrained, and should be assigned to the VC otherwise.[1] Joint control, an arrangement in which control is assigned simultaneously to both the manager and the VC, is predicted to be suboptimal,[2] except under some limited circumstances. Yet in practice, in most VC-financed firms, the control allocation resembles joint control; neither the VC nor the manager has exclusive authority over some of the key corporate decisions, and these decisions require the approval of both the agents.

With start-up firms, the authority to make corporate decisions primarily rests with the board; hence, board control is a key measure of control (Lerner (1995), Kaplan and Stromberg (2001,2002)). Kaplan and Stromberg (2001) find that in 61% of VC-financed firms, neither the VC nor the founder manager control a majority of the board seats.[3] Since the other members of the board are chosen by the mutual consent of the manager and the VC, this means that neither of the agents has exclusive authority to make crucial decisions. Apart from board representation, the contract also provides the manager and the VC with other safeguards to protect their interests. For example: the manager generally has a veto power against early sale of the firm, as long as the firm performs reasonably well; the VC usually has redemption rights that effectively give it the ability to threaten to walk out of the firm.

In this paper, I offer an explanation for why joint control is the optimal control allocation for most VC-financed firms. I argue that the optimal control allocation is determined by factors such as the firm's "financial slack" (the difference between its expected cash flows and its required investments and monitoring costs), collateral value, monitoring costs, and the VC's liquidity constraints. I show that for firms with low financial slack and a reasonable collateral value, joint control along with a credible threat of liquidation if the agents fail to reach an agreement, is strictly better than exclusive manager control or VC control.

My model is based on the idea that the contract between the manager and the VC is incomplete. The start-up firm will be faced with some important decisions, the nature and

[1] When control can be made contingent on future measures of performance, then contingent control – control switching from manager to VC following poor performance – is the optimal control allocation for financially constrained firms (Aghion and Bolton (1992))

[2] Aghion and Bolton (1992) predict that manager control achieves the same outcome as joint control, but at a lesser renegotiation cost; hence joint control can never dominate manager control. The property rights literature (see Hart (1995)) predicts that joint control is sub-optimal because it hurts the incentives of both agents to make relationship-specific investments.

[3] In fact, the VC and the manager have majority control of the board in only 25% and 14% of firms, respectively.

timing of which cannot be described in the contract. In this model, I focus on the decision that determines the manner of the VC's exit from the firm – should the firm undertake an IPO or be acquired by a larger rival?[4] The preferences of the manager and the VC are likely to differ significantly over this decision. The manager values her control rents in addition to financial returns; she may choose the IPO route to safeguard her control rents even when the financial returns from an acquisition are higher. On the other hand, the VC prefers an exit route that yields financial returns quickly, and requires less effort to monitor and manage the firm. So the VC may choose the acquisition route prematurely, especially when its liquidity constraints, and cost of continuing to monitor and manage the firm are high.

Since the contract cannot resolve this potential conflict of interest, it assigns control over the decision either to one of the agents (individual control) or to both the agents simultaneously (joint control). Under joint control, a decision is made only if both agents agree to it; if they fail to reach an agreement, the firm is liquidated, and both agents receive their respective liquidation claims. As will become apparent, this threat of inefficient liquidation under joint control is instrumental in getting the agents to agree to the efficient exit decision.

Under individual control, the agent who has control needs to be provided incentives to choose the efficient decision. Moreover, since the manager has no personal wealth and since the VC is constrained for funds, these incentives must come from the firm's cash flows. So when the manager is in control, she needs to be rewarded for choosing the acquisition route, in the form of a compensation for the control rents she forgoes. Similarly, when the VC is in control, it needs to be punished for choosing the acquisition route, in order to dissuade it from choosing such a route prematurely. So under both forms of individual control, the VC's payoff under the acquisition route must be low if the optimal exit route is to be chosen. But at the same time, the expected payoff to the VC must be high enough to persuade it to invest in the firm in the first place. These two conflicting objectives may be difficult to reconcile when the firm's "financial slack" is low, the VC's liquidity constraints are high, and the firm is costly to manage and monitor. For such firms, both manager control and VC control are sub-optimal.

Joint control, on the other hand, doesn't require punishing the VC (or rewarding the manager) when the firm chooses the acquisition route. The reasons are as follows: First, faced with the threat of inefficient liquidation, the manager will not insist on obtaining compensation for her control rents before agreeing to the choice of the acquisition route, when it is optimal. Second, since control is jointly held, the VC cannot force an acquisition on the firm prematurely. So joint control results in the optimal decision being made. The only remaining issue is whether the expected payoff to the VC is high enough to persuade it to invest in the firm, or not. This depends on the size of the VC's liquidation claim,

[4]Other possible examples are: Should it undertake a major expansion or not? Should it invest in R&D or in upgrading its marketing infrastructure?

which in turn cannot exceed the firm's collateral; the higher the VC's liquidation claim, the higher is its expected payoff under joint control.

The main prediction of the paper is that joint control strictly dominates both manager control and VC control when the following circumstances are met: (a) the firm has a low financial slack; (b) the VC's liquidity constraints, and cost of continuing to monitor and manage the firm are high; and (c) the firm has a reasonable collateral value. These circumstances are consistent with empirical evidence on the characteristics of firms that receive VC financing.

First, start-up firms that receive VC financing fit the description of low financial slack. Such firms tend to have risky return distributions, with a low probability of success. More than one-third of such investments result in an absolute loss for the VC, and more that two-thirds result in a realized return of less than 10% per annum (Sahlman (1990)). Only about 22% of firms that receive VC financing eventually undertake an IPO (Gompers (1995)). More importantly, even firms that eventually succeed may be financially constrained in the short-term.

Second, close to 70% of the firms that receive VC financing are in high-tech sectors like IT/software, telecom and biotechnology; the remaining 30% are in the healthcare, retail and other sectors (Kaplan and Stromberg (2001)). Since these firms are generally involved in developing a new product or service, there is considerable uncertainty surrounding the firm's business model, market conditions, etc.; so the cost to the VC of continuing to monitor and manage the firm is high.

Finally, a large percentage of VC financing goes to later-stage firms. These are firms that have a proven product with high sales growth, but are either unprofitable or marginally profitable, and require external capital to finance further expansion. Later-stage firms generally have collateral in the form of patents, marketing rights of existing products, fixed assets, inventories and receivables that might be seized in the event of liquidation.[5] Sahlman (1990) reports that only 15% of the capital disbursed by VCs went to ventures in early stages, whereas 65% was invested in later-stage firms; the remaining 20% was invested in leveraged buyout or acquisition deals.

As mentioned earlier, the threat of inefficient liquidation under joint control is instrumental in getting the agents to agree to the efficient exit decision. It is, therefore, important that the threat is perceived to be credible. One way in which the threat can be made credible is by providing the VC with a redemption right, i.e., a right to demand that the firm redeem its liquidation claim.[6] If the manager is unable to meet the redemption demand, the VC gets the right to liquidate the firm. Kaplan and Stromberg (2001) report that re-

[5] In contrast, early start-up firms are typically less than a year old, and are still at the product development or prototype testing stage; hence, they are unlikely to have much in the form of collateral (Sahlman (1990)).

[6] Alternatively, the firm may be financed with short-term securities.

demption rights are present in 79% of all VC contracts they survey, with a typical maturity of five years.

So the paper also offers an alternative explanation for the presence of redemption rights, namely, that they safeguard the bargaining power of the VC in future negotiations. Previous explanations of redemption rights have appealed to the abandonment option associated with debt financing. But as Kaplan and Stromberg (2002) note, "the abandonment option argument does not apply well to redemption rights that apply so far into the future." This is because poorly performing firms are generally liquidated in under four years.

Discussion of related literature: The property rights literature predicts that joint ownership[7] is suboptimal, except under some limited circumstances. Hart (1995) predicts that joint ownership is optimal when the returns from the investments made by the two agents are embodied in the physical capital of the firm, rather than in the human capital of the agents. On the contrary, I show that joint control is the optimal governance mechanism only when the manager's human capital is critical for the firm's success; if manager's control rents are low, then manager control is feasible and optimal.

Cai (2003) and Hauswald and Hege (2002) derive the optimality of joint ownership in a setting where two agents make relationship-specific investments that are non-contractable. Cai (2003) argues that when agents face a tradeoff between relationship-specific investments and general investments to promote their outside options, joint ownership acts as a mutual hostage, and promotes cooperation by committing the agents to the relationship. However, this does not explain the prevalence of joint control in VC-financed firms, because contracts between VCs and managers generally include mechanisms like non-compete clauses, time vesting of stock options etc. to restrict the manager's outside options, and to commit her to the relationship. Hauswald and Hege (2002) show that when agents face a tradeoff between investment and control rent seeking activities, 50-50 ownership may be optimal because it offers protection against rent seeking activities. In my paper, the focus is on the conflict of interests regarding future decisions, rather than the agents' investments in the venture.[8] The optimal control arrangement is determined by the firm's financial constraints, rather than by the attributes of the agents' investments into the firm.

This paper is also related to a number of studies that focus on the allocation of control rights in VC contracts. Berglof (1994) analyzes how an entrepreneur and a VC may allocate control rights so as to mitigate the distributional conflicts associated with a future sale of the firm. Hellmann (1998) analyzes the circumstances under which an entrepreneur voluntarily relinquishes control to a VC, including the right to fire the entrepreneur. In a model with double moral hazard, Cestone (2001) looks at the interaction of cash flow rights and control

[7] The property rights literature defines ownership of an asset as the possession of residual control rights over that asset (see Hart (1995)). So ownership and control have the same economic meaning.

[8] In my model, the VC's investment into the venture is contractible; the manager does not make any investment.

rights in VC contracts, and explains why riskier VC claims are often associated with weak VC control rights. In contrast to this paper, all the above mentioned papers treat control as an indivisible right that is held exclusively by the manager or the VC.

Kirilenko (2001) treats control as a continuous rather than a binary variable. Using a signaling framework, Kirilenko (2001) shows that VC control is increasing in the level of asymmetric information between the VC and the entrepreneur. However, his results are based on the assumption that the entrepreneur derives an exogenous utility that is increasing in her degree of control over the firm. I do not make such an exogenous assumption. Instead, my paper focuses on the endogenous relationship between control allocation, decision making, sharing of surplus, and overall firm value.

1 The Model

The model incorporates three dates; 0, 1 and 2. At date 0, an entrepreneur-manager sets up a firm by making an investment I. Being cash constrained herself, the manager raises this money from a venture capitalist (VC) by issuing claims against future cash flows of the firm. The cash flow from the firm, denoted $\tilde{x}$, is a random variable whose distribution on some positive interval X is determined by a future strategic decision to be made by the firm. $\tilde{x}$ is realized at date 2.

The strategic decision is modelled as an action choice a, where the firm chooses between two actions; a 'risky' action a_r and a 'safe' action a_s. The risky action is so named because the distribution of $\tilde{x}$ under this action, unlike under the safe action, depends on a state variable θ that is not known to either the manager or the VC at date 0. In this paper, the strategic decision is choosing the manner of the VC's exit from the firm, i.e., deciding whether the firm should undertake an IPO (risky action) or be acquired by a larger rival (safe action)? The IPO is labelled risky because it takes longer to yield returns, and because IPO returns are more uncertain than returns under the acquisition route.

The state θ is realized only at date 1. It may be viewed as summarizing the impact of factors such as the firm's technology, business model, market conditions, etc., that have a bearing on the profitability of the action a_r. θ can have two possible realizations; 'high' or 'low', denoted θ_h and θ_l. The expected cash flow at date 2 when the firm chooses action a_r is π_{rh} if the 'high' state is realized, and π_{rl} if the 'low' state is realized. The expected cash flow when the firm chooses action a_s is π_s, irrespective of state θ.

The project is sufficiently complex that the timing and nature of the future strategic decision cannot be described precisely at date 0. This is because outside parties, such as a court, cannot verify the realization of state θ and the timing of the action choice a. This assumption has the following implications: First, the initial contract cannot specify an action plan contingent on θ. Second, the contract also cannot include a clause to prevent the firm from choosing an action prematurely (i.e., before θ is observed). So the initial contract must specify which agent has control, i.e., the right to choose a. Control may be assigned exclusively to either the manager or the VC, or it may be assigned jointly to both agents. I discuss control allocations in greater detail in Section 2.

At date 0, both the manager and the VC believe that the 'high' state will be realized with probability q, where $q \in (0, 1)$ is a constant. When θ is realized at date 1, it is observed by both the agents. Thus, there is never any information asymmetry between the two agents. The action a and the cash flow $\tilde{x}$ are verifiable ex-post. Thus the initial contract can specify a rule for sharing $\tilde{x}$ that is contingent on the action chosen.

The manager and the VC are risk-neutral in their payoffs. While the VC is primarily interested in its monetary returns from the project, the manager also cares for her control rents. Control rents may be viewed as the non-pecuniary or otherwise non-contractible part of the future firm value that can accrue only to the manager, and cannot be seized by the VC.[9] They include less tangible things such as specific human capital, R&D capabilities, reputation, size, etc. that the manager cares for. I assume that the manager derives control rents valued at $C > 0$ when action a_r is chosen, and zero otherwise. For example: The manager prefers an IPO to an acquisition because she will be the CEO of an independent firm after an IPO, while she may be replaced or relegated to a less important position following an acquisition; she prefers investing in R&D because that would boost her reputation among her peers; she prefers a larger expansion because a larger firm offers greater managerial perquisites and prestige.

Assumption 1: The following conditions hold:

(a) $\pi_{rh} + C > \pi_s > \pi_{rl} + C$

(b) $\pi_{rh} \geq \pi_s$

The firm value is the sum of cash flows and the (monetary value of) manager's control rents. Assumption 1(a) says that firm value is maximized by choosing action a_r in state θ_h and action a_s in state θ_l. I refer to this as the *efficient action plan.* Assumption 1(a) implies that, in state θ_l, the manager may be reluctant to choose the efficient action, as she forgoes her control rents, C, by doing so. Assumption 1(b) says that, in state θ_h, the firm's cash flows are also maximized by choosing the efficient action a_r.

The analysis in this paper is simplified by defining,

$$\Delta_v \equiv \pi_{rh} + C - \pi_s \text{ and } \Delta \equiv \pi_{rh} - \pi_s \tag{1}$$

[9] See Diamond (1993)

Δ_v and Δ denote the incremental value and the incremental cash flow, respectively, to the firm from choosing action a_r over action a_s in state θ_h. Assumption 1(a) implies that $\Delta_v > 0$, and Assumption 1(b) implies that $\Delta > 0$. Overall, Assumption 1 says that the efficient action in each state also maximizes the cash flow in that state. If this were the end of the story, the efficient action plan could be implemented by giving the control right to the VC (see Aghion & Bolton (1992)).[10]

However, recall that the initial contract cannot contain a clause that prevents the safe action a_s being forced on the firm prematurely. When the VC is in control, it can force the safe action a_s irreversibly on the firm, before θ is revealed. To make the VC's choice non-trivial, I assume that the VC bears a cost m for forgoing the opportunity to force action a_s early on. m represents the following costs: (i) the opportunity cost of forgoing early liquidity by not forcing action a_s early on,[11] and (ii) the cost that the VC expects to expend on continuing to manage and monitor the firm.[12] I refer to m as the VC's opportunity cost of control.

Assumption 2: $q\Delta < m < q\Delta_v$.

Assumption 2 says that the incremental value if the firm waits for θ to be revealed, and then chooses the efficient action, is higher than the VC's opportunity cost of control; however, the incremental cash flow from doing so is less than the VC's opportunity cost of control. In other words, it is inefficient to choose a_s prematurely, but the VC may still do so because it doesn't care for the manager's control rents C, and wants to save on its own opportunity cost m.

The VC's preference for choosing action a_s early arises from its own liquidity constraints and the high cost it expends to manage and monitor the firm; the parameter m captures this preference. These costs arise due to the nature of the partnerships that VCs float to raise funds from investors such as pension funds, insurance companies, etc., and due to the structure of the partnership agreements.[13] First, partnerships have finite lives; so the VC must raise a new partnership once the funds from an existing partnership are fully invested, or about once every three to five years, in order to remain in business. So the VC has to divide its time and resources between monitoring existing portfolio firms, and

[10]To see this, consider a contract that gives away all of $\tilde{x}$ to the VC. If the VC is then asked to choose action a after θ has been observed, it will choose the efficient action in both states.

[11]Gompers (1995) reports that firms that went for an IPO were held for a longer time, and required more intense monitoring than firms that were acquired. Fenn, Liang and Prowse (1995) report that VCs may prefer private sale because it yields immediate liquidity, even though an IPO could result in a higher valuation for the company.

[12]Kaplan and Stromberg (2002) report that, while investing in small companies, most VCs worry that the monitoring costs and involvement costs may be too high. While VCs regularly play a monitoring and advisory role, they do not intend to become too involved in the company. Gorman and Sahlman (1989), Lerner (1995), Hellmann and Puri (2000) and Hellmann and Puri (2002) provide evidence that VCs spend substantial time and effort managing and supporting their investments.

[13]See Fenn, Liang and Prowse (1995) and Sahlman (1990).

raising new funds for making future investments. Second, the earlier the VC generates returns for its existing investors, the more likely it is that they will participate in future partnerships floated by the VC. Third, a large part of the VC's compensation is in the form of a percentage share, typically 20%, of the realized gains of the partnership. So the earlier the gains are realized, the better it is for the VC.

If θ were verifiable, the firm could have been directed to implement the efficient action plan. The expected firm value would then have been:

$$V^* = q\,(\pi_{rh} + C) + (1-q)\,\pi_s$$

The expected cash flow from the firm when the efficient action plan is chosen is $q\pi_{rh} + (1-q)\,\pi_s$. For financing to be viable, this must compensate for the investment I and the VC's opportunity cost of control, m.

Assumption 3: $q\pi_{rh} + (1-q)\,\pi_s \geq I + m$

2 The Contract & Control rights

As discussed earlier, the initial contract cannot specify an action plan contingent on the state θ. It can only assign control, i.e., the right to choose action a. I distinguish between two forms of control – individual control and joint control.

Individual control: Under individual control, the initial contract provides one of the agents – the manager or the VC – with the right to choose the action a. After observing the true state θ, the two agents might wish to renegotiate the initial contract. During renegotiation, the manager and the VC bargain over the action choice and cash flow rights. If renegotiation succeeds, action a is chosen by mutual consent, a new contract detailing fresh cash flow rights is signed, and the old contract is discarded. If renegotiation fails, the agent with the control right chooses the action that will maximize her payoff as per the cash flow rights laid down in the initial contract. So the initial contract defines the disagreement point of the renegotiation game; even if it is discarded, it is still not irrelevant ex-ante.

Joint control: Under joint control, action a can only be chosen by the approval of both agents. I model this as follows: If the manager proposes an action and the VC doesn't intervene, it is assumed that the VC approves of the manager's action choice. The manager and the VC then share the cash flow, $\tilde{x}$, as specified in the initial contract. Otherwise, if the VC intervenes, the manager and the VC bargain over the action a and the sharing of surplus. The two agents face the threat that in case they fail to reach an agreement, the firm is liquidated, and both agents receive their respective liquidation claims. If liquidated, the firm yields a value $L < \pi_s$. L is the collateral value of the firm, or the value of the firm's assets outside the firm. The payoffs to the manager and the VC in case of liquidation are $L - Y$ and Y, respectively, where Y denotes the liquidation claim of the VC. Limited liability requires that $0 \leq Y \leq L$.

As discussed in the introduction, the threat of liquidation can be made credible by providing the VC with a redemption right maturing at date 1, i.e., a right to demand that the firm redeem its liquidation claim at date 1. If the manager is unable to meet the redemption demand, the VC gets the right to liquidate the firm. In case the manager and the VC fail to reach an agreement, the VC exercises its redemption right. Since the firm has no cash flows prior to date 2, the manager cannot make the repayment; so this triggers liquidation of the firm.

The initial contract specifies cash flow rights, liquidation claims and the control allocation.

- Cash flow right: This is a function $S : X * \{a_s, a_r\} \to X$, where $S(x, a_i)$ denotes the payoff to the VC when the firm chooses action a_i and $\tilde{x} = x$. The manager being the residual claimant gets $x - S(x, a_i)$. Limited liability requires that $0 \leq S(x, a_i) \leq x$ for all x.
- Liquidation claim: This is a variable $Y \in [0, L]$, where Y denotes the payoff to the VC in the event of liquidation; the manager gets $L - Y$.
- Control allocation: This is a variable $\psi \in \{V, J, M\}$, where V, J and M stand for VC control, joint control and manager control, respectively.

The triple (S, Y, ψ) denotes the initial contract. Let S_{rh} and S_{rl}, respectively, denote the VC's expected payoffs when the firm chooses action a_r in the 'high' and 'low' states. Similarly, let S_s denote the VC's expected payoff when the firm chooses action a_s.

Let $E_{(S,Y,\psi)}(V)$ and $E_{(S,Y,\psi)}(S)$ denote the expected firm value and the expected cash flow to the VC, respectively, under the contract (S, Y, ψ). For simplicity, I assume that the manager gets to propose a contract to the VC, which the VC can either accept or reject. This implies that the manager obtains cash equal to $E_{(S,Y,\psi)}(S)$ from the VC at date 0. (This assumption simplifies analysis without affecting the qualitative results in this paper)

The manager's problem is

$$\max_{(S,Y,\psi)} E_{(S,Y,\psi)}(V)$$

subject to the financing constraint,

$$E_{(S,Y,\psi)}(S) \geq I + \phi m$$

where ϕ is an indicator variable that equals 1 if the VC waits till θ is revealed, and 0 if the VC chooses action a_s prematurely. If the financing constraint is not met, the manager will not be able to raise enough money at date 0 to finance the investment I; the contract is then said to be infeasible.

3 Renegotiation design

In this section, I analyze the renegotiation game between the manager and the VC at date 1. During renegotiation, the manager and the VC bargain over the action choice and sharing of cash flows. As mentioned earlier, renegotiation can take place irrespective of whether control is held by the manager or by the VC, or by both the agents jointly. The control allocation only determines the disagreement point, i.e., the outcome of the bargaining game in case the agents fail to reach an agreement: under individual control, the agent who is in control chooses the action that will maximize her payoff as per the cash flow rights laid down in the initial contract; under joint control, the firm is liquidated, and both agents receive their respective liquidation claims. I show that the efficient action plan is always implemented under manager control and joint control. The efficient action plan is also implemented under VC control, if the VC waits till θ to be revealed. When the manager has control, she agrees to choice of action a_s in the 'low' state, because her share of the renegotiation gains more than compensate for her lost control rents. Under joint control, the threat of inefficient liquidation causes both the agents to agree to the efficient action plan.

I model the renegotiation game between the manager and the VC as a Rubinstein game of alternating offers: Players make or respond to offers only at times t in the infinite set $T = \{0, 1, 2...\}$. At $t = 0$, the manager makes an offer which the VC can either accept or reject. In the event of rejection, the game proceeds to $t = 1$, at which time, the VC makes an offer. If the manager rejects the offer, game proceeds to $t = 2$, and so on. Let δ denote the time gap between times t and $t + 1$.

An offer in this game, denoted κ, takes the form $(a_i, (\pi_i + C_i - s, s))$. It consists of an action choice $a \in \{a_s, a_r\}$ and a payoff plan $(\pi_i + C_i - s, s)$, where π_i and C_i denote the cash flow and control rents generated under action a_i.[14] The payoffs to the manager and the VC if the offer is accepted are $\pi_i + C_i - s$ and s, respectively.[15]

To complete the description of the game, the disagreement point needs to be specified. The disagreement point, denoted D, is the outcome of the game if every offer in every time period is rejected. Under joint control, in case of failure to reach an agreement, the firm is

[14]So when action a_s is chosen, $\pi_i = \pi_s$, $C_i = 0$; when action a_r is chosen, $\pi_i \in \{\pi_{rh}, \pi_{rl}\}$, $C_i = C$.

[15]For the renegotiation game at date 1, the VC's opportunity cost m is not relevant because θ has already been observed.

liquidated. Therefore, $D = (a_l, (L - Y, Y))$, where a_l denotes liquidation. Under individual control, the disagreement point is determined by what the agent with the control right would have chosen in the absence of renegotiation. If the agent with the control right would have chosen action a_r in the absence of renegotiation, then $D = (a_r, (\pi_{rj} - S_{rj} + C, S_{rj}))$. Similarly, if the agent with the control right would have chosen action a_s in the absence of renegotiation, then $D = (a_s, (\pi_s - S_s, S_s))$.

Let $u_m(\kappa, t)$ and $u_{vc}(\kappa, t)$ denote the utilities of the manager and VC respectively if the offer κ is accepted at time t. $u_m(\kappa, t)$ and $u_{vc}(\kappa, t)$ are defined such that $u_m(D, .) = u_{vc}(D, .) = 0$, i.e., the manager and the VC value the payoffs they obtain over and above what they would obtain under the disagreement option. So if $D = (a_l, (L - Y, Y))$, then

$$u_m(\kappa, t) = \rho_m^{\delta t}(\pi_i + C_i - s - (L - Y))$$

$$u_{vc}(\kappa, t) = \rho_{vc}^{\delta t}(s - Y)$$

ρ_m and ρ_{vc} are constants satisfying $0 < \rho_m, \rho_{vc} < 1$. ρ_m and ρ_{vc} are referred to as the discount factors of the manager and the VC, respectively. They capture the impatience of the two players to reach an agreement sooner than later.

It is a well known fact that the structure of the Rubinstein game is asymmetric, i.e., the outcome of the game is different if the order of the players is reversed;[16] the player who proposes first has an advantage over the other player. The higher the δ, the more asymmetric is the game. The game becomes more and more symmetric as $\delta \to 0$. For convenience, I assume that $\delta \simeq 0$, although the results in this paper do not require this assumption.

Lemmas 1 and 2 below describe the subgame-perfect equilibrium (SPE) outcomes of the renegotiation game under the different disagreement points.

Define ρ and $\hat{Y}$ as follows,

$$\rho = \frac{\log \rho_m}{\log \rho_m + \log \rho_{vc}}$$

$$\hat{Y} \equiv (1 - \rho)\pi_{rh} - \rho(C - L) \tag{2}$$

Lemma 1 ***(SPE outcome of the renegotiation game with*** $D = (a_l, (L - Y, Y))$***):*** *The bargaining game has a unique SPE. In state θ_l, the VC and the manager reach an immediate agreement on action a_s; their respective payoffs are $Y + \rho(\pi_s - L)$ and $\pi_s - Y - \rho(\pi_s - L)$. In state θ_h, the VC and the manager reach an immediate agreement on action a_r; their respective payoffs are $\pi_{rh} - \max\left\{\hat{Y} - Y, 0\right\}$ and $\max\left\{\hat{Y} - Y, 0\right\} + C$.*

Proof. Please see appendix ■

[16] See Chapter 3 in Osborne and Rubinstein (1990)

In state θ_l, the manager and the VC agree on the choice of action a_s. The surplus from reaching this agreement is $\pi_s - L$. The renegotiation game determines how this surplus is divided. This situation is similar to two players bargaining over the division of a dollar. Just as in the "divide the dollar" game, this game too has a unique SPE in which the VC and the manager reach an immediate agreement on splitting the surplus $\pi_s - L$; their respective shares of this surplus are $\rho(\pi_s - L)$ and $(1-\rho)(\pi_s - L)$. The VC's payoff consists of its payoff under the disagreement option, Y, plus $\rho(\pi_s - L)$, and similarly for the manager. Notice that, as expected, ρ is decreasing in ρ_m and increasing in ρ_{vc}.[17] So as the manager becomes more patient relative to the VC, she captures a higher fraction of the surplus.

In state θ_h, the manager and the VC agree on the choice of action a_r. The surplus from reaching this agreement is $\pi_{rh} + C - L$. However, owing to the wealth constraint of the manager, a part of this surplus, consisting of control rents C, cannot be shared with the VC. In the unique SPE of this game, the VC's payoff is $Y + \rho(\pi_{rh} + C - L)$ or π_{rh}, whichever is lower. $\hat{Y}$ is the value of Y at which $Y + \rho(\pi_{rh} + C - L)$ equals π_{rh}. So if Y is higher than $\hat{Y}$, the VC captures the entire cash flow π_{rh}; the manager's payoff is her control rent C. Simple algebra shows that the VC's payoff can be written as $\pi_{rh} - \max\left\{\hat{Y} - Y, 0\right\}$.

Lemma 1 has the following important implications: First, joint control implements the efficient action in both states θ_l and θ_h. Second, the higher the VC's liquidation claim, Y, the higher (weakly) is its payoff. So liquidation claims, backed by the ability to force liquidation, enhance the VC's bargaining power under joint control. Other determinants of the VC's bargaining power are the relative values of the impatience parameters, ρ_m and ρ_{vc}. However, unlike the liquidation claim Y, these are exogenous parameters in this paper.

Lemma 2 below describes the equilibrium outcome of the following renegotiation games: (a) a game in which, in case of failure to reach an agreement, the firm chooses action a_s, and (b) a game in which, in case of failure to reach an agreement, the firm chooses action a_r. Define

$$\hat{S} \equiv (1-\rho)\,\pi_{rh} + \rho\,(\pi_s - C) \tag{3}$$

Lemma 2 ***(a) SPE outcome of the renegotiation game with*** $\mathbf{D} = (a_s, (\pi_s - S_s, S_s))$***:*** *The game has a unique SPE. In state θ_l, the VC and the manager reach an immediate agreement on action a_s; their respective payoffs are S_s and $\pi_s - S_s$. In state θ_h, the VC and the manager reach an immediate agreement on action a_r; their respective payoffs are $\pi_{rh} - \max\left\{\hat{S} - S_s, 0\right\}$ and $\max\left\{\hat{S} - S_s, 0\right\} + C$.*

(b) SPE outcome of the renegotiation game with $D = (a_r, (\pi_{rj} - S_{rj} + C, S_{rj}))$ ***in state*** θ_j***:*** *The game has a unique SPE. In state θ_h, the VC and the manager reach an immediate agreement on action a_r; their respective payoffs are S_{rh} and $\pi_{rh} - S_{rh} + C$.*

[17] ρ can be written as $\frac{1}{1+\left(\frac{|\log \delta_{vc}|}{|\log \delta_m|}\right)}$; $|\log \delta_m|$ and $|\log \delta_{vc}|$ are decreasing in δ_m and δ_{vc}, respectively, because $\delta_m, \delta_{vc} < 1$.

In state θ_l, the VC and the manager reach an immediate agreement on action a_s; their respective payoffs are $S_{rl} + \rho(\pi_s - \pi_{rl} - C)$ and $\pi_s - S_{rl} - \rho(\pi_s - \pi_{rl} - C)$.

Consider the renegotiation game in which, in case of failure to reach an agreement, the firm chooses action a_s. This situation arises when the VC has control, and the terms of the contract are such that it will choose action a_s in the absence of renegotiation.[18] In state θ_l, there is no scope for welfare improvement through renegotiation. This is because the VC will anyway choose the first-best action a_s. In state θ_h, renegotiation is welfare improving. The VC and the manager agree to a new contract and the firm chooses action a_r. The surplus from reaching this agreement is $(\pi_{rh} + C - \pi_s)$, which is shared by the VC and the manager. As explained above, not all of this surplus can be shared.

In the unique SPE of this game, the VC's payoff is $S_s + \rho(\pi_{rh} + C - \pi_s)$ or π_{rh}, whichever is lower. $\hat{S}$ is the value of S_s at which $S_s + \rho(\pi_{rh} + C - \pi_s)$ equals π_{rh}. So if S_s is higher than $\hat{S}$, the VC captures the entire cash flow π_{rh}; the manager's payoff is her control rent C. Simple algebra shows that the VC's payoff can be written as $\pi_{rh} - \max\left\{\hat{S} - S_s, 0\right\}$.

Next, consider the renegotiation game in which, in case of failure to reach an agreement, the firm chooses action a_r. This situation arises when the manager has control, and the terms of the contract are such that she will choose action a_r in the absence of renegotiation.[19] In state θ_h, there is no scope for welfare improvement through renegotiation. This is because the manager will anyway choose the first-best action a_r. In state θ_l, renegotiation is welfare improving. The VC and the manager agree to a new contract and the firm chooses action a_s. The surplus from reaching this agreement is $(\pi_s - \pi_{rl} - C)$, which is shared by the VC and the manager; their respective shares of this surplus are $\rho(\pi_s - \pi_{rl} - C)$ and $(1-\rho)(\pi_s - \pi_{rl} - C)$. The VC's payoff consists of its payoff under the disagreement option, S_{rl}, plus $\rho(\pi_s - \pi_{rl} - C)$, and similarly for the manager.

4 Optimal Control Allocation

An optimal control allocation is one that is feasible and maximizes the firm's value. Since the firm's value is maximized by implementing the efficient action plan, a control allocation is optimal if it is feasible and implements the efficient action plan. Note that feasibly implementing the efficient action plan is sufficient, but not necessary, for a control allocation to be optimal. As I discuss later in Section 5.1, for some firms, no control allocation can implement the efficient action plan. In this section, I discuss the circumstances under which the different control allocations can feasibly implement the efficient action plan.

[18] Alternatively, this situation can also arise if the initial contract explicitly directs the firm to choose action a_s in case of disagreement.

[19] Alternatively, this situation can also arise if the initial contract explicitly directs the firm to choose action a_r in case of a disagreement.

The main findings in this section are as follows: Manager control implements the efficient action plan, but is feasible only if the firm has sufficient financial slack, i.e., only if it is expected to generate enough cash to compensate the manager for her control rents and also compensate the VC for its investment in the firm; this requires a low C and a high q. VC control is efficient only if the VC's liquidity costs and monitoring costs are low, because otherwise it is impossible to dissuade the VC from choosing the safe action prematurely. Joint control implements the efficient action plan, but is feasible only if the firm has sufficient collateral. Overall, joint control is the optimal control allocation when the firm has low financial slack and a reasonable collateral value, and when the VC's liquidity constraints and cost of continuing to monitor and manage the firm are high.

4.1 Manager Control

When the manager is in control, she may be reluctant to choose the efficient action a_s in state θ_l, unless she obtains compensation for the control rents C that she forgoes by doing so. Such compensation may be provided to her in the initial contract; the manager then chooses a_s in state θ_l without necessitating renegotiation. Alternatively, the manager extracts her compensation for control rents C during ex-post renegotiation. A renegotiation-proof contract provides the manager with cash flow rights of at least C for choosing action a_s; this implies that the VC gets at most $\pi_s - C$ in state θ_l. Even if the initial contract is such that the manager's payoff is higher under action a_r in state θ_l, the contract will be renegotiated at date 1, and the firm will choose action a_s. Lemma 2(b) tells us that the VC's payoff after renegotiation is at most $(1-\rho)\,\pi_{rl} + \rho\,(\pi_s - C)$. Since $\pi_{rl} < \pi_s - C$ (Assumption 1(a)) and $\rho < 1$, the VC's payoff after renegotiation in state θ_l is less than $\pi_s - C$. Define

$$K_m \equiv q\pi_{rh} + (1-q)\,(\pi_s - C) \tag{4}$$

So under any contract with manager control, the VC's expected payoff is at most K_m. Clearly if $K_m < I + m$, the firm cannot raise enough money at date 0 to finance the investment I, by offering a contract with manager control. This result is stated in Lemma 3.

Lemma 3 *Manager control is feasible if and only if $K_m \geq I + m$. Whenever manager control is feasible, it implements the efficient action plan.*

The condition $K_m \geq I + m$ can be rewritten as $q\pi_{rh} + (1-q)\,\pi_s \geq I + m + (1-q)\,C$. So manager control is feasible only if the firm has sufficient financial slack, i.e., only if the firm is expected to comfortably generate enough cash flow to compensate the manager for her control rents, and also compensate the VC for its investment. This condition is unlikely to be met for firms with low q and high C. A low q means that the firm has a risky return distribution under the aggressive strategy a_r; there is a high probability that the 'low' state will be realized. A high C implies that the manager's human capital is critical for the firm's success.

These predictions are consistent with the empirical evidence on manager control in VC-financed firms. Kaplan and Stromberg (2001) report that the manager controlled a board majority in only 14% of all such firms they survey. Most VC-financed firms are start-up firms in high-tech sectors like information technology, telecom and biotechnology, that fit the description of a low q and a high C. Empirical and anecdotal evidence indicates that such firms generally do not have sufficient financial slack to support manager control. Sahlman (1990) reports that more than one-third of the investments made by VC funds result in absolute loss, and more that two-thirds result in a realized return of less than 10% per annum. More importantly, even firms that eventually succeed may be cash-strapped in the short-term.[20]

When $K_m \geq I + m$, the efficient action plan can be implemented by the following contract with manager control: $S(x, a_r) = x$, $S(x, a_s) = \left(\frac{\pi_s - C}{\pi_s}\right) x$, and $Y = L$. Under this contract, the manager gets a constant payoff of C irrespective of the action chosen. So the manager chooses the efficient action in both states.[21] This pattern of cash-flow rights and liquidation rights can be implemented by financing the venture using convertible preferred shares (or convertible debt), with the conversion ratio contingent on the action choice. Thus, the VC obtains higher cash flow rights if the firm chooses action a_r.

4.2 VC Control

When the VC is in control of the firm, it may force the safe action on the firm prematurely in order to save on its opportunity cost m. The VC's expected cash flow, if it waits for θ to be revealed, is $qS_{rh} + (1-q) S_s$. So it will choose action a_s prematurely if $S_s > qS_{rh} + (1-q) S_s - m$, i.e., if $S_s > S_{rh} - \frac{m}{q}$. The manager can renegotiate with the VC and offer to increase S_{rh} in order to persuade the VC to wait till θ is revealed. However, if $S_s > \pi_{rh} - \frac{m}{q}$, renegotiation cannot succeed, and the VC chooses a_s prematurely. Therefore, VC control is efficient only if $S_s \leq \pi_{rh} - \frac{m}{q}$. Define

$$K_{vc} \equiv q\pi_{rh} + (1-q)\left(\pi_{rh} - \frac{m}{q}\right) \tag{5}$$

For any contract with VC control that implements the efficient action plan, the expected payoff to the VC cannot exceed K_{vc}. So if $K_{vc} < I+m$, it is not possible to design a contract

[20] A prominent example mentioned in Sahlman (1990) is Federal Express. With the company behind plan and over budget, VCs had to infuse cash to rescue the company on two occasions.

[21] I am following the convention that if an agent is payoff indifferent between two actions, she will choose the efficient action.

with VC control that will implement the efficient action plan. This result is stated formally in Lemma 4.

Lemma 4 *VC control can implement the efficient action plan if and only if* $K_{vc} \geq I + m$.

The condition $K_{vc} \geq I+m$ can be rewritten as $q\pi_{rh}+(1-q)\,\pi_s \geq I+m+(1-q)\left(\frac{m}{q}-\Delta\right)$. Recall that $\frac{m}{q}-\Delta > 0$ (Assumption 2). So the requirement $K_{vc} \geq I+m$ may not be met for firms with low q, low Δ and high m. A low Δ means that the incremental cash flow from choosing the aggressive action over the safe action in the 'high' state, is low. A high m implies that the VC's liquidity constraints and cost of continuing to monitor and manage the firm are high. All else equal, the lower the q and Δ, and the higher the m, the more likely it is that the VC will choose the safe action prematurely. The only way to dissuade the VC from choosing action a_s prematurely is to reduce its payoff under action a_s; but then, the VC's expected payoff may be so low that it will refuse to invest in the firm at date 0.

When the condition $K_{vc} \geq I+m$ is met, then the following contract with VC control can implement the efficient action plan: $S\left(x,a_r\right) = x$, $S\left(x,a_s\right) = \frac{1}{\pi_s}\left(\pi_{rh}-\frac{m}{q}\right)x$ and $Y = \min\left\{L, \pi_{rh}-\frac{m}{q}\right\}$.Under such a contract $S_{rh} = \pi_{rh}$, $S_s = \pi_{rh}-\frac{m}{q}$ and $S_{rl} = \pi_{rl}$. Since $S_s \leq qS_{rh}+(1-q)\,S_s - m$, the VC will not force action a_s on the firm prematurely. Since $S_{rh} > S_s$, the VC will choose action a_r in state θ_h. Lastly, $\pi_{rh}-\frac{m}{q} > \pi_s - C$ (Assumption 2) and $\pi_s - C > \pi_{rl}$ (Assumption 1(a)) imply that $S_s > S_{rl}$; so the VC will choose action a_s in state θ_l. In other words, the VC waits till θ is revealed, and then chooses the efficient action in both states. As noted in the discussion following Lemma 3, this pattern of cash-flow rights and liquidation rights can be implemented by financing the venture using long-term convertible preferred shares (or convertible debt), with the conversion ratio contingent on the action choice.

Finally, $m < q\Delta_v$ (Assumption 2) implies that $K_{vc} > K_m$. So whenever manager control is feasible, the efficient action plan can also be implemented by giving control to the VC. Moreover, there might be firms for which $K_{vc} \geq I+m > K_m$; for such firms, manager control is infeasible but VC control can implement the efficient action plan. This result is stated as a corollary to Lemmas 3 and 4.

Corollary 5 *(to Lemmas 3 and 4)* $K_{vc} > K_m$. *So manager control cannot strictly dominate VC control.*

4.3 Joint Control

When the control right is held jointly by the manager and the VC, the VC cannot choose action a_s prematurely as the manager will not allow it to do so;[22] the firm then waits till θ is revealed. In state θ_l, the VC gets a payoff of $Y + \rho(\pi_s - L)$ if it intervenes, and a payoff of at most $\pi_s - C$ if it doesn't (see discussion preceding Lemma 3). So in state θ_l, the VC's payoff is at most $L + \rho(\pi_s - L)$ or $\pi_s - C$, whichever is higher.

Define

$$K_j \equiv q\pi_{rh} + (1-q)\max\{\pi_s - C, L + \rho(\pi_s - L)\} \tag{6}$$

The above discussion shows that under any contract with joint control, the VC's expected payoff is at most K_j. So if $K_j < I + m$, the firm will not be able to raise enough money at date 0 to finance the investment I, by offering a contract with joint control. Lemma 6 states this result.

Lemma 6 *Joint control is feasible if and only if $K_j \geq I + m$. Whenever joint control is feasible, it implements the efficient action plan.*

Lemma 6 implies that joint control is feasible if and only if the firm has sufficient collateral value. This is because K_j is increasing in L. The higher the L, the more likely it is that the condition $K_j \geq I + m$ is met. Also notice that $K_j \geq K_m$; $K_j = K_m$ unless $L + \rho(\pi_s - L) > \pi_s - C$. So whenever manager control is feasible, so is joint control, but not vice-versa.

When the condition $K_j \geq I + m$ is met, the efficient action plan can be implemented by the following contract with joint control: $S(x, a_r) = x$, $S(x, a_s) = \left(\frac{\pi_s - C}{\pi_a}\right)x$, $Y = L$. In state θ_h, the manager chooses the efficient action a_r; the VC does not intervene, and gets a payoff of π_{rh}. In state θ_l, the VC intervenes if and only if $L + \rho(\pi_s - L)$ exceeds $\pi_s - C$; its payoff is $\max\{(\pi_s - C), L + \rho(\pi_s - L)\}$. This pattern of cash flows and liquidation claims can be implemented by financing the firm with convertible debt or convertible preferred shares with face value equal to L, with a redemption right for the VC maturing at date 1. The crucial difference between joint control on one hand, and manager control and VC control on the other, is that the VC can now threaten to force liquidation of the firm at date 1. Of course, as shown in Lemma 1, liquidation never occurs; it is the credibility of the threat that matters.

[22]Note that the VC has a redemption right maturing at date 1. Prior to date 1, it cannot threaten liquidation of the firm in case of disagreement with the manager.

Joint control vs. individual control

Notice that under individual control, the agent with control needs to be provided incentives to choose the efficient action. Moreover, since the manager has no personal wealth and since the VC is constrained for funds, these incentives must come from the firm's cash flows. When control is held by the manager, the manager needs to be rewarded for choosing action a_s in the 'low' state, in the form of a compensation for the control rents she forgoes. When control is held by the VC, the VC needs to be punished for choosing action a_s, in order to dissuade it from choosing action a_s prematurely. So under both forms of individual control, the VC's payoff under action a_s must be low if the efficient action plan is to be implemented. At the same time, the expected payoff to the VC must be high enough to persuade it to invest in the firm. These two conflicting objectives may be difficult to reconcile for firms with low q, high C and high m.

Joint control, on the other hand, doesn't require punishing the VC (or rewarding the manager) in order to implement the efficient action plan. The reasons are as follows: First, faced with the threat of inefficient liquidation, the manager cannot insist on obtaining compensation for her control rents before agreeing to the choice of action a_s in the 'low' state. Second, the VC cannot choose action a_s prematurely because it needs the consent of the manager to do so, failing which the firm will wait till date 1. The only issue then is whether the expected payoff to the VC is high enough to persuade her to invest in the firm, or not. As shown earlier, this depends on the firm's collateral, L. The higher the L, the more likely it is that joint control is feasible.

Consider a firm for which $K_{vc} < I + m$. Then, VC control cannot implement the efficient action plan (Lemma 4). Moreover, manager control is also infeasible (Corollary 5). Suppose the firm has enough collateral that $K_j \geq I + m$. Then joint control is feasible and implements the efficient action plan (Lemma 6). For such a firm, joint control strictly dominates both VC control and manager control.

Proposition 7 below examines the circumstances under which joint control strictly dominates individual control. Define

$$L^{vc} = \pi_s - \frac{1}{(1-\rho)}\left(\frac{m}{q} - \Delta\right) \tag{7}$$

Proposition 7 *Joint control strictly dominates individual control if and only if the following conditions are satisfied:*

1. $L > L^{vc}$, *and*
2. $I + m$ *lies in the interval* $(K_{vc}, K_j]$

The requirement that $I + m$ must lie in the interval $(K_{vc}, K_j]$ (Condition 2) is obvious from the discussion above. But, for this to be possible, K_j must exceed K_{vc}. Since K_j is increasing in L, $K_j > K_{vc}$ if and only if L exceeds the threshold L^{vc} defined above (Condition 1).

Some explanatory comments regarding Proposition 7: First, the lower the q and Δ, and the higher the m, the lower is the threshold L^{vc}; hence, the more likely it is that the firm's collateral exceeds the threshold L^{vc}. As discussed earlier following Lemma 4, low q and Δ, and a high m also make it more likely that $I + m > K_{vc}$. Second, $m < q\Delta_v$ (Assumption 2) implies that $\frac{m}{q} - \Delta < C$. So $\frac{m}{q} - \Delta$ can be large only if C is large. Third, the greater the difference $(L - L^{vc})$, the wider is the interval $(K_{vc}, K_j]$; hence, the more likely it is that $I + m$ lies in this interval.

To summarize, joint control strictly dominates individual control when the following circumstances are met: (a) the firm has a low financial slack; (b) the VC's liquidity constraints, and cost of continuing to monitor and manage the firm are high (high m); and (c) the firm has a reasonable collateral value ($L > L^{vc}$). These predictions are consistent with empirical evidence on the characteristics of firms that receive VC financing.

As argued in the introduction, firms that receive VC financing fit the above description. VC-financed firms have low financial slack. They are costly to monitor and manage because they are generally involved in developing new products/ services, and so there is considerable uncertainty surrounding their business prospects. Finally, VC-financed firms are generally later-stage firms that have some collateral, in the form of fixed assets, inventories, receivables, marketing rights of existing products, patents etc., that can be seized in the event of liquidation.

Overall, the discussion in this section can be summarized in terms of the parameters K_m, K_{vc} and K_j as follows: If $K_m \geq I + m$, the initial contract can assign the control right to the manager. However, if $K_m < I + m$, the manager will either have to share control with the VC or give away control to the VC altogether, depending on whether K_j or K_{vc} (or both) is greater than $I + m$. If $\max\{K_m, K_{vc}, K_j\} < I + m$, then manager control and joint control are infeasible, and VC control cannot implement the efficient action plan. The only option then is to implement the inefficient action plan by giving away control to the VC altogether.

5 Contingent Control

So far, I have assumed that contracts may not be made contingent on θ, as it is not verifiable. In this section, I consider the possibility that contracts may be made contingent on a verifiable signal $\gamma \in \{l, h\}$, that is imperfectly correlated with θ. I use the modified notation $S^{\gamma}(x,a)$, Y^{γ} and ψ^{γ} to denote cash flow rights, liquidation rights and control rights contingent on the signal γ.

For simplicity, let $\Pr(\gamma = h|p = p_h) = \alpha = \Pr(\gamma = l|p = p_l)$, where α is a constant satisfying $\frac{1}{2} < \alpha < 1$. The variable α measures the informativeness of the signal γ. The higher the α, the more informative is γ regarding the state θ; $\alpha = \frac{1}{2}$ means that γ is completely uninformative, while $\alpha = 1$ means that γ is a perfect proxy for θ.

Let $q_l(\alpha)$ denote the posterior probability that the 'high' state will be realized, after $\gamma = l$ has been observed.

$$q_l(\alpha) \equiv \Pr(\theta = \theta_h|\gamma = l) = \frac{(1-\alpha)q}{(1-\alpha)q + \alpha(1-q)}$$

As expected, $q_l(\alpha) < q$, and $q_l(\alpha)$ is decreasing in α. In other words, when the signal γ is very precise, $\gamma = l$ most likely means that the 'low' state will be realized.

Let α^* denote the level of α at which $q_l(\alpha)\Delta_v = m$. So if $\alpha > \alpha^*$, then $q_l(\alpha)\Delta_v < m$, i.e., it is efficient to choose action a_s without waiting for θ to be revealed. But, the efficient action plan was defined as waiting for θ to be realized, and then choosing action a_r in the 'high' state, and action a_s in the 'low' state. In order to keep the definition of efficient action plan consistent with the earlier sections, I assume that:

Assumption 4: $\alpha \le \alpha^*$ (or alternatively, $q_l(\alpha)\Delta_v \ge m$)[23]

Consider the control allocation $\{\psi^h = M, \psi^l = V\}$, i.e., the manager has the control right, but if $\gamma = l$ is realized, control switches to the VC. A well known result from Aghion and Bolton (1992) is that when manager control is infeasible and VC control is inefficient, then the control allocation $\{\psi^h = M, \psi^l = V\}$ may be the optimal governance mechanism for the firm. In this section, I examine how the control allocation $\{\psi^h = M, \psi^l = V\}$ compares with joint control.

Remark 8 *When manager control is infeasible and VC control is inefficient, the control allocation $\{\psi^h = M, \psi^l = V\}$ cannot feasibly implement the efficient action plan.*

This is a fairly obvious result. A 'low' realization of the signal γ means that there is a higher probability of the 'low' state being realized. So if the VC cannot be dissuaded from choosing the safe action prematurely given its prior beliefs regarding θ, it cannot be dissuaded from choosing the safe action after observing a 'low' realization of γ. An

[23] This assumption does not mean that the optimality of joint control vanishes if $\alpha > \alpha^*$. When $\alpha > \alpha^*$, the definition of the efficient action plan itself changes. So, V^*, K_m, K_{vc}, and K_j will also change. Joint control could still turn out to be the optimal governance mechanism. However, as $\alpha \to 1$, the control allocation $\{\psi^h = M, \psi^l = V\}$ (contingent control) will dominate joint ontrol.

immediate implication of Remark 8 is that when joint control strictly dominates individual control, it strictly dominates the control allocation $\{\psi^h = M, \psi^l = V\}$ as well. So when the conditions in Proposition 7 are met, joint control is the strictly optimal governance mechanism for the firm.

5.1 Second-best Firm Value

Consider a firm for which $\max\{K_j, K_{vc}\} < I + m$. Then manager control and joint control are infeasible, and VC control cannot implement the efficient action plan. Moreover, Remark 8 says that the control allocation $\{\psi^h = M, \psi^l = V\}$ also cannot implement the efficient action plan. So the optimal firm value, V^*, cannot be achieved. This is because when the VC is in control, it forces the safe action on the firm prematurely.

When the efficient action plan cannot be implemented, the next best thing that the firm can do is to implement the second-best action plan. Consider a contract that gives control to the VC if $\gamma = l$ is realized. Under the second-best plan, the VC chooses action a_s without waiting for θ to be revealed whenever it is in control. To see why this is inefficient relative to the efficient action plan, consider the event $(\theta = \theta_h, \gamma = l)$. Under the efficient action plan, the firm would have chosen action a_r after observing $\theta = \theta_h$, but under the second-best plan the VC chooses action a_s after $\gamma = l$ is realized. So in the event $(\theta = \theta_h, \gamma = l)$, the firm's value is π_s under the second-best plan, instead of $\pi_{rh} + C$ under the efficient action plan. Define

$$V^{sb} = q\left[\alpha\left(\pi_{rh} + C\right) + (1-\alpha)\pi_s\right] + (1-q)\pi_s \tag{8}$$

V^{sb} is the firm's expected value when the second-best action plan is implemented. I refer to V^{sb} as the second-best firm value. Notice that $V^{sb} < V^*$, and that the difference $V^* - V^{sb}$ is decreasing in α. This is because the more precise the signal γ, the less likely is the event $(\theta = \theta_h, \gamma = l)$.

The second-best action plan can be implemented by a control allocation in which control switches to the VC only following a 'low' realization of the signal γ. Two such control allocations are: $\{\psi^h = M, \psi^l = V\}$ and $\{\psi^h = J, \psi^l = V\}$; in the latter allocation, control is jointly held by the manager and the VC, but the VC gets exclusive control if $\gamma = l$ is realized. I refer to the control allocation $\{\psi^h = J, \psi^l = V\}$ as partial joint control.

Define

$$K_{mv} = q\left[\alpha\pi_{rh} + (1-\alpha)\pi_s\right] + (1-q)\left[(1-\alpha)(\pi_s - C) + \alpha\pi_s\right] \tag{9}$$

$$K_{jv} = q\left[\alpha\pi_{rh} + (1-\alpha)\pi_s\right] + \\ (1-q)\left[(1-\alpha)\max\left\{L + \rho(\pi_s - L), (\pi_s - C)\right\} + \alpha\pi_s\right] \tag{10}$$

Under the control allocation $\left\{\psi^h = M, \psi^l = V\right\}$, the manager retains control of the firm if $\gamma = h$. So the manager has control in the event $(\theta = \theta_l, \gamma = h)$. Then, as discussed in Section 4.1, the manager chooses action a_s, and the VC's payoff is at most $\pi_s - C$. The event $(\theta = \theta_l, \gamma = h)$ occurs with probability $(1-q)(1-\alpha)$. So when the control allocation $\left\{\psi^h = M, \psi^l = V\right\}$ implements the second-best action plan, the VC's expected payoff cannot exceed K_{mv}.

Similarly, under the control allocation $\left\{\psi^h = J, \psi^l = V\right\}$, control is jointly held by the manager and the VC in the event $(\theta = \theta_l, \gamma = h)$. Then, as discussed in Section 4.3, the efficient action a_s is chosen, and the VC's payoff is at most $\max\left\{L + \rho(\pi_s - L), (\pi_s - C)\right\}$. So when partial joint control implements the second-best action plan, the VC's expected payoff cannot exceed K_{jv}.

A contract that implements the second-best action plan is feasible only if the VC's expected payoff exceeds $I + [q\alpha + (1-q)(1-\alpha)]\,m$. Recall that the VC incurs the cost m only when the firm waits for θ to be revealed. Under the second best plan, when the VC is in control, it chooses action a_s without waiting for θ to be revealed. So the cost m is incurred only if $\gamma = h$ is realized, because then the VC is not in control; this occurs with probability $q\alpha + (1-q)(1-\alpha)$.

Lemma 9 *(i) The control allocation* $\left\{\psi^h = M, \psi^l = V\right\}$ *can implement the second-best action plan if and only if* $K_{mv} \geq I + [q\alpha + (1-q)(1-\alpha)]\,m$.

(ii) Partial joint control can implement the second-best action plan if and only if $K_{jv} \geq I + [q\alpha + (1-q)(1-\alpha)]\,m$.

Lemma 9 says that when the efficient action plan cannot be implemented, it might be possible to implement the second-best action plan if α is sufficiently high, i.e., if the signal γ is precise enough. Notice that $K_{jv} \geq K_{mv}$. So partial joint control dominates the control allocation $\left\{\psi^h = M, \psi^l = V\right\}$; the dominance is strict only when $I + [q\alpha + (1-q)(1-\alpha)]\,m \in (K_{mv}, K_{jv}]$, which in turn is possible only when $L > \pi_s - \frac{1}{(1-\rho)}C \equiv L^m$. In other words, partial joint control strictly dominates the control allocation $\left\{\psi^h = M, \psi^l = V\right\}$ only when the manager's control rents are high, and the firm's collateral exceeds the threshold L^m.

Finally, if $K_{jv} < I + [q\alpha + (1-q)(1-\alpha)]\,m$, neither partial joint control nor the control allocation $\left\{\psi^h = M, \psi^l = V\right\}$ can implement the second-best action plan. The only option for the manager then is VC control, even though that is inefficient.

6 Conclusion

The main result in this paper is that assigning control jointly to the manager and the VC, and specifying a harsh penalty (inefficient liquidation, in this paper) if the agents fail to reach an agreement, is sometimes better than assigning control exclusively to either the manager or the VC. The threat of inefficient liquidation influences the agents to agree to the efficient decision; so unlike under individual control, no incentives need be provided to implement the efficient decision. So for firms that are highly financially constrained, joint control is optimal, provided the firm has sufficient collateral.

The paper predicts that the optimal control allocation is determined by factors such as the firm's financial slack, collateral value, VC's liquidity constraints and monitoring costs. For firms with high financial slack, manager control is feasible and optimal. For firms with somewhat lower financial slack, VC control is efficient provided the firm is easy to monitor and manage, and the VC is not liquidity constrained. For firms with very low financial slack but reasonable collateral value, joint control strictly dominates both VC control and manager control. Finally, for firms with very low financial slack and low collateral value, the only feasible option is inefficient VC control.

The predictions in this paper suggest avenues for future research. The paper provides the following empirically testable predictions regarding control allocations for start-up firms:

- VC control is more likely in early-stage start-up firms than in later-stage firms.
- Joint control is more likely in later-stage firms in high-tech industries.
- Manager control is more likely in later-stage firms, and in firms with higher capacity to raise external financing.

Another possible avenue for future research is to examine how the firm's choice of exit route depends on factors such as control allocation, VC's liquidity constraints, etc. For example: Are firms with VC in control less likely to undertake IPOs/ more likely to be sold sooner than firms with the manager in control? Is the exit decision influenced by the overall funds inflow into venture capital funds? Is it influenced by the number of other firms in the VC's portfolio?

References

Aghion, P. and Bolton, P. (1992). An incomplete contracts approach to financial contracting. *Review of Economic Studies*, *59*, 473–494.

Berglof, E. (1994). A control theory of venture capital finance. *Journal of Law, Economics and Organizations*, *10*(2), 247–267.

Cai, H. (2003). A theory of joint ownership. *Rand Journal of Economics*, *34*, 63–77.

Cestone, G. (2001). Venture capital meets contract theory: Risky claims or formal control? Working paper.

Diamond, D. (1993). Seniority and maturity of debt contracts. *Journal of Financial Economics*, *33*, 341–368.

Fenn, G. W., Liang, N., and Prowse, S. (1995). The economics of the private equity market. Board of Governors of the Federal Reserve System Publication.

Gompers, P. A. (1995). Optimal investment, monitoring, and the staging of venture capital. *Journal of Finance*, *50*, 1461–1489.

Gorman, M. and Sahlman, W. A. (1989). What do venture capitalists do? *Journal of Business Venturing*, *4*, 231–248.

Hart, O. (1995). *Firms contracts and financial structure*. Oxford University Press.

Hauswald, R. and Hege, U. (2002). Ownership and control in joint ventures: Theory and evidence. Working Paper.

Hellmann, T. (1998). The allocation of control rights in venture capital contracts. *Rand Journal of Economics*, *29*(1), 57–76.

Hellmann, T. and Puri, M. (2000). The interaction between product market and financing strategy: The role of venture capital. *Review of Financial Studies*, *13*(4), 959–984.

Hellmann, T. and Puri, M. (2002). Venture capital and the professionalization of start-up firms: Empirical evidence. *Journal of Finance*, *57*(1), 169–197.

Kaplan, S. and Stromberg, P. (2001). Financial contracting theory meets the real world: An empirical analysis of venture capital contracts. Working paper.

Kaplan, S. and Stromberg, P. (2002). Characteristics, contracts, and actions: Evidence from venture capital analyses. Working paper.

Kirilenko, A. (2001). Valuation and control in venture finance. *Journal of Finance*, *56*(2), 565–586.

Lerner, J. (1995). Venture capitalists and the oversight of private firms. *Journal of Finance*, *50*, 301–318.

Osborne, M. J. and Rubinstein, A. (1990). *Bargaining and Markets*. Academic Press.

Sahlman, W. A. (1990). The structure and governance of venture-capital organizations. *Journal of Financial Economics*, *27*, 473–521.

Appendix

Proof. (of Lemma 1) The strategy pair in a subgame perfect equilibrium (SPE) of an extensive game must induce a Nash equilibrium in every subgame of the extensive game. This implies that,

(i) In state θ_l, (in the SPE) neither the manager nor the VC will ever propose action a_r at any time. This is because the offer $(a_r, \pi_{rl} + C - s, s)$ can never be a best response, as it is clearly dominated by the offer $(a_s, (\pi_s - s - \varepsilon, s + \varepsilon))$ for some $\varepsilon > 0$. Therefore, both players agree on action a_s. The bargaining game then only determines how the surplus π_s is to be shared. As is well known (see Osborne and Rubinstein (1990)), this game has a unique SPE, that happens to be stationary. This stationary SPE, denoted $(\sigma_s^*, \hat{\sigma}_s)$, is characterized in Claim 10 below. If $\delta \simeq 0$, then the payoffs to the VC and the manager are $Y + \rho(\pi_s - L)$ and $\pi_s - Y - \rho(\pi_s - L)$, respectively.

(ii) Similarly, in state θ_h, (in the SPE) neither the manager nor the VC will ever propose action a_s at any time. This is because the offer $(a_s, (\pi_s - s, s))$ can never be a best response, as it is clearly dominated by the offer $(a_r, (\pi_{rh} + C - s, s))$. Therefore, both players agree on action a_r. The bargaining game then only determines how the surplus $\pi_{rh} + C$ is to be shared. As above, this game has a unique SPE, that happens to be stationary. This stationary SPE, denoted $(\sigma_r^*, \hat{\sigma}_r)$, is characterized in Claim 11 below. If $\delta \simeq 0$, then the payoffs to the VC and the manager are $\pi_{rh} - \max\left\{\hat{Y} - Y, 0\right\}$ and $\max\left\{\hat{Y} - Y, 0\right\} + C$, respectively, where $\hat{Y} \equiv (1 - \rho)\pi_{rh} - \rho(C - L)$. ■

Claim 10 *In the stationary SPE $(\sigma_s^*, \hat{\sigma}_s)$, the manager always proposes $\kappa^* = (a_s, (\pi_s - s_s^*, s_s^*))$ whenever it is her turn to propose, and accepts any offer $(a, (\pi_a + C_a - s, s))$ proposed by the VC if and only if $\pi_a + C_a - s \geq \pi_s - \hat{s}_s$; the VC always proposes $\hat{\kappa} = (a_s, (\pi_s - \hat{s}_s, \hat{s}_s))$ whenever it is its turn to propose, and accepts any offer $(a, (\pi_a + C_a - s, s))$ proposed by the manager if and only if $s \geq s_s^*$. The outcome is that the manager proposes κ^* at t=0, which the VC readily accepts. As $\delta \to 0$, $s_s^* \simeq \hat{s}_s \simeq Y + \rho(\pi_s - L)$, where $\rho = \left(\frac{\log \rho_m}{\log \rho_m + \log \rho_{vc}}\right)$.*

Proof. I now characterize s_s^* and $\hat{s}_s$.

(i) If the VC rejects an offer $(a_s, (\pi_s - s, s))$ at t=0, it proposes $\hat{\kappa}$ at t=1, which the manager accepts. Therefore, for the VC's rejection strategy to be credible it must be that $\rho_{vc}^{\delta}(\hat{s}_s - Y) > s - Y$, for all $s < s_s^*$. Similarly, for the VC's acceptance strategy to be credible, it must be that $\rho_{vc}^{\delta}(\hat{s}_s - Y) \leq s - Y$, for all $s \geq s_s^*$. Since $u_{vc}(., t)$ is continuous, we conclude that,

$$\rho_{vc}^{\delta}(\hat{s}_s - Y) = s_s^* - Y \tag{11}$$

(ii) Similarly, for the manager's acceptance and rejection strategies to be credible, the following condition must hold,

$$\rho_m^\delta (\pi_s - s_s^* - L + Y) = \pi_s - \hat{s}_s - L + Y \tag{12}$$

Solving equations (11) and (12), we obtain,

$$s_s^* = Y + \frac{\rho_{vc}^\delta (1 - \rho_m^\delta)}{(1 - \rho_m^\delta \rho_{vc}^\delta)} (\pi_s - L)$$

$$\hat{s}_s = Y + \frac{(1 - \rho_m^\delta)}{(1 - \rho_m^\delta \rho_{vc}^\delta)} (\pi_s - L)$$

Using L'Hospital rule,

$$\lim_{\delta \to 0} \hat{s} = \lim_{\delta \to 0} s^* = Y + \rho (\pi_s - L)$$

■

Claim 11 *In the stationary SPE* $(\sigma_r^*, \hat{\sigma}_r)$*, the manager always proposes* $\kappa^* = (a_r, (\pi_r - s_r^* + C, s_r^*))$ *whenever it is her turn to propose, and accepts any offer* $(a, (\pi_a + C_a - s, s))$ *proposed by the VC if and only if* $\pi_a + C_a - s \geq \pi_r + C - \hat{s}_r$*; the VC always proposes* $\hat{\kappa} = (a_r, (\pi_r + C - \hat{s}_r, \hat{s}_r))$ *whenever it is its turn to propose, and accepts any* $(a, (\pi_a + C_a - s, s))$ *proposed by the manager if and only if* $s \geq s_r^*$*. The outcome is that the manager proposes* κ^* *at t=0, which the VC readily accepts. If* $\delta \to 0$*,* $s^* \simeq \hat{s} \simeq \pi_r - \max\left\{\hat{Y} - Y, 0\right\}$*, where* $\hat{Y} \equiv (1 - \rho) \pi_r - \rho (C - L)$*.*

Proof. The players' strategies under the stationary SPE $(\sigma_r^*, \hat{\sigma}_r)$ are similar to that under $(\sigma_s^*, \hat{\sigma}_s)$ above. I now characterize s_r^* and $\hat{s}_r$. It must be true that $s_r^*, \hat{s}_r \leq \pi_{rh} + C - L + Y$, because otherwise the manager will be better off under the disagreement option where her payoff is $L - Y$. Combining this with limited liability, we require $s_r^*, \hat{s}_r \leq \min \{\pi_{rh}, \pi_{rh} + C - L + Y\} \equiv s_u$.

(i) If the VC rejects an offer at t=0, it proposes $\hat{\kappa}$ at t=1, which the manager accepts. Therefore, for the VC's rejection strategy to be credible it must be that $\rho_{vc}^\delta (\hat{s}_r - Y) > s - Y$, for all $s < s_r^*$. Similarly, for the VC's acceptance strategy to be credible, it must be that $\rho_{vc}^\delta (\hat{s}_r - Y) \leq s - Y$, for all $s \geq s_r^*$. Since $u_{vc}(., t)$ is continuous, we conclude that,

$$\rho_{vc}^\delta (\hat{s}_r - Y) = s_r^* - Y \tag{13}$$

(ii) Similarly, for the manager's rejection and acceptance strategies to be credible,

$$\rho_m^\delta (\pi_{rh} - s_r^* + C - L + Y) = \pi_{rh} - \hat{s}_r + C - L + Y \text{ or } \hat{s}_r = s_u \tag{14}$$

Let's conjecture that $\hat{s}_r$ is interior (need to verify this later). Then, solving the system of equations (13) and (14),

$$\hat{s}_r = Y + \frac{\left(1-\rho_m^\delta\right)}{\left(1-\rho_m^\delta\rho_{vc}^\delta\right)}\left(\pi_{rh}+C-L\right)$$

$$s_s^* = Y + \frac{\rho_{vc}^\delta\left(1-\rho_m^\delta\right)}{\left(1-\rho_m^\delta\rho_{vc}^\delta\right)}\left(\pi_{rh}+C-L\right)$$

Applying L' Hospital rule, we obtain that,

$$\lim_{\delta\to 0}\ \hat{s}_r = \lim_{\delta\to 0}\ s_r^* = Y + \rho\left(\pi_{rh}+C-L\right) \tag{15}$$

We still need to verify that $\hat{s}_r < s_u$.

Claim: $\hat{s}_r < s_u$ if and only if $Y < \hat{Y} \equiv (1-\rho)\,\pi_{rh} - \rho\,(C-L)$; otherwise $\hat{s} = \pi_{rh}$.

Proof: Recall that $s_u = \min\left[\pi_{rh}, \pi_{rh}+C-L+Y\right]$. (Necessity) Suppose $Y \geq \hat{Y} \equiv (1-\rho)\,\pi_{rh} - \rho\,(C-L)$. Now, $\pi_{rh}+C-L > 0 \Rightarrow \pi_{rh} > -(C-L)$. Therefore, $\hat{Y} > -(C-L)$. Since $Y > \hat{Y}$, this implies that $Y+C-L > 0 \Rightarrow s_u = \pi_{rh}$. From equation (15), it is then obvious that $\hat{s}_r \geq \hat{Y} + \rho\left(\pi_{rh}+C-L\right) = \pi_{rh}$, which can't be. Therefore, $\hat{s}_r = s_u = \pi_{rh}$, in this case. (Sufficiency) Suppose $Y < \hat{Y}$. Now, it is obvious from equations (15) that $\hat{s}_r < \pi_{rh}+C-L+Y$. Also, $\hat{s}_r < \hat{Y} + \rho\left(\pi_{rh}+C-L\right) = \pi_{rh}$. Therefore, $\hat{s}_r < s_u$. This concludes the proof of sufficiency.

Notice that $\hat{s} = \pi_{rh} - \left(\hat{Y}-Y\right)$ if $Y < \hat{Y}$ and $\hat{s} = \pi_{rh}$ otherwise. In other words, $\hat{s} = \pi_{rh} - \max\left\{\hat{Y}-Y, 0\right\}$. Therefore, the payoffs to the VC and the manager are $\pi_{rh} - \max\left\{\hat{Y}-Y, 0\right\}$ and $\max\left\{\hat{Y}-Y, 0\right\} + C$, respectively. ■

Proof. (of Lemma 2(a)) Given $D = (a_s, (\pi_s - S_s, S_s))$. In state θ_l, by the same argument used in the proof of Lemma 1, it is evident that the two players agree on action a_s. The outcome of the game then is that the manager proposes $D = (a_s, (\pi_s - S_s, S_s))$ at t=0, which the VC accepts. This is because for any $\kappa = (a_s, (\pi_s - s, s)) \neq D$, either $u_m(\kappa, .) < 0$ or $u_{vc}(\kappa, .) < 0$. The payoffs to the VC and the manager are S_s and $\pi_s - S_s$, respectively.

Similarly in state θ_h, the manager and the VC agree on action a_r (the key here is that $\pi_{rh} \geq \pi_s$. Suppose this weren't the case, then the players would have agreed on action a_r only if $S_s \leq \pi_{rh}$). The bargaining game then has a unique SPE that is also stationary. Following the same logic as in proof of Claim 11 above (with π_s replacing L, and S_s replacing Y), define $\hat{S} \equiv (1-\rho)\,\pi_{rh} - \rho\,(C-\pi_s)$. Payoffs to the VC and the manager are $\pi_{rh} - \max\left\{\hat{S}-S_s, 0\right\}$ and $\max\left\{\hat{S}-S_s, 0\right\}$, respectively. ■

Proof. (of Lemma 2(b)) Given $D = (a_r, (\pi_{rj} + C - S_{rj}, S_{rj}))$ in state θ_j. In state θ_l, by the same argument used in the proof of Lemma 1, it is evident that the two players agree on action a_s. The bargaining game then has a unique SPE that is also stationary. Following the same logic as in proof of Claim 10 above (with $\pi_{rl} + C$ replacing L, and S_{rl} replacing Y), we obtain that the payoffs to the VC and the manager are $S_{rl} + \rho(\pi_s - \pi_{rl} - C)$ and $\pi_s - S_{rl} - \rho(\pi_s - \pi_{rl} - C)$, respectively.

Similarly in state θ_h, the manager and the VC agree on action a_r.The outcome of the game then is that the manager proposes $D = (a_r, (\pi_{rh} + C - S_{rh}, S_{rh}))$ at t=0, which the VC accepts. This is because for any $\kappa = (a_r, (\pi_{rh} + C - s, s)) \neq D$, either $u_m(\kappa, .) < 0$ or $u_{vc}(\kappa, .) < 0$. The payoffs to the VC and the manager are S_{rh} and $\pi_{rh} + C - S_{rh}$, respectively. ■

Proof. (of Lemma 3) Contracts with manager control can be of the following types: (a) renegotiation-proof contracts, or (b) contracts requiring renegotiation in state θ_l.

(a) Renegotiation-proof contracts: Such a contract must induce the manager to choose action a_s in state θ_l and action a_r in state θ_h. Therefore,

$$\pi_{rh} - S_{rh} \ + C \geq \pi_s - S_s \ \geq \pi_{rl} - S_{rl} \ + C$$

The VC's ex-ante expected payoff, denoted $E(S) = qS_{rh} + (1-q) S_s$. From the second inequality above, $S_s \leq \pi_s - (\pi_{rl} - S_{rl}) - C \leq \pi_s - C$. Therefore, it must be that $E(S) \leq q\pi_{rh} + (1-q)(\pi_s - C)$.

Feasibility of the contract requires that $I + m \leq E(S)$. Therefore, such a contract is feasible only if

$$q\pi_{rh} + (1-q)(\pi_s - C) \geq I + m \tag{16}$$

(b) Contracts requiring renegotiation in state θ_l: The terms of this contract are such that the manager is always induced to choose action a_r. Therefore, $\pi_{rh} - S_{rh} + C \geq \pi_s - S_s$ and $\pi_{rl} - S_{rl} + C > \pi_s - S_s$. In state θ_l, the manager and the VC renegotiate the original contract, with $D = (a_r, (\pi_{rl} - S_{rl} + C, S_{rl}))$ as the disagreement point. From Lemma 2(b), the outcome of this renegotiation is that the firm chooses a_s. The payoffs to the VC and the manager are $S_{rl} + \rho(\pi_s - \pi_{rl} - C)$ and $\pi_s - S_{rl} - \rho(\pi_s - \pi_{rl} - C)$, respectively. Therefore, in this case, $E(S) \leq q\pi_{rh} + (1-q)[\pi_{rl} + \rho(\pi_s - \pi_{rl} - C)]$. So such a contract is feasible only if

$$q\pi_{rh} + (1-q)[\pi_{rl} + \rho(\pi_s - \pi_{rl} - C)] \geq I + m \tag{17}$$

Comparing conditions (16) and (17), and noting that $\pi_s - C > \pi_{rl} + \rho(\pi_s - \pi_{rl} - C)$, it is evident that manager control is feasible if and only if $q\pi_{rh} + (1-q)(\pi_s - C) = K_m \geq I+m$. (To see the sufficiency part, notice that if $K_m \geq I+m$, then the contract $S(x, a_r) = x$, $S(x, a_s) = \left(\frac{\pi_s - C}{\pi_s}\right) x$, with manager control is feasible and implements the first-best action plan.) ■

Proof. (of Lemma 4) When the VC is in control, it can force action a_s on the firm at date 1 before θ is revealed. The VC will be inclined to do so if $S_s > qS_{rh} + (1-q) S_s - m$, i.e., if $S_s > S_{rh} - \frac{m}{q}$. The manager and the VC can renegotiate to persuade the VC to wait till θ is revealed; this can be done by increasing S_{rh}. However, if $S_s > \pi_{rh} - \frac{m}{q}$, then renegotiation won't succeed, because limited liability requires that $S_{rh} \leq \pi_{rh}$. So it is necessary that $S_s \leq \pi_{rh} - \frac{m}{q}$. The expected payoff to the VC is given by,

$$\begin{aligned} E(S) &= qS_{rh} + (1-q) S_s \\ &\leq q\pi_{rh} + (1-q)\left(\pi_{rh} - \frac{m}{q}\right) = K_{vc} \end{aligned}$$

where the inequality follows from the above discussion. Feasibility of the contract requires that $I + m \leq E(S) \leq K_{vc}$. Hence, a contract with VC control is first-best efficient only if $K_{vc} \geq I + m$.

To see the sufficiency of the condition $K_{vc} \geq I + m$, consider the contract $S(x, a_r) = x$, $S(x, a_s) = \frac{\left(\pi_{rh} - \frac{m}{q}\right)}{\pi_s} x$. Under such a contract, $S_{rh} = \pi_{rh}$, $S_s = \pi_{rh} - \frac{m}{q}$ and $S_{rl} = \pi_{rl}$. Then, $S_s \leq qS_{rh} + (1-q) S_s - m$. Therefore, the VC will wait for θ to be revealed.

It only remains to be shown that the VC chooses the first-best action plan after θ is revealed, i.e., that $S_{rh} \geq S_s \geq S_{rl}$. It is obvious that $S_{rh} \geq S_s$. As for the second inequality, Assumption 2(a) implies that $\pi_{rh} - \frac{m}{q} > \pi_s - C$, and Assumption 1(a) implies that $\pi_s - C > \pi_{rl}$. Combining these two inequalities, we obtain $S_s = \pi_{rh} - \frac{m}{q} > \pi_{rl} = S_{rl}$. This concludes the proof of sufficiency as well. ■

Proof. (of Lemma 6) Lemma 1 says that if the VC forces renegotiation in state θ_h, it gets a payoff of $\pi_{rh} - \max\left\{\left(\hat{Y} - L\right), 0\right\}$. Instead, if it does not intervene, it gets a payoff of S_{rh}. So the VC will intervene in state θ_h if and only if $\pi_{rh} - \max\left\{\left(\hat{Y} - L\right), 0\right\}$ exceeds S_{rh}. Therefore, under joint control, the VC's payoff in state θ_h can be at most π_{rh}.

Similarly, if the VC intervenes in state θ_l, it gets a payoff of $Y + \rho(\pi_s - L)$. If $\pi_{rl} - S_{rl} + C > \pi_s - S_s$, the VC will intervene, because otherwise the manager will choose the inefficient action a_r. If $S_s \leq \pi_s - (\pi_{rl} - S_{rl}) - C \leq \pi_s - C$, the VC will intervene if and only if $Y + \rho(\pi_s - L)$ exceeds S_s. Therefore, under joint control, the VC's payoff in state θ_l can be at most $\max\{(\pi_s - C), L + \rho(\pi_s - L)\}$.

Therefore, the expected payoff to the VC under joint control is $E(S) \leq K_j = q\pi_{rh} + (1-q)\max\{(\pi_s - C), L + \rho(\pi_s - L)\}$. Feasibility of the contract requires that $K_j \geq I + m$.

To see sufficiency of the condition $K_j \geq I+m$, consider the following contract: $S^\gamma(x, a_r) = x$, $S^\gamma(x, a_s) = \left(\frac{\pi_s - C}{\pi_a}\right) x$, $Y^\gamma = L$. In state θ_h, the manager chooses the efficient action a_r; the VC does not intervene, and gets a payoff of π_{rh}. In state θ_l, the VC intervenes if and only if $L + \rho(\pi_s - L)$ exceeds $\pi_s - C$; its payoff is $\max\{(\pi_s - C), L + \rho(\pi_s - L)\}$. Since

$E(S) = K_j \geq I + m$, the contract is feasible. ■

Proof. (of Proposition 7) If $K_{vc} \geq I + m$, VC control implements the efficient action plan (Lemma 4). On the other hand, if $I + m > K_j$, then joint control is not feasible (Lemma 6). In either of these cases, joint control can't strictly dominate individual control. Hence, it is necessary that $I + m \in (K_{vc}, K_j]$ (Condition 2).

Next, the interval $(K_{vc}, K_j]$ is well defined if and only if $K_j > K_{vc}$, i.e., if and only if $\max\{(\pi_s - C), L + \rho(\pi_s - L)\} > \left(\pi_{rh} - \frac{m}{q}\right)$. Since $\pi_s - C < \pi_{rh} - \frac{m}{q}$ (by Assumption 2), this is equivalent to the condition $L + \rho(\pi_s - L) > \pi_{rh} - \frac{m}{q} \iff L > L^{vc} = \pi_s - \frac{1}{(1-\rho)}\left(\frac{m}{q} - \Delta\right)$ (Condition 1). ■

Proof. (of Remark 8) When manager control is infeasible and VC control is inefficient, it must be that $K_{vc} < I + m$ (Lemmas 3 and 4). I will prove by contradiction that the control allocation $\{\psi^h = M, \psi^l = V\}$ cannot feasibly implement the efficient action plan. Suppose it can.

(i) The manager retains control of the firm if $\gamma = h$. Consider the state $(\theta = \theta_l, \gamma = h)$. The manager needs to be compensated for her control rents C that she forgoes by choosing a_s. As discussed in Lemma 3, the VC's payoff in the state $(\theta = \theta_l, \gamma = h)$ cannot exceed $\pi_s - C$.

(ii) The VC gets control of the firm if $\gamma = l$ is realized. The VC will force action a_s on the firm unless $S_s^l \leq q_l(\alpha)\pi_{rh} + (1 - q_l(\alpha)) S_s^l - m$, i.e., unless $S_s^l \leq \pi_{rh} - \frac{m}{q_l(\alpha)} < \pi_{rh} - \frac{m}{q}$ (since $q_l(\alpha) < q$).

Therefore, it must be that,

$$\begin{aligned} E(S) &< q\pi_{rh} + (1-q)\left[(1-\alpha)(\pi_s - C) + \alpha\left(\pi_{rh} - \frac{m}{q}\right)\right] \\ &< K_{vc} < I + m \end{aligned}$$

Contradiction; the contract is infeasible. ■

Proof. (of Lemma 9) (i) Suppose the control allocation $\{\psi^h = M, \psi^l = V\}$ implements the second-best action plan. Consider the event $(\theta = \theta_l, \gamma = h)$ which occurs with probability $(1-q)(1-\alpha)$; since $\gamma = h$, the manager has the control right. The manager may be reluctant to choose the efficient action a_s, unless she is compensated for the control rents she forgoes by doing so. As discussed in Lemma 3, the VC can obtain a payoff of at most $\pi_s - C$ in the event $(\theta = \theta_l, \gamma = h)$.

Therefore,

$$\begin{aligned} E(S) &\leq q\left[\alpha\pi_{rh} + (1-\alpha)\pi_s\right] + (1-q)\left[(1-\alpha)(\pi_s - C) + \alpha\pi_s\right] \\ &= K_{mv} \end{aligned}$$

So if $K_{mv} < I + m$, then $E(S) < I + m$, and the contract is infeasible. Hence the control allocation $\left\{\psi^h = M, \psi^l = V\right\}$ implements the second-best action plan if and only if $K_{mv} \geq I + m$. (If $K_{mv} \geq I + m$, then the following contract implements the second-best action plan: $S^h(x, a_r) = S^l(x, a_r) = x$, $S^h(x, a_s) = \left(\frac{\pi_s - C}{\pi_s}\right)x$, and $S^l(x, a_s) = x$.)

(ii) Similarly, with the control allocation $\left\{\psi^h = J, \psi^l = V\right\}$, consider the event $(\theta = \theta_l, \gamma = h)$, when the control right is jointly held by the manager and the VC. Then, as discussed in Lemma 6, the VC's payoff in this event is at most $L + \rho(\pi_s - L)$ or $\pi_s - C$, whichever is higher.

Therefore,

$$\begin{aligned} E(S) &\leq q\left[\alpha\pi_{rh} + (1-\alpha)\pi_s\right] + \\ &\quad (1-q)\left[(1-\alpha)\max\left\{L + \rho(\pi_s - L), (\pi_s - C)\right\} + \alpha\pi_s\right] \\ &= K_{jv} \end{aligned}$$

So if $K_{jv} < I + m$, then $E(S) < I + m$, and the contract is infeasible. Hence the control allocation $\left\{\psi^h = J, \psi^l = V\right\}$ implements the second-best action plan if and only if $K_{jv} \geq I + m$. (If $K_{jv} \geq I + m$, then the following contract implements the second-best action plan: $S^h(x, a_r) = S^l(x, a_r) = x$, $S^h(x, a_s) = \left(\frac{\pi_s - C}{\pi_s}\right)x$, $S^l(x, a_s) = x$, and $Y^\gamma = L$)

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Financial assistance for the services of the editorial assistant of these series is provided by the European Commission through its RTN Programme on Understanding Financial Architecture: Legal and Political Frameworks and Economic Efficiency (Contract no. HPRN-CT-2000-00064).

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