Security Design for Asset Acquisitions*

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ABSTRACT

We propose a security design model in which a potential acquirer approaches a firm with a value-add plan. The target has a single owner, who possesses private information: he knows whether his firm is compatible with the plan, or not. The seller agrees the acquirer will add value but not as much as what the acquirer expects. Although the acquirer can choose any monotone limited liability security to offer along with cash, we show that, under general conditions, if a security is employed, it takes the form of non-recourse junior debt provided by the seller.

KEYWORDS: Security design, private capital, private asset acquisition, asymmetric information, seller financing, screening equilibrium, tranching.

JEL classification codes: G34, D86, D82, G32, G24

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1. Introduction

In 2016, Nasdaq-listed AMRI paid $315 million to acquire an Italian pharmaceutical company called Euticals, which was owned by LCS. $60 million of the compensation that LCS received was a debt claim with Euticals as sole collateral. Specifically, AMRI was due to pay $60 million to LCS over three years with the consequence of default being the return of Euticals back to LCS. Such non-recourse risky seller debt is part of the financing package in many acquisitions (Jansen, 2020; Everett, 2018) and this paper is the first to offer an explanation for its use.[1]

We propose a security design model in which an acquirer (she) makes an asset purchase offer to a company (the seller, he). The consideration offered to the seller is a mix of cash and a security. The seller possesses private information: he knows whether the asset is compatible with the acquirer's value-add plan, or not. The informed seller and the uninformed acquirer agree that the acquirer will add value if the asset is compatible, but the seller's expectation of the value add is less than that of the acquirer.[2] If the asset is incompatible, they agree that no value will be added.

We characterize optimal incentive-compatible offers in which the acquirer can pay using a mix of cash and securities. We show that, under general conditions, the acquirer always pays with cash and, if she uses a security in addition to cash, it is always non-recourse junior seller debt. Under these conditions, other securities (e.g., earnouts, or retained equity) are never optimal means of payment.

To illustrate our model, let us conjecture some additional details concerning the aforementioned acquisition. AMRI (the acquirer) can add value by improving the distribution of Euticals' products through better access to the American market. LCS (the seller) possesses private information concerning the probability

[1] Previous security design studies (e.g., DeMarzo and Duffie (1999), Axelson (2007a), Fulghieri et al. (2019), Malenko and Tsoy (2018)) do not explain why acquisitions are partly financed by a tranche of risky debt provided by the selling firm's owner and secured only by the acquired assets. Among non security design papers, only Fishman (1989) considers seller debt as a means of payment, but he proposes riskless debt, not risky debt. DeMarzo et al. (2005) study the design of seller securities but their results do not rationalize the use of seller debt financing. These studies are discussed at length in the next section. Note that seller debt is distinct from stapled finance, which refers to a loan commitment to prospective acquirers by sell-side investment banks (Povel and Singh, 2010; Aslan and Kumar, 2017). In stapled finance, the debt provider is a bank (not the seller) and the buyer is free to either accept this debt offer or to borrow from any other bank.

[2] This assumption is consistent with the finding that acquirer overconfidence has first-order effects in acquisitions of public firms (Malmendier and Tate, 2008). See also Camerer and Lovallo (1999)).

that the Food and Drug Administration (FDA) will approve Euticals' drug formulations. AMRI can only add value if the FDA approves the drug (i.e., Eutical's products are compatible with the value-add plan). Both parties agree that, absent FDA approval, AMRI adds no value because, then, AMRI's superior distribution network is of no avail. The two parties agree that Euticals will benefit from access to the American market, but AMRI expects that its distribution network will increase revenues more than what LCS expects (i.e., the acquirer is more confident than the seller about her value-add).

Because the acquirer is more confident than the seller about her value-add plan, she assigns a higher value than the seller does to any security claim on the acquired asset. The difference between the valuations of the acquirer and the seller is less with a debt claim than with more information sensitive securities such as equity. However, with an equity claim, the seller's compensation is more closely tied to the outcome of the acquisition. Thus, equity claims are better able to solve the adverse selection problem. The optimal security design is determined by the resolution of these two opposing forces: acquirer confidence pushing toward seller debt and adverse selection pushing toward more information sensitive securities like equity. This paper provides conditions under which the acquirer confidence dominates the adverse selection.

Given the two opposing forces in the model, it is natural that some conditions ensure the optimality of seller debt. The conditions are, however, not obvious. For example, seller debt optimality is independent of the magnitude of informational asymmetry relative to acquirer confidence. Seller debt financing can be optimal when the valuation difference produced by asset compatibility is large, and acquirers are only slightly more confident than sellers about the value-add plan. In fact, high acquirer confidence leads to all-cash offers. Furthermore, the conditions we develop are consistent with assumptions used in standard security design models, and are not particularly restrictive.[3]

The two key ingredients in our model are also standard in the literature. First, many models posit that sellers have private information and that this private information consists of a binary signal revealing

[3]Our conditions are restrictions on the "shape" of cash flow distributions. These conditions are consistent with standard Monotone Likelihood Ratio (MLR) ordering of the distributions of acquirer and seller beliefs for compatible and incompatible assets. As we show in Online Appendix A, any commonly used distributions satisfy these conditions. For example, the acquirer and seller agree that cash flows are log-normally distributed, with the log-mean ascribed to asset cash flows being a function of the information signal (compatible/incompatible) and of the agent (seller/acquirer).

whether the acquirer can add value or not (Fishman, 1989; Hansen, 1987). The second ingredient is rooted in a large body of work on overconfidence: agents believe they can add more value than other parties (e.g., Malmendier and Tate, 2008; Cai and Vijh, 2007; Hirshleifer et al., 2012). This difference in beliefs is particularly plausible in our setting because agents have little opportunity to learn about the quality of the value add plan (Morris, 1995, 1996). These two components are essential. Absent asymmetric information, optimal offers would be all-cash and never include a security. Absent differences in expectations regarding the value-add plan, seller debt is an optimal security but not the unique optimal security.[4]

We make two other important structural assumptions that are not necessary for the optimality of seller debt financing, but are standard in the acquisition literature (Hansen, 1987; Fishman, 1989). First, acquirers, rather than sellers, make acquisition offers. Second, acquirers are not cash constrained.[5]

Outside of standard security design frameworks, which presume that cash flows can be verified, cash constraints might play a role in determining acquisition offers. When an acquirer lacks the cash to complete the transaction and cash flows are non-verifiable, securities must be used as a means of payment. Absent verifiability, debt conjoined with creditor liquidation rights is the only enforceable contract. However, an explanation based on cash constraints and non-verifiable cash flows cannot explain the empirical fact that seller debt is non-recourse. Making the loan non-recourse reduces the collateral available to enforce repayment and thus increases the probability that the acquirer strategically defaults ex post, which increases ex ante financing costs.

Our analysis produces several empirical implications: The acquirer is *more* likely to propose a combination of seller debt and cash (compared to all-cash offer) when (a) a mismatch between the seller's assets and the acquirer's value-add plan is *more* likely; (b) the undervaluation of the seller debt claim, given the acquirer's beliefs is small; (c) the value of the asset under *seller* control is *more* sensitive to the seller's private information. Moreover, consistent with empirical evidence (Jansen, 2020; Everett, 2018), our model predicts that i) acquired assets are kept as standalone entities and not merged with the acquiring firm when

[4] In the event that the acquirer is less confident than the seller about his own abilities, there is no acquisition. The seller simply sells his talents to the seller through an employment contract. Online Appendix B provides an analysis of the underconfidence case.

[5] In the Online Appendix B, we show that seller debt can be an optimal security design in either case, i.e., when informed sellers propose acquisition offers, or when acquirers' ability to use cash is limited or absent.

seller debt is used; ii) seller debt is subordinated to other debt tranches; and iii) seller debt financing is ubiquitous in acquisitions of privately held firms and subsidiaries of public firms, but not for acquisition of public firms.[6]

Our analysis also makes a general contribution to the literature on security design. We cast our model within the standard asymmetric information security design paradigm: Monotone Likelihood Ratio (MLR) ordering and limited liability monotone securities (e.g., DeMarzo and Duffie, 1999; Nachman and Noe, 1994; Axelson, 2007b).[7] MLR ordering of cash flow distributions ensures that debt is the most informationally insensitive security. In this paper, we provide conditions under which the addition of acquirer relative confidence to this standard framework leads to a non-standard result: the informationally sensitive security is allocated to the *uninformed party*.[8]

Although our model setup seems realistic and describes many empirical settings, we explore how certain changes would affect our main result. Perhaps the most interesting outcome occurs when we assume that some value can also be added when the target firm is incompatible. In this case, another payment package can be optimal: a debt offering with two tranches, which are i) a non-recourse risky debt tranche, and ii) a subordinated seller debt tranche; i.e., seller debt is present but as mezzanine debt.

Our baseline model assumes that acquirers can withdraw offers, i.e., we only consider mechanisms that satisfy an ex-post participation constraint (Compte and Jehiel, 2007). Under this assumption, making a single offer targeted at the sellers with compatible assets or an offer targeted at the seller regardless of compatibility are the only optimal offer mechanisms. In the robustness section, we analyze the case where an acquirer cannot withdraw her offer. We show that, in this case, it can be optimal for the acquirer to make two offers: a profitable offer when assets are compatible, and a loss-making offer when assets are

[6] As the debt is used to separate asset types, the collateral is limited to the target asset. Also, since the risk-free component of the cash flows is information insensitive, it does not play a role in separating asset types. The risk-free component can therefore be financed by third parties (e.g., senior debt tranche provided by a bank) and the seller debt is therefore subordinated.

[7] Working within the standard framework sets our analysis apart from other papers that have examined the effect of relaxing MLR ordering (e.g., Fulghieri et al., 2019). In such papers, debt may not be the most informationally insensitive security.

[8] As discussed in Section 6.3, our conditions can be used in other settings where there is asymmetric information but the preferences and/or beliefs of agents vary: (1) when the informed party is more dual risk averse (Yaari, 1987); (2) when the uninformed party is *generally* more optimistic than the informed (contrasting our setting where belief divergence is limited to the value-add plan); and (3) when the acquirer and seller have different state dependent liquidity discount factors.

incompatible. In essence, the offer targeting the seller with incompatible assets is a "bribe" to induce him not to accept the offer targeted at sellers with compatible assets. However, we find that dropping the ex post participation constraint has no effect on the optimality of seller debt.

Our analysis focuses on assets owned and controlled by a single agent. The use of seller debt financing for assets with several shareholders is harder to analyze and outside the scope of this paper. If shareholders are widely dispersed, it is doubtful that they have any material private information. In this case, there would be no benefit from using seller debt financing. If share ownership was concentrated in the hands of a few strategic block holders, then shareholders could have significant private information. However, in this case, the analysis would be complicated by the strategic tendering behavior of these blockholders. In addition, because acceptance of the tender offer would lead to the target shareholders owning a large number of potentially liquid "seller-debt bonds," the effects of liquidity on the incentive compatibility of shareholders with incompatible assets rejecting the tender offer would lead to a difficult problem to solve.

In practice, seller debt financing is rare in acquisitions of publicly traded companies, but prevalent in acquisitions of companies with a single principal owner, as is typically the case for subsidiaries of public firms and for privately held firms.[9]

The next section reviews the related literature. Section 3 describes a simplified numerical version of the model. The full model follows in section 4. Section 5 provides the solution to the model. Section 6 discusses the model assumptions and their relaxation. Section 7 concludes.

[9]Empirically, seller debt financing is present in about half of the transactions targeting private firms (Jansen, 2020; Everett, 2018) and in acquisitions targeting public subsidiaries (Coates, 2012), but it is not used in acquisitions of public firms. The aggregate acquisition value of private firms and subsidiaries of public firms exceeded the aggregate acquisition value of public firms for the first time in the 2010s: $5 trillion versus $4 trillion (Source: SDC Platinum). We reviewed SEC 8-K filings and found instances of seller financing for carve-outs of public firms, but no instances of public firm acquisitions using seller financing.

2. Literature review

2.1. Branch 1: Optimal level of equity retention by an entrepreneur

This first branch of related literature was started by Leland and Pyle (1977), who consider the optimal level of equity to be retained by an entrepreneur. This literature has been expanded in many directions (e.g., Admati and Pfleiderer, 1994; Grinblatt and Hwang, 1989; Demange and Laroque, 1995; Casamatta, 2003; Edmans and Liu, 2011). The key features of these models are information asymmetry (entrepreneurs know more about value) and moral hazard (entrepreneurs do not work as hard if they retain less equity). In our setting, moral hazard is not an issue because buyers acquire and manage the whole firm, not part of it.

The following three papers in this branch are related to our work. First, DeMarzo and Duffie (1999), a security design paper, show that when issuers can signal via retention, issuing a debt-like claim collateralized by assets can be optimal. They allow for a variable scale of investment, which leads to a continuous equity retention decision and a separating equilibrium in which better firms retain more, and invest less. This contrasts with our setting: sellers do not choose the fraction of the company to be sold — they sell the entire company. Yet, sellers in our model are also revealing private information by retaining a debt claim. In DeMarzo and Duffie (1999), like in many security design models, the security issued to *uninformed* agents is debt. In contrast, our model shows that the security held by the *informed* agent is debt. Our results thus contrast with the standard intuition about the allocation of cash flows under asymmetric information.

Second, Coval and Thakor (2005) design a model with optimistic entrepreneurs who require funding from pessimistic investors. Their setting shares a common feature: a more confident agent buys an asset from a less confident agent; although in our setting the difference in confidence relates specifically to the acquisition value add. Their paper is not focused on security design, and they show that the optimistic agent (the entrepreneur) retains an equity tranche. The risky debt is held by a bank, not by the seller.

Third, Baldenius and Meng (2010) model the equity retention decision of an entrepreneur raising capital from an active agent whose effort can add value. They assume equity is the only financing option. Like in their model, we have an acquirer adding value and a seller with private information.

2.2. *Branch 2: Means of payments in acquisitions*

This second branch, started by Hansen (1987), considers the means of payment in acquisitions. Most related studies argue that the means of payment provides a signal (e.g., Eckbo et al. (1990), Rhodes-Kropf and Viswanathan (2004), Shleifer and Vishny (2003), Officer (2004)). These papers focus on the choice among an exogenously fixed set of securities and cash. In contrast, our model is that of a security design. We determine the optimal security for financing the control transfer under the most general conditions possible.

2.2.1. Debt as a means of payment

Fishman (1989) solves a model of preemptive bidding in acquisitions of public firms. In his setting, firms sometimes finance acquisitions with seller debt. Fishman's model assumes that the acquirer knows the payoff from the acquisition conditioned on seller type (compatible versus incompatible). Any seller debt issued to finance the acquisition will default if the assets are incompatible, and will never default if assets are compatible. As Fishman acknowledges, seller debt in this setting is equivalent to contracting directly on the seller's private information. In this framework, absent potential rival acquirers, seller debt is always an optimal security design and cash offers are never optimal. This contrasts with our setting in which acquirers and sellers do not know the future payoffs on assets. Instead, the agents rely on noisy signals of future payoffs. Consequently, no security choice can perfectly reveal future payoffs. Our model explains the conditions under which cash flow distributions will lead to the optimality of seller debt, which cannot be addressed in Fishman's setting. The driver of our results is the tension between disagreement about the size of the value add, and the seller private information about compatibility. [10]

Almeida et al. (2011) argue that when financially distressed firms are acquired by similar yet more liquid firms, the acquirer prefers to use credit lines over cash. Their key mechanism relates to the different levels of dilution attributable to securities, similar to our model. However, in their model, the credit line is provided by a bank, contrasting with the risky tranche of debt offered by the acquirer to the seller in our model.

[10] Fishman (1989) is commonly cited for showing that using a security, such as acquirer's stock, can be optimal (e.g., Table 4 Betton et al., 2008). The model is not cited for showing that debt is the optimal means of payment. Risk-free seller debt financing is an artifice of Fishman's model to simplify the analysis of preemptive bidding by implementing state contingent contracting.

2.2.2. Equity as a mean of payment

Gorbenko and Malenko (2017) study how acquirer's financial constraints affect the decision to pay in cash versus their own stock. They find that in the presence of competition among acquirers, the use of cash is more likely when (1) there are synergies; (2) acquirers can add value, and (3) there are fewer financial constraints. Our model also allows for the possibility of acquirer's value-add (and synergies). We show that seller debt financing occurs even absent financial constraints. Adding financial constraints should further increase the probability of using seller debt financing.

Hege et al. (2008) develop a two-sided asymmetric information model for the sale of an operating asset. They argue that payment in the form of acquirer's stock signals favorable information about the asset, and that investors obtain significant gains only in stock-based deals. Their model, like others in this branch, does not allow for the possibility of seller debt financing, despite this being a prominent means of payment in acquisitions.[11]

2.2.3. Securities as means of payment in auctions

In an auction setting where competing informed buyers make offers to a seller, DeMarzo et al. (2005) model bidding with securities. Buyers bid by offering the seller security claims on the cash flow produced by the auctioned good. Thus, if one reinterprets their model as a model of competing buyers for target firm assets, DeMarzo et al. (2005) model can be seen as a model of the optimal design of seller securities. In their model, the seller is uninformed. In our analysis, the buyer is uninformed. In DeMarzo et al. (2005) the seller moves first by committing to an auction mechanism and fixing a menu of seller securities. Buyers bid by selecting securities from the menu. In our analysis, the buyer (i.e., acquirer) moves first by offering security (and/or cash) to the seller in exchange for an asset. In DeMarzo et al. (2005), the objective is to maximize the revenue of the seller. In our setting, the objective is to maximize the payoff of the buyer.

[11] Papers discussing the frequency of seller debt in M&A transactions are Coates (2012) and Jansen (2020). Other types of claims on the acquired asset are rarely used in acquisitions. Only about 4% of M&A transactions have an earnout as part of the acquisition agreement. Earnouts are more prominent when (1) the targets are private; (2) the targets operate in industries with high growth or high return volatility; and (3) the acquirer and target are from different industries (Kohers and Ang, 2000; Cain et al., 2011; Cadman et al., 2014; Bates et al., 2018).

The results of their analysis and ours are different. DeMarzo et al. (2005) show that the revenue maximizing seller securities are "steep," e.g., levered equity. The worst security design in their framework is the "flattest" feasible security: seller debt. In contrast, in our baseline setting, optimal offers always consist of mixtures of cash and seller debt. Thus, if DeMarzo et al. (2005) is reinterpreted as a model of seller security-financed asset acquisitions, then their model fails to provide an explanation for the ubiquity of cash and seller debt financing in firm acquisitions.

2.2.4. Trade credit

There are some similarities between trade credit and seller debt financing: the seller receives a debt claim issued by the buyer. However, a trade creditor's claim is against the buyer's firm. Seller debt is secured by the asset purchased. Biais and Gollier (1997) develop a model of trade credit in which both suppliers and banks have private information about the quality of buyers. The empirical literature finds evidence that firms extend trade credit because of an information advantage relative to banks. In particular, Petersen and Rajan (1997) show that credit-rationed firms are more likely to use trade credit.

2.3. Branch 3: Capital raising and security choice

This third branch, started by Myers and Majluf (1984), considers the type of securities to issue when a firm wants to raise capital to invest in a profitable opportunity. Myers and Majluf (1984) suggest that firms should issue debt rather than equity because debt is less sensitive to private information. Subsequent work includes DeMarzo (2005), Inderst and Mueller (2006), and Axelson (2007a). In all of these papers, the only source of financing that is considered is external (i.e., institutional investors and banks provide the debt and equity). In addition, companies never raise capital in these models if they are not cash constrained. In our model, even a non-cash constrained company would raise some debt from the seller.

2.4. *Branch 4: Security design and belief disagreement*

Branch 4 considers the effect of belief disagreement on security design. Garmaise (2001) models security design in a rational beliefs setting in which agents need not have common priors over asset returns. Simsek (2013) analyzes the joint effects of belief disagreement and risk aversion on security design. Ortner and Schmalz (2018) model the effects of seller optimism on security tranching. Ellis et al. (2019) consider the effects of belief disagreement on security design from a general equilibrium perspective.

In contrast to our paper, these papers assume symmetric information while a main driver of our results is seller private information about the compatibility of the asset. The contextual framing of these papers is also different. The seller security in these models consists of a bundle of assets held by a large financial institution. Security issuance is mainly motivated by liquidity constraints. In our analysis, security issuance is motivated by adverse selection and sellers are not liquidity constrained. Also, the scope for belief disagreement in our analysis is more modest: acquirer and seller only disagree about returns when an asset is compatible and control of the assets is transferred to the acquirer.

2.5. *Branch 5: Informed agents holding informationally insensitive claims*

This branch of the literature focuses on providing conditions under which the standard security design conclusion — informed agents receive informationally sensitive claims (e.g., equity, warrants) and uninformed agents receive informationally insensitive claims (e.g., debt) is reversed. Yang (2020) considers uninformed liquidity constrained sellers who issue securities to sophisticated buyers that produce information about the seller's assets by investing in information production. To minimize the cost of information production, optimal security designs allocate the least informationally sensitive security (i.e., debt) to the informed buyers.[12] Yuan (2020) considers a similar problem in which buyers are endowed with private signals about the value of uninformed seller's assets. Because debt minimizes the winner's curse problem that results from bidding competitions between informed buyers, debt is the optimal security design.

Like the papers in this strand of the literature, our paper features a reversal of the standard security design

[12] Fulghieri and Lukin (2001) consider a similar problem in a debt/equity choice model.

result. In addition, both our paper and these papers are motivated by stylized facts that seem inconsistent with standard security design results. The existing literature examines debt financing provided by sophisticated investors to financially unsophisticated firm owners. This contrasts with our setting in which the seller debt financing is provided by asset owners who have privileged information about their own assets.

In addition, our setting and the drivers of our results are quite different. In our setting, the liquidity unconstrained acquirer issues a security only to facilitate the acquisition of a specific asset from its owner. Thus, buyer competition, the driver of results in these related papers, plays no role in our analysis. Nevertheless, we show that, like competition between sophisticated security buyers, minor belief disagreements between sellers and acquirers can reverse standard security design results.

2.6. Mechanism design for private equity fund acquisitions

Axelson et al. (2009) study a mechanism design for private equity fund acquisitions. They argue that target companies are levered in order to minimize the agency conflict between fund managers and their capital providers (i.e., the institutional investors). They do not distinguish between different sources of debt capital, but the lender's role in their model prevents the private equity firm from doing bad deals. Seller debt financing is not allowed in their model, but if it were, it might be optimal in their setting as well.

3. The simplest model

In this section we solve the simplest model possible to illustrate the intuition underlying our results. We consider only two security designs: seller debt and seller retained equity. The model is not a security design model because it simply compares to seller securities used in practice. However it does capture the basic intuition behind the results we formalize in the next section.

3.1. The setup

Consider a world with two dates, 0 and 1, populated by an acquirer and a seller that are risk neutral. The seller is endowed with an asset about which he has private information. This information can be either compatible, C, or incompatible, I. The acquirer assigns a prior probability of $\pi \in (0,1)$ that the state is C. Thus the acquirer's utility (i.e., payoff) equals her expected cash flow, X, from the asset at date 1.[13]

The cash flow from the asset at date 1 can take one of three values: low ($l = 0$), medium ($m = 1$), or high ($h = 2$). If the seller's payoff from accepting the acquirer's offer is less than his reservation value, the seller rejects the acquirer's offer. We represent the seller's reservation value in states C and I with R_S^C and R_S^I, respectively. We assume that $R_S^C > R_S^I$.[14] The acquirer makes a take-it-or-leave-it offer to the seller at date 0. The specific parameters used in this example are presented in Table 1.

Table 1
Parameters for the example.

Variable	Description	Value in example
X	Random cash flow under acquirer control	See Table 2
R_S^C	Seller's reservation demand when the state is C	$\frac{1}{2}$
R_S^I	Seller's reservation demand when the state is I	$\frac{5}{16}$
π	Acquirer's prior probability that the state is C	$\frac{1}{2}$

The seller and acquirer agree that the asset is more valuable in state C if control is transferred to the acquirer, and that the acquirer does not add value in state I. However, they disagree about how much value the seller can add in state C. The seller is more pessimistic than the acquirer, as shown in Table 2.

Note that the divergence in acquirer and seller expectations is larger with respect to the best outcome than with respect to the worst outcome. In state C, the acquirer assesses the odds of an incompatible outcome ($l = 0$) at 3 : 13, while the seller assesses the odds of an incompatible outcome at 4 : 12. In contrast, the acquirer is much more confident about the odds of the best outcome ($h = 2$): 9 : 7 (acquirer) vs. 3 : 13 (seller).

A purchase offer consists of a cash payment, p at date 0, and a security, s, issued on the date 1 asset cash

[13]The discount rate for future cash flows is set to one for all agents; i.e., there is no discounting.

[14]The reservation value of the seller can be interpreted as either the value of the asset under seller control or the price at which the asset could be sold to another acquirer with smaller value-add capacity.

Table 2

This table shows the probability distribution of cash flow under acquirer control, $X = l, m, h$, where $l = 0$, $m = 1$, and $h = 2$, under probability distributions for each state of world (C or I) based on the seller and acquirer beliefs.

	Agent					
State	*Seller Belief*			*Acquirer Belief*		
$\mathbb{P}[X =$	$l\,(=0)$	$m\,(=1)$	$h\,(=2)$	$l\,(=0)$	$m\,(=1)$	$h\,(=2)$
C	$\frac{4}{16}$	$\frac{9}{16}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{9}{16}$
I	$\frac{12}{16}$	$\frac{3}{16}$	$\frac{1}{16}$	$\frac{12}{16}$	$\frac{3}{16}$	$\frac{1}{16}$

flow. If the seller accepts security s, then for each possible asset cash flow, x, the seller receives $s(x)$.

3.2. *Payoffs with a cash-only offer*

A cash-only offer at the seller's reservation value in state C (R_S^C) induces the seller to accept the offer and thus transfer control in both states (C and I).[15] Because the acquirer is more confident about the future prospects of the asset, *ceteris paribus*, she prefers to compensate the seller with cash rather than with a share of the asset's payoffs. Computing the acquirer's payoff from an all-cash offer is then straightforward. Let $f_A(X|s)$ be the probability that the acquirer assigns to the cash flow X equals $x \in \{l, m, h\}$, given state $s \in \{C, I\}$. Using the parameters in Tables 1 and 2, the acquirer's payoff from the optimal pooling offer is the average of the acquirer's payoff in the two states, C and I. The acquirer's payoff in state C is given by

$$f_A(h|C)\,h + f_A(m|C)\,m + f_A(l|C)\,l - R_S^C = 0.875.$$

The acquirer's payoff in state I is given by

$$f_A(h|I)\,h + f_A(m|I)\,m + f_A(l|I)\,l - R_S^C = -0.1875.$$

[15] Note that because the seller's reservation value in state I is less than his reservation value in state C, any all-cash offer that the seller accepts in state C is also accepted in state I. Thus, any cash offer that is accepted when the acquirer can add value, i.e., in state C, is also accepted when the acquirer cannot add value, i.e., in state I. Also note that when the state is I, the acquirer pays more for the asset than she expects to generate from acquiring it.

Thus the acquirer's payoff from a pooling offer equals

$$\pi(0.875)+(1-\pi)(-0.1875)=0.34375.$$

3.3. Payoffs with a "cash plus seller debt" offer

Consider an offer consisting of cash, p, and a seller debt claim, $s_{\text{Dt.}}$, where $s_{\text{Dt.}}=\min[x,k]$ and k represents the face value of the debt claim. No offer with a security component can produce a higher payment than a cash offer if the offer is accepted in both states (C and I). Thus, a seller debt offer is preferred to a cash offer only when it induces acceptance by the seller in state C and rejection by the seller in state I.

The acquirer obtains the asset only in state C, and her payoff equals the residual equity claim on the cash flow: $x-s_{\text{Dt.}}(x)=\max[x-k,0]$. Thus, we can formulate the acquirer's problem as a simple programming problem:

$$\underset{p\geq 0,k\geq 0}{\text{Max}}\ \pi\left(f_A(h|C)\max[h-k,0]+f_A(m|C)\max[m-k,0]+f_A(l|C)\max[l-k,0]-p\right) \tag{1}$$

$$f_S(h|C)\min[h,k]+f_S(m|C)\min[m,k]+f_S(l|C)\min[l,k]+p\geq R_S^C \tag{2}$$

$$f_S(h|I)\min[h,k]+f_S(m|I)\min[m,k]+f_S(l|I)\min[l,k]+p\leq R_S^I \tag{3}$$

Equation (1) represents the objective function of the acquirer. Equations (2) and (3) represent the incentive-compatible constraints for the seller to accept the offer when the state is C and to reject the offer when the state is I. Solving this optimization problem produces a unique solution: debt with a face value of $k^*=\frac{3}{8}=0.375$ combined with a cash payment of $p^*=\frac{7}{32}=0.21875$. The payoff for the acquirer under this solution equals $\frac{109}{256}\approx 0.42578$. Thus, if the acquirer makes a cash-plus-debt offer, the expected payoff (with the optimal debt offer) is 0.42578, which is higher than with a cash-only offer.

3.4. *Payoffs with a "cash plus retained equity" offer*

Let $s_{\text{Dt.}} = \alpha\, x$, where α represents the fraction of equity retained by the seller. The problem of finding the optimal cash-plus-equity offer can be expressed as follows:

$$\underset{p\geq 0, 0\leq\alpha\leq 1}{\text{Max}}\ \pi\left(f_I(h|C)\,(1-\alpha)\,h + f_I(m|C)\,(1-\alpha)\,m + f_I(l|C)\,(1-\alpha)\,l - p\right)$$

$$f_S(h|C)\,\alpha\,h + f_S(m|C)\,\alpha\,m + f_S(l|C)\,\alpha\,l + p \geq R_S^C$$

$$f_S(h|I)\,\alpha\,h + f_S(m|I)\,\alpha\,m + f_S(l|I)\,\alpha\,l + p \leq R_S^I$$

The solution is a seller retained equity stake of $\alpha = 0.30$ combined with a cash payment of $p^* = \frac{7}{32} = 0.21875$. The expected payoff to the acquirer (with the optimal fraction of retained equity) is $\frac{119}{320} = 0.371875$, which is higher than with a cash-only offer. This result is due to the high probability of buying the asset at an inflated price in state I overwhelming the cost of issuing an undervalued equity claim (given the acquirer's beliefs). However, the acquirer prefers seller debt financing over retained equity.

Table 3

Summary of the results for the different offer structures for the numerical example

Panel A: Offer type

	Equity + Cash	Debt + Cash	All cash
Result:	Acquire in C only	Acquire in C only	Acquire in C and I
Terms:	$p = .219, \alpha = 0.30$	$p = .219, k = 0.375$	$p = 0.50$
Acquirer payoff:	0.372	0.426	0.344

Panel B: Value of seller security

	Equity + Cash	Debt + Cash
Acquirer: state C	0.41	0.30
Seller: state C	0.28	0.28
Seller: state I	0.09	0.09

3.5. *What drives the results?*

The Table 3, Panel A summarizes the characteristics of the three types of acquisition packages, along with the acquirer's payoff in each case. In Panel B, we report the valuations of the seller securities: retained equity in the first column and seller debt financing in the second column. In state C, the acquirer believes that the retained equity (21.9% of the shares) is worth 0.41. Because the seller assigns a lower probability to the high outcome, he believes that this equity stake is worth less: 0.28. An incompatible firm would not be sold, but if it were, the seller valuation of 21.9% of the equity would be 0.09.

The seller debt has a face value of 0.375, and the acquirer believes that the probability of the target not ending up with the bad outcome under her control is 13/16; hence, this debt is worth $(13/16) \times 0.375 = 0.30$ (since she will not repay anything if the firm is incompatible). The seller disagrees and believes that the incompatible outcome is less likely to be avoided; he assigns a probability of 12/16 to the likelihood of avoidance. Hence, the seller values the debt (when the firm is compatible) at $(12/16) \times 0.375 = 0.28$.

The seller believes the debt to be worth 0.28, whereas the acquirer believes it to be worth 0.30. Seller debt financing imposes a cost on the acquirer relative to the symmetric information scenario, where the seller could make a cash offer conditioned on the state being C. Yet, as the difference between the acquirer's debt valuation and the seller's debt valuation is smaller than the difference between their respective valuations of retained equity, they opt for seller debt financing.

In the limit case in which the seller and the acquirer are in agreement regarding the value-add plan, the difference in valuations for the debt contract and the equity contract is the same. As a result, when there is perfect agreement, seller debt may still be offered but is no longer uniquely optimal.

We do not develop the case in which the acquirer is more pessimistic than the seller because in this case, no transaction would occur.

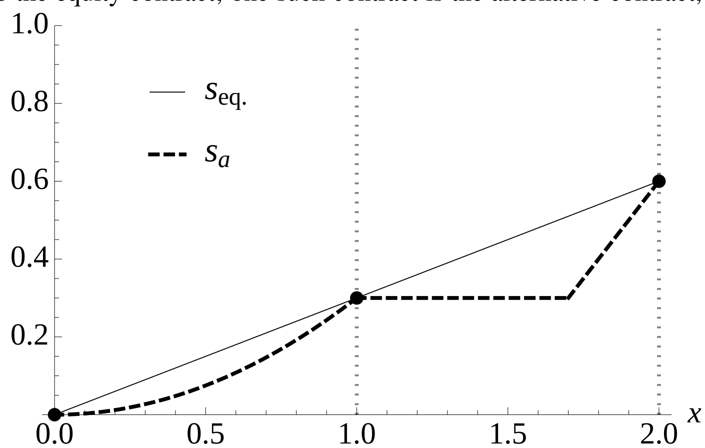
3.6. *Limitations*

Although this numerical example illustrates the fundamental drivers of the subsequent results, it leaves three issues unresolved. First, we only consider two security designs — seller debt and retained equity —

even though many other securities could be used to finance asset acquisitions. If one of these other securities dominates seller debt, then this analysis cannot reliably verify the optimality or predict the use of seller debt financing. Second, we have only established the dominance of debt financing for one parameterization and have not specified any general conditions for this dominance. Third, the distribution of cash flows is discrete, whereas the number of security designs that are perfectly equivalent (in that they produce exactly the same payoffs to both the acquirer and the seller) is infinite. We illustrate this fact in Figure 1. Consider security s_a, which assigns the same security payments to the seller as the equity security for cash flows $x = 0,1,2$. Because the acquirer and the seller, in this example, agree that cash flow equals 0, 1, or 2 with probability 1, s_a is equivalent to seller retained equity from both the acquirer's and seller's perspective. Because of this sort of equivalence, it is impossible to identify uniquely optimal security designs.

Figure 1

In the figure, the three dots represent the combinations of cash flows, x, and security payoffs, $s(x)$, when $x = 0,1,2$. The three points constitute the supports of cash flow distributions of the acquirer and seller. $s_{\text{Eq.}}$ represents the equity contract modeled in the example. Any contract specifying the same payment to the acquirer when $x = 0,1,2$ is equivalent to the equity contract; one such contract is the alternative contract, s_a, depicted in the figure.



4. The baseline model

The formal analysis we develop in this section and in section 5 addresses the limitations described in Section 3.6. We consider acquirer offers that feature not only debt and seller equity but all securities that satisfy the standard limited-liability and monotonicity conditions imposed in the security design literature. Second, we use continuous cash flow distributions under which unique optimal security designs can be identified. Working with continuous distributions also permits us to relate our conditions for the optimality of seller debt to the parametric distributions of cash flows and asset values commonly used in finance research. Third, we provide precise sufficient conditions for the optimality of seller debt financing.

4.1. Formal setup

We consider the problem of an acquirer, A, structuring the purchase of control rights over an asset from a seller, S. Both the seller and the acquirer are risk-neutral and have a discount rate of zero. The seller knows which of two possible states his firm is in: C, the "compatible" state, or I, the "incompatible" state. The acquirer's prior probability distribution over the two states assigns probability π to the state's being C. Negotiations between the acquirer and the seller occur at date 0 and concern the sale of the *cash flows* realized at date 1.[16]

Conditioned on control being transferred to the acquirer, the distribution of cash flows at date 1 is F_i^j, $i, = S, A$, $j = C, I$, where j represents the state of the world and i represents one of the parties to the transaction. For example, F_A^C represents the acquirer's distribution of cash flows conditioned on control being transferred and the state being C. Thus, the acquirer and seller have different subjective probability measures over the distribution of cash flows from the asset if control is transferred to the acquirer.[17]

Conditioned on control being retained by the seller, we represent the distribution of cash flows by H_i^j, $i, = S, A$, $j = C, I$. For example, H_S^C represents the seller's distribution of cash flow conditioned on the seller

[16]These cash flows represent the seller's claim on an asset's cash flow. If the seller's claim on the asset is unencumbered, then "cash flows" represents all cash flows from the asset. If, however, the seller's claim is subordinated to a senior debt claim, then "cash flows" represents cash flows in excess of the preexisting claim of senior debt on the asset's cash flows.

[17]The differences in acquirer and seller valuations in our formal analysis can also be interpreted in terms differences risk preferences. See Section 6.3.

retaining control of the asset and the state being C. Given risk neutrality, the seller's *reservation payoff* from rejecting an offer from the acquirer equals the seller's expected payoff from the asset under seller control. We represent this payoff by R_S^i, $j = I, C$, i.e.,

$$R_S^j = \int x dH_S^j(x).$$

Similarly, when control is transferred, we represent the expected cash flow, which equals the cash flow's value, given state j, $j = C, I$, given the beliefs of i, $i = A, S$ by μ_i^j, i.e.,

$$\mu_i^j = \int x dF_i^j(x).$$

At date 0, the acquirer makes a first and final offer to the seller which, if accepted, transfers control of the asset from the acquirer to the seller. The acquirer offers a package consisting of a cash payment, $p \geq 0$, and a security, s. The cash payment is made at date 0. The security is a claim on the date 1 cash flows produced by the asset. If the offer is accepted, control of the asset is transferred to the acquirer, and the seller receives a cash payment of p at date 0, and the security payoff, s, at date 1.

At date 1, the cash flow x is realized. If the offer was rejected at date 0, the entire cash flow accrues to the seller. If the offer was accepted, and the cash flow is x, the seller receives a payment of $s(x)$ and the acquirer receives $x - s(x)$. Thus, the distribution of cash flows depends on (i) the identity of the agent (acquirer, A or seller, S), (ii) the state of the world (C or I), and (iii) whether control is transferred to the acquirer.

We impose the standard assumptions in the security design literature on the admissible set of securities, i.e., we assume *monotonicity* ($x \hookrightarrow s(x)$ is non-decreasing, and $x \to x - s(x)$ is non-decreasing) and *limited liability* ($0 \leq s(x) \leq x$) (e.g., Nachman and Noe, 1994; DeMarzo and Duffie, 1999; Axelson, 2007b). We assume that all distributions — F_i^j, H_i^j, $i = S, A$, $j = C, I$ — have finite expectations, are supported by $\mathbb{R}^+$, and have continuous densities that are positive over $(0, \infty)$. For any distribution F, we denote $\bar{F}$ its complementary distribution, also known as the survival distribution ($\bar{F} = 1 - F$).

Note that in our setting, there is no riskless component of cash flows. Because the acquirer is not financially constrained and because the value of the riskless component of the cash flows does not vary with the acquirer's beliefs or with the seller's private information, a riskless component of cash flows has no effect on our analysis. To allow cash flows to have a riskless component is straightforward: let X represent the cash flow of the acquired asset and $x_r > 0$ represent the lowest possible cash flow generated by the asset. Our results below would simply become how security design distributes $X - x_r$ between the acquirer and the seller. In practice, in the presence of a riskless cash flow component, one would expect the acquisition of this component to be financed by a riskless debt claim underwritten by the acquirer's lead bank.

4.2. Security density

The monotonicity restrictions on s imply that for any two distinct cash flows, x' and x'', such that $x'' > x'$, it must be the case that $0 \leq s(x'') - s'(x) \leq x'' - x'$. In other words, the slope of s is bounded between 0 and 1. Limited liability implies that $s(0) = 0$. These conditions imply that s is absolutely continuous and, for this reason, s has a density. This density is a measurable function, which we call the *security density* w, where $w(x) \in [0,1]$ satisfies:

$$s(x) = \int_0^x w(t)\, dt.$$

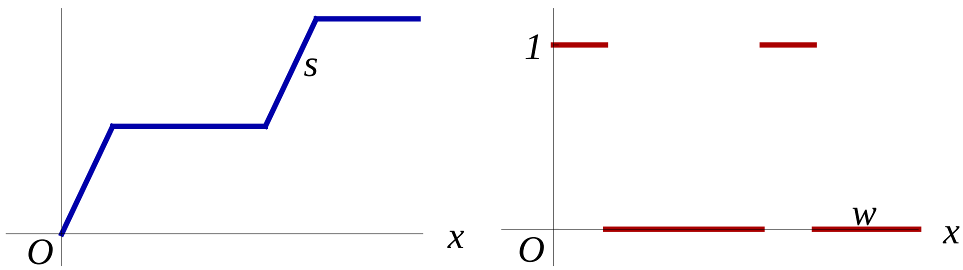
Security densities make it quite tractable to characterize optimal security designs and can be heuristically interpreted as marginal allocations of cash flows. If the cash flow equals x, $w(x)$ represents the proportion of cash flows between x and $x + dx$ ($dx \approx 0$) allocated to the seller. A debt security allocates all marginal cash flows to the seller, $w = 1$, until cash flows equal the debt face value, at which point the security allocates no marginal cash flows to the seller $w = 0$ (see Figure 2).

First note that, the value of a security under a given distribution of cash flows, F, equals

$$\int_0^\infty s(x)\, dF(x).$$

Result 2 in Appendix A shows that we can also express the value of a security s under distribution

Figure 2
The security density, w, and the security, s.



function F in terms of the security density and complementary distribution function, $\bar{F} = 1 - F$, via the identity

$$\int_0^\infty s(x)\,dF(x) = \int_0^\infty w(x)\,\bar{F}(x)\,dx. \tag{4}$$

Expressing the design problem as an optimization over security densities (and complementary distribution functions) simplifies the solution. Imposing monotonicity and limited liability constraints on the security issued, s, is equivalent to restricting the range security density, w, to $[0,1]$, i.e., $0 \le w(x) \le 1$.[18]

4.3. Stochastic orderings of cash flows

In our analysis, one distribution of cash flows is considered "better" than another if it dominates the other distribution in the Monotone likelihood Ratio (MLR) ordering. The MLR ordering is ubiquitous in the security design, auctions, and principal/agent literature (e.g., Milgrom, 1981).

A distribution MLR (strictly) dominates another distribution if the ratio between the probability density functions of the dominant and dominated distributions is (strictly) increasing. Under the dominant and dominated distributions, MLR ordering implies that the relative likelihood of cash flows exceeding a given

[18]The feasible set of densities equals all measurable functions bounded by 0 and 1, so expressing the feasible set of densities is much simpler than expressing the feasible set of monotone limited liability functions. Equality and order conditions related to security densities should be interpreted in the almost sure sense, e.g., $w = 0$ means that the Lebesgue measure of $\{x \in \mathbb{R}^+ : w \neq 0\}$ equals 0.

threshold increases as the threshold moves from the downside to the upside of the cash flow distribution.

We say that one distribution asymptotically dominates another if the density ratio converges to infinity as the cash flow at which the densities are evaluated converges to infinity. Interpreted in Bayesian terms, this implies that for any non-degenerate prior beliefs, a sufficiently large cash flow draw leads a decision maker to be nearly certain that the draw came from the dominant distribution. MLR dominance implies asymptotic dominance for many commonly employed distributions, e.g., the Lognormal, Gamma, Exponential, Weibull.

4.4. Definitions and assumptions

4.4.1. Definitions

Definition 1. Let $\mathscr{W}$ represent the set of measurable functions, $w : \mathbb{R}^+ \to [0,1]$, an *acquirer offer* is an ordered pair (w,p), $w \in \mathscr{W}$ and $p \geq 0$.

Definition 2. The value of the security offered by the acquirer is given by

$$v_i^j(w) = \int_0^\infty w(x)\,\bar{F}_i^j(x)\,dx, \quad i = A,S,\ j = C,I. \tag{5}$$

Definition 3. Let r_j, $j = A,C$ represent the ratio between right tail of the C and I state cash flow distributions under the beliefs of the acquirer and seller, i.e.,

$$r_A(x) = \frac{\bar{F}_A^C(x)}{\bar{F}_S^I(x)}, \quad r_C(x) = \frac{\bar{F}_S^C(x)}{\bar{F}_S^I(x)}.$$

The tail ratios, r_A and r_C represent the right tails, in state C, of the acquirer's and seller's cash flow distributions normalized by the right tail of the cash flow distribution in state I (i.e., the state where the acquirer cannot add value).

4.4.2. Assumptions

Assumption 1 is standard and states that state C is unambiguously better than state I for both parties.

Assumption 1. *Cash flows in the two states satisfy the following ordering conditions:*

1. F_A^C *MLR and asymptotically dominates* F_A^I*;*
2. F_S^C *MLR and asymptotically dominates* F_S^I*;*
3. $R_S^C > R_S^I$.

The following two assumptions are key to our framework. Assumption 2 states that both the acquirer and the seller agree that acquisition in state C adds value but the acquirer is more confident about her value-add capacity than the seller.

Assumption 2. F_A^C *strictly MLR and asymptotically dominates* F_S^C*, and* $\mu_S^C > R_S^C$.

Assumption 3. $F_A^I = F_S^I$ *and* $\mu_S^I = R_S^I$*;*

Assumption 3 implies that the acquirer's plan for increasing value critically depends on the target asset having specific characteristics known only by the seller. In state C, the asset has the characteristics necessary for value creation; but in state I, it does not. For example, suppose an acquirer seeks to acquire a lab that produces medical devices. The acquirer believes her superior distribution network can expand the market reach of the medical device, but this synergy will only be present if the target's product is reliable — information about which the seller will know the most.

All of the results of the baseline model go through, under the assumption that $\mu_S^I < R_S^I$, i.e., that it is common knowledge that acquisition destroys value when the value add plan is not compatible with the seller's assets. In fact, assuming that $\mu_S^I < R_S^I$ would favor the use of seller securities, by making the use of seller securities to separate incompatible and compatible sellers less costly to the acquirer and increasing the acquirer's incentive to avoid purchasing incompatible assets. The advantage of imposing Assumption 3 in the baseline model is that it ensures that the only channel through which the acquisition affects the seller's assets, and the only source of disagreement between the acquirer and seller, is the acquirer's value add plan.

Assumption 4. $r'_A(x)/r'_C(x)$ *is non-decreasing in x,*

Finally, Assumption 4 restricts the set of admissible cash flow distributions to those where the acquirer's expectations about the value add exceeds that of the seller in the sense that belief disagreement is primarily about the magnitude of the upside gains from a successful implementation of the value-add plan. This assumption is closely related to MLR ordering. We further further discuss this assumption and our other assumptions in section 6.1.

4.5. Optimal control transfer offers

We use a standard first-and-final offer mechanism: the acquirer makes an offer to the seller, which the seller accepts or rejects. As discussed in detail in section 6.2, permitting the acquirer to commit to a menu of contracts that includes a cash payment in excess of the value of the asset contingent on the seller reporting that the state is I can sometimes produce higher acquirer payoffs than a first-and-final offer. The first-and-final offer mechanism is optimal for the acquirer only when she *cannot* commit to paying more for the asset than asset's value, conditioned on the offer being accepted, i.e., an ex-post participation constraint is imposed on the mechanism design problem. However, the imposition of this constraint has no effect on the central result of this paper: the optimality of seller debt financing holds both in the no-commitment framework and in the commitment framework (section 6.2). Furthermore, as discussed in section 5.2 on the market for asset acquisition, no-commitment is the most realistic assumption.

Our offer mechanism also assumes that the acquirer possesses the bargaining power in the acquisition negotiations. This is consistent with the bargaining-power assumptions in the literature on cash vs. security acquisition offers (Hansen, 1987; Fishman, 1989). Although assigning the bargaining power to the seller (by assuming that the seller moves first) would make our analysis incompatible with most of the literature on acquisitions, it would not rule out the optimality of seller debt. As we show in Online Appendix B, if we formulate a signaling model in which the informed seller moves first and makes a first-and-final offer to the acquirer, qualitatively the tradeoffs determining the optimal security design would be similar to tradeoffs in our model. A seller with compatible assets would aim to achieve two objectives: (1) avoid being mimicked by the seller with incompatible assets, and (2) minimize the cost (given the seller's beliefs) of satisfying the acquirer's reservation constraint.

If the seller only considers objective (1), he would prefer to receive the most informationally sensitive claim possible, i.e., levered equity. However, the seller actually needs to consider both objectives. With respect to objective (2), seller debt is advantageous because the divergence of valuation between seller and acquirer is largest for the upside cash flows. Thus, the cheapest way for the seller to satisfy the acquirer's reservation constraint, is to assign the upside cash flows to the acquirer, implying that the seller receives a debt claim. Consequently, the optimal security design when sellers make first and final offers, as in our setting, depends on resolving the tension between the two objectives.

5. Solving the baseline model

In the first-and-final offer setting, the acquirer's optimization problem can be solved in three steps. First, solve for the optimal contract in the case of a *pooling offer*, i.e., the seller is willing to accept the offer regardless of the seller's private information. Second, solve for the optimal contract in the case of a *separating offer*, i.e., the seller is willing to accept the offer only if the seller has positive information about future cash flows. Third, compare the acquirer's respective payoffs from separating and pooling offers and identify the conditions under which each is optimal. This is what we do in the next three sub-sections. We then illustrate our findings with an example.

5.1. Solving for the optimal pooling offer

For pooling offers, i.e., offers that are designed to be accepted by the seller in both states C and I, the optimal contract is simply an all-cash offer equal to the reservation price of the seller when the state is C. Formally, for a pooling offer, the acquirer security design problem can be expressed as follows:

$$\underset{w\in\mathscr{W},p\geq 0}{\text{Min}}\ \pi\, v_A^C(w) + (1-\pi)\, v_A^I(w) + p,$$

$$\text{IC-C: } v_S^C(w) + p \geq R_S^C, \qquad \text{P-UCT}$$

$$\text{IC-I: } v_S^I(w) + p \geq R_S^I.$$

If IC-C is satisfied, then IC-I is also satisfied. Thus IC-I is a redundant constraint. Because of acquirer confidence in her own ability (Assumption 2) and the fact that the acquirer is not financially constrained, the cheapest way to meet the seller's reservation constraint in state C is to pay the seller with cash.

Proposition 1. *If (w^*, p^*) is an optimal pooling offer, $w^* = 0$ (the offer is an all cash offer) and $p^* = R_S^C$.*

5.2. *Solving for the optimal separating offer*

We define a *separating offer* as an offer under which it is incentive-compatible for the seller to accept the offer if the state is C and to reject the offer if the state is I.[19]

As the acquirer's claim on asset cash flows is given by $x - s(x)$, the acquirer payoff in state C is:

$$\int_0^\infty (x - s(x))\, dF_A^C(x) - p = \int_0^\infty x\, dF_A^C(x) - \int_0^\infty s(x)\, dF_A^C(x) - p =$$
$$\mu_A^C - \int_0^\infty s(x)\, dF_A^C(x) - p = \mu_A^C - \int_0^\infty w(x)\, \bar{F}_A^C(x)\, dx - p = \mu_A^C - (v_A^C(w) + p).$$

As the probability of the compatible state, C, is π, the expected acquirer payoff equals $\pi\,(\mu_A^C - (v_A^C(w) + p))$. Therefore, an optimal separating offer is a solution to the following problem:

$$\underset{w\in\mathscr{W},p\geq 0}{\text{Max}}\ \pi\,(\mu_A^C - (v_A^C(w) + p)),$$

$$\text{IC-C: } v_S^C(w) + p \geq R_S^C,$$

$$\text{IC-I: } v_S^I(w) + p \leq R_S^I.$$

[19]Under Assumption 3, the acquirer cannot add value in State I. Thus, the acquirer cannot profitably acquire the firm. We relax this assumption in Section 6.1.1. We also consider alternative sale mechanisms in Section 6.2.

Because π and μ_A^C are constants not affected by the offer, this problem is equivalent to the following minimization problem:

$$\begin{aligned} &\underset{w\in\mathscr{W},p\geq 0}{\text{Min}} v_A^C(w)+p, \\ &\text{IC-C: } v_S^C(w)+p \geq R_S^C, \qquad \text{P-CT} \\ &\text{IC-I: } v_S^I(w)+p \leq R_S^I. \end{aligned}$$

Consequently, the acquirer's problem can be thought of as minimizing the value of the total compensation (cash plus security) offered to the seller. This offer is conditioned on the acquirer's cash flow distribution in state C, subject to the constraint that her offer is incentive-compatible for the seller.

If information were symmetric, the acquirer could contract on the seller's asset being compatible. Thus, constraint IC-I in Problem P-CT would be eliminated. In this case, it is clear that IC-C would bind, i.e., $v_S^C(w)+p=R_S^C$ in any solution, implying that the objective function could be expressed as $v_A^C(w)-v_S^C(w)+R_S^C$. Because of acquirer confidence, $v_A^C(w)>v_S^C(w)$ for all $w>0$. Thus, the optimal solution to the minimization Problem P-CT would be the offer $w^*=0$ and $p^*=R_S^C$, i.e., the optimal offer would be all cash. Thus, absent asymmetric information, issuing seller securities would never be optimal.

In contrast, our first result shows that, in our asymmetric information setting, all separating offer have a security component. The proof of this proposition and the proofs of subsequent results are presented in the Appendix.

Lemma 1. *Separating-all-cash offers, i.e., offers* $(w=0,p)$*, are infeasible.*

Using the following simple lemmas (Lemma 2 and Lemma 3), the separating offer problem, P-CT, can be simplified to produce a more tractable formulation.

Lemma 2. *If* (w^*,p^*) *is an optimal solution to Problem* (P-CT)*, then IC-C binds, i.e.,* $v_S^C(w^*)+p^*=R_S^C$.

The intuition underlying Lemma 2 is that, if IC-C does not bind, it is possible to scale down the offer, either by reducing the security or cash component. If the scaling factor is sufficiently small, and IC-C is satisfied as a strict inequality, IC-C will still be satisfied after scaling down the offer. Scaling down the offer discourages the seller from accepting the offer when assets are incompatible, and therefore will not violate the IC-I constraint. Scaling down the offer will reduce the acquirer's valuation of the offer to the seller.

Lemma 3. *If* (w^*, p^*) *is an optimal solution to Problem* (P-CT)*, then IC-I binds, i.e.,* $v_S^I(w^*) + p^* = R_S^I$.

The intuition underlying Lemma 3 is that, if IC-I does not bind, an alternative offer is feasible under which the seller with compatible assets receives the same payoff from a offer with a larger cash component and a smaller security component. If the increased weight on cash in the alternative offer is sufficiently small, the IC-I constraint will still be satisfied as a strict inequality. By construction, the cash component increase will not affect the payoff to a seller with compatible assets. Thus, IC-C remains satisfied. Because of acquirer confidence, reducing the weight on the security component decreases the acquirer's valuation of the offer to the seller.

These two Lemmas imply that $v_S^C(w) - v_S^I(w) = R_S^C - R_S^I$, $p = R_S^I - v_S^I(w)$, and $p \geq 0$. Thus, (w, p) is optimal if and only if w is a solution to the following reduced problem.

$$\begin{aligned}
&\operatorname*{Min}_{w \in \mathscr{W}} v_A^C(w) + (R_S^I - v_S^I(w)), \\
&\text{VM: } v_S^C(w) - v_S^I(w) = R_S^C - R_S^I, \qquad \text{PR-CT} \\
&\text{SLL: } v_S^C(w) \leq R_S^C, \\
&\text{and } p = R_S^I - v_S^I(w).
\end{aligned}$$

The first constraint, value matching (VM), requires that up to the translation produced by the cash payment, the value of the security offered in the two states must equal the state contingent reservation valuations of the seller. The second constraint, the seller limited liability constraint (SLL), ensures that the translation produced by the cash payment can be supported by a nonnegative cash payment. A negative cash payment would involve the seller making a payment to the acquirer. Note that, given VM, which implies that $v_S^C(w) - v_S^C(w) = (R_S^C - R_S^I)$, the SLL constraint could equivalently be expressed as $v_S^C(w) \leq R_S^C$ and $p = R_S^C - v_S^C(w)$.

Inspection of Problem PR-CT illustrates the role of acquirer confidence in our analysis. It the acquirer were not more confident about the value add than the seller, then $v_A^C = v_S^C$, which implies that across all offers that satisfy VM, the objective function is constant. Thus, any offer that satisfies VM and SSL would be an optimal solution to Problem PR-CT.

In our setting, which presumes both acquirer confidence and adverse selection, the solution to Problem P-CT is less obvious. However, Problem PR-CT is an infinite dimensional linear programming problem and can be solved with standard optimization techniques. Forming the Lagrange associated with problem PR-CT yields

$$\mathscr{L}(w,\gamma,\eta) = v_A^C(w) + (R_S^I - v_S^I(w)) \\ - \gamma\Big((v_S^C(w) - v_S^I(w)) - (R_S^C - R_S^I)\Big) + \eta\left(v_S^I(w) - R_S^I\right), \quad \gamma \in \mathbb{R},\ \eta \geq 0. \quad (6)$$

In (6), γ is the multiplier associated with the equality constraint, VM, and η is the multiplier associated with the inequality constraint, SLL. Standard results in duality theory imply that if w^* is a security density that solves PR-CT, then for some $\gamma^* \in \mathbb{R}$ and $\eta^* \geq 0$, w^* solves the dual program

$$\operatorname*{Min}_{w \in \mathscr{W}} \mathscr{L}(w,\gamma^*,\lambda^*). \qquad \text{PRDual-CT}$$

Using the definitions of value provided by equation (5) and some algebraic simplifications yields the following version of the Lagrange function:

$$\mathscr{L}(w,\lambda,\eta) = \int_0^\infty w(x)\left((\bar{F}_S^C(x) - \bar{F}_S^I(x))\left(\frac{r_A(x) - (1-\eta)}{r_C(x) - 1} - \gamma\right)\right)dx + \\ \gamma(R_S^C - R_S^I) + (1-\eta)R_S^I, \quad (7)$$

$$\text{and } r_A(x) = \frac{\bar{F}_A^C(x)}{\bar{F}_S^I(x)}, \quad r_C(x) = \frac{\bar{F}_S^C(x)}{\bar{F}_S^I(x)}. \quad (8)$$

When the expression in the large parentheses is positive, minimization requires setting w at its smallest feasible value, $w = 0$. When the expression in the large parentheses is negative, minimization requires setting w at its largest feasible value, $w = 1$.

Assumption 2 implies that $\bar{F}_S^C(x) - \bar{F}_S^I(x) > 0$, for $x > 0$. The sign of the expression in the large parentheses thus depends on the sign of the ratio, $\mathscr{R}$, defined by

$$\mathscr{R}(x) = \frac{r_A(x) - (1-\eta)}{r_C(x) - 1} - \gamma, \quad x > 0. \tag{9}$$

Because $r_A > r_C$ and $\eta \geq 0$, the following lemma holds:

Lemma 4. *If w^* is an optimal security density, then there exists $\gamma^* > 1$ and $\eta^* \geq 0$ such that*

$$w^*(x) = \begin{cases} 0 & \mathscr{R}(x) - \gamma^* > 0, \\ 1 & \mathscr{R}(x) - \gamma^* < 0. \end{cases}, \tag{10}$$

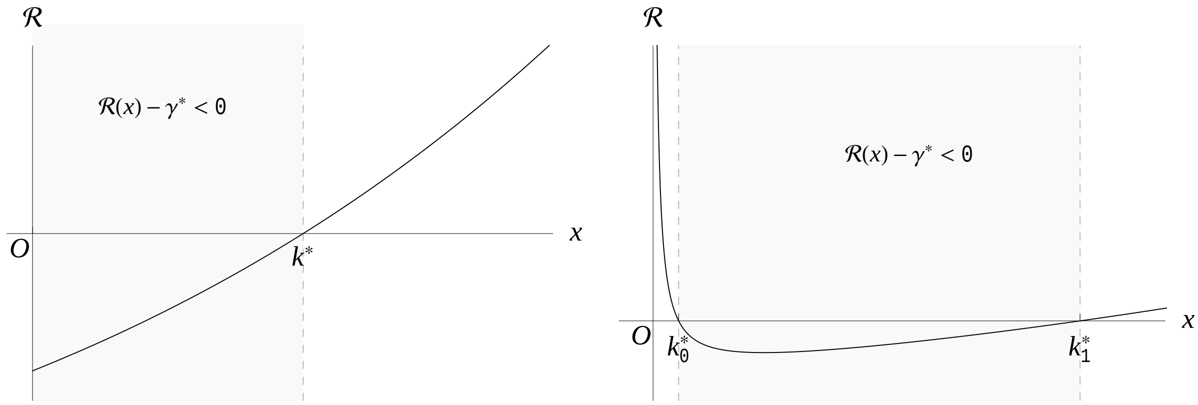
$$\text{where } \mathscr{R}(x) = \frac{r_A(x) - (1-\eta)}{r_C(x) - 1}. \tag{11}$$

The next lemma provides a technical result which is the foundation of all of our subsequent characterizations. Its implications are graphically represented in Figure 3.

Lemma 5. *$\mathscr{R}$, (defined by equation (11) in Lemma 4), has the following properties: (i) If $\eta \geq 0$, $\lim_{x\to\infty} \mathscr{R}(x) = \infty$. (ii) If $\eta = 0$, $\mathscr{R}$ is increasing. (iii) If $\eta > 0$, $\lim_{x\to 0^+} \mathscr{R}(x) = \infty$ and $\mathscr{R}$ is U-shaped.*

A key result for characterizing security designs in the baseline model is that the SLL constraint is redundant given Assumption 3.

Lemma 6. *All feasible offers that satisfy value-matching (VM) constraint strictly satisfy the seller limited liability (SLL) constraint.*



A. SLL does not bind, $\eta^* = 0$ B. SLL binds, $\eta^* > 0$

Figure 3

The ratio function, $\mathscr{R}$, and the seller limited liability constraint, SLL. In Panel A, SLL does not bind and $\mathscr{R}$ is strictly increasing. In Panel B, SLL is a binding constraint and $\mathscr{R}$ is U-shaped. The shaded area is the region where all marginal cash flows are paid to the seller, i.e., where $w = 1$ (Lemma 4).

Lemma 6 holds because Assumption 3 ensures that it is always possible to find a security that satisfies the limited liability constraint, (SLL) and satisfies VM. Under the seller's beliefs, value, even under seller control, is higher in state C than in state I. In state C, if control is transferred, value is even higher, but not in state I. For this reason, securities exist that make the gap between security value in state C and security value in state I, $v_S^C - v_S^I$, larger than the gap between the seller's reservation values, $R_S^C - R_S^I$, in the two states. By scaling down the security component of the offer, a value gap that is too large to satisfy VM can always be reduced until the VM condition is satisfied.

Lemma 6 ensures that the multiplier associated with the slack SLL constraint, η, is 0 at a solution to Problem PR-CT. When $\eta^* = 0$, Assumption 4 ensures that $\mathscr{R}$ is increasing and thus crosses the x-axis once from below (see Figure 3). This ensures (Lemma 4) the optimality of seller debt.

Proposition 2. *Optimal separating offers exist. An optimal separating offer consists of a cash component ($p^* > 0$) and a security component, s^*, with density given by $w*$,*

$$w^*(x) = \begin{cases} 1 & x < k^* \\ 0 & x > k^* \end{cases} \text{ and thus } s^*(x) = \min[x, k^*]; k^* > 0.$$

Thus, the optimal separating offer always consists of a mixture between cash and seller debt financing.

Proposition 2 states that the acquisition is financed by a mixture of cash and non-recourse risky seller debt that is senior to other *risky* claims, but junior to any non-risky debt (e.g., bank financing). In practice, the post acquisition capital structure of the target would consist of i) a tranche of senior debt, whose face value matches the riskless part of the cash flow provided by the acquirer's lead bank; ii) a subordinated non-recourse debt tranche provided by the seller; and iii) equity provided by the acquirer.

5.3. The choice between a pooling offer and a separating offer

The acquirer chooses between a pooling offer and a separating offer by comparing the respective payoffs. A separating offer ensures that the offer is accepted only in state C, which occurs with probability π. Thus, the payoff to the acquirer is equal to:

$$\pi\,(\mu_A^C - (v_A^C(w^*) + p^*)),$$

where (w^*, p^*) is the optimal separating offer. By Lemma 2, $v_S^C(w^*) + p^* = R_S^C$. Thus, the acquirer's payoff with the optimal separating offer is,

$$\pi\,(\mu_A^C - (v_A^C(w^*) - v_S^C(w^*)) - R_S^C). \tag{12}$$

The acquirer's payoff with the pooling offer is simply the expected value of the cash flows less the cash payment, $p = R_S^I$, to the seller. Thus, the acquirer's payoff from a pooling offer equals

$$\pi\,(\mu_A^C - R_S^C) + (1-\pi)\,(\mu_A^I - R_S^C).$$

Assumption 3 implies that $\mu_A^I = R_S^C$. Thus, we can express the acquirer's payoff in a pooling equilibrium as

$$\pi\,\mu_A^C + (1-\pi)\,R_S^I - R_S^C. \tag{13}$$

Equations (13) from (12) imply the following proposition.

Proposition 3. *A separating offer containing seller debt is optimal if and only if*

$$(1-\pi)\,(R_S^C - R_S^I) - \pi(v_A^C(w^*) - v_S^C(w^*)) \geq 0,$$

where w^* is the security specified in Proposition 2. If the inequality is reversed, an all-cash offer is optimal (with $p^* = R_S^C$).

Proposition 3 provides a simple characterization of the conditions under which the seller debt financing is used. Propositions 1 and 2 show that seller debt financing should always be used in a separating offer and never be used in a pooling offer. Thus, seller debt financing is used if and only if the offer is separating. Hence, the condition for the optimality of separating offers in Proposition 3 is also the condition that determines whether seller debt financing is optimal. Hence, Proposition 3 shows that seller debt financing is employed when (a) π (the likelihood that the state is C) is low, (b) when the "mispricing" conditioned on the acquirer's beliefs of the seller debt security is small, and (c) when the gap between the reservation values of the seller in the two states (C and I) is large.

5.4. Baseline example

To illustrate the design of optimal offers, we provide the following example. The parameters in this example are given in Table 4. We assume that cash flows are exponentially distributed in both states under the beliefs of both the acquirer and the seller. This implies that the complementary distribution functions are as follows:

$$\bar{F}_j^i(x) = \begin{cases} \exp[-\frac{x}{\mu_j^i}] & x \geq 0 \\ 1 & x < 0 \end{cases}, \quad i = C, I,\ j = A, S. \tag{14}$$

We start by verifying that the distributions in the example satisfy Assumptions 1, 2, 4, and 3. First, as cash flows are exponentially distributed and given that the assumed parameters satisfy $\mu_A^C \geq \mu_A^I$ and $\mu_S^C \geq \mu_A^I$, Assumption 1 is satisfied. Second, as cash flows are exponentially distributed and given that $\mu_A^C \geq \mu_S^C$ and

Table 4
Parameters for Example 5.4. All cash flow distributions are exponential.

		Valuations of the asset's cash flow		
		Control transferred		*Control not transferred*
State	Probability	Acquirer	Seller	Seller
C	$\pi = 0.25$	$\mu_A^C = 2.00$	$\mu_S^C = 1.50$	$R_S^C = 1.25$
I	$1-\pi = 0.75$	$\mu_A^I = 1.00$	$\mu_S^I = 1.00$	$R_S^I = 1.00$

$\mu_A^I \geq \mu_S^I$, Assumption 2 is satisfied. Third, from the data in Table 4, we can verify that Assumption 3 is satisfied. Finally, to see that Assumption 4 is satisfied, note that equation (14) shows that

$$r_A(x) = \exp\left(x\left(\frac{1}{\mu_A^I} - \frac{1}{\mu_A^C}\right)\right), \quad r_C(x) = \exp\left(x\left(\frac{1}{\mu_S^I} - \frac{1}{\mu_S^C}\right)\right).$$

Thus,

$$\begin{aligned} \log(r_A'(x)) &= x\left(\frac{1}{\mu_A^I} - \frac{1}{\mu_A^C}\right) + \log\left(\frac{1}{\mu_A^I} - \frac{1}{\mu_A^C}\right) \\ \log(r_C'(x)) &= x\left(\frac{1}{\mu_S^I} - \frac{1}{\mu_S^C}\right) + \log\left(\frac{1}{\mu_S^I} - \frac{1}{\mu_S^C}\right) \end{aligned}.$$

Thus,

$$x \hookrightarrow r_A'(x)/r_C'(x) \text{ is increasing } \iff \left(\frac{1}{\mu_A^I} - \frac{1}{\mu_A^C}\right) > \left(\frac{1}{\mu_S^I} - \frac{1}{\mu_S^C}\right).$$

Inspection of the parameters in Table 4 shows that this condition is satisfied.

Proposition 2 shows that the optimal security design is seller debt financing. Using the identity provided by equation (4), we see that the value of seller debt, to the seller, as a function of its face value in states $j = C, I$, is given by

$$\begin{aligned} \text{Vdebt}_i^j(k) = \int_0^\infty \min[x,k]\, dF_i^j(x) = \int_0^\infty w(x)\, \bar{F}_i^j(x)\, dx = \int_0^k \bar{F}_i^j(x)\, dx \\ = \mu_i^j \left(1 - \exp\left[-\tfrac{k}{\mu_i^j}\right]\right), \; i = A, S, \text{ and } j = I, C. \quad (15) \end{aligned}$$

Using equation (15) and solving for the value of debt that satisfies condition VM in Problem PR-CT yields

$k = k^* = 2.079$.

If the offer is accepted, which occurs if and only if the state is C, the acquirer's payoff conditioned on state C is the total value of cash flows, less the value of the seller's security and the cash payment. Thus, the acquirer's payoff conditioned on state C, $u_A^{\text{sep}}(C)$, is given by

$$u_A^{\text{sep}}(C) = \mu_A^C - v_A^C(w^*) - p^* = \mu_A^C - \text{Vdebt}_A^C(k^*) - p^* \approx 0.582.$$

Under a separating offer, the acquirer's payoff conditioned on state I equals 0. Thus, the acquirer's expected payoff using an optimal separating offer is given by

$$u_A^{\text{sep}} = \pi\, u_A^{\text{sep}}(C) \approx 0.146.$$

Note that if the seller's information was public, the acquirer would make a cash offer equal to the seller's reservation price in state C, R_S^C. Conditioned on state C, acceptance would result in an acquirer payoff of $\mu_A^C - R_S^C = 0.75$. Thus, conditioned on the state being C, ensuring that an optimal separating offer is incentive-compatible costs the acquirer $(\mu_A^C - R_S^C) - u_A^{\text{sep}}(C) = 0.75 - 0.582 = 0.168$. This cost results from the acquirer's higher valuation of state C cash flows. Consequently, the acquirer needs to issue securities to discourage the seller from accepting the offer in state I.

Next, we need to verify that this payoff is superior to the payoff obtained from a pooling offer. As shown in Section 5.1, under the parameters in the example, the acquirer's payoff from a pooling offer equals

$$u_A^{\text{pool}} = \pi\, \mu_A^C + (1 - \pi)\, \mu_A^I - R_S^C = 0.$$

Hence, the optimal policy for the acquirer is to make the separating offer: a seller debt claim with face value 2.079 plus a cash payment of 0.125. The offer is accepted if and only if the state is C.

6. Robustness

We have studied the financing of acquisitions through first-and-final offers when (1) acquirers are more confident about their value-add capacities if they gain control than the sellers, and (2) sellers have private information about the asset. We find that if the seller retains a claim on the asset, it will always be a non-recourse junior debt claim. In this section, we consider the robustness of our results along two dimensions. First, we discuss the relaxation of each of our assumptions, and in particular, relaxing Assumption 3. Second, we consider a different sale mechanism: commitment offer instead of first-and-final offer. Finally, we discuss alternative interpretations of the results.

6.1. Discussion of assumptions

6.1.1. The acquirer can always add value (modifying Assumption 3)

The most interesting assumption to relax is Assumption 3. The case we solve here is one in which the acquirer can also add value when the asset is incompatible (but less than when it is compatible). Formally, Assumption 3 is replaced by the following assumption :

Assumption 5. *F_A^I MLR and asymptotically dominates F_S^I, and $\mu_S^I > R_S^I$.*

Using the results already derived, specifically Lemma 5, we can examine the consequences of the change in assumptions. First, note that all results in the paper prior to Lemma 6 are valid under both Assumptions 3 and 5. As shown by Lemma 6, if the value-add is only in state C, (i.e., Assumption 3 is satisfied) then the seller limited liability constraint (SLL) is strictly satisfied. So, if the value-add in state I is sufficiently small, SLL continues to be strictly satisfied. Lemma 5 then implies that seller debt financing plus cash is the optimal security design.

However, when the value-add in state I becomes sufficiently large, SLL may become a binding constraint. In this case, a new offer design can emerge: optimal separating offers have no cash component and the seller's claim is subordinated to a risky debt debt claim, which would be provided by someone other than the seller (e.g., a private debt fund or a bank).

Proposition 4. *When Assumption 3 is replaced by Assumption 5* and *no debt contract exists which satisfies both value matching (VM) and seller limited liability (SLL) conditions of the offer design problem in Problem* (PR-CT)*, then optimal separating offers have no cash component, i.e., $p^* = 0$ and the acquisition is fully debt financed. A third party provides junior (risky) debt) and the seller provides subordinated debt, which we label "seller mezzanine debt,":*

$$w^*(x) = \begin{cases} 1 & x \in (k_0^*, k_1^*) \\ 0 & x \notin (k_0^*, k_1^*) \end{cases} \textit{ and thus } s^*(x) = \min[\max[x - k_0^*, 0], k_1^*]; 0 < k_0^* < k^*.$$

Proposition 4 states that the acquisition needs to be fully debt financed and that the seller's claim must be subordinated to a risky debt claim not held by the seller. In practice, this sort of financing would, in all likelihood, be implemented by i) a tranche of senior debt, whose face value matches the riskless component of the cash flows; ii) junior debt *not* provided by the seller, which is risky, and non-recourse (e.g., Stapled Finance); and iii) risky non-recourse debt subordinated to this junior debt held by the seller.

Absent Assumption 3, separating offers may not exist. In fact, when the baseline model assumptions about the relationship between the state and the value-add are reversed, the following result shows that no separating offers exist.

Result 1. If the value-add, conditioned on seller beliefs, is smaller in state C than in state I, $\mu_S^C - R_S^C < \mu_S^I - R_S^I$, then no separating offer exists.

When the conditions of Result 1 are satisfied, although the seller believes that asset value under both acquirer and seller control will be higher in state C than in state I, the seller believes that increase in value caused by control transfer will be higher in state I. In this case, the incentive of the seller to induce control transfer in state I so large that, seller securities cannot separate I and C.

The overall conclusion from the results in this section is that when the seller's private information is critical for the acquirer's plan to add value, offers of cash plus seller debt are always optimal. When the critical assumption is relaxed somewhat, seller debt plus cash is still the optimal offer. If the seller's information becomes even less critical, all debt offers with a mezzanine structure can be optimal. If Assumption (3) is reversed instead of relaxed, i.e., the seller believes that the value-add is concentrated in state I, separating offers of the sort we analyze, do not exist. Thus, we conjecture that asset acquisitions by acquirers whose value add potential is a substitute rather than a complement to the target's asset quality (e.g. turn-around artists) are less likely to rely on seller debt financing and more likely to opt for more complex seller financing options or all cash offers.

To illustrate, we use the same distributional assumptions as described in Example 5.4, i.e., cash flows are exponentially distributed. Thus, the complementary distribution functions for cash flows are defined by equation (14). The value of a subordinated debt claim s of the form $s(x) = \min[\max[x - k_1, 0], k_2]$ can be computed using the identity provided by equation (4), Thus yields the value Vsub_i^j of subordinated debt to $j = A, S$ and $i = C, I$, given k_1 and k_2.

$$\text{Vsub}_i^j(k_0, k_1) = \int_{k_0}^{k_1} \bar{F}_S^j(x) = \mu_S^j \left(\exp\left(-\frac{k_0}{\mu_S^j}\right) - \exp\left(-\frac{k_1}{\mu_S^j}\right)\right),\ i = A, S,\ \text{and}\ j = I, C \tag{16}$$

Parameter values for the example are provided by Table 5. The satisfaction of Assumption 4 follows from the Exponential distribution assumption as in Example 5.4. Satisfaction of Assumptions 1, 2, and 5 can be verified by inspecting Table 5

Table 5
Parameters for Example 5. All cash flow distributions are exponential

		Valuations of the asset's cash flow		
		Control transferred		*Control not transferred*
State	Probability	Acquirer	Seller	Seller
C	$\pi = 0.35$	$\mu_A^C = 1.75$	$\mu_S^C = 1.50$	$R_S^C = 1.074$
I	$1 - \pi = 0.65$	$\mu_A^I = 1.00$	$\mu_S^I = 1.00$	$R_S^I = 0.779$

Using equation (15) to solve for the amount of debt that satisfies condition VM of Problem PR-CT, we obtain the solution $k^o = 2.465$. The security density for this debt claim is given by $w^o(x) = 1$ if $x < k^o$

and $w^(x) = 0$ if $x > k^o$. Using equation (15) and computing the value of the debt claim to the seller in state I shows that $v_S^I(w^o) = 0.915 > 0.779 = R_S^I$; thus, the SLL condition in Problem PR-CT is violated which implies that the SLL constraint is binding. Proposition 4 implies that the optimal security design is subordinated debt, i.e., the form of the security is $\min[\max[x - k_0, 0], k_1$ and the optimal security density has the form $w(x) = 1$, if $x \in (k_0, k_1)$, and $w(x) = 0$ if $x \notin (k_0, k_1)$.

The value of a subordinated debt claim is given by equation (16). Solving for (k_0, k_1) that satisfy both VM and SLL with equality yields, $k_0^* = 0.15$ and $k_1^* = 2.5$. Because SLL binds, $p^* = 0$.

If the separating offer is accepted, which occurs if and only if the state is C, the acquirer's payoff conditioned on state C, $u_A^{\text{sep}}(C)$ and expected payoff are given by:

$$u_A^{\text{sep}}(C) = \pi\,(\mu_A^C - v_A^C(w^*) - p^* = \mu_A^C - \text{Vsub}_A^C(k_0^*, k_1^*) - p^*) \approx 0.563,$$
$$u_A^{\text{sep}} = \pi\, u_A^{\text{sep}}(C) \approx 0.197,$$

where Vsub is defined in equation (16). If the seller's information were public, conditioned on state C, the seller's payoff would equal $\mu_A^C - R_S^C = 0.676$. Thus, conditioned on state C, the cost the acquirer of making the separating offer incentive compatible equals $0.676 - 0.563 = 0.113$.

the distributional assumptions controlling signal and belief conditioned cash flow distributions (Assumptions 1, 1, and 4,

6.1.2. Distributional conditions (Assumptions 1, 2, and 4)

Assumption 1 simply defines compatible and incompatible states. Assumptions 2 and 4 capture our basic narrative: relative acquirer confidence concerning upside cash flows combined with seller private information lead to seller debt financing. We make no claim that, absent these assumptions, our results would hold.

Assumptions 1 and 2 are standard MLR ordering conditions used ubiquitously in the security design literature (Innes, 1990; DeMarzo and Duffie, 1999; Inderst and Mueller, 2006; Yang, 2020; Daley et al., 2016).

In our framework, these standard MLR ordering conditions imply that $x \hookrightarrow f_A^C(x)/f_S^C(x)$ $x \hookrightarrow f_S^C(x)/f_S^s(x)$ are strictly increasing. This implies the following right-tail ratio condition:

$$x \hookrightarrow \frac{1-F_A^C(x)}{1-F_S^C(x)} \text{ and } x \hookrightarrow \frac{1-F_S^C(x)}{1-F_S^I(x)} \text{are strictly increasing.} \tag{17}$$

Thus, we see from the definitions of r_A and r_I (Definition 3, that a consequence of imposing the standard MLR ordering conditions is that

$$x \hookrightarrow (r_A(x)/r_C(x)) \qquad \text{is increasing.} \tag{18}$$

Equation (17) implies *upside increasing belief divergence*: the divergence in beliefs between the acquirer and seller, and in the two states, C and I, is larger for events related to the upside of the cash flow distribution. For example, consider the seller's beliefs: if, conditioned on acquisition, the seller believes that the probability cash flows exceed 2 million is twice as large in state C as it is in state I, then seller's assessment of the probability that cash flows exceed 4 million must be more that twice as large. Now consider the divergence between acquirer and seller beliefs: if conditioned on acquisition in state C, the acquirer's assessment of the probability that cash flows exceed 2 million is twice as large as the seller's assessment, then the acquirer's assessment that cash flows will exceed 4 million must be more than twice as large.

Right-tail ratio ordering of signal-conditioned cash flow distributions is, in fact, the necessary and sufficient condition for debt to dominate all of other securities in a Myers-Majluf security design setting (Nachman and Noe, 1994). Thus, the fact that our model assumes that divergence in beliefs and information between the acquirer and seller is concentrated on upside cash flows is a consequence of the fact that our model makes a standard distributional assumption in the security design literature.

Our only "non-standard" distributional assumption is Assumption 4. We impose this assumption in order to ensure that belief divergence about the value add is also upside increasing, i.e., even after dividing the tail ratio's of the seller and acquirer in state C by the tail ratio in state I, the state in which the value add cannot be implemented.

Assumption 4 strengthens condition (18) — a consequence of standard MLR ordering conditions — by requiring $x \hookrightarrow r'_A(x)/r'_C(x)$ to also be increasing. This restriction is analogous to maximum likelihood order ordering. Assumption 4 requires the ratio between r'_A and r'_C to be non-decreasing. If F and G are any two absolutely continuous distributions with common supports, strict MLR ordering requires the ratio $\bar{F}'/\bar{G}' = f/g$ to be non-decreasing on the distributions' common support. Thus, one interpretation of Assumption 4 is that it is a requirement that the MLR ordering condition defining acquirer confidence (Assumption 2 is preserved when the complementary acquisition distributions of both the seller, $\bar{F}_S^C$, and acquirer, $\bar{F}_A^C$ are "normalized" by dividing them by the complementary acquisition distribution when assets are incompatible, $\bar{F}_S^I$.

As we show in the on-line supplement to this paper, Assumption 4 is a surprisingly weak restriction on cash flow distributions. Essentially all of the distribution families used in economics and finance that satisfy the standard security design assumptions (i.e., those that are supported by the non-negative real line, have finite expectations, and satisfy the MLR ordering conditions (i.e., Assumptions 1 and 2)) satisfy Assumption 4.[20]

However, Assumption 4 is not redundant. In the Appendix, we provide an example of distributions that satisfy our standard assumptions but fail to satisfy Assumption 4. In this example, the acquirer and seller largely agree on the upside return from the project conditioned on control transfer. The acquirer believes that the primary effect of acquisition is reducing the likelihood of low project returns. The seller thinks that the acquirer has overestimated the magnitude of this effect.

6.2. *Alternative sale mechanism*

We focused on first-and-final offers because they are optimal sale mechanisms from the perspective of the acquirer within the class of mechanisms that satisfy an "ex-post participation constraint" (Compte and Jehiel, 2007). An ex-post participation constraint rules out designs in which either party wants to renege

[20]The distribution families we consider in the on-line supplement include the Chi-squared, Exponential, Gamma, Gompertz, Half Normal, Lognormal, and Weibull. We were not able to analytically verify that the Log-logistic family satisfies Assumption 4. However, even in this case, numerically, we are not able to produce a violation of Assumption 4.

after an agreement has been reached. In other words, we restrict our attention to mechanisms that are *ex-post* incentive compatible.

In this sub-section, we investigate mechanisms that are not ex-post incentive compatible: acquirers and sellers commit to always follow through on the deal even if it is no longer in their best interest—we call these offers *commitment offers*.

We restrict our attention to mechanisms in which the seller reports the state of his company (compatible or incompatible) and receives an offer conditional on his report. The revelation principle (Myerson, 1979) shows that we need only consider truth-telling equilibria of direct mechanisms, i.e., mechanisms in which the seller reports the state, C or I, and receives an offer conditioned on his report. This restriction is therefore without loss of generality.

To solve the problem, we need to (a) determine the form of the offer (i.e., cash or security) conditioned on each of the two seller reports; and (b) determine the action to which the acquirer will commit in response to the seller's report.

Absent the ex-post participation constraint, an acquirer can offer a seller more than R_S^I, conditioned on the seller reporting that the state is I. Obviously, if the seller reports that the state is I the acquirer no longer wants to pay more than R_S^I since this is the value of the firm is in that state.

First consider (a). Assumption 3 implies that, in state I, the acquirer is indifferent between cash and security offers. However, cash offers, conditioned on the report of state I minimize mimicry incentives. To see this, consider two offers, one with a security component and the other all cash. If these offers, conditioned on I, have equal value to the seller, then Assumption 2 implies that the offer with a security component has a higher value to the seller when conditioned on C. Thus, the incentive of seller with state C to report I is minimized if the offer to the seller, conditioned on reporting I, is all cash.

Next, note that in order to satisfy the I-participation constraint, an offer conditioned on reporting I, that specifies control transfer must at least equal R_S^I. Moreover, an offer in excess of R_S^C to the seller conditioned on a report of I is never optimal. Offering the seller R_S^C in the form of cash, in response to both reports, supports a truth-telling equilibrium and costs the acquirer less than offers in excess of R_S^C. Thus, an offer, conditioned on reporting I and specifying control transfer, consists of a cash payment, p_I, where $R_S^I \leq p_I \leq R_S^C$.

Second, consider (b). The offer should specify control transfer conditioned on reporting C as the acquirer can only profit from the acquisition if the asset is acquired in state C. By Assumption 3, the value of the asset in state I is the same under acquirer and seller control, and equals R_S^I. Thus, absent incentive constraints, whether the asset is acquired in state I is matter of indifference.

However, mechanisms that permit the seller to retain the asset are not optimal, because they make the incentive compatibility conditions for the seller more difficult to satisfy. If the mechanism calls for the seller to retain control after a report of I, a seller reporting I when the state is C would receive the cash flows from the asset, worth R_S^C, plus the non-negative payment Δ associated with reporting I. If $\Delta > 0$, the seller's payoff from reporting C when the state is I, would be $R_S^C + \Delta_I > R_S^C$. If the offer mechanism specifies control transfer when the report is I, the seller's payoff from reporting I when the state is C equals $p_I \leq R_S^C$. Thus, a mechanism specifying that the seller retain control after a report of I, always makes satisfying the incentive constraints more difficult. Since, the value of the asset in state I is the same under buyer and seller control, under both acquirer and seller beliefs, mechanisms that permit the seller to retain control are strictly dominated by mechanisms that transfer control in state I. Of course, if $p_I = R_S^I$ (i.e., $\Delta = 0$) mechanisms that specify transfer in state I are completely equivalent to mechanisms that specify retention in state I.

When the mechanism specifies control transfer in both states and $p_I = R_S^I$, the mechanism is ex-post incentive compatible and is payoff equivalent to the separating offer in the baseline model. However, whenever $p_I > R_S^I$ the mechanism does not satisfy ex-post incentive compatibility. Thus, the question of how commitment affects security design can be reduced to the following question: How does permitting the acquirer to commit a cash payment, conditioned on the report of I, in excess of the state-I value, affect the optimal design of acquisition offers?

We now show that, in some cases, making a commitment offer by offering cash payment $p_I > R_S^I$ conditioned on a report of I can produce a higher payoff for the acquirer than first-and-final offers analyzed in the baseline model. Thus, in some cases, acquirers can gain from being able to credibly commit to offers that are not in their ex-post interest to follow through on. However, permitting commitment offers has no effect on the structure of optimal offers. As in the baseline model, seller debt financing is always an optimal security design, and all offers have a positive cash component.

Proposition 5. *Assuming acquirer commitment, an optimal offer mechanism consists of two offers, perhaps identical, one conditioned on the seller reporting state C, and the other conditioned on the seller reporting state I. Under both offers, acceptance of the offer leads to a transfer of the asset to the acquirer.*

The state-I offer is an all-cash and offer p_I. The state-C offer has a positive cash component, p_C, and, if the offer has a security component, the security offered is seller debt. An optimal offer might entail a state-I payment, p_I, in excess of the seller's state I reservation value, R_S^I, i.e., acquirer might commit to overpaying the seller when the seller reports state I.

Committing to overpay in state I can sometimes increase the ex-ante payoff of the borrower. Overpayment for reporting I is costly to the acquirer, but can reduce the seller's incentive to report C when the state is I. This permits the acquirer to reduce the size of the security component of the state C offer, which is undervalued by the seller given the acquirer's beliefs.

The question of whether such commitment offers are plausible reduces the question of whether sellers believe that acquirers will complete the acquisition when the very act of seller acceptance reveals that the acquirer is better off if the deal fails. In practice, merger agreements between the parties followed by the construction of the a final, legally binding, merger contract. It seems plausible that, if the acquirer wanted to thwart completion of the deal at this stage, the acquirer would be able to do so.

Comparing the asset sellers in our setting to "fly-by-night" general partners in private equity funds, as discussed in Axelson et al. (2009), one might argue that there are many potential acquirable assets which do not match the acquirer's business plans (i.e., type I assets). If it is common knowledge that acquirers buy type I assets at inflated prices, owners of type I assets, i.e., "fly-by night asset sellers," would be quick to offer their assets to the acquirer. Thus, committing to overpayment could increase the proportion of I types in the pool of potential acquired assets. And this effect might outweigh any gains from commitment.

The arguments considered here suggest that commitment offers are problematic. However, these arguments do no decisively settle the question of whether commitment offers are plausible. In fact, the question of whether acquirers can commit is difficult to resolve. Fortunately, we do not have to resolve this question because our results do not depend on the question's resolution: Commitment and no-commitment mechanisms produce the same optimal acquisition offers, cash plus seller debt.

6.3. Alternative Interpretations

In our framework, the acquirer and seller have *conditionally divergent valuations* — even when the acquirer and seller condition their expectations on the same state of the world, C and I, and control transfer decision, they value cash flows differently. We interpret this divergence as belief disagreement. However, belief disagreement is not the only interpretation of the model consistent with our formal results. With minor technical modifications, the model can also be interpreted as a model of a risk neutral acquirer attempting to acquire the assets of a Yarri dual risk-averse seller (Yaari, 1987).[21]

Although dual risk aversion is extensively employed to model auction mechanisms (Gershkov et al., 2020), insurance contracting (Attar et al., 2011), and is a foundation for some asset pricing models (Epstein and Zin, 1990), we prefer the belief disagreement interpretation. A large body of empirical evidence suggests that people are more optimistic about their ability to add value than others (Malmendier and Tate, 2008). We are aware of no evidence suggesting that acquirers are generally less dual risk-averse than sellers.

An alternative approach for producing conditional valuation divergence, used in the literature on tranching securities (e.g., Biais and Mariotti, 2005; Yang, 2020), is to assume that the sellers and buyers are risk-neutral, have common beliefs, but the seller faces liquidity constraints. In this case, buyer valuations equal expected cash flows, but seller valuations equal expected cash flows multiplied by a constant liquidity discount factor, $\delta \in (0,1)$. This liquidity-based approach applied to our problem, would yield very different conclusions than the conclusions reached by our model. Because of the liquidity discount factor, optimal designs would minimize the size (measured by expected value) of the seller's security claim, subject to in-

[21] Yaari's dual theory of choice under risk, which is otherwise identical to the Von Neumann-Morgenstern (VM) expected utility theory, replaces the VM axiom of independence with respect to convex probability mixtures with independence with respect to co-monotone payoff mixtures. For an intuitive presentation, see Gershkov et al. (2020).

centive compatibility, monotonicity, and participation constraints. Standard security design arguments show that size will be minimized by maximizing the information sensitivity of the seller's claim. MLR ordering implies that the maximally information-sensitive security design is an option on upside cash flows that grants the seller all marginal cash flows above the exercise price, i.e., the acquirer would receive a very large senior debt claim with control rights, and the seller a small highly levered equity claim with no control rights.

7. Conclusion

Acquisitions targeting companies with a single principal owner (e.g., subsidiaries of public firms and most private firms) have increased over time and have become more prominent than the acquisitions of public firms. These acquisitions are frequently motivated by the acquirer's expectation to increase the value of the target. In such cases, it seems reasonable to presume that (1) the acquirer's capability to add value critically depends on characteristics of the asset acquired; (2) the current asset owner has superior information about these characteristics; and (3) the acquirer is more confident than the seller about her ability to add value if the sellers assets are compatible with the acquirer's value add plan.

Our analysis shows that in this scenario, even when acquirers are not liquidity-constrained or limited to certain security designs, optimal offers always take the form of cash and non-recourse debt. Most importantly, the debt is junior, secured solely by the asset acquired, and issued by the owner of the firm. This result is consistent with the preponderance of seller debt financing in the acquisitions of privately held firms and subsidiaries of public firms. This phenomenon does not appear to have an alternative explanation that is grounded in direct applications of extant theoretical models to asset acquisitions.

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Appendix A. Definitions and Preliminary Results

These results and assume the restrictions imposed on distribution functions imposed in the model, i.e., all distributions are supported by the non-negative real line and are absolutely continuous.

Result 2.

$$\int_0^\infty s(x)\,dF(x) = \int_0^\infty w(x)\,\bar{F}(x)\,dx.$$

Proof. Note that definition of the security density implies that

$$\int_0^\infty s(x)\,dF(x) = \int_0^\infty \left(\int_0^x w(t)\,dt\right) dF(x) = \iint\limits_{\substack{0\le x<\infty \\ 0\le t\le x}} w(t)\,dt\,dF(x) = \iint\limits_{\substack{t\le x<\infty \\ 0\le t<\infty}} w(t)\,dF(x)\,dt =$$

$$\int_0^\infty w(t)\left(\int_{t-}^\infty dF(x)\right) dt = \int_0^\infty w(t)\,\bar{F}(t-)\,dt = \int_0^\infty w(t)\,\bar{F}(t)\,dt.$$

The first inequality follows from the definition of security densities. The second inequality follows from Fubini's theorem on the integration. The third inequality is just a restatement of the region of integration. The fourth inequality again follows from Fubini's theorem. The fifth inequality follows from the definition of $\bar{F}$. The final inequality follows from $\bar{F}$ being continuous and thus $\bar{F}(x-) = \bar{F}(x)$ for all x. Even if F was not assumed to be continuous, the inequality would still follow because $\bar{F}$ is nonincreasing and hence is measurable and discontinuous only on, at most, a countable set of points. Thus, the discontinuities of $\bar{F}$ have measure zero, and because, $w(t)\,dt$ is an absolutely continuous measure, the discontinuity points have no effect on the value of the integral. □

Definition 4. Let F_1 and F_2 be any two distributions over $\mathbb{R}^+$. If

$$\bar{F}_1(x) \le (<) \bar{F}_2(x), \quad x > 0,$$

then F_1 *(strictly) first-order stochastically dominates (FSD)* F_2.

Definition 5. Let F_1 and F_2 be any two distributions over $\mathbb{R}^+$. If for $x > 0$,

$$h_1(x) \le (<) h_2(x), \quad \text{where } h_1(x) = \frac{f_1(x)}{\bar{F}_1(x)} \text{ and } h_2(x) = \frac{f_2(x)}{\bar{F}_2(x)}.$$

then F_1 *(strictly) hazard rate* dominates F_2; differentiation shows that F_1 hazard rate dominates F_2 if and only if the function

$$x \hookrightarrow \frac{\bar{F}_1(x)}{\bar{F}_2(x)} \text{ is increasing}, \quad x \in [0,\infty).$$

Definition 6. If

$$\lim_{x\to\infty} \frac{\bar{F}_1(x)}{\bar{F}_2(x)} = \infty,$$

then F_1 *asymptotically hazard rate dominates* F_2.

Definition 7. Let f_i, $i = 1,2$, be the probability density functions of F_i, $i = 1,2$ If the function $x \hookrightarrow f_1(x)/f_2(x)$ is (strictly) increasing, then then F_1 (strictly) monotone likelihood ratio (MLR) dominates F_2.

Definition 8. Let f_i, $i = 1,2$, be the probability density functions of F_i, $i = 1,2$ If $\lim_{x\to\infty} f_1(x)/f_2(x) = \infty$, then F_1 *asymptotically MLR dominates* F_2 F_2.

Result 3. If F_1 (strictly) MLR dominates F_2 then F_1 (strictly) hazard rate dominates F_2

Proof. Theorem 1.C.1 in Shaked and Shanthikumar (2007). □

Result 4. F_1 *asymptotically MLR dominates* F_2 then F_1 *asymptotically hazard rate dominates* F_2.

Proof. because $\lim_{x\to\infty} \bar{F}_1(x) = \lim_{x\to\infty} \bar{F}_2(x) = 0$, this result follows from the L'Hospital's rule. □

Appendix B. Distributional Assumptions: Examples and Counterexamples

Preliminaries

Constructing distributions that do not satisfy Assumption 4 but satisfy all the other assumptions of our model is challenging. As shown in the online supplement, there appear to be no standard location/ scale families of of distributions that will serve as examples. Thus, we need to construct an example from first principles. We will use the probability transform function approach to stochastic orderings to construct our example. This requires explaining probability transforms and how they relate to stochastic orders. We provide this explanation in this section.

In this subsection, where we develop these results, we will use F and G to represent to arbitrary distributions that satisfy the technical conditions imposed by in the paper—the distributions are supported by the non-negative real line, have finite means, and are absolutely continuous.

Definition 9. A function $\tau : [0,1] \to [0,1]$ is a *survival probability transform function* if $\tau(0) = 0$, $\tau(1) = 1$, τ is strictly increasing, is continuous on $[0,1]$, and has a finite derivative on $(0,1)$.[22]

[22] Typically authors do not assume differentiability. All assertions in this section would go through without the differentiability assumption but the arguments would be come slightly more complicated. Since we are only interested in smooth survival probability transformations, we restrict our analysis to the differentiable case.

Remark 1. Note that. for any two distributions (satisfying the paper's technical conditions), F and G, the function $\tau(p) = \bar{F}\big(\bar{G}^{-1}(p)\big)$ (where $\bar{G}^{-1}$ represents the functional inverse of $\bar{G}$) is a survival probability transform function and $\bar{F}(x) = \tau\big(\bar{G}(x)\big)$. Thus, any two distributions can be related through a survival probability transform function.

Result 5. *F MLR dominates G if and only if there exists a concave survival probability transform function such that $\bar{F}(x) = \tau\big(\bar{G}(x)\big)$.*

Proof. This result is well known and fairly obvious (Lehmann and Rojo, 1992). The only part of this result that we will use is necessity. This is easy to establish. If $\bar{F}(x) = \tau\big(\bar{G}(x)\big)$ and τ is a survival probability transform function, then $F(x) = 1 - \tau\big(\bar{G}(x)\big)$, which implies that $f(x) = g(x)\tau'\big(\bar{G}(x)\big)$. Because $x \hookrightarrow \bar{G}(x)$ is decreasing, the likelihood ratio, f/g, can only be increasing if $p \hookrightarrow \tau'(p)$ is decreasing, i.e., τ is concave. □

Result 6. *If τ is a survival probability transform function, and $\bar{F}(x) = \tau\big(\bar{G}(x)\big)$, then F asymptotically dominates G, i.e., $\bar{F}(x)/\bar{G}(x) \to \infty$, if and only if $\tau'(0) = \infty$.*

Proof. $\bar{F}(x)/\bar{G}(x) = \tau\big(\bar{G}(x)\big)/\bar{G}(x)$, which can be expressed as $\bar{F}(x)/\bar{G}(x) = \phi\left(\bar{G}(x)\right)$ where $\phi(p) = \tau(p)/p$, $p \in (0,1]$. Thus, because $\bar{G}(x) \to 0$ as $x \to \infty$ and because $\bar{G}$ and ϕ is continuous, $\bar{F}(x)/\bar{G}(x) \to \infty$ as $x \to \infty$ if and only if $\phi(p) \to \infty$ as $p \to 0$, i.e., $\tau(p)/p \to \infty$ as $p \to 0$, which is equivalent to $\tau'(0) = \infty$. □

Result 7. *Let $h_F = f/\bar{F}$, and $h_G = g/\bar{G}$ represent the hazard rates of F and G respectively, if $\bar{F}(x) = \tau\big(\bar{G}(x)\big)$ and $\tau(p) = p^\gamma$, $\gamma > 0$, then $h_F(x)/h_G(x) = p$.*

Proof. This result follows by a straightforward calculation. □

The ratio condition–Assumption 4

Expanding the ratio definition in Assumption 4 shows that the condition can be expressed as

$$\begin{aligned} \frac{r'_A(x)}{r'_C(x)} &= \frac{\bar{F}^C_A(x)}{\bar{F}^C_S(x)}\,\Psi(x), \\ \Psi(x) &= \frac{h^I_S(x) - h^C_A(x)}{h^I_S(x) - h^C_S(x)}, \end{aligned} \tag{B1}$$

where h^j_i represents the hazard function for the cash flow distributions of agent $i = S, A$ in state $j = C, I$. Assumption 2 ensures that $x \hookrightarrow \bar{F}^C_A(x)/\bar{F}^C_S(x)$ is increasing. Assumption 1 and 2 also imply the distributions are hazard rate ordered, so $h^I_S(x) > h^C_S(x) > h^C_A(x)$, $x > 0$. Thus, $\Psi(x) > 1$, $x > 0$.

Remark 2. Thus, distributions related by a concave power law survival probability transform functions always satisfy Assumption 4. To see this, first note that inspecting equation (B1) shows see that a sufficient, but by no means necessary, condition for Assumption 4 to hold (i.e., $x \hookrightarrow r_A'(x)/r_C'(x)$ to be increasing) is that Ψ is non-decreasing. Using Result 7 and inspecting equation (B1), we see that this sufficient condition will be satisfied if tails of the three distributions are related by a power law survival probability transform function, i.e., $\bar{F}_S^C(x) = (\bar{F}_S^I(x))^\beta$ and $\bar{F}_S^C(x) = (\bar{F}_A^C(x))^\gamma$. In order for our other assumptions to hold, notably MLR dominance ordering (Assumptions 1 and 2, Remark 5 shows that must be the case that $\gamma < 1$, $\beta < 1$, i.e., the power law functions are concave.

Constructing the example

For ease of presentation we break the construction of the counterexample into a series of steps:

Step-1 *The κ function.* We start with a function that has its concavity concentrated around a single point, the κ function defined below.

$$\kappa(p;k,s) \;=\; -\frac{1}{s}\log\left[\exp(-s\,p)+\exp(-s\,k)\right], \quad p\in[0,1],\ k\in(0,1),\ s>0.$$

Because the Log-sum-exp function is strictly convex (pg. 72: Boyd and Vandenberghe, 2004) it is clear that κ function is strictly concave. Inspection shows that the function is strictly increasing. The κ function is also obviously smooth. Moreover, as the scale parameter, s, increases to infinity, the κ function converges pointwise to a kinked function $x \hookrightarrow \min[k,p]$. Thus, all the concavity of κ becomes concentrated around k as s grows without bound.

Step-2 *Normalizing κ.* κ is strictly increasing and strictly concave but κ does not satisfy $\kappa(0) = 0$ and $\kappa(1) = 1$; so κ is not a survival probability transform function. Thus, we add an affine function to κ to ensure that the resulting function, which we call τ_o, equals 0 at 0 and 1 at 1, i.e.,

$$\tau_o(p;k,s) \;=\; \alpha+\beta\,p+\kappa(p;k,s), \quad \alpha \;=\; -\kappa(0;k,s),\ \beta \;=1+\kappa(0;k,s)-\kappa(1;k,s). \tag{B1}$$

Step-3 *Ensuring an infinite derivative at 0.* Because adding an increasing affine function to a strictly concave strictly increasing function leaves the function strictly concave and strictly increasing, and $\tau_o(0;k,s) = 0$ and $\tau_o(1;k,s) = 1$, τ_o is a strictly concave survival probability transform function. However, it is not the case that $\tau_o'(0;k,s) = \infty$. To ensure this property we transform τ_o by a power transformation, to obtain τ, the general form of survival function we will use to construct our counterexample:

$$\tau(p;k,s,\gamma) \;=\; \tau_o(p;k,s)^\gamma = \Big(\alpha+\beta\,p+\kappa(p;k,s)\Big)^\gamma, \quad \gamma\in(0,1). \tag{B2}$$

Because the composition of a strictly concave, strictly increasing function with a strictly concave, strictly increasing function yields a strictly concave strictly increasing function, and a power function with a power between 0 and 1, is strictly concave and strictly increasing, τ is a strictly concave, strictly increasing survival transform function with the property that $\tau'(0;k,s) = \infty$.

Step-4 *Choosing the parameters and distributions.* We need the κ function to be "almost" kinked at k, so we picked a large scaling factor, $s = 20$. A low power coefficient makes it harder to construct our counterexample because lowering the coefficient increases concavity away from the kink point k. We only need the power coefficient to ensure that $\tau'(0;k,s) = \infty$ and any $p \in (0,1)$ will accomplish this objective. So we set $\gamma = 0.95$. To ensure that the concavity around k is isolated from the concavity on the upside produced by γ, we set k fairly low at $k = \frac{1}{3}$. Thus, we define τ^* as follows:

$$\tau^*(p) = \tau(p; k = \frac{1}{3}, s = 20, \gamma = 0.95). \tag{B3}$$

τ^* is depicted in Figure 4.

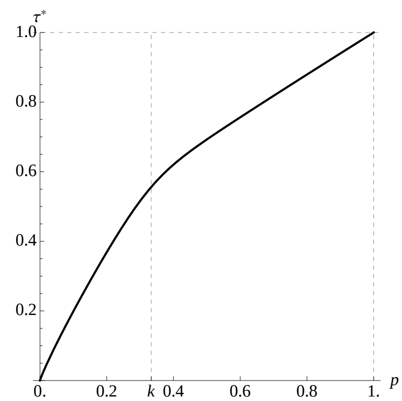

Figure 4

τ^*, the survival probability transform function used in the counterexample. The horizontal axis represents probability, p. The vertical axis represents the value of the survival probability transformation. The function τ^* is defined by equations (B1), (B2), and (B3)

Step-5 *The distributions.* Using τ^*, we define the distributions used in the counterexample as follows: first we set F_S^I equal to a Gamma distribution with shape parameter 2 and scale parameter $\frac{1}{2}$ and use survival probability transform functions to define the other distributions.[23]

$$\bar{F}_S^I(x) = 4\int_x^\infty t\,e^{-2t}\,dt, \quad \bar{F}_S^C(x) = \sqrt{\bar{F}_S^I(x)}, \quad \bar{F}_A^C(x) = \tau^*\big(\bar{F}_S^C(x)\big). \tag{B4}$$

Because the square-root function is strictly concave and strictly increasing over $[0,1]$ and has an infinite derivative at 0, Results 5 and 6 imply that F_S^C MLR and asymptotically dominates F_S^I. Step 3

[23]Because all of the restrictions on the distributions, other than the technical restrictions, are relational, the choice of any one of the three distributions is fairly arbitrary. We picked a Gamma distribution for F_S^I because the Gamma distribution leads to easily visualized effects in the graphs that follow.

and Results 5 and 6 imply that F_A^C MLR and asymptotically dominates F_S^C. Thus, Assumptions 1 and 2 are satisfied. However, as the next figure shows, the ratio r_A'/r_I' is not monotone, and thus Assumption 4 is not satisfied.

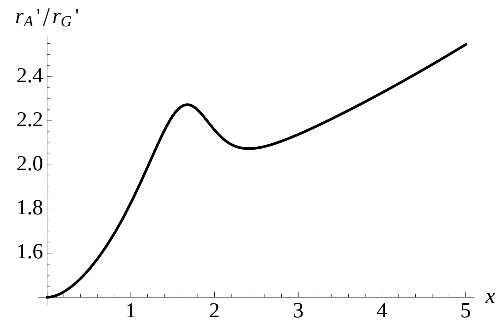

Figure 5

Counterexample. The figure presents an example of cash flow distributions that satisfy all of the conditions imposed in the baseline model except Assumption 4. The horizontal axis represents cash flows, x. The vertical axis represents the ratio, r_A'/r_C', which Assumption 4 requires to be increasing in x. The distributions used to construct the example are defined by equation (B4).

Some intuition for the counterexample can be provided by examining the PDFs of the probability density functions constructed for the counterexample. These densities are graphed in the next figure,

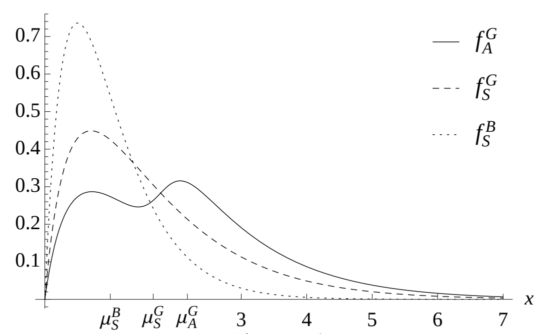

Figure 6

PDFs of the distributions used in the counterexample. The horizontal axis represents cash flows, x. The figure graphs the PDFs of the three cash flow distributions used in the counterexample: f_A^C (solid line), f_S^I (dashed line), and f_S^I (dotted line). These distributions are defined by equation (B4).

Figure 6 reveals that the source of disagreement in beliefs between the seller, S, and the acquirer, A, regarding the effects of control transfer in state C. Both believe that acquisition adds value on the upside. The acquirer is a bit more optimistic about the size of the value add. Disagreement between acquirer and seller largely results from the acquirer assigning a much smaller probability to extremely low cash flows post acquisition and a much higher probability to "mediocre" cash flows close to the mean cash flow anticipated by the acquirer, μ_A^C.

Appendix C. Proofs of Results

Proof of Proposition 1. Suppose an optimal offer contains a security component, i.e., it is not the case that $w^* = 0$ Define the offers (w_t, p_t), $t \in (0,1)$ as follows, $w_t = (1-t)w^*$ and $p_t = p^* + t\, v_S^C(w^*)$. Then $v_S^C(w_t) + p_t = v_S^C(w^*) + p^*$ and $v_S^I(w_t) + p_t > v_S^C(w^*) + p^*$ so both incentive constraints are satisfied by (w_t, p_t). At the same time the difference between value of the control transfer to the seller under (w^*, p^*), $v_A^C(w^*) + p^* - (v_A^C(w_t) + p_t) = t\,(v_A^C(w^*) - v_S^C(w^*)) > 0$, contradicting the optimality of (w^*, p^*).

To see that $p^* = R_S^C$, simply note that the seller must accept the offer in both states C and I. By the assumption that $R_S^C \geq R_S^I$, this implies that the cash offer must at least equal R_S^C. Any higher offer is clearly dominated by offering R_S^C. □

Proof of Lemma 1. Under an all cash offer, the incentive constraints reduce to $p \geq R_S^C$ and $p \leq R_S^I$. By Assumption 1, $R_S^I > R_S^C$. Thus, these conditions cannot be simultaneously satisfied. □

Proof of Lemma 2. To obtain a contradiction, suppose that IC-C does not bind in some optimal solution (w^*, p^*). In this case, $v_S^C(w^*) + p^* > R_S^C$. In order for IC-C to be satisfied by (w^*, p^*), it must be the case that $v_S^C(w^*) > 0$ or $p^* > 0$. First suppose that $v_S^C(w^*) > 0$. Consider offers of the form $(\lambda\, w^*, p^*)$, where $\lambda \in (0,1)$. $v_S^C(w^*) + p^* > R_S^C$ implies that, for λ sufficiently close to 1, $v_S^C(\lambda\, w^*) + p^* > R_S^C$. Thus $(\lambda\, w^*, p^*)$ satisfies IC-C. Because, $w^* \in [0,1]$, $\lambda\, w^* \in [0,1]$, thus $\lambda w^* \in \mathscr{W}$. Because, IC-I is satisfied by (w^*, p^*), $v_S^I(w^*) + p^* \leq R_S^I$. Thus, because $v_S^I(\lambda\, w^*) + p^* < v_S^I(w^*) + p^*$, IC-I is also satisfied by $(\lambda\, w^*, p^*)$. Thus, $(\lambda\, w^*, p)$ is a feasible offer.

Because $v_A^C(\lambda\, w^*) = \lambda\, v_A^C(w^*) < v_A^C(w^*)$, $v_A^C(\lambda\, w^*) + p^* < v_A^C(w^*) + p^*$, contradicting the optimality of (w^*, p^*). Now suppose that $p^* > 0$. Repeat the same argument by comparing (w^*, p^*) to $(w^*, p^* - \varepsilon)$, for $\varepsilon > 0$ but sufficiently small so that $v_S^C(w^*) + p^* - \varepsilon \geq R_S^C$. □

Proof of Lemma 3. To obtain a contradiction suppose that IC-I does not bind in some optimal solution (w^*, p^*).The hypotheses that w is not almost surely equal to 0, implies that $v_A^C(w^*) > 0$, $v_S^C(w^*) > 0$ and $v_S^I(w^*) > 0$. Define the offers (w_t, p_t), $t \in (0,1)$ as follows, $w_t = (1-t)w^*$ and $p_t = p^* + t\, v_S^C(w^*)$.

By Lemma 2, $v_S^C(w^*) + p^* = R_S^C$. By construction $v_S^C(w^*) + p^* = v_S^C(w_t) + p_t$. Thus under offer (w_t, p_t), IC-C binds. By hypothesis, IC-I does not bind, i.e, $v_S^I(w^*) + p^* < R_S^I$. For t sufficiently small, it will also be the case that $v_S^I(w_t) + p_t \leq R_S^I$. Thus (w_t, p_t) is feasible for $t > 0$ and sufficiently small. Now compare the values of the objective function under (w^*, p^*) and (w_t, p_t). This comparison yields,

$$(v_A^C(w^*) + p^*) - (v_A^C(w_t) + p_t) = t\,(v_A^C(w^*) - v_S^C(w^*)).$$

Because $t > 0$, Assumption 2 implies that $t\,(v_A^C(w^*) - v_S^C(w^*)) > 0$, contradicting the optimality of (w^*, p^*) □

Proof of Lemma 4. Derivation provided in main body of the paper. □

Proof of Lemma 5. Let N represent the numerator of the ratio defining $\mathscr{R}$ (equation 9) and let D represent the denominator, i.e.,

$$\mathscr{R}(x) = \frac{N(x)}{D(x)} = \frac{r_A(x) - (1-\eta)}{r_C(x) - 1} - \gamma, \quad x > 0.$$

Asymptotic dominance (Assumption 2 implies that

$$\lim_{x \to \infty} r_C(x) = \infty \text{ and } \lim_{x \to \infty} \frac{r_A(x)}{r_C(x)} = \infty.$$

This observation and inspection of the definition of $\mathscr{R}$ shows that, $\lim_{x\to\infty} \mathscr{R}(x) = \infty$.

Next note that hazard rate order dominance, ensured by Assumption 2, implies that $r_C(x) > 1$, for $x > 0$. Thus, $D(x) > 0$ for $x > 0$. Assumption 2 also ensures that $r'_C(x) > 0$, $x > 0$. Thus

$$D(x) > 0 \text{ and } D'(x) > 0, \quad x > 0 \tag{C1}$$

Next note that, if $\eta = 0$,

$$D(0^+) = N(0^+) = 0. \tag{C2}$$

By the Monotone L'Hospital Rule (Proposition 1.1: Pinelis, 2002) equations (C1) and (C2) imply that a sufficient condition for $N/D = \mathscr{R}$ to be increasing is that $x \hookrightarrow N'(x)/D'(x)$ is increasing. Next note that $N'(x)/D'(x) = r'_A(x)/r'_C(x)$. By hypothesis, $x \hookrightarrow r'_A(x)/r'_C(x)$ is increasing. Thus, $\mathscr{R}$ is increasing.

Now suppose that $\eta > 0$. In this case, as $x \to 0^+$, N is bounded away from 0 and, because $r_C(0) = 1$, $\lim_{x\to 0^+} D(x) = 0$; thus, $\mathscr{R}(x) = N(x)/D(x) \to \infty$ as $x \to 0^+$. Next, note that D is not affected by η Thus, condition (C1) is also satisfied in this case; however, when $\eta > 0$, $N(0^+) \neq 0$. Another variant of the monotone L'Hospital Rule (Table 1: Pinelis, 2006) nevertheless shows that $x \hookrightarrow N'(x)/D'(x) = r_A(x)/r_C(x)$ increasing is still a sufficient condition for $\mathscr{R} = N/D$ being strictly quasiconvex. Because $\lim_{x\to\infty} \mathscr{R}(x) = \infty$ and $\lim_{x\to 0^+} \mathscr{R}(x) = \infty$, strict quasiconvexity implies that $\mathscr{R}$ is U-shaped. □

Proof of Lemma 6. Let w, p be an offer satisfying VM. VM implies that $p = R_S^I - v_S^I(w) = R_S^C - v_S^C(w)$. So, if VM is satisfied, SLL binds if and only if $p = 0$. Assumption 3 implies that $\mu_S^I = R_S^I$. Therefore $v_S^I(w) \leq \mu_S^I = R_S^I$ and, unless the security density allocates the entire cash flow from the project to the seller, i.e. $w = 1$, $p = R_S^I - v_S^I(w^*) > 0$. Thus, if SLL binds then $w = 1$. However, Assumption 2 implies that $\mu_S^C > R_S^C$. Thus, if $w = 1$, $v_S^C(w) = \mu_S^C$. VM implies that $p = R_S^C - v_S^C(w) = R_S^C - \mu_S^C < 0$, which is impossible because, by definition, the cash component of a feasible offer is non-negative. □

Proof of Proposition 2. We will first show that a separating offer exists. Define the function $\Delta : \mathscr{W} \to \mathbb{R}$ by

$$\Delta(w) = v_S^C(w) - v_S^I(w)$$

Note that

$$\sup \Delta(\mathscr{W}) = \Delta(1) = \mu_S^C - \mu_S^I \quad \inf \Delta(\mathscr{W}) = \Delta(0) = 0. \tag{C3}$$

$\mathscr{W}$ is a convex and Δ is is a linear functional. The image of a convex set under a linear functional is convex and thus connected. Hence, $\Delta(\mathscr{W}) = [0, \mu_S^C - \mu_S^I]$. A security density satisfying VM exists if and only if there exists $w \in \mathscr{W}$ such that $\Delta(w) = R_S^C - R_S^I$. From these facts we can conclude that

$$\text{a security density, } w, \text{exists which satisfies VM} \iff 0 \leq R_S^C - R_S^I \leq \mu_S^C - \mu_S^I. \tag{C4}$$

Assumption 3 implies that $\mu_S^I = R_S^C$ and Assumption 2 implies that $\mu_S^C > R_S^C$. Thus, (C4) is satisfied and hence a security density exists that satisfied VM. Lemma 6 shows that SLL is satisfied by all $w \in \mathscr{W}$. Thus, a feasible solution to Problem PR-CT, i.e., a separating offer, exists.

Lemma 6, implies that even absent the SLL constraint, all feasible offers strictly satisfy SLL. Hence, constraint SLL in problem PR-CT is slack, which implies, by standard results in Duality theory, that its multiplier $\eta^* = 0$. By Lemma 5 this implies that $\mathscr{R}$ is strictly increasing. It is not possible for $\mathscr{R} - \gamma^* \geq 0$ for all $x > 0$, because, by Lemma 4 this would imply that $w^* = 0$, which is not possible by Lemma 1. As shown earlier in the proof, $w^* = 1$ is not consistent with the satisfaction of the VM constraint. Because $\mathscr{R}$ is continuous and strictly increasing, there exists a unique value of x which we call k^* such that $\mathscr{R}(x) - \gamma^* \leq 0$; for $x \leq k^*$, and $\mathscr{R}(x) - \gamma^* > 0$, for $x > k^*$. Thus Lemma 4 implies that all optimal security densities, w^*, have the form specified in the proposition. □

Proof of Proposition 3. Derivation provided in the manuscript. □

Proof of Proposition 5. Consider a truth-telling equilibrium of direct mechanism that specifies, $M(\hat{I}) = (0, p_I)$ and $M(\hat{C}) = (w, p_C)$. The objective function, incentive (IC) and participation (PC) constraints for this mechanism are given below:

$$\begin{aligned} &\operatorname*{Max}_M \pi \left(\mu_A^C - \left(v_A^C(w) + p_C\right)\right) + (1-\pi)(R_S^I - p_I), \\ &\text{ICRev-C: } v_S^C(w) + p_C \geq p_I, \\ &\text{ICRev-I: } p_I \geq v_S^I(w) + p_C, \\ &\text{PCRev-C: } v_S^C(w) + p_C \geq R_S^C, \\ &\text{PCRev-I: } p_I \geq R_S^I. \end{aligned} \tag{Rev-1}$$

It is apparent that a mechanism in which $p_I > R_S^C$ is never optimal. Consider the alternative mechanism, $M(\hat{C}) = (0, R_S^C)\, M(\hat{C}) = (0, R_S^C)$. Because the candidate mechanism satisfies PCRev-C, and because investor optimism implies that $v_A^C(w) - v_S^C(w) \geq 0$, and because $p_C \geq R_S^I$, $v_A^C(w) - v_S^C(w) + v_S^C(w) + p_C) > R_S^C$ and $R_S^I - p_I \leq 0$. The objective function is lower under the candidate mechanism than under the alternative mechanism. Because the payment to the firm type invariant and at least equal to the firms reservation value in both states the alternative mechanism is feasible. Thus the candidate mechanism is not optimal. Also note that PCRev-I implies that $p_I \geq R_S^I$, thus constraint ICRev-C is redundant. Using these observations we can reformulate the problem as follows:

$$
\begin{aligned}
&\underset{w, p_I, p_C}{\text{Max}}\ \pi \left(\mu_A^C - \left(v_A^C(w) + p_C \right) \right) + (1-\pi)\,(R_S^I - p_I), \\
&\text{ICRev-I: } p_I \geq v_S^I(w) + p_C, \\
&\text{PCRev-C: } v_S^C(w) + p_C \geq R_S^C, \\
&\text{PCRev-I: } p_I \in [R_S^I, R_S^C].
\end{aligned}
\qquad \text{Rev-2}
$$

We can simplify the problem by formulating it as a cost minimization problem. This program is provided below

$$
\begin{aligned}
&\underset{w, p_I, p_C}{\text{Min}}\ \pi \left(v_A^C(w) + p_C \right) + (1-\pi)\, p_I, \\
&\text{ICRev-I: } p_I \geq v_S^I(w) + p_C, \\
&\text{PCRev-C: } v_S^C(w) + p_C \geq R_S^C, \\
&\text{PCRev-I: } p_I \in [R_S^I, R_S^C].
\end{aligned}
\qquad \text{Rev-Cost}
$$

Forming the Lagrange dual for this problem yields:

$$
\mathscr{L}(w, p_C, p_I) = \pi \left(v_A^C(w) + p_C \right) + (1-\pi)\, p_I + \lambda_1 \left(v_S^I(w) + p_C - p_I \right) - \lambda_2 \left(v_S^C(w) + p_C - R_S^C \right). \qquad \text{(C5)}
$$

Thus, the dual problem can be expressed as

$$
\text{Min}\{\mathscr{L}(w, p_C, p_I) : p_C \geq 0, p_I \in [R_S^I, R_S^C], w \in [0,1]\}. \qquad \text{LRevMin}
$$

First note that

Result 8. In any solution to Problem Rev-Cost, constraint PCRev-C binds.

Proof. If PCRev-C were not binding then it would be possible to reduce either p_C or scale down the contract by some mount and leave the constraints satisfied and lower the cost function. □

Next note that any solution to Problem Rev-Cost, the cash payment to the seller conditioned on a report of C must be positive.

Result 9. If w, p_C, p_I is a solution to Rev-Cost then $p_C > 0$.

Proof. To obtain a contradiction, assume that (w, p_C, p_I) is a solution to Problem Rev-Cost and $p_C = 0$. Under this assumption, Result 8 implies that $v_S^C(w) = R_S^C$. Because $\mu_S^C > R_S^C$ (Assumption 2), it must be the case that the security s does not assign the entire cash flow to the seller $s(x) \neq x$, or equivalently $w \neq 1$. Therefore $v_S^I(w) < \mu_I^S = R_S^I$, where the last equality follows from Assumption 3. Therefore $v_S^C(w) - v_S^I(w) > R_S^C - R_S^I$. This implies that there exists a scaled down version of the security density, w, $\delta\, w$, which satisfies

$$v_S^C(\delta\, w) - v_S^I(\delta\, w) = R_S^C - R_S^I, \quad \delta \in (0,1). \tag{C6}$$

Define an alternative solution, (w^o, p_C^o, p_I^o), as follows: $p_I^o = R_S^C$, $w^o = \delta\, w$, and

$$p_C^o = R_S^C - v_S^C(w^o) = R_S^I - v_S^I(w^o), \tag{C7}$$

where the final equality follows from equation (C6). Note that, $p_C^o > 0$, $p_I^o \in [R_S^I, R_S^C]$, and that

$$p_I^o = v_S^I(w^o) + p_C^o \quad \text{and} \quad v_S^C(w^o) + p_C^o = R_S^C.$$

Thus (w^o, p_C^o, p_I^o) is feasible for Problem Rev-Cost.

Conditioned on the state being C, Result 8 implies that the cost to the acquirer under (w, p_C, p_I) equals

$$v_A^C(w) + p_C = v_S^C(w) + p_C + (v_A^C(w) - v_S^C(w)) = R_S^C + (v_A^C(w) - v_S^C(w)). \tag{C8}$$

Conditioned on the state being C, the cost to the acquirer under (w^o, p_C^o, p_I^o) equals

$$v_A^C(w^o) + p_C^o = R_S^C + (v_A^C(w^o) - v_S^C(w^o)) = R_S^C + (v_A^C(\delta\, w) - v_S^C(\delta\, w)) = R_S^C + \delta\,(v_A^C(w) - v_S^C(w)). \tag{C9}$$

Thus, comparing equations (C8) and (C9), using Assumption 2 and the fact that $\delta < 1$, shows that, conditioned on state C, cost under (w^o, p_C^o, p_I^o), cost is less than than under (w, p_C, p_I). Conditioned on the state being I, the cost under (w, p_C, p_I) equals $p_I \geq R_S^C = p_I^o$. Thus, cost conditioned on state I under (w^o, p_C^o, p_I^o) is weakly less than cost under (w, p_C, p_I). Hence (w^o, p_C^o, p_I^o) is not a solution to Problem (Rev-Cost). □

Next note that Result 9 implies that in any optimal solution of the dual problem defined by (LRevMin) must satisfy $\partial \mathscr{L} / \partial p_C = 0$, inspecting equation (C5) shows that $\partial \mathscr{L} / \partial p_C = 0$ is equivalent $\pi + \lambda_1 - \lambda_2 = 0$, or $\lambda_2 = \lambda_1 + \pi$. Substituting this relationship be between the multipliers into the right-hand side of equation (C5) yields

$$\left(\pi\left(v_A^C(w)-v_S^C(w)\right)-\lambda_1\left(v_S^C(w)-v_S^I(w)\right)\right)+(\pi+\lambda_1)R_S^C+p_I\left(1-(\lambda_1+\pi)\right).$$

Next note that the approach to simplification used in the proof of Proposition 2 shows that

$$\pi\left(v_A^C(w)-v_S^C(w)\right)-\lambda_1\left(v_S^C(w)-v_S^I(w)\right)=\pi\int w(x)\left(\left(\bar{F}_S^C(x)-\bar{F}_S^I(x)\right)\left(\frac{r_A(x)-1}{r_C(x)-1}-\frac{\lambda_1}{\pi}\right)\right)dx. \quad \text{(C10)}$$

The sign of the expression in the large parentheses on the right-hand side of equation C10 is determined by the sign of $x \hookrightarrow (r_A(x)-1)/(r_C(x)-1)$ which equals $\mathscr{R}$ (defined in equation 9) when $\eta = 0$. Thus, by Lemma 5, $\mathscr{R}$ is increasing and crosses the x-axis once from above. Lemma 4 thus shows that a density of the form $w(x) = 1$ if $x < k$ and $w(x) = 0$ otherwise minimizes the Lagrange, i.e. the optimal security design is debt. This observation and Result 9 establish the Proposition. □

Proof of Proposition 4. First note that only three results derived in this appendix—Lemma 6, Proposition 5, and Proposition 2—use Assumption 3. Thus, we can use all other preceding results in this proof without risking circularity. Next note that, absent Assumption 3, the conclusion of 6 need not hold. An offer solves (PR-CT) without the SLL constraint being imposed if and only if it solves the dual problem with the multiplier for the SLL constraint, η, set to 0. Lemma 5 shows that when $\eta^* = 0$, the optimal security design is seller debt. Thus, condition (4) implies that the optimal design with the SLL constraint not imposed, is not feasible. Hence, in an optimal solution to the dual problem, Problem PRDual-CT, the multiplier associated with the SLL constraint $\eta^* > 0$, Lemma 5 thus implies that $\mathscr{R}$ is U-shaped, decreasing and then increasing. Lemma 5 also implies that $\lim_{x\to 0^+}\mathscr{R}(x) = \infty$ and $\lim_{x\to\infty}\mathscr{R}(x) = \infty$. Thus, the set $\{x \in \mathbb{R}^+ : \mathscr{R} - \gamma^* < 0\}$ is an interval not including 0 and $\{x \in \mathbb{R}^+ : \mathscr{R} - \gamma^* = 0\}$ has measure 0. Let $k_0^* > 0$ represent the lower boundary of this interval and let $k_1^* > k_0^*$ represent its upper boundary. Lemma 4 implies that $w^* = 1$ on the interval and (almost surely) equals 0 off the interval. Thus Lemma 4 shows that optimal security densities have the form specified in the proposition. □

Proof of Result 1. See equation (C4) in the proof of Proposition 2.

□