ESG Investing: How to Optimize Impact?

Augustin Landier*
Stefano Lovo †

January 30, 2020

Abstract

This paper develops a general equilibrium model of a productive economy with negative externalities. Investors are not willing to accept lower returns than their best investment alternatives and entrepreneurs maximize profits. If capital markets are subject to a search friction, an ESG fund can raise assets and improve social welfare despite the selfishness of all agents. The presence of the ESG fund forces companies to partially internalize externalities. We derive the fund's optimal policy in terms of industry allocation and pollution limits imposed to portfolio companies. The fund prioritizes investments in companies where (i) the inefficiency induced by the externality is particularly acute and (ii) the capital search friction is strong. We also show that the ESG fund can take advantage of the supply-chain network: It can amplify its impact by imposing restrictions on the suppliers of the firms where it invests.

*HEC Paris
†HEC Paris

1 Introduction

Negative externalities generated by corporations, such as pollution, are a central theme in current policy debates. The traditional economic prescription to solve such externalities is regulation: Via Pigouvian taxes or tradable pollution permits ("cap-and-trade"), governments can influence the decisions of firms, thereby forcing them to internalize externalities (Weitzman (1974); Cropper and Oates (1992)). Due to political economy constraints, this approach has sometimes delivered disappointing results. Consider the example of carbon emissions: Free-riding among countries, political short-termism, and lobbying frictions, have strongly inhibited the regulatory response to climate change (see e.g. Tirole (2012)).

An alternative channel to curb firms' behavior is the financing channel: The participation of socially responsible investors to financial markets might decrease the cost of capital for companies that act responsibly, hence providing incentives to behave better. More and more investors do actually use sustainability criteria in their investment policy: According to the *The Forum for Sustainable and Responsible Investment*, as of year-end 2017, about 25% of U.S. professionally managed assets can be categorized as "socially responsible". Broadly speaking, one can identify two reasons inducing an individual to invest via a responsible fund. First, a *non-consequentialist* view that consists of an intrinsic preference for financing responsible firms regardless on whether this has an impact or not on the level of negative externality in the economy. Second, a *consequentialist* approach that aims at investing into a fund whose objective is to have a real impact in the economy by reducing negative externality, regardless on the firms in which the fund actually invests.

This paper embraces the consequentialist view and amis at answering the following question. Consider a responsible fund whose objective is to have a real impact in reducing externalities to maximize social welfare. How should this fund choose the composition of its portfolio, and what should and can the fund impose on the firms it finances. The answer is not obvious for two main reasons. The first one is substitution: Companies that are not compliant with the restrictions imposed by responsible investors might simply seek capital from non-responsible investors.[1]. The

[1]So, unless they constitute a large majority, the impact of responsible investors on the cost of capital in equilibrium might be small. In an interview(Edgecliffe-Johnson and Billy (2019)), Bill Gates summarizes this view quite bluntly: "Divestment, to date, probably has reduced about zero tonnes of emissions. It's not like you have capital-starved

second reason is that most responsible investors insist on generating returns that are competitive with non-responsible alternatives, which severely restricts their investment strategies.[2]

To answer this question, we model a multi-sector competitive productive economy where the two constraints mentioned above are carefully taken into account. There is a continuum of atomistic entrepreneurs and investors. Investors can either invest in companies started by entrepreneurs via profit-maximizing funds or via a responsible fund (the "ESGF"), which cares about aggregate welfare. Entrepreneurs raise capital to produce and they can choose the amount of pollution involved in their production process. Pollution increases production, and comes at no direct cost to the individual polluting firms. However, the aggregate level of pollution affects individual welfare negatively. To be conservative, we impose that no investor is willing to accept lower returns than her best investment alternatives. Hence, a responsible fund cannot raise capital if its expected returns are less than what investors can achieve via other funds. An additional difficulty for the ESGF is that companies can raise capital from non-responsible investors: This substitutability makes it hard for the ESGF to affect companies. We introduce a matching friction (a la Duffie et al. (2005)) in capital markets, so that we can parametrize how easy it is for companies to finance themselves without recourse to the ESGF. The optimal policy of the ESGF is defined by its capital allocation across sectors and the pollution requirements it imposes on companies if they decide to accept its capital. We compute the optimal policy of an ESGF, as a functions of its assets under management. In equilibrium, the presence of the ESGF increases aggregate welfare but reduces aggregate production and consumption.

We find several results, which have concrete normative implications for the sustainable finance industry. First we show that an ESGF that just defines its strategy as a cross-sector capital allocation has no impact on social welfare. To have an impact, the ESGF must impose some binding pollution caps to the firms it finances. Second, we show that the ESGF applies a pecking order: It prioritizes investment in sectors where the laissez-faire equilibrium externality level is

the people making steel and gasoline. I don't know the mechanism of action where divestment keeps emissions from going up every year."

[2]This might be due to both preferences and to legal constraints: For instance, in the US, investors subject to the fiduciary duties defined by the Employee Retirement Income Security Act cannot invest in a manner that hurts expected risk-adjusted returns.

particularly inefficient and where the search friction is particularly acute. Due to the search friction, concentrating ESG capital in one sector makes it more costly for companies to not comply with the restrictions of the ESGF. The prioritized sector typically does not coincide with the least polluting sector. Above a critical threshold of assets under management, the ESGF diversifies into a second sector. If the ESGF is large enough, first-best can be achieved. Third, we show that the responsible fund can take advantage of the economy's supply-chain network by imposing to the firms it finances restrictions on the choice of their suppliers. This strategy is particularly effective when the sector where reduction in emission would be the most beneficial, say sector i, is also the least subject to the financial friction. Firms in this sector can easily substitute ESGF's capital with "non-responsible" capital, which limits the *direct* impact of the ESGF. It is then optimal for the ESGF to invest all its capital in sector j and impose to firms in that sector to purchase their input from clean producers in sector i, the ESGF has then an *indirect* impact on sector i. Despite receiving no direct funds from the ESGF, a fraction of the firms in sector i reduce their emissions to be able to supply their ESGF financed customers in sector j. In equilibrium, sector i produces both "clean" and "dirty" goods, trading at different prices. This mechanism is in line with empirical results by Dai et al. (2019) and Schiller (2018) who document propagation of ESG standards along the supply chain network.

Literature Review. Our paper is related to different strands of the literature. On the empirical side, several papers explore the performance and preferences of socially responsible investors. On performance, the evidence is quite mixed. Hong and Kacperczyk (2009) and El Ghoul et al. (2011) document that "sin stocks" have positive abnormal returns suggesting their cost of capital is higher. Bolton and Kacperczyk (2019) also find that stocks of companies with higher CO_2 emission intensity earn higher returns. Barber et al. (2018) finds that impact investing private equity earns lower returns; Zerbib (2019) and Baker et al. (2018) find that green bonds are issued at a premium (controlling for risk), hence deliver lower returns. However, there is also evidence in the opposite direction, arguing that a company's ESG performance predicts positively its stock-returns. A possible explanation is market under-reaction to ESG information. For example, Edmans (2011) documents that firms that treat employees relatively well have positive abnormal returns. Derwall et al. (2005) find that more socially responsible portfolios provide higher average returns. Gibson and Krueger (2018) and Henke (2016) find a link between a portfolio sustainability footprint and its performance in the equity and bond markets respectively. Andersson et al. (2016) report over-performance of decarbonized stock indices and predict such green indices will out-perform further in the future: They argue that the market fails to fully recognize the impact of future restrictions on CO2 emissions[3]. In a broad meta-analysis of the empirical literature on responsible investing, Margolis et al. (2007) concludes that there is an ambiguous correlations between social responsibility and financial returns.

Regarding the motivations of socially responsible investors, Krueger et al. (2018) use a large-scale survey of institutional investors and find that they believe that screening companies based on environmental information can enhance risk-adjusted returns because equity valuations do not fully reflect climate risks. Hartzmark and Sussman (2018) reports a causal link between the flows into mutual funds and the publication of their sustainability ratings. Riedl and Smeets (2017) collect survey data and find that moral preferences are important factors for decisions by this type of investors. In our model, as we want to be conservative, we do not assume that investors are willing to bear lower returns for doing good. In particular, this allows our normative results to be agnostic about the existence or non-existence of a temporary under-reaction of markets to ESG information.

On the theory side, several papers model the implications of the existence of socially responsible investors. For instance, Heinkel et al. (2001) develop a model where a fraction of investors boycott firms that are not clean. "Dirty" companies trade at a discount compared to their "clean" peers, because in equilibrium, their shareholders (i.e. those that have no moral concerns) are more concentrated in "dirty" companies. In our paper, there is no uncertainty which shuts down the channel explored by Heinkel et al. (2001). Morgan and Tumlinson (2019) develop a theory where firms internalize externalities in that they solve a free-rider problem experienced in the production of a public good by maximizing shareholder welfare. Chowdhry et al. (2014) studies optimal contracting in the presence of externalities, when some investors are willing to pay for public goods, providing a foundation for impact securities. In the same spirit, Oehmke and Opp (2019) offer a theory

[3]This view is congruent with that of central bankers such as Matt Carney who have repeatedly warned that climate risks are not fully reflected in asset valuations yet.

of responsible investing where a moral hazard problem creates financial constraints that interact with externalities. By internalizing social externalities, responsible investors facilitate the scaling of virtuous projects and they are complementary to regular financiers. Different from Oehmke and Opp (2019), in our model, responsible investors have the same returns than regular investors. Our model emphasizes general equilibrium forces and a search friction that endows investors with some bargaining power.

In the following, Section 2 describes our analytical framework. Section 3 compares the laissez-faire equilibrium with the social optimum. Section 4 analyzes the ESGF optimal portfolio and policy when the fund focuses on reducing the emissions solely of the firms it finance. Section 5 analyzes the impact of ESGF can have exploiting the supply chain to curb the emission the firms it finance and/or of their suppliers. Section 6 concludes.

2 Model

We consider a competitive general equilibrium economy where agents are atomistic, enjoy consumption, but suffer from the toxic emissions generated by production of goods. The population of agents is composed of a mass 1 of capitalists and a mass 1 of entrepreneurs. Each capitalist is endowed with one unit of capital but lacks the skill to run a company. Each entrepreneur has the skill to run a company but has no capital. There are 2 goods; each good can be consumed or used as an input to produce the other good. Each good is produced in an industry, $i = 1, 2$, consisting of a continuum of competitive firms (with endogenous mass).

Technology. Let firms of industry i be indexed by $f \in [0, K_i]$, where K_i is the (endogenous) capitalization of industry i. The quantity $y_{i,f}$ of good i produced by a single firm f from the unit of capital depends on the firm's input quantity $x_{j,f} \geq 0$ of good j and the level $e_{i,f} \in [0, 1]$ of toxic emission the firm releases during production:

$$y_{i,f} = e_{i,f}^{\beta_i} x_{j,f}^{\alpha_{ij}} \tag{1}$$

where $\beta_i \in (0,1)$ and $\alpha_i \in (0,1)$. The industry's aggregate emission is $E_i = \int_0^{K_i} e_{i,f} df$.

Preferences. Individuals derive utility from the consumption of both goods, but suffer from the aggregate amount of emissions in the economy. Namely an individual utility from a consumption plan (c_1, c_2) is

$$u(c_1, c_2, E_1, E_2) = \frac{c_1^{\gamma_1} c_2^{\gamma_2}}{(1+E_1)^{\delta_1}(1+E_2)^{\delta_2}} \tag{2}$$

where E_i is the industry i aggregate emission and $\gamma_1 + \gamma_2 = 1$.

Goods markets Goods are exchanged in competitive markets, at prices that we denote p_i, $i = 1, 2$.

ESG policy and compliance conditions. Within this framework we introduce three mutual funds: a fund investing in industry 1, a fund investing in industry 2, and an ESG fund (ESGF henceforth) that can invest in both industries. The ESGF can commit to policies specifying maximal emissions thresholds specific to each industry. Namely, we denote with $(\hat{e}_1, \hat{e}_2)$ the ESG policy. An entrepreneur in industry i *complies* with the ESGF requirements only if her firm's emission $e_i(f)$ does not exceed $\hat{e}_i$. In any given industry only the entrepreneur who comply can be financed by the ESGF.

Search for capital. We introduce a search friction on capital markets. Let K_i denote the aggregate amount of capital invested in industry i and $k_{F,i}$ be the amount of capital that the ESGF invests into industry i. We note $\Phi(e, \hat{e}_i)$ the probability of being financed for an entrepreneur in industry i, given the emission level of her firm $e_i(f) = e$ and the ESG policy $\hat{e}_i$ in sector i. We assume

$$\Phi(e, \hat{e}_i) := \begin{cases} 1 \text{ if } e \leq \hat{e}_i \\ \frac{K_i - k_{F,i}}{K_i - \eta_i k_{F,i}} \text{ if } e > \hat{e}_i \end{cases},$$

where $\eta_i \in [0,1]$ is an industry specific parameter measuring the fluidity of the capital-entrepreneur matching market. $\Phi(e, \hat{e}_i)$ is 1 for a compliant entrepreneur, reflecting the fact that a compliant entrepreneur can be financed by all types of capitalists. $\Phi(e, \hat{e}_i)$ decreases with $k_{F,i}/K_i$ when $\hat{e}_i < e$, reflecting the fact that it becomes more difficult for a non-compliant entrepreneur in industry i to find financing if a larger fraction of the pool of capital dedicated to this industry is ESG. We provide in appendix a micro-foundation for the function $\Phi(e, \hat{e}_i)$, based on an explicit search game. What is important to note is that it spans two intuitive polar cases: For $\eta_i = 1$, $\Phi(e, \hat{e}_i)$ is 1, which means that the matching market is frictionless. When $\eta_i = 0$, $\Phi(e, \hat{e}_i)$ becomes $\frac{K_i - k_{F,i}}{K_i}$, which is the fraction of non-ESG capital invested in industry i. Intuitively, it is as if the entrepreneur just had one draw from the pool of capitalists to find a match. The intensity of the matching friction is measured by $1 - \eta_i \in [0, 1]$. If $\eta_i < \eta_j$ then friction capital matching market are more severe in industry i than in industry j. In this case we will say that industry i is the *friction industry.*

Sequence of play. The following actions unfold sequentially :[4]

1. The ESGF announces its policy.
2. Each capitalist choose how to allocate their capital among the three funds.
3. Each entrepreneur chooses irreversibly the good i that she wants to produce and a technology that determines the firm's emissions.
4. Entrepreneurs search for capital.
5. Production happens and output is sold. Profits are split between the entrepreneur and the capitalists: a fraction λ of profits is paid to the entrepreneur and the rest is paid to the capitalists who financed the firm.
6. Individuals spend their revenues to consume.

Having this timing in mind we can solve the model by backward induction.

Consumption choices. Consider an individual whose revenue is w. Her consumption choice solves:

[4] This timing of actions is given for expositional clarity. Because this is a single period general equilibrium economy where production and consumption are simultaneous, strictly speaking the agents interaction is modeled as a simultaneous move game, where all agents correctly anticipate the other agents's strategies.

$$\max_{c_1,c_2} \quad \frac{c_1^{\gamma_1} c_2^{\gamma_2}}{(1+E_1)^{\delta_1}(1+E_2)^{\delta_2}} \tag{3}$$

$$s.t. \quad p_1 c_1 + p_2 c_2 \leq w \tag{4}$$

Note that, since they are atomistic, agents take aggregate emissions (E_1, E_2) as exogenously given. Taking the first order condition, the individual's demand for good i is

$$c_i = \frac{\gamma_i w}{p_i}, \tag{5}$$

that brings to her a level of utility

$$u^*(w, E_1, E_2) = w \frac{\left(\frac{\gamma_1}{p_1}\right)^{\gamma_1} \left(\frac{\gamma_2}{p_2}\right)^{\gamma_2}}{(1+E_1)^{\delta_1}(1+E_2)^{\delta_2}}. \tag{6}$$

Production choices. Consider a firm in industry i with a technology inducing emissions $e \in [0, 1]$. Then the firm's demand for good j, solves

$$\text{argmax}_{x_j} \quad p_i y_i - p_j x_j \tag{7}$$

$$s.t. \quad y_i = e^{\beta_i} x_j^{\alpha_{ij}} \tag{8}$$

The resulting demand of good j from this firm is

$$x_j = \frac{\alpha_{ij} p_i y_i}{p_j} \tag{9}$$

and firm's profit is

$$\pi_i(e) = p_i y_i (1 - \alpha_{ij}) = \left(p_i e^{\beta_i} \left(\frac{\alpha_{ij}}{p_j} \right)^{\alpha_{ij}} \right)^{\frac{1}{1-\alpha_{ij}}} (1 - \alpha_{ij}) \tag{10}$$

which is increasing in the level of emission e.

Entrepreneur's choice: sector and technology. An entrepreneur has to choose ex-ante (before raising capital and producing) her firm's sector i and emission level $e_i \in [0,1]$. Complying with the standards of the ESGF in industry i means that $e_i \leq \hat{e}_i$. The entrepreneur spends her revenue to consume. From expression (6), the level of utility she will achieve is linear in her revenue. Thus, an entrepreneur chooses the industry i and the emissions level e_i such as to maximize expected revenues. Conditional on being financed, the entrepreneur gains a revenue equal to an (exogenous) fraction λ of the firm's profit $\pi_i(e_i)$ and zero otherwise.[5] The probability of finding capital is $\Phi(e, \hat{e}_i)$, which depends on emissions choice e. Hence the maximization program that describes the choice by the entrepreneur of her sector and emission level writes:

$$\max_{i \in \{1,2\}, e \in [0,1]} \Phi(e, \hat{e}_i) \lambda \pi_i(e) \tag{11}$$

This maximization trades off between (1) the fact that profits conditional on being financed increase in emissions and (2) the fact that finding financing is less likely if the firm does not comply.

Capitalists' portfolio choice. Consider now a capitalist who has to choose how allocate his unit of capital between the three funds. As each capitalist is atomistic, he takes the aggregate level of emissions as exogenous. Also, from equation (6), he chooses his portfolio such as to maximize his revenue. Let r_1, r_2 and r_F denote the respective returns on fund 1, fund 2 and the ESGF. Then a capitalist portfolio choice solves

$$\begin{aligned} \max_{\omega_1, \omega_2, \omega_F} \quad & r_1\omega_1 + r_2\omega_2 + r_F\omega_F \\ s.t. \quad & \\ & \omega_1 + \omega_2 + \omega_F \leq 1 \end{aligned}$$

We assume that an exogenous mass K_F of capitalists is ESG sensitive in the sense that they will invest all their capital in the ESGF if and only if $r_F \geq r_1, r_2$. The remaining $1 - K_F$ capitalists invest in the ESGF if and only if $r_F > r_1, r_2$.

[5] Here $\lambda \in (0,1)$ can be seen as the result of Nash bargaining between the entrepreneurs and the capitalists

We can now define a competitive equilibrium of this economy

Definition 1 *An equilibrium is a set of prices* (p_1, p_2) *and fund returns* (r_1, r_2, r_F)*, such that all agents maximize their utility taking the prices and the ESG policy as given; prices are such that the markets for goods and for capital clear; the ESGF chooses its policy to maximize agents' utility.*

The equilibrium is said to be symmetric *if all firms in the same industry choose the same technology.*

We normalize prices such that agents aggregate wealth is 1. The following proposition describes some of the common features to all equilibria of this economy.

Proposition 1 *In a symmetric equilibrium of the economy:*

1. *In every industry* i *either all firms comply or no firm complies. All firms are financed.*

2. *The total sales revenue of industry* i *is equal to*

$$Z_i := \frac{\gamma_i + \alpha_{ji}\gamma_j}{1 - \alpha_{ij}\alpha_{ji}} \tag{12}$$

3. *The capitalization of industry* i *is* $K_i = Z_i(1 - \alpha_{ij})$.

4. *The return on capital equals* $r = 1 - \lambda$*, no matter the firm in which the capital is invested.*

5. *A financed firm operating in industry* i *realizes profits* $\pi_i = 1$*. This makes entrepreneurs indifferent between producing in industry* 1 *or* 2.

6. *Individual revenues are* $1 - \lambda$ *for a capitalist and* λ *for an entrepreneur.*

7. *Let* $e_i := \frac{E_i}{K_i}$ *denote the average per-firm emission in industry* i*. Then the equilibrium level of utility of an individual with revenue* w *is equal to* $U(e_1, e_2)Cw$*, where* C *is a strictly positive constant and*

$$U(e_1, e_2) := \frac{e_1^{\beta_1 Z_1} e_2^{\beta_2 Z_2}}{(1 + K_1 e_1)^{\delta_1}(1 + K_2 e_2)^{\delta_2}} \tag{13}$$

The proposition shows that the equilibrium has three remarkable properties. First, the equilibrium composition of the market portfolio, and hence the size K_i of each industry $i = 1, 2$, only depends on consumers' taste for the two goods (γ_1 and γ_2) and the goods productivity as intermediary goods (α_{12} and $\alpha_{2,1}$). Note that the equilibrium formulae imply $K_1 + K_2 = 1$. Second, the equilibrium level of utility equals $U(e_1, e_2)C\lambda$ for an entrepreneur and $U(e_1, e_2)C(1 - \lambda)$ for a capitalist hence we can identify social welfare with $U(e_1, e_2)$. Third, all funds provide exactly the same return no matter whether they are ESG or not. Hence in equilibrium the amount of capital invested through the ESGF is $K_F \in [0, 1]$.

3 Levels of Emission: Laissez-faire vs. First best

3.1 Laissez-Faire

We call laissez-faire the equilibrium that prevails absent the ESG fund. Because firms are price-takers, each firm's profit is increasing in the amount of its direct emission. Hence, absent any incentive or regulation, all firms will set direct emissions to its maximum, that is $e_1 = e_2 = 1$ for all firms. The social welfare is then $U(1, 1)$.

3.2 First-Best

If consumers suffer strongly enough from an industry's aggregate emission, it is socially optimal to put a cap on firm's emission. Note that because emissions are necessary for production, a 0-emission level cannot be socially optimal. Consider a benevolent planner who can choose the level of emission in each firm as to maximize agents' utility. Formally the first best social optimum solves

$$\max_{e_1, e_2} U(e_1, e_2)$$

In order to guarantee that reducing emissions in both industries is socially optimal we assume that δ_1 and δ_2 are large enough.

Assumption 1 *For $i = 1, 2$,*

$$\delta_i \geq \frac{\beta_i(1 + K_i)}{1 - \alpha_{ij}}.$$

Then we have

Proposition 2 *Under Assumption 1 the social optimum is attained iff each firm in industry i has a level of emission equal to*

$$e_i^* = \frac{\beta_i}{(\delta_i - \beta_i Z_i)(1 - \alpha_{ij})} < 1 \tag{14}$$

The socially optimal level of emission results from the tradeoff between the discomfort of emission on consumer's utility, measured by δ_i, and the productive advantage of emission for good i. The latter increases with β_i, the production elasticity of emission, with α_{ij} and α_{ji}, the production elasticity in the input output matrix, and with γ_i, the utility elasticity from consuming good i.

4 Impact and Optimal ESG strategy

In this section we characterize the optimal strategy that the ESGF should implement to maximize agents utility.

Footprint of the ESGF. A notion that is often used in practice is the ESG footprint of a portfolio, which measures if a portfolio is tilted towards companies that have important levels of externalities. For instance, the "carbon footprint" of a portfolio measures the average level of emissions per unit of capital of companies in the portfolio: A green portfolio can be defined as a portfolio that has a low carbon footprint.

Formally, we define the toxic footprint of a portfolio allocated with industry weights (ω_1, ω_2) as: $\delta_1\omega_1 e_1 + \delta_2\omega_2 e_2$. The definition of the toxic footprint can be understood by going back to the utility function defined earlier.[6]

Impact of the ESGF. We define the impact of a policy of the ESGF as the difference in social welfare when the fund applies this policy vs. when the fund does not exist (or equivalently when the fund does not impose restrictions). This consequentialist definition of impact is in line with

[6]Fix the consumption level; the log utility is up to a constant $\delta_1 ln(1 + K_1 e_1) + \delta_2 ln(1 + K_1 e_1)$; The marginal impact on this quantity of a portfolio dk allocated with weights (ω_1, ω_2) is up to a scaling factor $(\sum \delta_i \omega_i e_i)dK$.

Brest and Born (2013) : “An impact investor seeks to produce beneficial social outcomes that would not occur but for his investment in a social enterprise. [...] Having impact implies causation, and therefore depends on the idea of the counterfactual.” We first show that tilting the sector allocation of the fund has by itself no impact on the economy. To have impact, the ESGF needs to impose limits to the emissions of firms where it invests.

4.1 Can industry tilting have impact by itself?

In our model, the answer is no: The mere shifting of a portfolio toward less polluting industries has no impact. There are two reasons for this: First, consider the situation where the tilt of the ESGF is small enough, such that capital allocated to a given industry does not exceed its laissez-faire equilibrium level. This tilt would be perfectly neutralized by the substitution by other sources of capital. Second, consider the situation where the ESG fund invests in an industry an amount that is large (larger than the equilibrium level characterized in Proposition 1); then it would actually be unable to provide returns that match those of the competing funds, which could take advantage of the scarcity of capital in other sectors to increase returns.

Corollary 1 *Suppose the ESGF imposes no emission restriction to the firms it finances, i.e., $\hat{e}_1 = \hat{e}_2 = 1$. Then, no matter the portfolio composition of the ESGF, $e_1 = e_2 = 1$ and individual utility is $U(1, 1)$.*

The ineffectiveness of mere portfolio choice is a direct consequence of Proposition 1. From point 2 of the proposition the composition of the market portfolio, and hence the relative size of each industry only depends on the consumer taste γ_1, γ_2 and the input-output matrix $(\alpha_{12}, \alpha_{21})$. Shall the ESGF choose a portfolio whose composition differ from the market portfolio to favor one industry over the other, the flow of non-ESG capital will undo the ESG one.[7] Also, if to be financed

[7] How is this possible? For example suppose that capitalists invest in the ESG a total amount $K_F > K_2$ and for some reason the ESG invest all this capital in industry 2. Why does not the market capitalization of industry 2 exceed K_2? This cannot happen in equilibrium. In fact there would be an abnormally large supply of good 2, profits of industry 2 firms will be lower than for industry 1 firms and hence the return on the ESGF will be strictly smaller than the return on fund 1. But then capitalists anticipating this will strictly prefer investing in fund 1 rather than in the ESGF. Thus a contradiction.

by ESGF, the entrepreneurs do not need to reduce its emission, they will just choose to maximize profit by setting direct emission to its maximum.

Corollary 1 implies that to have an impact the ESGF has to impose a restrictive emission policy on the firms it finances. The emission caps that the ESGF can impose, as well as the optimal composition of its portfolio, both vary with the size of its portfolio. This is what we want to characterize next.

4.2 Can the ESGF impose limits to emissions?

We now characterize how far the ESGF can go in imposing limits to emissions, as a function of the capital amount it invests in a given industry. The intuition is that the fund should set the max emission threshold $\hat{e}_i$ such that entrepreneurs are indifferent between complying or not. This happens if expected profits without complying (and thus setting emissions to 1) equal expected profits conditional on compliance:

$$\Phi(1, \hat{e}_i)\pi_i(1) = \pi_i(\hat{e}_i), \tag{15}$$

Equation (16) highlights the key role played by the matching friction in the ability of the ESGF to impose emission limits: if capital markets are perfectly fluid, $\Phi(1, \hat{e}_i) = 1$, then $e_i = 1$ is the only solution of (16). The economic intuition is that matching frictions make entrepreneurs worry about being compatible with the ESGF, in case they are matched with it. Also, note that the matching friction enables the ESGF to affect the behavior of *all* firms, even though it finances only a fraction of them. This is because emission choices are made ex-ante. In turn, this guarantees that the returns from the ESGF are competitive with those of other funds: In a sense, non-ESG investors are involuntarily ESG-compliant in our model, as the ESGF affects the behavior of all entrepreneurs in the same industry.

By developing equation (16), we can explicit the optimal emission limits in each industry as a function of capital invested by the ESGF in that industry:

Lemma 1 *The minimum amount of capital that the ESGF needs to invest in industry i to success-*

fully impose a limit to emissions $\hat{e}_i$ *is:*

$$K_{F,i}(\hat{e}_i) = \frac{1 - \hat{e}_i^{\frac{\beta_i}{1-\alpha_{ij}}}}{1 - \eta_i \hat{e}_i^{\frac{\beta_i}{1-\alpha_{ij}}}} K_i \tag{16}$$

By pledging $K_{F,i}(\hat{e}_i)$ *to industry* i *and committing to finance firms in that industry only if their emission does not exceed* $\hat{e}_i$*, the ESGF induces* all *firms in industry* i *to reduce their emissions to* $\hat{e}_i$*.*

$K_{F,i}(\hat{e}_i)$ is larger when $\hat{e}_i$ is smaller. This means that when the ESGF increases the capital it invests in an industry, it can impose tighter emission requirements in that industry: This is because entrepreneurs know they are more likely to be matched with the ESG investor and hence are more inclined to comply. The ability to reduce an industry's emission is stronger in industries where the capital matching friction is high (i.e. η_i is small), because, for a given level of capital invested, the ESGF has more ability to convince entrepreneurs to comply. It is also easier to reduce emissions when β_i is low, as the entrepreneur sacrifices less output by complying. We can express the constrained maximization problem of the ESG fund managing an amount of capital K_F as follows:

$$\begin{aligned} \max_{e_1, e_2} \quad & U(e_1, e_2) \\ s.t. \quad & K_{F,1}(e_1) + K_{F,2}(e_2) \leq K_F \end{aligned}$$

This makes apparent that there is a tradeoff between limiting emissions in one industry versus the other. The tradeoff comes from the fact that to impose stronger limitation to industry i emission, the ESGF needs to increase the capital it allocates to industry i at the expenses of industry j, reducing in this way its grip on industry j's emissions.

What should the ESGF do when it manages a small fraction of the total capital? One can see that, instead of spreading capital thin on the two sectors, it should instead concentrate capital in one sector. The sector to be prioritized is the one where the marginal impact of capital is the strongest.

Lemma 2 ***priority to the highest impact sector***. *If K_F is small, the ESGF invests all its capital in only one sector. This sector is the one where capital has the highest marginal impact on welfare:* $i_0 = \text{argmax}_{i\in\{1,2\}} \left(\frac{1-e_i^*}{e_i^*}\right)\left(\frac{1-\eta_i}{1+K_i}\right)$

To see this, consider an ESG managing a small infinitesimal amount dK_F, allocated across sectors as $(dK_{F,1}, dK_{F,2})$, where $dK_{F,1} + dK_{F,2} = dK_F$. This allocation triggers an impact on welfare, given by the following Taylor expansion:

$$\Delta U = U(1-de_1, 1-de_2) - U(1,1) = -\sum_{i=1,2} \frac{\partial U}{\partial e_i}\frac{\partial e_i}{\partial K_{F,i}} dK_{F,i} = \sum_{i=1,2} \frac{-\frac{\partial U}{\partial e_i}}{\frac{\partial K_{F,i}}{\partial e_i}} dK_{F,i} > 0$$

It follows that the welfare impact is maximized by investing all available capital in the industry i_0 where capital has the highest marginal impact on welfare:

$$i_0 := \text{argmax}_{i\in\{1,2\}} \frac{-\left.\frac{\partial U}{\partial e_i}\right|_{e_i=e_j=1}}{\left.\frac{\partial K_{F,i}}{\partial e_i}\right|_{e_i=1}}$$

The expression for i_0 follows from simple computations, and has a clear economic interpretation: To determine i_0, the industry on which small ESGF should focus, two elements need to be considered, the social desirability in reducing emission, measured by $(1-e_i^*)/e_i^*$, and the effectiveness of ESG incentives on entrepreneurial choice, measured by $(1-\eta_i)/(1+K_i)$. Given the same first best emission level, i.e., $e_1^* = e_2^*$, the ESGF should first focus on where its investment is most influential. Given the same effectiveness, the ESGF should first focus on the industry in which reduction of emission is most desirable, that is the critical industry, i.e. where e_i^* is the smallest. This is in line with the intuition in Brest and Born (2013): "Impact investing typically does not take place in large cap public markets, however, but rather in domains subject to market frictions. While some of these frictions impose barriers to socially neutral investors, socially motivated impact investors may exploit them". This suggests that rather than focussing on liquid shares of companies, impact investing should prioritize primary offerings, private equity, as well as less liquid stocks.

We can now characterise fully the ESGF's portfolio composition and policy that must be chosen in order to maximize social welfare:

Proposition 3 *Let K_F be the size of the fund and consider ESG policies that aim to maximize social welfare by only constraining firms direct emissions. There is $\underline{K}, \in (0, K_{F,i}(e_1^*) + K_{F,i}(e_2^*))$ such that all firms comply with the ESG policy, and:*

1. *If $K_F \leq \underline{K}$, then the ESGF invests only in industry i_0 and imposes emissions in that industry to be lower than $K_{F,i}^{-1}(K_F) = \left(\frac{K_i - K_F}{K_i - \eta_i K_F}\right)^{\frac{1-\alpha_{ij}}{\beta_i}}$.*

2. *If $\underline{K} < K_F < K_{F,i}(e_1^*) + K_{F,i}(e_2^*)$, then the then the ESGF invests in both industries, the optimal policy $(\hat{e}_1, \hat{e}_2)$ satisfies:*
$$\frac{\frac{\partial U}{\partial e_1}}{\frac{\partial K_{F,1}}{\partial e_1}} = \frac{\frac{\partial U}{\partial e_2}}{\frac{\partial K_{F,2}}{\partial e_2}}$$

3. *If $K_F \geq K_{F,i}(e_1^*) + K_{F,i}(e_2^*)$, then the ESGF invests in each industry i at least $K_{F,i}(e_1^*)$ its policy imposes first-best emissions: $(\hat{e}_1, \hat{e}_2) = (e_1^*, e_2^*)$.*

Let us interpret the different elements of Proposition 3:

When the fund size is particularly small, that is $K_F < \underline{K}$, the ESGF concentrates capital in industry i_0, as was discussed above in lemma 2. Firms in industry i_0 will comply, whereas in the other industry all firms will set emission at the maximum, $e_j = 1$. As more and more capital is invested in i_0, the marginal impact of capital in that industry goes down (since $\hat{e}_i$ gets closer to e_i^*), getting closer to that in the other sector. The threshold $\underline{K}$ corresponds to the mass of ESG capital that needs to be invested in i_0, such that the marginal impact of incremental ESG capital is the same in each sector.

For $\underline{K} < K_F < K_1^* + K_2^*$, the ESGF invests in both sectors. It equalizes the marginal impact of capital in each of the two sectors. The size of ESGF is however not sufficient to bring emission to the first best. The best the ESG can do is to fully exploit the financial constraint, given its size. The policy on each industry i converges toward the weights inducing e_i^* as K_F approaches $K_{F,1}(e_1^*) + K_{F,2}(e_2^*)$.

When the size of the ESGF $K_F > K_{F,i}(e_1^*) + K_{F,i}(e_2^*)$, the fraction of the total capital managed by the ESGF is large enough for the fund to be able to induce all firms to comply with the first best levels of emission, that is e_1^* for industry 1, and e_2^* for industry 2. The ESGF invests in both industries an amount sufficient to make the policy (e_1^*, e_2^*) acceptable to entrepreneurs. The first best is achieved. Note that when $K_F > K_{F,i}(e_1^*) + K_{F,i}(e_2^*)$, the marginal impact of additional ESG capital is zero, as the first-best is already implemented. An increase in the level of the capital market friction, reduces the total amount of ESG capital that is necessary to reach the first best.

Figure 1, illustrates the ESG constrained maximization problem in the plane $(\hat{e}_1, \hat{e}_2)$. Figure 2 depicts the socially optimal weight of industry 1 in the ESG portfolio as a function of K_F.[8]

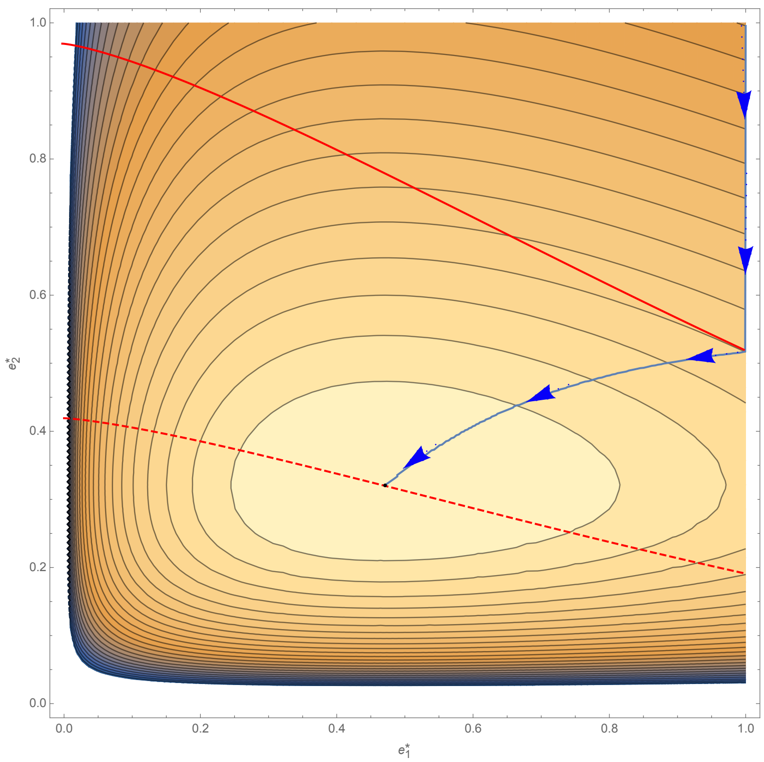

Figure 1: This figure shows ESG maximization problem in the plane (e_1, e_2). The black curves are iso-utility curves and the global maximum level of utility is achieved for (e_1^*, e_2^*). The continuous red curve indicates the minimum levels of (e_1, e_2) that can be achieved when $K_F = \underline{K}$. The dashed red curve indicates the minimum levels of (e_1, e_2) that can be achieved when $K_F = K_1^* + K_2^*$. The blu line indicate the constraint socially optimum level of emission for the different $K_F \in [0, 1]$ where arrows move from $K_F = 0$ toward $K_F \geq K_1^* + K_2^*$.

The next corollary relates the size of the ESGF with the level of utility, the level of consumption

[8] The parameters' values for these figures are $\alpha_{12} = 0.7$, $\alpha_{21} = 0.2$, $\beta_1 = 0.2$, $\beta_2 = 0.2$, $\gamma_1 = 0.2$, $\delta_1 = 1.5$, $\delta_2 = 1$, $\mu_1 = 0.5$, $\mu_1 = 1$.

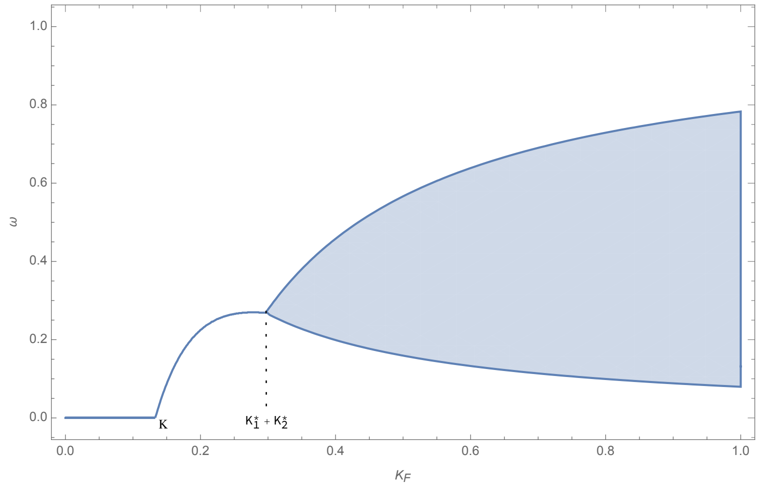

Figure 2: This figure presents the weight of industry 1 in the ESG socially optimal portfolio as a function K_F. Here, $i_0 = 2$, and hence for low enough K_F all ESG capital goes in industry 2. For $\underline{K} < K_F < K_1^* + K_2^*$, the weight of industry 1 increases with K_F. For $K_F > K_1^* + K_2^*$ many portfolio compositions are consistent with the first best social optimum as the only constraints are $\omega_i K_F \geq K_i^*$, for $i = 1, 2$.

and the level of price.

Corollary 2 *As long as the size of the ESGF $K_F < K_1^* + K_2^*$, an increase in K_F brings an increase in the individuals 'utility level and in the goods prices. It decreases the production and consumption of each good and weakly decreases level of emission in each industry. For $K_F \geq K_1^* + K_2^*$, utility and prices are maximal, whereas production and consumption are minimal.*

This result sheds light on the fact that in our model social welfare and aggregate consumption are decoupled: the ESGF helps reaching a higher level of welfare by implementing a lower level of aggregate consumption. The reason is that reducing emissions leads to a loss of productive efficiency, hence to lower aggregate output.

4.3 Footprint of the ESG fund

In real-world implementation of ESG investing, a relatively usual approach consists in limiting the "Carbon footprint" of the investment portfolio. This type of approach is also sometimes recommended by academics: For instance Gollier (2019) proposes that ESG funds should report their performance by subtracting from financial returns a multiple of the carbon emissions, where the multiple would be an explicit carbon price. However, in our set-up, this approach is potentially highly misleading. In fact, it turns out that there are cases where the toxic footprint of the ESG fund would be *higher* rather than lower than that of regular funds. The reason is that to maximize their impact, ESG funds should focus their investments in industries where they can convince managers to implement changes that are highly beneficial for welfare. In particular, investing in an industry that does not pollute is simply useless in terms of impact and consumes some of the ESG fund impact capacity in other sectors. This can be summarized in the following proposition. We have defined earlier the toxic footprint of a portfolio allocated with industry weights (ω_1, ω_2) as: $\delta_1\omega_1 e_1 + \delta_2\omega_2 e_2$.

Proposition 4 *The ESG fund does not necessarily have a better footprint than that of a regular fund.*

The proposition highlights that it is important to distinguish between footprint and impact, a distinction that is not always clear in the debate. The proof consists in finding a simple example: For instance, if $\delta_1 = 0$, the ESG fund will be all invested in in sector 2 (emissions in sector 1 are not harmful); whereas the "regular" investor is diversified across both sectors.

5 Using both direct and indirect emissions caps

In this section we explore what happens when the ESGF can express restrictions not only on the emissions of the firms where it invests, but also on their suppliers.

5.1 Internally consistent policies

Consider a firm in industry i. Beside its emission e_i directly resulting from the firms production process, the firm's economic activity is associated with another type of emissions: the direct emission of the firm's supplier of good $j \neq i$. We call this *indirect emission* and denote it with $\hat{e}_{Ui}$. In this section we study the impact the ESGF can have when eligibility to ESG capital requires a reduction in both direct and indirect emissions. That is, an entrepreneur in industry i complies with the ESGF requirements only if her firm's direct and indirect emissions do not exceed the caps set by the ESGF.

We focus on ESGF policies that are internally consistent, meaning that a firm in industry i which complies, is able to sell its output and purchase its input to and from compliant firms in industry j, respectively. Formally,

Definition 2 *A consistent policy is a quadruple* $\hat{e} = \{\hat{e}_1, \hat{e}_{U1}, \hat{e}_2, \hat{e}_{U2}\} \in [0,1]^4$*, such that* $\hat{e}_i \leq \hat{e}_{jU}$*, for all* $i = 1, 2$ *and all* $j \neq i$

Because compliant firms in industry i can only buy from industry j producers whose direct emissions do not exceed $\hat{e}_{Ui}$, a consistent policy implies the presence of goods markets that are specific to the producers' direct emissions levels. To take this into account we amend our base model in three dimensions. First, goods are exchanged in competitive dedicated' markets. We assume that if within an industry i a strictly positive mass of firms choose the same level e of emission, these firms will sell their outputs in a dedicated competitive market at a price that we denote $p_i(e)$. Note that a firm whose direct emission differs from all other firms has no dedicated market for its output. Because firms are atomistic, we assume that such a firm will be able to smuggle its production and buy its input in any of the markets for the corresponding goods. Second, when the same good is available in more than one market, consumers have the choice of where to purchase the good. Because a single individual's choice has no impact on aggregate emissions, each agent will purchase her consumption goods in the markets where they are the cheapest. Third, because eligibility to ESG capital concerns both direct and indirect emission, when searching for capital the entrepreneur has to specify both its firm direct and indirect emissions.

The following proposition shows that the use of indirect emission caps gives another reason for the ESGF to concentrate capital in only one of the two industries:

Proposition 5 *By adopting a consistent policy and investing in both industries, the ESGF cannot have more impact than when adopting an appropriate direct emission policy.*

The logic behind this result is as follows. When the ESGF invests in both industries all firms in each industry comply to the internally consistent policy $\hat{e}$. If all firms in the other industry j comply, then the only thing firms in industry i need to do in order to comply is to reduce their direct emissions. Hence we are back to the case where the ESGF does solely focus on direct emissions.

5.1.1 Indirect incentives

Can the ESGF have a stronger impact by investing all its capital in a single industry, but requiring the firms it finances to reduce both direct and indirect emissions?

Such ESGF's strategy provides to each industry incentives of different nature. By focusing its capital on a single industry i, the ESGF maximizes that industry's financial incentive to reduce direct and/or indirect emissions. In equilibrium, the presence of compliant firms in industry i gives rise to an endogenous mass of firms in industry j who choose to reduce their direct emissions. They do not do this to have better chances to be financed, but rather because good j produced with a low emission technology can be sold for a higher price than the same good produced with high emission. As we show in the next Proposition, in equilibrium the industry that receives ESGF capital will only be composed of compliant firms, whereas in the other industry both low-emission and high-emission firms co-exist.

Proposition 6 *Suppose ESG only invests in industry i, requiring compliant firms to reduce their direct and indirect emissions to $\hat{e}_i$ and $\hat{e}_{Ui}$, respectively, with:*

$$e_i^{\beta_i} \hat{e}_{Ui}^{\beta_j \alpha_{ij}} \geq \left(\max \left\{ 0, \frac{K_i - K_F}{K_i - \eta_i K_F} \right\} \right)^{1-\alpha_{ij}} \tag{17}$$

Then, in equilibrium

1. *In industry i all firms comply by setting their direct emission at $e_i = \hat{e}_i$ and buying from industry j firms with direct emission of $e_j = \hat{e}_{Ui}$.*

2. *Industry j is split into a mass of size $K_j\theta_j$ of high-emission firms, and a mass of size $K_j(1-\theta_j)$ of low-emission firms, where $\theta_j := \frac{\gamma_j(1-a_{12}a_{21})}{\gamma_j + a_{ij}\gamma_i} \in (0,1)$. A typical high-emission firm's direct emission equals 1, whereas a typical low-emission firm's direct emission equals $\hat{e}_{Ui}$.*

3. *Good j equilibrium prices satisfy $p_j(1) = (\hat{e}_{Ui})^{\beta_j} p_j(\hat{e}_{Ui}) \leq p_j(\hat{e}_{Ui})$.*

4. *Consumers buy good j exclusively from high emission firms, whereas industry i firms buy input j exclusively from low emission firms.*

5. Per firm average emissions in industry i and j are $e_i = \hat{e}_i$ and $e_j = \theta_j + (1-\theta_j)\hat{e}_{UI}$.

6. Social welfare is proportional to

$$U_I(e_i, e_j) := \frac{e_i^{\beta_i Z_i}}{(1+\hat{e}_i K_i)^{\delta_1}} \frac{\left(\frac{e_j-\theta_j}{1-\theta_j}\right)^{\beta_j \alpha_{ij} Z_i}}{(1+e_j K_j)^{\delta_2}} \tag{18}$$

By providing capital only to industry i and requiring compliant firms in this industry to reduce their direct and indirect emissions to $\hat{e}_i$ and $\hat{e}_{Ui}$, respectively, the ESGFs brings the direct emission to each individual firm in industry i to $\hat{e}_i$. Note however that the indirect emission cap on industry i only affects the direct emissions of a fraction $1-\theta_j$ of industry j. The remaining $K_j\theta_j$ firms will set their emission to 1. Thus, the average per-firm emission for industry j is qual to $\theta_j + (1-\theta_j)\hat{e}_{Ui}$. We can use this expression to translate inequality (17) into the constraint on the average per-firm emission that the policy can induce. This brings to the following maximization problem for the ESGF:

$$\begin{aligned} \max_{e_1, e_2, i \in \{1,2\}} \quad & U_I(e_i, e_j) && (19) \\ s.t. \quad & && \\ e_j \quad & \geq \quad \theta_j && (20) \\ e_i^{\beta_i} \left(\frac{e_j - \theta j}{1-\theta_j}\right)^{\beta_j \alpha_{ij}} \quad & \geq \quad \left(\max\left\{0, \frac{K_i - K_F}{K_i - \eta_i K_F}\right\}\right)^{1-\alpha_{ij}} && (21) \end{aligned}$$

It is worth interpreting constraints (20) and (21). No matter the policy $\hat{e}$, a fraction θ_j of firms in industry j will set emissions to 1. Thus, the minimum average emission in industry j cannot fall below θ_j, thus constraint (20). Note that if (21) holds with equality, then e_i must be decreasing in e_j. That is, the stricter are the restrictions on industry j emission the softer needs to be the emission restrictions in industry i. The tradeoff between the emission caps in the two industries is of different nature than that resulting from the purely direct emission policy we explored in Section 4. Here, in order to decrease average emission of industry j, e_j, the ESGF has to lower $\hat{e}_{Ui}$. This decreases the direct emission for a low-emission firms of industry j. To choose to lower their emissions, these firms must be compensates with a bigger selling price for their product, $p_j(\hat{e}_{Ui})$, compared to the price $p_j(1)$, at which high-emission firms in the same industry can sell theirs. That is, the lower e_j, the larger the relative price $p_j(\hat{e}_{Ui})/p_j(1)$. Consider now a firm of industry i when all other firms in the same industry comply. If such firm deviates and does not comply, it will maximize its production by setting its direct emission at 1, purchase input at low price $p_j(1)$ and then smuggle its production at price $p_i(\hat{e}_i)$. However it will risk not being financed. If instead the firm complies, it will certainly be financed but it will have to both reduce its direct emission, and hence production volume, and purchase good j input at the high price $p_j(\hat{e}_{Ui})$. Complying is an equilibrium only if the reduction of direct emission is commensurate with the increase in the relative price of input $p_j(\hat{e}_{Ui})/p_j(1)$. Hence the negative relation between e_i and e_j that result from the l.h.s. of (21). The r.h.s. of (21) shows how the grip of ESGF on emissions increases with K_F. For example if $K_F = 0$ or $\eta_i = 1$, then industry i firms have no benefit from complying because they can equally likely be financed without complying. If there are no compliant firms in industry i, no low-emission firm will be created in industry j.

5.1.2 Direct incentives vs indirect incentives for a small size ESGF

We have seen in Lemma 2 that for K_F small enough when the ESGF focuses on direct policies, it will invest all its capital in a single industry and use all its financial weight to curb that industry direct emissions. Depending on the values of the parameters this could be the the friction industry (i.e. the one with the smallest η_i) or not. If $\eta_1 = \eta_2$, then the ESGF should invest in the industry where $\frac{1-e_i^*}{e_i^*(1+K_i)}$ is the smallest. Lets call this industry the *critical industry*. In the following lemma we show that when ESGF can use indirect incentives this is not necessarily the most effective way of reducing industry j emissions ?

Lemma 3 *Consider a ESGF with K_F close to 0. To maximize its impact the ESG should invest all its capital in the friction industry and adopt a policy that amis at reducing only the critical industry's emission.*

When the friction industry and the critical industry are the same industry i, this is achieved by *imposing the friction industry only direct emission cap*

$$\hat{e}_i = \left(\frac{K_i - K_F}{K_i - \eta_i K_F} \right)^{\frac{\beta_i}{1-\alpha_{ij}}}.$$

When the friction industry is i and the critical industry is $j \neq i$, this is achieved by imposing to the friction industry only indirect emissions cap

$$\hat{e}_{Ui} = \left(\max \left\{ 0, \frac{K_i - K_F}{K_i - \eta_i K_F} \right\} \right)^{\frac{1-\alpha_{ij}}{\beta_j \alpha_{ij}}}.$$

5.1.3 Direct incentives vs indirect incentives for a medium size ESGF

Can focusing on a single industry direct and indirect emission have more impact than focusing direct emissions only but on both industries?

In the following proposition we provide sufficient conditions for which the answer is positive.

Proposition 7 *Suppose $K_F < K_{F,1}(e_1^*) + K_{F,2}(e_2^*)$. If $\eta_j > \eta_i$ with η_j large enough, or $\alpha_{ij} - \gamma_j$ is large enough, then to maximize its impact the ESG has to invest K_F into industry i and impose to this industry direct and indirect emission caps.*

Clearly if $K_F \geq K_{F1}(e_1^*) + K_{F2}(e_2^*)$, then the first best can be achieved by investing in both industries. Thus focusing on a single industry can maximize the ESGF's impact only if its size K_F is relatively small. The case $\eta_j > \eta_i$ with η_j large corresponds to a situation in which there is some friction in the matching capital market of industry i whereas in industry j capital market is virtually frictionless. In this case the ESGF capital in industry i provides substantially stronger financial incentives than in industry j. To maximize impact, the ESGF should invest all its capital where it can affect firms production choices and ask these firms to purchase from clean suppliers. A fraction $1 - \theta_j$ of industry j will then choose to reduce its emission to profit from the higher prices of good j in the clean supply chain.

The case $\alpha_{ij} - \gamma_j$ large correspond to a situation in which $\alpha_{ij} \simeq 1$ and $\gamma_i \simeq 0$. This reflects a situation in which consumers derive utility mostly from good i rather than from j, but good j

represents a substantial input for good i production. Observe that K_i is a decreasing function of a_{ij}. When K_i is small, by investing all its capital in industry i then ESGF acquires a substantial control of the industry and can impose strong limit to direct and indirect emission. The limit on industry i indirect emission will affect only a fraction $1-\theta_j$ of firms in industry j. However for γ_j close to 0, industry j good is mostly used as an input for industry i, implying that θ_j is close to 0.

6 Conclusion

This paper develops a general equilibrium model of a productive economy with negative externalities. We analyze the strategy of an ESG fund who amis at maximizing social welfare, when all individuals and firm do not internalize the externalities of their choices. We show that if capital markets are subject to a search friction, the ESG fund, can raise assets and improve social welfare despite the selfishness of all agents. The ESG fund has an impact on companies' behavior, forcing them to partially internalize externalities. We derive the fund's optimal policy in terms of industry allocation and pollution limits imposed to portfolio companies. This policy can be detailed in three steps. First for each industry one should determine the socially optimal level of emission, keeping into account the utility that consumption of that industry good generates as well as the disutility implied but that industry's emission. Second, the ESGF should consider in what industry its capital can have the strongest effect in reducing emissions. Third, the ESGF allocates its capital across the two industries keeping into account both which industries emission mostly need to be reduced and in which industry the ESGF financial incentive are most effective. We show that the fund applies a pecking order: It prioritizes investments in companies where (i) the inefficiency induced by the externality is particularly acute and (ii) the capital search friction is strong. We also show how the ESG fund can take advantage of the supply-chain network: It can amplify its impact by focusing on a single industry but imposing restrictions on the suppliers of the firms where it invests. This is particularly useful when the industry whose emission need most to be reduced is also the industry where the effectiveness of ESGF direct investment is the least. In general, to have any impact at all, the ESGF should finance firms in exchange of some reduction in their direct and/or indirect emission. An ESGF that only invest in the less polluting industries has no impact on the level of emission nor on the social welfare. We show that there can be little relation between a fund "carbon footprint" and its actual impact on improving social welfare.

References

Andersson, Mats, Patrick Bolton, and Frédéric Samama, "Hedging climate risk," *Financial Analysts Journal*, 2016, *72* (3), 13–32.

Baker, Malcolm, Daniel Bergstresser, George Serafeim, and Jeffrey Wurgler, "Financing the response to climate change: The pricing and ownership of US green bonds," Technical Report, National Bureau of Economic Research 2018.

Barber, Brad M, Adair Morse, and Ayako Yasuda, "Impact investing," *Available at SSRN 2705556*, 2018.

Bolton, Patrick and Marcin T Kacperczyk, "Do Investors Care about Carbon Risk?," *Available at SSRN 3398441*, 2019.

Brest, Paul and Kelly Born, "When can impact investing create real impact," *Stanford Social Innovation Review*, 2013, *11* (4), 22–31.

Chowdhry, Bhagwan, Shaun William Davies, and Brian Waters, "Incentivizing impact investing," in "Conference on the Impact of Responsible and Sustainable Investing, Hong Kong University of Science and Technology, Honk Kong, China" 2014.

Cropper, Maureen L and Wallace E Oates, "Environmental economics: a survey," *Journal of economic literature*, 1992, *30* (2), 675–740.

Dai, Rui, Hao Liang, and Lilian K Ng, "Socially responsible corporate customers," *Journal of Financial Economics (JFE), Forthcoming*, 2019.

Derwall, Jeroen, Nadja Guenster, Rob Bauer, and Kees Koedijk, "The eco-efficiency premium puzzle," *Financial Analysts Journal*, 2005, *61* (2), 51–63.

Duffie, Darrell, Nicolae Gârleanu, and Lasse Heje Pedersen, "Over-the-counter markets," *Econometrica*, 2005, *73* (6), 1815–1847.

Edgecliffe-Johnson, Andrew and Nauman Billy, "Fossil fuel divestment has zero climate impact, says Bill Gates," *Financial Times*, 2019, *sept. 17.*

Edmans, Alex, "Does the stock market fully value intangibles? Employee satisfaction and equity prices," *Journal of Financial economics*, 2011, *101* (3), 621–640.

Ghoul, Sadok El, Omrane Guedhami, Chuck CY Kwok, and Dev R Mishra, "Does corporate social responsibility affect the cost of capital?," *Journal of Banking & Finance*, 2011, *35* (9), 2388–2406.

Gibson, Rajna and Philipp Krueger, "The sustainability footprint of institutional investors," *Swiss Finance Institute Research Paper*, 2018, (17-05).

Gollier, Christian, *Le climat après la fin du mois*, Presses Universitaires de France, 2019.

Hartzmark, Samuel M and Abigail B Sussman, "Do investors value sustainability? A natural experiment examining ranking and fund flows," 2018.

Heinkel, Robert, Alan Kraus, and Josef Zechner, "The effect of green investment on corporate behavior," *Journal of financial and quantitative analysis*, 2001, *36* (4), 431–449.

Henke, Hans-Martin, "The effect of social screening on bond mutual fund performance," *Journal of Banking & Finance*, 2016, *67*, 69–84.

Hong, Harrison and Marcin Kacperczyk, "The price of sin: The effects of social norms on markets," *Journal of Financial Economics*, 2009, *93* (1), 15–36.

Krueger, Philipp, Zacharias Sautner, and Laura T Starks, "The importance of climate risks for institutional investors," 2018.

Margolis, Joshua D, Hillary Anger Elfenbein, and James P Walsh, "Does it pay to be good? A meta-analysis and redirection of research on the relationship between corporate social and financial performance," *Ann Arbor*, 2007, *1001*, 48109–1234.

Morgan, John and Justin Tumlinson, "Corporate provision of public goods," *Management Science*, 2019.

Oehmke, Martin and Marcus Opp, "A theory of socially responsible investment," 2019.

Riedl, Arno and Paul Smeets, "Why do investors hold socially responsible mutual funds?," *The Journal of Finance*, 2017, *72* (6), 2505–2550.

Schiller, Christoph, "Global supply-chain networks and corporate social responsibility," in "13th Annual Mid-Atlantic Research Conference in Finance (MARC) Paper" 2018.

Tirole, Jean, "Some political economy of global warming," *Economics of Energy & Environmental Policy*, 2012, *1* (1), 121–132.

Weitzman, Martin L, "Prices vs. quantities," *The review of economic studies*, 1974, *41* (4), 477–491.

Zerbib, Olivier David, "The effect of pro-environmental preferences on bond prices: Evidence from green bonds," *Journal of Banking & Finance*, 2019, *98*, 39–60.

Appendix

Micro-fundation of the matching model

Suppose that matching occurs as follows. Consider industry i. K_i is the total amount of capital in the industry and $k_{F,i}$ is the amount of capital coming from ESGF. We can see the matching as the result of the following dynamic process. In every period an entrepreneur in industry i that has not been financed meets a capital provider. With probability $k_{F,i}/K_i$, the capital provider offers ESGFs. With the complement probability the capital provider offers non-ESGFs. A non-compliant entrepreneur can only be financed with non-ESG capital whereas a compliant entrepreneur can be financed by either type of capital. We denote with $1 - \eta_i \in [0, 1]$ the hazard rate that the capital matching process stops before the entrepreneur finds an appropriate capital provider.

At the beginning of the matching phase we have that the expected payoff from an entrepreneur who chose a non-compliant technology is

$$V_{id} = \frac{K_i - k_{F,i}}{K_i} \lambda \pi_{id} + \frac{K_i - k_{F,i}}{K_i} \eta_i V_{id}$$

That is, with probability if the $\frac{K_i - k_{F,i}}{K_i}$, the entrepreneur is matched with non-ESG capital. She starts the firm immediately and retain a fraction λ of a non-compliant firm profit π_{id}. With probability $\frac{K_i - k_{F,i}}{K_i}$ the entrepreneur faces a ESG capital provider, she is not financed and has to go back to the matching market that will provide with a new capital provider with probability η_i. This implies

$$V_{id} = \lambda \pi_{id} \left(\frac{K_i - K_{Fi}}{K_i - \eta_i k_{F,i}} \right)$$

The expected payoff for an entrepreneur who chose a compliant technology is

$$V_{ic} = \left(\frac{K_i - k_{F,i}}{K_i} + \frac{K_i - k_{F,i}}{K_i} \right) \lambda \pi_{ic} = \lambda \pi_{ic}$$

That is, a compliant entrepreneur is certain to be immediately financed no matter the type of the capital provider she meets. The entrepreneur will choose a compliant technology only if $V_{id} \leq V_{ic}$, that is

$$\pi_{id}\left(\frac{K_i - K_{Fi}}{K_i - \eta_i k_{F,i}}\right) \leq \pi_{ic}$$

Note that when $\eta_1 = 1$, there are no friction and $\left(\frac{K_i - K_{Fi}}{K_i - \eta_i k_{F,i}}\right) = 1$. For η_1 the friction are maximal and $\left(\frac{K_i - K_{Fi}}{K_i - \eta_i k_{F,i}}\right) = \left(\frac{K_i - K_{Fi}}{K_i}\right)$. Equation (16), then can be written

$$\hat{e}_i \geq \Phi(K_F, \omega_i) := \left(\max\left\{0, \frac{K_i - \omega_i K_F}{K_i - \eta_i \omega_i K_F}\right\}\right)^{\frac{(1-\alpha_{ij})}{\beta_i}}, i = 1, 2 \tag{22}$$

Proofs

Proof of Proposition 1.

1. Let first prove that in equilibrium in any given industry either all firms comply or no firm comply. Suppose that there is an equilibrium where in industry i compliant and non-compliant firms coexist. Note first that each compliant firm in industry i will set its emission at $e_i(f) = \hat{e}_i$. In facts setting emission below $\hat{e}$ will not increase the entrepreneur probability of being financed whereas it will decrease the firm's total output and profit. For the same reason a non-compliant firm in industry i will set its emission to $e_i(f) = 1$, rather than to any other level $e_i(f) \in (\hat{e}_i, 1)$. Let denote with $\pi_{i,c}$ and $\pi_{i,n}$ the profits or a compliant and a non-compliant firm respectively. Note that from (10), one has $\pi_{i,c}/\pi_{i,n} = \hat{e}_i^{\beta_i} \leq 1$. If $\hat{e}_i = 1$, then there is no difference between a compliant and a non-compliant firm. If $\hat{e}_i < 1$, then $\pi_{i,c} < \pi_{i,c}$. Because one unit of capital invested in a firm with profit π provides $\pi(1-\lambda)$ to the investor, the return on capital on non-compliant firms will be strictly larger than the return on capital on compliant firms. This implies that the average return on industry i will be strictly larger than the return on compliant firms in industry i. As a result the return on the ESGF will be lower than the return a capitalist can obtain from a portfolio composed of fund 1 and fund 2. Capitalist will then not invest into the ESGF and there will be no compliant firms in the economy. Thus a contradiction.

   To see that in equilibrium all firms are financed, notice that if no firm complies in industry i, then the industry is fully financed by non-ESG capital and hence a non compliant firm is certain to be financed. If all firms in industry i comply, then each firm can be financed both with ESG and non-ESG capital and hence will be financed with certainty.

2. Taking into account consumers demand and firms demand given in (5) and (9), respectively, we can write the equilibrium condition on the goods markets:

$$\begin{cases} Y_1 = \frac{\gamma_1}{p_1} + \frac{p_2 Y_2}{p_1} a_{12} \\ Y_2 = \frac{\gamma_2}{p_2} + \frac{p_1 Y_1}{p_2} a_{21} \end{cases}$$

where $Y_i := Z_i / p_i$ is the aggregate production of good i and we claimed that consumer's aggregate wealth is equal to 1. The solution of this system provides expression (12).

2.-3. Note that in equilibrium the return of the three funds must be the same. Namely there is some $r > 0$ such that

$$r_1 = r_2 = r_F = r. \tag{23}$$

This because, first, if $r_1 \neq r_2$ then one of the two sector would receive no capital, but this would lead to no production of one of the goods, and this cannot occur in equilibrium. Second the ESGF portfolio is obtained from investing in either one or both industries. Because in each industry firms are homogenous, it must be that $r_F = r$.

Observe that industry i's aggregate profit is equal to $Z_i(1-\alpha_{ij})$. A fraction $1-\lambda$ of this profit will be distributed prorata to the capitalists. Hence, if the total amount of capital invested in industry i is K_i, then the return on investing in industry i must satisfy

$$r = \frac{Z_i(1-\alpha_{ij})(1-\lambda)}{K_i}$$

or equivalently

$$K_i = \frac{Z_i(1-\alpha_{ij})(1-\lambda)}{r}$$

Note that $Z_1(1-\alpha_{12}) + Z_2(1-\alpha_{21}) = \gamma_1 + \gamma_2 = 1$ and that the total amount of capital in the economy sum up to 1, i.e., $K_1 + K_2 = 1$. These equalities are satisfied only if $r = 1 - \lambda$ and

$K_i = Z_i(1 - \alpha_{ij})$.

4. Take a firm in industry i, then its profit is equal to $\pi_i = \frac{p_i Y_i (1-\alpha_{ij})}{K_i} = 1$, where the second equality follows from the fact that $p_i Y_i = Z_i$.

5. We already know from 2. that by investing his unit of capital, each capitalist gets $r = 1 - \lambda$. A typical entrepreneur's revenue is $\lambda \pi_i = \lambda$ because of 4. Consistently with our claim to prove point 2., consumers aggregate wealth equals $\int_0^1 \lambda de + \int_0^1 1 - \lambda dc = 1$.

3. From point 4. we know that a firm equilibrium profit π_i equals 1. Using expression (10) we can write

$$\begin{cases} \frac{p_1}{p_2^{\alpha_{12}}} e_1^{\beta_1} = \frac{1}{\alpha_{12}^{\alpha_{12}}} \frac{1}{(1-\alpha_{12})^{(1-\alpha_{12})}} \\ \frac{p_2}{p_1^{\alpha_{21}}} e_2^{\beta_2} = \frac{1}{\alpha_{21}^{\alpha_{21}}} \frac{1}{(1-\alpha_{21})^{(1-\alpha_{21})}} \end{cases}$$

implying,

$$\begin{cases} p_1^{1-\alpha_{12}\alpha_{21}} e_1^{\beta_1} e_2^{\alpha_{12}\beta_2} = \frac{1}{A_1} \\ p_2^{1-\alpha_{21}\alpha_{12}} e_2^{\beta_2} e_1^{\alpha_{21}\beta_1} = \frac{1}{A_2} \end{cases}$$

where $A_i := \alpha_{ij}^{\alpha_{ij}} (1-\alpha_{ij})^{(1-\alpha_{ij})} \alpha_{ji}^{\alpha_{ji}\alpha_{ij}} (1-\alpha_{ji})^{(1-\alpha_{ji})\alpha_{ij}}$. Observe that

$$p_1^{-\gamma_1} p_2^{-\gamma_2} = \Theta e_1^{\beta_1 Z_1} e_2^{\beta_2 Z_2}$$

where $\Theta := A_1^{\frac{\gamma_1}{1-\alpha_{12}\alpha_{21}}} A_2^{\frac{\gamma_2}{1-\alpha_{12}\alpha_{21}}}$. Replacing this expression of the prices into (6), and considering that $E_i = e_i K_i$, we have that the equilibrium level of an individual utility is proportional to the following function of the levels of emissions

$$U(e_1, e_2) = C \frac{e_1^{\beta_1 Z_1} e_2^{\beta_2 Z_2}}{(1 + e_1 K_1)^{\delta_1} (1 + e_2 K_2)^{\delta_2}}$$

where $C := \Theta \gamma_1^{\gamma_1} \gamma_2^{\gamma_2}$.

■

Proof of Proposition 2. Consider a social planner who can fix the average emission level in each industry e_i, e_2 in order to maximize the individuals utility. Note that in equilibrium each individual's utility is proportional to

$$U(e_1, e_2) = C\frac{e_1^{\beta_1 Z_1} e_2^{\beta_2 Z_2}}{(1+e_1K_1)^{\delta_1}(1+e_2K_2)^{\delta_2}}$$

So the social optimum maximizes separately for each $i \in \{1,2\}$. Namely, the socially optimal level of emission in industry i is the minimum between 1 and the e_i solving the following first order condition:

$$\beta_i Z_i(1+E_i) - \delta_i E_i = 0$$

The maximization problem has an internal solution only if $\delta_i < \frac{\beta_i(1+K_i)}{1-\alpha_{ij}}$. Otherwise it is optimal to let $e_i = 1$. ■

Proof of Lemma 1. Suppose that $\hat{e}_i$ satisfy inequality (16). Note first that if $\eta_i = 1$ then $\hat{e}_i = 1$ and the ESGF induce no constraint on industry i firms. So the Lemma is trivially satisfied. Hence, take $\hat{e}_i < 1$. Consider a single entrepreneur in industry i who expects all other entrepreneurs in the same industry to comply by setting their individual emission level at $\hat{e}_i$. We have seen in the proof of Proposition 1 that conditionally on complying he will set $e_i(f) = \hat{e}_i$. Second, let show that choosing a non-compliant technology cannot be optimal. The best a non-compliant firm f can do is to maximize production by maximizing emission, $e_i(f) = 1$, buy the input good j at the cheapest price and smuggling its product with the one of the compliant firms. By choosing not to comply however, the entrepreneur takes the risk of not being financed. Let K_{Fi} be the amount of capital that the ESGF invest in industry i. From the definition of $\Phi(e, \hat{e}_i)$ and expression (10), here expected payoff is

$$\left(\max\{0, \frac{K_i - k_{F,i}}{K_i - \eta_i k_{F,i}}\}\right) \lambda \left(p_i(\hat{e}_i)\left(\frac{\alpha_{ij}}{p_j}\right)^{\alpha_{ij}}\right)^{\frac{1}{1-\alpha_{ij}}} (1-\alpha_{ij}) \tag{24}$$

that is not larger than

$$\lambda \left(p_i(\hat{e}_i)\hat{e}_i^{\beta_i}\left(\frac{\alpha_{ij}}{p_j}\right)^{\alpha_{ij}}\right)^{\frac{1}{1-\alpha_{ij}}} (1-\alpha_{ij}) \tag{25}$$

only if $k_{F,i}$ is not smaller than $K_{F,i}(\hat{e}_i)$.

We already know from point 1 of Proposition 1 that all firm in the industry will either comply or not comply. If $\hat{e}_i < 1$ and the ESGF receive capitalist fund it must be that this capital is invested in compliant firms. Hence all firms in industry i comply. Another way of seeing this is that to make sure that all firms in industry i comply to a level of emission $\hat{e}_i$, the ESGF needs to invest in this industry strictly more than $K_{F,i}(\hat{e}_i)$. ■

Proof of Proposition 3.

The program of the ESGF is

$$\max_{e_1,e_2} \quad U(e_1, e_2) \tag{26}$$

$$s.t. \quad K_{F,1}(e_1) + K_{F,2}(e_2) \leq K_F \tag{27}$$

which we rewrite taking log of the objective function:

$$\max_{e_1,e_2} \quad \sum_{i=1,2} \beta_i Z_i ln(e_i) - \delta_i ln(1 + K_i e_i) \tag{28}$$

$$s.t. \quad \sum_{i=1,2} \frac{1 - e_i^{\frac{\beta_i}{1-\alpha_{ij}}}}{1 - \eta_i e_i^{\frac{\beta_i}{1-\alpha_{ij}}}} K_i \leq K_F \tag{29}$$

which we can see as:

$$\max_{e_1,e_2} \quad \sum_{i=1,2} f_i(e_i) \tag{30}$$

$$s.t. \quad \sum_{i=1,2} g_i(e_i) \leq K_F \tag{31}$$

1. First consider the case where K_F is small, preventing the fund to have high impact. Each e_i will be in the neighborhood of 1. The ESGF should prioritize the sector for which $-f_i'(1)/g_i'(1)$ is the highest. Now, $f_i'(1) = \beta_i Z_i - \delta_i \frac{K_i}{1+K_i}$ and $g_i'(1) = -\frac{1}{1-\eta_i}\beta_i Z_i$. We use $K_i = (1-\alpha_{ij})Z_i$; we get the pecking-oder rule: $i_0 = \text{argmax}_{\,i\in\{1,2\}}(1-\eta_i)[\frac{\delta_i(1-\alpha_{ij})}{\beta_i(1+(1-\alpha_{ij})Z_i)} - 1]$. From $e_i^* = \frac{\beta_i}{(\delta_i - \beta_i Z_i)(1-a_{ij})}$, it is easy to verify that $[\frac{\delta_i(1-\alpha_{ij})}{\beta_i(1+(1-\alpha_{ij})Z_i)} - 1] = \frac{1-e_i^*}{e_i^*(1+K_i)}$. Let $e_i(k_{F,i}) = K_{F,i}^{-1}(e_i) = \frac{K_i - k_{F,i}}{K_i - \eta_i k_{F,i}}$. Let $\underline{K}$ such that $f_{i_0}'(e_{i_0}(\underline{K})) = f_{-i_0}'(1)$. Then investing all ESGF capital in industry i_0 is optimal as long as $K_F \leq \underline{K}$.

2. For $\underline{K} < K_F < K_{F,1}(e_1^*) + K_{F,2}(e_2^*)$, there is an interior solution that is determined by the system of equations $f_i'(e_i) = \lambda g_i'(e_i)$, where λ is the Lagrangian multiplier.

3. When the ESGF size $K_F > K_{F,1}(e_1^*) + K_{F,2}(e_2^*)$, the fund manages a capital that is large enough to induce the first best socially optimal behavior. That is, by investing at least $K_{F,1}(e_i^*)$ in industry i, for $i = 1, 2$, the ESGF can guarantee that the average emission in each industry equals the socially optimal level.

■

Proof of Proposition 5. Suppose that the ESGF invests amounts $K_{F,1} > 0$ and $k_{F,2} > 0$ in industries 1 and 2, respectively. Then we have that $\left(\frac{K_i - k_{F,i}}{K_i}\right)^{\mu_i} < 1$ for $i = 1, 2$. From point 1 of Proposition 1, it follows that in any given industry i, all firms comply. With internally consistent policies, $\hat{e}_j \leq \hat{e}_{U,i}$, implying all possible suppliers of a firm in industry i have emissions that are compatible with the firm indirect emission cap implied by the policy. Thus, to comply the firm a firm in industry i need just to have its direct emission not exceeding $\hat{e}_i$. Because profits are decreasing in direct emission, each firm will set its emission to $\hat{e}_i$. Thus all firms in industry comply by setting $e_i = \hat{e}_i$ and we are back to the situation analyzed in section Section 4.2 where policies only focus on firms' direct emissions. ■

Proof of Proposition 6.

If the ESGF invests all its capital K_F in industry i, then $k_{F,i} = K_F$ and $k_{F,j} = 0$, and so $\left(\frac{K_i - k_{F,i}}{K_i - \eta_i k_{F,i}}\right) < 1$ and $\left(\frac{K_j - k_{F,j}}{K_j - \eta_j k_{F,j}}\right) = 1$. Hence from point 1 of Proposition 1, all firms in industry i comply. Consider hence industry j. Clearly there must be a non-nil fraction of firms in this industry that comply by setting their direct emission at $e_j(f) \leq \hat{e}_{Ui}$, otherwise there would be no supplier to firms in industry i that would allow these firms to comply. Let us show that not all firms in industry j will set their emissions below $\hat{e}_{Ui}$. Suppose instead that each firm f in industry j sets $e_j(f) \leq \hat{e}_{Ui}$. Then, an entrepreneur in industry j would profit by setting up the only firm whose direct emission are $e_j = 1$ and then smuggling its product in the only market of good j. This will allow the firm to generate bigger profit compared to the other firms in the same industry. Because there is no ESG capital invested in industry j, by choosing a non-compliant technology the entrepreneur will not reduce the chance of being financed. Thus, a profitable deviation. Let show now that compliant firms in industry i, low emission firms in industry j, and high emission firms in industry j will set their direct emission at $\hat{e}_i$, $\hat{e}_i^D$, and 1, respectively. Suppose that compliant firms in industry i set direct emission at $e_i < \hat{e}_i$. Then an entrepreneur of this industry could choose a technology with direct emission at $e_i = \hat{e}_i$, without reducing the probability of being financed and being able to smuggle its production in the same market of the other firms in the industry, thus having a larger profit than other firms in the same industry. The same argument applies to low emission firms in industry j. If these firms set their emission at $e_j < \hat{e}_i^D$, a profitable deviation would consist in technology where $e_j = \hat{e}_i^D$ and smuggling the production in the market for low emission good j. Such technology would be financed because there is no ESG capital in industry j and the firm profit would exceed the one of the other low emission firms of industry j. The same argument for high emission firms: if high emission firms were to set $e_j < 1$, then a firm would profit from setting $e_j = 1$.

To show 3., note that because both high emission and low emission firms exist in industry j, then industry j entrepreneurs must be indifferent between high and low emission. Because $\left(\frac{K_j - k_{F,j}}{K_j - \eta_j K_{Fj}}\right) = 1$, emission level does not affect the probability of being financed. Hence we must have that high and low emission firms generate the same profit. That is,

$$\left(p_j(1)\left(\frac{\alpha_{ji}}{p_i(\hat{e}_i)}\right)^{\alpha_{ji}}\right)^{\frac{1}{1-\alpha_{ji}}}(1-\alpha_{ji}) = \left(p_j(\hat{e}_{Ui})\hat{e}_{Ui}^{\beta_j}\left(\frac{\alpha_{ji}}{p_i(\hat{e}_i)}\right)^{\alpha_{ji}}\right)^{\frac{1}{1-\alpha_{ji}}}(1-\alpha_{ji})$$

that is is possible only if $p_j(1) = \hat{e}_{Ui}^{\beta_j} p_j(\hat{e}_{Ui}) \le p_j(\hat{e}_{Ui})$, where the inequality follows from $\hat{e}_{Ui} \le 1$.

Result 4. follows from the fact that consumers buy goods from the firms selling at the lowest prices, and $p_j(1) \le p_j(\hat{e}_{Ui})$. Thus, they will buy good j only from high emission firms. Because all firms in industry i comply, these firms can only buy their input from low emission firms in industry j.

We can now write the equilibrium condition on good i market, and on good j markets for high and low emission levels.

$$\begin{cases} Y_i = \frac{\gamma_i}{p_i(\hat{e}_i)} + \left(\frac{p_j(\hat{e}_{Ui})Y_{jL}}{p_i(\hat{e}_i)} + \frac{p_j(1)Y_{jH}}{p_i(\hat{e}_i)}\right)\alpha_{ji} \\ Y_{jL} = \frac{p_i(\hat{e}_i)Y_i}{p_j(\hat{e}_{Ui})}\alpha_{ij} \\ Y_{jH} = \frac{\gamma_j}{p_j(1)} \end{cases}$$

where, for industry j we used the subscrpit jL and jH to denote distinguish low emission and high emission variables, respectively. Solving this system one gets the following levels of sales revenues:

$$\begin{cases} Z_i = \frac{\gamma_i + \alpha_{ji}\gamma_j}{1-\alpha_{12}\alpha_{21}} \\ Z_{jL} = \frac{\gamma_i + \alpha_{ji}\gamma_j}{1-\alpha_{12}\alpha_{21}}\alpha_{ij} \\ Z_{jH} = \gamma_j \end{cases}$$

From these equilibrium level of sales, using the same argument as for the proof of point 2 in Proposition 1 we have that $K_i = Z_i(1-\alpha_{ij})$, $K_{jL} = Z_{jL}(1-\alpha_{ji})$ and $K_{jH} = Z_{jH}(1-\alpha_{ji})$.

We can now identify the level of $(\hat{e}_i, \hat{e}_{Ui})$ that can be achieved with such a strategy. First, note that if all other firms in industry i comply, then the best a non-compliant firm in industry i can do is to set its direct emission at 1, buy good j at the low price, $p_j(1)$, from high emission firms, and smuggle its production for $p_i(\hat{e}_i)$. This will result in a profit of

$$\left(p_i(\hat{e}_i)\left(\frac{\alpha_{ij}}{p_j(1)}\right)^{\alpha_{ji}}\right)^{\frac{1}{1-\alpha_{ji}}}(1-\alpha_{ji}).$$

The profit of a compliant firm in industry i is

$$\left(p_i(\hat{e}_i)\hat{e}_i^{\beta_i}\left(\frac{\alpha_{ij}}{p_j(\hat{e}_{Ui})}\right)^{\alpha_{ij}}\right)^{\frac{1}{1-\alpha_{ij}}}(1-\alpha_{ij}).$$

From equation (11), we have that the minimum emissions the ESG can induce in industry i are such that

$$\left(p_i(\hat{e}_i)\hat{e}_i^{\beta_i}\left(\frac{\alpha_{ij}}{p_j(\hat{e}_{Ui})}\right)^{\alpha_{ij}}\right)^{\frac{1}{1-\alpha_{ij}}}(1-\alpha_{ij}) \geq$$

$$= \left(p_i(\hat{e}_i)\left(\frac{\alpha_{ij}}{p_j(1)}\right)^{\alpha_{ij}}\right)^{\frac{1}{1-\alpha_{ij}}}(1-\alpha_{ij})\left(max\left\{0, \frac{K_i - K_F}{K_i - \eta_i K_F}\right\}\right)$$

By replacing $p_j(1)$ with $(\hat{e}_{Ui})^{\beta_j} p_j(\hat{e}_{Ui})$ and simplifying one gets inequality (17).

The level of an individual's utility is given by equation (6). Considering that consumers purchase good j at price $p_j(1)$ we have that social welfare is proportional to

$$\frac{\left(\frac{\gamma_i}{p_i(\hat{e}_i)}\right)^{\gamma_i}\left(\frac{\gamma_j}{p_j(1)}\right)^{\gamma_j}}{(1+E_1)^{\delta_1}(1+E_2)^{\delta_2}}$$

Considering that $\pi_i = \pi_{jL} = 1$ and following similar steps as for the proof of Proposition 1, we have

$$p_i(\hat{e}_i)^{-\gamma_i} p_j(\hat{e}_{Ui})^{-\gamma_j} = \Theta \hat{e}_i^{\beta_i Z_i} \hat{e}_{Ui}^{\beta_j Z_j}$$

where $\Theta = A_1^{\frac{\gamma_1}{1-\alpha_{12}\alpha_{21}}} A_2^{\frac{\gamma_2}{1-\alpha_{12}\alpha_{21}}}$ and $A_i = \alpha_{ij}^{\alpha_{ij}}(1-\alpha_{ij})^{(1-\alpha_{ij})}\alpha_{ji}^{\alpha_{ji}\alpha_{ij}}(1-\alpha_{ji})^{(1-\alpha_{ji})\alpha_{ij}}$. Recalling from point 3 above that $p_j(1) = \hat{e}_{Ui}^{\beta_j} p_j(\hat{e}_{Ui})$, we can write

$$p_i(\hat{e}_i)^{-\gamma_i} p_j(1)^{-\gamma_j} = \Theta \hat{e}_i^{\beta_i Z_i} \hat{e}_{Ui}^{\beta_j (Z_j - \gamma_j)}$$

Observing that $Z_j - \gamma_j = \alpha_{ij} Z_i$, we have that social welfare is proportional to

$$\frac{\hat{e}_i^{\beta_i Z_i} \hat{e}_{Ui}^{\beta_j \alpha_{ij} Z_i}}{(1+\hat{e}_i K_i)^{\delta_1}(1+(\theta_j + (1-\theta_j)\hat{e}_{Ui})K_j)^{\delta_2}},$$

or, in term of average emission per firm in each industry, e_i, e_j,

$$\frac{e_i^{\beta_i Z_i}\left(\frac{e_j - \theta_j}{1-\theta_j}\right)^{\beta_j \alpha_{ij} Z_i}}{(1+\hat{e}_i K_i)^{\delta_1}(1+e_j K_j)^{\delta_2}}$$

Because in industry i all the K_i firms comply, we have $E_i = \hat{e}_i K_i$. In industry j aggregate emission are the sum of the $\hat{e}_j K_{jL}$ from low emission firms and K_{jH} from high-emission firms, thus $E_j = \hat{e}_j K_{jL} + K_{jH}$. Replacing the expressions for p_i, p_j, E_i, E_j in (6), we find the expression in the proposition where $C = \gamma_1^{\gamma_1}\gamma_2^{\gamma_2}\Theta$.

■

Proof of Lemma 3. The proof is in two steps. First we show that if small size ESG is willing to reduce the average emission on a given industry, say industry 2, the most effective way is to put all its capital in the friction industry, i.e. the industry with the smallest η_i. Second, given that maximum impact is achieved by investing all capital in the friction industry, we show that the ESGF should focus on reducing the emission of the critical industry, i.e. the one with the smallest e_i^*.

Step 1: Suppose ESGF wants to reduce emission in industry 2. By investing all its capital in this industry it can brings its direct emissions to

$$e_{2,Dir}(K_F) := \left(\frac{K_2 - K_F}{K_2 - \eta_2 K_F}\right)^{\frac{1-\alpha_{21}}{\beta_2}}$$

If instead it invests all its capital in industry 1, imposes no cap on industry 1's direct emission, to focus on industry 1's indirect emission, it can bring industry 2 emission down to:

$$e_{2,Ind}(K_F) := \theta_2 + (1-\theta_2)\left(\frac{K_1 - K_F}{K_1 - \eta_1 K_F}\right)^{\frac{1-\alpha_{12}}{\alpha_{12}\beta_2}}$$

Note that $e_{2,Dir}(0) = e_{2,Ind}(0) = 1$. With some algebra we get

$$\left.\frac{\partial e_{2,Dir}}{\partial K_F}\right|_{K_F=0} = -\frac{1-\eta_2}{\beta_2 Z_2}$$

and

$$\left.\frac{\partial e_{2,Ind}}{\partial K_F}\right|_{K_F=0} = -\frac{1-\eta_1}{\beta_2 Z_2}.$$

Hence, the marginal impact of ESGF capital in reducing e_2 is stronger when investing in the industry with the smaller η_i. The same argument applies if the ESGF wanted to reduce industry 1's emission.

Step 2: Without loss of generality let assume that $\eta_1 < \eta_2$, implying the the fund must invest all its capital in industry 1. For K_F small the marginal gain in (the log of) social welfare from using the capital to reducing industry 1 or industry 2 emission is

$$\frac{\partial U(e_1,e_2)}{\partial e_1}\left.\frac{\partial e_{1,Dir}}{\partial K_F}\right|_{K_F=0} = -\left(\beta_1 Z_1 - \frac{\delta_1 K_1}{1+K_1}\right)\frac{1-\eta_1}{\beta_1 Z_1} = \frac{1-e_1^*}{e_1^*(1+K_1)}(1-\eta_1)$$

$$\frac{\partial U_I(e_1,e_2)}{\partial e_2}\left.\frac{\partial e_{2,Ind}}{\partial K_F}\right|_{K_F=0} = -\left(\beta_2 Z_2 - \frac{\delta_2 K_2}{1+K_2}\right)\frac{1-\eta_1}{\beta_2 Z_2} = \frac{1-e_2^*}{e_2^*(1+K_2)}(1-\eta_1),$$

respectively. Where, we used $e_1 = e_2 = 1$ for $K_F = 0$ and we have replaced $\left(\beta_i Z_i - \frac{\delta_i K_i}{1+K_i}\right)$ with $\frac{\beta_i Z_i(e_i^*-1)}{e_i^*(1+K_i)}$, $i = 1, 2$, using the definition of e_i^*. ■

Proof of Proposition 7. Let call the direct policy the case in which the ESGF focus on limiting each industry direct emission by investing in each of them. Let call indirect policy the case where all ESGF capital goes in industry i and imposes to this industry direct an indirect emissions caps. To fix idea we set $i = 1$ and $j = 2$. That is, the ESGF invests all its capital (up to K_i) in industry i requiring direct and indirect emission caps to firms it finances.[9] The argument of the proof unfolds as follows. Fix an arbitrary level of $e_1 \in [0, 1]$ and and let $K_F < K_{F,1}(e_1^*) + K_{F,2}(e_2^*)$. Consider the minimum level of e_2 that an ESGF of size K_F can impose to industry 2 while imposing e_1 to industry 1. Namely let $e_{2D}(e_1, K_F)$ and $e_{2I}(e_1, K_F)$ be these levels for the direct and the indirect policy, respectively. For the value of parameters in the proposition $e_{2D}(e_1, K_F) > e_{2I}(e_1, K_F)$ for almost all feasible e_1. Hence the set of (e_1, e_2) that can be implemented with the direct policy is strictly included in the set of (e_1, e_2) that can be implemented with the indirect policy. Thus, the impact of an appropriate direct policy is greater than the impact of the best of direct policies.

For the direct policy, rearranging $K_{F,1}(e_1) + K_{F,2}(e_2) \le K_F$, we have that

$$e_{2D}(e_1, K_F) = \max\left\{0, \frac{K_2 - (K_F - K_{F,1}(e_1))}{K_2 - \eta_2(K_F - K_{F,1}(e_1))}\right\}^{\frac{1-\alpha_{21}}{\beta_2}}.$$

[9]If $K_F > K_i$, the remaining ESGF's remaining capital is invested in industry j without requiring any emission cap to this industry.

For the indirect policy, eearranging (21) we have that

$$e_{2I}(e_1, K_F) = \min\left\{1, \theta_2 + (1-\theta_2)e_1^{-\frac{\beta_1}{\alpha_{12}\beta_2}} \max\left\{0, \frac{K_1 - K_F}{K_1 - \eta_1 K_F}\right\}^{\frac{1-\alpha_{12}}{\alpha_{12}\beta_j}}\right\}$$

Observe that for $e_1 \leq \max\{0, \frac{K_1-K_F}{K_1-\eta_1 K_F}\}^{\frac{1-\alpha_{12}}{\beta_1}}$, one has $e_{2D}(e_1, K_F) = e_{2I}(e_1, K_F) = 1$. For $e_1 > \max\{0, \frac{K_1-K_F}{K_1-\eta_1 K_F}\}^{\frac{1-\alpha_{12}}{\beta_1}}$ one has that $e_{2D}(e_1, K_F) < 1$ and $e_{2I}(e_1, K_F) < 1$. Hence without loss of generality let consider $e_1 > \max\{0, \frac{K_1-K_F}{K_1-\eta_1 K_F}\}^{\frac{1-\alpha_{12}}{\beta_1}}$

Fix $\eta_1 < 1$ and let $\eta_2 = 1 - \varepsilon > \eta_1$, then

$$\lim_{\varepsilon \to 0} e_{2D}(e_1, K_F) = 1 > e_{2D}(e_1, K_F)$$

hence, for any K_F there is $\eta_2 \in (\eta_1, 1)$ such that $e_{2D}(e_1, K_F) > e_{2I}(e_1, K_F)$, for all $e_1 > \max\{0, \frac{K_1-K_F}{K_1-\eta_1 K_F}\}^{\frac{1-\alpha_{12}}{\beta_1}}$.

Observe that because $0 < \gamma_2, \alpha_{1,2} < 1$, if $\alpha_{1,2} - \gamma_2$ is large, then there is some ε small such that $\gamma_2 \leq \varepsilon$ and $\alpha_{1,2} \geq 1 - \varepsilon$. Note that because $\alpha_{1,2} > 1 - \varepsilon$, one has that $\lim_{\varepsilon \to 0} K_{F,1}(e_1) = K_1$ and $\lim_{\varepsilon \to 0} K_1 = 0$. Hence,

$$\lim_{\varepsilon \to 0} e_{2D}(e_1, K_F) =, \left(\frac{K_2 - K_F}{K_2 - \eta_2 K_F}\right)^{\frac{1-\alpha_{21}}{\beta_2}} > 0$$

where the inequality follows from the fact that $K_F < K_{F1}(e_1^*) + K_{F2}(e_2^*)$, $\lim_{\varepsilon \to 0} K_{F1}(e_1^*) = 0$ and $K_{F2}(e_2^*) < K_2$. Also

$$\lim_{\varepsilon \to 0} \max\left\{0, \frac{K_1 - K_F}{K_1 - \eta_1 K_F}\right\}^{\frac{1-\alpha_{12}}{\alpha_{12}\beta_j}} = 0$$

Consider now $e_{2I}(e_1, K_F)$. Because $\gamma_2 < \varepsilon$ one has that $\lim_{\varepsilon \to 0} \theta_2 = 0$. Thus $\lim_{\varepsilon \to 0} e_{2I}(e_1, K_F) = 0$. By continuity there is $\varepsilon > 0$ small enough such that $e_{2I}(e_1, K_F) < e_{2D}(e_1, K_F)$.

■