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Ownership, Investment and Governance: The Costs and Benefits of Dual Class Shares

Finance Working Paper N° 352/2013
Revised August 2017

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This paper can be downloaded without charge from:
https://doi.org/10.67929/ecgi-fin-352-2013

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ECGI Working Paper Series in Finance

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Abstract

We show that dual-class shares can be a solution to agency conflicts rather than a result of agency conflicts. When firms with a controlling shareholder issue voting shares to fund projects, the risk of losing control rises, which can threaten the controller's private benefits. Thus, incumbents may forgo positive NPV investments to maintain control. Non-voting shares allow firms to fund projects without diluting an incumbent's voting rights; which alleviates the underinvestment problem. But, issuing non-voting shares dilutes dividends per share and facilitates entrenchment, reducing value-enhancing takeover bids. We derive conditions when the benefits from using non-voting shares outweigh its costs.

Keywords: JEL Classification: G32, G34, G38, K20

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Abstract

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Keywords: Blockholders, Controlling shareholders, Dual-class shares, Hostile takeovers, Ownership structure, Private benefits of control, Non-voting shares, Shareholder welfare, Takeover defenses, Underinvestment, Voting rights.

JEL Classification Code: G32, G34, G38, K20

*Corresponding author's tel: +61293855860 and email: ron.masulis@unsw.edu.au. Suman Banerjee (sbanerj2@stevens.edu) gratefully acknowledges financial support (SUG Tier 1) from the Ministry of Education, Singapore. We would like to specially thank Jennifer Arlen, Harry DeAngelo, David Hirshleifer, Clive Lennox, Prem Jain, Chang-Mo Kang, Thomas H. Noe, Pavle Radicevic and Laura Stark for insightful comments. We would also like to thank participants at FIRN Conference 2012 participants and in particular Jordan Neyland for their helpful comments. Also, we thank participants at the Indian School of Business Summer Conference 2014 and in particular discussant Abhiroop Mukherjee for helpful comments. Further, we would like to thank participants at ECGI Corporate Finance Symposium 2015 participants and in particular discussant Li Jin for their helpful comments. All remaining errors are our own.

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1 Introduction

In recent years, equalizing the voting power and influence of common shares has become a touchstone of good governance and shareholder democracy in much of the world. Corporate charter provisions that explicitly limit the rights of minority shareholder are completely antithetical to this viewpoint, although they widely exist globally.[1] Thus, it is not surprising that dual-class provisions, which create a second class of common stock with reduced or no voting power, have come under fire. More generally, activist shareholders have voiced serious concerns about the disenfranchising of investors holding shares with inferior voting rights.[2] The corporate democracy movement has led policy makers, most vocal among them are a high-level group of EU company law experts and Indian corporate activists, to warn of the threats posed by dual-class ownership structure.

Yet, the theoretical literature on dual-class shares is quite sparse. Models by Grossman and Hart (1988), Harris and Raviv (1988) and Ruback (1988) and finance and law literature by Bebchuk and Kastiel (2017) and Bebchuk and Hamdani (2017), have analyzed non-voting equity in the context of control contests and found that a dual-class share structure yields a negative shareholder wealth effect. These authors trace this negative wealth effect to the unbundling of voting rights and cash flow rights – arguing that the unbundled votes can act as an anti-takeover device. When the likelihood of a successful takeover is significantly diminished, it allows managers to deviate from actions that enhance shareholder wealth. For example, Shleifer and Wolfenzon (2002) show that firms with weaker shareholder protection have lower valuation, which is consistent with investors anticipating that some profits or

[1] Companies like Berkshire Hathaway, Blackstone Group, Clearwire, Dolby, Echostar, Facebook, Ford, Fox News, Google, MasterCard, News Corp, Rosetta Stone, VISA, VMWare, and WebMD have multiple classes of shares. Dual-class structures are widely used in countries like Brazil, Canada, Denmark, Finland, Germany, Italy, Norway, Sweden, and Switzerland. In Canada 5 to 6% of listed companies, like Metro, Bombardier, Gluskin Sheff, Air Canada, Exfo, Cossette, and Celestica, have multiple classes of shares.

[2] Institutional Shareholder Services (ISS) recently recommended that dual-class structures be eliminated entirely for all newly listed companies. Also, ISS wants corporate laws to be changed to require sunset provisions for companies with dual class structure, such that all shares will revert to common shares after a pre-specified time, unless a majority of inferior-class shareholders vote to reaffirm the dual-class structure.

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valuable assets are likely to be diverted. As the market for corporate control weakens as a disciplining mechanism, investors recognize that managers have greater latitude to extract private benefits. Thus, according to the existing literature dual class share structures hurt firm value and shareholder wealth.

We argue for a more nuanced view of the role of dual-class shares: Although a dual-class structure weakens the incentives associated with the market for corporate control, it helps to mitigate the underinvestment problem resulting from the non-contractibility of a firm's investment policy. For example, a scale-expanding investment project generally requires a sizable issue of new shares. If a manager is forced to issue voting shares, then these newly issued votes dilute a manager's proportional voting power and severely impede his ability to resist future takeover attempts. DeAngelo and DeAngelo (1985) observe that an underinvestment problem can exist if managers face an increasing risk of losing control when funding new projects with voting equity. Thus, a rational manager owning voting shares and extracting valuable private benefits of control has an incentive to reject many profitable scale-expanding investments.[3] For example, Wruck (1989) finds average management-controlled share ownership fall by 1.5% around the time of a typical private equity sale, while public offers are likely to dilute manager voting rights much more, given SEOs are typically of larger size. Likewise, Faccio and Masulis (2005) find that firms with controlling shareholders are often reluctant to finance acquisitions with stock for a similar reason.[4] On the other hand, newly issued non-voting shares do not affect a manager's control rights and thus, do not discourage value maximizing investments.

We show that an incumbent manager, who has the option to use non-voting shares, does

[3]Also, Field and Lowry (2017) find little evidence that the recent tendency of IPO firms to adopt classified boards is driven by agency problem.

[4]According to the Library of Canadian Parliament, "Undeniably, some of the best-performing companies in Canada have multiple-voting shares. Thus, not all shareholders are concerned with the voting rights attached to a share. They may be more interested in the potential of sharing the company's wealth or trading on future prospects by buying cheaper, subordinated shares." Also, according to Barry Reiter of Bennett Jones LLP, "some dual-class firms are created to favor Canadian ownership in strategic or culturally sensitive fields. Many foreign investors have happily bought into structures of this sort."

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not automatically use non-voting shares to fund all new investments. This is because issuance of non-voting shares comes at a substantial cost: Non-voting shares dilute the cash flow rights of all existing shareholders, including the incumbent manager, as more non-voting shares relative to voting shares must be issued to raise the same amount of project funding. This causes the aggregate dividend payout to be divided among more shares, thereby reducing the per share dividend. For example, Faccio and Masulis (2005) find incentives to use non-voting shares to fund an acquisition, which tend to be large investments, are strongest when a target firm's ownership is concentrated and a bidder's controlling shareholder has an intermediate level of voting power – a range where the incumbent manager is most vulnerable to a loss of control under a stock-financed acquisition.

Similarly, an incumbent who has no option but to use voting shares to fund new investments will not necessarily underinvest. Because the manager is also a shareholder, he bears part of the cash flow loss from the firm forgoing positive NPV projects. Thus, it is possible for this cash flow loss to outweigh his expected control-related benefits from this underinvestment. Hence, an incumbent manager faces a clear trade-off when choosing the security class to use to fund the new investment projects. The manager faces the choice of dilution of his cash flow rights from forgoing profitable investment projects and dilution of control rights from accepting the project.

To add yet another wrinkle to the decision process, we observe that outside shareholders, who are assumed to have rights to approve for dual class shares, also face a trade-off: the expected costs of greater management entrenchment against a higher firm value from avoiding under-investment. If the outside shareholders allow an incumbent manager to issue either voting and or non-voting shares, then the incumbent invests in all available positive NPV projects; but outside shareholders also know that when this additional investment is funded by issuing non-voting shares, the incumbent manager becomes relatively more entrenched because his private benefits play an enhanced role in potential takeover contests. Hence, future takeover attempts are made more costly, which results in lower expected takeover

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premiums and, consequently, lower firm values. However, if outside shareholders force an incumbent to use only voting shares to fund new investments, then the incumbent may forgo some positive NPV projects, which is also costly for outside shareholders.[5]

Taking into consideration all of these possibilities, we propose that differences in investment opportunity sets may help to explain the considerable variation in the effects of dual-class share issues both within and among firms. Others have shown that deviations from "one share-one vote" can be optimal, but our model is the first to integrate the dual-class decision (heretofore viewed simply as problem for outside shareholders to overcome in the control literature) into the rich body of research on capital structure and the underinvestment problem. We specifically focus on a firm's decision to forgo positive NPV investment opportunities.[6]

In a control context, the problem of underinvestment differs considerably from standard underinvestment scenarios: Here underinvestment results from a manager's fear of diluting his control rights – which reduces his likelihood of retaining control. For example, Masulis et al. (2009) find quite a few firms with dual-class shares, where the largest shareholder does not own a majority of the voting rights. However, if the manager owns a tiny block of voting shares or owns a majority of votes well in excess of 50%, it is easy to show that there is no scope for significant control dilution and hence, there is no underinvestment problem. Thus, only when manager voting power is in an intermediate range does an underinvestment problem exist.[7] Likewise, this is only a concern when the project's size is substantial relative

[5]For example, DeAngelo and DeAngelo (1985) indicate that an underinvestment problem can exist if the managers are faced with increasing risk of losing control when they fund new projects with voting equity.

[6]For example, Neeman and Orosel (2006) show that a voting contest for votes in addition to a contest for shares can have efficiency advantages; Blair et al. (1989) show that a market for votes can increase efficiency during control contests in the presence of taxes; Edmans and Manso (2011) show that shareholders who hold non-voting shares can exert influence through the threat of exit. Also, Masulis et al. (2011) show that control can also be maintained by pyramid structured business groups, which may act as substitute for non-voting shares. Further, Laux (2012) shows that a suboptimal vesting period in CEO incentive compensation contracts can induce myopic investment behavior similar to ours.

[7]Underinvestment and its causes have been studied in a number of papers: Debt-induced underinvestment is considered by Galai and Masulis (1976), Henderson (1993), Myers (1977), and Berkovitch and Kim (1990).

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to a firm's equity capitalization.

One possible solution to the underinvestment problem is to issue debt rather than equity. However, debt does not solve the underinvestment problem, because it carries with it the risk of bankruptcy. Issuing more debt generally requires stricter covenants, which in turn raises the risks of loss of control to creditors for the incumbent manager. This is because when a covenant is violated, the creditors have the ability to demand that the managers be replaced before agreeing to either a covenant revision or a voluntary debt restructuring. For example, consistent with this concern DeAngelo and DeAngelo (1985) find evidence that dual-class firms infrequently resort to increasing leverage to retain control. Instead, dual-class firms seek to keep leverage low, consistent with their desire to minimize the risk of creditors taking control of the firm. The issuance of non-voting stock to fund new investment does not result in dilution of a manager's control rights; nor does it have any adverse consequences in terms of stricter or additional debt covenants.

A dual-class structure can be particularly helpful for a smaller firm facing large, profitable investment opportunities. It enables a firm to significantly increase shareholder wealth by not passing up profitable investments and thereby improves a firm's overall economic efficiency. However, as mentioned earlier the costs associated with issuing non-voting equity limit its effectiveness in solving the underinvestment problem for firms with a controlling shareholder.[8]

Our model predicts that high growth firms, rather than firms whose value is dominated by assets-in-place, are more likely to use dual-class shares. This prediction is consistent with existing empirical findings in Lehn et al. (1990) and Dimitrov and Jain (2006) which states

Myers and Majluf (1984) and Cooney and Kalay (1993) derive conditions under which an undervalued firm forgoes positive NPV investments.

[8]If shareholders collectively make the firm's investment decisions, then underinvestment is no longer an issue. However, this solution would undercut efficient investment decision making because it makes it next to impossible to prevent competitors from gaining access to important proprietary information. If less stringent information requirements are imposed, then a manager can insure underinvestment by withholding crucial information from shareholders. Also, if it is possible to directly contract with a manager across all states of the world, again underinvestment can be avoided. However, such a contractual solution would require a firm's investment opportunities to be known to its shareholders. Furthermore, these investment oppoWrtunities must be verifiable - imposing added verification costs on shareholders.

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that high growth firms are more likely to adopt dual class share structure.[9] Our analysis produces a number of novel testable implications:

- First, restrictions on equity security design can reduce shareholder value. Such restrictions can lead to severe underinvestment and at times may outweigh the positive value effects that requiring the issuing of voting shares have on control contests.
- Second, the likelihood that voting shares is initially rising and then falling in a manager's shareholdings. If manager shareholdings are small, then there is no scope for control dilution. If his shareholding size is relatively large, then he bears a large fraction of the cost of underinvestment and dividend dilution.
- Third, the extent to which the likelihood of a takeover is reduced by having dual class shares.
- Fourth, the expected change in the takeover premium with the issuance or redemption of dual class shares.
- Lastly, the expected change in the consumption of private benefits of control with the issuance or redemption of dual class shares.

The remainder of this article is organized as follows: In Section 2 we develop a detailed numerical example capturing the essence of our theoretical model. In Sections 3, 4 and 5, we show that the basic intuition of our numerical example can be formally modeled and analyzed. In these sections, we fully characterize the underinvestment problem and further analyze the effect of underinvestment on outside shareholders and incumbent managers. Possible extensions are discussed in Section 6. Conclusions are presented in Section 7. Some of the more cumbersome results and an extensive numerical exercise are delegated to the

[9]There are a few more empirical implications that are partly or fully tested in prior studies papers. For example, our model corroborates that, conditional on the same level of assets and investment activities, a dual-class firm is less valuable than a comparable single-class firm – a prediction partly tested in Claessens et al. (2002), Boone and Mulherin (2007) and Gompers et al. (2010). Also, our model shows that a dual-class firm is less likely to become a takeover target, but conditional on a takeover bid surfacing, the premium offered for voting shares is likely to be higher. Papers like Studies by Seligman (1986), Jarrell and Poulsen (1988), Ambrose and Megginson (1992), Smart and Zutter (2003) and Krishnan and Masulis (2011) have empirically tested these implications and reported supportive evidence.

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appendix.

2 Numerical Example

Consider a firm that has a public value of \$2.00 million, generates a private value for the incumbent of \$0.20 million and has 100 shares outstanding. The value of the existing firm, both public and private, is the same under both the incumbent and the rival manager. The incumbent owns 50 shares in the firm and the incumbent manager is wealth constrained. Given our assumption that the incumbent owns half the shares in the firm, there is a zero probability of a change in control of the firm, ($\phi = 1$), without the incumbent's consent.

The expected value of the incumbent's stake in the firm is the sum of the expected public value of the shares that he owns, plus the expected private benefits of control; that is, the value of the incumbent's stake in the firm is \$1.20 million ($= \frac{1}{2} \times \$2.00 + \phi \times 0.20 = \$1.00 + 1.0 \times 0.2$). The value of the shares owned by outside shareholders is the probability of the incumbent's retaining control times the public value of the firm under the incumbent, plus the probability of the rival's gaining control times the price paid by the rival. Thus, the value of the shares owned by the existing outside shareholders is \$1.00 million.

To keep the numerical example simple, we assume that the incumbent has to choose from three discrete investment levels: invest \$0, invest \$1.00 million, or invest \$2.00 million. If the incumbent invests nothing, there is no addition to the value of the firm and no new shares are issued. If the incumbent invest \$1.00 or \$2.00 million in positive net present value (NPV) projects, the resulting value of the firm, and the additional public and private value generated under the incumbent and a rival manager are summarized in Table 1 below.

Investment in the projects adds to the public value of the firm and to the private benefits of the manager-in-control at the end of the investment horizon. We assume that the rival manager is strictly superior to the incumbent: The rival manager can generate a higher public value than the sum of the public and private values that the incumbent can generate.

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	Initial number of shares outstanding: 100 Managerial ownership: 50 shares						
Existing Firm Value ($ million)			Value of New Investment Opportunity ($ million)				
Public Value	Private benefit	Number of New Shares Issued	Investment	Under Incumbent Control		Under Rival Control	
				Addition to public value	Addition to private benefit	Addition to public value	Addition to private benefit
2	0.2	0	0	0	0	0	0
2	0.2	50	1	1.1	0.01	1.18	0
2	0.2	100	2	2.12	0.011	2.21	0

Table. 1: This table summarizes the value of the existing firm and the additional public and private value created by the new investments under the incumbent and the rival manager. The first row depicts the no investment case; the second and the third rows depict the cases where the incumbent invests \$1.00 and \$2.00.

For example, if the incumbent invests \$1.00 million and he is the manager-in-control at the end of the investment horizon, then the public value is \$1.10 million and his private benefit is \$0.06 million, giving an aggregate value \$1.16. Whereas, if the incumbent invests \$1.00 million and the rival manager is the manager-in-control at the end of the investment horizon, then the public value is $\$1.18 > \$1.10 + \$0.06$ million.

If non-voting equity is used to finance the investment, the incumbent's proportional ownership of the control rights (votes) remains at 50% and the incumbent retains the ability to prevail in all control contests. If voting shares are issued to fund the investment, the incumbent's proportional ownership of the control rights drops to either 33% ($= {}^{50}/_{150}$) or 25% ($= {}^{50}/_{200}$) depending on the level of investment. We assign exogenous probabilities $\phi^1(\text{x=1}) = 0.95$ and $\phi^1(\text{x=2}) = 0.79$ that the incumbent retains control.[10] These probabilities reflect the ability of the incumbent to prevail in a control contest when he owns 33% and 25% of the voting shares. Basically, if the firm under the incumbent invests \$1 million in the new project and issues 33% new voting shares to finance the investment, then the likelihood he is able to buy these outside shares and retain control drops by 5%. Table 2 summarizes this information.

[10]We generalize these probabilities to enable them to be endogenously determined in the model below.

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Voting Shares Issued to Finance Investments ($ million)		Non-Voting Shares Issued to Finance Investments ($ million)	
Managerial Ownership of Voting Rights	Probability of Retaining Control	Managerial Ownership of Voting Rights	Probability of Retaining Control
50%	1	50%	1
33.3%	0.8	50%	1
25%	0.6	50%	1

Table. 2: The first half of the table shows the likelihood of retaining control if voting shares are used to fund the new investment. The second half of the table shows the likelihood of retaining control if non-voting shares are used to fund the new investment.

The expected value of the incumbent's stake in the firm is the sum of the expected public value of his shares plus the expected private benefits of control. The expected public value of a share in the firm is the probability that the incumbent retains control times the public value of the firm under the incumbent, plus the probability that the rival manager gains control times the public value of the firm under the rival manager. For an investment level of $1 million, the expected public value is equal to the firm's existing value without investment plus the expected NPV under the incumbent plus the expected NPV under the rival manager; that is, $2 + (0.95 \times (1.1 - 1) + 0.05 \times (1.18 - 1)]$ or $2.104 million.

The expected private benefit extracted from the firm by the incumbent is the private benefit of control times the probability of remaining in control. For an investment level of $1.00 million, the expected private benefit is 0.95×0.26 or $0.247 million. Therefore, the expected value of the incumbent's stake if he invests $1.00 million is $0.5 \times (2.104) + 0.247$ or $1.299 million. For the investment level $1.00, the outside shareholders' expected wealth is 0.5×2.104 or $1.052 million. Further, if the incumbent has a choice regarding the class of equity to issue to finance the project, then for the same level of investment, the incumbent issues non-voting equity to invest $1.00 million and the expected value of the shares owned by existing outside shareholders is $1.05 million. For an investment level of $2.00 million, we follow the same logic to obtain the expected public value of the firm's stock, $2 + 0.79 \times (2.12 - 2) + 0.21 \times (2.20 - 2)$ or $2.1368 million. The expected private

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benefit extracted by the incumbent is 0.79×0.27 or \$0.2133 million. Therefore, the expected value of the incumbent's stake given an investment level of \$2.00 is $0.5 \times 2.1368 + 0.2133$ or \$1.2817 million. The expected wealth of the outside shareholders under the incumbent is 0.5×2.1368 or \$1.0684 million. Table 3 summarizes this information.

	Voting Shares Issued to Finance Investments (\$ million)		Non-Voting Shares Issued to Finance Investments (\$ million)	
Investments	Manager	Shareholders	Manager	Shareholders
0	1.200	1.000	1.200	1.000
1	1.226	1.058	1.260	1.050
2	1.205	1.078	1.271	1.060

Table. 3: The first half of the table shows the expected payoff of the incumbent and outside shareholders when voting shares are issued to finance the investment, and the second half of the table shows payoff of the incumbent and outside shareholders when non-voting shares are issued to finance the investment. Outside shareholders always want the manager to invests in all positive NPV projects. But if forced to use voting shares, then the incumbent's optimal response is to invest \$1.00 rather than \$2.00 given his payoff from investing \$1.00 is \$1.2990 which is strictly greater than \$1.2817 – incumbent's payoff from investing \$2.00. It also dominates not investing in the project.

The incumbent's expected wealth is maximized at an investment level of \$1.00 million when the investment is financed with voting shares and at an investment level of \$2.00 million when the investment is financed with non-voting shares. The decision of the existing outside shareholders is over which classes of shares the firm can use to finance the new investment. If the incumbent is required to issue voting shares, then the incumbent invests \$1.00 million and the expected value of the shares owned by existing outside shareholders is \$1.052 million. From the table, we see that situations exist in which it is optimal for outside shareholders to allow the incumbent to finance with non-voting shares since this raises outside shareholder wealth from \$1.052 million to \$1.06 million. This is true regardless of the fact that non-voting shares can entrench the incumbent and prevent better rivals from taking over the firm. Thus, the difference in the value of the shares owned by existing outside shareholders when voting versus non-voting shares are used to finance the investment reflects the change

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in the cost of entrenchment (for investment level \$1 million, the cost of entrenchment is $\$1.052 - \$1.05 = \$0.002$) per million dollar of investment.

Allowing the manager to issue non-voting shares raises the value of the shares owned by existing outside shareholders when the loss in value from underinvestment is larger than the loss in value from increased entrenchment. Examples of this situation are firms that have large growth opportunities and firms in relatively new industries. For firms, that have relatively small growth opportunities, the above condition is unlikely to hold. In these firms, underinvestment is less likely to be a problem and constraining these firms to issue voting shares has a smaller negative impact on firms' values from underinvestment.

Does a contractual solution to the underinvestment problem work? Often it may be possible to make a side payment to the manager to induce him to undertake the investment. This alternative requires outside shareholders to compensate the manager for the decrease in expected wealth due to fewer private benefits associated with an investment of \$2 million financed using voting shares. In the above scenario, the contractual solution does not work. The increase in outside shareholder expected wealth, $\$1.0684 - \$1.052 = \$0.0164$ million, if the investment level is raised from \$1.00 to \$2.00 million, is dominated by the drop in the incumbent's expected wealth, $\$1.2817 - \$1.299 = -\$0.0173$ million. Hence, side payments are not a feasible solution in this case.

3 Model Preliminaries

The model considers a firm that faces a profitable investment opportunity. We assume a typical publicly traded firm with a sizable insider shareholding. Initially, our firm has only one class of shares, the "commons." Each common share has an equal percentage claim to a firm's total cash flows as well as to its total voting rights. We assume that shareholders make decisions under a simple majority voting rule concerning broad corporate objectives and policies such as changes in the board of directors, changes in control of the firm, and

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the menu of securities that the firm can issue to raise new capital. We highlight four players in our model – (i) the incumbent manager (I), (ii) outside shareholders, (iii) potential new investors, and (iv) the manager of a potential rival firm (R).

The incumbent manager searches for new investment opportunities, makes the initial project evaluations, and decides which investment projects to undertake. Like Jensen and Meckling (1976), Myers (1977), Cooney and Kalay (1993) and Zwiebel (1996), we assume the incumbent maximizes the market value of the firm plus the private benefits he derives from being in control. In addition, we assume that the incumbent owns a large "block" of shares in the firm, representing β fraction of the existing N commons (or voting shares).[11]

There can be two classes of "closely held" ownership structures: The incumbent has a large minority block, which exceeds that of any other shareholder; or the incumbent owns an absolute majority of the votes. Initially, we consider an incumbent who has a large minority block which exceeds that of any other shareholder; that is, $0 < \beta < 1/2$ implying that the incumbent has effective control, rather than absolute control.[12] The remaining $1-\beta$ fraction of the common shares are diffusely held by outside shareholders. Each individual outside shareholder wants to maximize the value of his holdings.[13]

The incumbent manager needs to issue equity to raise investment funds. New and existing investors buy any securities that the firm issues, if any, to finance a new investment project. We do not restrict the existing outside shareholders from purchasing the newly

[11] Although only about 20% of the major exchange-listed public firms are closely held in the United States, a vast majority of U.S. corporations are closely held. Also, a study of top 27 stock markets finds that only 36% of the largest publicly traded firms are widely held – that is, there is no single shareholder controlling more than 20% of the total votes. Most large publicly traded firms (64%) have a controlling shareholder, which may be a family (30%), the state (19%), or another firm (15%). Among smaller companies the share of closely held firms is even higher. For detailed discussion see, for example, La Porta et al. (1999) and Claessens et al. (1999).

[12] All of our results can be reproduced if we consider the case where the incumbent has absolute control; that is, he owns a simple majority of the votes ($\approx 50\%+$). We develop this case as a numerical example in the Appendix 8.

[13] We do not consider implications of legal provisions like Delaware 203, which permits a controlling shareholder to block a hostile takeover bid with much less than 50% stock ownership. Specifically, Delaware 203 works if only the board is classified, and we do not consider classified boards.

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issued securities, although we do assume that the incumbent manager is wealth constrained and cannot buy enough newly issued shares to maintain his existing ownership percentage. Thus, if the firm invests by issuing common shares, the incumbent's ownership fraction declines. Also, we rule out preemptive rights offers and similar shareholder priority rules when new shares are sold.

The final player is the rival manager, who controls the rival firm. The rival manager, if he values our firm higher than the incumbent, offers to buy the firm. We rule out rival toeholds and a "manager-rival negotiated" takeover: The only way to acquire the firm is through a market transaction, specifically through an open market purchase of at least 50% plus of the voting shares. All participants are risk-neutral and the discount rate is zero; thus, all securities have prices equal to their expected payoffs.

The temporal evolution of events is as follows: Shareholders decide on the classes of securities that a firm can issue to finance a new investment opportunity. Next, the incumbent decides the level of investment, x, and if $x > 0$, then the firm issues securities to finance the new investment. A rival arrives, and if he can take over the firm, i.e. pay a higher price for a majority of the shares, then he bids for the firm and takes control. The actual investment is undertaken by the winning manager. In the final period, the firm is liquidated and the public value is paid out to existing shareholders as a dividend, while the manager-in-control extracts his private benefits. The quality of the rival is uncertain at the beginning of the scenario, but is revealed at the time of his bid. The figure below depicts the timeline described above:

3.1 New Project

The project generates a public value for the firm's shareholders and a private benefit that accrues to the firm's manager. The realized value of the project is $x + a_i\, P(x) + \varepsilon_x$. The random variable, ε_x, is uniformly distributed over the interval $(-\sigma_x,\ +\sigma_x)$, with a mean

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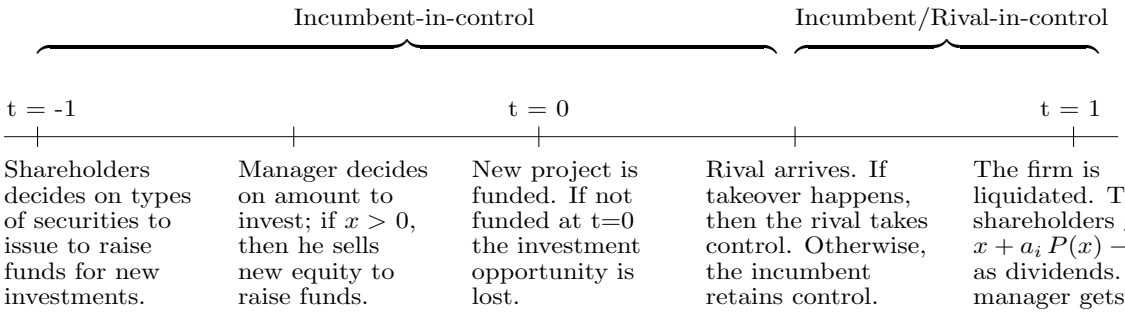


Figure. 1: This figure shows the evolution of events in our model.

zero and variance $\sigma_x^2/3$. $P(x)$ is a concave function of the investment level, differentiable everywhere with a unique maximum at $\bar{x}$. Thus, the maximized expected value of the new project is $\bar{x} + a_i\, P(\bar{x})$.

The parameter a_i is a measure of the manager-in-control's ability to generate cash flows at the end of the investment process from the new project. Henceforth, we call the parameter a_i, the "public quality" of the manager-in-control, where the manager-in-control is either the incumbent (I) or the rival manager (R). We assume that the public quality of the incumbent is common knowledge and $a_I \in [0,\, 1]$. Initially, the rival manager's public quality is unknown; thus, we assume that a_R is a random variable drawn from a *uniform distribution* with support at 0 and 1. The lowest public quality for a manager is $a_i = 0$, and the resulting NPV of the new project is 0. The highest public quality for a manager is the one with $a_i = 1$, and the resulting NPV of the new project is $P(x)$.

Also, we assume that the manager-in-control (whether incumbent or rival) can appropriate some benefits that are not shared by outside shareholders – a private benefit of control. This private benefit is not verifiable; otherwise it would be relatively easy for outside shareholders to stop the manager from appropriating it. The realized value of the private benefit is $B_i = b_i\; a_i\, P(x)$, where $i = I,\, R$. The parameter b_i measures the manager-in-control's ability to convert one unit of NPV into his private benefit. Henceforth, we will call the

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parameter b_i the "ability to extract private benefits" of the manager-in-control. We avoid the problem of over-investment by assuming that private benefits are also maximized at $\bar{x}$.[14] Like his public quality, we assume that the incumbent's ability to extract private benefits, $b_I \in [0, 1]$, is common knowledge and the potential rival manager's ability to extract private benefits b_R is a random variable drawn from a uniform distribution with support at 0 and 1.

3.2 Firm Value

We normalize the initial value of the firm, as $V_0 = 0$; hence, the only source of future dividends is the present value of all the cash flows generated from the new investment adjusted for the cost of the private benefits of control. What are the costs of private benefits? There is a direct loss to shareholders as a dollar worth of private benefit equals a dollar less for the outside shareholders. We call this loss the "value effect" of the private benefits. There is also an important indirect loss to the shareholders: This second private benefit effect does not directly reduce value of the firm, but allows the manager to use the private benefits to stall a potential value-enhancing takeover bid. We call this the entrenchment effect of private benefits. In both cases, these private benefits are direct gains for the manager. We focus on both these effects of private benefits.[15]

If the firm invests in this new project and the manager-in-control at the liquidation date is of (a_i, b_i) type, then the expected firm value, denoted by FV_i, is

$$FV_i = \text{Investment} + \text{NPV} - \text{Private Benefits} = x + a_i\, P(x) - b_i\; a_i\, P(x) = x + a_i\,(1 - b_i\,)\, P(x), \tag{1}$$

where $i = I, R$. We assume the rival's public quality and private benefits,$\tilde{a}_R$ and $\tilde{b}_R$ are

[14] Otherwise, the ability to issue nonvoting shares to fund new investments may encourage managers to invest in negative NPV projects that help to enhance their private benefits.

[15] In Section 8 of the paper, we develop an extensive numerical example that deals exclusively with the "entrenchment effect" of private benefits on investment decision. In a separate paper titled "Strategic Underinvestment and Ownership Structure of a Firm" we formally developed the case that considers only the entrenchment effect of private benefits on new investments.

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independent random variables. Outside shareholders want to maximize firm value; hence, they want a manager who has the highest public quality and has the least ability to convert shareholder value into private benefits; that is to say, the ideal manager has $a_i = 1$ and $b_i = 0$. If $a_i = 1$ and $b_i = 0$, then the expected firm value, $x + P(x)$ for any level of investment x, is maximized. If $a_i = 1$ and $b_i = 1$, that is, the manager-in-control is the best manager in terms of public quality, but also extracts the most private benefits; hence, the resultant firm value is x, which is strictly less than $x + P(x)$. Thus, if $a_1 = 1$ and $1 > b_i > 0$, then there must exist a rival manager with the same/lower public quality and lower ability to extract private benefits than the incumbent such that $a_R\,(1 - b_R\,) > (1 - b_I)$. This condition implies that if such a rival takes control of the firm, then he can generate a higher firm value than the incumbent can. For example, using an average estimate of 14% shareholder wealth appropriation by the manager reported by Dyck and Zingales (2004), we find that a rival with public quality, $a_R = 0.50$ and private benefits $b_R = 0.5$, can generate higher cash flows than the incumbent with $a_I = 1$ and $b_I = 0.86$.

4 Potential Control Contest

The potential control contest is a critical element in the model. To gain control of the firm, the rival has to offer outside shareholders a higher price for their shares than the incumbent can offer. If the rival cannot offer more, then he does not bid and the incumbent retains control. If the rival can offer a greater amount, then he pays shareholders an amount slightly higher than what the incumbent can offer and the rival takes control of the firm. Initially, we assume that the incumbent does not tender shares in the control contest.[16] In Subsection *6.1* we relax this assumption and allow the manager to tender if a rival makes a dominating

[16]This is justified because firm insiders' stock sales are subject to restrictions by securities regulatory authorities and these restrictions may severely affect the incumbent manager's ability to tender in a control contest. For example, the incumbent manger may hold "restricted voting" shares, which will create hurdles for tendering in a control contest.

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offer. We show that the results are qualitatively similar if the incumbent is allowed to tender in the control contest.

At this stage we need to introduce some additional notation: let

- $j = 0$ for non voting shares and $j = 1$ for voting shares;
- n^j = number of new shares issued if j class shares are issued to finance the investment;
- ϕ^j = probability of no takeover, if j class shares are issued to finance the investment;
- V_D^j = value of pure dividend per share if j class shares are issued to finance the investment;
- V_{vote}^j = value of a pure vote claim if j class shares are issued to finance the investment.

If the new investment is financed with voting shares, only one class of share is outstanding and its aggregate value is denoted by V_1^1 and the total number of voting shares (old plus new) is donated by $N + n^1$. If non-voting shares are issued to fund the new investments, then two different classes of shares are outstanding and their values are given by V_1^0, for the old voting shares, and V_0^0, for the newly issued non-voting shares. The value of the voting shares is equal to the value of the dividend received plus the value of the vote, while the value of the non-voting shares is simply equal to the value of the dividend received. This implies the following equalities for (1) the value of the voting shares when the new shares issued are also voting shares, (2) the value of the voting shares when the new shares issued are non-voting shares and (3) the value of the newly issued non-voting shares respectively

$$V_1^1 = V_D^1 + V_{vote}^1, \quad V_1^0 = V_D^0 + V_{vote}^0, \quad \text{and } V_0^0 = V_D^0 \tag{2}$$

We assume that all voting and non-voting shares are paid the same dividend, but the per-share dividend is different depending on whether the newly issued shares have voting rights or not. The reason is that the non-voting shares have a lower price, which requires issuing more dividend paying shares to finance the same investment, so $n^0 > n^1$. Thus, $V_D^1 > V_D^0$.

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Proposition 1. *The number of new non-voting shares, n^0, needed to finance x dollars of investment is always at least weakly greater than the number of voting shares, n^1, needed to finance the same dollar amount of investment.*

Proof. Follows directly from Equation (2). □

By design, voting and non-voting shares receive the same dividend. Since the value of a vote is nonnegative (the vote premium is similar to an option premium), the value of one voting share, which is equal to the expected value of the dividend plus the value of the vote, has to be at least weakly greater than value of one non-voting share, which is just the expected value of the dividend. See, for example, Smith and Amoako-Adu (1995) for a detailed discussion of relative prices of voting versus non-voting shares.

4.1 The Decision Problems

Both the incumbent manager and the outside shareholders are assumed to maximize their expected wealth. For the manager, the decision variable is the level of investment, x. Given that the incumbent manager does not tender his shares to the rival, this level is equivalent to

$$\max_x W_I(x) = \max_x \{\beta\, N\, V_D^j\,(x) + \phi^j\, b_I\ a_I\, P(x)\}. \tag{3}$$

where is ϕ^j is defined as the probability of the incumbent retaining control. The objective function above has two parts: The first term on the RHS is the product of the manager's fractional ownership of the outstanding common shares, βN, and the value of the per share dividend. Thus, it is related to the firm's public value and reflects the fact that the manager receives the same per share dividend paid as any other shareholder.[17] The second term is the incumbent manager's expected private benefit of control, and is realized only if the manager retains control of the firm. This private benefit is a function of the product of the controlling

[17]Recall that the incumbent manager does not tender his shares, so his voting rights have no direct value, but they do affect his expected private benefits.

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manager's public value and his private benefit as well as the size of the new investment. The solution to the manager's problem yields the manager's optimal response to restrictions on the class of security that shareholders allow the firm to issue.

Let $\hat{x}^j$ be the solution to the manager's optimization problem given that he issues $j-$voting shares to finance the investment. Outside shareholders maximize the value of their shares, picking the class of security that the manager can issue, taking the manager's optimal response function as given. Thus, the decision problem of the outside shareholders is

$$\max_{j=0,1} V_1^j\left(\hat{x}^j\right). \tag{4}$$

To solve the incumbent's and outside shareholders' optimization problems, given by Equations (3) and (4), we need to know (i) the probability that there is no takeover, (ii) the value of the dividend, and (iii) the value of a vote, for the two cases when the investment is financed with voting shares and non-voting shares respectively.

4.2 Potential Control Payoffs

A change in control occurs when the rival can offer a higher per-share price for the outside voting shares than the incumbent. The incumbent retains control only if he can offer a weakly higher price for the outside voting shares than the rival. We assume that the rival only bids if he is sure to win. We separately consider the cases of financing the new investment with voting shares and non-voting shares. The question that the rival asks: What is the maximum price the incumbent can pay to buy the outside voting shares? Obviously, the incumbent has to pay the public value of the shares held by outside voting shareholders, FV_I. The incumbent can also be forced to pay a significant part of the present value of all his private benefits as a premium per share to prevent the rival from gaining control of the firm. The maximum price the incumbent can offer for the outside voting shares equals

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$$\text{Bid price}^I = \text{Current share price} + \text{Premium per share}^I, \tag{5}$$

where

$$\text{Current market price}^I = \frac{\text{Expected value of the firm}}{\text{Number of outstanding cash flow claims}} = \frac{FV_I}{N + n^1}$$

and

$$\text{Premium per share}^I = \frac{\text{Incumbent's total private benefits}}{\text{Number of outside voting shares}} = \frac{b_I \; a_I \, P(x)}{(1-\beta)\, N + n^1}.$$

The incumbent retains control of the firm only if the value shown in Equation (5) is weakly greater than the potential rival's maximum offer, Bid^R. If n^1 voting shares are issued to finance the investment, this condition is equivalent to $\text{Bid}_1^I \geq \text{Bid}_1^R$ or

$$\frac{FV_I}{N + n^1} + \frac{b_I \; a_I \, P(x)}{(1-\beta)\, N + n^1} \geqslant \frac{FV_R}{N + n^1} + \frac{b_R \; a_R \, P(x)}{(1-\beta)\, N + n^1}. \tag{6}$$

The first term on the LHS of Equation (6) describes the per-share public value that is generated with the incumbent in control. The second term on the LHS is related to the incumbent's private benefits: the denominator of the second term is smaller than the denominator of the first term because the incumbent's private benefits are only offered to outside voting shareholders, since the incumbent is assumed not to tender. Alternatively, the first term represents the per share dividend paid to each shareholder if the incumbent is in control and the second terms is the private benefit of control that the incumbent can pay as a premium for each minority voting share. The RHS terms are analogously defined as the public and private benefits per-share generated under the rival's control. For example, if we have a situation where $b_I = b_R$ and $a_I < a_R$, then the rival will win the control contest.

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Substituting Equation (1) into (6) and simplifying, we obtain

$$a_I \, + \, \kappa^1 \, b_I \, a_I \geqslant a_R \, + \, \kappa^1 \, b_R \, a_R, \tag{7}$$

where $\kappa^1 = \frac{N\,\beta}{(1-\beta)N+n^1}$. Notice that κ^1 represents the ratio of incumbent voting shares to outside voting shares. Inequality (7) shows that if public benefits + private benefits $\times\kappa^1$ is greater for the incumbent than the rival, then the incumbent will retain control. If n^0 non-voting shares are issued to finance new investments, then the incumbent retains control of the firm if $\text{Bid}_0^I \geq \text{Bid}^R$ or

$$\frac{FV_I}{N+n^0} + \frac{b_I \; a_I \, P(x)}{(1-\beta)\, N} \geqslant \frac{FV_R}{N+n^0} + \frac{b_R \; a_R \, P(x)}{(1-\beta)\, N}. \tag{8}$$

where the denominator of the second term is the initial voting shares outstanding. In a successful bidding contest, the private benefits are only captured by those outside shareholders who own voting shares. The key differences from equation (6) are that the number of new shares is greater so the dividend per share is less and the number of minority voting shares is now much less since no new voting shares are issued. Thus, the manager's private benefits of control per voting share rises. So an incumbent manager's public value per share gets diluted, while the private benefits per share that he can pay for minority voting shares rises. Simplifying Equation (8) we obtain

$$a_I \, + \, \kappa^0 \, b_I \, a_I \geqslant a_R \, + \, \kappa^0 \, b_R \, a_R, \tag{9}$$

where $\kappa^0 = \frac{N\,\beta+n^0}{(1-\beta)N}$, which represents the ratio of incumbent voting shares plus new non-voting shares to outside voting shares. Inequality (9) shows that if (public benefits + private benefits $\times\kappa^0$) is greater for the incumbent than the rival, then the incumbent retains control.[18]

[18]From Equations (6) and (8) we see that the rival can take over the firm from an incumbent whose public quality, $a_I = 1$ and ability to extract private benefits, $b_I = 1$. This is because the incumbent can always

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While the public quality and private benefit traits of the incumbent are known, the rival's traits are unknown prior to a bid, but the range of possible values and their distributions are known. So we can now determine is the range of the rival's public values and private benefits where the rival successfully takes over the firm. For this purpose, we derive "upper" and "lower" bounds on the range of the public and private benefits of the unknown rival where the rival will be successful in taking over the firms and thus, also the boundaries where the rival will be unsuccessful. After simplifying, Inequalities (7) and/or (9) can be expressed as

$$b_R \leqslant \underline{b}_R^j = \frac{1}{\kappa^j}\left(\frac{a_I}{a_R} - 1\right) + b_I\,\frac{a_I}{a_R}, \quad j = 0,\, 1, \tag{10}$$

where $\underline{b}_R^j$ is the lowest value of b_R such that a takeover is not possible. Given that $\underline{b}_R^j \in [0,\, 1]$, we simplify and rearrange Equation (10) to derive $\underline{a}_R^j$ and $\bar{a}_R^j$:

$$\bar{a}_R^j = a_I(1 + \kappa^j\, b_I) \;\text{ and }\; \underline{a}_R^j = \frac{a_I(1 + \kappa^j\, b_I)}{1 + \kappa^j} = \frac{\bar{a}_R^j}{1 + \kappa^j}. \tag{11}$$

The likelihood of a takeover and the role of private benefits in a takeover contest depends on $\bar{a}_R^j$, $\underline{a}_R^j$ and $\underline{b}_R^j$, as well as the incumbent's public value and private benefits. The proposition below formally states these observations.

Proposition 2. *(i) Rivals with public quality a_R higher than $\bar{a}_R^j$ can gain control of the firm regardless of their ability to extract private benefits (i.e., even if $b_R = 0$); whereas (ii) rivals with public quality lower than $\underline{a}_R^j$ cannot gain control of the firm, even if they have the highest possible ability to extract private benefits (i.e., even if $b_R = 1$).*

Proof. Directly follows from Equations (10) and (11). □

If the rival's public quality is significantly higher (lower) than the public quality of the

offer at least as much as any rival and hence, keep control of the firm. The corporate control market fails to work – firm value is lower than what it could be under a host of rivals! Also, this result is independent of the class of security that the incumbent uses to fund the new investments.

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incumbent, then the control contest will be decided based only on the public quality of the contestants, and their private benefits will not play a role in the control contest. If the rival manager's public quality is particularly high ($a_R > \bar{a}_R^j$), then the rival manager gains control; whereas, if the rival manager's public quality is especially low ($a_R < \underline{a}_R^j$), then the incumbent retains control.[19] Ability to extract private benefits plays a role in the control contest only when the rival's public quality is in an intermediate range, $a_R \in [\underline{a}_R^j, \bar{a}_R^j]$. Rival managers with public quality, $a_R \in [\underline{a}_R^j, \bar{a}_R^j]$ and with the ability to extract private benefits, $b_R \in [\underline{b}_R^j, 1]$ can rest control of the firm from the incumbent. These control regions, which are based on the potential rival's public quality and his ability to extract private benefits, are depicted in Figure 2.

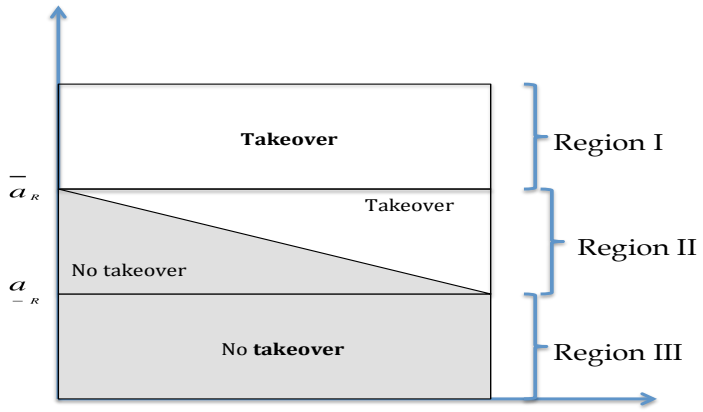


Figure. 2: This figure depicts critical control regions as a function of potential rival's public quality, a_R and ability to extract private benefits, b_R. Note that $\bar{a}_R^j$ and $\underline{a}_R^j$ and b_R^j are all functions of the incumbent's a_I and b_I. Private benefits do not play any role in the takeover contest if the potential rival's public quality is sufficiently high or sufficiently low so that either $[\bar{a}_R^j, 1]$ (blue box) or $[0, \underline{a}_R^j]$ (yellow box) for any given pair of the incumbent's qualities. If the potential rival's public quality is drawn from the intermediate range $[\underline{a}_R^j, \bar{a}_R^j]$, then the private benefits of the rival manager do play a role in the takeover contest. If the potential rival's ability to extract private benefits is $b_R > \underline{b}_R^j$ (dashed line), then the incumbent loses control of the firm; otherwise, incumbent retains control.

[19]There can be many (a_I, b_I) combinations such that $\bar{a}_R^j > 1$. This simply implies that there exists no rival who can take over the firm based only on his public quality, a_R.

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4.3 Effect of Investment on the Control Contest

Next, consider the effects of increasing investment, x, on the bounds $\underline{a}_R^j$, $\bar{a}_R^j$, and $\underline{b}_R^j$. These bounds determine the outcome of the control contest: The likelihood that the incumbent retains control of the firm after the investment depends on the depends on the investment level. We formally state these results in the Propositions 3 below.

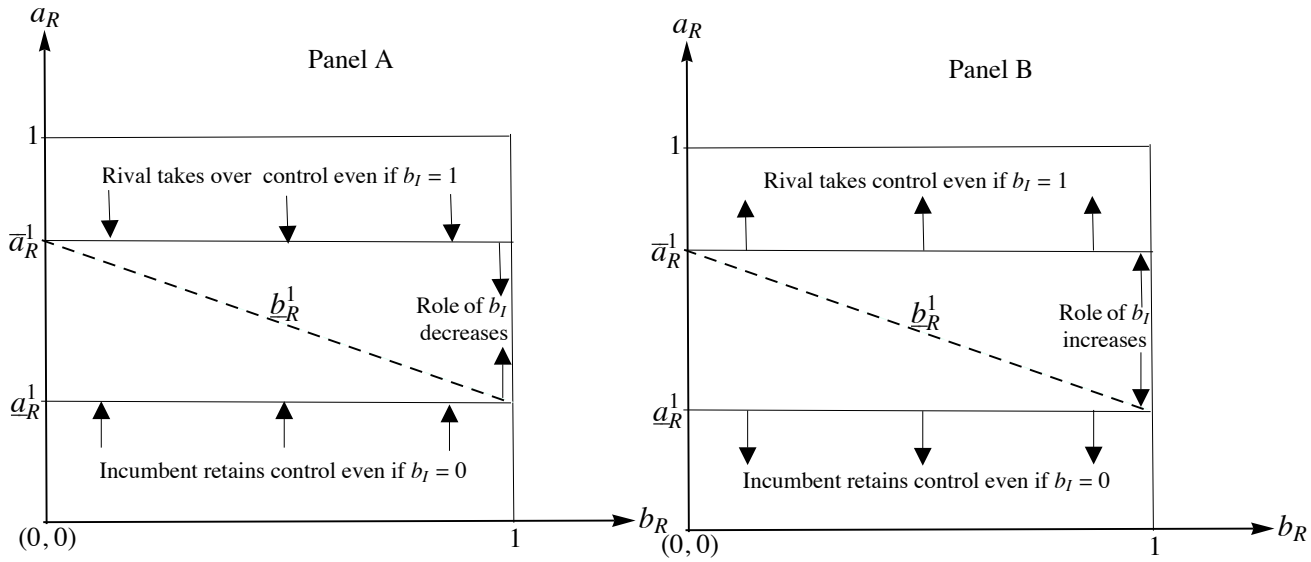
Proposition 3. *(i) If the firm issues voting shares to fund new investments, then the set of rivals who can take over the firm using only their public quality,* $[\bar{a}_R^1, 1]$ *increases; whereas (ii) if the firm issues non-voting shares to fund new investments, then the set of rivals who can take over the firm using only their public quality,* $[\bar{a}_R^0, 1]$*, decreases.*

Proof. Follows directly from Equation (BR-7). □

If the firm under the incumbent uses voting shares to fund the new investments, then the larger the investment, x, the larger is the likelihood that a rival can gain control regardless of his ability to extract private benefits (i.e., even if $b_R = 0$). An incumbent with relatively high ability to extract private benefits, b_I, will be especially concerned! As he issues more and more voting shares, his private benefits become less and less useful in the control contest! This is because issuing new voting shares shift the relative weights from the vote premium to the per share dividend in any control contest. When voting shares are issued, the vote premium is divided among a larger number of outside vote holders, $(1-\beta)N + n^1$, and as a result the per-share vote premium falls. On the other hand the number of new shares issued is less than when non-voting shares are issued, so the per share dividend rises, causing the public value of the manager to be more important. In contrast, if the firm issues non-voting shares to raise funds for investments, the role of private benefits remains unchanged; the incumbent can use his private benefits to buy just the existing outside votes, $(1-\beta)N$. However, more new non-voting shares must be issued, so the per share dividend is reduced, reducing the public value of the manager to be less important. In Appendix *8.1.1* we derive

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the comparative statics of these bounds with respect to changes in investment level. We find that if the firm under the incumbent issues voting shares to fund the new investment, then the region over which the incumbent retains control regardless of a rival's ability to extract private benefits, $[0, \underline{a}_R^1]$, increases; whereas (ii) if the firm issues non-voting shares, then the region over which the incumbent retains control of the firm regardless of a rival's ability to extract private benefits, $[0, \underline{a}_R^0]$, decreases. The intuition is similar; if the firm uses voting shares, then the role of an incumbent's private benefits in the control contest are reduced (region II gets smaller). On the other hand, if the firm uses non-voting shares, then the role of an incumbent's private benefits in the control contest increases (region II expands). Panel A and B of Figure 3 depicts the results stated these intuitions.



Panel A. Voting Shares

Panel B. Non-voting Shares

Figure. 3: The effect of increasing x on $\bar{a}_R^j$, $\underline{a}_R^j$ and $\underline{b}_R^j$. Panel A depicts the case when the incumbent issues voting shares to fund the firm's new investment. Panel B depicts the case when the incumbent issues non-voting shares to fund the firm's new investment. In both cases, arrows show the direction of movement of the upper bound, $\bar{a}_R^j$, and lower bound $\underline{a}_R^j$, as x, increases. In panel A, as x increases region II shrinks, implying that the private benefit plays a relatively lesser role in the control contest when the firm's investment is finance using voting shares. In panel B, as x increases, region II expands, implying that private benefit plays a more important role in the control contest when the firm's investment is finance using non-voting shares.

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The effect of increasing investment, x, on the likelihood of incumbent's retaining control is not unambiguous: *Region I*, where the incumbent loses control for certain, expands. But *Region III*, where the incumbent retains control for sure, expands too. Thus, the net effect of increasing x on the probability of the incumbent retaining control, ϕ^j depends on the incumbent's public and private benefits relative to the average public and private benefits of a potential rival. Recall that we define $j = 1$ for voting share and $j = 0$ for non voting share.

4.4 Probability of Control

The incumbent's probability of retaining control of the firm after issuing j-th class shares is

$$\phi^j = \int_0^{\underline{a}_R^j} da_R + \int_{\underline{a}_R^j}^{\bar{a}_R^j} \int_0^{\underline{b}_R^j} db_R \, da_R \quad \text{where } j = 0,\, 1. \tag{12}$$

The first term in Equation (12) is the region where the potential rival's public quality is very low. In this region, the rival has no hope of gaining control regardless of his ability to extract private benefits. The second term is the region where the rival's public quality is such that the incumbent retains control if the rival's ability to extract private benefits is lower than $\underline{b}_R^j$; otherwise, the rival gains control. Over the final range $[\bar{a}_R^j,\, 1]$, the incumbent has no hope of retaining control, regardless of the rival's ability to extract private benefits. Integrating the expression in Equation (12) and further simplifying, we obtain the incumbent's probability of retaining control,

$$\phi^j(a_I,\, b_I,\, \beta,\, N,\, ,\, x) = a_I(1 + b_I\, k^j)\, \frac{\log(1 + k^j\,)}{k^j} = a_I \times \phi^{j'}(b_I,\, \beta,\, N,\, ,\, x). \tag{13}$$

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We use the chain rule to differentiate Equation (13) with respect to x and obtain

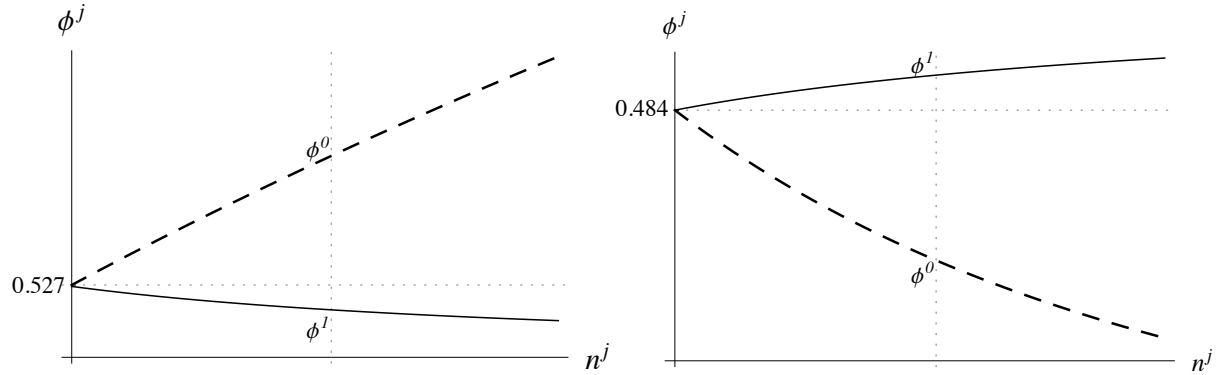
$$\frac{\partial \phi^j(x)}{\partial x} = a_I \times \frac{\partial \phi^{j'}(x)}{\partial k^j} \times \frac{\partial k^j}{\partial x} = a_I \times \frac{\frac{k^j(1+b_I\, k^j)}{1+k^j} - \frac{\log(1+k^j)}{}}{(k^j)^2} \times \frac{\partial k^j}{\partial x}. \tag{14}$$

Thus, the public quality of the manager only has a "level effect" on the incumbent manager's likelihood of retaining control, given that the firm invests x. For any given value of a_I the change in the incumbent's likelihood of retaining control after this investment is function of the incumbent's ability to extract private benefits, b_I. For $b_I \geqslant E(b_R)$, $\frac{\partial \phi^j(x)}{\partial k^j}$ is nonnegative. For all $b_I < E(b_R)$, $\frac{\partial \phi^j(x)}{\partial k^j}$ is strictly negative. Hence, the sign of $\frac{\partial \phi^j(x)}{\partial x}$ depends on the sign of $\frac{\partial k^j}{\partial x}$ and the value of b_I. Recall that $E(b_R) = 1/2$. In Appendix *8.1.2* we derive the relationship between investment level and the incumbent's likelihood of retaining control. Suppose that the incumbent's ability to extract private benefits is greater than an average rival's ability to extract private benefits; that is, $b_I \geqslant E(b_R) = 1/2$. If an incumbent issues voting shares to fund the new investment, then the likelihood of the incumbent retaining control, ϕ^1, decreases in the firm's investment level, x; whereas, if the incumbent issues non-voting shares to fund the new investment, then the likelihood of incumbent retaining control, ϕ^0, increases in the firm's investment level, x. This happens because issuing voting shares makes the *dividend per share* relatively more important in a control contest than the *vote premium*.

On the other hand, non-voting shares make the *vote premium* relatively more important than the *dividend per share* in a control contest. This is because a smaller number of shares are entitled to receive the *vote premium* relative to number of shares entitled to the *dividends*. Hence, an incumbent who has relatively weak ability to extract private benefits, $b_I < 1/2$ prefers to issue voting shares to fund the new investment, thus shifting the weight away from the *vote premium* towards the *dividend per share*. The main intuition of these results are depicted in the Figure 4 below.[20]

[20] As x increases so does n^j – not linearly, but strictly monotonically; hence, we use n^j instead of x.

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Panel A. $b_I > E(b_R)$ Panel B. $b_I < E(b_R)$

Figure. 4: This figure depicts the likelihood that the incumbent retains control of the firm, ϕ^j, as a function of the size of the new equity issue, n^j. The solid black lines in both panels A and B correspond to the case where the new investment is financed using voting shares. The dashed lines and dotted lines, in both panels A and B, correspond to the case where the new investment is financed using non-voting shares.

4.5 Value of One Dividend Claim

The value of a pure dividend claim is equal to the expected dividend that the holder of the claim gets. This value depends on the manager-in-control's public quality and the class of security issued to finance the new investments. Thus, the value of the per-share pure dividend claim is

$$V_D^j = \phi^j \frac{FV_I}{N+n^j} + \int_{\underline{a}_R^j}^{\bar{a}_R^j} \int_{\underline{b}_R^j}^{1} \frac{FV_R}{N+n^j}\, db_R\, da_R + \int_{\bar{a}_R^j}^{1} \int_{0}^{1} \frac{FV_R}{N+n^j}\, db_R\, da_R, \tag{15}$$

where $j = 0, 1$. The first term is the probability that the incumbent retains control times the per-share public value of the firm under the incumbent. The second and third terms give the expected dividend under the rival. Rivals of intermediate public quality and relatively high ($> \underline{b}_R^j$) ability to extract private benefits populate the second region. Rivals of very high public quality, who can take over the firm regardless of their ability to extract private benefits, populate the third region. If ϕ^j decreases, then $1 - \phi^j$ increases and vice versa.

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Given our assumption that all available investment opportunities are positive NPV projects, the dividend level increases in the investment level, x. If non-voting shares are used to fund the new investment, then the average dividend per-share increases less relative to the increase in dividends if voting shares are used.

4.6 Value of One Vote Claim

Voting rights matter because they allow stockholders to have a say in *who runs the company* and *how it is run*.[21] Voting power becomes important, especially at badly managed companies, when a challenge is mounted against the incumbent either from within (activist stockholders) or from outside (hostile acquisitions).[22]

The value of a pure vote claim is related to the extraction of private benefits from the rival in the form of a vote premium. To obtain an expression for the value of the vote, we classify rival managers into one of three types: The first type represents rivals who cannot gain control of the firm because they have very low public quality ($a_R < \underline{a}_R^j$). If this type of rival is drawn, no private benefit is extracted and the value of the vote is zero. Next consider rivals who can gain control of the firm without having to pay out any of their private benefit – those with very high public quality ($a_R > \bar{a}_R^j$). Again, it is not necessary for the rival manager to give up any of his private benefits. Hence, the private benefit is only relevant when a rival manager is of an intermediate type ($\underline{a}_R^j < a_R^j < \bar{a}_R^j$). In this case, the rival needs to pay a "takeover" premium that is greater than the dividend it will produce to prevail in

[21] Zingales (1995b) and Nenova (2003) estimate the value of a vote based on the price difference of shares in firms with unequal voting rights that have both classes publicly traded. They find that the value of the vote is positive and varies across countries. See also Smart et al. (2008) for implications of "vote" on IPO valuation.

[22] Institutional investors' benign neglect of different voting share classes at Google is rationalized by the fact that they think the company is well managed and that control is therefore worth little or nothing. There is a kernel of truth to this statement: The expected value of control (and voting rights) is greater in badly managed companies than in well managed ones. However, if you are an investor for the long term, you have to worry about whether managers who are perceived as good managers today could be perceived otherwise in a few years.

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the control contest. Hence, the payoff to the vote claim when the firm funds the project with voting shares can be written as:

$$\begin{cases} \frac{FV_I}{N+n^1} + \frac{B_I}{N(1-\beta)+n^1} - \frac{FV_R}{N+n^1} & \text{if } \underline{a}_R^1 \leqslant a_R \leqslant \overline{a}_R^1 \\ 0 & \text{otherwise.} \end{cases} \tag{16}$$

Similarly, the payoff on the vote claim when the firm funds the project with non-voting shares can be written as:

$$\begin{cases} \frac{FV_I}{N+n^0} + \frac{B_I}{N(1-\beta)} - \frac{FV_R}{N+n^0} & \text{if } \underline{a}_R^0 \leqslant a_R \leqslant \overline{a}_R^0 \\ 0 & \text{otherwise,} \end{cases} \tag{17}$$

The value of the vote is simply the expectation of these two values in Equations (16) and (17),

$$V_{vote}^1 = \left(\frac{FV_I}{N+n^1} + \frac{B_I}{N(1-\beta)+n^1} \right) \int_{\underline{a}_R^1}^{\overline{a}_R^1} \int_{\underline{b}_R^1}^{1} db_R \, da_R - \int_{\underline{a}_R^1}^{\overline{a}_R^1} \int_{\underline{b}_R^1}^{1} \frac{FV_R}{N+n^1} \, db_R \, da_R \tag{18}$$

and

$$V_{vote}^0 = \left(\frac{FV_I}{N+n^0} + \frac{B_I}{N(1-\beta)} \right) \int_{\underline{a}_R^0}^{\overline{a}_R^0} \int_{\underline{b}_R^0}^{1} db_R \, da_R - \int_{\underline{a}_R^0}^{\overline{a}_R^0} \int_{\underline{b}_R^0}^{1} \frac{FV_R}{N+n^0} \, db_R \, da_R. \tag{19}$$

Using the expressions in Equation (2), we can derive the value of the voting shares when the project is funded with new voting shares, as well as when the project is funded with new nonvoting shares. Next, we depict the type of manager who underinvests if forced to finance the firm's investment using voting shares.

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5 Entrenchment and Investment

The manager chooses the investment level to maximize his expected wealth. There are three terms in the manager's objective function that depend on the level of investment chosen: The value of the dividend, the probability of retaining control, and the private benefits of control. The value of the dividend increases with investment as we consider only *positive NPV* opportunities. By design, the private benefits of control also increase with investment. We also see, from Equations (BR-9) and (BR-10), that the likelihood that the incumbent retains control of the firm depends primarily on two variables: the investment level (x) and the incumbent's inherent ability to extract private benefits (b_I).

5.1 Low Quality Managers and the Control Contest

First, we turn to the issue of economic efficiency explored in the existing literature. Grossman and Hart (1988) and other subsequent studies show that non-voting shares allow control of the firm to remain with or pass to the hands of inferior managers, lowering economic efficiency. We begin by showing that non-voting shares allow inferior managers to win control contests.[23]

Our result is similar to the Grossman and Hart (1988) result. The principal difference between voting and non-voting shares is that non-voting shares cause the private benefits of managers to have a larger impact on the control contest's outcomes. Consider a rival with an ability to extract higher private benefits relative to the incumbent, that is, $b_R > b_I$. Nonvoting shares favor the rival in a control contest, making it easier for him to gain control of the firm; that is, he can gain control for lower values of a_R, values where he would otherwise lose the control contest if instead the investment was financed with voting shares. Similarly,

[23] A statement on economic efficiency requires an analysis of a trade-off between the costs of underinvestment and the cost of inefficient management. This assessment requires assumptions regarding the ability of other firms to undertake projects that the firm under consideration has forgone. We leave this aspect of the problem to future research.

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if $b_R < b_I$; that is, if the incumbent has greater ability to extract private benefits than the rival, then non-voting shares would favor the incumbent in a control contest, making it easier for him to retain control of the firm. That is, an incumbent can keep control of the firm with lower levels of a_I, values where he would otherwise lose control if the investment was financed with voting shares. The following proposition formalizes this result.

Proposition 4. *The minimum public quality required for an incumbent manager to retain control of the firm is lower in firms financed with dual-class shares.*

Proof. See the proof stated in Section *8.2*. □

The fact that a manager of lower public quality can gain control of firms should be a serious concern for market regulators. However, if other mechanisms can be used to discipline managers, then the cost of this problem can be small. For example, Moyer et al. (1992) find that an alternative monitoring mechanisms such as an independent board are often present in firms after they issue dual-class shares.[24]

5.2 Incumbent's Ownership and Underinvestment

Next, consider the case in which the manager does not own any existing shares in the firm; that is, $\beta = 0$. Suppose investments are financed by issuing voting shares; because the incumbent has no shareholding in the firm, there is no possibility of dilution in his control rights. Thus, the likelihood of the incumbent retaining control of firm is unaffected by the firm's investment level. In fact, the likelihood that the incumbent retains control depends only on his public quality, a_I. Thus, the incumbent's objective function is strictly increasing in x. Since the incumbent does not underinvest, there is no need to allow him to issue non-voting shares. The proposition below formalizes this result.

[24]Hollinger International presents a good example of the negative effects of dual-class shares. Former CEO Conrad Black controlled all of the company's class-B shares, which gave him 30% of the firm's equity and 73% of its voting power. He ran the company as if he were the sole owner, exacting huge management fees, consulting payments, and personal dividends.

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Proposition 5. *When investments are financed by issuing voting shares and the incumbent does not own any equity in the firm (i.e., $\beta = 0$), then the incumbent invests in all available positive NPV projects.*

Proof. See the proof stated in Section *8.3*. □

This may appear to be a counter-intuitive result: When the incumbent owns part of the firm's equity, he bears part of the cost of underinvestment. The larger the incumbent's percentage ownership level, β, the larger is his share of the underinvestment cost. Thus, it may seem that if $\beta = 0$, then the incumbent should only care about the probability of retaining control and his private benefits! But Proposition 5 shows that if $\beta = 0$, then the incumbent always invests in all available positive NPV projects. From Equation (6), we see that if $\beta = 0$, then it does not matter whether the incumbent pays more dividends or pays an equivalent vote premium, because the number of outside votes, $N + n^1$, is equal to number of dividend claims, $N + n^1$. Hence, the control contest depends only on how much cash flow the incumbent generates vis-á-vis the rival. Thus, he invests $\bar{x}$ to maximizes the cash flows and consequently, maximizes his chance of retaining control.

5.3 Investment using Voting and Non-voting Shares

Next, we consider a firm's financing decision and its effect on the incumbent's likelihood of retaining control after the new investments are funded. From Equations (BR-9) and (BR-10) we know that the incumbent's ability to extract private benefits, b_I, determines whether the probability of retaining control, ϕ^j, increases or decreases with the investment level. Given an incumbent with any level of public quality, a_I, we can isolate two types of incumbents: An incumbent with relatively high ability to extract private benefits, $E(b_R) \leqslant b_I \leqslant 1$; and an incumbent with relatively low ability to extract private benefits, $0 \leqslant b_I < E(b_R)$ relative to the rival's expected ability to extract private benefits.

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If the incumbent has relatively high ability to extract private benefits and uses voting shares to fund the new investments, then two implications arise for the incumbent's objective function: First, his expected private benefits decline because the likelihood of retaining control decreases in x. But, the new investment increases his total dividend payments, $N\,\beta\,V_D^1$, as well as raising his private benefits, $b_I\ a_I\,P(x)$. Hence, the firm's investment level depends on the net effect of investment x on his expected wealth. This result is formalized in the following proposition.

Proposition 6. *If voting shares are issued to fund new investments, then the necessary conditions for the incumbent manager to forgo some positive NPV projects are (i) $\beta > 0$ and (ii) $b_I \gtrless E(b_R)$.*

Proof. See the proof stated in Section *8.4*. □

Even if the incumbent's expected private benefit falls as the firm invests more by financing it with voting shares, the more valuable dividend claim that results from higher investments still can drive the incumbent's willingness to invest. In addition, by design the level of private benefit increases with the investment level.

Thus, when the negative impact of the investment on the likelihood of retaining control outweighs the positive effects of the investment on the incumbent's dividends and private benefits, then the incumbent will underinvest. Next, we derive the sufficiency condition for underinvestment.

Proposition 7. *When voting shares are issued to fund new investment and the incumbent manager owns equity in the firm ($\beta > 0$), then the incumbent manager forgoes some positive NPV projects if his ability to extract private benefits, b_I, is weakly greater than $\hat{b}_I$, where*

$$\hat{b}_I = \min\left[\frac{(1-\beta)\left(\frac{2(1-\beta)^2\,\log\left(\frac{1}{1-\beta}\right)}{2-(4-\beta)\beta} - \beta\right)}{\beta^2}, 1\right].$$

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Proof. See the proof stated in Section *8.5*. □

The condition provided in Proposition 7 is a sufficient condition for underinvestment: The incumbent manager always forgoes some positive NPV investments if he is forced to fund the new investments with voting shares and his ability to extract private benefits is greater than $\hat{b}_I$. The incumbent manager may forgo some positive NPV projects, even when he has weaker ability to extract private benefits, $E(b_R) \leqslant b_I \leqslant \hat{b}_I$. One can interpret $\hat{b}_1$ as a proxy for the likelihood of underinvestment: Incumbent managers with private benefits greater than $\hat{b}_I$ are sure to forgo some positive NPV investments. As $\hat{b}_I$ gets larger and approaches one, it becomes less likely that a rival with the ability to extract private benefits greater than $\hat{b}_I$ will appear; hence, it also becomes less likely that the incumbent will underinvest. As a shareholder, the incumbent bears part of any underinvestment opportunity cost. The larger an incumbent's ownership fraction, the larger is his share of the opportunity costs of underinvestment. Thus, the condition given in Proposition 7 depends on β. If the manager owns 8% of the firm's equity, then the incumbent forgoes some positive NPV projects only if $b_I > 0.8689$. This implies that if we collect a sample of firms with an average 8% managerial ownership level, then we should expect to find underinvestment in 13.11% of the sample. Figure 5 depicts the likelihood of underinvestment, $1 - \hat{b}_I$, as a function of an incumbent's ownership fraction, β, and his ability to extract private benefits, b_I.

5.4 Welfare of Existing Shareholders and the Incumbent Manager

So far we obtain conditions under which investment increases or decreases if voting shares or non-voting shares are used to fund the investment. Increased investment financed with non-voting shares is not always in the best interests of either outside shareholders or the incumbent. There are costs to issuing non-voting equity as detailed below.

Since investors who buy non-voting shares are not entitled to any private benefits paid in a control contest as a *vote premium*, they demand a lower per-share price to purchase

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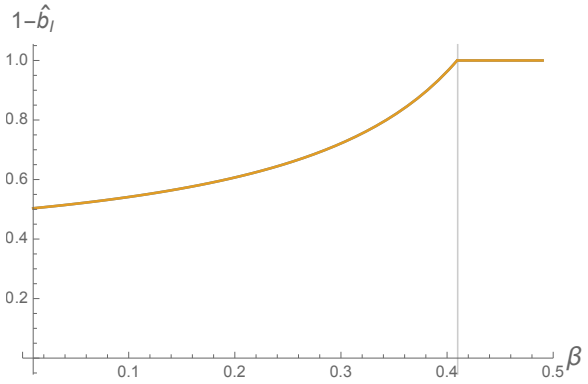


Figure. 5: This figure depicts the likelihood of an incumbent manager underinvesting, $1 - \hat{b}_I$ as function of his shareholdings, β. In this figure, we plot the likelihood of incumbent manager underinvestment as function of a manager's initial ownership level, β. For any $\beta > 0.41$, an incumbent manager will not underinvest.

non-voting shares relative to otherwise comparable voting shares. This means that a larger number of non-voting shares, n^0, relative to voting shares, n^1, must be issued to finance a given level of investment, x, reducing the per-share dividend that is available to existing shareholders. This is called the "dividend dilution" effect. Also, issuance of non-voting shares decreases the likelihood of a successful takeover bid, which we call the "entrenchment" effect. Thus, holding constant the incumbent's private benefits, a drop in the likelihood of a bid reduces the value of the voting rights. There is a partially offsetting gain, namely a larger share of the extracted private benefits paid out as a *vote premium* to a smaller number shares. To summarize, issuance of non-voting shares affects existing shareholders and the manager in three ways: (i) a lower per-share dividend; (ii) a lower probability of a change in control; and, (iii) a higher per-share takeover premium conditional on a successful takeover.

Recall that the firm's value for any incumbent type (a_I, b_I) given an investment level, $x \in (0, \bar{x}]$, financed with voting shares is, $FV(x^1 = x) \geqslant FV(x^0 = x)$. So voting shares dominates the value of the same firm given the same level of investment, but financed with non-voting shares. Thus, existing outside shareholders will voluntarily allow an incumbent to issue non-voting shares only if the investment level, $\bar{x} - \Delta\bar{x}$, financed with voting shares

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leads to a lower firm value, $FV(x^1 = \bar{x} - \Delta\,\bar{x}) < FV(x^0 = \bar{x})$, than if financed with non-voting shares. The next proposition presents conditions under which the value of the existing voting shares is higher if new non-voting shares are used to fund the investment project.

Proposition 8. *For all $b_I \geqslant \hat{b}_I$ and $\bar{x}$ such that $n^0(\bar{x}) \leqslant N$, existing outside shareholders prefer the firm to finance its investment financed using non-voting shares if the underinvestment cost meets the condition*

$$1 - \frac{P(x)}{P(\bar{x})} \geqslant \frac{a_I^2\; b_I(2 + b_I(2\,\beta + 1) - 2\,\beta)}{2\,a_I(1-\beta)^2\,(1 -\; b_I) - a_I^2\,(1 - \beta(1 -\; b_I))^2 + (1-\beta)^2}.$$

Proof. See the proof stated in Section *8.6*. □

Proposition 8 gives the underinvestment level needed before existing shareholders voluntarily allow the incumbent manager to raise funds by issuing non-voting shares. The LHS of the above inequality is a measure of outside shareholder loses due to underinvestment, while the RHS is a measure of the costs of issuing non-voting shares. Outside shareholders find it optimal to allow the manager to issue non-voting shares only when the gains realized by reducing underinvestment outweigh the costs of issuing non-voting shares. If $a_I = 0.5$, $b_I = 0.75$ and $\beta = 0.05$, outside shareholders find issuance of non-voting shares to finance an investment optimal, even if the underinvestment level is roughly 6.8% and the likelihood of underinvestment, $1 - \hat{b}_I$ is 27%.

Next, we consider the incumbent's expected wealth maximizing investment choice. Use of non-voting shares to fund the new investments lowers the per-share dividend and thus, negatively affects the incumbent's wealth. If $b_I \geqslant E(b_R)$, then non-voting shares lower the probability of a successful takeover and thus, increase the expected wealth of the incumbent.[25] Given the incumbent's type (a_I, b_I) and his objective function, underinvestment occurs if the incumbent is forced to only issue voting shares to fund the new investment such that

[25]The increase in the takeover premium does not affect the manager since he is assumed not to tender in a takeover.

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$W_I(x^1 = \bar{x}) < W_I(x^0 = \bar{x})$ and there exists a $\Delta\,\bar{x}$ such that $W_I(x^1 = \bar{x} - \Delta\,\bar{x}) = W_I(x^0 = \bar{x})$. The propositions below provide conditions on the types of managers who are better off if the firm issues non-voting stock.

Proposition 9. *For all $E(b_R) \leqslant b_I \leqslant 1$ and $\bar{x}$ such that $n^0(\bar{x}) \leqslant N$, the incumbent prefers investment financed by non-voting shares if $b_I \geqslant \hat{\hat{b}}_I$, where*

$$\hat{\hat{b}}_I = \frac{(1-\beta)\left(\beta^2(1+\beta) + 2(1-\beta)^2\left((1+\beta)\log\left(1+\frac{\beta}{1-\beta}\right) - \beta\log\left(1+\frac{(1+\beta)}{1-\beta}\right)\right)\right)}{\beta(1+\beta)\left(4(1-\beta)^2\log\left(1+\frac{(1+\beta)}{1-\beta}\right) - \beta(1+\beta) - 4(1-\beta)^2\log\left(1+\frac{\beta}{1-\beta}\right)\right)}.$$

Proof. See the proof stated in Section *8.7*. □

From Equation (BR-10), we see that investment using non-voting shares increases the likelihood that the incumbent retains control if $b_I \geqslant E(b_R)$. From Proposition 9 we know that the incumbent is better off if non-voting shares are issued and $b_I \geqslant \hat{\hat{b}}_I$. The divergence exists because of the dividend dilution caused by the issuance of lower priced non-voting shares. If non-voting shares are issued, then the aggregate dividend is divided among $N + n^0$ shares, which is strictly greater than the $N + n^1$ shares outstanding if voting shares are issued. If $\beta = 0.2$, the manager prefers non-voting shares if b_I is greater than 0.59, even though the incumbent's likelihood of retaining control increases with the investment level for all $b_I \geqslant 0.5$. For all $\beta > 0.22$, the value of $\hat{\hat{b}}_I \geqslant 1$, implies that the incumbent never prefers non-voting shares over this range of parameter values.

From Propositions 7, 8, and 9, we can derive some interesting observations: Consider the case when $b_I \in [E(b_R),\, 1]$ and for all β such that $\hat{b}_I$ and $\hat{\hat{b}}_I$ are within the range of $E(b_R)$ and 1, we have $\hat{b}_I > \hat{\hat{b}}_I$. Thus, we can divide the range of the incumbent's private benefits into four regions: $[0,\, E(b_R))$, $[E(b_R), \hat{\hat{b}}_I)$, $[\hat{\hat{b}}_I,\, \hat{b}_I)$, and $[\hat{b}_I,\, 1]$. If the incumbent is constrained to use voting shares, then the incumbent with $b_I \in [\hat{b}_I,\, 1]$ underinvests.[26] Otherwise, he

[26] If $b_I \in [\hat{\hat{b}}_I,\, \hat{b}_I]$, incumbent may or may not underinvest. This is because we are only able to solve the sufficient condition for underinvestment along with the incumbent's choice of voting vs. non-voting shares.

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invests in all available positive NPV projects. If the incumbent is given a choice of using either voting shares or non-voting shares, then the incumbent always invests in all available positive NPV projects. The manager uses voting shares to fund the investment, if his ability to extract private benefits is $b_I \in [0, \hat{\hat{b}}_I)$ and uses only non-voting shares, if $b_I \in [\hat{\hat{b}}_I, 1]$.

The next section discusses extensions to our model, along with the effect of relaxing some of our initial assumptions.

6 Extensions

Next, we consider three related issues. First, we allow the incumbent to tender his holdings in a control contest. This is important because it helps to further entrench the incumbent. Second, we address the issuance of shares with fewer than one vote per-share. Issuing such shares can lower the dividend dilution costs relative to issuing pure non-voting shares. Also, firms in many countries allow firms to issue multiple classes of shares with different voting rights such as dual class shares. Third, we discuss the costs and benefits of issuing multiple classes of shares to finance new investment.

6.1 Entrenchment and investment when incumbent managers can tender shares

Change in control occurs when the rival manager offers a higher per-share value to outside shareholders than the incumbent can offer. Earlier, we assumed that the incumbent does not tender his shares in the control contest. We now relax this assumption. If the incumbent tenders his shares, then the rival's private benefit paid as a takeover premium muct be divided over a larger number of shares, $N + n$ as opposed to $(1 - \beta)N + n$ outside voting shares when the incumbent manager can not tender. Thus, the takeover premium must be shared across more voting shares. So, for a fixed total takeover premium, each voting share

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receives a smaller takeover premium. This puts the rival at a disadvantage relative to the incumbent.[27] The probability of a takeover is obtained separately for the cases of non-voting shares and voting shares. As before, the incumbent retains control if he offers a higher per share price than the rival. If voting shares are used to finance the investment, this contest is equivalent to

$$\frac{FV_I}{N+n^1} + \frac{b_I\ a_I\, P(x)}{(1-\beta)\, N+n^1} \geqslant \frac{FV_R}{N+n^1} + \frac{b_R\ a_R\, P(x)}{N+n^1}. \tag{20}$$

The first two terms on the LHS of Equation (20) represents the per-share public value that is generated with the incumbent in control. The third term on the LHS is related to the incumbent's private value. The denominator is smaller here than in the first two terms because the private benefits are distributed only to the outside shareholders and excludes the incumbent. The RHS terms are related to the public and private benefits per-share realized under the rival. The private benefits of the potential rival are now divided among all the firm's shareholders including the incumbent. Comparing Equation (20) to Equation (6), we see that LHS of (20) is smaller which implies that the rival has a lower probability of success. Simplifying Equation (20) in the same way that we simplify Equation (6) to obtain Equation (7), gives us the condition when the incumbent retains control.

$$a_I + a_I\, \kappa^1\, b_I\ \geqslant a_R, \tag{21}$$

where $\kappa^1 = \frac{N\,\beta}{(1-\beta)N+n^1}$.

From Equation (21), we conclude that the upper bound on the rival's public benefit where the incumbent retains control is $\bar{a}_R^1 = a_I\ (1 +\ \kappa^1\, b_I\,)$. Thus, the incumbent retains control of the firm if $a_R \in [0,\, \bar{a}_R^1]$. The ability to extract the rival's private benefits plays no role in the control contest, whereas the ability to extract the incumbent's private benefits plays a significant role in the control contest. When the incumbent can tender, the range of the

[27] See, for example, Burkart et al. (1998) for more discussions.

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rival's public quality over which the incumbent retains certain control, $[0,\ \bar{a}_R^1]$, is much higher than $[0,\ \underline{a}_R^1]$, the range when the incumbent does not tender. Since the range over which the incumbent loses control, $[\bar{a}_R^1,\ 1]$, remains the same, the likelihood that the incumbent retains control after investing x, when the incumbent can tender, defined as $\hat{\phi}^1(x)$, is greater than $\phi^1(x)$, the likelihood that the incumbent retains control after investing x, when the incumbent can not tender.

What happens to the incumbent's likelihood of retaining control if x increases? Because $\frac{\partial \kappa^1}{\partial x}$ is negative, $\frac{\partial \hat{\phi}^1}{\partial x}$ is also negative. Hence, the incumbent's likelihood of retaining control decreases as the investment level rises. If non-voting shares are issued to finance the investment and if the manager can tender his shares in a control contest, then the rival's private benefit is divided over a larger number of shares, N rather than only $(1-\beta)N$. It follows that the incumbent retains control if

$$\frac{FV_I}{N+n^0} + \frac{b_I\ a_I\, P(x)}{(1-\beta)\, N} \geqslant \frac{FV_R}{N+n^0} + \frac{b_R\ a_R\, P(x)}{N}. \tag{22}$$

The private value is distributed equally across all shareholders who own voting shares. The owners of the non-voting shares do not share in the private value since they have no impact on the outcome of the control contest. Simplifying Equation (22), as we did for Equation (8), we obtain,

$$a_I\ \left(1+\ \kappa^0\, b_I\ \right) \geqslant a_R\ \left(1+\ \hat{\kappa}^0\, b_R\ \right), \tag{23}$$

where $\kappa^0 = \frac{N\,\beta + n^0}{(1-\beta)N}$ and $\hat{\kappa}^0 = \frac{n^0}{N}$. Given that $\kappa^0 - \hat{\kappa}^0 = \frac{\beta(N+n^0)}{(1-\beta)N} > 0$ and $\hat{\bar{a}}_R^0 = \bar{a}_R^0$, it follows that

$$\hat{\underline{a}}_R^0 = \frac{a_I(1+\ \kappa^0\, b_I)}{1+\ \hat{\kappa}^0} = \frac{\bar{a}_R^0}{1+\ \hat{\kappa}^0} > \underline{a}_R^0. \tag{24}$$

From Equation (24), we see that the rival's range of public quality where the incumbent retains control when the incumbent can tender $[0,\ \hat{\underline{a}}_R^0]$, is larger than $[0,\ \underline{a}_R^0]$, the rival's range

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of public quality where the incumbent retains control when the incumbent cannot tender. Since the range over which the incumbent loses control remains the same, i.e. $[\bar{a}_R^0, 1]$, the incumbent's likelihood of retaining control for a given investment level, $\hat{\phi}^0(x)$, is weakly greater when the incumbent can tender, relative to when he cannot tender. This is formally stated in the proposition below.

Proposition 10. *When an incumbent can tender shares in the control contest, the incumbent's ability to extract private benefits, b_I, irrespective of the financing choice plays a relatively more decisive role in a control contest, which raises the incumbent's entrenchment level.*

Proof. Follows directly from Equations (21) or (24). □

Like our initial setup, these bounds determine the outcome of the control contest: The likelihood of the incumbent retaining control after the investment depends on the investment's size, x and that the incumbent chooses x. Since the number of new shares needed to finance the investment rises with the investment size, it follows that $\frac{\partial n^0}{\partial x} > 0$, so $\frac{\partial \hat{\kappa}^0}{\partial x} > 0$. Hence, differentiating Equation (24), we obtain

$$\frac{\partial}{\partial x}\,\underline{\hat{a}}_R^0 = \frac{\partial}{\partial n^0}\,\underline{\hat{a}}_R^j \times \frac{\partial n^0}{\partial x} = -\frac{a_I N(1-\beta-\frac{b_I}{2}(1-\beta))}{(N+n^0)^2(1-\beta)} \times \frac{\partial n^0}{\partial x} < 0. \tag{25}$$

Thus, the implications of raising investment on the incumbent manager's likelihood of retaining control when he can tender are qualitatively similar to the case when the incumbent cannot tender.

6.2 Optimal vote-dividend combination

The optimal vote-dividend combination can be viewed from two perspectives. The first perspective is to consider shares that have one unit of dividend and θ votes, and to find the

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optimal value of θ. The second perspective is to allow the firm to simultaneously issue both voting and non-voting shares. We first consider perspective one, where the optimal θ-votes are determined.

The optimality of θ-vote shares, with $0 < \theta < 1$, depends on the size of the firm's investment opportunity. For a class of shares, the vote has value only if a sufficient mass of votes in that class exists, so that these shares can be used by the incumbent manager to block a takeover. This means that managers issue θ-vote shares only if the investment opportunity is large enough that the condition below holds

$$\beta N + n\,\theta \geqslant \frac{1}{2}\left(N + n\,\theta\right). \tag{26}$$

The explanation for the above inequality is as follows: Consider a firm with two outstanding share classes, full-vote shares and θ-votes shares. Suppose $n\,\theta$ is small so that the above weak inequality is not met. In this case, the manager has no incentive to bid for the θ-vote class of shares; blocking the rival only requires the incumbent manager to bid for the full-vote shares. Likewise the rival has no incentive to bid for the θ-vote shares either. The outcome of the control contest is determined solely by the owners of the full voting shares. This causes the value of the votes to be zero for the θ-vote shares, giving the manager no incentive to issue θ-vote shares.

If $\bar{x}$ is small, $\theta = 0$ is likely to be optimal. This is because the number of shares that are issued is going to be small for a small $\bar{x}$, and the total number of votes held by shareholders in that class will be insufficient to meet the condition in inequality (25). Our model assumes that shareholders are homogeneous. However, heterogeneity among shareholders may result in cases where θ-vote shares may become optimal even when $\bar{x}$ is small.

Allowing firms to simultaneously issue both non-voting and voting shares will increases the set of firms that find it optimal to issue dual-class shares. This assertion is based on the following line of reasoning: Existing one-vote shareholders prefer non-voting shares when

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the level of underinvestment is high; that is, $1 - \frac{P(x)}{P(\bar{x})} \geqslant g(, \beta, \kappa^0, b_I, a_I)$. From Equation (P11-5) we know that the RHS of the above inequality is an increasing function of κ^0, which itself is an increasing function of $n^0(\bar{x})$. Hence, as $n^0(\bar{x})$ falls, the outside shareholders find it optimal to allow the manager to finance investments using non-voting shares, even for low levels of underinvestment. Suppose that the investment size is $\bar{x}$. If a portion of the investments, say $0.75\,\bar{x}$, is partly financed using one-vote shares and our analysis is carried out over the remaining unfunded projects, i.e., $0.25\,\bar{x}$, then the relevant $n^0(0.25\,\bar{x})$ will have a smaller value, implying that the owners of the voting shares would be willing to allow managers to issue non-voting shares over a wider range of conditions. In this case, the existing shareholders could find it optimal to allow the manager the choice of issuing full voting shares or a mix of θ-voting shares per non-voting share issued.

6.3 Multiple classes of shares

We previously considered a firm that issues only two classes of shares: Voting and non-voting shares. One logical extension of this model is to consider multiple classes of shares. But, is it optimal either for the manager or for existing shareholders to issue multiple classes of shares? Consider shares that give their owners fractional voting rights. Now, the firm can simultaneously issue shares with θ_0, θ_1, θ_2, and θ_3 votes (an example can be $\theta_0 = 0, \theta_1 = 0.33, \theta_2 = 0.5$, and $\theta_3 = 1$). In the above framework, the shares with fractional votes are only issued if the fractional votes have value. If there is a sufficient mass of each of these share classes outstanding, so that the rival is forced to buy them to take control of the firm, then the fractional votes will have value.

The manager can raise the cost of a takeover for the rival by issuing multiple classes of shares. However, this does not mean that it is optimal for the manager to issue multiple share classes. The manager bears a cost when he issues multiple share classes, which is in the form of lower dividends per share. Thus, the existing shareholders are likely to find multiple

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classes of shares detrimental to their interest. As the number of share classes increases, the probability of a change in control is likely to decrease very quickly. The compensating factor, greater investment, is unlikely to go up fast enough to increase the value of the shares held by outside shareholders. Thus, multiple share classes are unlikely to be optimal for outside shareholders.

7 Conclusions

This study provides a theoretical justification for easing the prohibitions on the issuance of dual-class shares, which have recently been proposed or enacted in a number of developed and developing countries. We analyze a firm's decision problem when a set of positive NPV projects are available. We show that if a firm requires outside equity financing to undertake profitable investment projects, then there are cases when managers find separation of voting and dividend claims optimal. Raising equity capital has two effects: (i) The value of the firm increases as more positive NPV projects are undertaken and (ii) the proportion of the firm's shares owned by the manager decreases, raising the likelihood that the incumbent manager loses control of the firm. Thus, a manager, who values control because of the private benefits it offers, can find it optimal to forgo some positive NPV projects. Non-voting shares enable the manager to finance the investment without diluting his voting power, and thus, increasing his chances of retaining control, which in turn increases his willingness to undertake all the positive NPV projects. As a consequence, outside shareholders can be made better off by allowing the firm to issue non-voting shares, if the profitability of the project is high relative to the forgone takeover premium and the higher are the expected private benefits of control.

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8 Appendix A

8.1 Basic Results

Before we present the proofs of the propositions, we first show some basic results that we will use repeatedly. By definition we have

$$\kappa^0(x) = \frac{\beta\, N + n^0(x)}{(1-\beta)N} \text{ and } \kappa^1(x) = \frac{\beta\, N}{(1-\beta)N + n^1(x)}, \tag{BR-1}$$

where $n^j(x) = 0$ at $x = 0$, and $n^j(x) > 0$, $\forall\, x \in (0,\, \bar{x}]$, and $j = 0,\, 1$. Also, $\frac{\partial n^j(x)}{\partial x} > 0$. Thus,

$$\kappa^0(0) = \kappa^1(0) = \frac{\beta}{1-\beta}. \tag{BR-2}$$

For $x > 0$ we get

$$\kappa^0(x) = \frac{\beta}{1-\beta} + \frac{n^0(x)}{(1-\beta)N} > \frac{\beta}{1-\beta} = \kappa^0(0),$$

If $n^0(\bar{x}) \leq N$, then

$$\kappa^0(x) \;\leq\; \frac{\beta}{1-\beta} + \frac{1}{(1-\beta)}. \tag{BR-3}$$

$$\kappa^1(x) \;=\; \frac{\beta}{1-\beta} - \frac{\beta\, n^1(x)}{(1-\beta)N\,[(1-\beta)N + n^1]} < \frac{\beta}{1-\beta} = \kappa^1(0). \tag{BR-4}$$

8.1.1 Comparative statics: Upper and Lower Bounds w.r.t. investment level

Thus $\kappa^0(x) \geqslant \kappa^1(x)\ \forall\, x \in [0,\, \bar{x}]$. Differentiating $\kappa^0(x)$ and $\kappa^1(x)$ with respect to x we get

$$\frac{\partial \kappa^0}{\partial x} = \frac{\partial}{\partial x}\left(\frac{N\,\beta + n^0}{(1-\beta)N}\right) = \frac{1}{(1-\beta)N}\frac{\partial n^0}{\partial x} > 0. \tag{BR-5}$$

and

$$\frac{\partial \kappa^1}{\partial x} = \frac{\partial}{\partial x}\left(\frac{N\,\beta}{(1-\beta)N + n^1}\right) = \frac{-1}{((1-\beta)N + n^1)^2}\frac{\partial n^1}{\partial x} < 0. \tag{BR-6}$$

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Using the results that $\partial n^j/\partial x > 0$, $\forall j$ implies that $\frac{\partial \kappa^0}{\partial x} > 0$ and $\frac{\partial \kappa^1}{\partial x} < 0$. Thus,

$$\frac{\partial}{\partial x}\,\bar{a}_R^j = a_I\, b_I \frac{\partial \kappa^j}{\partial x} \begin{cases} > 0 & \text{if } j = 0 \\ < 0 & \text{if } j = 1, \end{cases} \tag{BR-7}$$

and

$$\frac{\partial}{\partial x}\,\underline{a}_R^j = \frac{\partial}{\partial \kappa^j}\,\underline{a}_R^j \times \frac{\partial \kappa^j}{\partial x} = -\frac{a_I(1-b_I)}{(1+\kappa^j)^2} \times \frac{\partial \kappa^j}{\partial x} \begin{cases} < 0 & \text{if } j = 0 \\ > 0 & \text{if } j = 1 \end{cases} \tag{BR-8}$$

8.1.2 Comparative statics: Probability of retaining control w.r.t. investment level

Taking derivative of $l\phi^1(x)$ with respect to investment level, x we obtain

$$\frac{\partial \phi^1(x)}{\partial x} = \frac{\partial \phi^1(x)}{\partial k^1} \times \frac{\partial k^1}{\partial x} \begin{cases} < 0 & \text{if } b_I \geqslant E(b_R) \\ > 0 & \text{if } b_I < E(b_R). \end{cases} \tag{BR-9}$$

Similarly, taking derivative of $l\phi^0(x)$ with respect to investment level, x we obtain

$$\frac{\partial \phi^0(x)}{\partial x} = \frac{\partial \phi^0(x)}{\partial k^0} \times \frac{\partial k^0}{\partial x} \begin{cases} > 0 & \text{if } b_I \geqslant E(b_R) \\ < 0 & \text{if } b_I < E(b_R) \end{cases} \tag{BR-10}$$

8.1.3 Dividend claims

We can simplify Equation (15) and rewrite as

$$\begin{aligned} (N+n^j)V_D^j = \quad & x + \phi^j \times a_I(1-b_I)P(x) + \int_{\underline{a}_R^j}^{\bar{a}_R^j} \int_{\underline{b}_R^j}^{1} (a_R(1-b_R)P(x))\, db_R\, da_R \\ + \quad & \int_{\bar{a}_R^j}^{1} \int_{0}^{1} (a_R(1-b_R)P(x))\, db_R\, da_R. \end{aligned} \tag{BR-11}$$

Integrating the third term of the above expression we get

$$\int_{\bar{a}_R^j}^{1} \int_{0}^{1} (a_R(1-b_R)P(x))\, db_R\, da_R = \frac{1}{4}\left(1-(\bar{a}_R^j)^2\right) P(x). \tag{BR-12}$$

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Integrating the second term of the above expression we get

$$\int_{\underline{a}_R^j}^{\bar{a}_R^j} \int_{\underline{b}_R^j}^{1} (a_R(1 - b_R)P(x))\, db_R\, da_R = A(x,\, \kappa^j) + B(x,\, \kappa^j), \tag{BR-13}$$

where $A(x,\, \kappa^j) = \frac{(\bar{a}_R^j)^2\left(2-\kappa^j+5\,(\kappa^j)^2\right)P(x)}{4\,\kappa^j(1+\kappa^j)}$ and $B(x,\, \kappa^j) = \frac{\bar{a}_R^j\,P(x)}{2\,\kappa^j}\ln(1+\kappa^j)$.

8.1.4 Value of existing voting shares when voting shares are issued

Using Equations (15) and (18) we can express total value of all voting shares when voting shares are issued to finance the new projects:

$$(N + n^1)V_1^1 = (N + n^1)V_D^1 + (N + n^1)V_{\text{vote}}^1 \tag{BR-14}$$

$$\begin{aligned}
(N + n^1)V_1^1 &= \int_0^{\bar{a}_R^1} \int_0^1 FV_I\, db_R\, da_R + \int_{\bar{a}_R^1}^1 \int_0^1 FV_R\, db_R\, da_R + \int_{\underline{a}_R^1}^{\bar{a}_R^1} \int_{\underline{b}_R^1}^1 \frac{(N + n^1)B_I}{(1 - \beta)N + n^1}\, db_R\, da_R \\
&= x + \frac{1}{2}P(x)\left((a_I(1 - b_I) + (1 + \kappa^1)\left(1 - \frac{\ln(1 + \kappa^1)}{\kappa^1}\right)\bar{a}_R^1\right) \\
&+ \frac{1}{2}P(x)\,\frac{1}{2}\left(1 - (\bar{a}_R^1)^2\right).
\end{aligned} \tag{BR-15}$$

If the firm issues voting stock to finance the investment, and because all new securities are issued at zero expected profit, $n^1V_1^1 = x$, we get

$$N\,V_1^1 = \frac{1}{2}P(x)\left((a_I(1 - b_I) + (1 + \kappa^1)\left(1 - \frac{\ln(1 + \kappa^1)}{\kappa^1}\right)\bar{a}_R^1 + \frac{1}{2}\,(1 - (\bar{a}_R^1)^2)\right). \tag{BR-16}$$

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8.1.5 Value of existing voting shares when non-voting shares are issued

If non-voting shares are used to finance the investment, then using Equations (15) and (18), the value of an existing voting share is

$$\begin{aligned}(N+n^0)V_1^0 &= (N+n^0)V_D^0 + (N+n^0)V_{\text{vote}}^0 \\ &= \frac{P(x)}{2}\left((a_I(1-b_I) + \left((1+\kappa^0) - \frac{1}{1-\beta}\right)\left(1 - \frac{\ln(1+\kappa^0)}{\kappa^0}\right)\bar{a}_R^0\right) \\ &+ \frac{P(x)}{2}\frac{1}{2}\left(1-(\bar{a}_R^0)^2\right).\end{aligned} \tag{BR-17}$$

8.2 Proof of Proposition 4

Using Equations (7) and (9) we get the incumbent's minimum public quality required to win the control costest: If financed using voting shares, it is

$$\underline{a}_I^1 \geqslant \frac{a_R\,(1+\kappa^1\,b_R)}{(1+\kappa^1\,b_I)}, \tag{P7-1}$$

and if financed using non-voting shares, it is

$$\underline{a}_I^0 \geqslant \frac{a_R\,(1+\kappa^0\,b_R)}{(1+\kappa^0\,b_I)}. \tag{P7-2}$$

We need to show that $\underline{a}_I^0 \leqslant \underline{a}_I^1$. Thus, we obtain

$$\underline{a}_I^0 - \underline{a}_I^1 = -\frac{a_R\;(b_I - b_R)\,(\kappa^0 - \kappa^1)}{(1+\kappa^1\,b_I)\,(1+\kappa^0\,b_I)} \leqslant 0. \tag{P7-3}$$

From Equation (BR-1) we know that $\kappa^0 - \kappa^1 > 0$. Thus, if $b_I \geqslant b_R$ then $\underline{a}_I^0 < \underline{a}_I^1$. Hence, the proof.

8.3 Proof of Proposition 5

The manager chooses the investment level to maximize his objective function. We show that the first derivative of the manager's objective function evaluated at $\bar{x}$ is nonnegative, which

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implies that the incumbent invests $\bar{x}$.

When investments financed using voting shares and $\beta = 0$, then from Equation (11) we get $\underline{a}_R^1 = \bar{a}_R^1 = a_I$. This is because $\kappa^1 = \frac{\beta N}{(1-\beta)N+n^1} = 0$. Thus,

$$\phi^1(x) = \int_0^{\underline{a}_R^1} da_R + \int_{\underline{a}_R^1}^{\bar{a}_R^1} \int_0^{\underline{b}_R^1} db_R \, da_R = \int_0^{a_I} da_R = a_I. \tag{P8-1}$$

Incumbent manager's objective function if he finances the new investment issuing j-type shares, say MO^j, is

$$MO^j(x) = \beta \, N \, V_D^j(x) + \phi^j(x) \, b_I \, a_I \, P(x). \tag{P8-2}$$

Differentiating the incumbent's objective function with respect to x gives

$$\frac{\partial \, MO^j(x)}{\partial \, x} = \beta \, N \, \frac{\partial \, V_D^j(x)}{\partial \, x} + \, b_I \, a_I \left(\frac{\partial \, \phi^j(x)}{\partial \, x} \, P(x) + \phi^j(x) \, \frac{\partial \, P(x)}{\partial \, x} \right). \tag{P8-3}$$

Substituting $j = 1$ and $\beta = 0$ we get

$$\frac{\partial \, MO^1(x)}{\partial \, x} = \, b_I \, a_I \left(\frac{\partial \, \phi^1(x)}{\partial \, x} \, P(x) + \phi^1(x) \, \frac{\partial \, P(x)}{\partial \, x} \right). \tag{P8-4}$$

But $\frac{\partial \, \phi^1(x)}{\partial \, x}|_{\beta=0} = \frac{\partial \, a_I}{\partial \, x} = 0$. Hence,

$$\frac{\partial \, MO^1(x)}{\partial \, x}|_{\beta=0} = \, b_I \, a_I \, \phi^1(x) \, \frac{\partial \, P(x)}{\partial \, x}. \tag{P8-5}$$

Since by definition $\frac{\partial \, P(x)}{\partial \, x} > 0$ for all $x \in [0, \, \bar{x})$ and $\frac{\partial \, P(x)}{\partial \, x} = 0$ for $x = \bar{x}$, it must be the case that $\frac{\partial \, MO^1(x)}{\partial \, x} > 0$ for all $x \in [0, \, \bar{x})$ and $\frac{\partial \, MO^1(x)}{\partial \, x} = 0$ for $x = \bar{x}$. Thus, the incumbent manager invests in all available positive NPV projects. Hence, the proof.

From Equation (BR-9) we know that if voting shares are issued and $b_I < 1/2$, then $\frac{\partial \, \phi^1(x)}{\partial \, x} > 0$ for all x. Hence, the sufficient condition for the incumbent to invest $\bar{x}$ using voting shares is $b_I < 1/2$. Similarly, from Equation (BR-10) we know that if non-voting shares are issued and $b_I \geqslant 1/2$, then $\frac{\partial \, \phi^0(x)}{\partial \, x} > 0$ for all x. Hence, the sufficient condition for the incumbent to invest $\bar{x}$ using non-voting shares is $b_I \geqslant 1/2$. Hence, the proof.

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8.4 Proof of Proposition 6

Differentiating Equation P8-2, substituting $j = 1$ and further simplifying we obtain

$$\frac{\partial\, MO^1(x)}{\partial\, x} = \beta\, N\, \frac{\partial\, V_D^1(x)}{\partial\, x} + \, b_I\, a_I \left(\frac{\partial\, \phi^1(x)}{\partial\, x}\, P(x) + \phi^1(x)\, \frac{\partial\, P(x)}{\partial\, x} \right). \tag{P9-1}$$

Let us consider following situations:

1. $\beta = 0$ and $b_I \in [0,\, 1]$;
2. $\beta > 0$ and $b_I \in [0,\, 1/2)$; and,
3. $\beta > 0$ and $b_I \in [1/2,\, 1]$.

We have shown in Proposition 6 that regardless of the incumbent manager's ability to extract private benefits, the incumbent will fund all available positive NPV projects if $\beta = 0$. Hence, $\beta > 0$ is a necessary condition for underinvestment. If $\beta > 0$, then $\frac{\partial\, \phi^1(x)}{\partial\, x} < 0$ if $b_I \geqslant 1/2$ and $\frac{\partial\, \phi^1(x)}{\partial\, x} > 0$ if $b_I < 1/2$.

Thus, if $b_I < 1/2$, then all three, $\frac{\partial\, V_D^1(x)}{\partial\, x}$, $\frac{\partial\, P(x)}{\partial\, x}$, and $\frac{\partial\, \phi^1(x)}{\partial\, x}$ in Equation P9-1, are increasing in all $x \in [0,\, \bar{x}]$; hence, $\frac{\partial\, MO^1(x)}{\partial\, x}$ is increasing in x. If $b_I \geqslant 1/2$, then $\frac{\partial\, V_D^1(x)}{\partial\, x}$ and $\frac{\partial\, P(x)}{\partial\, x}$ are still increasing in x, but $\frac{\partial\, \phi^1(x)}{\partial\, x}$ is decreasing in x. Thus, only way $\frac{\partial\, MO^1(x)}{\partial\, x}$ can be negative is when $\frac{\partial\, \phi^1(x)}{\partial\, x} < 0$. But the only way $\frac{\partial\, \phi^1(x)}{\partial\, x} < 0$ is if $\beta > 0$ and $b_I \geqslant 1/2$. Hence, the proof.

8.5 Proof of Proposition 7

Using Equations (2) and (BR-15) we find that the total value of voting shares,

$$N\, V_1^1 = \frac{1}{2} P(x) \left((a_I(1 - b_I) + (1 + \kappa^1) \left(1 - \frac{\ln(1 + \kappa^1)}{\kappa^1} \right) \bar{a}_R^1 + \frac{1}{2} \left(1 - (\bar{a}_R^1)^2 \right) \right), \tag{P10-1}$$

and the total value of vote is

$$N\, V_{\text{vote}}^1 = \frac{P(x)}{2} (1 + \kappa^1) \left(\frac{1 - \ln(1 + \kappa^1}{\kappa^1} \right) \bar{a}_R^1. \tag{P10-2}$$

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Hence, the total value of the dividend claim is

$$N\,V_D^1 = N\,V_1^1 - N\,V_{\text{vote}}^1 = \frac{1}{2}P(x)\left((a_I(1-b_I) + \frac{1}{2}\,(1-(\bar{a}_R^1)^2))\right). \tag{P10-3}$$

Thus, MO^1 can be expressed as

$$MO^1 = \beta\,N\,V_D^1 + \phi^1\,b_I\,a_I\,P(x) = P(x)\left(\frac{\beta}{2}\left((a_I(1-b_I) + \frac{1}{2}\,(1-(\bar{a}_R^1)^2))\right) + \phi^1\,b_I\,a_I\right). \tag{P10-4}$$

Let $A_{p11} = \frac{\beta}{2}\left((a_I(1-b_I) + \frac{1}{2}(1-(\bar{a}_R^1)^2))\right) + \phi^1\,b_I\,a_I$. Differentiating Equation (P10-4) with respect to x and rearranging terms, we have

$$\frac{\partial MO^1}{\partial x} = A_{p11}(\kappa^1)\frac{\partial P(x)}{\partial x} + P\,(x)\,\frac{\partial A_{p11}(\kappa^1)}{\partial k^1}\frac{\partial \kappa^1}{\partial x}. \tag{P10-5}$$

From Equation (P10-5) we know that $\frac{\partial MO^1}{\partial x} < 0$ at $x = \bar{x}$ implies that $\frac{\partial A_{p11}(\kappa^1)}{\partial k^1}\frac{\partial \kappa^1}{\partial x} < 0$ because $\frac{\partial P(x)}{\partial x} = 0$ at $x = \bar{x}$. Also, $\frac{\partial A_{p11}(\kappa^1)}{\partial k^1}\frac{\partial \kappa^1}{\partial x} < 0$ when $\frac{\partial \kappa^1}{\partial x} < 0$ implies that $\frac{\partial A_{p11}(\kappa^1)}{\partial k^1} > 0$. Differentiating A_{p11} with respect to κ^1 and rearranging terms, we have

$$\frac{\partial A_{p11}(\kappa^1)}{\partial \kappa^1} = \frac{(1+b_I\,\kappa^1)}{\kappa^1(1+\kappa^1)} + \frac{b_I\,\log(1+\kappa^1)}{\kappa^1} - \frac{(1+b_I\,\kappa^1)\log(1+\kappa^1)}{(\kappa^1)^2} - \frac{1}{2}\beta(1+b_I\,\kappa^1). \tag{P10-6}$$

For $\frac{\partial A_{p11}(\kappa^1)}{\partial \kappa^1} > 0$, we need

$$\frac{(1+b_I\,\kappa^1)}{\kappa^1(1+\kappa^1)} + \frac{b_I\,\log(1+\kappa^1)}{\kappa^1} > \frac{(1+b_I\,\kappa^1)\log(1+\kappa^1)}{(\kappa^1)^2} + \frac{1}{2}\beta(1+b_I\,\kappa^1). \tag{P10-7}$$

Solving for b_I such that the condition in Equation (P10-7) is satisfied,

$$b_I > \frac{\frac{2\log(1+\kappa^1)}{(\kappa^1)^2} + \beta - \frac{2}{\kappa^1(1+\kappa^1)}}{\left(\frac{2}{1+\kappa^1} - \beta\,\kappa^1\right)}. \tag{P10-8}$$

We do not know the exact value of κ^1, but we do know the RHS of expression (P10-8) is strictly increasing in κ^1. Hence, we replace κ^1 by its maximum value $\frac{\beta}{1-\beta}$, and get the

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sufficient condition for underinvestment as follows:

$$b_I > \frac{(1-\beta)\left(\frac{2(1-\beta)^2\beta)\log\left(1+\frac{\beta}{1-\beta}\right)}{2-\beta(4-\beta)} - \beta\right)}{\beta^2}. \tag{P10-9}$$

8.6 Proof of Proposition 8

Consider the values of b_I for which the manager invests in all available positive NPV projects if non-voting equity is used to finance the investment. Assume that the manager invests some x if voting equity is used to finance the investment. We have to obtain conditions such that $V_1^0(\bar{x}) \geqslant V_1^1(x)$. Substituting for $V_1^0(\bar{x})$ and $V_1^1(x)$ from Equation (BR-16), we get

$$\begin{aligned} V_1^0(\bar{x}) - V_1^1(x) &= \frac{a_I(1-b_I)}{2}\left(P(\bar{x}) - P(x)\right) + \frac{1}{4}\left(1 - a_I^2 - a_I^2\, b_I\, \kappa^0(2 + b_I\, \kappa^0)\right) P(\bar{x}) \\ &- \frac{1}{4}\left(1 - a_I^2 - a_I^2\, b_I\, \kappa^1(2 + b_I\, \kappa^1)\right) P(x) - \frac{P(x)}{2}(1+\kappa^1)\left(1 - \frac{\ln(1+\kappa^1)}{\kappa^1}\right)\bar{a}_R^1 \\ &+ \frac{P(\bar{x})}{2}\left((1+\kappa^0) - \frac{1}{1-\beta}\right)\left(1 - \frac{\ln(1+\kappa^0)}{\kappa^0}\right)\bar{a}_R^0 \end{aligned} \tag{P11-1}$$

But $\bar{a}_R^0 - \bar{a}_R^1 = b_I\, a_I(\kappa^0 - \kappa^1) > 0$ and $P(\bar{x}) - P(x) > 0\ \forall x \neq \bar{x}$. Thus, ignoring these terms we substitute maximum value of κ^1 and minimum value of κ^0, as both are strictly increasing in κ, and simplifying we get

$$\begin{aligned} &\left((1+\kappa^0) - \frac{1}{1-\beta}\right)\left(1 - \frac{\ln(1+\kappa^0)}{\kappa^0}\right) - (1+\kappa^1)\left(1 - \frac{\ln(1+\kappa^1)}{\kappa^1}\right) \\ = \ &\frac{(1-\beta)\log\left(\frac{\beta}{1-\beta}+1\right) - \beta^2 - \beta(1-\beta)^2\log\left(\frac{1}{1-\beta}+1\right)}{(1-\beta)\beta} > 0, \end{aligned} \tag{P11-2}$$

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for all $\alpha \in [0, 1]$ and for all $\beta \in (0, 1/2)$. Using this result we can ignore these terms and rewrite Equation P11-1 as

$$V_1^0(\bar{x}) - V_1^1(x) = \frac{a_I(1-b_I)}{2}\left(P(\bar{x}) - P(x)\right) + \frac{1}{4}\left(1 - a_I^2 - a_I^2\, b_I\, \kappa^0(2 + b_I\ \kappa^0)\right) P(\bar{x})$$

$$- \frac{1}{4}\left(1 - a_I^2 - a_I^2\, b_I\, \kappa^1(2 + b_I\, \kappa^1)\right) P(x). \tag{P11-3}$$

After simplifying and rearranging we get

$$1 - \frac{P(x)}{P(\bar{x})} \geqslant \frac{\frac{1}{4}\left((1 - a_I^2 - a_I^2\, b_I\, \kappa^1(2 + b_I\, \kappa^1)) - (1 - a_I^2 - a_I^2\, b_I\, \kappa^0(2 + b_I\, \kappa^0))\right)}{\frac{1}{4}\left(1 - a_I^2 - a_I^2\, b_I\, \kappa^1(2 + b_I\, \kappa^1)\right) + \frac{a_I(1-b_I)}{2}}. \tag{P11-4}$$

The expression, say $J_{p11}^1 = \frac{1}{4}\left(1 - a_I^2 - a_I^2\, b_I\, \kappa^1(2 + b_I\, \kappa^1)\right)$, is a decreasing function of κ^1 and the entire expression is an increasing function of J_{p11}^1. Thus, we substitute minimum value of κ^1 into the above expression. Similarly, the expression, say $J_{p11}^0 = \frac{1}{4}\left(1 - a_I^2 - a_I^2\, b_I\, \kappa^0(2 + b_I\, \kappa^0)\right)$, is a decreasing function of κ^0, but the entire expression is a decreasing function of J_{p11}^0. Thus, substituting maximum value of $\kappa^0 = \frac{\beta}{1-\beta} + \frac{1}{1-\beta}$ and simplifying we get

$$1 - \frac{P(x)}{P(\bar{x})} \geqslant \frac{a_I^2\, b_I(2 + b_I(2\,\beta + 1) - 2\,\beta)}{2\, a_I(1-\beta)^2\,(1-b_I) - (a_I - a_I\,\beta(1-b_I))^2 + (1-\beta)^2}. \tag{P11-5}$$

8.7 Proof of Proposition 9

We prove this proposition in two parts: First we hold the investment level fixed and obtain conditions under which the manager is better off if the investment is financed using non-voting shares; then, we show that the manager remains better off if the investment level is increased. From Proposition 8 we know that $MO^j = A_{p1j}\, P(x)$, where j stands for the type of shares issued. To show that $MO^0 \geqslant MO^1$, we need to obtain conditions under which $A(k^0) \geqslant A_{p11}$. Substituting for A_{p10} and A_{p11} from Equations and simplifying we get

$$\frac{\beta}{2}\left(a_I(1-b_I) + \frac{1}{2}(1 - (\bar{a}_R^0)^2)\right) + \phi^0\, b_I\, a_I > \frac{\beta}{2}\left(\frac{1}{2}(1 - (\bar{a}_R^1)^2)\right) + \phi^1\, b_I\, a_I. \tag{P12-1}$$

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Rearranging the terms we get

$$\phi^0\, b_I\, a_I - \phi^1\, b_I\, a_I > \frac{\beta}{2}\left(\frac{1}{2}(1-(\bar{a}_R^1)^2))\right) - \frac{\beta}{2}\left(\frac{1}{2}(1-(\bar{a}_R^0)^2))\right). \tag{P12-2}$$

After substituting for $\bar{a}_R^j$ from Equation (11) and ϕ^j from Equation (13) and on further simplification,

$$4\,\kappa^1(1+b_I\,\kappa^0)\log(1+\kappa^0) - 4\,\kappa^0(1+b_I\,\kappa^1)\log(1+\kappa^1) - \beta\,\kappa^0\,\kappa^1\,(\kappa^0-\kappa^1)\left(2+b_I(\kappa^0+\kappa^1)\right) > 0. \tag{P12-3}$$

Next, we solve for b_I such that the above inequality holds:

$$b_I \geqslant \hat{\hat{b}}_I = \frac{\beta(\kappa^0-\kappa^1) + 4\,\frac{1}{\kappa^0}\log(1+\alpha\,\kappa^0) - 4\,\frac{1}{\kappa^1}\log(1+\alpha\,\kappa^1)}{\alpha(4\log\frac{1+\alpha\,\kappa^1}{1+\kappa^0} + \beta(\kappa^0-\kappa^1)(\kappa^1+\kappa^0))}. \tag{P12-4}$$

As κ^1 increases for $\hat{\hat{b}}_I$, so we substitute the maximum value of $\kappa^1 = \frac{\beta}{1-\beta}$ into the expression. But $\hat{\hat{b}}_I$ is also an increasing function of κ^0 and unfortunately κ^0 is unbounded:

$$\kappa^0 = \frac{\beta\,N + n^0}{(1-\beta)N} = \frac{\beta}{1-\beta} + \frac{n^0}{(1-\beta)N}. \tag{P12-5}$$

If we assume that $\bar{x}$ is such that $n^0(\bar{x}) \leqslant N$, (i.e., if new shares issued are no greater than the existing number of shares), then the maximum value of $\kappa^0 = \frac{\beta}{1-\beta} + \frac{1}{1-\beta}$. Substituting maximum value of $\kappa^1 = \frac{\beta}{1-\beta}$ and assumed maximum value of $\kappa^0 = \frac{\beta}{1-\beta} + \frac{1}{1-\beta}$ we obtain

$$\hat{\hat{b}}_I = \frac{(1-\beta)\left(\beta^2(1+\beta) + 2(1-\beta)^2\left((1+\beta)\log\left(1+\frac{\beta}{1-\beta}\right) - \beta\log\left(1+\frac{(1+\beta)}{1-\beta}\right)\right)\right)}{\beta(1+\beta)\left(4(1-\beta)^2\log\left(1+\frac{(1+\beta)}{1-\beta}\right) - \beta(1+\beta) - 4(1-\beta)^2\log\left(1+\frac{\beta}{1-\beta}\right)\right)}. \tag{P12-6}$$

Equation P12-6 ensures that the manager is better off if the level of investment remains the same. The second part of the proof requires us to show that the manager is better off if the level of investment increases. This requires that $MO^0(x_2) > MO^0(x_1)$, where $x_2 > x_1$. But we know that if $b_I \geqslant \frac{1}{2}$, then $\frac{\partial MO^0(x)}{\partial x} > 0$. Hence, $\hat{\hat{b}}_I \geqslant \frac{1}{2}$ is sufficient condition for full investment. If $b_I \geqslant \hat{\hat{b}}_I$, then the incumbent is better off if he is allowed to use non-voting shares to fund new projects.

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