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Advising the Management: A Theory of Shareholder Engagement

Finance Working Paper N° 829/2022
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Abstract

We study the effectiveness of shareholder engagement, that is, shareholders communicating their views to management. When shareholders and management have different beliefs, each shareholder engages more effectively when other shareholders engage as well. A limited shareholder base can thus prevent effective engagement. However, a limited shareholder base naturally arises under heterogeneous beliefs because investors who most disagree with management do not become shareholders. Passive funds, which own the firm regardless of their beliefs, can counteract these effects and improve engagement. When shareholders' and management's preferences are strongly misaligned, shareholders' engagement decisions become substitutes and the role of ownership structure declines.

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Abstract

We study the effectiveness of shareholder engagement, that is, shareholders communicating their views to management. When shareholders and management have different beliefs, each shareholder engages more effectively when other shareholders engage as well. A limited shareholder base can thus prevent effective engagement. However, a limited shareholder base naturally arises under heterogeneous beliefs because investors who most disagree with management do not become shareholders. Passive funds, which own the firm regardless of their beliefs, can counteract these effects and improve engagement. When shareholders' and management's preferences are strongly misaligned, shareholders' engagement decisions become substitutes and the role of ownership structure declines. (*JEL* D71, D74, D82, D83, G34)

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> Shareholder engagement has become one of the most talked-about issues in corporate governance, and with good reason.
>
> —Equilar, March 30, 2016[1]

Shareholder engagement, that is, shareholders communicating to management their views on corporate policies and strategy, has become a central component of corporate governance. According to former SEC chairman Mary Schapiro (2010), it is vital that shareholders and companies "move beyond the minimum required communications and become truly engaged" because management can "benefit from access to the ideas and the concerns investors may have." While communication between managers and shareholders took place in the past (e.g., Carleton, Nelson, and Weisbach 1998; Becht et al. 2009), this exchange has become particularly important and widespread in recent years.[2] In a survey of institutional investors, McCahery, Sautner, and Starks (2016) found that 63% of the respondents had engaged in direct discussions with top management over the previous five years. Actively managed investors, as well as passive funds are advising management, with the Big Three (BlackRock, Vanguard, and State Street) being particularly involved. For example, BlackRock's Investment Stewardship Annual Report states that in 2020, BlackRock "had over 3,000 in-depth conversations with corporate leadership," including "more than 1,000 engagements on corporate strategy and 400 engagements on the impact of COVID-19."

In addition to the growth in direct engagements, shareholders' communication with management has become more prevalent due to the increase in the number and breadth of issues that are brought up for nonbinding, that is, advisory, shareholder votes. The Dodd-Frank requirement of a regular advisory vote on executive compensation (say-on-pay), as well as nonbinding proposals submitted by shareholders via Rule 14a-8, allow the entire shareholder base to express their views and advise management on multiple issues concerning the firm's governance and strategy.

In light of the increasing prevalence and attention given to shareholder communication with management, it is important to understand when this communication is effective and what factors can enhance it. How does managerial learning from the shareholders interact with the firm's ownership structure? How does the growing ownership by passively managed

[1] (Equilar Blog 2016). Equilar is the leading provider of governance tools and executive compensation data for corporations, institutional investors, and the media.

[2] The fraction of S&P 100 companies that discuss their communication with shareholders in proxy statements increased from 2% in 2011 to 55% in 2015 (Equilar Blog 2016).

funds affect shareholder engagement? And how can firms improve shareholder communication, for example, by putting more issues up for advisory votes, adding shareholders to their boards, or changing managerial incentives? This paper provides a theory of equilibrium ownership structure and shareholder engagement that studies these questions.

In our model, the firm needs to make a decision, the value of which depends on an unknown state. Investors first decide which stakes to acquire, and their decisions determine the firm's ownership structure. We view this trading stage as shaping the firm's long-term shareholder base, for example, at the time of the initial public offering (IPO). Then, shareholders of the firm observe private signals about the state and communicate them to the manager by sending nonverifiable messages ("cheap talk"), and the manager decides which action to take. Thus, information about the state is dispersed among investors, which creates value from shareholder engagement.

The manager can be prevented from learning the most from investors for two reasons. The first are frictions in communication: if a shareholder has different preferences or beliefs from those of the manager, he may have incentives to misrepresent his information. For example, consider a firm deciding on the scale of production in a new market. The manager may have misaligned preferences and prefer a larger scale due to empire-building motives, giving the shareholder incentives to report more negative information than he privately has. Furthermore, the shareholder may have incentives to report more negative information if the manager has more optimistic prior beliefs about the growth of the new market. Indeed, substantial evidence suggests that heterogeneous beliefs are important in explaining corporate finance decisions and the dynamics of asset prices and volume,[3] and Li, Schwartz-Ziv, and Maug (2022) show that heterogeneity in beliefs affects how shareholders vote and trade around shareholder meetings.[4] Both differences in beliefs and misaligned preferences prevent effective communication between the manager and shareholders.

Differences in beliefs and misaligned preferences can also create another impediment to managerial learning: many potentially informed investors may choose not to become share-

[3]See Kandel and Pearson (1995), Diether, Malloy, and Scherbina (2002), Malmendier and Tate (2005), Dittmar and Thakor (2007), and Thakor and Whited (2011), among others; see Hong and Stein (2007) for a survey.

[4]The survey evidence by Edmans, Gosling, and Jenter (2022) suggests that boards see themselves as maximizing shareholder value but having different beliefs from investors about what CEO compensation contracts should look like. Likewise, there is often substantial disagreement about the effect of other governance policies, even among parties with similar interests, such as shareholders with similar portfolios. See, for example, Solomon (2015), who discusses shareholder disagreements on the issue of CEO-chairman separation and Solomon (2009), who discusses disagreements about the optimal terms of proxy access.

holders in the first place, especially if their views about the firm's strategy are not aligned with those of the manager. Such investors have no incentives or ability to communicate with the manager, so their information does not affect corporate decision-making.

We show that these two sources of inefficiencies – communication frictions and a limited shareholder base – interact and can exacerbate each other. First, a limited shareholder base can lead to less effective engagement by those investors who own the firm. This is because in the presence of differences in beliefs, and if the manager's preferences are not too misaligned, shareholders' engagement decisions are complements: any individual shareholder's engagement with management is more effective when more other shareholders engage as well. In particular, a shareholder is more likely to communicate his information truthfully if he expects other shareholders to do so. Intuitively, if the shareholder expects the manager to get advice from many other investors, he anticipates the manager's posterior beliefs about the optimal strategy to be more congruent with his own, which improves communication between them. In contrast, if the shareholder base is limited and the manager gets advice from a small selected set of investors, then the shareholder expects their differences in beliefs to persist and has little incentive to convey his information truthfully. This complementarity is consistent with the evidence in Doidge et al. (2019) and Dimson, Karakas, and Li (2015, 2021), who conclude that shareholder engagement is particularly effective when multiple investors collaborate with each other in their engagements.

Even though effective shareholder engagement requires a wide shareholder base, the shareholder base that arises under heterogeneous beliefs is often more limited. Investors whose views about the firm's optimal strategy are different from those of the manager expect the manager to make incorrect decisions (according to their beliefs), so they have low valuations of the stock and choose not to become shareholders. Instead, the firm is held by a limited subset of investors whose views are relatively more aligned with those of the manager.

Moreover, a limited shareholder base is especially likely to emerge when shareholder engagement is less effective: anticipating inefficiencies in shareholder-manager communication, investors whose views are misaligned with those of the manager expect their belief disagreements to remain strong and thus are less likely to become shareholders. Thus, both frictions in communication and a limited shareholder base inhibit managerial learning, and these inefficiencies amplify each other. We show that because of this two-way interaction between the ownership structure and the effectiveness of shareholder engagement, multiple equilibria can arise. An equilibrium where many informed investors become shareholders and communicate

with the manager can coexist with an equilibrium where only a small subset of investors become shareholders and managerial learning is highly limited. Intuitively, if an investor expects the firm to have few shareholders and the manager's decisions to be thus primarily based on the manager's prior beliefs, he expects to disagree with the manager's decisions ex post, and hence does not invest in the firm in the first place, making this equilibrium self-fulfilling.

Our results suggest an important role of passively managed funds. The unique feature of passive funds is that they are required to hold most public stocks regardless of whether their fund managers agree or disagree with the firms' policies. We show that as a result, the growth in passive funds can enhance managerial learning from shareholders, increase the informativeness of corporate decisions, and raise the share price. First, passive funds become shareholders and thus can engage with management even when active funds in their position (with the same preferences and beliefs) would have not taken a stake in the firm. Moreover, when shareholders' engagement decisions are complements, the presence of passive funds has a positive spillover effect on the engagement of actively managed funds: a larger number of active funds communicate their views to management when more passive funds are present. Finally, in the presence of equilibrium multiplicity, the growth in passive funds breaks the feedback loop between the ownership structure and managerial learning, and can eliminate the inefficient equilibrium.

We also show that ownership structure is less important for engagement when differences in beliefs are accompanied by misaligned preferences between the manager and shareholders (i.e., conflicts of interest). This is because if conflicts of interest are sufficiently strong, shareholders' engagement decisions become substitutes: as more shareholders share their views with management, each shareholder's incentives to communicate truthfully decline. Intuitively, when a shareholder misrepresents his information to push the manager closer to his preferred decision, he is afraid to make too big of an impact and move the manager's decision too much, away even from the shareholder's preferred decision. This concern constrains misreporting if the manager reacts strongly to the shareholder's advice. However, if many other shareholders provide advice, the manager reacts less to each individual shareholder's advice, so this concern no longer constrains misreporting. If conflicts of interest are strong, this effect dominates the complementarity effect described above: even if the manager learns a lot from shareholders and their posterior beliefs converge, strong conflicts of interest introduce a wedge between the shareholders' and manager's preferred decisions and limit how congruent the manager and shareholders can become. We show that when the substitution effect dominates, adding more

shareholders who can communicate their views does not improve the effectiveness of shareholder engagement. As a result, a limited shareholder base is no longer a key impediment to managerial learning, and the presence of passive funds plays a less important role.

This contrast between the complementarity and substitution effects also has implications for governance policies that aim to promote shareholder-manager communication. One such policy is the use of advisory shareholder votes. Advisory votes may create more problems than they solve if these votes do not provide useful information: for example, both the say-on-pay provision of the Dodd-Frank Act and Rule 14a-8 have been highly debated because of their potential downsides, such as distractions, time, and resources they may require from management.[5] Our results suggest that introducing an advisory vote on a certain decision is likely to have a particularly positive effect on managerial learning when there is substantial heterogeneity in beliefs about this decision. In this case, the advisory vote allows a cost-effective way to get the views of a large number of shareholders, especially those who would not be able to engage with management individually. Furthermore, due to complementarities in communication, it may encourage more effective communication by other shareholders, who would otherwise not share their views because of belief disagreements with the manager. In contrast, for decisions involving strong conflicts of interest, the substitution effect dominates, and shareholder-manager communication is not enhanced by additional shareholders who can convey their views. Hence, introducing an advisory vote may not improve managerial learning at all, and the potential downsides of such a vote become of first-order importance. For a similar reason, increasing board size by adding shareholders to the board (e.g., venture capitalists, activist investors, or other blockholders) is more likely to enhance managerial learning and increase value if there are strong differences in beliefs about the firm's strategy, but is more likely to decrease value if the manager's and shareholders' interests are strongly misaligned.

Overall, our paper highlights a new informational channel through which financial markets affect the quality of managerial decisions, and thus contributes to the feedback literature (e.g., Dow and Gorton 1997; Subrahmanyam and Titman 1999; Goldstein and Guembel 2008; for a survey, see Bond, Edmans, and Goldstein 2012). We emphasize that financial markets influence decision-making not only by the information contained in the prices but also by determining the firm's ownership structure and thus affecting which investors provide advice to management

[5] For example, if a certain advisory vote is not informative, then its only effect is that it "requires companies to devote significant time and resources... and distracts management and shareholders" (see the February 3, 2020 letter to the SEC from the Corporate Governance Coalition for Investor Value). See Maiden (2020) and Thomas, Palmiter, and Cotter (2012) for discussions of these debates.

via direct engagements, advisory voting, or joining the board.

The literature has analyzed the role of ownership structure for other channels of shareholder influence, namely, intervention ("voice") and selling shares ("exit") (for a survey, see Edmans and Holderness 2017). Most related to our paper is the subset of this literature that analyzes multiple blockholders and interactions between them (Winton 1993; Noe 2002; Edmans and Manso 2011; Doidge, Dyck, and Yang 2021; Brav, Dasgupta, and Mathews 2021; Cvijanovic, Dasgupta, and Zachariadis 2022). In papers that focus on governance through voice, dispersed ownership typically reduces the effectiveness of shareholder intervention due to the free-rider problem (e.g., Winton 1993; Edmans and Manso 2011). In contrast, the literature on governance through exit shows that ownership dispersion can be beneficial, either because multiple blockholders trade more aggressively to compete for profits (Edmans and Manso 2011) or because they react to each other's exits due to reputational concerns (Cvijanovic, Dasgupta, and Zachariadis 2022). Our paper highlights another reason ownership dispersion can improve governance: it can enhance communication between shareholders and management.

Several other papers have studied communication from shareholders to management (Levit 2019, 2020), as well as from the board of directors to management (e.g., Adams and Ferreira 2007; Harris and Raviv 2008; Baldenius, Melumad, and Meng 2014; Chakraborty and Yilmaz 2017). These papers analyze communication by a single agent, whereas our focus is on how communication decisions of multiple agents interact.[6] Thus, our paper contributes to the literature on cheap talk communication (Crawford and Sobel 1982) by multiple imperfectly informed senders (Austen-Smith 1993; Battaglini 2004; Morgan and Stocken 2008; Levit and Malenko 2011; Galeotti et al. 2013). The substitution effect that arises in our model when the misalignment of preferences is substantial is related to the result in Morgan and Stocken (2008) that full information revelation is an equilibrium in a poll with a small sample, but not with a large one. While this literature only studies heterogeneous preferences, we also introduce heterogeneous beliefs and show that when differences in beliefs are substantial, the results are the opposite of those under heterogeneous preferences. In addition, we highlight how in a corporate setting, the set of agents who communicate with management (i.e., the shareholder base) is itself endogenously determined by agents' preferences and beliefs, and how this, in turn, affects communication.

Finally, our paper is related to the literature on heterogeneous priors. Morris (1995) pro-

[6] Malenko (2014), Khanna and Schroder (2015), and Chemmanur and Fedaseyeu (2018) study communication between multiple members of the board or committee, but do not analyze their communication to the manager.

vides an overview of the heterogeneous prior assumption and discusses why it is consistent with rationality. Our model also features rational agents: although they have different priors, they are not dogmatic and update their beliefs in a Bayesian way after receiving new information. A large theoretical literature studies the implications of heterogeneous beliefs for trading in financial markets.[7] The contribution of our paper is to examine how heterogeneous beliefs affect not only shareholders' trading decisions but also their subsequent communication with management and the interaction between these two decisions. Boot, Gopalan, and Thakor (2006, 2008) also study differences in beliefs between shareholders and the manager, but from a very different perspective: these papers analyze the firm's choice between public and private ownership and do not feature asymmetric information and communication, which are the focus of our paper. Che and Kartik (2009), Van den Steen (2010), and Alonso and Camara (2016) study communication under heterogeneous beliefs but with only one sender and not via cheap talk, and thus do not consider the forces highlighted in our paper.[8]

1 Setup

In this section, we present a simple model, which captures a conflict of interest between the manager and shareholders, heterogeneous beliefs, and dispersed private information, and has tractable and intuitive solutions.

The environment consists of a firm, which is run by the manager, and a set of N investors (potential shareholders) indexed by i, $i \in \{1, ..., N\}$. The firm needs to make a decision, denoted by a continuous action $a \in \mathbb{R}$, whose value depends on the unknown state Z. If the manager takes action a in state Z, the firm delivers per-share value of

$$U(a, Z) = u_0 - (a - Z)^2, \tag{1}$$

where $u_0 > 0$ is sufficiently high, so that the equilibrium share price is always positive. The manager's interests may not be fully aligned with shareholders: his utility is

$$U_m(a, Z) = u_0 - (a - Z - b)^2, \tag{2}$$

where $b \geq 0$ measures the extent of conflicts of interest. Thus, from the shareholders' point of view, the optimal action is $a = Z$, whereas the manager's preferred action is $a = Z + b$. For

[7] See, for example, Harris and Raviv (1993), Kandel and Pearson (1995), Banerjee, Kaniel, and Kremer (2009), Banerjee and Kremer (2010), and Kyle, Obizhaeva, and Wang (2018), among many others.

[8] Garlappi, Giammarino, and Lazrak (2017, 2021) examine group decision-making under heterogeneous beliefs but without private information and communication.

example, if a refers to the firm's investment decision or how much to bid for a potential target, then b can capture the extent of empire-building preferences of the manager.

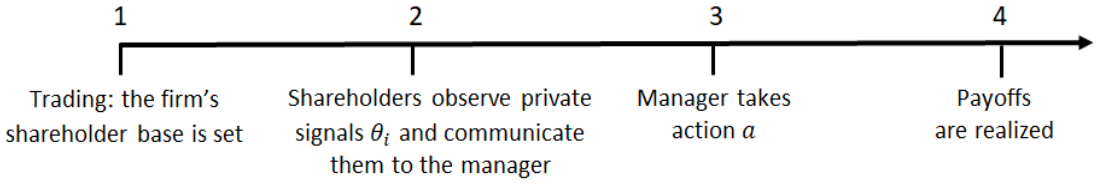


Figure 1. Timeline of the model

The timing, illustrated by Figure 1, is as follows. At the initial stage, all N investors participate in the market for the firm's shares, during which the total stock of the firm is sold by the original owner (seller) in a competitive market. The stock is in unit supply, so holding α_i shares is equivalent to holding fraction α_i of the firm. Each investor submits a demand schedule that specifies the quantity he wants to buy for various prices, $\{\alpha_i(p)\}$, and the equilibrium price p^* is set to clear the market. Suppose (e.g., as in Vives 1993) that investor i's utility from buying stake α_i is given by

$$\alpha_i \left(\mathbb{E}_i[U(a, Z)] - p\right) - \frac{\lambda}{2}\alpha_i^2, \tag{3}$$

where $U(a, Z)$ is his utility from each share and is given by (1), p is the share price, and $\lambda > 0$ captures either the holding cost due to limited diversification and risk aversion or the transaction cost due to limited liquidity. For example, λ is likely to increase with firm size and volatility because holding a given fraction of the firm is costlier when the firm is larger and more risky. Subscript i in the expectation operator captures the fact that investors could have heterogeneous beliefs, as will be described below.

Trading determines the firm's shareholder base, $S \subseteq \{1, ..., N\}$, which consists of all investors who hold a positive number of shares after the trading stage: $S = \{i : \alpha_i > 0\}$. After trading, each shareholder $i \in S$ learns a private signal θ_i about the state and sends a non-verifiable cheap-talk message to the manager. Investors who do not become shareholders do not communicate with the manager. After the communication stage, the manager chooses action $a \in \mathbb{R}$, and the payoffs are realized. To describe these stages in more detail, we will next define the information structure of the model.

The state of the world is equal to the sum of $K \geq N$ signals

$$Z = \sum_{i=1}^{K} \theta_i, \tag{4}$$

where $\theta_i \in \{0, 1\}$ are identically distributed binary signals: θ_i equals one with probability φ and zero with probability $1 - \varphi$. These signals are independent conditional on φ, but unconditionally correlated since φ is unknown, as described below. Signals θ_i can be thought of as different factors relevant to the decision. Information about these factors is dispersed among investors: if investor i becomes a shareholder, he privately observes θ_i and is uncertain about other signals. Such information structure is common in the literature (e.g., Harris and Raviv 2008; Chakraborty and Yilmaz 2017) and captures the idea that investors may have different areas of expertise and thus be informed about different aspects of the decision. For example, in the context of M&A decisions, a could be the choice of how much to bid for a potential target, and signals θ_i could capture the synergies from the merger, the intrinsic value of the target, the number of potential competing bidders and their bids, the costs of integrating the two companies, and other relevant factors. Since $K \geq N$, the model allows for residual uncertainty: all investors collectively can observe N signals at most, so $K - N$ of payoff-relevant signals always remain unknown at the decision-making stage. For simplicity, we assume that the manager is uninformed and that all investors' signals are equally important for the decision. In Section A.1 of the Internet Appendix, we show that the model can be easily extended to incorporate heterogenous importance of investors' signals and private information of the manager without changing the results.

Investors and the manager have heterogeneous beliefs about the state: some agents are ex ante more optimistic, whereas others are more pessimistic. In particular, the agents disagree about φ, the probability that each signal θ_i is equal to one: optimists have a higher expectation of φ than pessimists. The manager's prior is that φ is drawn from the Beta distribution with parameters $(\rho_m, \tau - \rho_m)$, whereas investor i's prior is that φ is drawn from the Beta distribution $(\rho_i, \tau - \rho_i)$.[9] Since the expected value of this Beta distribution is $\frac{\rho_i}{\tau}$, investors with a higher ρ_i are more optimistic.[10] Note that optimism in our model does not mean more positive beliefs about the value of the shares (if the action is fully informed, all agents agree that firm value is u_0), but rather beliefs that a higher action should be taken. While agents may have different

[9] That is, agent i believes that the density of φ is $f_i(\varphi) = \varphi^{\rho_i - 1}(1 - \varphi)^{\tau - \rho_i - 1} \frac{\Gamma(\tau)}{\Gamma(\rho_i)\Gamma(\tau - \rho_i)}$, where $\Gamma(\cdot)$ is the gamma function.

[10] See Auxiliary Lemma A.1 in the appendix. Notice that ρ_i also affects other moments, not only the mean. In Section A.2 of the Internet Appendix, we consider a more flexible specification in which agent i's prior of φ is characterized by the Beta distribution $(\rho_i, \tau_i - \rho_i)$. The main results extend to this setting.

prior beliefs, they update their beliefs rationally (according to Bayes' rule) when they receive new information.

We look for equilibria in pure strategies at the communication stage (see Section 4 for a discussion of mixed strategy equilibria). Because signals are binary, it is without loss of generality to consider a binary message space: the communication strategy of shareholder i is a mapping from his signal $\theta_i \in \{0,1\}$ into a binary nonverifiable message $\mu_i \in \{0,1\}$. Thus, in equilibrium, each shareholder either communicates his information truthfully (i.e., $\mu_i(\theta_i) = \theta_i$ up to relabeling) or sends an uninformative (babbling) message (i.e., $\mu_i(0) = \mu_i(1)$). If there are multiple equilibria that can be Pareto-ranked in the communication subgame, we assume that the more efficient equilibrium is played.

Discussion of the model. We assume that investors trade based on their prior beliefs, but do not trade again ex post, after learning their private signals. This simplifying assumption greatly enhances tractability: a model in which investors both trade on private information and decide how to communicate it is very difficult to analyze. There are two arguments for this assumption. First, as we discussed in the literature review, prior research has extensively studied how trading incorporates investors' private information into real decisions through its impact on prices (Bond, Edmans, and Goldstein 2012). In contrast, our contribution is to examine how trading incorporates investors' information into real decisions through a different channel, communication: trading determines the firm's shareholder base and thus, determines which investors communicate their information to the manager via engagement, advisory voting, or being on the board. Assuming that investors do not trade based on private information allows us to abstract from the price channel and focus on the more novel communication channel. Second, we view our trading stage as determining the firm's long-term shareholder base (e.g., at the time of the IPO) and ρ_i as capturing investors' beliefs at that point, for example, how congruent they are with the overall strategic direction the management is pursuing. One can then reasonably assume that such long-term shareholders' ownership stakes are not affected by more transitory private information that arrives later.

To make the analysis tractable, we also assume a specific communication protocol and make several assumptions about the information structure. We will discuss the robustness of our results to these assumptions in Section 4.

2 Analysis of the Model

2.1 Communication stage

We first characterize the action taken by the manager for a given outcome of the communication stage. Suppose that after communicating with the shareholders, the manager knows subset $R \subseteq \{1, ..., K\}$ of signals ("revealed" signals) and does not know all the other signals, $-R \equiv \{1, ..., K\} \backslash R$. We use R and $-R$ to represent the signal indexes and $\theta_R \equiv \{\theta_i, i \in R\}$ and $\theta_{-R} \equiv \{\theta_i, i \in -R\}$ to represent the corresponding subsets of signal realizations.

Given the quadratic payoff function, the optimal action of the manager is the sum of his bias b and his expectation of the state given his prior ρ_m and the signals he learned θ_R:

$$a_m(\theta_R) = b + \mathbb{E}_m\left(Z \,|\, \theta_R\right) = b + \sum_{i \in R} \theta_i + \sum_{j \in -R} \mathbb{E}_m\left[\theta_j | \theta_R\right]. \tag{5}$$

The subscript m in the expectation operator $\mathbb{E}_m$ highlights that the manager uses his own prior ρ_m to update his beliefs. In the appendix, using the properties of the Beta distribution, we show that the manager's posterior belief is that $\mathbb{E}_m(\varphi|\theta_R) = \frac{\rho_m + \sum_{i \in R} \theta_i}{\tau + |R|}$, where $|R|$ is the number of signals in R. This gives the following result:

Lemma 1 (Optimal action of the manager). *Suppose that after the communication stage, the manager knows subset R of signals. Then his optimal action is*

$$a_m(\theta_R) = b + \sum_{i \in R} \theta_i + \frac{\rho_m + \sum_{i \in R} \theta_i}{\tau + |R|} \left(K - |R|\right). \tag{6}$$

For any given information set θ_R, a higher bias b and a higher prior belief ρ_m both induce the manager to take a higher action. However, while the effect of b does not depend on the manager's information, the prior ρ_m becomes less important as the manager becomes more informed and updates his beliefs. In particular, as the set R expands, the term $\frac{K-|R|}{2\rho+|R|}$, and hence the effect of ρ_m decreases. The manager's action coincides with the optimal action from the perspective of shareholder i if $b = 0$, $K = N$, and $R = \{1, ..., N\}$.

Using Lemma 1, we will next examine when shareholders will truthfully communicate their information to the manager. Consider any shareholder i and suppose that the manager knows subset $R_i \subset \{1, ..., K\}$ of signals, where R_i does not include shareholder i's signal θ_i. The manager does not know all the other signals, that is, θ_i and all signals in the subset $-R_i \backslash \{i\}$, where as before, $-R_i \equiv \{1, ..., K\} \backslash R_i$. Suppose the manager believes the shareholder's message and uses it to update his belief about the state according to (6). If shareholder i reveals his

signal truthfully, the manager's action is

$$a_m\left(\theta_{R_i},\theta_i\right) \equiv b+\theta_i+\sum_{j\in R_i}\theta_j \ + \frac{\rho_m+\theta_i+\sum_{j\in R_i}\theta_j}{\tau+|R_i|+1}\left(K-|R_i|-1\right). \tag{7}$$

If shareholder i misreports and claims that his signal is $1-\theta_i$, the manager's action is

$$a_m\left(\theta_{R_i},1-\theta_i\right) \equiv b+(1-\theta_i)+\sum_{j\in R_i}\theta_j \ + \frac{\rho_m+(1-\theta_i)+\sum_{j\in R_i}\theta_j}{\tau+|R_i|+1}\left(K-|R_i|-1\right). \tag{8}$$

Shareholder i only knows his signal θ_i and does not know the set of all the other $K-1$ signals, which we denote by θ_{-i}. Thus, he compares his expected payoff from actions $a_m\left(\theta_{R_i},\theta_i\right)$ and $a_m\left(\theta_{R_i},1-\theta_i\right)$ given θ_i and his own prior belief about the distribution of those signals, and reports his signal truthfully if and only if

$$\sum_{\theta_{-i}\in\{0,1\}^{K-1}}\left[\left(a_m\left(\theta_{R_i},\theta_i\right)-Z\right)^2-\left(a_m\left(\theta_{R_i},1-\theta_i\right)-Z\right)^2\right]P_i(\theta_{-i}|\theta_i)\leq 0, \tag{9}$$

where $P_i\left(\theta_{-i}|\theta_i\right)$ is shareholder i's belief about θ_{-i} given θ_i and his prior ρ_i. The next result characterizes the necessary and sufficient conditions for (9) to hold.

Proposition 1 (IC constraint for truthful reporting). *Suppose that the manager learns subset R_i of signals (which does not include θ_i) and does not learn all the other signals, $-R_i$. Then shareholder i reports his signal truthfully if and only if*

$$\left|\left(\tau+|R_i|+1\right)b+\left(K-|R_i|-1\right)\left(\rho_m-\rho_i\right)\right|\leq\frac{\tau+K}{2}. \tag{10}$$

As is standard in cheap talk games, communication is ineffective if the manager's preferences are sufficiently different from those of the shareholder: (10) is violated if b is large. The misalignment of preferences creates incentives to misreport, as the shareholder wants to tilt the manager's action in the direction away from the manager's bias. Similarly, communication is ineffective if the manager and shareholders have very different prior beliefs: (10) is violated if $|\rho_m-\rho_i|$ is large. For example, if the shareholder thinks that the manager is too optimistic, he wants to correct this "bias in beliefs" by reporting a more negative signal.

Thus, with a single shareholder, disagreements due to differences in preferences and differences in beliefs have similar effects. However, this is no longer true with multiple shareholders. In this case, there are communication externalities – a shareholder's incentives to communicate truthfully depend on how much the manager is expected to learn from other shareholders (i.e., $|R_i|$) – and these externalities are very different depending on the source of disagreements. To

explain the intuition, we rewrite (10) in the following form:

$$2\left|b+\frac{K-|R_i|-1}{\tau+|R_i|+1}(\rho_m-\rho_i)\right|\leq 1+\frac{K-|R_i|-1}{\tau+|R_i|+1}. \tag{11}$$

The left-hand side of (11) captures the incongruence between the manager and the shareholder. For example, if the shareholder is more pessimistic than the manager ($\rho_m > \rho_i$), then the manager's preferred action is higher than that of the shareholder, both due to the manager's bias in preferences ($b > 0$) and due to his too optimistic beliefs. The right-hand side of (11) measures the manager's reaction to the shareholder's advice, that is, by how much the manager's action changes if the shareholder misreports his signal θ_i.[11] Intuitively, the shareholder faces a trade-off: while he wants to tilt the manager in the direction of his own preferred action (the benefit of misreporting, captured by the left-hand side of (11)), he is also afraid to tilt it too much, away even from his own optimal action, that is, to "overshoot" (the cost of misreporting). This concern makes the shareholder reluctant to misreport if the manager reacts strongly to the shareholder's advice (the right-hand side of (11) is large enough), but not otherwise. One can now easily see the two opposite forces through which $|R_i|$ affects the shareholder's IC constraint:

1. Complementarity in shareholders' communication decisions. The first force is that the heterogeneity in prior beliefs becomes less important as the manager becomes more informed. This leads to shareholders' communication decisions being *complements*: the more information the manager is expected to learn from others (i.e., the higher is $|R_i|$), the more likely it is that shareholder i will also truthfully communicate his signal. Intuitively, the shareholder expects the manager to become more congruent with him as the manager learns more: the term $\frac{K-|R_i|-1}{\tau+|R_i|+1}(\rho_m-\rho_i)$ in (11) decreases in $|R_i|$. This happens due to two related effects. First, once a signal is revealed, agents update their posteriors about the distribution of the state. Hence, even if the shareholder's and manager's initial beliefs are very different, the shareholder expects them to become closer following the revelation of information by other investors. Second, heterogeneous beliefs generate disagreement only over the information that is still unknown — once a signal gets revealed, all parties agree about it.[12] To see the com-

[11] Both of these statements follow from (7) to (8). From (7), the term under the absolute value sign on the left-hand side is the difference between the preferred actions of the manager and the shareholder given information θ_{R_i} and θ_i. From (7) to (8), the right-hand side equals $a_m(\theta_{R_i},\theta_i)-a_m(\theta_{R_i},1-\theta_i)$.

[12] The first effect is captured by the denominator, $\tau+|R_i|+1$: the manager updates his beliefs about φ and hence signals θ_{-R_i} after learning signals θ_{R_i}. The second effect is captured by the numerator, $K-|R_i|-1$: the manager learns signals θ_{R_i} out of $\theta_{\{1,\ldots,K\}}$.

plementarity effect most starkly, consider the extreme case of $b = 0$. Suppose that there is no residual uncertainty ($K = N$), and the manager knows all the signals except shareholder i's signal: $R_i = \{1, ..., N\}\backslash\{i\}$. Then, truthfully reporting the last remaining signal θ_i results in the manager taking the action that is optimal from the perspective of the shareholder, and hence is always incentive compatible.

The complementarity effect only arises in the presence of heterogeneous beliefs. If agents have common priors ($\rho_i = \rho_m$ for all i) and $b = 0$, then (10) is always satisfied; that is, each shareholder has incentives to communicate his signal truthfully regardless of how many other shareholders communicate with the manager.

2. Substitution in shareholders' communication decisions. The second force is that as the manager learns from a larger number of shareholders, he reacts less to each individual shareholder's advice: the right-hand side of (11) decreases in $|R_i|$. As a result, the shareholder is less worried that misreporting his signal will tilt the manager's action too far away from the shareholder's own optimal action, that is, the cost of misreporting declines. Hence, the shareholder is more likely to misreport when more other shareholders communicate with management, leading shareholders' communication decisions to be *substitutes*.

Proposition 1 shows that which of these two forces dominates depends on the relation between $|\rho_m - \rho_i|$ and b. If $b = 0$, the left-hand side of (10) always decreases in $|R_i|$: if heterogeneous beliefs are the only communication friction, shareholder's communication decisions are always complements. More generally, (10) implies that the complementarity effect dominates if b is sufficiently small relative to $|\rho_m - \rho_i|$. However, as b increases, the complementarity effect is eventually dominated by the substitution effect: the left-hand side of (10) increases in $|R_i|$ once b becomes sufficiently large relative to $|\rho_m - \rho_i|$. Intuitively, the misalignment of preferences limits how congruent the manager and shareholders can become due to learning: even if the manager learns a lot and his beliefs converge to those of the shareholders, strong preference misalignments introduce a wedge between the shareholders' and manager's preferred decisions.

To simplify the exposition and derive a complete, closed-form characterization of the equilibria, we assume in the remainder of the paper that there are only two types of investors:

Assumption: *Suppose $\rho_m = \rho$, $\tau = 2\rho$, and there are two types of investors: N_o optimists with $\rho_i = \rho + \Delta$, and $N_p \equiv N - N_o$ pessimists with $\rho_i = \rho - \Delta$, where $\rho > \Delta$.*

Thus, the manager believes that φ is drawn from $Beta(\rho, \rho)$, whereas optimists (pessimists) are more (less) optimistic than the manager and believe that φ is drawn from $Beta(\rho + \Delta, \rho - \Delta)$ ($Beta(\rho - \Delta, \rho + \Delta)$). The case $\Delta = 0$ captures common priors: for example, if $\Delta = 0$ and $\rho = 1$, all agents believe that φ is uniformly distributed on $[0, 1]$. All of the parameters are publicly known.

Next, we will use (10) to characterize the most informative equilibrium at the communication stage given any shareholder base S (as we will show in the next section, the most informative equilibrium is Pareto efficient if K is large enough). Since $b \geq 0$, then for any given $|R_i|$, if (10) holds for a pessimistic shareholder ($\rho_m - \rho_i = \Delta$), it also holds for an optimistic shareholder ($\rho_m - \rho_i = -\Delta$). Intuitively, a pessimistic shareholder is worried that the manager's action will be too high both because the manager has a preference for a higher action and because he is more optimistic than the shareholder. In contrast, from an optimistic shareholder's point of view, the manager's preference for a higher action counterbalances the manager's pessimism. Essentially, optimists are more aligned with the manager than pessimists, and thus have lower incentives to misreport. This implies that without loss of generality, we can focus on equilibria in which pessimists communicate truthfully only if all optimists communicate truthfully.[13] These equilibria have the following properties:

Proposition 2 (Equilibrium at the communication stage). *The most informative equilibrium features the largest (in terms of the number of shareholders) subset of S that satisfies (10). Without loss of generality, it takes one of the following three forms: (1) all shareholders communicate truthfully; (2) all optimists communicate truthfully and some (potentially zero) pessimists communicate truthfully; or (3) some (potentially zero) optimists communicate truthfully and no pessimist communicates truthfully. In addition:*

(i) If b is sufficiently small, then either all shareholders communicate truthfully (if $|S| > K - \frac{\rho + K/2}{\Delta}$) or no shareholder does (if $|S| < K - \frac{\rho + K/2}{\Delta}$).

(ii) If Δ is sufficiently small, the number of signals communicated is either $|S|$ or the floor of $\frac{\rho + K/2}{b} - 2\rho$, whichever is lower.

Statement (i) follows from shareholders' communication decisions being complements when b is small: if there exists an equilibrium in which at least one shareholder communicates truth-

[13] Formally, as we show in the proof of Proposition 2, if there exists an equilibrium in which n_o optimists and n_p pessimists communicate truthfully, and S_o is the number of optimistic shareholders, then there exists a payoff-equivalent (in the sense of ex ante payoffs of all shareholders and the manager) equilibrium in which $\min\{n_o + n_p, S_o\}$ optimists and $\max\{0, n_o + n_p - S_o\}$ pessimists communicate truthfully.

fully, there also exists a (more informative) equilibrium in which all shareholders communicate truthfully. Moreover, because of complementarities, an equilibrium with truthful communication does not exist unless the number of shareholders $|S|$ is large enough.

Statement (ii) follows from the substitution effect, which dominates when differences in beliefs (Δ) are small relative to b. The fact that shareholders' communication decisions are substitutes implies that truthful communication by all shareholders is not possible unless their number $|S|$ is sufficiently small. Notice that the effect of $|S|$ in this case is the opposite of its effect in (i), where $|S|$ has to be sufficiently large for truthful communication to occur.

2.2 Trading stage

To solve for the equilibrium in the trading game, we first derive each investor's ex ante expected utility from holding one share (not accounting for his holding costs) as a function of the set of signals learned by the manager at the communication stage. We refer to this utility as the investor's valuation.

Lemma 2 (Ex ante payoffs). *Suppose that in equilibrium, the manager learns subset R of the signals and does not learn all the other signals, $-R$. Then investor i's valuation of each share is given by:*

$$\begin{aligned}\mathbb{E}_i[U|R] &= u_0 - b^2 - \frac{2b(\rho-\rho_i)}{2\rho+|R|}\left(K-|R|\right) \\ &\quad -\frac{\rho^2-\Delta^2}{2\rho(2\rho+1)}\left(K-|R|\right)\frac{2\rho+K}{2\rho+|R|} - \left[\frac{\Delta\left(K-|R|\right)}{2\rho+|R|}\right]^2.\end{aligned} \tag{12}$$

If $K > \bar{K}$, where $\bar{K} \equiv \frac{4b\Delta\rho(2\rho+1)}{\rho^2-\Delta^2} - 2\rho$, then $\mathbb{E}_i[U|R]$ is increasing in $|R|$ for every agent.

Intuitively, if the decision were fully informed and unbiased from an investor's perspective, his valuation would be u_0. However, the decision is biased from the investor's perspective due to the manager's misaligned preferences ($b > 0$) and different beliefs ($\rho \neq \rho_i$), which is captured by the second and third terms in (12). In addition, even if the manager has the same preferences and beliefs as the shareholder but does not have full information ($|R| < K$), the shareholder's valuation is below u_0 because the manager's decision is not fully informed. The forth and fifth terms in (12) capture this effect.

Recall that from an optimistic shareholder's point of view, the manager's preference for a higher action counterbalances the manager's pessimism. Because of this, an optimistic shareholder could even, under some circumstances, benefit from a less informed manager, that is,

a lower $|R|$. However, focusing on a large enough K (above $\bar{K}$ given by the lemma) ensures that this effect is not too strong, so that the more direct, beneficial, effect of managerial learning dominates: if $K > \bar{K}$, then all investors benefit from a more informed manager.[14] In what follows, we assume that this condition on K is satisfied, so that the most informative communication equilibrium is Pareto efficient and hence is played.

Given (3) and (12), we can calculate the demand for shares from each investor i for any set of signals R that the investor expects to be communicated to the manager. Maximizing (3) with respect to α, the optimal ownership stake of investor i given share price p is

$$\alpha_i(p) = \max\left\{\frac{\mathbb{E}_i[U|R] - p}{\lambda}, 0\right\}. \tag{13}$$

A larger holding cost λ decreases the investor's demand for shares, while higher expected utility $\mathbb{E}_i[U|R]$ from each share increases his demand. Given (13) and the unit supply of shares, market clearing implies $1 = \sum_{i=1}^{N} \alpha_i(p) = \sum_{i \in S} \frac{\mathbb{E}_i[U_i] - p}{\lambda}$. Hence, the equilibrium price for a given R satisfies

$$p^* = \frac{1}{|S|}\left(\sum_{i \in S} \mathbb{E}_i[U|R] - \lambda\right). \tag{14}$$

Because the manager's bias toward a higher action amplifies his disagreements with the pessimists but weakens his disagreements with the optimists, the pessimists have a lower valuation of the firm than the optimists (see the third term in (12)). Thus, the pessimists hold smaller stakes than the optimists and, if the differences in their valuations are substantial, do not hold any shares at all, resulting in more concentrated ownership.[15] The result that differences in beliefs generate more concentrated ownership would also hold for a more general distribution of investors' beliefs, either if the manager's bias is sufficiently large or if there is residual uncertainty and investors' differences in beliefs with the manager are not symmetric across investors.[16]

[14] The reason the positive effect dominates for large K is that by learning the signals θ_R, the manager updates his beliefs about φ and hence the signals θ_{-R}, and the set θ_{-R} is larger when K is larger.

[15] Recall that in our model, optimism does not mean a higher valuation of shares (each investor's valuation is u_0 if $a = Z$), but rather a belief that a higher action should be taken. This means that if the manager were biased toward a lower action ($b < 0$), then optimists would have lower valuations than pessimists, and hence would hold smaller stakes.

[16] To see this, consider a general distribution of investor beliefs $\{\rho_i, i = 1, ..., N\}$. If preferences are aligned ($b = 0$), and there is no residual uncertainty ($K = N$), there exists an equilibrium in which ownership is fully dispersed and the manager learns all the signals. Indeed, in this equilibrium, there are no disagreements in posterior beliefs, making truthful communication incentive compatible and, in turn, making the ex ante valuations of all investors the same. However, if b is large enough (so that not all information is communicated), then ex post disagreements remain, and some investors will be naturally closer to the manager than others.

2.2.1 Equilibria of the game. According to (13), all investors with the same prior beliefs own the same number of shares in equilibrium. Furthermore, because pessimists' valuations are lower than those of the optimists', pessimists only become shareholders if all optimists also become shareholders. As a result, the equilibria of the game take two possible forms.

The first case is that both pessimistic and optimistic investors become shareholders. This happens if the holding cost λ is sufficiently high, so that the demand for shares from the optimists declines relatively fast. Then, the shareholder base S consists of all investors, with optimists generally holding larger ownership stakes than pessimists. The set of shareholders that communicate truthfully is the largest subset of all investors for which the IC constraint (10) is satisfied; it is characterized by Proposition 2.

The second case is that only optimistic investors become shareholders, whereas pessimistic investors do not. This happens if the holding cost λ is sufficiently low. Then, the demand for shares from optimists does not decline very fast, and their high demand increases the share price to the level at which pessimists do not want to become shareholders. In this case, each optimist holds stake $\frac{1}{N_o}$, and the number of shareholders that communicate truthfully is the highest number in $[0, N_o]$ for which the IC constraint (10) for optimists is satisfied.

2.2.2 Sources of inefficiencies. Two sources of inefficiencies emerge in equilibrium. One is suboptimal quality of decision-making if the manager does not learn all the available information. We will say that an equilibrium features *more informative communication* if the manager learns more signals from investors, that is, $|R|$ is higher. Lemma 2 and the assumption $K > \bar{K}$ guarantee that if the manager learns more signals, then the expected valuation of the shares from the perspective of each investor, as well as the utility of the manager, is higher. In this sense, a greater number of signals learned by the manager means more informed and efficient decision-making. Note that managerial learning can be limited both because the firm's shareholders do not convey their views truthfully and because some potentially informed investors (pessimists in our setting) do not become shareholders in the first place.

The second source of inefficiency is suboptimal diversification by investors: the aggregate holding costs would be minimized if each investor's stake were $\frac{1}{N}$. Both inefficiencies reduce investors' combined utility from holding the stock, as well as the share price. The following proposition provides sufficient conditions under which these inefficiencies do not arise:

Similarly, ex post disagreements in beliefs remain if there is residual uncertainty ($K > N$, so that agents disagree even if all investors' information is shared with the manager), and then an asymmetry in investors' ex ante differences in beliefs with the manager will also lead to differences in their valuations of shares.

Proposition 3. *If* $b < \frac{\rho+K/2}{2\rho+N}$, *there exists* $\rho(b,\Delta) > 0$, *which is decreasing in* b *and* Δ, *such that if* $K - N \leq \rho(b,\Delta)$, *there exists an equilibrium where all investors become shareholders and truthfully communicate their information to the manager. Moreover, if* $K = N$, *then for any* Δ, *all investors acquire the same number of shares, achieving full diversification and truthful communication. If, in addition,* $b = 0$, *the equilibrium achieves first-best.*

The logic is as follows. Suppose that all investors indeed become shareholders. Condition $b < \frac{\rho+K/2}{2\rho+N}$ ensures that preferences are sufficiently aligned, so that all investors communicate truthfully if beliefs are aligned as well. The condition that the residual uncertainty is low, $K - N \leq \rho(b,\Delta)$, guarantees that if the manager is expected to learn all investors' information ($|R| = N$), his remaining belief disagreements with the shareholders are small, so that it is indeed incentive compatible for all investors to communicate truthfully. Together, these two conditions imply that the manager's decision is both relatively unbiased and sufficiently informed, so both optimists' and pessimists' valuations are high and they all become shareholders. Thus, an equilibrium with $S = \{1, ..., N\}$ and full communication indeed exists. Moreover, if $K = N$, then even if the original differences in beliefs are very large, the posterior beliefs of all agents are the same. In this case, the optimists and pessimists have the same ex ante valuations and hence acquire equal stakes, achieving optimal diversification. If $b = 0$, the equilibrium achieves first-best: a social planner who maximizes the combined utility of all players would pick the same allocation of shares ($\frac{1}{N}$ to each investor) and the same corporate action ($a = Z$) as those that arise in equilibrium.

We fully characterize the set of equilibria in the appendix, after the proof of Lemma 2. In general, either one or both of the two inefficiencies are present in equilibrium. Moreover, these inefficiencies are interrelated and amplify each other. On the one hand, the fact that the manager does not learn all the information implies that ex post, his beliefs about the state are different from those of the shareholders. Anticipating these disagreements, shareholders who are relatively less aligned with the manager (i.e., the pessimists) acquire a lower stake than the optimists and, potentially, do not acquire any shares at all. Thus, imperfect shareholder communication leads to suboptimal diversification across investors. On the other hand, the fact that some investors do not become shareholders in the first place implies that they do not engage and communicate with the manager, which leads to less informed managerial decision-making. We explore these interactions and their implications next.

3 Implications

In this section, we derive our key implications. Section 3.1 highlights the role of the firm's ownership structure for shareholder engagement. Section 3.2 analyzes the effect of passive investors. Section 3.3 discusses advisory shareholder voting and the advisory role of the board. We also present the testable implications of the model.

3.1 Ownership structure and shareholder engagement

The analysis in Section 2 shows that differences in beliefs and misaligned preferences may prevent effective communication between shareholders and management, resulting in less informed corporate decisions. These frictions can be exacerbated by the fact that the ownership structure is itself endogenous: investors who disagree with management may choose not to become shareholders of the firm. Such investors then have no incentives or no ability to communicate their information, leading to a loss of potentially valuable information for decision-making. Instead, ownership is concentrated among investors who are relatively more aligned with the management, and only they provide their advice. This concentration of ownership is more likely to happen when holding costs are relatively small, for example, when the firm is smaller or less risky. The following proposition formalizes this intuition.

Proposition 4. *Suppose* $(2\rho + N_o + 1)\, b + (K - N_o - 1)\, \Delta \leq \rho + \frac{K}{2}$. *Then there exists* $\hat{\lambda}$ *such that for any* $\lambda > \hat{\lambda}$, *there is an equilibrium that features a more dispersed ownership structure and more informative communication than any equilibrium for* $\lambda < \hat{\lambda}$. *The equilibrium stock price is nonmonotone in* λ: *it is decreasing in* λ *for* $\lambda < \hat{\lambda}$ *and* $\lambda > \hat{\lambda}$, *but increases discontinuously at* $\lambda = \hat{\lambda}$.

Intuitively, if the holding costs are small, $\lambda < \hat{\lambda}$, the optimistic investors' demand is high and increases the share price to the level that exceeds the pessimistic investors' valuation of the stock. Thus, the firm is entirely held by the optimists, and pessimists do not communicate their information even if they would have incentives to do so, had they owned the firm. An increase in holding costs prevents this ownership concentration and encourages more investors to hold the firm and communicate their information (the condition on b and Δ in the statement of the proposition ensures that not only optimists, but also some pessimists, have incentives to communicate truthfully). As a result, while higher holding costs generally decrease the share price (p^* decreases in λ for a given S and R in (14)), this may no longer be the case when

learning from shareholders is important. As the last statement of the proposition shows, the wider shareholder base and the resultant improvement in corporate decision-making can lead the share price to increase in λ.

The result that more dispersed ownership strictly improves communication relies on the substitution effect not being too strong. If there is a large misalignment in preferences and the substitution effect is substantial, then increasing ownership dispersion does not hurt communication, but does not improve it either.[17]

Overall, in our setting, ownership dispersion always weakly improves shareholder engagement. In general, however, an important drawback of dispersed ownership is the free-rider problem: dispersed ownership discourages each individual shareholder from monitoring (e.g., Winton 1993; Admati, Pfleiderer, and Zechner 1994; Edmans and Manso 2011) or acquiring information (e.g., Malenko and Malenko 2019). A related effect would arise in our model if information acquisition were costly for shareholders. In Section A.3 of the Internet Appendix, we introduce costs of information acquisition and show that more dispersed ownership decreases each shareholder's incentives to acquire information. Combined, the two effects (on information acquisition and on communication) imply that ownership should be neither too concentrated nor too dispersed for shareholder engagement to be most effective. Because the free-rider problem is well-understood in the literature, we abstract from costly information acquisition in the basic model and focus on the more novel effects coming from complementarities in shareholders' communication decisions.

3.2 Role of passively managed funds

As the previous discussion shows, when investors optimally pick their holdings in the firm, the shareholder base can become too limited, and the information of investors who disagree with management can be lost. This suggests an interesting distinction between the advisory role of actively managed versus passively managed (index) funds. In recent decades, an increasing fraction of firms' ownership is comprised by passive funds (e.g., Appel, Gormley, and Keim 2016). Passively managed funds become shareholders even if they disagree with the firm's

[17] To see this, suppose that Δ is small. Then part (ii) of Proposition 2 implies that the number of signals learned by the manager is limited by the number of shareholders $|S|$ and the floor of $\frac{\rho+K/2}{b} - 2\rho$, whichever is smaller. If b is small, the binding constraint comes from ownership, so increasing $|S|$ improves managerial learning. However, if b is large, then increasing $|S|$ does not change the quality of communication. For example, if only optimistic investors own the firm, but not all of them communicate truthfully, then adding pessimists to the shareholder base (e.g., by increasing holding costs λ) does not change the communication equilibrium.

management: as William McNabb III, former chairman and CEO of Vanguard put it, "We're going to hold your stock if we like you. And if we don't" (McNabb 2015). This requirement to hold the stock regardless of the fund manager's views about the company implies that the growth in passive funds can make shareholder-manager communication more effective. First, passive funds hold stock and engage with management when active funds in their position would not have. Moreover, because of complementarities in communication, engagement by passive funds can have a positive spillover effect on the engagement by other firm's shareholders.

To show these implications, we make a small modification of the basic model. Suppose that of N investors, L investors are required to hold $\frac{1}{N}$ shares of the firm regardless of the market price of the shares or their valuations. We refer to these investors as "passive." The remaining $\frac{N-L}{N}$ shares are sold in the market to the remaining $N-L$ investors, who we call "active." Suppose that optimists and pessimists are equally represented among passive and active investors: the number of optimists among passive (active) investors is $N_o\frac{L}{N}$ $(N_o\frac{N-L}{N})$. This guarantees that we keep investors' beliefs the same as we change L; that is, there are always N_o optimists and $N-N_o$ pessimists, regardless of the number of passive investors. The basic model corresponds to $L=0$.

The assumption that each passive investor holds $\frac{1}{N}$ shares ensures that passive investors do not have price effects by changing the residual supply of shares: as we show in the proof of Proposition 5, given the same equilibrium at the communication stage, the stock price with L passive investors is the same as in the basic model without passive investors.[18] However, shareholder communication is improved by the presence of passive investors: the manager learns more information than in the basic model. As a result, managerial decisions are more informed, and the share price is higher:

Proposition 5. *Suppose* $(2\rho+N_o+1)\,b+(K-N_o-1)\,\Delta\leq\rho+\frac{K}{2}$ *and* $\lambda<\hat{\lambda}$, *as defined in Proposition 4. Then, the equilibrium with* $L>0$ *passive investors features more informative communication and a higher share price than the equilibrium without passive investors. Informativeness of communication and the share price are weakly increasing in* L.

Intuitively, an active investor may choose not to become a shareholder if he is pessimistic and the stock price exceeds his valuation of the shares. Such active investors do not communicate with the manager, even though they would do so if they were forced to become shareholders

[18] If passive investors' stakes were lower (higher) than $\frac{1}{N}$, active investors would hold larger (smaller) stakes than in the basic model, leading to a lower (higher) stock price because of their holding costs.

(the parameter restrictions in Proposition 5 guarantee that the IC constraint of at least one pessimist would be satisfied). In contrast, passive investors own the shares regardless of their beliefs, that is, even if they are pessimistic. Thus, passive investors become shareholders and provide advice to the manager even if active investors in their position (with exactly the same beliefs and preferences) would have stayed away from the firm. Overall, passive fund growth (i.e., an increase in L) makes corporate decisions more informed and increases the stock price.[19]

Moreover, the presence of passive funds can enhance communication between the manager and *other shareholders*. This spillover effect occurs when shareholders' communication decisions are complements:

Proposition 6. *Suppose* $\lambda < \frac{4b\Delta(K-N)N_o}{2\rho+N}$ *and* $(K-N_o)\Delta - (2\rho+N_o)b > \rho + \frac{K}{2}$. *Then, without passive investors, only* N_o *optimistic investors become shareholders and not all of these optimists communicate truthfully, that is,* $|R| < N_o$. *If, in addition,* $\left(2\rho + N_o + N_p\frac{L}{N}\right)b + \left(K - N_o - N_p\frac{L}{N}\right)\Delta \leq \rho + \frac{K}{2}$, *then in the model with* L *passive investors,* N_o *optimists and* $N_p\frac{L}{N}$ *pessimists become shareholders, and all* $N_o + N_p\frac{L}{N}$ *shareholders communicate truthfully.*

Intuitively, by engaging with the manager and making him more informed, passive investors reduce belief disagreements between the manager and other investors, encouraging them to communicate their information as well. As a result, whereas only a subset of optimists communicate with the manager when $L = 0$, all of the optimists (active and passive) communicate when passive investors are present. While Proposition 6 is specific in its conditions, the general intuition is that if there are substantial disagreements in beliefs, so that the complementarity effect in communication dominates the substitution effect, the presence of passive investors could facilitate communication by all shareholders, both active and passive, and thus lead to more informative decisions of the management.

3.2.1 Empirical implications. The model predicts that greater passive fund ownership will be associated with more effective communication between shareholders and management, especially when differences of beliefs are substantial. This prediction can be tested both in the time series and in the cross-section. In the time series, the rise in passive fund ownership over the last decades has coincided with the increased impact of advisory votes on firms' decisions, as

[19] Note, however, that the equilibrium does not necessarily become more efficient as L increases. This is because even though the manager's decision becomes more informed, passive investors' diversification is suboptimal: unless the manager becomes fully informed ($|R| = K$), optimistic (pessimistic) passive investors are restricted to holding a strictly smaller (larger) stake than they would have chosen optimally.

well as the rise in shareholder engagement campaigns and management responsiveness to them. For example, Ferri (2012) discusses the evolution of advisory voting and concludes that until early 2000s, it was "low-impact" and that such votes "were largely ignored" by management, but that it "has become a more powerful tool" in recent years. Importantly, votes by passive funds and engagement campaigns by large index fund managers are a significant part of this overall improvement in shareholder-manager communication.[20] While these contemporaneous trends in no way show causality, to establish a more causal link between passive fund ownership and the effectiveness of shareholder communication, one could conduct cross-sectional analysis similar to the Russell-3000 reconstitution studies (e.g., Appel, Gormley, and Keim 2016). To measure the effectiveness of shareholder communication in this setting, one could look at managerial responsiveness to advisory votes (as, e.g., in Ertimur, Ferri, and Stubben 2010; Cuñat, Gine, and Guadalupe 2012; Ferri 2012) and to shareholder engagement campaigns (as in, e.g., Gormley et al. 2021).

3.2.2 Incentives to become a passive investor. Since the presence of passive funds improves communication and the effectiveness of corporate decisions, a natural question is whether passive funds can arise endogenously in the model. In other words, is it optimal for some investors to commit to a passive strategy under which they become shareholders despite potential disagreements with management? In our current model, making such a commitment is suboptimal for investors because they do not capture the benefits of improved communication (we formally show this in the Internet Appendix). Intuitively, other investors anticipate the resultant improvements in the firm's performance, so these improvements are reflected in the higher stock price that the passive investor pays for the shares. All the benefits from improved communication thus accrue to the original owner of the shares. For passive ownership to arise endogenously, investors needs to capture a sufficient amount of value that they create by becoming passive.

One mechanism through which investors can capture their contribution to firm value is by having an initial endowment of shares. As long as an investor's endowment is sufficiently large, the appreciation in the value of his endowment outweighs the loss from paying a high price for the shares. We formalize this intuition in Section A.10 of the Internet Appendix, where we consider an example in which all investors hold a stake $\frac{\alpha}{N}$ at the beginning of the game. We show that if α is sufficiently high, passive ownership arises endogenously, and the magnitude

[20] See, for example, Fichtner, Heemskerk, and Garcia-Bernardo (2017). According to Larry Fink, the CEO of BlackRock, "we are taking a more active dialogue with our companies" (see Financial Times 2016).

of α determines how many investors become passive funds. Having an endowment of shares is not the only mechanism through which a passive investor can capture some value from more effective communication. Another such mechanism is IPO underpricing, whereby initial owners strategically sell shares below the market price to give some ownership to passive investors.[21]

3.2.3 Equilibrium multiplicity. While the above results emphasize how the firm's ownership structure affects managerial learning and decision-making, there is an effect in the other direction as well: shareholders' anticipation of the firm's decisions affects their valuation of the shares and the stakes they acquire. If management is expected to choose a strategy that is suboptimal from most investors' perspective, the firm will only attract investors whose views are aligned with this strategy. This creates a feedback loop between the ownership structure and managerial decision-making, which may lead to multiple equilibria: one where the firm is widely held and management gets advice from a large set of investors, and another where the firm is held by a subset of shareholders and managerial learning is limited. The first equilibrium features both a higher share price and higher welfare, which we define as the combined utility of all players (investors, the original owner of the shares, and the manager).

Proposition 7. *Suppose that* $0 < b \leq \frac{1}{2}$, *and there is no residual uncertainty,* $K = N$.

(i) There always exists an equilibrium in which all N *investors become shareholders, acquire equal stakes* $\alpha_i = \frac{1}{N}$ *in the firm, and the most informed action* $a = b + Z$ *is taken.*

(ii) Suppose, in addition, that $\frac{2\rho+N_o}{\Delta}b \leq K - N_o \leq \frac{2\rho+N_o}{\Delta}b + \frac{\rho+K/2}{\Delta}$ *and* $\lambda < 4b\Delta\frac{(K-N_o)N_o}{2\rho+N_o}$. *If* $L = 0$, *there also exists an equilibrium in which only optimistic investors become shareholders, the manager's action is not fully informed (*$a \neq b+Z$*), and both welfare and the share price are lower than in the first equilibrium. If* L *is sufficiently large, the equilibrium in (i) is unique.*

Equilibrium multiplicity arises due to complementarity in shareholders' communication decisions. Statement (i) is a direct consequence of Proposition 3: if preferences are sufficiently aligned ($b \leq \frac{1}{2}$) and $K = N$, then regardless of how strong differences in beliefs are, there exists an equilibrium in which all investors become shareholders and the manager's action reflects all available information. This equilibrium features the highest welfare and share price,

[21] The intuition we rely on in this discussion is similar to that in Grossman and Hart (1980) and Admati, Pfleiderer, and Zechner (1994), where the benefit of a bidder's (activist's) future value improvement is reflected in the price. The literature has proposed initial endowments (Shleifer and Vishny 1986; Admati, Pfleiderer, and Zechner 1994) and IPO underpricing (DeMarzo and Urosevic 2006) as ways to incentivize investors to engage in value improvement.

both because the firm's decision is most informed and because investors' total holding costs are minimized since the stock is evenly divided among them. Statement (ii) shows that when there are no passive investors ($L = 0$), this equilibrium can coexist with an equilibrium in which the manager's decision is not based on all the available information and total holding costs are larger. Intuitively, if only a subset of investors (optimists in our setting) are expected to become shareholders and provide advice to the manager, there are still ex post differences in beliefs between the manager and the shareholders. Anticipating this at the trading stage, investors who are less aligned with the manager (pessimists in our setting) do not buy shares in the first place, making this equilibrium self-fulfilling. Thus, in the presence of equilibrium multiplicity, there is yet another reason the presence of passive investors ($L > 0$) can enhance the effectiveness of shareholder communication. Since passive funds become shareholders regardless of whether their fund managers agree with the firm's CEO, their presence breaks the feedback loop between the ownership structure and managerial decision-making, and can eliminate the less efficient equilibrium.[22]

3.3 Nonbinding voting and the advisory role of the board

In this section, we discuss the implications of our analysis for two channels of shareholder communication with management that have been extensively explored in the empirical literature: nonbinding (i.e., advisory) shareholder voting and the company's board of directors.

3.3.1 When does nonbinding voting enhance managerial learning? Shareholders of the firm have different means of communicating with management. First, they can meet and engage with management directly. Second, they can join the company's board and express their views in board meetings. However, these channels of communication are only available to the largest shareholders, as it is not feasible and worthwhile for management to meet with all of the firm's investors, or to add all of them to the board. In this sense, advisory voting offers a low-cost way to collect the views of all of the shareholders. Thus, the Dodd-Frank requirement of regular advisory votes on executive compensation, as well as investors' ability to submit proposals for an advisory vote via Rule 14a-8, can be viewed as helping expand the set of shareholders who can communicate their views to management.

Both the mandatory say-on-pay requirement and Rule 14a-8 have been hotly debated be-

[22] The result that the equilibrium is unique for large L relies on the equilibrium selection criterion described above: the most informative equilibrium at the communication stage is Pareto efficient and hence is played.

cause of their potential downsides, such as the time and resources they may require from management and potential distractions they can cause. Thus, to judge the overall effects of these policies, it is important to understand the extent to which expanding the set of communicating shareholders enhances managerial learning. Our results suggest that whether managerial learning is substantially improved or not depends on the extent of disagreements in prior beliefs about the decision, as well as how much shareholders' and manager's preferences regarding the decision are aligned. If belief disagreements are large and preferences are relatively aligned (Δ is large relative to b), shareholders' communication decisions are complements. In this case, expanding the set of shareholders who can convey their views has an amplified positive effect. First, it allows communication by shareholders who would not be able to convey their views otherwise. Second, it may also have a spillover effect and encourage truthful communication by shareholders who could convey their views (e.g., those on the board) but would not do so truthfully because of strong belief disagreements with the manager.[23] In contrast, if belief disagreements over a decision are small or conflicts of interest are substantial (Δ is small relative to b), shareholders' communication decisions are substitutes. Part (*ii*) of Proposition 2 then shows that the information the manager can learn is limited. Thus, expanding the set of shareholders who can communicate with management, for example, through mandatory advisory voting on this decision, may not improve managerial learning at all, and the downsides of such votes may become first-order.

3.3.2 Advisory role of the board and board size. For large shareholders, joining the board of directors can be another way to communicate with managers, as advising the management is one of the most important functions of the board. For example, venture capitalists (VCs) and activist investors commonly take board seats (e.g., Field, Lowry, and Mkrtchyan 2013; Bebchuk et al. 2020). Moreover, VCs often assume a "board observer" role: they attend board meetings and offer their views, but do not have board voting rights.

Increasing board size to include more shareholders who can provide advice is not always beneficial, as it brings the problems of coordination and the costs of new directors' compensation. Our results suggest that adding more advisory directors is beneficial when differences

[23] Many decisions that involve large disagreements in beliefs are also the decisions that involve a large degree of uncertainty. In Section A.4 of the Internet Appendix, we show that a change in parameters that increases the variance of the state from the perspective of each investor (i.e., increases uncertainty), while keeping its expected value fixed, makes the IC condition (10) more likely to be satisfied. Intuitively, higher variance means relatively uninformative priors, inducing the manager to react more strongly to the shareholder's advice. This makes it more costly for the shareholder to misreport, inducing truthful communication.

in beliefs are substantial: in this case, the complementarity effect implies that a larger board improves managerial learning. However, if conflicts of interest are substantial, the substitution effect dominates, and a larger board is more likely to decrease value. Thus, our results have implications for the literature on board size (e.g., Yermack 1996; Coles, Daniel, and Naveen 2008; Jenter, Schmid, and Urban 2019), with the caveat that they are more first-order in situations where the board's primary role is to provide advice.

3.3.3 Empirical implications. Analyzing advisory voting and the advisory role of the board can help test our model. Our results imply that in the presence of differences in beliefs, a shareholder's (or director's) ability to influence the manager with his views is enhanced by the expertise of other shareholders (directors). However, as the preferences of the manager and the shareholders (directors) become less aligned, this effect weakens and is eventually reversed.

The empirical literature examines the effectiveness of the board's advisory role by studying how the presence of directors with a certain type of expertise is related to corporate policies and performance. For example, Dass et al. (2014) analyze directors' expertise in related industries; Güner, Malmendier, and Tate (2008) study financial expertise; and Harford and Schonlau (2013) focus on directors' experience in mergers and acquisitions. Our model can be tested in a similar way, but our unique prediction is that the advisory role of a director (i.e., whether his information influences the manager's decisions) should not be viewed in isolation, but depends on the expertise of other directors in the way described above. Another way to test our predictions is by studying managerial responsiveness to the advisory vote tally (as in Ertimur, Ferri, and Stubben 2010; Cuñat, Gine, and Guadalupe 2012; Ferri 2012) and analyzing how it varies with the firm's ownership structure (number and sophistication of the shareholders). To measure the extent of differences in beliefs, one could rely on several measures of belief heterogeneity proposed by the literature (e.g., Thakor and Whited 2011; Diether, Malloy, and Scherbina 2002; Malmendier and Tate 2005), whereas the misalignment in preferences between shareholders and management will vary with the firm's executive compensation structure and corporate governance characteristics.

4 Discussion and Robustness

In this section, we discuss the key assumptions of the model and their role for the results.

4.1 Information structure

To make the analysis tractable and derive simple, closed-form solutions, we make specific assumptions about the information structure. As we will discuss next, the complementarity and substitution effects in shareholders' communication decisions arise under many other information structures, although not all of them.

In particular, the property that drives the complementarity effect is that communication by other shareholders brings the manager's and shareholder's posterior beliefs closer to each other. Below we discuss the robustness of this property.

4.1.1 Heterogeneous interpretation of information. While in our model agents interpret information (i.e., signals θ_i) the same way, it is also natural to expect that they might interpret information differently. To explore this, in Section A.5 of the Internet Appendix, we follow models of differences of opinion in which agents disagree about the precision of signals (e.g., Banerjee, Kaniel, and Kremer 2009; Kyle, Obizhaeva, and Wang 2018) and assume that each shareholder overestimates the importance of his own signal. As we show, communication decisions are complements even though agents now interpret information differently. This is because for any given realization of the shareholder's own signal, communication by other shareholders still moves the manager's posterior belief closer to that of the shareholder's. This property holds in a large class of models of different beliefs, although not in all of them.

4.1.2 Mixed strategy equilibria. Our focus is on pure strategy equilibria, which simplifies the analysis by making communication of each shareholder either truthful or uninformative. Under mixed strategy equilibria, communication of each shareholder can be partially informative, making the model less tractable. In unreported results, we analyze a symmetric mixed strategy equilibrium in a setting with $b = 0$ and two investors, one optimist and one pessimist. We show that the IC constraint of a shareholder is more likely to be satisfied if the probability with which the other shareholder communicates truthfully increases, suggesting that shareholders' decisions are again complements. Intuitively, even if communication of other shareholders is only partially informative, it still brings the manager's and shareholder's beliefs closer to each other.

The property that drives the substitution effect is that each subsequent signal has a smaller effect on the action of the manager, and thus communication by other shareholders decreases the manager's reaction to the shareholder's message. This property holds in a large class of

models, but not in all of them. In particular:

4.1.3 Complementarity versus substitutability of signals. Börgers, Hernando-Veciana, and Krähmer (2013) introduce the notion of substitutability versus complementarity of signals and show that it may affect strategic interactions between agents. They call signals substitutes (complements) if the marginal impact of an additional signal on the agent's utility decreases (increases) in the number of signals.[24] In Section A.6 of the Internet Appendix, we show that in our model, signals θ_i are substitutes under this definition. Intuitively, this is because learning each additional signal leads to increasingly smaller updating of beliefs about φ. As Börgers, Hernando-Veciana, and Krähmer (2013) highlight, this property is not without loss of generality. However, the substitutability of signals is a very common feature in the literature and is natural in many applications, so we believe that our conclusions are applicable in many settings.

To see why the substitutability of signals may play a role, suppose that φ is a commonly known parameter (i.e., there is no learning about φ, unlike in the basic model). We consider this scenario in Section A.6 of the Internet Appendix and show that signals θ_i are then neither substitutes nor complements under the definition of Börgers, Hernando-Veciana, and Krähmer (2013): the extra benefit from an additional signal does not depend on the total number of signals received. As we also show, the substitution effect in communication decisions does not arise in this case because the manager's reaction to a shareholder's advice does not depend on how many other signals he learns. Thus, the substitution effect in communication is somewhat tied to the substitutability between agents' signals. An interesting implication of this result is an amplified beneficial effect of the diversity of expertise. If shareholders' expertise is diverse, in that they have information about different aspects of the decision (i.e., have unconditionally independent signals), then asking more shareholders for advice is useful for two reasons. First, the added value from an additional signal does not decline with the number of shareholders, and second, asking more shareholders for advice does not inhibit the communication of other shareholders.

[24] Formally, Börgers, Hernando-Veciana, and Krähmer (2013) define signals as substitutes (complements) if the decision-maker's added utility from having two signals relative to having one signal, assuming he takes the optimal action given these signals, is smaller (larger) than his added utility from having one signal relative to no signal at all. Despite the similarity in terminology, the two notions of complementarity/substitutability (in our paper and in Börgers, Hernando-Veciana, and Krähmer 2013) are very different: their paper focuses on complementarity/substitutability of *signals* in a *single* decision-maker's problem, whereas our paper studies complementarity/substitutability of *actions* (communicating truthfully or not) in a game with *multiple* players.

4.1.4 Multiple dimensions of expertise. The previous discussion suggests, more generally, that with multiple dimensions of expertise, the substitution effect is likely to be weaker than with a single dimension of expertise. Intuitively, when a shareholder's signal not only provides noisy information about some common underlying state but also provides information about a different, independent aspect of the decision, the manager is likely to react relatively strongly to the shareholder's advice, even if he receives advice from many other shareholders. As a result, the costs of misreporting do not decrease as much, weakening the substitution effect. We confirm this intuition in Section A.7 of the Internet Appendix, where we analyze a variation of our model with only one dimension of expertise: each shareholder receives a noisy signal about the state and has no independent expertise beyond that.[25] We show that both the complementarity effect and the substitution effect arise as well, as in the basic model. However, the substitution effect is stronger relative to the basic model, so that when $b = 0$, it offsets the complementarity effect. While the offsetting result is special to the Beta distribution, the intuition that the presence of multiple dimensions of expertise weakens the substitution effect is more general.

4.2 Communication protocols

4.2.1 Communication among shareholders. Our model assumes that shareholders do not communicate privately among themselves prior to communicating with management. In practice, limitations indeed hinder such communication. First, communication with other shareholders can be viewed as "forming a group," which could require the shareholders to file form 13D or could trigger a poison pill. According to the report by Dechert (2011), "shareholder concern about unintentionally forming a group has chilled communications among large holders of shares in U.S. public companies." Second, shareholders often avoid such communication as it could be considered by management as running an activist campaign and lead to managerial retaliation. In Section A.8 of the Internet Appendix, we partly relax this assumption by considering the following change of the communication stage: first, all shareholders of the same type (i.e., with the same prior beliefs) share their signals among themselves, and then, one representative of each group communicates with the manager via cheap talk. We show that the necessary and sufficient conditions for the existence of an equilibrium where all shareholders communicate truthfully are the same as in the basic model. Thus, the results of Proposition 2

[25] In contrast, in our basic model, each shareholder has both a noisy signal about the state (because θ_i provides noisy information about φ) and independent expertise beyond that (because conditional on φ, θ_i is independent of other shareholders' signals and is informative about the optimal decision).

continue to hold: an equilibrium with all shareholders communicating truthfully exists if and only if the shareholder base $|S|$ is large enough when b is small, and if and only if $|S|$ is small enough when Δ is small.

4.2.2 Sequential communication. Our model assumes simultaneous (or, equivalently, private sequential) communication by shareholders to the manager. Instead, some shareholders could publicly announce their views, so that other shareholders can update their beliefs before communicating with the manager themselves. In Section A.9 of the Internet Appendix, we consider a variation of the model in which shareholders send public messages in a known sequence to all other shareholders and the manager. We show that the IC conditions for truthful communication are the same as in the basic model, and hence for any sequence, the equilibrium at the communication stage is the same as in the model with simultaneous communication. Intuitively, this is because what matters for the shareholder's incentives is the combined (ex post) set of signals that the manager learns before taking his action, as this combined set of signals determines both the manager's reaction to the shareholder's advice and the congruence between the manager and the shareholder at the decision-making stage.

5 Conclusion

Shareholder engagement, that is, shareholders communicating their views about the firm's policies to management, has become increasingly important in recent years. This paper develops a theory of shareholder engagement to understand what affects managerial learning from shareholders, how it can be enhanced, and examine the role of ownership structure in particular.

We show that when shareholders and management have different beliefs, shareholders have incentives to misrepresent their information, which makes shareholder engagement less effective. However, since differences in beliefs decrease as more shareholders convey their views to management, the engagement of each individual shareholder is more effective when more other shareholders engage with management as well. This complementarity suggests an important benefit of a wide shareholder base: it can enhance the quality of shareholder communication with management. However, differences in beliefs naturally lead to a more limited shareholder base: investors whose views about the optimal strategy disagree with those of management have lower valuations and thus do not become shareholders in the first place. Thus, both communication frictions and a limited shareholder base inhibit managerial learning from shareholders,

and these inefficiencies amplify each other. We show that the presence of passively managed institutional investors, who become shareholders even if they disagree with management, can counteract these effects and enhance shareholder engagement.

Finally, we highlight that when shareholders and management also have different preferences, and these preference misalignments are substantial, shareholders' communication decisions become substitutes. As a consequence, the role of ownership structure for shareholder engagement becomes less important. These results suggest that introducing advisory voting and adding shareholders to the firm's board can significantly improve managerial learning for decisions involving differences in beliefs, but these policies are likely to be less effective for decisions involving large conflicts of interest.

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Appendix

Proof of Lemma 1

Since θ_i is a binary signal equal to one with probability φ or zero with probability $1-\varphi$, the manager's optimal action (6) can be written as

$$a_m(\theta_R) = b + \sum_{i\in R}\theta_i + \mathbb{E}_m\left[\varphi|\theta_i, i\in R\right](K-|R|). \tag{A1}$$

Let $\mathbf{1}_R \equiv \sum_{i\in R}\theta_i$ be the number of signals in R equal to one. The conditional probability that $\mathbf{1}_R$ signals out of $|R|$ are equal to one given φ is $P(\mathbf{1}_R|\varphi) = \binom{|R|}{\mathbf{1}_R}\varphi^{\mathbf{1}_R}(1-\varphi)^{|R|-\mathbf{1}_R}$. Since the prior distribution is Beta and the likelihood function is Binomial, the posterior distribution is also Beta but with different parameters (this is a known property of the Beta distribution). Formally, let $P_i(\mathbf{1}_R)$ be agent i's assessed probability that $\mathbf{1}_R$ signals out of $|R|$ are equal to one (over all possible values of φ). Using Bayes' rule, agent i's posterior belief of φ, $P_i(\varphi|\mathbf{1}_R)$, is

$$\begin{aligned} P_i(\varphi|\mathbf{1}_R) &= \frac{f_i(\varphi)P(\mathbf{1}_R|\varphi)}{P_i(\mathbf{1}_R)} = \frac{\varphi^{\rho_i-1}(1-\varphi)^{\tau-\rho_i-1}}{Beta(\rho_i,\tau-\rho_i)}\frac{1}{P_i(\mathbf{1}_R)}\binom{|R|}{\mathbf{1}_R}\varphi^{\mathbf{1}_R}(1-\varphi)^{|R|-\mathbf{1}_R} \\ &= \frac{1}{Beta(\rho_i,\tau-\rho_i)P_i(\mathbf{1}_R)}\binom{|R|}{\mathbf{1}_R}\times\varphi^{\rho_i+\mathbf{1}_R-1}(1-\varphi)^{\tau-\rho_i+|R|-\mathbf{1}_R-1}, \end{aligned} \tag{A2}$$

which is some constant that does not depend on φ times $\varphi^{\rho_i+\mathbf{1}_R-1}(1-\varphi)^{\tau-\rho_i+|R|-\mathbf{1}_R-1}$. Since the posterior beliefs must integrate to one over possible values of φ, this automatically implies that the posterior belief also follows a Beta distribution with parameters $(\rho_i+\mathbf{1}_R, \tau-\rho_i+|R|-\mathbf{1}_R)$ and density

$$P_i(\varphi|\mathbf{1}_R) = \frac{1}{Beta(\rho_i+\mathbf{1}_R,\tau-\rho_i+|R|-\mathbf{1}_R)}\varphi^{\rho_i+\mathbf{1}_R-1}(1-\varphi)^{\tau-\rho_i+|R|-\mathbf{1}_R-1}. \tag{A3}$$

It is known that the mean of a Beta distribution with parameters (α,β) is $\frac{\alpha}{\alpha+\beta}$. Therefore, using these expressions and the above posterior distribution, agent i's expected value of φ is $\mathbb{E}_i(\varphi|\mathbf{1}_R) = \frac{\rho_i+\mathbf{1}_R}{\tau+|R|}$, which proves the lemma.

Auxiliary Lemma A.1

Suppose $\varphi \sim Beta(\rho,\tau-\rho)$ and $X = \{x_1, x_2, \cdots, x_n\}$, where $x_i \in \{0,1\}$ are independent draws with $x_i = 1$ with probability φ. Let $\mathbf{1}_X \equiv \sum_{i=1}^n x_i$. Then

$$\mathbb{E}_X[\mathbf{1}_X] = n\frac{\rho}{\tau} \text{ and } \mathbb{E}_X[\mathbf{1}_X^2] = n\rho\,\frac{\tau-\rho+n(\rho+1)}{\tau(\tau+1)}. \tag{A4}$$

Proof. It is known that the first two moments of a random variable X distributed according to a Beta distribution with parameters α and β are $\mathbb{E}[X] = \frac{\alpha}{\alpha+\beta}$ and $\mathbb{E}[X^2] = \frac{\alpha(\alpha+1)}{(\alpha+\beta)(\alpha+\beta+1)}$. Hence, $\mathbb{E}[\varphi] = \frac{\rho}{\tau}$ and $\mathbb{E}[\varphi^2] = \frac{\rho(\rho+1)}{\tau(\tau+1)}$. Using this, we get

$$\begin{aligned}\mathbb{E}\left[1_X\right] &= \mathbb{E}\left[\sum_{i=1}^{n} x_i\right] = n\mathbb{E}\left[x_i\right] = n\mathbb{E}\left[\varphi\right] = n\frac{\rho}{\tau}; \qquad \text{(A5)}\\ \mathbb{E}\left[1_X^2\right] &= \mathbb{E}\left[\sum_{i=1}^{n} x_i^2 + \sum_{i\neq j} x_i x_j\right] = \mathbb{E}\left(n\mathbb{E}\left[x_i^2|\varphi\right] + n\left(n-1\right)\mathbb{E}\left[x_i|\varphi\right]^2\right)\\ &= n\mathbb{E}\left[\varphi\right] + n\left(n-1\right)\mathbb{E}\left[\varphi^2\right] = \frac{n\rho}{\tau\left(\tau+1\right)}\left(\tau-\rho+n\left(\rho+1\right)\right).\end{aligned}$$

Proof of Proposition 1

Plugging (7) and (8) into (9) gives

$$\begin{aligned}0 \geq \sum_{\theta_{-i}\in\{0,1\}^{K-1}} &\left[2\theta_i - 1 + \left(K-|R_i|-1\right)\cdot\frac{2\theta_i-1}{\tau+|R_i|+1}\right] \qquad \text{(A6)}\\ &\times\left[2b + \left(1-2\theta_i\right) - 2\sum_{j\in -R_i\setminus\{i\}}\theta_j + \frac{2(\rho_m+\mathbf{1}_{R_i})+1}{\tau+|R_i|+1}\left(K-|R_i|-1\right)\right]P_i(\theta_{-i}|\theta_i).\end{aligned}$$

Since the first multiple in each term equals $(2\theta_i-1)\frac{\tau+K}{\tau+|R_i|+1}$, this is equivalent to

$$0 \geq \left(2\theta_i-1\right)\sum_{\theta_{-i}} P_i(\theta_{-i}|\theta_i)\left(2b + \left(1-2\theta_i\right) - 2\sum_{j\in -R_i\setminus\{i\}}\theta_j + \frac{2(\rho_m+\mathbf{1}_{R_i})+1}{\tau+|R_i|+1}\left(K-|R_i|-1\right)\right). \qquad \text{(A7)}$$

Since $\sum_{\theta_{-R_i\setminus\{i\}}}\left(\sum_{j\in -R_i\setminus\{i\}}\theta_j\right)P_i(\theta_{-R_i\setminus\{i\}}|\theta_i,\theta_{R_i}) = \frac{\rho_i+\mathbf{1}_{R_i}+\theta_i}{\tau+|R_i|+1}\left(K-|R_i|-1\right)$, we can further simplify it to

$$\left(2\theta_i-1\right)\left[2b + \left(1-2\theta_i\right) + \frac{2(\rho_m-\rho_i)+1-2\theta_i}{\tau+|R_i|+1}\left(K-|R_i|-1\right)\right] \leq 0. \qquad \text{(A8)}$$

We consider two separate cases. If $\theta_i = 0$, the above inequality becomes

$$2b + 1 + \frac{2(\rho_m-\rho_i)+1}{\tau+|R_i|+1}\left(K-|R_i|-1\right) \geq 0, \qquad \text{(A9)}$$

and if $\theta_i = 1$, it becomes

$$2b - 1 + \frac{2(\rho_m-\rho_i)-1}{\tau+|R_i|+1}\left(K-|R_i|-1\right) \leq 0. \qquad \text{(A10)}$$

Together we get (10), which completes the proof.

Proof of Proposition 2

Notice that the IC constraint is more lax for optimists than for pessimists. This is because

$$\begin{aligned} \left|\left(2\rho+|R_i|+1\right)b-\left(K-|R_i|-1\right)\Delta\right| &\leq \left(2\rho+|R_i|+1\right)b+\left(K-|R_i|-1\right)\Delta \\ &= \left|\left(2\rho+|R_i|+1\right)b+\left(K-|R_i|-1\right)\Delta\right| \end{aligned} \tag{A11}$$

for any $b \geq 0$ and $\Delta \geq 0$. Given this, we next show that without loss of generality, the equilibrium in the communication subgame is as described in the statement of the proposition.

Consider a firm owned by S_o optimistic shareholders and S_p pessimistic shareholders. We show that if there is an equilibrium $\mathcal{E}$ in which n_o optimists and n_p pessimists communicate truthfully, then there must be a payoff-equivalent equilibrium $\mathcal{E}'$ in which $\min\{n_o+n_p, S_o\}$ optimistic shareholders and $\max\{0, n_o+n_p-S_o\}$ pessimistic shareholders communicate truthfully. Notice that the statement holds trivially if either $n_o = S_o$ or $n_p = 0$. Therefore, we consider the case $n_o < S_o$ and $n_p > 0$. The existence of equilibrium $\mathcal{E}$ implies that the pessimists' IC constraint (10) is satisfied for $|R_i| = n_o + n_p - 1$:

$$\left(2\rho+n_o+n_p\right)b+\left(K-n_o-n_p\right)\Delta \leq \rho+\frac{K}{2}. \tag{A12}$$

Consider equilibrium $\mathcal{E}'$ and show that both optimists and pessimists have incentives to communicate truthfully if the manager learns n_o+n_p-1 other signals. Since (10) only depends on R_i through $|R_i|$, then for any pessimist, his IC constraint in $\mathcal{E}'$ is the same as his IC constraint in $\mathcal{E}$ (i.e., (A12)), and thus holds. For any optimist, his IC constraint for $|R_i| = n_o + n_p - 1$ is satisfied as well because it is more lax than for pessimists, and the pessimists' IC constraint is satisfied for $|R_i| = n_o + n_p - 1$ given (A12). Thus, if $\mathcal{E}$ is an equilibrium, then $\mathcal{E}'$ is also an equilibrium. Note that the reverse is generally not true: for example, if $n_o + n_p < S_o$, then equilibrium E' requires only the IC constraint for the optimists to hold, while E requires the IC constraints for both optimists and pessimists to hold, and the latter may be violated. Finally, note that equilibria $\mathcal{E}$ and $\mathcal{E}'$ are payoff-equivalent, in the sense that the ex ante payoffs of all players (before they learn their signals) are the same in the two equilibria. This is because as shown in Lemma 2, the valuation of shares by each investor only depends on the set R of signals that were communicated through $|R|$.

Next, we prove the other statements of the proposition. Consider statement (i). If $b = 0$, the IC constraint (10) reduces to

$$\left(K-|R_i|-1\right)\Delta \leq \rho+\frac{K}{2}. \tag{A13}$$

This constraint becomes more lax as the set of shareholders that communicate truthfully expands. Thus, in the most informative equilibrium, either all shareholders communicate truthfully (which happens if $K-|S| \leq \frac{\rho+K/2}{\Delta}$) or no shareholder does (if $K-|S| > \frac{\rho+K/2}{\Delta}$). By continuity, the same is true for small enough $b > 0$ if the corresponding inequalities are satisfied strongly.

Next, consider statement (ii). If $\Delta = 0$, the IC constraint (10) reduces to

$$(2\rho + |R_i| + 1)\, b \leq \rho + \frac{K}{2}. \tag{A14}$$

If (A14) holds for $|R_i| = |S| - 1$, then the most informative equilibrium has all shareholders communicating truthfully. In particular, if it holds strongly, that is, $|S| < \frac{\rho + K/2}{b} - 2\rho$, then by continuity in Δ, all $|S|$ shareholders communicate truthfully for small enough $\Delta > 0$. If $|S| > \frac{\rho + K/2}{b} - 2\rho$, then given that (A14) becomes tighter as $|R_i|$ increases, the number of investors that communicate in the most informative equilibrium is one plus the highest $|R_i|$ for which (A14) is satisfied, that is, the floor of $\frac{\rho + K/2}{b} - 2\rho$, and by continuity, the same is true for small enough $\Delta > 0$. Taken together, this proves statement (ii).

Proof of Lemma 2

Let $\mathbf{1}_R = \sum_{i \in R} \theta_i$ denote the number of signals equal to one in set R. Using Lemma 1, we obtain agent i's ex ante payoff, $\mathbb{E}_i(a_m(\theta_R) - Z)^2$, as follows:

$$\mathbb{E}_i\left[U_i | R\right] = u_0 - b^2 - U_1 - U_2, \tag{A15}$$

where

$$\begin{aligned} U_1 &\equiv 2b\mathbb{E}_i\left[\left(\tfrac{\rho + \mathbf{1}_R}{\tau + |R|}\left(K - |R|\right) - \textstyle\sum_{j \in -R} \theta_j\right) | R\right], \\ U_2 &\equiv \mathbb{E}_i\left[\left(\tfrac{\rho + \mathbf{1}_R}{\tau + |R|}\left(K - |R|\right) - \textstyle\sum_{j \in -R} \theta_j\right)^2 | R\right]. \end{aligned} \tag{A16}$$

Using independence of θ_j conditional on φ, and Auxiliary Lemma A.1, U_1 simplifies to

$$U_1 = 2b\frac{\rho - \rho_i}{\tau + |R|}\left(K - |R|\right). \tag{A17}$$

To simplify U_2, we use the law of iterated expectations:

$$\begin{aligned} U_2 = \mathbb{E}_i\left[\left(\tfrac{(\rho + \mathbf{1}_R)(K - |R|)}{\tau + |R|}\right)^2 - 2\tfrac{(\rho + \mathbf{1}_R)(\rho_i + 1_R)(K - |R|)^2}{(\tau + |R|)^2} | R\right] \\ + \mathbb{E}_i\left[\mathbb{E}_i\left[\left(\textstyle\sum_{j \in -R} \theta_j\right)^2 | \theta_R, R\right] | R\right], \end{aligned} \tag{A18}$$

where we used $\mathbb{E}_i\left[\sum_{j \in -R} \theta_j | \theta_R, R\right] = (K - |R|)\,\mathbb{E}_i\left[\varphi | \theta_R, R\right] = \left((K - |R|)\right)\frac{\rho_i + 1_R}{\tau + |R|}$. Consider the last term under the expectation sign:

$$\begin{aligned} \mathbb{E}_i\left[\left(\textstyle\sum_{j \in -R} \theta_j\right)^2 | \theta_R, R\right] &= \mathbb{E}_i\left[\textstyle\sum_{j \in -R} Var_i\left[\theta_j | \varphi, R\right] + \varphi^2\left(K - |R|\right)^2 | \theta_R, R\right] \\ &= \mathbb{E}_i\left[\textstyle\sum_{j \in -R} \varphi\left(1 - \varphi\right) + \varphi^2\left(K - |R|\right)^2 | \theta_R, R\right] \\ &= \tfrac{\rho_i + 1_R}{\tau + |R|}\left(\left(K - |R|\right) + \left(\left(K - |R|\right)^2 - \left(K - |R|\right)\right)\tfrac{\rho_i + 1_R + 1}{\tau + |R| + 1}\right) \\ &= \tfrac{\rho_i + 1_R}{\tau + |R|}\left(K - |R|\right)\left(1 + \left(K - |R| - 1\right)\tfrac{\rho_i + 1_R + 1}{\tau + |R| + 1}\right), \end{aligned} \tag{A19}$$

where the second equality is due to $Var_i[\theta_j|\varphi, R] = \varphi(1-\varphi)$ and the third equality is due to the fact that the agent i's posterior distribution of φ conditional on θ_R is Beta with parameters $\rho_i + 1_R$ and $\tau + |R| - \rho_i - 1_R$, whose first and second moments are, respectively, $\frac{\rho_i+1_R}{\tau+|R|}$ and $\frac{(\rho_i+1_R)(\rho_i+1_R+1)}{(\tau+|R|)(\tau+|R|+1)}$. Plugging this expression into (A18) and simplifying using Auxiliary Lemma A.1, we get

$$\begin{aligned} U_2 - \left[\tfrac{\Delta(K-|R|)}{\tau+|R|}\right]^2 &= \mathbb{E}_i\left[\tfrac{(K-|R|)(\rho_i+1_R)}{\tau+|R|} - \left(\tfrac{(K-|R|)(\rho_i+1_R)}{\tau+|R|}\right)^2 |R\right] \\ &+ \left((K-|R|)^2 - (K-|R|)\right)\mathbb{E}_i\left[\tfrac{(\rho_i+1_R+1)(\rho_i+1_R)}{(\tau+|R|+1)(\tau+|R|)}|R\right] \\ &= \left(\tfrac{(K-|R|)^2}{\tau+|R|} + (K-|R|)\right)\mathbb{E}_i\left[\tfrac{(\rho_i+1_R)(\tau+|R|-\rho_i-1_R)}{(\tau+|R|)(\tau+|R|+1)}\right] \\ &= \left(\tfrac{(K-|R|)^2}{\tau+|R|} + (K-|R|)\right)\tfrac{\rho_i(\tau-\rho_i)}{\tau(\tau+1)} = (K-|R|)\tfrac{\tau+K}{\tau+|R|}\tfrac{\frac{\tau^2}{4}-\Delta^2}{\tau(\tau+1)}. \end{aligned} \tag{A20}$$

Combining with (A15) and (A17) gives (12).

Next, we study how the ex ante payoff $\mathbb{E}_i[U|R]$ of each agent depends on $|R|$. We denote $z = \frac{K-|R|}{\tau+|R|}$ and note that $\mathbb{E}_i[U|R] = u_0 - b^2 + u(z, \rho_i)$, where

$$u(z, \rho_i) = -2b(\frac{\tau}{2} - \rho_i)z - \frac{\left(\frac{\tau^2}{4} - \Delta^2\right)(\tau + K)}{\tau(\tau+1)}z - \Delta^2 z^2. \tag{A21}$$

Note that $\mathbb{E}_i[U|R]$ is increasing in $|R|$ if $u(z, \rho_i)$ is decreasing in $z \in \left[\frac{K-N}{\tau+N}, \frac{K}{\tau}\right]$. Differentiating with respect to z yields

$$u'(z, \rho_i) = -2b(\frac{\tau}{2} - \rho_i) - \frac{\left(\frac{\tau^2}{4} - \Delta^2\right)(\tau + K)}{\tau(\tau+1)} - 2z\Delta^2. \tag{A22}$$

For pessimists, $\rho_i = \frac{\tau}{2} - \Delta$, and $u'\left(z, \frac{\tau}{2} - \Delta\right) < 0$. Similarly, for the manager, $\rho_i = \frac{\tau}{2}$, and $u'\left(z, \frac{\tau}{2}\right) < 0$. For optimists, $\rho_i = \frac{\tau}{2} + \Delta$. Therefore,

$$u'\left(z, \frac{\tau}{2} + \Delta\right) = 2(b - z\Delta)\Delta - \frac{\left(\frac{\tau^2}{4} - \Delta^2\right)(\tau + K)}{\tau(\tau+1)} < 2b\Delta - \frac{\left(\frac{\tau^2}{4} - \Delta^2\right)(\tau + K)}{\tau(\tau+1)}. \tag{A23}$$

Thus, a sufficient condition for $u'(z, \rho_i) < 0$ for optimists is that $K \geq \bar{K}$, where $\bar{K} \equiv \frac{2b\Delta\tau(\tau+1)}{\frac{\tau^2}{4}-\Delta^2} - \tau$. Thus, if $K \geq \bar{K}$, then the ex ante payoff of any agent (any investor and the manager) is increasing in $|R|$.

A.1 Characterization of Equilibria in the Trading Game

Here, we characterize all possible equilibria in the trading game. Given that the demand function (13) is the same for all shareholders with the same belief (optimistic or pessimistic) and that it is strictly higher for optimists than pessimists (unless $K = N$ and $b = 0$, in which the two are equal), the equilibria take two possible forms:

1. **Both pessimistic and optimistic investors become shareholders**. Let R denote the subset of signals learned by the manager in the most informative equilibrium of the communication subgame as characterized by Proposition 2. Then (14) implies that the equilibrium share price is

$$p^* = \frac{N_o}{N}\mathbb{E}_o[U|R] + \frac{N_p}{N}\mathbb{E}_p[U|R] - \frac{\lambda}{N}, \tag{A24}$$

where $\mathbb{E}_o[U|R]$ and $\mathbb{E}_p[U|R]$ denote the valuations of the shares (12) for the optimists and pessimists, respectively. The existence condition for this equilibrium is that the price (A24) is weakly below the valuation of the shares by the pessimists. Using (12) and (A24), we get

$$\frac{\lambda}{N_o} \geq \mathbb{E}_o[U|R] - \mathbb{E}_p[U|R] = \frac{4b\Delta\left(K - |R|\right)}{2\rho + |R|}. \tag{A25}$$

2. **Only optimistic investors become shareholders, while pessimistic investors do not**. Let R denote the subset of signals learned by the manager in the most informative equilibrium of the communication subgame as characterized by Proposition 2; $|R|$ is the highest number in $[0, N_o]$ at which the IC constraint (10) for optimists is satisfied. Given that only optimists become shareholders, (14) implies that the equilibrium share price in this case is

$$p^* = \mathbb{E}_o[U|R] - \frac{\lambda}{N_o}. \tag{A26}$$

The existence condition for this equilibrium is that the price (A26) strictly exceeds the valuation of the shares by the pessimists. Using (12) and (A26), we get:

$$\frac{\lambda}{N_o} < \mathbb{E}_o[U|R] - \mathbb{E}_p[U|R] = \frac{4b\Delta\left(K - |R|\right)}{2\rho + |R|}. \tag{A27}$$

Within each of these two types of equilibria, the most informative equilibrium of the communication subgame could feature communication by either a strict subset of the shareholders or all shareholders. Next, we will characterize all possible cases.

a. $|S| = |R| = N$: **all investors become shareholders; all shareholders communicate truthfully.** This equilibrium exists if and only if (1) the IC constraint (10) is satisfied for a pessimist if he expects all other shareholders to communicate truthfully ($|R_i| = N - 1$), and (2) each investor prefers to become a shareholder given that he expects all shareholders to communicate truthfully, that is, (A25) holds for $|R| = N$:

$$\begin{aligned} (2\rho + N)\, b + (K - N)\, \Delta &\leq \rho + \frac{K}{2}, & \text{(A28)} \\ \frac{\lambda}{N_o} &\geq \frac{4b\Delta\left(K - N\right)}{2\rho + N}. & \text{(A29)} \end{aligned}$$

b. $|S| = N$, $|R| \in [N_o{+}1, N{-}1]$: **all investors become shareholders; all optimists and some, but not all, pessimists communicate truthfully.** This equilibrium exists if and only if (1) the IC constraint (10) for a pessimist is violated for $|R_i| = N - 1$, that is,

(A31), (2) the IC constraint (10) for a pessimist is satisfied for some $|R_i| \in [N_o, N-2]$, and (3) each investor prefers to become a shareholder given that he expects the manager to learn $|R_i|$ signals, that is, (A25) holds for such $|R_i|$. Note that (10) for a pessimist simplifies to

$$(2\rho + |R_i| + 1)\, b + (K - |R_i| - 1)\, \Delta \leq \rho + \frac{K}{2}. \tag{A30}$$

If $b < \Delta$, the left-hand side is decreasing in $|R_i|$. Thus, if this inequality is violated for $|R_i| = N-1$ (i.e., equilibrium with all investors communicating truthfully does not exist), it is also violated for any lower $|R_i|$ due to the complementarity effect in communication, so equilibrium with $|R| \in [N_o + 1, N-1]$ does not exist. If $b > \Delta$, the left-hand side is increasing in $|R_i|$. Hence, in this case, there exists $|R_i| \in [N_o, N-2]$ such that (A30) is satisfied for this $|R_i|$ if and only if (A30) is satisfied for $|R_i| = N_o$, that is, (A32). Finally, (A25) is the least restrictive when $|R|$ is the highest possible within the set of $|R| \in [N_o+1, N-1]$ for which it is incentive compatible for $|R|$ investors to communicate. Thus, the conditions for this type of equilibrium are:

$$(2\rho + N)\, b + (K - N)\, \Delta > \rho + \frac{K}{2}, \tag{A31}$$

$$(2\rho + N_o + 1)\, b + (K - N_o - 1)\, \Delta \leq \rho + \frac{K}{2}, \tag{A32}$$

$$\frac{\lambda}{N_o} \geq \frac{4b\Delta\,(K - |R|)}{2\rho + |R|}, \tag{A33}$$

where $|R|$ is the highest integer in $[N_o + 1, N-1]$ for which it is incentive compatible for $|R|$ investors to communicate, or equivalently, the lowest integer $|R_i|$ in $[N_o + 1, N-1]$ for which the IC condition for the pessimist (A30) stops holding.

c. $|S| = N$, $|R| = N_o$: **all investors become shareholders; all optimists but no pessimists communicate truthfully.** This equilibrium exists if and only if (1) the IC constraint (10) is satisfied for an optimist if he expects all other optimistic shareholders and no pessimistic shareholder to communicate truthfully ($|R_i| = N_o - 1$), that is, (A34), (2) the constraint (10) for a pessimist is violated for all $|R_i| \in [N_o, N-1]$, and (3) each investor prefers to become a shareholder given that he expects the manager to learn N_o signals, that is, (A25) holds for $|R| = N_o$, giving (A36). Since the left-hand side of (A30) increases (decreases) in $|R_i|$ if $b > \Delta$ ($b < \Delta$), the second condition holds if and only if (A30) is violated for $|R_i| = N_o$ if $b \geq \Delta$, and for $|R_i| = N-1$ if $b < \Delta$. Thus, the conditions for this equilibrium are

$$|(2\rho + N_o)\, b - (K - N_o)\, \Delta| \leq \rho + \frac{K}{2}, \tag{A34}$$

$$2\rho b + K\Delta + (b - \Delta)\left((N_o + 1)\, \mathbf{1}\{b \geq \Delta\} + N \mathbf{1}\{b < \Delta\}\right) > \rho + \frac{K}{2}, \tag{A35}$$

$$\frac{\lambda}{N_o} \geq \frac{4b\Delta\,(K - N_o)}{2\rho + N_o}. \tag{A36}$$

d. $|S| = N$, $|R| \in [0, N_o - 1]$: **all investors become shareholders; not all optimists communicate truthfully.** This equilibrium exists if and only if the IC constraint (10) is violated for an optimistic shareholder if he expects all other optimistic shareholders to communicate truthfully ($|R_i| = N_o - 1$) and if each investor prefers to become a shareholder given that he expects the manager to learn $|R|$ signals:

$$|(2\rho + N_o)\, b - (K - N_o)\, \Delta| > \rho + \frac{K}{2}, \tag{A37}$$

$$\frac{\lambda}{N_o} \geq \frac{4b\Delta\,(K - |R|)}{2\rho + |R|}, \tag{A38}$$

where $|R|$ is one plus the highest integer $|R_i|$ in $[0, N_o - 2]$ for which (10) for an optimist is satisfied.

e. $|S| = |R| = N_o$: **only optimists become shareholders; all shareholders communicate.** This equilibrium exists if and only if the IC constraint (10) is satisfied for an optimistic shareholder if he expects all other optimistic shareholders to communicate truthfully ($|R_i| = N_o - 1$) and if a pessimistic investor prefers to not become a shareholder under the equilibrium stock price (i.e., (A27) is satisfied for $|R| = N_o$):

$$|(2\rho + N_o)\, b - (K - N_o)\, \Delta| \leq \rho + \frac{K}{2}, \tag{A39}$$

$$\frac{\lambda}{N_o} < \frac{4b\Delta\,(K - N_o)}{2\rho + N_o}. \tag{A40}$$

f. $|S| = N$, $|R| \in [0, N_o - 1]$: **only optimists become shareholders; not all shareholders communicate.** This equilibrium exists if and only if the IC constraint (10) is violated for an optimistic shareholder if he expects all other optimistic shareholders to communicate truthfully ($|R_i| = N_o - 1$) and if a pessimistic investor prefers to not become a shareholder under the equilibrium stock price:

$$|(2\rho + N_o)\, b - (K - N_o)\, \Delta| > \rho + \frac{K}{2}, \tag{A41}$$

$$\frac{\lambda}{N_o} < \frac{4b\Delta\,(K - |R|)}{2\rho + |R|}, \tag{A42}$$

where $|R|$ is one plus the highest integer $|R_i|$ in $[0, N_o - 2]$ for which (10) for an optimist is satisfied.

Proof of Proposition 3

Applying (A25) for $|R| = N$, if all shareholders are expected to communicate information to the manager truthfully, then all investors choose to become shareholders if and only if $K - N \leq \frac{\lambda}{N_o} \frac{\rho + N/2}{2b\Delta}$. Using the fact that the IC condition for pessimists is more difficult to satisfy than for optimists, and applying (10) for $|R_i| = N - 1$ and $\rho_i = \rho - \Delta$, the equilibrium in which all shareholders communicate truthfully exists if and only if

$$K - N \leq \frac{\rho + K/2}{\Delta} - (2\rho + N)\frac{b}{\Delta}. \tag{A43}$$

Thus, for any $b < \frac{\rho+K/2}{2\rho+N}$, if $K - N \leq \min\left\{\frac{\lambda}{N_o}\frac{\rho+N/2}{2b\Delta}, \frac{\rho+K/2}{\Delta} - (2\rho+N)\frac{b}{\Delta}\right\}$, there exists an equilibrium in which all investors become shareholders and communicate information to the manager truthfully. Note that if $K = N$, this equilibrium exists if $b < \frac{1}{2}$. In this case, the manager's action is $a = b + Z$, and as follows from (12), both optimistic and pessimistic investors have the same valuation of shares. Hence, in equilibrium they acquire the same number of shares, $\frac{1}{N}$. Finally, when $b = 0$, the equilibrium achieves first-best: it features the same allocation of shares and corporate action as would be chosen by the social planner who maximizes the combined expected utility of all players.

Proof of Proposition 4

Notice that condition $(2\rho + N_o + 1)\, b + (K - N_o - 1)\,\Delta \leq \rho + \frac{K}{2}$ implies that if the shareholder base includes all investors, then the communication stage has an equilibrium in which all optimists and at least one pessimist communicate truthfully to the manager. This follows directly from (10) by plugging in $|R_i| = N_o$. Let $\hat{r}$ denote the number of signals communicated to the manager if all investors become shareholders (i.e., one plus the highest $|R_i|$ at which the IC constraint (10) for pessimistic investors is satisfied):

$$\hat{r} = \max_{|R|\in[N_o+1,N],|R|\in\mathbb{N}} \left\{ |R| : (2\rho + |R|)\, b + (K - |R|)\,\Delta \leq \rho + \frac{K}{2} \right\}. \tag{A44}$$

Then, using (A25), if

$$\lambda > \hat{\lambda} \equiv \frac{4b\Delta\,(K - \hat{r})\,N_o}{2\rho + \hat{r}}, \tag{A45}$$

then there exists an equilibrium in which all investors become shareholders and all optimistic shareholders and either some or all pessimistic shareholders communicate truthfully. Using (A24), the equilibrium stock price is

$$p^* = \frac{N_o}{N}\mathbb{E}_o[U|\hat{r}] + \frac{N_p}{N}\mathbb{E}_p[U|\hat{r}] - \frac{\lambda}{N}, \tag{A46}$$

where $\mathbb{E}_i[U|r]$ for an integer r denotes the valuation of investor i if the manager learns r signals in equilibrium (by (12)), investors' valuation only depends on R through $|R|$).

Next, consider $\lambda < \hat{\lambda}$. Since $\frac{4b\Delta(K-|R|)}{2\rho+|R|}$ is strictly decreasing in $|R|$, the fact that $\lambda < \hat{\lambda}$ implies that (A25) is violated for any R such that $|R| \leq \hat{r}$. Thus, no equilibrium in which all investors become shareholders exists. Hence, in equilibrium, only optimistic investors become shareholders and thus at most N_o shareholders communicate truthfully. Therefore, the ownership structure is less dispersed than for $\lambda > \hat{\lambda}$: if $\lambda < \hat{\lambda}$, each optimist holds $\frac{1}{N_o}$ shares and each pessimist holds zero shares, whereas if $\lambda > \hat{\lambda}$, each optimist holds fewer than $\frac{1}{N_o}$ shares and each pessimist holds a positive number of shares. Since $N_o < \hat{r}$, the manager's decision is less informed compared to $\lambda > \hat{\lambda}$, which is manifested in lower utility (12) from each investor's point of view. Using (A26), the equilibrium stock price is

$$p^* = \mathbb{E}_o[U|R] - \frac{\lambda}{N_o}, \tag{A47}$$

where R is the set of shareholders that communicate truthfully when only optimistic investors become shareholders.

Finally, we examine how the stock price depends on λ, as we increase it from zero. Note that λ does not enter the IC constraints of shareholders at the communication stage, so it affects the stock price only via the holding cost and via the ownership structure. Holding the ownership structure fixed, the stock price is decreasing in λ: both (A46) and (A47) are decreasing in λ. However, when λ crosses $\hat{\lambda}$ from below, the ownership structure changes from only optimistic investors becoming shareholders to all investors becoming shareholders. For a given price p, the demand for shares (13) of each investor i increases discontinuously due to a jump in $E_i[U|R]$ due to an increase in the number of signals that the manager learns. Hence, the market clearing price jumps up discontinuously at $\lambda = \hat{\lambda}$.

Proof of Proposition 5

Consider $\lambda < \hat{\lambda}$, where $\hat{\lambda}$ is defined by (A45). By the argument in the second paragraph of the proof of Proposition 4, in equilibrium, only optimistic investors become shareholders and thus at most N_o shareholders communicate truthfully. Let $\hat{r}_1 \in [0, N_o]$ the number of shareholders that communicate their signals truthfully in this case. The stock price is given by (A26):

$$p^* = \mathbb{E}_o[U|\hat{r}_1] - \frac{\lambda}{N_o}, \tag{A48}$$

where $\mathbb{E}_i[U|r]$ is investor i's valuation if the manager learns r signals in equilibrium.

Consider a model with $L > 0$ passive investors. We outline two potential cases: (1) only optimistic active investors become shareholders; (2) all $N - L$ active investors become shareholders. We will show that only the first case can arise in equilibrium given that $\lambda < \hat{\lambda}$.

Consider the first case. Then, the firm has $N_o + \frac{N_p}{N} L$ shareholders. Among them, N_o are optimistic and $\frac{N_p}{N} L$ are pessimistic. By assumption in the statement of the proposition,

$$(2\rho + N_o + 1)\, b + (K - N_o - 1)\, \Delta \leq \rho + \frac{K}{2}, \tag{A49}$$

and hence (10) implies that all optimistic shareholders and at least one pessimistic shareholder communicate truthfully. Therefore, the equilibrium number of signals communicated to the manager, $\hat{r}_2(L)$, is at least $N_o + 1$. Hence, the equilibrium features more informative communication (in the sense of a higher number of signals learned by the manager) and more informed corporate decision-making (in the sense of a higher expected utility (12) for each shareholder). Consider the share price. The demand from each of the $\frac{N-L}{N} N_o$ optimistic active investors is given by (13). The demand from each of the L passive investors is given by $\frac{1}{N}$. Hence, the market clearing condition is:

$$N_o \frac{N - L}{N} \left(\frac{\mathbb{E}_o[U|\hat{r}_2(L)] - p^*}{\lambda} \right) = 1 - \frac{L}{N}, \tag{A50}$$

which yields

$$p^* = \mathbb{E}_o[U|\hat{r}_2(L)] - \frac{\lambda}{N_o}. \tag{A51}$$

Since $\mathbb{E}_o[U|r]$ is strictly increasing in r, the equilibrium stock price with L passive investors, (A51), exceeds the equilibrium stock price without passive investors, (A51). Notice also that the presence and number of passive investors only affects the price by affecting how many signals the manager learns in equilibrium, but not by changing the residual supply of shares (because of the assumption that each passive investor demands $\frac{1}{N}$ shares): the price (A51) only depends on L through $\hat{r}_2(L)$ and coincides with the price (A26) without passive investors if $|R|$ is the same.

Consider the second case. Then, the firm has N shareholders, among them, N_o are optimistic and N_p are pessimistic. Given (A49), all optimists and at least some pessimists communicate truthfully, and the total number of signals communicated to the manager is given by $\hat{r}$, defined by (A44). Then, the market-clearing condition is:

$$\frac{N-L}{N}\left(N_o\frac{\mathbb{E}_o[U|\hat{r}]-p}{\lambda}+N_p\frac{\mathbb{E}_p[U|\hat{r}]-p}{\lambda}\right)=1-\frac{L}{N}, \tag{A52}$$

which yields

$$p^* = \frac{N_o}{N}\mathbb{E}_o[U|\hat{r}] + \frac{N_p}{N}\mathbb{E}_p[U|\hat{r}] - \frac{\lambda}{N}. \tag{A53}$$

Notice again that for a given number of signals learned by the manager, the price is not affected by L and is the same as in the model without passive investors. The existence condition for this equilibrium is that the price (A53) is weakly below the valuation of the shares by the pessimists. Using (12) and (A53), we get:

$$\lambda \geq \frac{4b\Delta(K-\hat{r})N_o}{2\rho+\hat{r}} = \hat{\lambda}, \tag{A54}$$

which contradicts the assumption $\lambda < \hat{\lambda}$. Hence, the second case is indeed not possible.

Finally, we examine comparative statics in L. Given L, the ownership structure has N_o optimistic shareholders and $\frac{N_p}{N}L$ pessimistic shareholders. The equilibrium number of signals communicated to the manager is given by

$$\hat{r}_2(L) = \max_{|R|\in\left[N_o+1,N_o+\frac{N_p}{N}L\right],|R|\in\mathbb{N}}\left\{|R| : (2\rho+|R|)b+(K-|R|)\Delta \leq \rho+\frac{K}{2}\right\}, \tag{A55}$$

that is, it is determined by the highest number of signals for which the IC constraint for a pessimistic shareholder is still satisfied. Given (A49), $\hat{r}_2(L) \geq N_o+1$. Notice that $\hat{r}_2(L)$ is weakly increasing in the number of passive investors L, and once it reaches $\hat{r}$, it stays constant at this level as L further increases. Since $\hat{r}_2(L)$ is weakly increasing in L and $\mathbb{E}_i[U|r]$ is increasing in r, the informativeness of decision-making (evaluated from the optimist's, the pessimist's, or the manager's point of view) is weakly increasing in L. As a consequence, the equilibrium stock price (A51) is also weakly increasing in L.

Proof of Proposition 6

First, consider the case without passive investors. Notice that conditions $\lambda < \frac{4b\Delta(K-N)N_o}{\tau+N}$ and $(K-N_o)\Delta - (2\rho+N_o)b > \rho + \frac{K}{2}$ imply (A41)-(A42), and thus the equilibrium without passive investors is such that only optimistic investors become shareholders and not all of them communicate truthfully.

Next, consider the case with $L > 0$ passive investors. Since $\lambda < \frac{4b\Delta(K-N)N_o}{\tau+N}$, we have

$$\lambda < \frac{4b\Delta\left(K-|R|\right)N_o}{\tau+|R|} \ \forall\, |R| \in [0,N]\,. \tag{A56}$$

As shown in the proof of Proposition 5, the price for a given $|R|$ is the same as in the model without passive investors, and since (A56) coincides with (A27), the price strictly exceeds the valuation of the shares by the pessimists. Hence, only optimistic investors among active investors become shareholders. Thus, the ownership structure consists of N_o optimistic shareholders ($N_o\frac{L}{N}$ passive and the rest active) and $N_p\frac{L}{N}$ pessimistic shareholders (all passive). Given (10), the condition that all $N_o + N_p\frac{L}{N}$ investors communicate truthfully is

$$\left(2\rho + N_o + N_p\frac{L}{N}\right)b + \left(K - N_o - N_p\frac{L}{N}\right)\Delta \leq \rho + \frac{K}{2}. \tag{A57}$$

Finally, we show that the inequalities in the statement of the proposition define a nonempty set of parameters. This is the case when

$$\begin{aligned}\left(2\rho + N_o + N_p\frac{L}{N}\right)b + \left(K - N_o - N_p\frac{L}{N}\right)\Delta \ &< \ (K-N_o)\Delta - (2\rho+N_o)b \qquad \text{(A58)}\\ &\Leftrightarrow \ \Delta > \left(\frac{2\left(2\rho+N_o\right)N}{N_pL}+1\right)b,\end{aligned}$$

which holds for a large enough Δ.

Proof of Proposition 7

We first note that the conditions in the statement of the proposition describe a nonempty set of parameters. For example, these conditions, as well as the assumption $K > \bar{K}$, are satisfied by choosing a sufficiently large K, N_o that is close to K, and sufficiently small b and λ. The existence of the first equilibrium follows from Proposition 3: $b < \frac{\rho+K/2}{2\rho+N}$ is equivalent to $b < \frac{1}{2}$ for $K = N$. Next, we will prove that under the additional conditions in the statement of the proposition, and if the number of passive investors $L = 0$, there exists an equilibrium where only optimists become shareholders and acquire a stake $\frac{1}{N_o}$ each, and they all truthfully communicate to the manager. First, note that if $|R| = N_0$ (i.e., the manager learns all optimists' signals), then (14) implies $p^* = \mathbb{E}_o[U|R] - \frac{\lambda}{N_o}$. The existence condition for the equilibrium in which only optimists become shareholders is that this equilibrium price strictly exceeds the value of the share by the pessimistic investor, that is, (A27). This condition is satisfied for $|R| = N_o$ by the assumption on λ in the statement of the proposition. Finally, we prove that

the IC constraint (10) holds for all optimists, that is,

$$|(2\rho + N_o)b - (K - N_o)\Delta| \leq \rho + \frac{K}{2}. \tag{A59}$$

Since $\frac{2\rho+N_o}{\Delta}b \leq K - N_o$ by the assumption in the statement of the proposition, this can be rewritten as

$$(K - N_o)\Delta - (2\rho + N_o)b \leq \rho + \frac{K}{2} \Leftrightarrow K - N_o \leq \frac{\rho + K/2}{\Delta} + \frac{2\rho + N_o}{\Delta}b, \tag{A60}$$

which is satisfied by the other assumption in the statement of the proposition, completing the proof. In contrast, if the number of passive investors L is large, this equilibrium does not exist. In particular, if $L = N$, all N investors are restricted to holding $\frac{1}{N}$ shares, so only the first equilibrium remains.

To prove the price and welfare comparisons between the two equilibria, recall that under our assumption that $K > \bar{K}$, $\mathbb{E}_i[U|R]$ is increasing in $|R|$ for every agent (by Lemma 2). Hence, both the manager's expected utility and each investor's valuation of the share, $\mathbb{E}_i[U|R]$, are higher in the first equilibrium than in the second. Let p_1 and p_2 denote the share price in the first and second equilibrium. Then (14) implies

$$p_1 = \frac{N_o}{N}\mathbb{E}_o[U|N] + \frac{N_p}{N}\mathbb{E}_p[U|N] - \frac{\lambda}{N}. \tag{A61}$$

Since pessimists hold shares, it must be that $p_1 < \mathbb{E}_p[U|N]$. Hence, (A61) implies

$$p_1 > \frac{N_o}{N}\mathbb{E}_o[U|N] + \frac{N_p}{N}p_1 - \frac{\lambda}{N} \Leftrightarrow p_1 > \mathbb{E}_o[U|N] - \frac{\lambda}{N_o}, \tag{A62}$$

so

$$p_2 = \mathbb{E}_o[U|N_o] - \frac{\lambda}{N_o} < \mathbb{E}_o[U|N] - \frac{\lambda}{N_o} < p_1, \tag{A63}$$

where the equality follows from (14), the first inequality follows from the fact that $\mathbb{E}_i[U|R]$ increases in $|R|$, and the second inequality is equivalent to (A62). Thus, the share price is higher in the first equilibrium. The combined utility of the initial owner and N investors is $\sum_{i=1}^{N}\alpha_i\mathbb{E}_i[U|R] - \frac{\lambda}{2}\sum_{i=1}^{N}\alpha_i^2$, which is higher in the first equilibrium:

$$\frac{N_o}{N}\mathbb{E}_o[U|N] + \frac{N_p}{N}\mathbb{E}_p[U|N] - \frac{\lambda}{2N} > \mathbb{E}_o[U|N_o] - \frac{\lambda}{2N_o} \Leftrightarrow p_1 - \frac{\lambda}{N} > p_2 - \frac{\lambda}{N_o}, \tag{A64}$$

which is satisfied because $p_1 > p_2$ and $N_o < N$. Thus, welfare (the combined utility of investors, original owner, and the manager) is also higher in the first equilibrium.

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