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Appointment Thresholds, Human Capital Investments, and Top Management Diversity

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Abstract

We study how systematic biases in recruitment affect top management diversity. Individuals differ in talent and invest in human capital before appointment. When appointment thresholds differ systematically across groups, a higher threshold lowers entry-level representation but can raise representation at the top. Intuitively, higher thresholds motivate more talented individuals to invest more, which generates a thicker upper tail of the ability distribution. The model challenges the "pipeline argument": a diversity initiative or an algorithmic bias correction that lowers the appointment threshold for a group raises its entry-level representation but can lower its representation at the top in meritocratic firms.

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Abstract

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1 Introduction

The representation of demographic groups within firms has received substantial attention and motivated initiatives aimed at correcting historical imbalances. Some of these imbalances can be attributed to biased appointment processes (Goldin and Rouse (2000), Bertrand and Duflo (2017)). In response, many firms have modified appointment criteria to account for individuals' demographic characteristics. A large scale study found that two-thirds of US companies have strategies for hiring under-represented groups, and 70% track gender representation in hiring.[1] Thus, the differential treatment of different groups in the appointment process is systematic, i.e. it is common to a large set of firms.

Systematic biases can originate from the diffusion of industry practices or the use of the same screening technologies, including recruitment algorithms. In turn, greater hiring of certain groups in entry-level positions can become self-reinforcing when recruitment processes rely on algorithms or artificial intelligence (AI), which are being adopted with increasing frequency (Köchling and Wehner (2020), Fabris et al. (2025)). Indeed, if an algorithm or AI learns that some type of candidate was more likely to be successfully hired in the past, it will tend to favor this type of candidate in the future. Moreover, it can be viewed as a black box: while the resulting outcomes are observable, the internal logic that generates them is typically opaque.[2] As a result, the differential appointment thresholds implicitly applied to different groups are effectively systematic but exogenous.[3]

This has implications for top management representation. In principle, increasing the proportion of employees from under-represented groups should ultimately improve the representation of these groups in top management positions. Indeed, companies often blame an

[1] Source: Women in the Workplace 2017, by McKinsey and Lean In.

[2] Proprietary recruitment algorithms often rely on complex models trained on large datasets, making it difficult to trace how candidate characteristics affect appointment probabilities (Raghavan et al. (2020), Cowgill and Tucker (2020)). Moreover, organizations rarely disclose the structure or training data of these algorithms, both for intellectual property reasons and due to the inherent complexity of modern machine-learning systems (Köchling and Wehner (2020)).

[3] The analysis in this paper applies to a systematic bias in either direction, since the results depend on the comparison of appointment thresholds across groups. Several studies document algorithmic biases at the expense of historically under-represented groups (Chen (2023)).

insufficient "pipeline" of suitably qualified candidates to justify a lack of senior level diversity.[4] For example, a shortage of suitable candidates explains the rarity of female CEOs (He and Whited (2026)). In this perspective, it is surprising that the increase in historically under-represented groups' representation in entry-level positions has not led to similar increases in upper-level representation (Fernandez-Mateo and Fernandez (2016)).[5] Larcker and Tayan (2020) note the lack of "tangible progress in increasing the prevalence of diverse executives in corporate leadership positions."

This paper develops a simple theoretical model to study how group-contingent appointment thresholds affect group representation in junior and senior positions. Unlike Coate and Loury (1993), individuals differ in talent, invest in human capital ("effort") to improve their ability, and are appointed to entry-level positions based on noisy estimates of their ability. The assumption of heterogeneous innate talent results in heterogeneous effort incentives and leads to novel predictions. We show that a lower appointment threshold for a particular group can increase group representation at the entry level while reducing it at the senior level. This challenges the common "pipeline argument" (Helfat et al., 2006), which would predict that greater representation at the junior level automatically translates into improved representation at the top. The model thus provides a potential explanation for observed patterns in which certain groups are relatively well represented at the entry level but remain under-represented in top management, even absent initial differences between groups.

We assume that a candidate will only be appointed to an entry-level position if her estimated ability exceeds a given "appointment threshold". In our model, there can be a different

[4] "One reason companies say they find it difficult to appoint more women to their boards is the lack of a "pipeline" of suitably qualified candidates." Source: *Financial Times*, Women still mind the gap on pay and power, July 14 2015.

[5] For example, in the US, women hold 47% of entry-level positions, but they account for only 37% of managers, 29% of Vice Presidents, 21% of C-suite leaders, and only 6% of Fortune 500 CEOs – a proportion that has been stable for years. Source: Women in the Workplace 2017, by McKinsey and Lean In. The survey included 222 companies employing more than 12 million people. Likewise, the *Financial Times* notes that "black employees have only seen significant growth in the most junior roles in finance" with a decline in senior role representation from 2007 to 2018. Source: *Financial Times*, Share of black employees in senior US finance roles falls despite diversity efforts, March 31 2021. Similarly, using data from US firms with at least 100 employees, Gee and Peck (2017) find that Asian Americans are the largest minority group among professionals but the least likely to be promoted to executive positions.

appointment threshold for each group. We consider systematic appointment biases, including algorithmic biases, over which firms have little control.[6] We study their implications for the probability that the firm CEO will belong to a particular group. To this end, we first assume that the firm will select its CEO among the subset of appointed individuals with a minimum level of ability. This minimum level of ability required is interpreted as the level of "meritocracy" in the firm. In our model, different firms can be more or less meritocratic. We later generalize this selection process by allowing factors other than ability, such as connections, to influence the choice of the CEO, so that the weight of ability in CEO selection is an additional measure of meritocracy.

As a first step, we study the implications of these appointment thresholds for human capital accumulation decisions optimally made by individuals. In our model, individuals are atomistic, so that their effort decisions depend on the appointment threshold to entry-level positions, not on the probability of being appointed CEO since it is equal to zero. For example, undergraduate students who implicitly make an effort decision when choosing their courses and their majors will consider the effects of these choices on the probability of getting a job at a prestigious firm, but they will not consider how they change the probability of being appointed CEO of that firm.[7]

A candidate will be especially motivated to exert "effort" by independently acquiring additional skills if she believes that enhancing her talent will have a sufficiently important effect on the appointment decision. At one end of the spectrum, individuals who are very talented will view such effort as superfluous, because they expect their estimated ability to be above the threshold regardless. At the other end of the spectrum, untalented individuals

[6]The model specifies an objective function for the firm, which determines the appointment threshold applied in the absence of bias. When all groups have the same talent distribution, this threshold is the same for all groups, so that group-contingent thresholds correspond to deviations from this benchmark caused by systematic biases.

[7]More generally, the effort decision in the model represents human capital investments made before entry into the labor market, such as the choice of a field of study or of professional certifications. At the time of these investments, the probability of reaching a specific top position at a specific firm is negligible. Introducing career concerns after appointment would reinforce effort incentives for individuals who expect to be appointed, without removing the effect of the appointment threshold on the effort decisions of marginal candidates, which drives our results.

will think that effort is useless, because they expect their estimated ability to be below the threshold regardless. Thus, only individuals who believe that their talent is close to the appointment threshold will be motivated to enhance it by exerting effort. This mechanism is consistent with the empirical evidence in Huesmann et al. (2026).

Consequently, as in Coate and Loury (1993) and the large literature that followed, raising the threshold can either increase or decrease human capital investment by an individual.[8] In our model, raising the threshold increases the probability that a given individual will exert effort if and only if the threshold is below a certain level. Furthermore, this level is increasing in the individual's talent: talented individuals are best motivated by high appointment thresholds, i.e., when it is hard to be appointed to entry-level positions.

We use this framework to study the implications of group-contingent appointment thresholds for entry-level appointments and CEO selection. First of all, we consider the case when information is perfect and symmetric: the ability of any individual is common knowledge, as in Krishna and Tarasov (2016). Our first main result is that, in any given group, the measure of appointed individuals with ability high enough to be CEO is nondecreasing in the appointment threshold for this group – as long as the threshold is lower than the minimum level of ability required to be CEO. Thus, in firms where CEO selection is highly meritocratic, i.e. firms where this minimum level is high, the CEO is more likely to be from a group with a relatively high appointment threshold, even though the measure of appointed individuals from this group will be relatively low.

Intuitively, in a group that faces a relatively high appointment threshold, only talented individuals will exert effort to improve their ability. These individuals will then be very able, since they are initially talented and they also improved their human capital. By contrast, in a group that faces a relatively low appointment threshold, only untalented individuals will exert effort to improve their ability. Many individuals from this group will be appointed, but most

[8] Coate and Loury (1993) show that changing the threshold affects individual's effort decisions. In their model, however, individuals do not have heterogeneous "talent": only their effort decision ("investment") matters for employability purposes. This assumption is widely used in the literature that followed (e.g. Moro and Norman (2003, 2004), Fryer and Loury (2013)).

will not be very able.

Next, we consider the case when individuals receive noisy signals about their talent before making their effort decisions, and are appointed based on their imperfectly measured ability. To isolate the effect of each information friction, we first study each friction separately, and we establish two analytical results. When individuals are uncertain about their own talent but screening is perfect, the measure of appointed individuals who are eligible for the CEO position reaches its maximum at a finite appointment threshold: a very high threshold discourages effort, including effort by talented individuals. When individuals know their talent but screening is noisy, a strictly positive measure of individuals with ability above the CEO eligibility level is rejected at any threshold. The first friction creates a discouragement effect, and the second friction creates a rejection effect. We then combine both frictions.

First, we derive numerically the measure of appointed individuals from a group who are eligible for the CEO position as a function of the appointment threshold for this group, for several values of the minimum level of ability necessary to be eligible. We find that this measure is non-monotonic in the appointment threshold, and reaches its maximum for an interior value of the appointment threshold. Intuitively, only talented individuals have a chance to be selected as CEO, and setting the threshold either too low or too high discourages them from exerting effort.

Second, we compute numerically the appointment threshold that maximizes this measure. We find that this threshold is an increasing function of the level of meritocracy. Intuitively, highly meritocratic firms will only consider individuals with very high ability for the CEO position. Very few individuals from any group will have the minimum ability required on the basis of talent only. Moreover, when individuals are (imperfectly) informed about their own talent, a high appointment threshold tends to elicit effort among highly talented individuals.

Third, we assume that the population is equally divided into two groups with the same talent distribution, and we compute numerically the probability that the firm CEO will be from one group or the other. As long as appointment thresholds are not too high, the CEO is

most likely to be from the group with the highest appointment threshold, which will also be under-represented in entry-level jobs. The more meritocratic the firm, the larger the subset of appointment thresholds such that this result holds. On the contrary, in firms which are not very meritocratic, this tradeoff between group representation at the junior and senior level may not arise. Then, a bias against a group may simply reduce its representation both at the junior and senior levels.

Our main prediction is that applying different appointment thresholds to different groups will lead to opposite effects on group representation at the junior and senior level when the selection of senior management is highly meritocratic. This is in contrast to a naïve interpretation of the pipeline argument, which would predict that group representation at the senior level is facilitated by enhancing group representation at more junior levels.

In practice, well-governed large firms are likely to be meritocratic. When CEOs have a multiplicative effect on firm value (Gabaix and Landier (2008)), even a small reduction in managerial ability can destroy billions of dollars in value for a large firm. Of course, this does not imply that all large firms are meritocratic. First, those with weak governance and severe agency problems may still fail to act in shareholders' interests (Kaplan and Minton (2012), Pagano and Picariello (2025)). Second, family firms may prioritize control over profitability (Burkart, Panunzi, and Shleifer (2003)). Overall, large non-family firms with effective governance mechanisms tend to operate meritocratically, although there are significant differences across countries (Bandiera et al., 2024).

In summary, when appointment thresholds remain moderate, the model predicts that large, well-governed firms will be more likely to be led by individuals from groups that face mild but systematic discrimination in recruitment processes. In contrast, it predicts that less meritocratic or poorly governed firms exhibit the opposite pattern. To the extent that some groups tend to receive favorable treatment at the entry level, this prediction is consistent with their persistent under-representation at the top of meritocratic firms. Thus, the analysis reveals a potential unintended consequence of entry-level policies that lower the appointment

threshold for a group.

2 Related literature

This paper contributes to a large literature on management diversity (Adams and Ferreira (2009), Nielsen (2010), Matsa and Miller (2011), Neely et al. (2020)) with a focus on top management appointments. For brevity and simplicity, it abstracts from several important factors that affect diversity in top management appointments (Smith et al. (2013), Cook and Glass (2014), Dezsö et al. (2016)). It is also not the first to highlight limitations of the pipeline argument (Castleman and Allen (1998), Cannady et al. (2014)), although to our knowledge it is the first to highlight the limitation based on human capital investments that we study. This limitation arises from the two-tier structure of the model: a change in the appointment threshold at the entry level affects group representation at the CEO level via the effort incentives of talented individuals, so that it can have the opposite effect at these two different levels. This paper is also related to the large economics literature on diversity policies (Fang and Moro (2011), Fryer and Loury (2013), Krishna and Tarasov (2016), Schaede and Mankki (2026)), including the literature that focuses on the private sector (Coate and Loury (1993), Peck (2017)). Finally, it is related to the large literature on human capital accumulation, especially in a context when groups are treated differently (Cotton et al. (2026)). Sethi and Somanathan (2023) and Chowdhury, Esteve-González, and Mukherjee (2023) review this literature.

Our theoretical model builds on a literature that follows Coate and Loury (1993) and which is surveyed by Fang and Moro (2011). Coate and Loury (1993) and many papers that followed, including Fryer (2007), do not assume heterogeneous talent. In these models, changes in the threshold do not have different implications for individuals with different talent – which is crucial in our analysis. Moreover, as opposed to Coate and Loury (1993), Fryer (2007), and other models of "statistical discrimination", the mechanism at play in our model

does not involve multiple equilibria based on self-fulfilling beliefs that result in statistical discrimination.

We now discuss other closely related papers. Krishna and Tarasov (2016) focus on the propensity of individuals with different ability to exert effort to pass an admission exam, and study excessive effort due to the "rat race." As in our model, innate talent is identically distributed across groups; however, unlike our model, individuals in different groups vary in their capacity to improve their ability. In their model, selection occurs at a single stage. In our model, the tradeoff between group representation at the entry level and at the CEO level results from the combination of an appointment threshold and a subsequent ability cutoff. In a model of search, Fershtman and Pavan (2021) assume that the evaluation of minority candidates is noisier than that of non-minorities, and show that this can make soft affirmative action policies counterproductive. Blair and Chung (2021) study the value of a signal of ability ("occupational licensing") in a model with workers with heterogeneous ability when the cost of obtaining this signal varies across groups. Wages are determined by firms that compete for workers and use observable licensing to update their beliefs about a worker's ability, similar to Spence (1973). Sethi and Somanathan (2018) study a setting where individuals with heterogeneous ability can enhance their ability through observable effort ("training"). When different groups do not have the same access to training opportunities, they show that threshold policies are not necessarily meritocratic.

In the models of Pikulina and Ferreira (2026) and Ferreira, Nikolowa, and Pikulina (2025), intrinsically biased employers can discriminate between equally qualified candidates. In these models, a candidate can be either skilled or unskilled, whereas two different candidates will not be equally qualified in our model. Pikulina and Ferreira (2026) study the consequences for human capital accumulation decisions across groups, and show that the bias can lead to either overinvestment or underinvestment depending on the stakes. Ferreira, Nikolowa, and Pikulina (2025) study the consequences for equilibrium outcomes, including labor market segmentation, firm profitability and economic efficiency.

3 The model

Each individual belongs to an identifiable group, denoted by k. An individual i has "talent" a_i.[9] It is drawn from the distribution $\tilde{a}_i \sim \mathcal{N}(\mu_k, \sigma_k^2)$. Thus, the parameters μ_k and σ_k^2 are respectively the mean and the variance of talent within group k. We assume that all groups have the same average talent, and we normalize $\mu_k = 0 \ \forall k$. An individual privately receives an imperfectly informative signal s_i of her talent a_i: given talent a_i, the signal is $\tilde{s}_i = a_i + \tilde{\epsilon}_i^s$, where $\tilde{\epsilon}_i^s \sim \mathcal{N}(0, \sigma_s^2)$. Let s_i refer to the realization of the signal observed by individual i.

After observing signal s_i, each individual i can exert unobservable effort e_i to improve her ability. As in Coate and Loury (1993) and Krishna and Tarasov (2016), this effort captures human capital accumulation decisions privately made by individuals such as acquiring additional skills. Individual i chooses $e_i \in \{0, \overline{e}\}$ at cost $C(e_i)$, with $C(0) = 0$ and $C(\overline{e}) = c \in (0, 1)$. We henceforth distinguish between an individual's talent, a_i, and ability $a_i + e_i$.[10]

Appointments are based on noisy nonverifiable signals of ability. The signal on individual i's ability is $\tilde{q}_i = a_i + e_i + \tilde{\epsilon}_i^q$, where $\tilde{\epsilon}_i^q \sim \mathcal{N}(0, \sigma_q^2)$. An individual i from group k is appointed if and only if $q_i \geq t_k$. This assumption is standard following the seminal paper of Coate and Loury (1993). Thus, the appointment threshold can be contingent on the individual's group, and the threshold for group k is denoted by t_k. We are studying the impact of exogenous systematic biases, including algorithmic biases (Chen (2023)), on effort and appointment decisions.[11]

Individual i derives private benefits B from being appointed, which we normalize at $B = 1$,

[9] We use the standard definition of "talent" as natural aptitude or skill.

[10] Thus, talent and effort are perfect substitutes. In the model of Coate and Loury (1993), by contrast, an individual is "qualified" only if she has exerted effort.

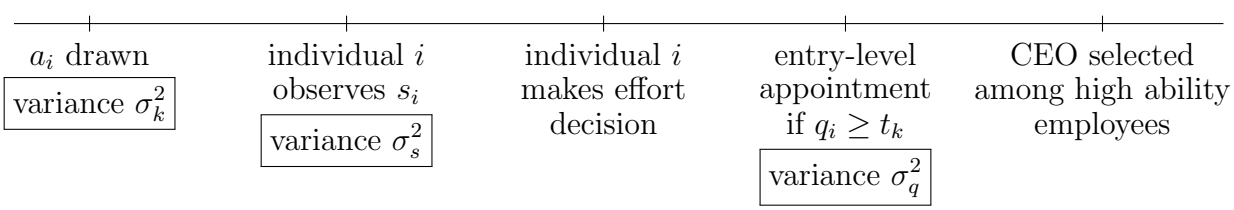
[11] Algorithmic decision-making in hiring is often a black box: while the resulting outcomes are observable, the underlying decision rules are typically opaque and proprietary (Raghavan et al. (2020), Cowgill and Tucker (2020)). Accordingly, we treat group-dependent appointment thresholds as systematic but exogenous (Köchling and Wehner (2020), Fabris et al. (2025)). It is also noteworthy that idiosyncratic biases, which are specific to a single organization, would not have the same effect on individual effort decisions.

and chooses effort e_i to maximize:[12]

$$\mathbb{P}\left(\tilde{q}_i \geq t_k | s_i\right) - C(e_i) \tag{1}$$

The firm's CEO will be selected among all appointed individuals with ability greater than a target level $\bar{a}$: even though CEO appointments involve many different factors outside the scope of this paper, only high-ability individuals are considered for this position.[13] This is consistent with the notion that a firm can only get noisy signals about the ability of outsiders (which results in noisy appointments to entry-level positions), but can learn the ability of its employees with high potential. Indeed, the empirical evidence suggests that large firms mostly rely on insiders for CEO appointments because of information frictions (Cziraki and Jenter (2022)). Section 5.5 generalizes this selection process by allowing factors other than ability to influence the choice of the CEO.

We normalize the measure of individuals in each group to a continuum $[0, 1]$. The timeline of the model for each individual i from group k is as follows:



In the model of Coate and Loury (1993), there is no heterogeneity in "talent" among individuals ($\sigma_k = 0$). In the model of Krishna and Tarasov (2016), individuals know their talent ($\sigma_s = 0$) and individual performance reveals their ability ($\sigma_q = 0$). Our results in section 5.1 involve a similar setting. Sections 5.2 and 5.3 introduce individual uncertainty about one's talent ($\sigma_s > 0$) and imperfect screening ($\sigma_q > 0$), and section 5.4 combines both.

The appointment thresholds set by various organizations depend on many considerations outside the scope of this paper, including the need to fill positions, the importance of ability in

[12] As we make clear later, only the ratio c/B matters for effort decisions.

[13] Selecting the appointed individual with the highest ability, as a firm with a finite number of employees would do, would place even more weight on the right tail of the ability distribution of appointed individuals. Effort incentives matter especially for this part of the distribution.

junior-level positions, and the composition of the available talent pool. Accordingly, the analysis below takes appointment thresholds as given. A bias is defined relative to the threshold that would be applied in its absence, consistently with the empirical literature that identifies discrimination from the differential treatment of otherwise identical candidates (Goldin and Rouse (2000), Bertrand and Duflo (2017)). To specify this benchmark, suppose that an appointed individual generates output equal to her ability, and is paid a wage w by the firm. The firm would then set the threshold t to maximize $W(t) = \mathbb{E}\left[(\tilde{a}_i + e_i - w)\,\mathbf{1}_{\tilde{q}_i \geq t}\right]$, where $\mathbf{1}_{\cdot}$ is the indicator function and effort decisions follow Lemma 1. We denote by t^{**} a threshold that maximizes this objective function.[14] The specification of W is only used to define the benchmark t^{**}.

More importantly, since all groups have the same talent distribution, the threshold that maximizes the firm's objective is the same for every group: in the absence of bias, the firm would apply a uniform threshold t^{**} to all candidates. Group-contingent thresholds can then be written as $t_k = t^{**} + b_k$, where b_k is a systematic bias in the screening technology, such as an algorithmic bias. In this perspective, the differential treatment of groups does not result from optimization by the firm: the bias b_k is not chosen by the firm – the opacity of recruitment algorithms limits its ability to measure and correct this bias. The analysis below describes the consequences of the bias for group representation at the entry level and at the CEO level.

4 Effort decision and appointments

4.1 Effort decision

We study the effort decision for a given appointment threshold. With a continuum of candidates, the probability for any individual to be appointed CEO is zero. Therefore, an individual does not consider a potential CEO appointment when she chooses her effort. Individuals update their beliefs after observing a signal of their talent. Individual i from group k's

[14] Under the parameter values used in section 5.4, numerical computations give $t^{**} = 1.15$ for $w = 1$, $t^{**} = 0.82$ for $w = 0.5$, and $t^{**} = 1.66$ for $w = 1.5$.

updated belief about her talent is normally distributed with the following mean and variance:

$$\mu_{a_i|s_i} \equiv \mathbb{E}\left[\tilde{a}_i|s_i\right] = \mu_k + \frac{cov(\tilde{a}_i, \tilde{s}_i)}{var(\tilde{s}_i)}\left[s_i - \mathbb{E}[\tilde{s}_i]\right] = \mu_k + \frac{\sigma_k^2}{\sigma_k^2 + \sigma_s^2}\left[s_i - \mu_k\right] \quad (2)$$

$$\sigma_{a_i|s_i}^2 \equiv var(\tilde{a}_i|s_i) = \sigma_k^2 - \frac{\left(cov(\tilde{a}_i, \tilde{s}_i)\right)^2}{var(\tilde{s}_i)} = \frac{\sigma_k^2\sigma_s^2}{\sigma_k^2 + \sigma_s^2} \quad (3)$$

Conditional on s_i and e_i, the individual's performance $\tilde{q}_i$ is normally distributed with mean $e_i + \mu_k + \frac{\sigma_k^2}{\sigma_k^2+\sigma_s^2}\left[s_i - \mu_k\right]$, and variance $\Sigma^2 \equiv \frac{\sigma_k^2\sigma_s^2}{\sigma_k^2+\sigma_s^2} + \sigma_q^2$. Denoting by Φ the standard normal CDF, let s^* be the solution to:

$$\Phi\left(\Sigma^{-1}\left(t_k - \mu_k - \frac{\sigma_k^2}{\sigma_k^2+\sigma_s^2}\left[s^* - \mu_k\right]\right)\right) - \Phi\left(\Sigma^{-1}\left(t_k - \overline{e} - \mu_k - \frac{\sigma_k^2}{\sigma_k^2+\sigma_s^2}\left[s^* - \mu_k\right]\right)\right) = c. \quad (4)$$

The LHS of equation (4) reaches its maximum, $2\Phi\left(\frac{\overline{e}}{2\Sigma}\right) - 1$, when the expected performance without effort, $\mu_k + \frac{\sigma_k^2}{\sigma_k^2+\sigma_s^2}\left[s^* - \mu_k\right]$, is equal to $t_k - \frac{\overline{e}}{2}$ (see the proof of Lemma 1). We assume that $c < 2\Phi\left(\frac{\overline{e}}{2\Sigma}\right) - 1$, a condition which does not depend on t_k, so that equation (4) has two solutions s^* (otherwise no individual would exert effort). We denote these solutions by $\underline{s}(t_k)$ and $\overline{s}(t_k)$, with $\underline{s}(t_k) < \overline{s}(t_k)$, and we refer to them as the "effort bounds."

Lemma 1 *An individual i exerts effort if and only if her private signal $s_i \in [\underline{s}(t_k), \overline{s}(t_k)]$, and the sensitivity of these effort bounds to t_k is equal to $\frac{\sigma_k^2+\sigma_s^2}{\sigma_k^2}$.*

Lemma 1 shows that the decision to exert effort is nonmonotonic in the signal that each individual receives about her talent. An individual who receives a very high signal ($s_i > \overline{s}$) is very confident about her ability to exceed the threshold, even without effort, while an individual who receives a very low signal ($s_i < \underline{s}$) believes that she will not exceed the threshold even if she exerts effort, so she does not. On the contrary, for individuals who receive signals within the bounds ($s_i \in [\underline{s}, \overline{s}]$), exerting effort has a sufficiently important effect on the likelihood to exceed the threshold, i.e., returns to effort are high, and these individuals optimally exert effort. Lemma 1 also shows how moving the threshold changes effort decisions: the effort bounds are linearly increasing in the appointment threshold t_k.

4.2 Appointments

We now study the appointment of individuals from group k given the appointment threshold t_k.

We take into account individuals' effort decisions, as derived in Lemma 1. Letting ϕ be the PDF of the signal distribution conditional on talent, the probability that an individual with talent a_i exceeds the threshold and gets appointed is:

$$\begin{aligned}
\mathbb{P}\left(\tilde{q}_i \geq t_k | a_i\right) &= \int_{-\infty}^{\infty} \mathbb{P}\left(a_i + e_i + \tilde{\epsilon}_i^q \geq t_k | a_i, s_i\right) \phi(s_i|a_i) ds_i \\
&= \int_{-\infty}^{\underline{s}(0)+\frac{\sigma_k^2+\sigma_s^2}{\sigma_k^2}t_k} \left(1 - \Phi\left(\frac{1}{\sigma_q}\left(t_k - a_i\right)\right)\right) \phi(s_i|a_i) ds_i \\
&\quad + \int_{\underline{s}(0)+\frac{\sigma_k^2+\sigma_s^2}{\sigma_k^2}t_k}^{\overline{s}(0)+\frac{\sigma_k^2+\sigma_s^2}{\sigma_k^2}t_k} \left(1 - \Phi\left(\frac{1}{\sigma_q}\left(t_k - a_i - \overline{e}\right)\right)\right) \phi(s_i|a_i) ds_i \\
&\quad + \int_{\overline{s}(0)+\frac{\sigma_k^2+\sigma_s^2}{\sigma_k^2}t_k}^{\infty} \left(1 - \Phi\left(\frac{1}{\sigma_q}\left(t_k - a_i\right)\right)\right) \phi(s_i|a_i) ds_i \qquad (5)
\end{aligned}$$

An increase in the threshold has two effects on the probability of appointment in equation (5). The first is a direct effect, which represents the change in the probability that performance exceeds the threshold for a given effort. This direct effect is strictly negative when $\sigma_q > 0$: an increase in the threshold lowers the probability that performance exceeds the threshold, ceteris paribus. The second is an incentive effect, which represents the change in the probability that performance exceeds the threshold due to a change in the effort decision. Formally:

$$\begin{aligned}
\frac{d}{dt_k}\mathbb{P}\left(\tilde{q}_i \geq t_k | a_i\right) &= \underbrace{\int_{-\infty}^{\infty} \frac{\partial}{\partial t_k}\mathbb{P}\left(a_i + e_i + \tilde{\epsilon}_i^q \geq t_k | a_i, s_i\right) \phi(s_i|a_i) ds_i}_{\text{direct effect}} \\
&+ \underbrace{\frac{\sigma_k^2+\sigma_s^2}{\sigma_k^2}\left[\Phi\left(\frac{t_k - a_i}{\sigma_q}\right) - \Phi\left(\frac{t_k - a_i - \overline{e}}{\sigma_q}\right)\right]\left[\phi\left(\overline{s}(t_k)|a_i\right) - \phi\left(\underline{s}(t_k)|a_i\right)\right]}_{\text{incentive effect}}, \qquad (6)
\end{aligned}$$

where the incentive effect follows from the Leibniz rule applied to the bounds of integration in equation (5), using $\frac{\partial \underline{s}(t_k)}{\partial t_k} = \frac{\partial \overline{s}(t_k)}{\partial t_k} = \frac{\sigma_k^2+\sigma_s^2}{\sigma_k^2}$ from Lemma 1. The first bracket in the incentive

effect is the increase in the probability of appointment due to effort, and the second bracket is the difference between the conditional densities of the signal at the two effort bounds.

This incentive effect can be either positive or negative. Proposition 1 studies its sign, and also shows how the level of the threshold that maximizes this incentive effect depends on the individual's talent.

Proposition 1 *For an individual with talent a_i, the incentive effect is positive if and only if t_k is sufficiently low. Moreover, the level of the threshold t_k that maximizes the incentive effect for an individual with talent a_i is increasing in a_i.*

As established in Lemma 1, an increase in the threshold t_k increases the effort bounds $\underline{s}$ and $\overline{s}$. This increases the probability that an individual with talent a_i exerts effort if and only if $\phi(s_i|a_i)$, the conditional density of s_i, is larger at $s_i = \overline{s}$ than at $s_i = \underline{s}$. In turn, this is the case if and only if the threshold is sufficiently low. In this case, the "stimulant effect" of a higher threshold (effort inducement for individuals whose signal is close to $s_i = \overline{s}$) dominates the "discouragement effect" (effort withdrawal for individuals whose signal is close to $s_i = \underline{s}$). The crucial assumption underlying this result is that the talent distribution is single-peaked.[15] In sum, higher thresholds tend to elicit more effort from more talented individuals, but less effort from less talented individuals.

5 The talent pool

We now consider the measure of appointed individuals from a given group. In particular, we study the effect of the threshold on the pipeline of eligible CEO candidates. The firm CEO is selected among appointed individuals whose ability is higher than a target level denoted by $\bar{a}$. With a continuum $[0, 1]$ of candidates, the ex-ante probability for a member of group k to be appointed is also the measure of appointed individuals from this group.

[15] More precisely, the first part of Proposition 1 uses the symmetry and single-peakedness of the density of the signal conditional on talent, and the second part uses the functional form of the density function of the normal distribution, notably its log-concavity. Propositions 2, 3, and 4 use the talent distribution only via the continuity and the full support of its density.

5.1 Perfect information

In this subsection, we focus on the case when individual abilities are common knowledge, i.e. $\sigma_s = 0$ and $\sigma_q = 0$. By abstracting from uncertainty, these results also serve as stepping stones to better understand the effects at play in more complex settings.

Proposition 2 studies analytically the effect of the threshold on the measure of appointed candidates whose ability is sufficiently high to be CEO-eligible.

Proposition 2 *When individuals know their talent and screening is perfect, the measure of appointed individuals with ability larger than $\bar{a}$ is nondecreasing in the threshold t_k if $t_k < \bar{a}$, and strictly increasing if $t_k \in [\bar{a} - \overline{e}, \bar{a})$.*

When individual abilities are known and the targeted level of ability $\bar{a}$ is above the appointment threshold, there is no direct effect. Indeed, with a perfect appointment process, an individual exceeds the threshold if and only if her ability is sufficiently high, so that raising the threshold does not result in the unintended rejection of high-ability candidates. In addition, the incentive effect is then always nonnegative because we focus on individuals whose ability is above the threshold, and the appointment process is perfect. In this case, the stimulant effect is relevant, whereas the discouragement effect is not. In sum, in the case considered in Proposition 2, raising the appointment threshold never decreases the measure of appointed individuals with sufficiently high ability, and it increases this measure when $t_k \in [\bar{a} - \overline{e}, \bar{a})$.

This analytical result implies that, when there is no uncertainty about individual ability, a "pipeline" of very high ability individuals from a particular group is best cultivated by setting a very high appointment threshold. Indeed, this policy maximizes incentives for individuals with high talent to exert effort, which thickens the right tail of the ability distribution.

Proposition 2 implies that the effects of a higher appointment threshold on group representation at the entry level and at the CEO level can have opposite signs. Corollary 1 states this tradeoff for two groups subject to different thresholds.

Corollary 1 *When individuals know their talent and screening is perfect, consider two groups A and B with appointment thresholds $t_A < t_B < \bar{a}$. The measure of appointed individuals is strictly lower for group B than for group A, whereas the measure of appointed individuals with ability larger than $\bar{a}$ is weakly higher for group B than for group A, and strictly higher if $t_B > \bar{a} - \bar{e}$.*

Under the CEO selection process described in section 3, the CEO is drawn from the set of appointed individuals with ability higher than $\bar{a}$. Corollary 1 therefore implies that the CEO is weakly more likely to be from group B than from group A, and strictly more likely if $t_B \in (\bar{a} - \bar{e}, \bar{a})$, even though group B is under-represented at the entry level.

These results rely on two informational assumptions, which the next subsections relax in turn. When individuals are imperfectly aware of their own talent ($\sigma_s > 0$), setting a very high threshold could backfire because some individuals with high talent will not exert effort. Likewise, when the appointment process is not perfect ($\sigma_q > 0$), setting a very high threshold could backfire because some candidates with high ability will not be appointed.

5.2 Uncertainty about own talent

We first isolate the effect of individuals not knowing their own talent: each individual receives a noisy signal of her talent ($\sigma_s > 0$), while the screening process remains perfect ($\sigma_q = 0$). In this case, an individual is appointed if and only if her ability $a_i + e_i$ is at least as high as the threshold t_k.

Proposition 3 *When $\sigma_s > 0$, $\sigma_q = 0$, and c is sufficiently low for the effort bounds in Lemma 1 to exist, the measure of appointed individuals with ability higher than $\bar{a}$ converges to $\mathbb{P}(\tilde{a}_i \geq \bar{a})$ as $t_k \to -\infty$, converges to zero as $t_k \to \infty$, and is strictly higher than $\mathbb{P}(\tilde{a}_i \geq \bar{a})$ for every $t_k < \bar{a}$. This measure therefore reaches its maximum for a finite t_k.*

Proposition 3 contrasts with Proposition 2. Under perfect information, the measure of appointed individuals with ability higher than $\bar{a}$ is nondecreasing in the threshold for any

$t_k < \bar{a}$, so that a very high threshold is desirable when $\bar{a}$ is very high. Instead, with noisy individual signals ($\sigma_s > 0$), a very high threshold discourages effort: individuals rely on their prior beliefs about their talent, and few individuals receive a signal high enough to justify effort when the effort bounds are very high. As the threshold becomes very high, the measure of appointed individuals with ability higher than $\bar{a}$ declines. Thus, when individuals are uncertain about their own talent, the discouragement effect described in section 4.2 also applies to talented individuals, who may receive a low signal. This effect dominates the stimulant effect when the threshold is high, so that the threshold that maximizes the measure of CEO-eligible appointees is finite even when $\bar{a}$ is arbitrarily high.

5.3 Imperfect screening

We next isolate the effect of imperfect screening: individuals know their talent ($\sigma_s = 0$), while the appointment decision is based on a noisy measure of ability ($\sigma_q > 0$).

Proposition 4 *When $\sigma_s = 0$ and $\sigma_q > 0$, for every threshold t_k, a strictly positive measure of individuals with ability higher than $\bar{a}$ is rejected, and a strictly positive measure of individuals with ability lower than $\bar{a}$ is appointed. In addition, the measure of appointed individuals with ability higher than $\bar{a}$ converges to $\mathbb{P}(\tilde{a}_i \geq \bar{a})$ as $t_k \to -\infty$ and converges to zero as $t_k \to \infty$.*

Imperfect screening creates a rejection effect: appointment mistakes occur at every threshold, and an increase in the threshold lowers the appointment probability of every individual, including individuals with ability higher than $\bar{a}$. In the decomposition of section 4.2, the direct effect of an increase in the threshold is strictly negative for every individual when $\sigma_q > 0$. Under perfect screening, by contrast, an increase in the threshold with $t_k < \bar{a}$ does not affect the appointment of individuals with ability higher than $\bar{a}$. The rejection effect explains why the measure of appointed individuals with ability higher than $\bar{a}$ can be decreasing in the threshold even for $t_k < \bar{a}$.

5.4 Imperfect information

We now study the case when individuals imperfectly know their own talent ($\sigma_s > 0$) and the screening process is imperfect ($\sigma_q > 0$). Allowing for $\sigma_s > 0$ and $\sigma_q > 0$ introduces the discouragement of effort among talented individuals described in section 5.2 as well as the rejection effect described in section 5.3.

First, the less precise the signal that individuals receive about their talent (high σ_s), the more they rely on their prior belief when making their effort decision. In particular, individuals whose talent is significantly higher than the mean $\mu_k = 0$ in their group should in principle exert effort when the recruitment threshold is high. However, if they are not aware of their high talent, they will tend to make effort decisions similar to the average individual in their group, and thus have the strongest propensity to exert effort when the threshold is intermediate rather than high. In this case, high thresholds will discourage effort even among talented individuals.

Second, a more noisy appointment process results in more mistakes, including appointments of low ability individuals and rejections of high ability individuals. When the appointment process is substantially noisy (high σ_q), appointment decisions are largely random, which discourages effort.[16] Then, maximizing the measure of appointed individuals with ability higher than a cutoff $\bar{a}$ tends to be achieved with a lower threshold, as Figure 1 and Appendix B illustrate. On the contrary, when the appointment process is high-quality (low σ_q), increasing the threshold allows the organization to reject more individuals with low ability (direct effect), and it also encourages high ability individuals to exert effort (incentive effect) – all the more that these individuals receive a precise signal about their type.

We now consider the measure of individuals from group k who are appointed to entry-level positions and whose ability is higher than a cutoff $\bar{a}$. This measure is given by integrating over a_i such that $a_i + e_i \geq \bar{a}$ the measure of appointed individuals with talent a_i derived in

[16] Considering the effort incentive constraint for a given s_i in equation (10) in the Appendix, it is not satisfied if σ_q is sufficiently large. This means that an individual optimally decides not to exert effort if appointments are sufficiently random.

equation (5):

$$\begin{aligned}\mathbb{P}\left(\tilde{q}_i \geq t_k \cap \tilde{a}_i + e_i \geq \bar{a}\right) &= \int_{\bar{a}}^{\infty} \int_{-\infty}^{\underline{s}(0)+\frac{\sigma_k^2+\sigma_s^2}{\sigma_k^2}t_k} \left(1 - \Phi\left(\frac{1}{\sigma_q}\left(t_k - a_i\right)\right)\right) \phi(s_i|a_i)\varphi(a_i) ds_i da_i \\ &+ \int_{\bar{a}-\bar{e}}^{\infty} \int_{\underline{s}(0)+\frac{\sigma_k^2+\sigma_s^2}{\sigma_k^2}t_k}^{\overline{s}(0)+\frac{\sigma_k^2+\sigma_s^2}{\sigma_k^2}t_k} \left(1 - \Phi\left(\frac{1}{\sigma_q}\left(t_k - a_i - \bar{e}\right)\right)\right) \phi(s_i|a_i)\varphi(a_i) ds_i da_i \\ &+ \int_{\bar{a}}^{\infty} \int_{\overline{s}(0)+\frac{\sigma_k^2+\sigma_s^2}{\sigma_k^2}t_k}^{\infty} \left(1 - \Phi\left(\frac{1}{\sigma_q}\left(t_k - a_i\right)\right)\right) \phi(s_i|a_i)\varphi(a_i) ds_i da_i \quad (7)\end{aligned}$$

where φ is the PDF of the standard normal distribution. The integrals on the first and third lines account for individuals who do not exert effort, whose ability is simply equal to their talent. The integrals on the second line account for individuals who exert effort, whose ability is equal to the sum of their talent and effort. From Lemma 1, we know that only individuals who receive a signal s_i in-between the "effort bounds" will exert effort.

We now consider the calibration of model parameters, which will matter for numerical results. We normalize the standard deviation of the talent distribution as $\sigma_k = 1$.

First, meta-analyses find that the best combination of test scores used by firms for screening explains between 26% and 42% of the variance of actual work performance, a fraction which has been increasing over time (see Schmidt, Oh, and Shaffer (2016), which updates the seminal paper of Schmidt and Hunter (1998)). This suggests that $\sigma_q \approx 1.2$.[17]

Second, meta-analyses of the correlation ρ between students' self-assessments of their own ability and the marks they actually receive suggest that $\rho \approx 0.45$ (Falchikov and Boud (1989), León, Panadero, and García-Martínez (2023)),[18] which given $\sigma_q \approx 1.2$ suggests that $\sigma_s \approx 1$. When individuals are more educated or receive feedback, the correlation coefficient is typically around $\rho \approx 0.5$ or higher (Ackerman, Kanfer, and Goff (1995), Ober et al. (2022), Atrash, Katz-Leurer, and Shahar (2023), Patzl, Oberleiter, and Pietschnig (2024)), which given $\sigma_q \approx$

[17] Back of the envelope calculations that abstract from effort show that: $R^2 = \left(\text{corr}\left(\tilde{a}_i, \tilde{q}_i\right)\right)^2 = \frac{var(\tilde{a}_i)}{var(\tilde{q}_i)} = \frac{1}{1+\sigma_q^2}$. With $R^2 = 0.42$, this gives $\sigma_q = 1.18$; with $R^2 = 0.26$, this gives $\sigma_q = 1.69$.

[18] Falchikov and Boud (1989) find a distribution of correlation coefficients across studies that peaks at $\rho = 0.45$, while León, Panadero, and García-Martínez (2023) find a Fisher z-transformation of 0.472, corresponding to a correlation coefficient of 0.44.

1.2 suggests that $\sigma_s \approx 0.8$.[19] Accordingly, we set $\sigma_s = 0.8$ and $\sigma_q = 1.2$ in our baseline numerical computations, and we repeat the same computations with different values for σ_s and σ_q in the section below and in Appendix B.

Third, the cost of effort must be strictly less than the benefit B of being appointed, otherwise no one would exert effort. Moreover, because of imperfect information, the cost of effort cannot be too high. For example, with $\sigma_s = 0.8$ and $\sigma_q = 1.2$, the highest cost of effort such that a positive measure of individuals exerts effort is $c = 0.288$, whereas it is $c = 0.317$ with $\sigma_s = 1$ and $\sigma_q = 1$, and $c = 0.9996$ with $\sigma_s = 0.1$ and $\sigma_q = 0.1$. The effect of the threshold on effort decisions is strongest when effort is a marginal decision for a substantial measure of individuals, i.e. when the cost of effort is close to this upper bound. As the cost of effort decreases, the measure of individuals who exert effort regardless of the threshold increases, and the effect of the threshold on the composition of the pool of appointed individuals weakens. Accordingly, we assume a cost of effort of $c = 0.25$ when we set $\sigma_s = 0.8$ and $\sigma_q = 1.2$, and $c = 0.3$ when we set either $\sigma_s = 1$ and $\sigma_q = 1$ or $\sigma_s = 0.1$ and $\sigma_q = 0.1$. We also set the effect of effort on ability equal to one standard deviation of the talent distribution: $\overline{e} = \sigma_k = 1$. This similar magnitude implies that both talent and effort are important to achieve a very high level of ability.

We consider the effect of the threshold on the measure of appointed individuals with high ability. Figure 1 depicts the measure of individuals from group k who get appointed ($q_i \geq t_k$) and whose ability is higher than a cutoff $\bar{a}$, as a function of the threshold t_k. The target $\bar{a}$, the minimum level of ability required to be selected as CEO, can be viewed as a measure of the meritocracy of the CEO selection process: the higher $\bar{a}$, the more ability matters; the lower it is, the more other factors matter.[20] These other factors are modeled explicitly in section 5.5. Figure 1 presents numerical computations based on our baseline parameter values, as

[19] Using the updated belief about own ability in equation (2), back of the envelope calculations that abstract from effort show that a correlation of ρ means that: $\rho = \text{corr}\left(\mathbb{E}[\tilde{a}_i|\tilde{s}_i], q_i\right) = \text{corr}\left(\frac{\sigma_k^2}{\sigma_k^2+\sigma_s^2}(\tilde{a}_i + \tilde{\epsilon}_i^s), \tilde{a}_i + \tilde{\epsilon}_i^q\right) = \frac{1}{\sqrt{1+\sigma_s^2}\sqrt{1+\sigma_q^2}}$.

[20] These other factors may include connections both inside and outside the firm, including linkages to the previous CEO and linkages to directors (Wiersema, Nishimura, and Suzuki (2018), Bandiera et al. (2024)).

well as alternative calibrations (see also Appendix B for additional calibrations based on other parameter values). We can draw two main lessons from these numerical computations.

First, increasing the appointment threshold for a group can substantially increase the measure of appointed individuals with high ability: the function $\mathbb{P}\left(\tilde{q}_i \geq t_k \cap \tilde{a}_i + e_i \geq \bar{a}\right)$ is increasing in the appointment threshold t_k when the threshold is low.[21] Thus, the forces at play in Proposition 2 remain relevant and can be economically significant even with $\sigma_s > 0$ and $\sigma_q > 0$.

Second, these numerical computations also show that increasing the threshold reduces the measure of appointed individuals with high ability when the threshold is high: the measure $\mathbb{P}\left(\tilde{q}_i \geq t_k \cap \tilde{a}_i + e_i \geq \bar{a}\right)$ is decreasing in the appointment threshold t_k when it is sufficiently high. This can happen even with $t_k < \bar{a}$, which is in contrast to the result in Proposition 2. When there is uncertainty about individual abilities, the appointment threshold that maximizes the measure of high-ability employees is not necessarily as high as the targeted level of ability.

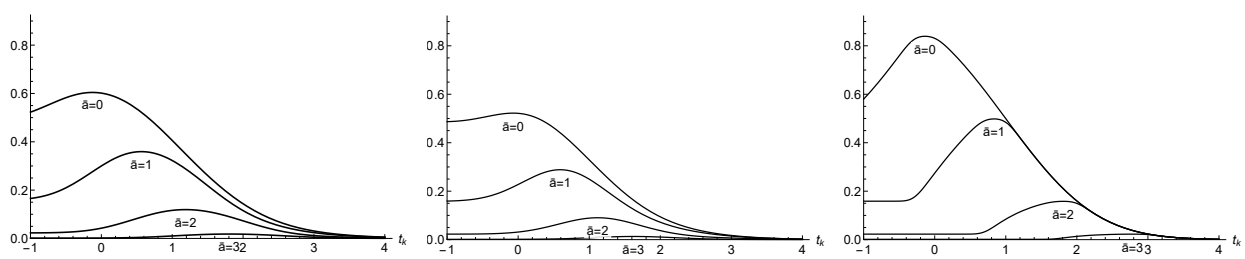


Figure 1: Measure of appointed high-ability employees $\mathbb{P}\left(\tilde{q}_i \geq t_k \cap \tilde{a}_i + e_i \geq \bar{a}\right)$ as a function of the appointment threshold t_k for $\mu_k = 0$, $\sigma_k = 1$, $\overline{e} = 1$.
Left panel: $\sigma_s = 0.8$, $\sigma_q = 1.2$, $c = 0.25$; middle panel: $\sigma_s = 1$, $\sigma_q = 1$, $c = 0.3$; right panel: $\sigma_s = 0.1$, $\sigma_q = 0.1$, $c = 0.3$.

Next, we consider the threshold that maximizes the measure of appointed individuals with ability above the minimum level necessary to be selected as CEO. Figure 2 depicts the threshold t_k^* that maximizes the measure of appointed individuals with ability above $\bar{a}$, for

[21] This is only true when $\overline{a}$ is sufficiently high. For example, for $\overline{a} \to -\infty$, this function would be strictly decreasing in the appointment threshold at any point.

a range of values of $\bar{a}$, as computed numerically. For the range considered, this threshold is increasing in the minimum level of ability $\bar{a}$. Intuitively, changing the threshold shifts the subset of individuals who exert effort. When the threshold increases, less talented individuals no longer exert effort (the discouragement effect), whereas more talented individuals start exerting effort (the stimulant effect). Since the talent distribution is single-peaked with an upper tail that gets thinner and thinner, increasing the threshold from a high level involves a tradeoff between eliciting effort from more talented individuals and eliciting effort from fewer individuals. As the cutoff $\bar{a}$ increases, it becomes even more important to elicit effort from more talented individuals, so that the threshold that maximizes the measure increases.

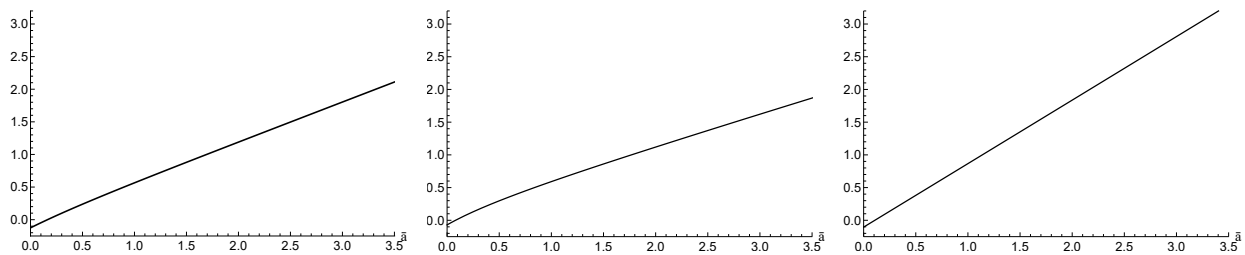


Figure 2: Appointment threshold t_k^* that maximizes the measure of appointed high-ability employees $\mathbb{P}\left(\tilde{q}_i \geq t_k \cap \tilde{a}_i + e_i \geq \bar{a}\right)$ as a function of $\bar{a}$ for $\mu_k = 0$, $\sigma_k = 1$, $\overline{e} = 1$.
Left panel: $\sigma_s = 0.8$, $\sigma_q = 1.2$, $c = 0.25$; middle panel: $\sigma_s = 1$, $\sigma_q = 1$, $c = 0.3$; right panel: $\sigma_s = 0.1$, $\sigma_q = 0.1$, $c = 0.3$.

5.5 CEO selection and group representation

The effect of the appointment threshold on the probability for a member of a group to become CEO is relative: it depends not only on the threshold applied to this group, but also on the threshold applied to other groups. In Figure 3, we consider two groups, A and B, subject to different appointment thresholds, t_A and t_B, and we ask whether the firm CEO is more likely to be from group A or from group B, depending on the thresholds. As long as the appointment thresholds are sufficiently low, we find that the CEO is more likely to be from the group with the highest appointment threshold. On the contrary, when the thresholds are

sufficiently high, the CEO is more likely to be from the group with the lowest appointment threshold. Moreover, the more meritocratic the firm (i.e. the higher $\bar{a}$), the larger is the subset of thresholds such that the CEO is more likely to be from the group with the highest threshold. For example, for $t_A = 1.25$ and $t_B = 1.5$, the CEO is more likely to be from group A if $\bar{a} = 2$, but she is more likely to be from group B if $\bar{a} = 3$.

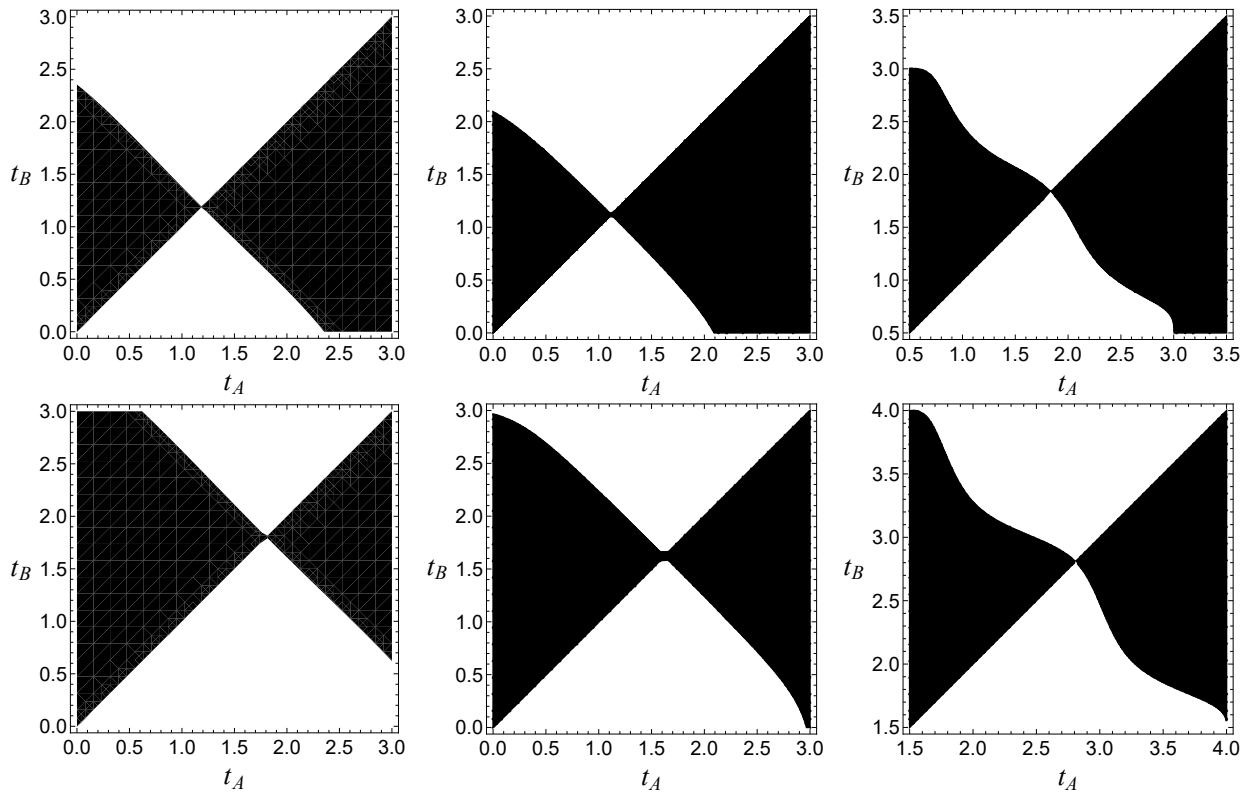


Figure 3: The black area is the subset of group-contingent appointment thresholds $\{t_A, t_B\}$ such that the measure of appointed high-ability employees $\mathbb{P}\left(\tilde{q}_i \geq t_k \cap \tilde{a}_i + e_i \geq \bar{a}\right)$ is higher for group B than for group A, for $\mu_k = 0$, $\sigma_k = 1$, $\overline{e} = 1$.
First row: $\bar{a} = 2$. Second row: $\bar{a} = 3$. On each row: left panel: $\sigma_s = 0.8$, $\sigma_q = 1.2$, $c = 0.25$; middle panel: $\sigma_s = 1$, $\sigma_q = 1$, $c = 0.3$; right panel: $\sigma_s = 0.1$, $\sigma_q = 0.1$, $c = 0.3$.

Finally, to quantify the magnitude of the effects at play, we present numerical examples that involve two firms with the same group-contingent appointment thresholds (i.e. biases are systematic) but different levels of meritocracy. We consider two different cases. Example

1 relies on our baseline parameter values, and considers moderate appointment thresholds. Example 2 considers the hypothetical case when abilities are very precisely estimated and appointment thresholds are high.

Example 1 *Individual talent is normally distributed with mean* $\mu_k = 0$ *and variance* $\sigma_k^2 = 1$. *The cost of effort is* $c = 0.25$, *and the effect of effort on ability is equal to one standard deviation of the talent distribution:* $\overline{e} = 1$. *There are two firms. The first is highly meritocratic: it only considers very high-ability individuals as potential CEOs, with* $\overline{a} = 3$. *This is two standard deviations above the mean of the ability distribution for individuals who exert effort. By contrast, the second firm considers a broader pool of individuals as potential CEOs, with* $\overline{a} = 2$. *This is one standard deviation above the mean of the ability distribution for individuals who exert effort. There are two groups in the population, group A and group B, with the same measure of individuals from each group. The appointment threshold is biased in favor of individuals from group A:* $t_A = 1.0$ *and* $t_B = 1.5$. *Using our baseline calibration for* σ_s *and* σ_q, *we have:*

$\sigma_s = 0.8$, $\sigma_q = 1.2$	*Proportion of individuals from group B in the firm*	*Probability that the CEO is from group B*
More meritocratic firm: $\bar{a} = 3$	*39.0%*	*63.8%*
Less meritocratic firm: $\bar{a} = 2$	*39.0%*	*49.2%*

Example 2 *Consider the same parameter values as in Example 1 except that* $\sigma_s = 0.1$, $\sigma_q = 0.1$, $c = 0.3$, $t_B = 2.5$, *and* $t_A = 2.0$. *Then we have:*

$\sigma_s = 0.1$, $\sigma_q = 0.1$	*Proportion of individuals from group B in the firm*	*Probability that the CEO is from group B*
More meritocratic firm: $\bar{a} = 3$	*29.4%*	*76.0%*
Less meritocratic firm: $\bar{a} = 2$	*29.4%*	*30.2%*

In the cases considered in Examples 1 and 2, we find that the probability that the CEO is from the group with the higher appointment threshold is higher than the representation of

this group in the firm. Intuitively, its talented members will have stronger effort incentives than talented members from the other group, and its less talented members will most likely not get appointed.

The magnitude of this effect depends on the cost of effort. In Example 1 with $\bar{a} = 3$, the probability that the CEO is from group B is equal to 63.8% for $c = 0.25$, 55.5% for $c = 0.2$, 51.1% for $c = 0.15$, and 49.4% for $c = 0.1$, and it converges to 48.9% when no individual exerts effort. A lower cost of effort increases the measure of individuals who exert effort regardless of the threshold, which weakens the effect of the threshold on the composition of the pool of appointed individuals.

The selection process considered so far assumes that the CEO is drawn from the set of appointed individuals with ability higher than $\bar{a}$, with equal probabilities within this set. In practice, factors other than ability, such as connections to the previous CEO or to directors, also influence CEO selection (Wiersema, Nishimura, and Suzuki (2018), Bandiera et al. (2024)). To take this into account, now suppose that the CEO is drawn from the set of appointed individuals with ability higher than $\bar{a}$ with probability $\lambda \in [0, 1]$, and from the set of all appointed individuals with probability $1 - \lambda$. The parameter λ measures the weight of ability in CEO selection, and complements $\bar{a}$ as a measure of the meritocracy of the selection process. The specification used so far corresponds to $\lambda = 1$. For two groups A and B, let N_A and N_B denote the measures of appointed individuals from each group, and let Λ_A and Λ_B denote the measures of appointed individuals from each group with ability higher than $\bar{a}$.

Proposition 5 *The probability that the CEO is from group B is:*

$$P_B(\lambda) = \lambda \frac{\Lambda_B}{\Lambda_A + \Lambda_B} + (1 - \lambda) \frac{N_B}{N_A + N_B}. \tag{8}$$

If $\frac{\Lambda_B}{\Lambda_A + \Lambda_B} > \frac{1}{2} > \frac{N_B}{N_A + N_B}$, *then there exists* $\hat{\lambda} \in (0, 1)$ *such that the CEO is more likely to be*

from group B than from group A if and only if $\lambda > \hat{\lambda}$, with:

$$\hat{\lambda} = \frac{\frac{1}{2} - \frac{N_B}{N_A+N_B}}{\frac{\Lambda_B}{\Lambda_A+\Lambda_B} - \frac{N_B}{N_A+N_B}}. \tag{9}$$

Proposition 5 shows that the tradeoff between group representation at the junior level and group representation at the senior level requires a sufficiently meritocratic CEO selection process. The condition $\frac{\Lambda_B}{\Lambda_A+\Lambda_B} > \frac{1}{2} > \frac{N_B}{N_A+N_B}$, i.e. $\Lambda_B > \Lambda_A$ and $N_B < N_A$, states that group B is over-represented in the pool of high-ability appointees ($a_i > \overline{a}$) but under-represented at the entry level. Under perfect information, Corollary 1 shows that this condition holds whenever $\bar{a} - \overline{e} < t_B < \bar{a}$ and $t_A < t_B$. Under the baseline parameter values, numerical computations show that the measure of appointed individuals N_k decreases with t_k, so that the second inequality is equivalent to $t_B > t_A$. Under this condition, the CEO is more likely to be from group B than from group A if and only if the weight of ability in CEO selection is higher than $\hat{\lambda}$. In Example 1 with $\bar{a} = 3$, equation (9) gives $\hat{\lambda} = 0.44$. For lower values of λ, group B is under-represented at the CEO level even though group B has a larger pool of high-ability appointees.

In summary, appointment biases against a group do not automatically increase the probability that the firm CEO is a member of this group. This can be for two reasons. First, in the less meritocratic firm, the probability that the CEO is from group B is less than one-half in both Examples, even though group B accounts for one-half of the population. These firms do not require their CEO to have very high ability, so that the probability that the CEO is from a particular group mostly depends on the measure of appointed individuals from this group. Second, if individuals receive a very noisy signal about their own talent, talented individuals from a group subject to a high appointment threshold will fail to exert effort because they put a lot of weight on their prior belief μ_k about their talent. Formally:

Remark 1 *If $t_k > \overline{e} + \sqrt{\sigma_k^2 + \sigma_q^2}\, \max\{0, \Phi^{-1}(1-c)\}$, the measure of individuals from group k who exert effort converges to zero as $\sigma_s \to \infty$.*

When almost no individual exerts effort, the "incentive effect" of changing the threshold (as described in section 4.2) becomes negligible, so that a higher threshold simply reduces the number of individuals from a group recruited by the firm. For example, with $\mu_k = 0$, $\sigma_k = 1$, $\overline{e} = 1$, $c = 0.3$, $\sigma_q = 1$, and $t_k = 3$, the measure of individuals who exert effort is lower than 10^{-8} if $\sigma_s > 1.5$.

6 Discussion and implications

The model developed in this paper highlights an important tradeoff in recruitment practices. Algorithmic or procedural biases that result in higher appointment thresholds for some demographic groups can have opposing effects at different levels of the corporate hierarchy. While a higher threshold reduces the representation of that group at the entry level, it can increase the share of high-ability individuals from the same group at the top, especially in meritocratic firms where promotions are tightly linked to individual ability.

The model generates testable predictions. Most notably, for groups that face discrimination at the hiring stage, their representation at the CEO level should be positively correlated with measures of meritocracy across firms – as proxied by governance quality and shareholder orientation. To take into account firm fixed effects in group representation, an empirical test could alternatively consider senior level representation relative to junior level representation across firms. Measures of the meritocracy of CEO selection include board independence, institutional ownership, and the management practice scores of Bandiera et al. (2024).

The tradeoff described above has direct implications for the use of AI in recruitment. Many hiring algorithms are trained on historical data that reflect prior human biases and thus systematically vary thresholds across groups. The model implies that any change that lowers the appointment threshold for a group, whether a diversity initiative or the correction of a bias at the expense of this group, raises its entry-level representation but weakens the effort incentives of its talented members, and can consequently reduce its representation at the top

of meritocratic organizations.

The results also emphasize that diversity initiatives and AI auditing practices cannot be evaluated solely based on short-term hiring outcomes. Algorithmic systems influence human capital investment long before promotion decisions occur. More demanding selection processes can strengthen motivation and effort among talented individuals, especially in meritocratic organizations. However, this mechanism depends on firm-level meritocracy and the quality of information available. In less meritocratic firms or when individuals have poor feedback about their own talent, higher thresholds suppress participation at all levels. By contrast, as AI-driven recruitment becomes more prevalent, improvements in screening accuracy and in individuals' ability to assess their own talent will effectively reduce the values of the parameters σ_q and σ_s, which are the key frictions in the model. This technological progress may amplify the tradeoff between diversity at junior and senior levels.

Finally, the structure of hiring and promotion incentives can have substantial effects on the distribution of talent within the firm and, consequently, on firm value. In well-governed firms, even small improvements in the quality of the internal talent pool can translate into substantial value creation, especially in large firms where CEO ability matters more (Gabaix and Landier (2008)).

7 Conclusion

In this paper, we study the implications of group-dependent appointment thresholds for human capital accumulation decisions optimally made by individuals, for entry-level appointments, and for CEO selection. A higher appointment threshold for a group motivates more talented individuals in this group to exert effort. As a result, even though it decreases entry-level representation, it can increase the number of high-ability individuals from this group who are appointed. In turn, assuming that eligibility for the CEO position requires an ability level exceeding a cutoff, a higher appointment threshold for a particular group may increase the

probability that the firm's CEO is selected from that group. We find that this tradeoff arises when individuals are sufficiently informed about their own talent and when CEO selection is sufficiently meritocratic – when both the ability cutoff and the weight placed on ability are high. In summary, systematic biases that impede the appointment of individuals from a group can thicken the upper tail of the ability distribution in this group, which enlarges the pool of candidates from which top management is selected.

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Appendix

A Ability distribution

Figure 4 depicts the distribution of the ability of appointed individuals for four different levels of the appointment threshold t_k. These graphs allow to better understand how different values of t_k affect effort decisions and the ability distribution of appointed individuals. First, a low threshold ($t_k = 0$) results in a very high measure of appointed individuals with average and moderately high ability, but a very low measure of appointed individuals with very high ability (above $\mu_k + 3\sigma_k = 3$). Second, a high threshold ($t_k \geq 3$) results in a low measure of appointed individuals, but a relatively high measure of appointed individuals with very high ability when both σ_s and σ_q are low (see the bottom row in Figure 4).

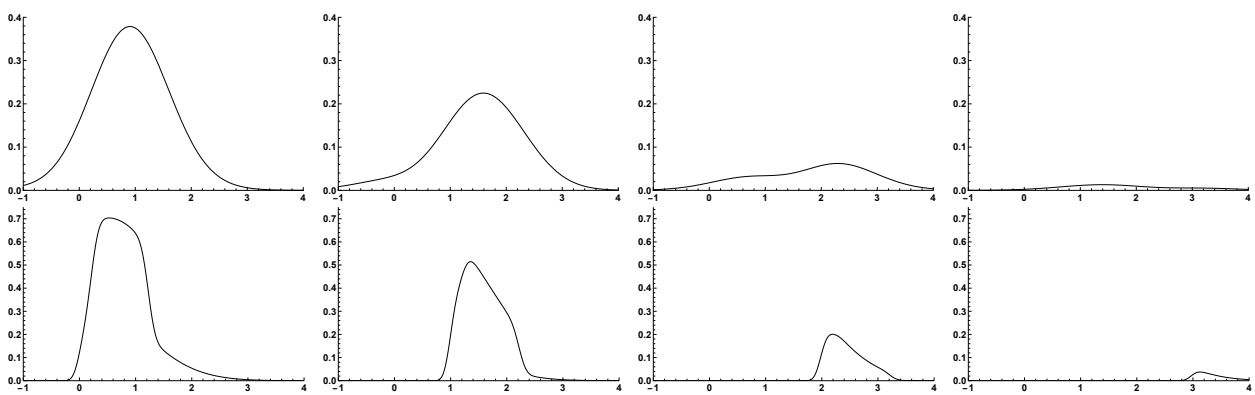

Figure 4: Measure of the ability $\tilde{a}_i + e_i$ of appointed individuals, for $\mu_k = 0$, $\sigma_k^2 = 1$, $\overline{e} = 1$. Top row: $\sigma_s = 0.8$, $\sigma_q = 1.2$, $c = 0.25$ and from left to right: $t_k = 0$, $t_k = 1$, $t_k = 2$, $t_k = 3$. Bottom row: $\sigma_s = 0.1$, $\sigma_q = 0.1$, $c = 0.3$ and from left to right: $t_k = 0$, $t_k = 1$, $t_k = 2$, $t_k = 3$.

B Alternative parameter values

This section presents numerical findings based on alternative parameter values for robustness purposes. The main text already presented findings with lower values for σ_q and σ_s. Figure 5 presents findings with higher values for these two key parameters.

A positive measure of individuals exerts effort only if the cost of effort c is lower than $2\Phi\left(\frac{\bar{e}}{2\Sigma}\right) - 1$ (see the proof of Remark 1). In the case $\sigma_q = 1.5$, $\sigma_s = 1$, this upper bound is equal to 0.237, so that no individual exerts effort when $c = 0.25$ or $c = 0.3$. Accordingly, in this case we set a lower cost of effort: $c = 0.22$. In the case $\sigma_q = 1$, $\sigma_s = 1.2$, this upper bound is equal to 0.308, and we set $c = 0.3$.

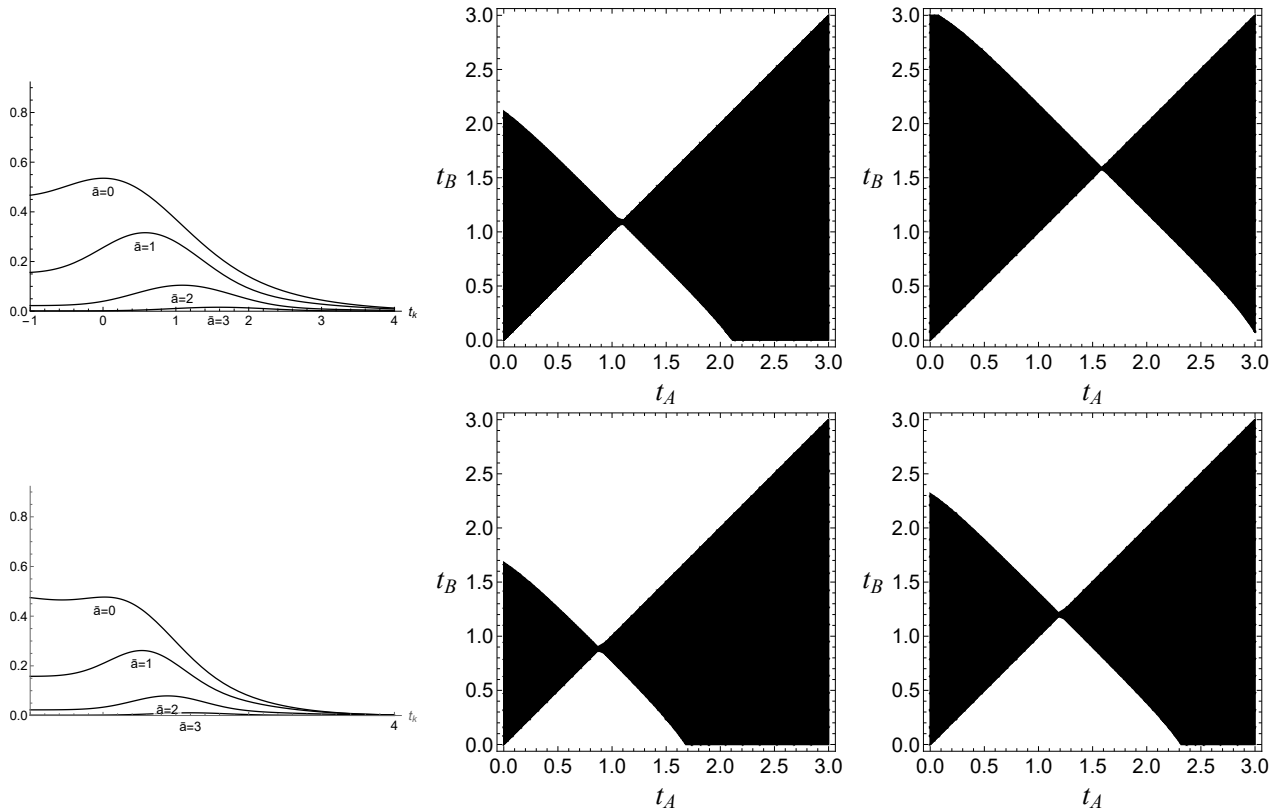

Figure 5: Left panel: $\mathbb{P}\left(\tilde{q}_i \geq t_k \cap \tilde{a}_i + e_i \geq \bar{a}\right)$ as a function of t_k for different values of $\bar{a}$. Middle and right panels: the black area is the subset of $\{t_A, t_B\}$ such that the measure $\mathbb{P}\left(\tilde{q}_i \geq t_k \cap \tilde{a}_i + e_i \geq \bar{a}\right)$ is higher for group B than for group A; middle panel: $\bar{a} = 2$; right panel: $\bar{a} = 3$. In all panels, $\mu_k = 0$, $\sigma_k^2 = 1$, $\overline{e} = 1$. Top row: $\sigma_q = 1.5$, $\sigma_s = 1$, $c = 0.22$. Bottom row: $\sigma_q = 1$, $\sigma_s = 1.2$, $c = 0.3$.

C Proofs

Proof of Lemma 1:

Conditional on the signal s_i, the random variable $\tilde{a}_i + \tilde{\epsilon}_i^q$ is normally distributed with mean

$$\mu_k + \frac{\sigma_k^2}{\sigma_k^2 + \sigma_s^2}\left[s_i - \mu_k\right]$$

and variance

$$\Sigma^2 \equiv \frac{\sigma_k^2 \sigma_s^2}{\sigma_k^2 + \sigma_s^2} + \sigma_q^2.$$

Based on her belief about her own talent described in equations (2) and (3), individual i optimally exerts effort if and only if:

$$\begin{aligned}
& \mathbb{P}(\tilde{q}_i \geq t_k | e_i = \overline{e}, s_i) - \mathbb{P}(\tilde{q}_i \geq t_k | e_i = 0, s_i) \geq c \\
\Leftrightarrow \quad & \mathbb{P}(\tilde{a}_i + \tilde{\epsilon}_i^q \geq t_k - \overline{e} | s_i) - \mathbb{P}(\tilde{a}_i + \tilde{\epsilon}_i^q \geq t_k | s_i) \geq c \\
\Leftrightarrow \quad & \mathbb{P}\left(\tilde{\varepsilon} \geq \left(\sqrt{\frac{\sigma_k^2 \sigma_s^2}{\sigma_k^2 + \sigma_s^2} + \sigma_q^2}\right)^{-1} \left(t_k - \overline{e} - \mu_k - \frac{\sigma_k^2}{\sigma_k^2 + \sigma_s^2}\left[s_i - \mu_k\right]\right)\right) \\
& -\mathbb{P}\left(\tilde{\varepsilon} \geq \left(\sqrt{\frac{\sigma_k^2 \sigma_s^2}{\sigma_k^2 + \sigma_s^2} + \sigma_q^2}\right)^{-1} \left(t_k - \mu_k - \frac{\sigma_k^2}{\sigma_k^2 + \sigma_s^2}\left[s_i - \mu_k\right]\right)\right) \geq c \\
\Leftrightarrow \quad & 1 - \Phi\left(\left(\sqrt{\frac{\sigma_k^2 \sigma_s^2}{\sigma_k^2 + \sigma_s^2} + \sigma_q^2}\right)^{-1} \left(t_k - \overline{e} - \mu_k - \frac{\sigma_k^2}{\sigma_k^2 + \sigma_s^2}\left[s_i - \mu_k\right]\right)\right) \\
& -1 + \Phi\left(\left(\sqrt{\frac{\sigma_k^2 \sigma_s^2}{\sigma_k^2 + \sigma_s^2} + \sigma_q^2}\right)^{-1} \left(t_k - \mu_k - \frac{\sigma_k^2}{\sigma_k^2 + \sigma_s^2}\left[s_i - \mu_k\right]\right)\right) \geq c, \qquad (10)
\end{aligned}$$

where the random variable $\tilde{\varepsilon}$ follows a standard normal distribution with CDF Φ. The LHS of equation (10), which measures the expected benefits from effort, is positive since $\Phi' > 0$. Let s^* be implicitly defined by:

$$\Phi\left(\Sigma^{-1}\left(t_k - \mu_k - \tfrac{\sigma_k^2}{\sigma_k^2 + \sigma_s^2}\left[s^* - \mu_k\right]\right)\right) - \Phi\left(\Sigma^{-1}\left(t_k - \overline{e} - \mu_k - \tfrac{\sigma_k^2}{\sigma_k^2 + \sigma_s^2}\left[s^* - \mu_k\right]\right)\right) = c. \quad (11)$$

The LHS of equation (11) depends on s^* only via $u \equiv t_k - \mu_k - \frac{\sigma_k^2}{\sigma_k^2+\sigma_s^2}[s^* - \mu_k]$, and it is equal to $\Phi\left(\frac{u}{\Sigma}\right) - \Phi\left(\frac{u-\overline{e}}{\Sigma}\right)$. Since the standard normal density is symmetric and single-peaked, $\Phi'\left(\frac{u}{\Sigma}\right) > \Phi'\left(\frac{u-\overline{e}}{\Sigma}\right)$ if and only if $u < \frac{\overline{e}}{2}$, so that the LHS is strictly increasing in u for $u < \frac{\overline{e}}{2}$ and strictly decreasing for $u > \frac{\overline{e}}{2}$, with maximum $2\Phi\left(\frac{\overline{e}}{2\Sigma}\right) - 1$ at $u = \frac{\overline{e}}{2}$. Equation (11) therefore has two solutions if $c < 2\Phi\left(\frac{\overline{e}}{2\Sigma}\right) - 1$ and no solution if $c > 2\Phi\left(\frac{\overline{e}}{2\Sigma}\right) - 1$. Under the assumption $c < 2\Phi\left(\frac{\overline{e}}{2\Sigma}\right) - 1$ made in the main text, denote by $\underline{s}$ and $\overline{s}$ as respectively the lowest and highest value of s^* that solve equation (11). We have:

$$\underline{s}(t_k) = \underline{s}(0) + \frac{\sigma_k^2 + \sigma_s^2}{\sigma_k^2} t_k, \quad \text{and} \quad \overline{s}(t_k) = \overline{s}(0) + \frac{\sigma_k^2 + \sigma_s^2}{\sigma_k^2} t_k. \tag{12}$$

■

Proof of Proposition 1:

We have:

$$\begin{aligned}
\frac{d}{dt_k}\mathbb{P}\left(\tilde{q}_i \geq t_k | a_i\right) \quad = \quad & -\int_{-\infty}^{\underline{s}(0)+\frac{\sigma_k^2+\sigma_s^2}{\sigma_k^2}t_k} \frac{1}{\sigma_q}\Phi'\left(\frac{1}{\sigma_q}\left(t_k - a_i\right)\right)\phi(s_i|a_i)ds_i \\
& -\int_{\underline{s}(0)+\frac{\sigma_k^2+\sigma_s^2}{\sigma_k^2}t_k}^{\overline{s}(0)+\frac{\sigma_k^2+\sigma_s^2}{\sigma_k^2}t_k} \frac{1}{\sigma_q}\Phi'\left(\frac{1}{\sigma_q}\left(t_k - a_i - \overline{e}\right)\right)\phi(s_i|a_i)ds_i \\
& -\int_{\overline{s}(0)+\frac{\sigma_k^2+\sigma_s^2}{\sigma_k^2}t_k}^{\infty} \frac{1}{\sigma_q}\Phi'\left(\frac{1}{\sigma_q}\left(t_k - a_i\right)\right)\phi(s_i|a_i)ds_i \\
& +\frac{\sigma_k^2+\sigma_s^2}{\sigma_k^2}\left[\Phi\left(\frac{1}{\sigma_q}\left(t_k - a_i\right)\right) - \Phi\left(\frac{1}{\sigma_q}\left(t_k - a_i - \overline{e}\right)\right)\right] \\
& \times\frac{1}{\sigma_s}\left[\varphi\left(\frac{1}{\sigma_s}\left(\overline{s}(0) + \frac{\sigma_k^2+\sigma_s^2}{\sigma_k^2}t_k - a_i\right)\right) - \varphi\left(\frac{1}{\sigma_s}\left(\underline{s}(0) + \frac{\sigma_k^2+\sigma_s^2}{\sigma_k^2}t_k - a_i\right)\right)\right]
\end{aligned} \tag{13}$$

where φ is the PDF of the standard normal distribution. We study the sign of the last two lines on the RHS of equation (13). With $\Phi' > 0$, we have $\Phi\left(\frac{1}{\sigma_q}\left(t_k - a_i\right)\right) - \Phi\left(\frac{1}{\sigma_q}\left(t_k - a_i - \overline{e}\right)\right) > 0$.

Therefore, the incentive effect is positive if and only if

$$\varphi\left(\frac{1}{\sigma_s}\left(\overline{s}(0)+\frac{\sigma_k^2+\sigma_s^2}{\sigma_k^2}t_k-a_i\right)\right) \;\geq\; \varphi\left(\frac{1}{\sigma_s}\left(\underline{s}(0)+\frac{\sigma_k^2+\sigma_s^2}{\sigma_k^2}t_k-a_i\right)\right). \tag{14}$$

Therefore, for given $\underline{s}(0) < \overline{s}(0)$, the condition in (14) holds if and only if

$$\frac{1}{2}\left[\underline{s}(0)+\frac{\sigma_k^2+\sigma_s^2}{\sigma_k^2}t_k+\overline{s}(0)+\frac{\sigma_k^2+\sigma_s^2}{\sigma_k^2}t_k\right]\leq a_i \quad\Leftrightarrow\quad t_k\leq\frac{\sigma_k^2}{\sigma_k^2+\sigma_s^2}\left[a_i-\frac{\underline{s}(0)+\overline{s}(0)}{2}\right]\equiv\hat{t}_k(a_i).$$

This establishes the first part of the Proposition.

For the second part, define:

$$G(x)\equiv\Phi\left(\frac{x}{\sigma_q}\right)-\Phi\left(\frac{x-\overline{e}}{\sigma_q}\right),\qquad H(y)\equiv\varphi\left(y+\Delta\right)-\varphi\left(y\right),\qquad \Delta\equiv\frac{\overline{s}(0)-\underline{s}(0)}{\sigma_s}>0.$$

With this notation, the incentive effect in equation (13) is:

$$I(t_k,a_i)=\frac{\sigma_k^2+\sigma_s^2}{\sigma_k^2\sigma_s}G(t_k-a_i)\,H\left(y(t_k,a_i)\right),\quad \text{where}\quad y(t_k,a_i)\equiv\frac{1}{\sigma_s}\left(\underline{s}(0)+\frac{\sigma_k^2+\sigma_s^2}{\sigma_k^2}t_k-a_i\right).$$

The function G is strictly positive. From the first part of the proof, $H\left(y(t_k,a_i)\right)>0$, and therefore $I(t_k,a_i)>0$, if and only if $t_k<\hat{t}_k(a_i)$. As $t_k\to-\infty$, $G(t_k-a_i)$ converges to zero while H remains bounded, so $I(t_k,a_i)$ converges to zero. In addition, $I\left(\hat{t}_k(a_i),a_i\right)=0$. Therefore, the level of t_k that maximizes the incentive effect belongs to the interval $\left(-\infty,\hat{t}_k(a_i)\right)$, on which $I>0$.

We now calculate the derivatives of $\ln G$ and $\ln H$. Using $\varphi'(u)=-u\varphi(u)$, the first two derivatives of G are:

$$\begin{aligned}G'(x) &= \frac{1}{\sigma_q}\left[\varphi\left(\frac{x}{\sigma_q}\right)-\varphi\left(\frac{x-\overline{e}}{\sigma_q}\right)\right],\\ G''(x) &= \frac{1}{\sigma_q^3}\left[(x-\overline{e})\,\varphi\left(\frac{x-\overline{e}}{\sigma_q}\right)-x\,\varphi\left(\frac{x}{\sigma_q}\right)\right].\end{aligned}$$

Therefore:

$$\frac{d\ln G(x)}{dx} = \frac{G'(x)}{G(x)}, \qquad \frac{d^2\ln G(x)}{dx^2} = \frac{G''(x)}{G(x)} - \left(\frac{G'(x)}{G(x)}\right)^2. \tag{15}$$

The change of variable $m = x - \sigma_q u$ gives $G(x) = \int_0^{\overline{e}} \frac{1}{\sigma_q}\varphi\left(\frac{x-m}{\sigma_q}\right) dm$, which is the convolution of a normal density with the indicator function of the interval $[0, \overline{e}]$. Both functions are log-concave, and the convolution of two log-concave functions is log-concave (Prékopa (1973)). The expression in (15) is therefore nonpositive for every x:

$$\frac{d^2\ln G(x)}{dx^2} \leq 0. \tag{16}$$

For the function H, the ratio $\frac{\varphi(y+\Delta)}{\varphi(y)}$ is equal to $e^{\tau(y)}$, where $\tau(y) \equiv -y\Delta - \frac{\Delta^2}{2}$, so that $H(y) = \varphi(y)\left[e^{\tau(y)} - 1\right]$. Since $\Delta > 0$, the function H is strictly positive if and only if $\tau(y) > 0$, i.e. if and only if $y < -\frac{\Delta}{2}$. On this region, $\ln H(y) = -\frac{y^2}{2} - \ln\sqrt{2\pi} + \ln\left(e^{\tau(y)} - 1\right)$, and $\frac{d\tau(y)}{dy} = -\Delta$, so that:

$$\begin{aligned} \frac{d\ln H(y)}{dy} &= -y - \frac{\Delta\, e^{\tau(y)}}{e^{\tau(y)} - 1}, \\ \frac{d^2\ln H(y)}{dy^2} &= -1 - \frac{\Delta^2 e^{\tau(y)}}{\left(e^{\tau(y)} - 1\right)^2} \;<\; 0. \end{aligned} \tag{17}$$

On the region where $I > 0$, consider $\ln I(t_k, a_i)$, and let $\kappa \equiv \frac{\sigma_k^2 + \sigma_s^2}{\sigma_k^2}$ denote the sensitivity of the effort bounds to the threshold, as established in Lemma 1. The factorization of I gives:

$$\ln I(t_k, a_i) = \ln\kappa - \ln\sigma_s + \ln G\left(x(t_k, a_i)\right) + \ln H\left(y(t_k, a_i)\right), \quad \text{where} \quad x(t_k, a_i) \equiv t_k - a_i. \tag{18}$$

In the derivatives below, $\frac{d\ln G(x)}{dx}$ is evaluated at $x = x(t_k, a_i)$, and $\frac{d\ln H(y)}{dy}$ is evaluated at $y = y(t_k, a_i)$. The function x has derivative 1 with respect to t_k and -1 with respect to a_i, and the function y has derivative $\frac{\kappa}{\sigma_s}$ with respect to t_k and $-\frac{1}{\sigma_s}$ with respect to a_i. The first

derivatives of $\ln I$ are therefore:

$$\begin{aligned}\frac{\partial \ln I(t_k, a_i)}{\partial t_k} &= \frac{d \ln G(x)}{dx} + \frac{\kappa}{\sigma_s} \frac{d \ln H(y)}{dy}, \\ \frac{\partial \ln I(t_k, a_i)}{\partial a_i} &= -\frac{d \ln G(x)}{dx} - \frac{1}{\sigma_s} \frac{d \ln H(y)}{dy}.\end{aligned} \tag{19}$$

Substituting the closed forms of $\frac{d \ln G(x)}{dx}$ and $\frac{d \ln H(y)}{dy}$ into equation (19), the first-order condition $\frac{\partial \ln I(t_k, a_i)}{\partial t_k} = 0$ is:

$$\frac{\varphi\left(\frac{t_k - a_i}{\sigma_q}\right) - \varphi\left(\frac{t_k - a_i - \overline{e}}{\sigma_q}\right)}{\sigma_q \, G\left(t_k - a_i\right)} = \frac{\kappa}{\sigma_s}\left[y(t_k, a_i) + \frac{\Delta \, e^{\tau(y(t_k, a_i))}}{e^{\tau(y(t_k, a_i))} - 1}\right]. \tag{20}$$

Writing $\gamma \equiv \frac{d^2 \ln G(x)}{dx^2}$ and $\eta \equiv \frac{d^2 \ln H(y)}{dy^2}$ for brevity, with the same evaluation convention, and differentiating equation (19) once more:

$$\frac{\partial^2 \ln I(t_k, a_i)}{\partial t_k^2} = \gamma + \frac{\kappa^2}{\sigma_s^2}\eta \; < \; 0, \tag{21}$$

$$\frac{\partial^2 \ln I(t_k, a_i)}{\partial t_k \partial a_i} = -\gamma - \frac{\kappa}{\sigma_s^2}\eta \; > \; 0, \tag{22}$$

where γ and η are given in closed form by equations (15) and (17). The inequalities in (21) and (22) follow from $\gamma \leq 0$ in (16) and $\eta < 0$ in (17). Therefore, $\ln I$ is strictly concave in t_k on the region where $I > 0$, and the maximizer $\bar{t}_k(a_i)$ is the unique solution to equation (20). The implicit function theorem then gives:

$$\begin{aligned}\frac{d\,\bar{t}_k(a_i)}{da_i} &= -\left(\frac{\partial^2 \ln I(t_k, a_i)}{\partial t_k \partial a_i}\right)\left(\frac{\partial^2 \ln I(t_k, a_i)}{\partial t_k^2}\right)^{-1} \\ &= \frac{\gamma + \frac{\kappa}{\sigma_s^2}\eta}{\gamma + \frac{\kappa^2}{\sigma_s^2}\eta} \; > \; 0,\end{aligned}$$

where the numerator and the denominator are both strictly negative. ■

Proof of Proposition 2:

With $\sigma_q = 0$, an individual exceeds the threshold if and only if $a_i + e_i \geq t_k$. Thus, with $\sigma_s = 0$, an individual will exert effort if and only if $a_i \in [t_k - \overline{e}, t_k)$ so that we have $\underline{s} = t_k - \overline{e}$ and $\overline{s} = t_k$. In sum:

$$\mathbb{P}\left(\tilde{q}_i \geq t_k | a_i\right) = \begin{cases} 1 & \text{if } t_k - a_i - e_i < 0, \\ 0 & \text{if } t_k - a_i - e_i > 0. \end{cases} \tag{23}$$

Let ψ be the PDF of $\tilde{a}_i$, and assume $t_k < \bar{a}$. There are two cases to consider.

First, if $t_k + \bar{e} < \bar{a}$, the ability of all individuals who optimally exert effort is still below $\bar{a}$. The measure of appointed individuals with ability above $\bar{a}$ is then simply:

$$\Lambda(\bar{a}) = \int_{\bar{a}}^{\infty} \psi\left(a\right) da \tag{24}$$

Second, if $t_k + \bar{e} \geq \bar{a}$, individuals with talent $a_i \in (\bar{a} - \bar{e}, t_k)$ exert effort and have an ability over $\bar{a}$. The measure of appointed individuals with ability above $\bar{a}$ is then:

$$\Lambda(\bar{a}) = \int_{\bar{a}-\bar{e}}^{t_k} \psi\left(a\right) da + \int_{\bar{a}}^{\infty} \psi\left(a\right) da. \tag{25}$$

In sum, for a given $t_k < \bar{a}$, the effect of a change in the threshold t_k on $\Lambda(\bar{a})$ is:

$$\frac{d}{dt_k}\Lambda(\bar{a}) = \begin{cases} \psi(t_k) & \text{if } \bar{a} \in (t_k, t_k + \overline{e}], \\ 0 & \text{if } \bar{a} > t_k + \overline{e}. \end{cases} \tag{26}$$

That is, for $t_k < \bar{a}$, we have $\frac{d}{dt_k}\Lambda(\bar{a}) \geq 0$. ■

Proof of Corollary 1:

With $\sigma_s = 0$ and $\sigma_q = 0$, the proof of Proposition 2 shows that an individual with talent a_i exerts effort if and only if $a_i \in [t_k - \overline{e}, t_k)$, and is appointed if and only if $a_i + e_i \geq t_k$. Individuals with talent $a_i \geq t_k$ are appointed without effort, and individuals with talent $a_i \in [t_k - \overline{e}, t_k)$

exert effort and are appointed. Let ψ be the PDF of the talent distribution. The measure of appointed individuals from group k is therefore:

$$N_k = \int_{t_k - \overline{e}}^{\infty} \psi(a) da, \tag{27}$$

which is strictly decreasing in t_k since the talent distribution has full support. With $t_A < t_B$, equation (27) gives $N_B < N_A$.

Let Λ_k denote the measure of appointed individuals from group k with ability larger than $\bar{a}$. From the proof of Proposition 2, for $t_k < \bar{a}$ we have $\Lambda_k = \int_{\bar{a}}^{\infty} \psi(a) da$ if $t_k < \bar{a} - \overline{e}$, and $\Lambda_k = \int_{\bar{a}-\overline{e}}^{t_k} \psi(a) da + \int_{\bar{a}}^{\infty} \psi(a) da$ if $\bar{a} - \overline{e} \leq t_k < \bar{a}$. As a function of t_k, this measure is constant on $(-\infty, \bar{a} - \overline{e})$ and strictly increasing on $[\bar{a} - \overline{e}, \bar{a})$. With $t_A < t_B < \bar{a}$, this gives $\Lambda_B \geq \Lambda_A$. If in addition $t_B > \bar{a} - \overline{e}$, then either $t_A < \bar{a} - \overline{e}$, in which case $\Lambda_B - \Lambda_A = \int_{\bar{a}-\overline{e}}^{t_B} \psi(a) da > 0$, or $t_A \geq \bar{a} - \overline{e}$, in which case $\Lambda_B - \Lambda_A = \int_{t_A}^{t_B} \psi(a) da > 0$. ■

Proof of Proposition 3:

With $\sigma_q = 0$, an individual is appointed if and only if $a_i + e_i \geq t_k$. From Lemma 1, an individual exerts effort if and only if $s_i \in [\underline{s}(t_k), \overline{s}(t_k)]$, where both effort bounds are increasing in t_k and the difference $\overline{s}(t_k) - \underline{s}(t_k)$ does not depend on t_k. Let ψ be the PDF of the talent distribution, and let $\Lambda(t_k)$ denote the measure of appointed individuals with ability higher than $\bar{a}$.

Consider first $t_k < \bar{a}$. Every individual with talent $a_i \geq \bar{a}$ is appointed and has ability higher than $\bar{a}$, with or without effort. An individual with talent $a_i \in [\bar{a} - \overline{e}, \bar{a})$ is appointed with ability higher than $\bar{a}$ if and only if this individual exerts effort, since $a_i + \overline{e} \geq \bar{a} > t_k$. An individual with talent $a_i < \bar{a} - \overline{e}$ cannot have ability higher than $\bar{a}$. Therefore:

$$\Lambda(t_k) = \mathbb{P}\left(\tilde{a}_i \geq \bar{a}\right) + \int_{\bar{a}-\overline{e}}^{\bar{a}} \left[\Phi\left(\frac{\overline{s}(t_k) - a}{\sigma_s}\right) - \Phi\left(\frac{\underline{s}(t_k) - a}{\sigma_s}\right)\right] \psi(a) da. \tag{28}$$

The probability that an individual with talent a exerts effort is strictly positive for every a,

since $\overline{s}(t_k) > \underline{s}(t_k)$ and the normal distribution has full support. Therefore, $\Lambda(t_k) > \mathbb{P}(\tilde{a}_i \geq \bar{a})$ for every $t_k < \bar{a}$.

As $t_k \to -\infty$, both effort bounds tend to $-\infty$, so this probability converges to zero for every a. It is bounded by one, so the integral converges to zero by dominated convergence, and $\Lambda(t_k)$ converges to $\mathbb{P}(\tilde{a}_i \geq \bar{a})$.

Consider next $t_k \geq \bar{a}$. Every individual with talent $a_i \geq t_k$ is appointed, with or without effort, and has ability higher than $\bar{a}$. An individual with talent $a_i \in [t_k - \overline{e}, t_k)$ is appointed if and only if this individual exerts effort, in which case the ability $a_i + \overline{e}$ is higher than $\bar{a}$. Therefore:

$$\Lambda(t_k) = \mathbb{P}\left(\tilde{a}_i \geq t_k\right) + \int_{t_k - \overline{e}}^{t_k} \left[\Phi\left(\frac{\overline{s}(t_k) - a}{\sigma_s}\right) - \Phi\left(\frac{\underline{s}(t_k) - a}{\sigma_s}\right)\right] \psi(a) da \leq \mathbb{P}\left(\tilde{a}_i \geq t_k - \overline{e}\right), \quad (29)$$

which converges to zero as $t_k \to \infty$.

The function Λ is continuous, since the effort bounds are continuous in t_k and these probabilities are bounded. Fix any $t_0 < \bar{a}$, so that $\Lambda(t_0) > \mathbb{P}(\tilde{a}_i \geq \bar{a})$. The limits derived above imply that $\Lambda(t_k) < \Lambda(t_0)$ outside of a sufficiently large compact interval. On this compact interval, the continuous function Λ reaches its maximum, which is therefore a global maximum reached at a finite threshold. ■

Proof of Proposition 4:

With $\sigma_s = 0$, the signal s_i reveals the talent a_i. From equation (10) with $\Sigma = \sigma_q$, an individual with talent a_i exerts effort if and only if:

$$\Phi\left(\frac{t_k - a_i}{\sigma_q}\right) - \Phi\left(\frac{t_k - \overline{e} - a_i}{\sigma_q}\right) \geq c. \quad (30)$$

The LHS of equation (30) depends only on the difference $t_k - a_i$, converges to zero as $t_k - a_i \to -\infty$ and as $t_k - a_i \to \infty$, and its derivative with respect to $t_k - a_i$ is positive if and only if $t_k - a_i < \frac{\overline{e}}{2}$. Therefore, the set of talent levels such that an individual exerts effort is either

empty or an interval $[t_k - x_2, t_k - x_1]$, where the constants $x_1 < x_2$ do not depend on t_k.

Conditional on talent a_i and effort e_i, the probability of appointment is $1 - \Phi\left(\frac{t_k - a_i - e_i}{\sigma_q}\right)$, which is strictly between zero and one for every t_k, since the normal distribution has full support. The set of individuals with ability higher than $\bar{a}$ includes all individuals with talent higher than $\bar{a}$, and each of these individuals is rejected with a strictly positive probability, so a strictly positive measure of individuals with ability higher than $\bar{a}$ is rejected. Likewise, the set of individuals with ability lower than $\bar{a}$ includes all individuals with talent lower than $\bar{a} - \overline{e}$, and each of these individuals is appointed with a strictly positive probability, so a strictly positive measure of individuals with ability lower than $\bar{a}$ is appointed.

Let ψ be the PDF of the talent distribution, let $e(a) \in \{0, \overline{e}\}$ denote the effort decision of an individual with talent a given the threshold t_k, and let $\Lambda(t_k)$ denote the measure of appointed individuals with ability higher than $\bar{a}$:

$$\Lambda(t_k) = \int_{\{a\,:\,a+e(a)\geq\bar{a}\}} \left[1 - \Phi\left(\frac{t_k - a - e(a)}{\sigma_q}\right)\right] \psi(a)da. \tag{31}$$

As $t_k \to -\infty$, the interval $[t_k - x_2, t_k - x_1]$ tends to $-\infty$, so the measure of individuals who exert effort and reach an ability higher than $\bar{a}$ converges to zero, and the appointment probability of every individual converges to one. Dominated convergence then implies that $\Lambda(t_k)$ converges to $\mathbb{P}(\tilde{a}_i \geq \bar{a})$.

As $t_k \to \infty$, the appointment probability of every individual who does not exert effort converges to zero pointwise, and the measure of individuals who exert effort is at most $\mathbb{P}(\tilde{a}_i \geq t_k - x_2)$, which converges to zero. Dominated convergence then implies that $\Lambda(t_k)$ converges to zero. ■

Proof of Proposition 5:

With probability λ, the CEO is drawn from the set of appointed individuals with ability higher than $\bar{a}$, with equal probabilities within this set. Conditional on this event, the prob-

ability that the CEO is from group B is $\frac{\Lambda_B}{\Lambda_A+\Lambda_B}$. With probability $1-\lambda$, the CEO is drawn from the set of all appointed individuals, with equal probabilities within this set. Conditional on this event, the probability that the CEO is from group B is $\frac{N_B}{N_A+N_B}$. This gives equation (8).

The function P_B is linear in λ, with $P_B(0) = \frac{N_B}{N_A+N_B} < \frac{1}{2}$ and $P_B(1) = \frac{\Lambda_B}{\Lambda_A+\Lambda_B} > \frac{1}{2}$ under the condition in the Proposition. Therefore, P_B is strictly increasing in λ, and there exists a unique $\hat{\lambda} \in (0,1)$ such that $P_B(\hat{\lambda}) = \frac{1}{2}$. The CEO is more likely to be from group B than from group A if and only if $P_B(\lambda) > \frac{1}{2}$, i.e. if and only if $\lambda > \hat{\lambda}$. Solving $P_B(\hat{\lambda}) = \frac{1}{2}$ for $\hat{\lambda}$ gives equation (9). ■

Proof of Remark 1:

The effort bounds are defined when equation (4) has two solutions s^*. The LHS of equation (4), maximized over s^*, is equal to $2\Phi\left(\frac{\overline{e}}{2\Sigma}\right)-1$. If $2\Phi\left(\frac{\overline{e}}{2\Sigma}\right)-1 < c$, equation (4) has no solution, no individual exerts effort, and the measure of individuals who exert effort is equal to zero.

Now suppose that equation (4) has two solutions. With $\mu_k = 0$, the change of variable $u = t_k - \frac{\sigma_k^2}{\sigma_k^2+\sigma_s^2} s^*$ rewrites equation (4) as:

$$\Phi\left(\frac{u}{\Sigma}\right) - \Phi\left(\frac{u-\overline{e}}{\Sigma}\right) = c, \tag{32}$$

with two solutions $u_1 < u_2$, so that $\underline{s}(t_k) = \frac{\sigma_k^2+\sigma_s^2}{\sigma_k^2}(t_k - u_2)$ and $\overline{s}(t_k) = \frac{\sigma_k^2+\sigma_s^2}{\sigma_k^2}(t_k - u_1)$. Equation (32) implies $1 - \Phi\left(\frac{u_2-\overline{e}}{\Sigma}\right) \geq c$, so that:

$$u_2 \leq \overline{e} + \Sigma\,\Phi^{-1}(1-c) \leq \overline{e} + \sqrt{\sigma_k^2+\sigma_q^2}\,\max\left\{0, \Phi^{-1}(1-c)\right\}, \tag{33}$$

where the second inequality uses $\Sigma^2 = \frac{\sigma_k^2\sigma_s^2}{\sigma_k^2+\sigma_s^2} + \sigma_q^2 \leq \sigma_k^2 + \sigma_q^2$.

Under the condition of the Remark, $t_k - u_2 \geq \delta$ for a constant $\delta > 0$ which does not depend on σ_s. The signal $\tilde{s}_i$ is normally distributed with mean zero and variance $\sigma_k^2 + \sigma_s^2$. The

measure of individuals who exert effort is therefore bounded above as follows:

$$\mathbb{P}\left(\tilde{s}_i \in [\underline{s}(t_k), \overline{s}(t_k)]\right) \leq 1 - \Phi\left(\frac{\underline{s}(t_k)}{\sqrt{\sigma_k^2 + \sigma_s^2}}\right) = 1 - \Phi\left(\frac{(t_k - u_2)\sqrt{\sigma_k^2 + \sigma_s^2}}{\sigma_k^2}\right) \leq 1 - \Phi\left(\frac{\delta\sqrt{\sigma_k^2 + \sigma_s^2}}{\sigma_k^2}\right), \quad (34)$$

which converges to zero as $\sigma_s \to \infty$. ■

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