Socially Responsible Divestment

Finance Working Paper N° 823/2022
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Responsible investing – the incorporation of environmental and social ("ES") factors into investment decisions – is becoming both increasingly important and increasingly contested. In 2006, the United Nations established the Principles for Responsible Investment, which was signed by 63 investors managing a total of $6.5 trillion. By 2025, this had grown to 5,261 investors with $140 trillion. However, despite the substantial amount of funds in responsible investing strategies, its real-world impact is debated. The strategies used in practice may not be the most effective at reducing externalities; moreover, investors may announce a strategy and not follow through with it. This paper studies both the effectiveness of different responsible investment strategies and their credibility when investors cannot commit to future trades.

A primary channel through which responsible investors aim to reduce externalities is divestment: selling "brown" companies that exert negative externalities, increasing their cost of capital and hindering their expansion. Under the cost of capital channel, the most powerful divestment strategy is blanket exclusion of externality-producing industries, a strategy pursued by many investors. The Net Zero Asset Managers Initiative originally required members to disclose net-zero aligned portfolios, which are most easily achieved through divesting emitting companies. Beyond climate, $4 trillion of assets worldwide are invested in exclusionary screening strategies. Turning to asset owners, 1,731, collectively managing $40 trillion, state that they will not invest in fossil fuels.[1] Many asset owners expect asset managers to exclude at least some sectors, and ask about such exclusions in their due diligence questionnaires. Activists have organized numerous campaigns and protests against banks and asset managers because of their holdings in fossil fuel companies.

However, this argument considers only one channel through which divestment can affect a company's real actions – the primary markets channel, whereby divestment affects new capital raising. As the survey of Bond, Edmans, and Goldstein (2012) points out, investors can also have real effects through a secondary markets channel. Specifically, buying or selling shares leads to the stock price reflecting a manager's real actions, thus rewarding or punishing him for taking them. Even if a firm is in an irremediably brown sector, where externalities are always negative, the manager may be able to take corrective actions to mitigate these externalities. Blanket exclusion fails to reward such actions because the firm is divested no matter what. Thus, it may be more effective for a responsible investor to pursue a conditional strategy

[1] Global Sustainable Investment Review (2022); Global Fossil Fuel Divestment Commitments Database, accessed June 2026.

that invests more in brown firms that have reduced their per-unit externalities – even though doing so allows such firms to expand and emit externalities over a larger scale. Practitioners sometimes refer to the latter as a "tilting" strategy, where the investor tilts away from a brown industry but does not exclude it entirely.

In practice, however, tilting faces two significant challenges. First, the corrective action may not be contractible or even observable. A fossil fuel company may invest in a clean technology whose eventual success is uncertain; an alcohol company may adopt new advertising practices whose effects are difficult to verify; a hard-to-abate industrial firm may undertake environmental initiatives whose effectiveness will only be learned over time. While investors observe ES disclosures, ratings, and other public signals, they are only imperfectly informative: for example, Berg, Kölbel, and Rigobon (2022) find that ES ratings differ significantly across rating agencies. Second, investors may be unable to credibly commit to a tilting strategy, given the non-contractibility of corrective actions. Once the signal has been observed, the action has already been taken (or not taken), and so the investor can only affect the firm's scale. She may therefore renege and not buy to limit its expansion. Thus, tilting is credible only if the investor will voluntarily buy upon a positive signal of reform.

We build a model in which conditional investment is hindered by both of the above frictions. In addition, our theory features both the primary and secondary markets channels through which responsible investment affects firm behavior, and so tilting has the cost of allowing the firm to expand and generate additional externalities. Despite incorporating the costs of tilting and the two frictions, we show that a conditional strategy can nevertheless be both effective in reducing externalities and credible, in contrast to the widespread use of blanket exclusion and criticism of investors that hold brown firms.

Our model features a company that emits negative externalities. The firm's manager privately observes whether a corrective action would be effective, and then chooses whether to take it. The action reduces firm value, and also lowers externalities if it is effective. It is unobservable, but investors observe a non-contractible public signal of whether the action was both taken and effective. The firm also raises capital to fund an expansion, which increases both firm value and externalities. A lower stock price reduces the amount raised and thus the scale of the expansion.

The firm's shares are traded by a continuum of atomistic, profit-motivated investors ("households") and a responsible investor. Households face trading costs, giving the responsible investor downward-sloping residual demand. The responsible investor may also face trading costs, which limit how much she buys. She differs from households in two ways. First, she can take a large position and have price impact. Second, her objective function depends on externalities as well as trading profits.

The responsible investor trades after observing the public signal. A conditional strategy is one in which she buys more after a favorable signal; an unconditional strategy is one in which she buys the same amount regardless of the signal. Conditioning gives the manager incentives to take the corrective action, thus reducing externalities, but also helps the brown firm expand, which increases externalities. Since the responsible investor cannot commit, however, the chosen strategy is not simply the one that minimizes externalities, or even maximizes her objective function ex ante. Instead, it depends on which one is credible, and a conditional strategy is feasible only if the responsible investor's trades are optimal ex post.

Our first result is that the corrective action can be induced only when the responsible investor's concern for externalities is intermediate. If it is too high, then her concern for profits is too low. She then has little incentive to buy shares even after a favorable public signal. At that point, the action has already been taken (or not taken), and the responsible investor can affect only the firm's scale. Buying shares allows the firm to expand and generate additional externalities, which she is unwilling to do due to her social concerns. Thus, a promise to reward a favorable signal is not credible, and the only equilibrium is one in which the manager never takes the action, even when it is effective (the "no-action" equilibrium).

If externality concerns are intermediate, then the responsible investor's profit motive gives her an ex-post reason to buy shares, making it credible to reward a favorable signal. Her concern for externalities leads her to buy more after a positive than negative signal: expansion generates fewer externalities if the firm has been reformed. The additional purchases, and thus the higher stock price, upon a positive signal give the manager incentives to take the action. Thus, there is an "action equilibrium" in which the manager takes the action with positive probability if it is effective. Within this equilibrium, the probability of taking the action is weakly increasing in the responsible investor's externality concerns.

If externality concerns are too low, then the responsible investor's profit concerns are too high, and she buys a large number of shares irrespective of the signal. Since she places little weight on externalities, an unfavorable signal does not deter her from buying. The additional purchases following a favorable signal are therefore too small to compensate the manager for taking the costly action, and the action equilibrium is unsustainable.

This result has nuanced implications for how policymakers define and enforce fiduciary duty. It is often claimed that a narrow definition – focused exclusively on financial returns – deters investors from having societal impact; many fund managers report that fiduciary duty constrains their ability to pursue ES objectives (Edmans, Gosling, and Jenter, 2025). However, a fund that cares only about externalities may be unable to induce the action. This is not for the standard reason that corrective actions reduce firm value: their expected cost is reflected in the stock price and does not affect trading profits. Instead, it is because such a fund has no reason to buy after a favorable signal, making conditional investment non-credible. Ensuring that funds pay attention to financial returns may support, rather than hinder, societal impact. However, if the profit motive is too strong, the investor barely conditions on the signal and so fails to induce reform.

Beyond the investor's objective, other comparative statics clarify when the action is more likely to be taken. One case is where the action is more effective or less costly. Thus, conditioning is less likely to be feasible in industries such as controversial weapons, where it is difficult to reduce the harm produced, and more likely in sectors where firms can take meaningful corrective actions such as fossil fuel companies developing clean energy.[2] The probability of reform is also higher when household trading costs are greater, as this increases the responsible investor's price impact, and when the manager places more weight on the stock price as he is more influenced by the price reward from acting. This observation contrasts common concerns that managerial short-termism is always socially undesirable.

A particularly important determinant of the equilibrium is the precision of the public signal. We first treat signal precision as a parameter, affected by improvements in ES ratings, disclosure standards, and anti-greenwashing enforcement. A more precise signal means that the action is more likely to lead to a favorable signal and thus additional responsible investor purchases, strengthening the manager's incentives. Indeed, the action equilibrium cannot exist when disclosure quality is too low.

However, there is a countervailing force. As the signal becomes more precise, a favorable signal causes the stock price to more fully reflect the expected cost of the action, reducing the net price reward from acting. After a point, the signal is sufficiently informative that

[2] Cohen, Gurun, and Nguyen (2024) show that the fossil fuel industry produces more green patents than nearly any other sector, suggesting that companies within this industry can take corrective actions.

the responsible investor buys the minimum possible (zero) after an unfavorable signal. Then the trading benefit of further precision is limited – the responsible investor cannot reduce her purchases further after bad news – and the cost-pricing effect now dominates. A more precise signal can thus eliminate the action equilibrium.

This result builds on prior research on the effects of ES disclosure. Dhaliwal et al. (2011) show that it reduces the cost of capital and Matsumura, Prakash, and Vera-Muñoz (2014) find that it increases market valuations. Krueger, Sautner, Tang, and Zhong (2024) find that mandatory ES disclosure improves stock liquidity, while Christensen, Hail, and Leuz (2021) survey its effects on capital markets, stakeholders, and firm behavior. Our model studies a specific incentive channel: disclosure determines whether investor trading can induce non-contractible corrective actions.

We then allow the manager to choose disclosure precision. A minimum level of disclosure is necessary to sustain the action equilibrium. However, the manager may not prefer it to the no-action equilibrium: while the responsible investor buys more shares, increasing the stock price and firm value, the cost of the action reduces both. The manager's disclosure choice has a threshold structure. If the action is sufficiently inexpensive, he prefers the action equilibrium and chooses high (potentially full) disclosure to sustain it. If the action is sufficiently costly, he chooses minimal disclosure to rule out the action equilibrium. Our model thus offers a new explanation for imperfect ES disclosure. In Xue (2025), firms voluntarily disclose less because investors value opacity. In our model, the manager may instead choose imperfect disclosure because greater disclosure would hold him accountable for taking a costly corrective action.

We finally conduct a welfare analysis, studying the effect of the equilibrium on expected externalities, firm value, responsible investor utility, and household utility. Externalities need not be lower in the action equilibrium: even though the action lowers per-unit externalities, it also causes the responsible investor to buy more shares, helping the firm expand. If the firm is sufficiently small, the per-unit reduction applies to a small base, and so externalities are higher in the action equilibrium. Since stronger externality concerns can make the action equilibrium sustainable, they may paradoxically increase expected externalities.

Combining the disclosure and welfare analyses, private and social preferences over disclosure can diverge in both directions. The manager may choose low disclosure to eliminate the action equilibrium, even when that equilibrium would lower expected externalities. More surprisingly, he may choose disclosure that sustains the action equilibrium even when that equilibrium increases expected externalities, because disclosure allows him to be rewarded by additional responsible investor purchases. Thus, optimal disclosure policy may involve ceilings as well as floors on disclosure quality. The optimal level of disclosure is higher for larger firms, not for the standard reason that they are more able to bear the cost (disclosure is costless in our model) but because corrective actions have a larger impact when applied to a larger base.

1 Related Literature

In addition to the disclosure literature already cited, this paper is related to responsible investment theories in which conditioning capital on a corrective action induces the action. Heinkel, Kraus, and Zechner (2001) feature a set of responsible investors that refuse to hold brown stocks. Non-investment by them requires financial investors to hold more stocks, increasing the risk they bear and lowering the stock price. As a result, some firms take corrective actions to obtain a higher price. In Oehmke and Opp (2025), a responsible investor induces a clean technology by purchasing, at a financial loss, a green bond that stipulates this technology.[3] They also show that the responsible investor's impact is monotonically decreasing in her profit motives. In our paper, the responsible investor does not need to make losses to induce reform; indeed, profit motives make a conditional strategy credible. In Landier and Lovo (2025), a responsible fund sets a cap to externalities which limits output and thus firm profits, and withholds financing from any firm that exceeds this cap.

These papers require responsible investors to be able to specify the investment rule ex ante and follow it ex post. Firms know what they must do to become eligible for responsible capital, and investors can commit to providing this capital. In reality, investors may not know what corrective actions a company can take ex ante, or verify them sufficiently well ex post to be stipulated in a contract. In the language of Dow and Gorton (1997), investors may have retrospective but not prospective information – their expertise is evaluating rather than running companies, or stock selection rather than activism. Moreover, even if investors observe a favorable signal of reform, it is difficult to credibly commit to additional purchases. Mutual fund mandates rarely stipulate positive investment in particular firms: a fund can always claim that a stock is overvalued or uninvestible for other reasons, or that it has better investment

[3] In a similar vein, in Chowdhry, Davies, and Waters (2019), the impact investor makes a donation in return for the entrepreneur committing to pursue social goals.

opportunities elsewhere. We show that, even when corrective actions are non-contractible and the responsible investor cannot commit, she can still induce reform. However, her ability to do so depends on her profit motives and disclosure precision, both in non-monotonic ways. Another difference is that Heinkel, Kraus, and Zechner (2001) and Oehmke and Opp (2025) do not consider the trade-off between conditional and unconditional strategies that is central to our analysis, and Landier and Lovo (2025) do not do so when they allow investors to choose funds.[4] While we study how different responsible investment strategies affect a given firm, Green and Roth (2025) study which firms to invest in to maximize impact. They find that, if responsible investors buy companies that create positive externalities, their impact may be limited as they displace traditional investors.

Some empirical studies examine the effectiveness of divestment, finding mixed results. Teoh, Welch, and Wazzan (1999) show that the South Africa exclusion campaign had a negligible effect on company valuations. The model and calibration of Berk and van Binsbergen (2025) show that ES-motivated exclusion has little effect on the cost of capital, because other investors can buy the underpriced stocks. Green and Vallée (2025) find that bank divestment policies reduce coal firms' total assets because substitution by other lenders is limited. Pastor, Stambaugh, and Taylor (2022) document a significant return differential between German green bonds and otherwise-equivalent non-green twins, also implying limited substitution. More broadly, Koijen and Yogo (2019) find a significant effect of institutional investor demand on asset prices. Our contribution is not to ask whether divestment has price impact in general, but how a responsible investor uses whatever price impact she has: unconditionally to starve the firm of capital, or conditionally to reward reform.

[4]In Heinkel, Kraus, and Zechner (2001), responsible investors' demands are deterministic, so they do not choose between strategies. Oehmke and Opp (2025) deliberately shut down the exclusion channel by assuming risk-neutral financial investors and a perfectly elastic supply of profit-motivated capital. In Landier and Lovo (2025), exclusion is only effective if it leads to significantly lower returns than conditional strategies. Thus, when they allow clients to choose funds, they consider only conditional strategies. Models of governance through exit, such as Admati and Pfleiderer (2009), Edmans (2009), and Edmans and Manso (2011), also show that making divestment conditional upon an action induces the action. These papers do not feature externalities and thus there is no role for exclusion.

2 The Model

We consider a single publicly traded firm operating in a brown industry. The firm is managed by a risk-neutral manager and has $\overline{q} > 0$ shares outstanding. There are two types of risk-neutral investors: a continuum of atomistic, profit-motivated investors ("households"), indexed by $i \in [0, 1]$, and a large "responsible investor" whose preferences depend on both financial returns and the firm's externalities. Households hold the entire initial supply of shares $\overline{q}$.[5]

There are three dates, $t \in \{1, 2, 3\}$. At $t = 1$, the manager decides whether to undertake a corrective action $a \in \{0, 1\}$, at a cost $c > 0$, that mitigates the firm's negative externalities. These externalities are denoted f and specified below. The manager's action is unobservable to investors. Examples of such actions include a fossil-fuel company investing in clean-energy technologies, an alcohol producer curbing youth-targeted advertising, or a firm in a hard-to-abate sector (e.g., steel, cement, or ammonia) investing in technologies that reduce emissions, pollution, or biodiversity loss.

The action reduces the firm's externalities by a fraction ae, where $e \in \{0, \overline{e}\}$ measures its effectiveness. It is effective with probability $\xi \in (0, 1)$, in which case $e = \overline{e} \in (0, 1)$, and ineffective with probability $1 - \xi$, in which case $e = 0$. Importantly, because $\overline{e} < 1$, the firm continues to generate negative externalities even when the action is undertaken and effective. Examples include fossil-fuel companies that retain carbon-intensive assets while investing in clean energy, alcohol producers whose products generate negative health consequences despite responsible advertising practices, and hard-to-abate industries that continue to pollute even after adopting emissions-reduction technologies.[6] As a result, even if the action is undertaken whenever it is effective, a responsible investor may still prefer exclusion to tilting.

The manager privately observes the effectiveness of the corrective action, e, before choosing a. He then selects the action to maximize

$$U_M = \omega p + (1 - \omega)v, \tag{1}$$

[5]The responsible investor's initial stake is zero, but all of our results would be unchanged if she had a positive initial stake, in which case "exclusion" would involve divestment, not just non-investment.

[6]The same logic applies in a multi-period setting. Certain industries generate negative externalities regardless of how many corrective actions they undertake or over how many periods. Consequently, unconditional and conditional investment strategies remain distinct. Full exclusion entails never holding a brown firm because it inevitably generates externalities, whereas conditioning entails increasing investment in a brown firm provided that it continues to undertake corrective actions.

where p is the market share price, v is the per-share long-term firm value, and $\omega \in [0,1]$ is the weight the manager places on the share price. This concern for the short-term price is standard in the literature and can arise from a variety of sources, such as takeover threats (Stein, 1988) or reputational concerns (Narayanan, 1985; Scharfstein and Stein, 1990). All our results continue to hold when $\omega = 0$.

At $t = 2$, the firm raises external financing by issuing $q > 0$ new shares. Given the market price p, the firm raises $I(p) = pq$ to finance an investment project that yields a gross return $r > 1$ per dollar invested. The firm's long-term (per-share) value is therefore

$$v = \frac{\mu + rI(p) - ca}{\overline{q} + q}, \tag{2}$$

where μ represents the expected cash flow generated by the firm's assets in place.[7]

To focus on the responsible investor's trade-off between incentivizing corrective actions and restricting access to capital, we take the firm's share issuance q as exogenous. One interpretation is that q is constrained by the amount of equity that can be raised before the agency costs of outside equity become prohibitively large (e.g., Jensen and Meckling, 1976; Leland and Pyle, 1977). Conditional on q, a lower stock price reduces the amount of capital raised and therefore the scale of investment.[8] The parameter q captures the firm's dependence on external equity financing. Firms that rely primarily on retained earnings require relatively little external capital and therefore have low q, whereas firms with substantial financing needs have high q.

Once shares are issued, and before trade takes place, all investors observe a non-contractible public signal $s \in \{0,1\}$ about the firm's expected externalities and trade claims to the firm's terminal value. The signal s is distributed according to

$$\Pr[s = 1 | ae = \overline{e}] = \Pr[s = 0 | ae = 0] = \tau, \tag{3}$$

[7]We assume the cost c does not reduce the funds available for investment. For example, it could represent the choice of a greener technology, which requires no up-front investment but reduces future profits. If the cost c reduced investment I, then this would create an additional benefit of conditioning – the firm uses some capital to reduce its externalities rather than to expand.

[8]In a richer model, the firm could optimally choose q. A lower stock price p would then reduce the optimal issuance level, leading to lower investment just as in the present setup. Our specification assumes constant returns to scale, so that the firm can invest any amount I and earn a constant gross return r per dollar invested. An alternative approach would be to assume decreasing returns to scale and endogenize q. While more realistic, doing so would substantially complicate the analysis without altering the qualitative mechanisms or results.

where $\tau \in \left[\frac{1}{2}, 1\right]$ denotes signal precision. The signal provides information about the reduction in externalities ae, rather than the action and its effectiveness separately. This captures the idea that ES ratings and related public disclosures are typically informative about outcomes (such as carbon emissions), rather than the specific actions taken to achieve them or their effectiveness.

Households submit price-dependent demand schedules x_i for the newly issued shares, conditional on the public signal s, to maximize trading profits net of a quadratic trading cost:

$$\max_{x_i} \; x_i \left(\mathbb{E}[v|s] - p\right) - \frac{\kappa_H}{2} x_i^2, \tag{4}$$

where $\kappa_H > 0$ captures trading frictions, such as illiquidity, transaction costs, and wealth constraints, that limit households' ability to build large positions in the firm (e.g., Vives, 1993).[9]

The responsible investor submits demand $x_R \geq 0$ to maximize expected utility $\mathbb{E}[U_R|s]$, where

$$U_R = x_R(v - p) - \frac{\kappa_R}{2} x_R^2 - \varphi f. \tag{5}$$

Here $\kappa_R \geq 0$ parametrizes her trading cost and $\varphi \geq 0$ her concern for externalities. Importantly, because she is non-atomistic, the responsible investor has price impact: she faces a downward-sloping residual demand curve generated by households' trading costs. This source of price impact does not require $\kappa_R < \kappa_H$: it arises from her size rather than lower trading costs.[10]

We assume that $x_R \geq 0$. This restriction captures short-sale constraints, which are standard in the "governance through trading" literature (Admati and Pfleiderer, 2009; Edmans, 2009). Allowing short sales would not qualitatively affect the main results, provided the responsible investor faces either short-sale costs or a limit on how much she can costlessly short.

As shown below, the responsible investor optimally conditions her trade on the public signal s, i.e., $x_R(s)$. The "conditional" strategy $x_R(1) > x_R(0)$ involves her holding more of

[9]Because each household has zero mass, its trade x_i and endowment are infinitesimal.

[10]Our results would be qualitatively unchanged if the large responsible investor were replaced by a continuum $j \in [0, 1]$ of small responsible investors with "warm-glow" (non-consequentialist) preferences, $U_{S,j} = x_{S,j}\left(v - p - \varphi_S f\right) - \frac{\kappa_S}{2} x_{S,j}^2$, under which each investor's disutility scales with her own holding of the firm times the externality, rather than her impact on the total externality. Although each investor's trade has no effect on aggregate quantities, her optimal trade depends on the signal through $\mathbb{E}[f|s]$. The manager's incentive depends on the aggregate demand across small responsible investors, which is similar to the large responsible investor's demand in the current model.

the stock if $s = 1$. The "unconditional" strategy $x_R(1) = x_R(0)$ does not depend on the signal. We refer to the unconditional strategy of $x_R(1) = x_R(0) = 0$ as "full exclusion". The key distinction is that the unconditional strategy increases the firm's cost of capital, which limits its expansion; the conditional strategy provides incentives to take corrective actions.[11] The latter rewards the manager through additional purchases upon a positive signal. This appears analogous to an incentive contract, but with an important difference. The incentive contracting literature assumes that the principal can commit to the contract, otherwise she will not make the promised payment. Here, the responsible investor is unable to commit to a trading strategy ex ante; instead, she trades optimally to maximize her objective function ex post. Thus, it is not obvious that a conditional strategy can be sustained in equilibrium.

Finally, at $t = 3$, the firm generates both a terminal cash flow $\mu + rI(p) - ca$ and externalities. The latter are given by

$$f = \lambda(\mu + rI(p))(1 - ae), \tag{6}$$

where $\lambda > 0$ determines the magnitude of the externality, $\mu + rI(p)$ represents the firm's scale, and ae captures the externality reduction through the action. Externalities stem from both assets in place (μ) and the expansion from the new investment (rI); for example, an oil and gas company's emissions arise from both its existing and new production capacity. The functional form for f implies that the action reduces the externality in a multiplicative way. For example, if the action involves developing a less polluting technology, it is implemented firm-wide, resulting in a greater effect on larger firms. We only require externalities to be increasing in firm size, not necessarily linear. If externalities were entirely independent of size, exclusion would have no effect because limiting the firm's expansion would not reduce externalities.

As mentioned in the introduction, exclusion is ineffective in some papers because externalities do not scale with the firm. Our paper models exclusion by featuring an externality that scales with firm size and households who face convex trading costs, so that divestment by the responsible investor has price impact. This captures the primary argument for exclusion used in practice. Despite incorporating the very features that make exclusion effective and condi-

[11] Practitioners sometimes refer to such a conditional strategy as "tilting" or "best-in-class." Because our model is parsimonious and features a single firm, the best-in-class concept does not apply literally; were the model extended to multiple firms, however, buying more of those that take corrective actions would amount to a best-in-class strategy.

tional investment difficult – the responsible investor cannot commit to a strategy or observe the action – we show that conditional investment may nevertheless be optimal.

3 Analysis

3.1 Trading strategies and the stock price

We first derive the market-clearing stock price. Households maximize their objective function (4) leading to demand of:

$$x_i(s) = \frac{\mathbb{E}[v|s] - p}{\kappa_H}. \tag{7}$$

Market clearing requires that total demand equals the supply of newly issued shares:

$$x_R(s) + \int_0^1 x_i(s)di = q. \tag{8}$$

Solving for p yields:

$$p = \mathbb{E}[v|s] - \kappa_H \left(q - x_R(s)\right) \tag{9}$$

with

$$\mathbb{E}[v|s] = \frac{\mu + rqp - \mathbb{E}[ac|s]}{\overline{q} + q}. \tag{10}$$

The stock price p equals the per-share value of the firm, net of a discount that reflects households' trading costs. We will later show that $q > x_R(s)$: households are net buyers of the stock and must receive a discount to incentivize them to incur the trading cost and buy.

Substituting (10) into equation (9) and solving for p gives:

$$p\left(x_R, s\right) = \frac{\mu - c \cdot \mathbb{E}[a|s] - \left(q - x_R\right)\left(\overline{q} + q\right)\kappa_H}{\overline{q} + q - rq}. \tag{11}$$

Ignoring the returns on the cash raised from the equity issue, the stock price would equal expected firm value net of a trading-cost discount, divided by the number of shares $\overline{q} + q$. One may think that the returns from investing this cash should add an additional term to the firm value in the numerator. However, since the value of the investment is rqp, it effectively reduces the number of shares by rq in the denominator.

We assume that $\overline{q} + q - rq > 0$, which can be rewritten as $\theta \equiv \frac{rq}{\overline{q}+q} < 1$. If r were too large, households' demand could become upward-sloping: a higher price would finance such a profitable investment opportunity that it would further increase value. Setting μ sufficiently large ensures that $p\left(x_R, s\right)$ is positive.

The next result solves for the responsible investor's optimal trade conditional on the public signal s. All proofs are given in the Appendix.

Lemma 1 *The responsible investor's optimal trade given the signal s is*

$$x_R(s) \equiv \frac{q}{\kappa_R/\kappa_H + 2} \max\left\{1 - \varphi \frac{r\lambda}{1-\theta}\left(1 - \mathbb{E}[ae|s]\right), 0\right\}. \tag{12}$$

Moreover, $x_R(1) \geq x_R(0)$.

If the responsible investor is purely profit-motivated ($\varphi = 0$), the optimal trade is $\frac{q}{\kappa_R/\kappa_H+2}$. She purchases a fraction of the newly issued shares that depends on her trading costs relative to those of households. Since $x_R(s) \leq q/2$, the responsible investor always buys less than the newly issued supply so households are net buyers in equilibrium. In addition, $x_R(s)$ is increasing in κ_H: she buys more when household trading costs are high, since households are then less willing to absorb the new shares.

If the responsible investor also has externality concerns (i.e., $\varphi > 0$), she buys fewer shares because she partially internalizes that her purchases increase the stock price, the funds raised, and thus the externalities emitted. If her externality concerns are sufficiently strong, then she engages in full exclusion, i.e., $x_R\left(1\right) = x_R\left(0\right) = 0$. Interestingly, her optimal trade is increasing in the expected externality reduction, $\mathbb{E}[ae|s]$, even though the externality-reducing action lowers firm value. The reason is that the expected cost of the action is reflected in the stock price and thus does not affect trading profits. Buying shares thus has the same trading benefit to the responsible investor but a lower externality cost, because helping the firm expand increases externalities by less when the firm is more likely to have reformed. Since the signal $s = 1$ is positive news about ae, we have $x_R(1) \geq x_R(0)$.

3.2 Managerial actions

Conditional on observing the realized effectiveness e, the manager strictly prefers to take the action if and only if $\mathbb{E}[U_M|a=1,e] > \mathbb{E}[U_M|a=0,e]$. Using (1), this condition becomes:

$$\mathbb{E}[p(x_R(s),s)|a=1,e] - \mathbb{E}[p(x_R(s),s)|a=0,e] \;> \frac{c}{\overline{q}+q}\frac{1-\omega}{\omega+(1-\omega)\theta}. \tag{13}$$

The left-hand side is the price benefit from taking the action, which depends on how much the action affects the likelihood of the good signal. A good signal affects the stock price in two ways: it increases it by raising the responsible investor's purchases if she follows a conditional strategy (i.e., $x_R(1) > x_R(0)$), and reduces it as the expected cost of the action is reflected in the price. A higher stock price always benefits the manager because it enables the firm to raise more capital and thereby increases its long-term value v; this benefit is greater the higher is ω, since the manager then also places more direct weight on the stock price. The right-hand side captures the loss in per-share firm value from the costly action. The manager takes the action only if the expected benefit exceeds this cost.

When $e=0$, the action is ineffective, and so the distribution of the signal is unaffected by whether the manager takes the action. Since the stock price depends only on the signal, the action has no impact on the stock price; its only effect is to reduce firm value due to its cost. As a result, the manager always chooses $a=0$ if $e=0$, regardless of the market's expectations.[12]

Lemma 2 *In equilibrium, the manager chooses $a=0$ if $e=0$.*

When $e=\overline{e}$, the action is effective. Suppose the manager chooses $a=1$ with probability $\eta \in [0,1]$ and $a=0$ otherwise. If $\eta \in (0,1)$, the manager randomizes, and so must be indifferent between taking the action and not taking it. The unconditional probability that an effective action is taken is thus given by $\xi\eta$.

Whenever $\eta > 0$, investors cannot perfectly anticipate whether the manager will take the action because the realization of e is the manager's private information. They use the public signal s to update their beliefs about the expected reduction in externalities, ae. Let $\alpha(s;\eta)$

[12]Under the knife-edge case of $\omega=1$, there exists an equilibrium in which $a=1$ even when $e=0$. This equilibrium, however, is non-generic: it unravels for any arbitrarily small personal effort cost to the manager of taking the action. We therefore ignore it from this point on.

denote the posterior probability, given signal s, that an effective action was taken:

$$\alpha(s;\eta) \;\equiv\; \Pr[ae=\overline{e}|s] \;=\; \begin{cases} \dfrac{\xi\eta\tau}{\xi\eta\tau+(1-\xi\eta)(1-\tau)} & \text{if } s=1, \\ \dfrac{\xi\eta(1-\tau)}{\xi\eta(1-\tau)+(1-\xi\eta)\tau} & \text{if } s=0. \end{cases} \tag{14}$$

For $\eta > 0$, $\alpha(1;\eta) > \alpha(0;\eta)$. A favorable signal raises the market's belief that an effective action was taken, influencing both the responsible investor's trade and the households' pricing of the action cost.

3.3 Equilibrium

A "no-action" equilibrium in which the manager never takes the action, $\eta^* = 0$, always exists. If $\eta = 0$, the signal is uninformative: the posteriors after each signal in equation (14) both equal zero. All investors therefore trade the same quantities after both signals, and the stock price does not depend on the manager's action. He thus does not act, even when $e = \overline{e}$.

We are interested in when "action" equilibria with $\eta^* > 0$ exist. As shown in Lemma 1, the responsible investor's optimal trade depends on the expected reduction in externalities, $\mathbb{E}[ae|s] = \alpha(s;\eta^*)\overline{e}$. Since the latter depends on the manager's equilibrium strategy η^*, we denote the responsible investor's trading strategy as $x_R(s;\eta^*)$. Define $\mathcal{B}(\eta)$ as the manager's net benefit from taking the action when it is effective, i.e.:

$$\mathcal{B}(\eta) \;\equiv\; x_R(1;\eta) - x_R(0;\eta) - \mathcal{C}(\eta), \tag{15}$$

where $x_R(1;\eta) - x_R(0;\eta)$ captures the responsible investor's additional purchases upon a good signal and

$$\mathcal{C}(\eta) \;=\; \frac{c}{\kappa_H(\overline{q}+q)}\left(\frac{(1-\omega)(1-\theta)}{\omega+(1-\omega)\theta}\frac{1}{2\tau-1} + \alpha(1;\eta) - \alpha(0;\eta)\right) \tag{16}$$

captures the effective cost of the action to the manager. The first term inside the parentheses reflects the manager's concern for firm value and is present only when $\omega < 1$; we focus on this case in what follows and discuss the knife-edge case $\omega = 1$ in the Appendix. The second and third terms capture the stock price adjustment to the inferred cost of the action.[13]

[13] We use the convention that $\mathcal{C}(\eta) = \infty$ whenever $\tau = \frac{1}{2}$.

For $e = \overline{e}$, condition (13) can be rewritten as

$$\mathcal{B}(\eta) \geq 0. \tag{17}$$

An equilibrium with $\eta \in (0,1)$ requires $\mathcal{B}(\eta) = 0$, since the manager must be indifferent. An equilibrium with $\eta = 1$ requires $\mathcal{B}(1) \geq 0$.

We impose a standard stability criterion to reduce, but not eliminate, equilibrium multiplicity. The criterion is the usual notion of dynamic (best-response) stability that dates back to Samuelson (1941) and is widely used to refine equilibria in models with strategic complementarities (e.g., Cooper and John, 1988). An equilibrium is stable if a small increase (decrease) in investors' beliefs (responsible investor and households) about the likelihood that the manager takes the action when it is effective decreases (increases) the manager's net benefit from taking the action. Intuitively, the equilibrium is then robust to small perturbations in investors' beliefs. Graphically, $\mathcal{B}(\eta^*)$ must cross zero from above.[14] Formally:

Stability condition: An equilibrium η^* is stable if for all $\epsilon > 0$ sufficiently small: either $\eta^* = 0$ or $\mathcal{B}(\eta^* - \epsilon) > 0$; and either $\eta^* = 1$ or $\mathcal{B}(\eta^* + \epsilon) < 0$.

Hereafter, we refer to a stable equilibrium simply as an equilibrium.

Proposition 1 *An equilibrium with $\eta^* = 0$ always exists; if $\tau = \frac{1}{2}$, it is the unique equilibrium. If $\tau > \frac{1}{2}$, there exists $\hat{c} > 0$ such that an equilibrium with $\eta^* > 0$ exists if and only if $c < \hat{c}$ and $\varphi \in (\underline{\varphi}, \overline{\varphi})$, for some $0 < \underline{\varphi} < \overline{\varphi} < \infty$. Whenever an equilibrium with $\eta^* > 0$ exists, it is unique among such equilibria, and η^* is weakly increasing in φ.*

Proposition 1 is our central result. It shows that the manager's incentive to act is non-monotonic in the responsible investor's concern for externalities: the action is taken with positive probability only for intermediate values of φ.

If φ is too low (i.e., $\varphi < \underline{\varphi}$), the responsible investor is primarily profit-motivated and buys large amounts under both signals, since the shares trade at a discount. She buys somewhat

[14] For instance, suppose $\mathcal{B}$ crosses zero from below at some interior point $\eta^* \in (0,1)$, i.e. $\mathcal{B}(\eta^* - \epsilon) < 0 < \mathcal{B}(\eta^* + \epsilon)$. This point is a fixed point of the belief–action mapping but fails our criterion. It is not robust: a marginal increase in investors' belief that the manager acts when effective raises his net benefit from acting, so he acts with even higher probability, which raises investors' belief further. Beliefs and actions thus spiral away from η^* toward the adjacent stable equilibrium at $\eta = 1$, while a marginal decrease unravels symmetrically toward $\eta = 0$. Our criterion discards this knife-edge equilibrium and retains the stable equilibria on either side.

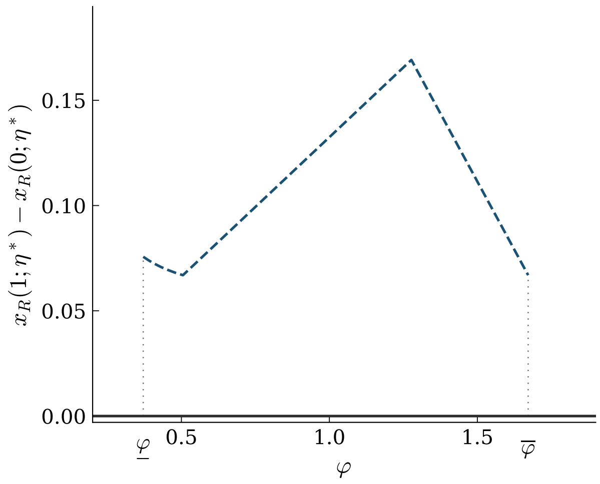
less following a negative signal, because expansion raises externalities by more when the firm is unlikely to have reformed. However, when externality concerns are weak, this reduction is small. The resulting price differential across signals is too low to compensate the manager for the action cost c.

If instead φ is too high (i.e., $\varphi > \overline{\varphi}$), the responsible investor buys few shares under either signal. Purchases lower the firm's cost of capital, facilitating its expansion and increasing its externalities. Thus, higher externality concerns reduce her purchases under both signals. After a point, the short-sale constraint binds and she buys the minimum possible (zero) after a negative signal. This constraint compresses the difference between the responsible investor's trades after favorable and unfavorable signals, thereby dampening the manager's price reward from acting. When φ is sufficiently large, the constraint binds after both signals: she fully excludes the firm, and the manager's incentives to act disappear.

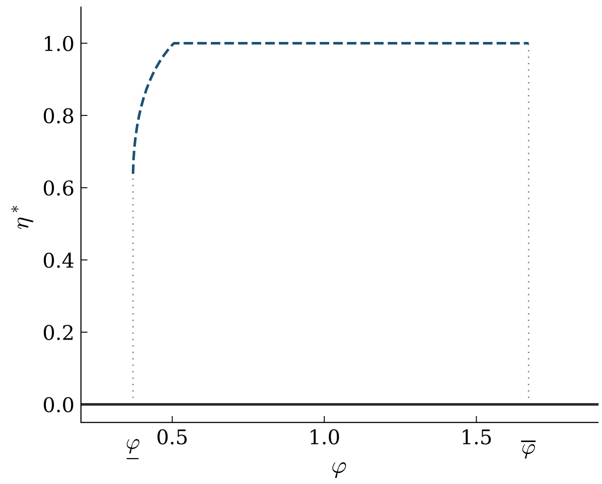
For intermediate φ, she is willing to buy after a favorable signal, because the firm is likely to have reformed and so additional purchases generate fewer externalities, but reluctant to buy after an unfavorable signal. This price differential incentivizes the manager to take the action. The condition $c < \hat{c}$ ensures that the action cannot be too costly, otherwise no feasible price reward can compensate the manager. Within the window $(\underline{\varphi}, \overline{\varphi})$, the equilibrium with $\eta^* > 0$ is unique and η^* is weakly increasing in φ: stronger externality concerns increase the equilibrium probability that the manager acts.

This result has implications for how fiduciary duty is defined and enforced. It is often argued that requiring investors to focus exclusively on financial returns limits their ability to generate social impact. Our model shows that this concern is incomplete. If fiduciary duty permits investors to place very high weight on externalities, they may exclude even firms that undertake corrective actions, making conditional investment non-credible and reducing their ability to induce reform.

Figure 1 illustrates the equilibrium structure characterized by Proposition 1. Panel (a) shows the responsible investor's trade differential $x_R(1;\eta^*) - x_R(0;\eta^*)$, which is zero in the no-action equilibrium (solid line) and positive in the action equilibrium (dashed line). The action equilibrium exists over the range of φ where this spread exceeds the manager's effective cost $\mathcal{C}(\eta^*)$, yielding the window $(\underline{\varphi}, \overline{\varphi})$. Panel (b) shows that, at $\underline{\varphi}$, η^* is strictly below 1. As φ rises, the responsible investor conditions more strongly on the signal, increasing the manager's incentive to act and thus raising η^* until it reaches 1 at $\varphi = 0.51$; the action equilibrium



(a) $x_R(1;\eta^*) - x_R(0;\eta^*)$ vs. φ



(b) η^* vs. φ

Figure 1: Panel (a) plots $x_R(1;\eta^*) - x_R(0;\eta^*)$, the spread between the responsible investor's demand after the two signals, as a function of her social concerns φ. The solid line is the equilibrium with $\eta^* = 0$ and the dashed line is the equilibrium with $\eta^* > 0$. Panel (b) plots the equilibrium probability of the action η^* as a function of φ. Parameters: $r = 1.7$, $c = 0.25$, $q = \overline{q} = 1$, $\kappa_H = 1$, $\kappa_R = 0$, $\lambda = 0.2$, $\overline{e} = 0.9$, $\xi = 0.8$, $\tau = 0.6$, $\omega = 0.5$. The action equilibrium exists for $\varphi \in (\underline{\varphi}, \overline{\varphi})$ with $\underline{\varphi} = 0.37$ and $\overline{\varphi} = 1.67$.

then continues to exist, with $\eta^* = 1$, until $\overline{\varphi}$. Outside the window, the manager's incentive is insufficient and only the equilibrium with $\eta^* = 0$ survives.

Panel (a) shows that the trade differential depends on the responsible investor's externality concerns in a non-monotonic way. A rise in φ from $\underline{\varphi}$ has two effects. First, holding η^* fixed, it makes the responsible investor condition more strongly on the signal, because the signal affects the expected externalities facilitated by her purchases, increasing $x_R(1;\eta^*) - x_R(0;\eta^*)$. At the initial value of η^*, this strengthens the manager's incentive to act, increasing η^*. Second, when η^* rises further, the action is expected to be taken more often in equilibrium. Thus, an unfavorable signal provides less negative news about the action, so the posterior spread $\alpha(1;\eta^*) - \alpha(0;\eta^*)$ falls. As shown by Lemma 3 in the Appendix, this endogenous decline in signal informativeness more than offsets the direct effect of a higher φ, reducing the equilibrium value of $x_R(1;\eta^*) - x_R(0;\eta^*)$.

Once $\eta^* = 1$ (at $\varphi = 0.51$), beliefs no longer adjust. Further increases in φ then have only the first effect: the responsible investor reduces $x_R(0;1)$ relative to $x_R(1;1)$, increasing the trade differential. Eventually (at $\varphi = 1.28$), $x_R(0;1)$ reaches zero and cannot decline further with φ, but $x_R(1;1)$ continues to fall because expected externalities remain positive even if reform is likely. The trade differential then falls, and eventually becomes too small to sustain the action equilibrium.

Corollary 1 gives comparative statics for the equilibrium in Proposition 1.

Corollary 1 *An equilibrium with $\eta^* = 1$ is more likely to exist when the corrective action is more effective (higher $\bar{e}$) and less costly (lower c), when the manager places greater weight on the stock price (higher ω), when households face higher trading costs (higher κ_H), and when the responsible investor faces lower trading costs (lower κ_R). Whenever an equilibrium with $\eta^* \in (0,1)$ exists, the same comparative statics applies to the probability that the action is taken.*

When the action is more effective (higher $\bar{e}$), a favorable signal implies a greater externality reduction. As a result, buying more shares is significantly less costly for the responsible investor after good than bad news. This widens her trade differential $x_R(1;\eta^*) - x_R(0;\eta^*)$ and increases the manager's price reward from acting.

As long as $\omega < 1$, the manager internalizes the negative impact of the action on firm value, so a lower cost c increases his incentive to act. A higher ω shifts the manager's payoff towards the stock price and away from firm value, making him less sensitive to the action's negative firm value impact and more responsive to the price reward generated by the responsible investor's trading. This increases his incentive to act.

Higher household trading costs κ_H make households less willing to absorb newly issued shares at any given price. As a result, each share not acquired by the responsible investor requires a larger price discount for households to absorb the remaining supply. This increases the responsible investor's price impact and thus the manager's price reward from taking the action. Lower responsible investor trading costs κ_R increase the responsible investor's purchases overall, but the effect is larger upon signals with greater desired purchases because marginal trading costs are increasing. This again widens the trade differential and increases the manager's price reward for reform.

3.4 Disclosure

We now study how the precision of the public signal, τ, affects both the existence of the action equilibrium and the probability of taking the action. We first treat τ as an exogenous parameter, reflecting factors such as improvements in ES ratings, disclosure standards, and anti-greenwashing enforcement. We then turn to the manager's endogenous choice of τ.

3.4.1 Exogenous disclosure quality

Define

$$\mathcal{T}_A \equiv \left\{\tau \in \left[\tfrac{1}{2}, 1\right] : \eta^* > 0\right\} \tag{18}$$

as the set of signal precisions for which there exists an action equilibrium. From Proposition 1, for every signal precision $\tau \in [\frac{1}{2}, 1]$, there exists $\bar{c}_\tau \geq 0$ such that $\tau \in \mathcal{T}_A$ if and only if $c < \bar{c}_\tau$. If $\bar{c}_\tau = 0$, then necessarily $\tau \notin \mathcal{T}_A$.

Proposition 2 characterizes $\mathcal{T}_A$ and how the signal precision affects the equilibrium probability that an action is taken.

Proposition 2

(i) *There exists $\underline{\tau} > \frac{1}{2}$ such that $\mathcal{T}_A \subseteq (\underline{\tau}, 1]$. Furthermore, suppose the short-sale constraint binds in the no-action equilibrium and $\bar{c}_1 > 0$. Then, there exists $\bar{\xi} \in (0,1)$ such that, for all $\xi > \bar{\xi}$, the cutoff $\bar{c}_\tau$ is strictly decreasing in τ to the left of $\tau = 1$.*

(ii) *Wherever an action equilibrium exists, the equilibrium probability of taking the action, η^*, is weakly increasing in τ.*

Part (i) of Proposition 2 first establishes that an action equilibrium exists only when the signal is sufficiently precise: only if $\tau > \underline{\tau}$. When the signal is too noisy, the responsible investor's trading responds weakly to the signal realization, providing the manager with insufficient incentives to act. The same intuition underlies part (ii). In a mixed-strategy equilibrium, greater signal precision amplifies the responsible investor's trading response to the signal, increasing the price reward from acting. This "trading effect" induces the manager to act more frequently.

More surprisingly, part (i) shows that the action equilibrium may disappear when the signal becomes too precise. The intuition is that the signal affects not only the responsible investor's trading but also households' valuation of the firm. As τ increases, a favorable signal provides stronger evidence that the manager has taken the action, so households increasingly incorporate the action's cost, c, into the stock price, which erodes the manager's price reward from acting. This "cost-pricing effect" counteracts the trading effect.

Crucially, the two effects need not scale symmetrically with precision. Once the responsible investor's demand following an unfavorable signal has fallen to zero, short-sale constraints preclude any further response to bad news: the trading effect can then strengthen only through additional purchases after a favorable signal, and is therefore bounded.[15] The cost-pricing effect, by contrast, continues to rise with τ. If the action cost is sufficiently low ($c < \bar{c}_1$), the cost-pricing effect never outweighs the trading effect, and the action equilibrium survives under full disclosure: $1 \in \mathcal{T}_A$. If instead $c > \bar{c}_1$ but $\mathcal{T}_A$ is non-empty, the cost-pricing effect eventually dominates, the manager's incentives decline with signal precision, and the action equilibrium disappears before full disclosure is reached: $1 \notin \mathcal{T}_A$. Part (i) shows that this occurs when the corrective action is likely to be effective (high ξ), since households then assign a high probability to reform following a favorable signal, increasing the cost-pricing effect. In that case, $\bar{c}_\tau$ is decreasing in τ near $\tau = 1$. Hence, for costs just above $\bar{c}_1$, an action equilibrium exists at sufficiently high but imperfect precision, yet is eliminated by full disclosure.

Figure 2 illustrates. The gray region shows the pairs (τ, c) for which the action equilibrium exists; for the chosen parameter values, the equilibrium involves $\eta^* = 1$. When $c \approx 1$, the action equilibrium exists only for intermediate levels of signal precision and fails under full disclosure ($\tau = 1$). The cutoff $\bar{c}_\tau$ determining existence is non-monotonic in τ: it initially increases because the trading effect dominates, then decreases once the short-sale constraint binds.

This mechanism differs from the standard argument that short-sale constraints weaken governance through exit. Here, their effect depends critically on disclosure: more precise signals would strengthen incentives if the responsible investor could respond more aggressively to unfavorable news, but short-sale constraints cap that response. Improving disclosure without relaxing trading constraints can therefore backfire, suggesting that disclosure policy and trading constraints should be designed jointly.

3.4.2 Endogenous disclosure quality

We now let the manager publicly choose the precision τ at $t = 0$, before observing any private information, to maximize his expected utility, $\mathbb{E}[U_M]$. We assume henceforth that whenever

[15] Formally, Proposition 2 states the sufficient condition in terms of the no-action equilibrium. Under full disclosure ($\tau = 1$), an unfavorable signal in the action equilibrium induces the same posterior beliefs – and thus the same responsible investor demand – as in the no-action equilibrium.

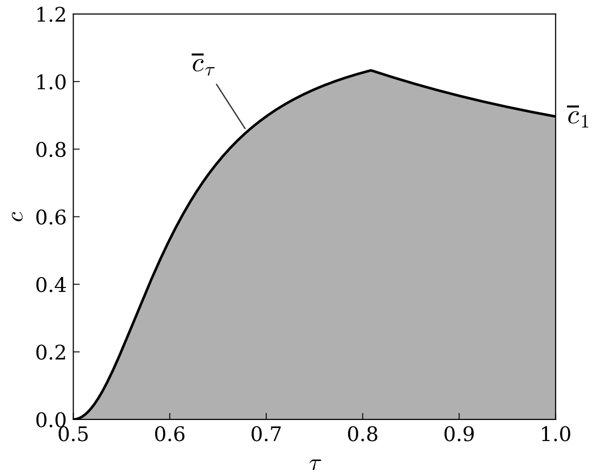


Figure 2: Existence of the action equilibrium ($\eta^* > 0$) as a function of signal precision τ and action cost c. The gray region shows the pairs (τ, c) for which the action equilibrium exists; in the white region only the no-action equilibrium ($\eta^* = 0$) prevails. Parameters: $\overline{e} = 0.95$, $\xi = 0.5$, $\kappa_H = 1$, $\kappa_R = 0$, $q = \overline{q} = 1$, $r = 1.1$, $\lambda = 1$, $\omega = 0.9$, $\mu = 2.2$, and $\varphi = 0.5$.

an equilibrium with $\eta^* > 0$ exists, it is played with strictly positive probability. We further assume that when the manager is indifferent among several choices of τ, he selects the lowest one whenever it exists.[16] If disclosure were costly (e.g., Verrecchia, 1983), even an arbitrarily small disclosure cost would make the lowest precision strictly optimal.

Proposition 3

(i) The manager is indifferent to τ in the no-action equilibrium and weakly prefers larger τ in any action equilibrium. Moreover, this preference is strict if and only if the responsible investor's expected purchases are strictly increasing in τ.

(ii) If $1 \in \mathcal{T}_A$ (equivalently, $c < \overline{c}_1$), then the manager strictly prefers the action equilibrium to the no-action equilibrium; he thus chooses $\tau^* \in \mathcal{T}_A$. Otherwise, the manager weakly prefers the no-action equilibrium (strictly if $c > \overline{c}_1$) and chooses $\tau^* = \frac{1}{2}$, even when $\mathcal{T}_A$ is non-empty.

In the no-action equilibrium ($\eta^* = 0$), all investors ignore the signal, since they expect the manager never to take the action, and thus interpret favorable signals ($s = 1$) as pure noise.

[16]If the set of payoff-maximizing precisions has no lowest element, the choice among them is immaterial for our results because they all yield the same payoff.

Thus, neither the stock price nor firm value depends on τ, and so the manager is indifferent to signal precision. In contrast, in the action equilibrium ($\eta^* > 0$), a more precise signal weakly raises the responsible investor's expected purchases. If short-sale constraints do not bind and $\eta^* = 1$, an increase in τ causes the responsible investor to buy more upon a positive signal and less upon a negative signal, and so expected purchases are unchanged. However, if short-sale constraints bind, then only the first effect arises and expected purchases increase; in a mixed-strategy equilibrium, expected purchases also rise because η^* increases with τ. As a result, the manager's expected utility weakly increases in τ.

Part (ii) of Proposition 3 examines which equilibrium the manager prefers. On the one hand, the action reduces firm value due to its cost, in turn lowering the stock price. On the other hand, it increases the responsible investor's trades, raising the stock price and thus firm value. When the cost c is sufficiently low, the latter effect dominates.

The threshold separating the two cases is the same $\bar{c}_1$ that determines the existence of the action equilibrium at $\tau = 1$ in Proposition 2. The intuition is as follows. Under full disclosure, a manager who deviates from the action equilibrium by not acting generates $s = 0$ with certainty. Investors therefore form the same beliefs as in the no-action equilibrium, so the responsible investor's trade, the stock price, and firm value all coincide with their no-action values. Thus, deviating from the action equilibrium yields exactly the manager's payoff in the no-action equilibrium. The incentive constraint for sustaining the action equilibrium therefore reduces to comparing the manager's payoff in the action equilibrium with his payoff in the no-action equilibrium. Hence, the manager strictly prefers the action equilibrium if and only if $c < \bar{c}_1$.

Part (i) implies that, among action equilibria, the manager's utility is weakly increasing in τ. Hence, if $c < \bar{c}_1$, the manager prefers the action equilibrium and chooses a disclosure precision in $\mathcal{T}_A$; if the responsible investor's expected purchases are strictly increasing in τ, he strictly prefers $\tau = 1$. If $c \geq \bar{c}_1$, even the best attainable payoff from an action equilibrium does not exceed the no-action payoff, so the manager chooses $\tau^* = \frac{1}{2}$ to preclude the action equilibrium. Thus, even though disclosure is costless, the manager sometimes strictly prefers no disclosure.

Figure 3 plots the manager's expected utility as a function of τ, with the solid line corresponding to the no-action equilibrium and the dashed line to the action equilibrium. For $\tau < 0.545$, the action equilibrium does not exist. For $\tau > 0.545$, it exists but initially yields

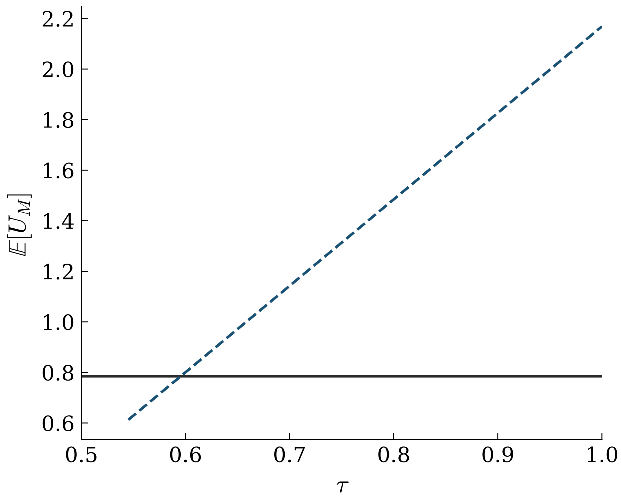


Figure 3: Manager's expected utility $\mathbb{E}[U_M]$ as a function of signal precision τ. The solid line is the equilibrium with $\eta^* = 0$ and the dashed line is the equilibrium with $\eta^* > 0$. Parameters: $q = 1$, $\overline{q} = 0.65$, $r = 1.57$, $\kappa_H = 1$, $\kappa_R = 0$, $\lambda = 0.38$, $\overline{e} = 0.5$, $\xi = 0.5$, $\omega = 0.59$, $\varphi = 0.11$, $\mu = 1.68$, $c = 0.04$. For these parameter values, $\mathcal{T}_A = (0.545, 1]$.

the manager lower utility than the no-action equilibrium, because the responsible investor's expected purchases are small. As τ increases further, expected purchases grow, raising the manager's utility in the action equilibrium until it eventually exceeds that in the no-action equilibrium ($\tau > 0.60$). Two features of the parameterization in Figure 3 are worth noting. First, $c < \bar{c}_1$, so the manager prefers the action equilibrium at $\tau = 1$. Second, the short-sale constraint binds after $s = 0$ throughout the region where the action equilibrium exists, so higher precision strictly raises expected purchases and hence the manager's utility.

The manager's conflicting incentives over disclosure imply that firms have incentives to tailor disclosure across investor groups. In particular, they prefer to provide relatively little ES information to profit-motivated households, thereby limiting the extent to which the stock price reflects the costly corrective action, while providing more informative disclosures to responsible investors, whose trading rewards firms for taking the action. Consequently, firms may have incentives to circumvent the spirit of Regulation FD by selectively communicating ES-related information to responsible investors. In practice, many large investors regularly meet with companies to gather qualitative information that is not fully captured by public disclosures, such as ES performance. This is known as "engaging for insight", as opposed to "engaging for influence", where investors seek to encourage specific actions.

3.5 Welfare effects

We finally exploit the multiplicity of equilibria to study the welfare effects of the corrective action. We analyze four potential components of a social welfare function: expected externalities, firm value, responsible investor utility, and household utility.

Proposition 4 *Suppose an equilibrium with $\eta^* > 0$ exists. Then:*

(i) There exists $\overline{\mu} > 0$ such that, if and only if $\mu > \overline{\mu}$, expected externalities $\mathbb{E}[f]$ are lower in the action equilibrium than in the no-action equilibrium.

(ii) Expected firm value may be either higher or lower in the action equilibrium than in the no-action equilibrium. There exists $\underline{c} > 0$ such that for all $c < \underline{c}$, we have $\eta^* = 1$ and expected firm value is strictly higher in the action equilibrium.

(iii) The responsible investor strictly prefers an action equilibrium to a no-action equilibrium.

(iv) Household net trading profits are lower in the action equilibrium than in the no-action equilibrium, but household utility can be either higher or lower. If κ_H is sufficiently large, household utility is higher in the action equilibrium with $\eta^* = 1$.

Part (i) shows that the corrective action does not necessarily reduce the firm's externalities. The action equilibrium reduces per-unit externalities, but also induces the responsible investor to trade more and help the firm expand. If the firm is small ($\mu < \overline{\mu}$), the per-unit reduction applies to a relatively small base, and the cost of capital channel dominates. Expected externalities are then higher in the action equilibrium. If the firm is large ($\mu > \overline{\mu}$), the per-unit reduction applies to a large base and expected externalities are lower.

Recall from Proposition 1 that the responsible investor's externality concerns affect the sustainability of the action equilibrium. Combining it with Proposition 4, part (i), shows that stronger externality concerns can paradoxically increase externalities. Suppose $\mu < \overline{\mu}$ and $\varphi \approx \underline{\varphi}$. Then, an increase in φ allows the action equilibrium to be supported; since this equilibrium leads to higher externalities when $\mu < \overline{\mu}$, expected externalities rise. In addition, suppose $\mu > \overline{\mu}$ and $\varphi \approx \overline{\varphi}$. Then, an increase in φ causes the action equilibrium to disappear; since this equilibrium leads to lower externalities when $\mu > \overline{\mu}$, expected externalities again rise. Thus, policies or campaigns that seek to increase investors' concern for externalities are not unambiguously beneficial and can worsen the outcome they seek to improve. This result is summarized in Corollary 2 below:

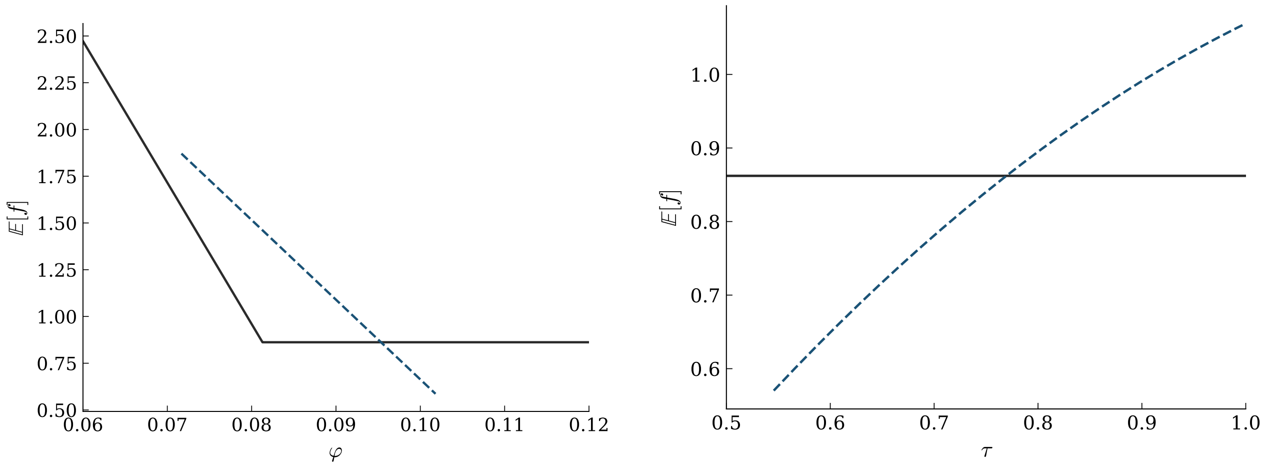
Corollary 2 *Suppose $\mu < \overline{\mu}$ and $\varphi \approx \underline{\varphi}$, or $\mu > \overline{\mu}$ and $\varphi \approx \overline{\varphi}$. Then a marginal increase in the responsible investor's social concerns φ raises expected externalities $\mathbb{E}[f]$.*

Figure 4 illustrates these effects. For the chosen parameter values, the action equilibrium involves $\eta^* = 1$. Panel (a) plots the expected externalities as a function of the responsible investor's externality concerns φ. At $\varphi = 0.072$, the action equilibrium begins to exist, as shown by the appearance of the dashed line. At this point, it has higher expected externalities than the no-action equilibrium ($\mu < \overline{\mu}$). Thus, $\mathbb{E}[f]$ jumps upward: while the action is taken with positive probability, the responsible investor trades substantially more.

As φ continues to rise, expected externalities fall within the action equilibrium, since the responsible investor's demand falls and the firm's cost of capital increases. As a result, the threshold $\overline{\mu}$ declines with φ. For $\varphi > 0.095$, we have $\mu > \overline{\mu}$: the firm's externalities are lower under the action equilibrium. At $\varphi = 0.104$, the action equilibrium ceases to exist and the no-action equilibrium has higher expected externalities. Thus, expected externalities again jump upward. At this level of φ, the responsible investor buys few shares in the action equilibrium, and so the corrective action dominates the cost of capital channel.

Recall also that Proposition 3 studied the manager's choice of disclosure. Combining it with Proposition 4, part (i), shows that privately chosen disclosure may be too low from the perspective of externality reduction. If the action equilibrium lowers expected externalities (because $\mu > \overline{\mu}$) but the manager prefers the no-action equilibrium, he chooses $\tau = \frac{1}{2}$ even though some $\tau \in \mathcal{T}_A$ would sustain the action equilibrium. Intuitively, disclosure would hold the manager accountable for taking the action, which he does not want to do because it is costly. Thus, a minimum level of ES disclosure may be valuable in encouraging firms to undertake corrective actions and reduce externalities.

More surprisingly, privately chosen disclosure may also be socially excessive. When the manager prefers the action equilibrium, he chooses high disclosure to sustain it, even if the equilibrium increases expected externalities because $\mu < \overline{\mu}$. Intuitively, he wants to disclose the action so that he is rewarded by additional responsible investor purchases, even though they increase externalities. A policymaker concerned with externality reduction may thus limit disclosure precision to rule out the action equilibrium.



(a) $\mathbb{E}[f]$ vs. φ

(b) $\mathbb{E}[f]$ vs. τ

Figure 4: Expected externality $\mathbb{E}[f]$ as a function of her social concerns φ (Panel (a)) and signal precision τ (Panel (b)). The solid line is the equilibrium with $\eta^* = 0$ and the dashed line is the equilibrium with $\eta^* > 0$. Common parameters: $q = 1$, $\overline{q} = 0.65$, $r = 1.57$, $\kappa_H = 1$, $\kappa_R = 0$, $\lambda = 0.38$, $\overline{e} = 0.5$, $\xi = 0.5$, $\omega = 0.59$, $\mu = 1.68$. Panel (a): $c = 0.12$ and $\tau = 0.55$. Panel (b): $c = 0.04$ and $\varphi = 0.11$. For these parameter values, $\underline{\varphi} = 0.072$, $\overline{\varphi} = 0.104$, and $\mathcal{T}_A = (0.545, 1]$.

Overall, optimal disclosure policy may involve ceilings as well as floors on disclosure quality: more disclosure is not always socially desirable, in contrast to the conventional view that privately chosen disclosure is typically too low (Leuz and Wysocki, 2016). The optimal level of disclosure depends on firm size and is higher for larger firms. However, this is not for the standard reason that disclosure is too costly for small firms: in our model, disclosure is costless. Instead, it is because ES disclosure encourages actions that improve ES performance, and such actions are more impactful in larger firms.

When the manager chooses disclosure, we have seen that the regulator may limit disclosure to rule out the action equilibrium. A different rationale for imperfect disclosure arises when disclosure is instead chosen by the regulator, as in Proposition 2. Here, the regulator may limit disclosure to *enable* the action equilibrium. If the action equilibrium involves lower externalities and can be sustained only under imperfect disclosure, the regulator will choose imperfect disclosure to sustain it. This rationale does not exist when the manager chooses disclosure, since if $1 \notin \mathcal{T}_A$, the action equilibrium would give him lower utility. He therefore chooses $\tau = \frac{1}{2}$ and there is no need for a disclosure ceiling.

Panel (b) of Figure 4 illustrates these nuances, studying how disclosure τ affects expected externalities. Within the action equilibrium, higher τ generally has two opposing effects. On the one hand, it increases the responsible investor's trading response to the signal and thus the manager's reward from acting. This increases his frequency of acting in a mixed-strategy equilibrium, reducing externalities. On the other hand, it weakly increases the responsible investor's purchases, increasing externalities. For the parameter values in Figure 4, we have $\eta^* = 1$ in the action equilibrium so the first effect disappears; as a result, externalities are increasing in signal precision. For low τ, $\mu > \overline{\mu}$ and externalities are lower in the action equilibrium than in the no-action equilibrium; for high τ, $\mu < \overline{\mu}$ and externalities are higher in the action equilibrium.

This panel should be compared with Figure 3, which has the same parameter values and illustrates how τ affects the manager's utility. Figure 3 showed that the manager prefers the action equilibrium if and only if $\tau > 0.60$, whereas Figure 4 shows that the action equilibrium leads to higher externalities if and only if $\tau > 0.77$. Thus, for $\tau \in [0.60, 0.77]$, the manager prefers the action equilibrium and this equilibrium reduces externalities: private and social incentives coincide. However, for $\tau > 0.77$, the manager prefers the action equilibrium even though it increases externalities. He has the incentive to increase disclosure above 0.77 (all the way to 1, if he can), even though this would increase expected externalities.

Part (ii) of Proposition 4 shows that the action has a nuanced effect on firm value. It directly reduces firm value by c, but induces the responsible investor to purchase more shares, which lowers the cost of capital. When c is sufficiently small, this latter effect dominates, and the action increases firm value.

Part (iii) of Proposition 4 states that, all else equal, the responsible investor strictly prefers the action equilibrium, even when it results in higher externalities. The intuition is as follows. First, the action directly reduces externalities. Second, she can reoptimize her trading strategy. Since she could always continue trading as before, reoptimization cannot make her worse off. In fact, because each purchase now facilitates fewer externalities, she optimally buys more shares and is therefore strictly better off.

The policy implication is that responsible investor welfare is not a proxy for social welfare. The responsible investor always prefers the action equilibrium, even when it raises externalities, and thus may advocate outcomes that are socially undesirable. Moreover, the solution is not necessarily to increase the responsible investor's social concerns: doing so may increase expected externalities by affecting the sustainability of the action equilibrium.

Finally, part (iv) analyzes the effect on households' utility. It is given by

$$x_i\,(v-p) - \frac{\kappa_H}{2}x_i^2 + \overline{q}v. \tag{19}$$

The first two terms capture households' net trading profits. In the proof, we show that in equilibrium, this term simplifies to $\frac{\kappa_H}{2}\mathbb{E}[(q - x_R(s))^2]$. By Lemma 1, the responsible investor always purchases fewer than q shares, implying that households are net buyers in equilibrium. Moreover, the responsible investor purchases more shares when the action is more likely to have been taken. This increases the stock price and lowers household trading profits.

The third term, $\overline{q}v$, represents the value of households' initial endowment. As noted in part (ii), expected firm value in equilibrium can be either higher or lower when the action is taken, which makes the effect of the action on household welfare ambiguous in general. However, when κ_H is sufficiently large, households are less willing to absorb the shares issued by the firm that are not purchased by the responsible investor. As a result, the responsible investor's additional purchases in the action equilibrium have greater price impact, strengthening the cost-of-capital effect so that it outweighs the cost of the action and firm value rises overall. For high κ_H, the resulting gain in the value of households' initial endowment outweighs the reduction in their trading profits, and so they prefer the action equilibrium.

Overall, the responsible investor always prefers the action equilibrium, but its broader effects are more nuanced. The action equilibrium reduces externalities if and only if the firm is sufficiently large, and it raises firm value only if the action's cost is sufficiently low. For households, it always lowers trading profits, yet it raises their overall welfare when their trading costs are sufficiently high.

4 Conclusion

This paper has analyzed equilibrium divestment strategies of a responsible investor who is concerned with both trading profits and externalities, but cannot observe externality-reducing actions or commit to a trading strategy. While unconditional exclusion minimizes the stock price and thus the amount of externality-enhancing investment the firm can undertake, it provides no incentives to undertake a corrective action. A conditional strategy can encourage the action, but it does so by buying the firm's stock after positive news about the action and thus allowing the firm to expand.

We show that the action is taken with positive probability only when the responsible investor's concern for externalities is intermediate. If it is too low, her trade is not sufficiently sensitive to the signal to reward the action. If it is too high, she buys few shares after a favorable signal because doing so expands the firm, again reducing incentives to act. Thus, up to a point, profit motives can support social impact by making conditional investment credible; social motives can undermine impact by making the investor unable to reward reform. This result suggests caution in interpreting exclusion as the most responsible strategy – even if it is widely undertaken by investors who care about externalities. A fund may exclude not because exclusion is most effective at reducing externalities, but because it cannot credibly commit to the more impactful conditional strategy.

The analysis also highlights the critical role of disclosure. More precise disclosure is needed for conditional investment to work, since a noisy signal generates too little price reward for acting. However, more disclosure is not always socially desirable. Greater precision can cause the action cost to be priced in more sharply, eliminating the action equilibrium. Even when it induces the action, expected externalities can rise if the resulting increase in firm size outweighs the per-unit reduction in externalities. Thus, disclosure regulation may involve ceilings as well as floors.

Our results have a number of potential implications for policymakers. Regulators should not automatically treat holdings of brown stocks by a sustainable fund as evidence of irresponsibility: such holdings may be the means through which it rewards corrective action. Under the EU's Sustainable Finance Disclosure Regulation, Article 9 funds are viewed as the most sustainable, and are preferred by many clients, but the common interpretation of the requirements is that such funds cannot hold fossil fuel stocks even if they are best-in-class. Similarly, the EU's Non-Financial Reporting Directive requires institutional investors to report the percentage of their portfolio that is environmentally sustainable, which also encourages investors to choose unconditional over conditional strategies. In contrast, regulators should potentially police the opposite behavior – sustainable funds that claim to be actively managed but unconditionally exclude certain sectors. Just as regulators are scrutinizing actively-managed mainstream funds that act like closet indexers, they could also scrutinize actively-managed sustainable funds that engage in unconditional exclusion.

At the same time, exclusion should not be presumed inferior to conditional investment (e.g. because it ignores public signals of reform). Its desirability depends on the effectiveness and cost of reform, existing firm size and the amount of capital raised, and the quality of disclosure. Similarly, fiduciary duty can either support or hinder externality reduction: a concern for returns can make conditional investment credible, but too strong a concern for returns can lead to the responsible investor buying even upon a negative signal, reducing the manager's incentives to reform.

While our analysis has focused on brown industries, a similar result likely applies for green industries. Some sustainable funds will be significantly more likely to own a company if it is in a green sector. However, such an unconditional policy provides few incentives for companies within this sector to increase their positive externalities, since they will likely be held anyway. A conditional strategy, which overweights green industries but avoids "worst-in-class" companies within such sectors, may be more effective than unconditional inclusion. Similarly, while we consider equity investors, our results will continue to hold for debt investors. Some banks have policies not to finance new oil and gas projects, but being willing to lend to brown firms that are best-in-class may encourage such companies to reform.

The model suggests several avenues for future research. One is to study multiple firms, which would allow the conditional strategy to be literally best-in-class and would introduce product-market interactions. A second is to study multiple responsible investors. Their collective price impact may make conditioning more powerful, but competition for client flows may encourage simple exclusion if clients view it as more responsible. A third is a dynamic setting, where past actions affect the effectiveness of future ones and where reputation may help investors commit. Finally, future work could build a full regulatory model of disclosure mandates. This would require specifying the regulator's welfare criterion, the costs and enforceability of disclosure, and how a mandate applies across heterogeneous firms. Our analysis provides a building block by showing why disclosure mandates are subtle: more information can be necessary to induce reform, but too much information may undermine reform incentives; even when disclosure allows reform, it may increase externalities.

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A Proofs

Proof of Lemma 1. We start with the specification for R's utility in equation (5) and plug in the expression for the externality and the short-term stock price in equations (6) and (11), respectively. This gives

$$\begin{aligned}\mathbb{E}[U_R(x_R,s)] &= x_R\left(\mathbb{E}[v|s]-p\left(x_R,s\right)\right)-\frac{\kappa_R}{2}x_R^2-\varphi\lambda\left(\mu+qrp\left(x_R,s\right)\right)\left(1-\mathbb{E}[ae|s]\right)\\ &= x_R\kappa_H(q-x_R)-\frac{\kappa_R}{2}x_R^2\\ &\quad -\varphi\lambda\left(\mu+qr\frac{\mu-\mathbb{E}[ac|s]-(q-x_R)(\overline{q}+q)\kappa_H}{\overline{q}+q-rq}\right)\left(1-\mathbb{E}[ae|s]\right) \qquad (20)\end{aligned}$$

which is concave in x_R. The FOC with respect to x_R is

$$\kappa_H(q-2x_R)-\kappa_R x_R-\varphi\lambda qr\frac{\kappa_H(\overline{q}+q)}{\overline{q}+q-rq}\left(1-\mathbb{E}[ae|s]\right)=0, \qquad (21)$$

and rearranging terms gives the interior solution in (12). Since the objective (20) is strictly concave in x_R, whenever the solution to the first-order condition is negative, the constraint $x_R\geq 0$ binds and the optimum is $x_R=0$; this yields the $\max\{\cdot,0\}$ operator in (12). ■

Proof of Lemma 2. Consider the expected price difference in condition (13) and suppose $e=0$. In this case, $s=0$ with probability τ and $s=1$ with probability $1-\tau$ irrespective of the action. Hence, both $a=1$ and $a=0$ lead to $p(x_R,0)$ with probability τ and $p(x_R,1)$ with probability $1-\tau$. It follows that $\mathbb{E}[p(x_R,s)|a=1]-\mathbb{E}[p(x_R,s)|a=0]=0$. When $\omega<1$, the manager bears a direct cost from taking the action (through its impact on v), so $a=0$ is strictly optimal. When $\omega=1$, the manager is indifferent, since his payoff depends only on the stock price, which is unaffected by a when $e=0$. We set $a^*=0$ in this case: the product ae equals zero regardless of a when $e=0$, so the choice has no effect on signal distributions, beliefs, or prices. ■

Proof of Proposition 1. The proof proceeds in three steps. First, we show that $\eta^*=0$ is always an equilibrium. Second, we establish that any stable equilibrium with $\eta^*>0$ is unique. Third, we characterize the set of φ supporting $\eta^*>0$ as an interval $(\underline{\varphi},\overline{\varphi})$ and identify the threshold $\hat{c}$. Throughout Steps 2 and 3 we further assume $\omega<1$; the knife-edge case $\omega=1$ is addressed separately at the end of the proof.

Substituting Lemma 1 into the trade differential gives:

$$x_R(1;\eta) - x_R(0;\eta) \;=\; \frac{q}{\kappa_R/\kappa_H + 2} \times \begin{cases} \dfrac{\varphi r\lambda}{1-\theta}\big[\alpha(1;\eta) - \alpha(0;\eta)\big]\bar{e} & \text{if } x_R(1) > x_R(0) > 0, \\ 1 - \dfrac{\varphi r\lambda}{1-\theta}\big(1 - \alpha(1;\eta)\,\bar{e}\big) & \text{if } x_R(1) > x_R(0) = 0, \\ 0 & \text{if } x_R(1) = x_R(0) = 0. \end{cases} \quad (22)$$

Suppose $x_R(1;\eta) > 0$. Then, $\mathcal{B}(\eta;\varphi)$ from (15) takes two forms depending on whether $x_R(0;\eta)$ is strictly positive (*int*) or zero (*tilt*):

$$\mathcal{B}_{\text{int}}(\eta;\varphi) = \varphi \frac{q}{\kappa_R/\kappa_H + 2} \frac{r\lambda}{1-\theta}[\alpha(1;\eta) - \alpha(0;\eta)]\bar{e} - \mathcal{C}(\eta), \quad (23)$$

$$\mathcal{B}_{\text{tilt}}(\eta;\varphi) = \frac{q}{\kappa_R/\kappa_H + 2}\left[1 - \frac{\varphi r\lambda}{1-\theta}[1 - \alpha(1;\eta)\bar{e}]\right] - \mathcal{C}(\eta). \quad (24)$$

Note that $\mathcal{B}_{\text{int}}$ is strictly increasing in φ and $\mathcal{B}_{\text{tilt}}$ is strictly decreasing in φ.

Step 1: Existence of $\eta^* = 0$**.** At $\eta = 0$, $\alpha(1;0) = \alpha(0;0) = 0$, so $x_R(1;0) = x_R(0;0)$ and $\mathcal{B}(0;\varphi) = -\mathcal{C}(0) < 0$ for every φ. Stability additionally requires $\mathcal{B}(\epsilon;\varphi) < 0$ for all sufficiently small $\epsilon > 0$, which holds by continuity of $\mathcal{B}$ in η since $\mathcal{C}(0) > 0$ under $\omega < 1$. Hence $\eta^* = 0$ is a stable equilibrium.

Step 2: Uniqueness. A stable equilibrium with $\eta^* > 0$ satisfies $\mathcal{B}(\eta^*;\varphi) \geq 0$, ruling out full exclusion (where $\mathcal{B} = -\mathcal{C}(\eta^*) < 0$). So either the *int* or *tilt* regime applies. In the *int* regime, substituting $\mathcal{C}(\eta)$ from (16) into (23) yields a linear expression in $\alpha(1;\eta) - \alpha(0;\eta)$ with coefficient $\varphi q r\lambda\bar{e}/[(\kappa_R/\kappa_H + 2)(1-\theta)] - c/(\kappa_H(\bar{q}+q))$. If this coefficient is non-positive, $\mathcal{B}_{\text{int}} < 0$ everywhere; otherwise $\mathcal{B}_{\text{int}}$ is a positive multiple of $\alpha(1;\eta) - \alpha(0;\eta)$ minus a constant. Since the spread $\alpha(1;\eta) - \alpha(0;\eta)$ is strictly concave with unconstrained maximum at $\eta = 1/(2\xi)$, $\mathcal{B}_{\text{int}}$ is strictly concave with maximizer

$$\eta_{\max} \equiv \min\left\{\frac{1}{2\xi}, 1\right\}. \quad (25)$$

Strict concavity gives at most one downward-crossing zero, so any stable equilibrium with $\eta^* > 0$ in the *int* regime is unique: $\eta^* \in (0,1)$ when $\mathcal{B}_{\text{int}}(1;\varphi) < 0$ and $\eta^* = 1$ when $\mathcal{B}_{\text{int}}(1;\varphi) > 0$. In the boundary case $\mathcal{B}_{\text{int}}(1;\varphi) = 0$, the equilibrium condition holds at $\eta = 1$, but stability requires $\mathcal{B}_{\text{int}}(1-\epsilon;\varphi) > 0$: this holds if $\eta_{\max} < 1$, since $\mathcal{B}_{\text{int}}$ is then strictly decreasing on $(\eta_{\max}, 1]$, and fails if $\eta_{\max} = 1$ (i.e., $\xi \leq 1/2$), since $\mathcal{B}_{\text{int}}$ is then strictly increasing below $\eta = 1$. So $\eta^* = 1$ with $\mathcal{B}_{\text{int}}(1;\varphi) = 0$ is a stable equilibrium if and only if $\eta_{\max} < 1$.

In the *tilt* regime, $\mathcal{B}_{\text{tilt}}$ is strictly increasing in η when the same coefficient as before is non-negative, and convex in η when it is negative (since $\alpha(1;\eta)$ is concave and $\alpha(0;\eta)$ is convex). The tilt regime at any η requires $\varphi \geq (1-\theta)/(r\lambda)$, so evaluating (24) at $\eta = 0$ gives $\mathcal{B}_{\text{tilt}}(0;\varphi) \leq -\mathcal{C}(0) < 0$. Hence, any interior zero is an ascending crossing, ruling out stable interior equilibria. So a stable equilibrium with $\eta^* > 0$ in the *tilt* regime must be $\eta^* = 1$ with $\mathcal{B}_{\text{tilt}}(1;\varphi) > 0$.

To rule out the coexistence of a pure tilt equilibrium and a mixed int equilibrium, suppose both exist. Then

$$\mathcal{B}_{\text{int}}(1;\varphi) < 0 < \mathcal{B}_{\text{tilt}}(1;\varphi). \tag{26}$$

Evaluating (23) and (24) at $\eta = 1$,

$$\mathcal{B}_{\text{int}}(1;\varphi) - \mathcal{B}_{\text{tilt}}(1;\varphi) = \frac{q}{\kappa_R/\kappa_H + 2}\left[\frac{\varphi r\lambda}{1-\theta}(1-\alpha(0;1)\bar{e}) - 1\right]. \tag{27}$$

Hence $\mathcal{B}_{\text{int}}(1;\varphi) < \mathcal{B}_{\text{tilt}}(1;\varphi)$ requires $\varphi r\lambda(1-\alpha(0;1)\bar{e})/(1-\theta) < 1$, i.e., $x_R(0;1) > 0$. But $x_R(0;1) > 0$ places us in the *int* regime at $\eta = 1$, which yields a contradiction.

Step 3: The φ-window. We characterize the set of φ that support an equilibrium with $\eta^* > 0$. The responsible investor's position after $s = 0$ is positive if and only if

$$x_R\left(0;\eta\right) > 0 \Leftrightarrow \varphi < \varphi_l\left(\eta\right) \equiv \frac{1}{\frac{r\lambda}{1-\theta}\left(1-\alpha(0;\eta)\bar{e}\right)}, \tag{28}$$

where $\varphi_l\left(\eta\right) > 0$. Similarly, the position after $s = 1$ is positive if and only if

$$x_R\left(1;\eta\right) > 0 \Leftrightarrow \varphi < \varphi_u\left(\eta\right) \equiv \frac{1}{\frac{r\lambda}{1-\theta}\left(1-\alpha(1;\eta)\bar{e}\right)}. \tag{29}$$

Since $\alpha(0;\eta) < \alpha(1;\eta)$, we have $\varphi_l(\eta) < \varphi_u(\eta)$ for every $\eta > 0$ (and equality for $\eta = 0$). The two relevant regimes correspond to $\varphi < \varphi_l(\eta)$ (*int*) and $\varphi_l(\eta) \leq \varphi < \varphi_u(\eta)$ (*tilt*). Any stable equilibrium with $\eta^* > 0$ requires $\varphi < \varphi_u(\eta^*)$, since otherwise we have full exclusion and $\mathcal{B} < 0$.

We first characterize when a pure-strategy equilibrium ($\eta^* = 1$) exists, then turn to mixed-strategy equilibria ($\eta^* \in (0,1)$).

(a) *Pure-strategy equilibrium in the tilt regime.* This requires $\varphi_l(1) \leq \varphi < \varphi_u(1)$ (the *tilt* regime at $\eta = 1$) and $\mathcal{B}_{\text{tilt}}(1;\varphi) > 0$. From (24),

$$\mathcal{B}_{\text{tilt}}(1;\varphi) > 0 \Leftrightarrow \varphi < \varphi_{\text{tilt}} \equiv \varphi_u(1)\left(1 - \mathcal{C}(1)\frac{\kappa_R/\kappa_H + 2}{q}\right). \tag{30}$$

Since $\mathcal{C}(1) > 0$, we have $\varphi_{\text{tilt}} < \varphi_u(1)$, so the binding upper bound is φ_{tilt}. A pure-strategy equilibrium in the *tilt* regime therefore exists if and only if $\varphi \in \Phi_a$, where

$$\Phi_a \equiv [\varphi_l(1), \varphi_{\text{tilt}}). \tag{31}$$

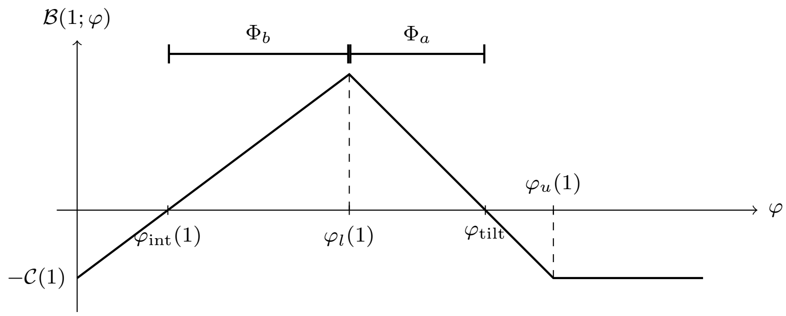
(b) *Pure-strategy equilibrium in the int regime.* This requires $\varphi < \varphi_l(1)$ (the *int* regime at $\eta = 1$) and $\mathcal{B}_{\text{int}}(1;\varphi) > 0$. From (23),

$$\mathcal{B}_{\text{int}}(\eta;\varphi) > 0 \Leftrightarrow \varphi > \varphi_{\text{int}}(\eta) \equiv \frac{1}{\frac{r\lambda}{1-\theta}[\alpha(1;\eta) - \alpha(0;\eta)]\bar{e}}\mathcal{C}(\eta)\frac{\kappa_R/\kappa_H + 2}{q} > 0. \tag{32}$$

Evaluating this condition at $\eta = 1$ implies that a pure-strategy equilibrium in the *int* regime exists if and only if $\varphi \in \Phi_b$, where

$$\Phi_b \equiv (\varphi_{\text{int}}(1), \varphi_l(1)). \tag{33}$$

Φ_a and Φ_b non-empty together. Fix $\eta = 1$. $\mathcal{B}_{\text{int}}(1;\varphi)$ is linear and strictly increasing in φ on the int side, with zero at $\varphi_{\text{int}}(1)$; $\mathcal{B}_{\text{tilt}}(1;\varphi)$ is linear and strictly decreasing in φ on the tilt side, with zero at φ_{tilt}. The two pieces coincide at $\varphi = \varphi_l(1)$ (where $x_R(0;1) = 0$). Hence $\mathcal{B}(1;\varphi)$ on $[0, \varphi_u(1))$ is piecewise linear with an inverted-V shape: rising on $(0, \varphi_l(1))$, peak at $\varphi_l(1)$, falling on $(\varphi_l(1), \varphi_u(1))$.



Both regions are non-empty iff the peak value $\mathcal{B}(1;\varphi_l(1))$ is strictly positive, equivalently

$$\varphi_{\text{int}}(1) < \varphi_l(1) < \varphi_{\text{tilt}}; \tag{34}$$

in this case $\eta^* = 1$ is a pure-strategy equilibrium if $\varphi \in (\varphi_{\text{int}}(1), \varphi_{\text{tilt}})$. Otherwise both regions are empty and no pure-strategy equilibrium with $\eta^* = 1$ exists.

(c) *Mixed-strategy equilibrium ($\eta^* \in (0,1)$).* By Step 2, such an equilibrium requires $\mathcal{B}_{\text{int}}(\eta^*;\varphi) = 0$ with $\frac{\partial \mathcal{B}_{\text{int}}}{\partial \eta}|_{\eta^*} < 0$. By strict concavity, the downward crossing must lie past the maximum, so $\eta^* > \eta_{\max}$. Mixed equilibria therefore require $\eta_{\max} < 1$ (equivalently, $\xi > 1/2$). If $\eta_{\max} \geq 1$, then Φ_c is empty and the analysis reduces to $\Phi_a \cup \Phi_b$ as handled in the combine step below. Hereafter, we assume $\eta_{\max} < 1$ in this subsection.

$\mathcal{B}_{\text{int}}(\eta;\varphi) = 0$ has a solution $\eta^* \in (\eta_{\max}, 1)$ if and only if (i) $\mathcal{B}_{\text{int}}(1;\varphi) < 0 \Leftrightarrow \varphi < \varphi_{\text{int}}(1)$, and (ii) $\mathcal{B}_{\text{int}}$ is positive somewhere in the int regime. Since φ_l is strictly increasing in η, we can define[17]

$$\eta_l(\varphi) \equiv \varphi_l^{-1}(\varphi). \tag{35}$$

Then

$$x_R(0;\eta) > 0 \;\Leftrightarrow\; \eta > \eta_l(\varphi) \;\Leftrightarrow\; \varphi < \varphi_l(\eta), \tag{36}$$

so the int regime for fixed φ is $(\eta_l(\varphi), 1]$.

To characterize the mixed-equilibrium set, note that $\mathcal{B}_{\text{int}}(\eta;\varphi) = 0 \Leftrightarrow \varphi = \varphi_{\text{int}}(\eta)$. Combined with (36), a mixed strategy $\eta^* \in (\eta_{\max}, 1)$ requires $\varphi = \varphi_{\text{int}}(\eta^*)$ and

$$\varphi_l(\eta^*) > \varphi_{\text{int}}(\eta^*). \tag{37}$$

[17]For φ outside the range of φ_l, we set $\eta_l(\varphi) = 0$ if $\varphi < \varphi_l(0)$ and $\eta_l(\varphi) = 1$ if $\varphi > \varphi_l(1)$, so that η_l is defined on all of $\varphi > 0$.

This recovers conditions (i)–(ii): for (i), φ_{int} is strictly increasing on $(\eta_{\max}, 1]$ — implicit differentiation of $\mathcal{B}_{\text{int}}(\eta; \varphi_{\text{int}}(\eta)) = 0$ gives $\varphi'_{\text{int}}(\eta) = -\frac{\partial \mathcal{B}_{\text{int}}/\partial \eta}{\partial \mathcal{B}_{\text{int}}/\partial \varphi} > 0$ past the peak $\eta_{\max}$ — so $\eta^* < 1$ gives $\varphi = \varphi_{\text{int}}(\eta^*) < \varphi_{\text{int}}(1)$; for (ii), (37) places η^* strictly inside the int regime at $\varphi = \varphi_{\text{int}}(\eta^*)$, so a left neighborhood of η^* lies in the int regime as well; strict concavity (peak at $\eta_{\max} < \eta^*$, zero at η^*) then gives $\mathcal{B}_{\text{int}}(\eta^* - \varepsilon; \varphi_{\text{int}}(\eta^*)) > 0$ for $\varepsilon > 0$ sufficiently small.

Hence the set of φ-values for which a mixed-strategy equilibrium exists is

$$\Phi_c \equiv \{\varphi_{\text{int}}(\eta) : \eta \in \Theta\}, \tag{38}$$

where

$$\Theta \equiv \big\{\eta \in (\eta_{\max}, 1) : \varphi_l(\eta) > \varphi_{\text{int}}(\eta)\big\}. \tag{39}$$

Since φ_{int} is strictly increasing on $[\eta_{\max}, 1]$, the set Φ_c is an interval in φ if and only if the set Θ is an interval in η. So it suffices to show the latter.

Define $F(\eta) \equiv \mathcal{B}_{\text{int}}(\eta; \varphi_l(\eta))$. Using the definition of $\varphi_l(\eta)$ and the expression (23) gives

$$F(\eta) = \frac{q}{\kappa_R/\kappa_H + 2} \frac{\alpha(1;\eta) - \alpha(0;\eta)}{\frac{1}{e} - \alpha(0;\eta)} - \mathcal{C}(\eta). \tag{40}$$

Since $\mathcal{B}_{\text{int}}(\eta; \cdot)$ is strictly increasing in φ with zero at $\varphi_{\text{int}}(\eta)$, $F(\eta) > 0 \Leftrightarrow \varphi_l(\eta) > \varphi_{\text{int}}(\eta)$. So $\Theta = \{F(\eta) > 0\} \cap (\eta_{\max}, 1)$.

Clearing positive denominators ($\Pr[s = 0]$, $\Pr[s = 1]$, and $\frac{1}{e} - \alpha(0;\eta)$ are all strictly positive on the relevant domain; we additionally multiply through by $\Pr[s = 0]$), $F(\eta) > 0$ is equivalent to:

$$\begin{aligned}
\frac{q}{\kappa_R/\kappa_H + 2} \frac{\alpha(1;\eta) - \alpha(0;\eta)}{\frac{1}{e} - \alpha(0;\eta)} - \mathcal{C}(\eta) &> 0 \Leftrightarrow \\
\frac{q}{\kappa_R/\kappa_H + 2}[\alpha(1;\eta) - \alpha(0;\eta)] - \left(\frac{1}{e} - \alpha(0;\eta)\right)\mathcal{C}(\eta) &> 0 \Leftrightarrow \\
\frac{q}{\kappa_R/\kappa_H + 2}\left(\frac{\tau}{\Pr[s=1]} - \frac{(1-\tau)}{\Pr[s=0]}\right)\xi\eta - \left(\frac{1}{e} - \frac{\xi\eta(1-\tau)}{\Pr[s=0]}\right)\mathcal{C}(\eta) &> 0 \Leftrightarrow \\
\left[\begin{array}{c} \frac{q}{\kappa_R/\kappa_H + 2}(\tau\Pr[s=0] - (1-\tau)\Pr[s=1])(\xi\eta)\Pr[s=0] \\ -\left(\frac{1}{e}\Pr[s=0] - \xi\eta(1-\tau)\right)\mathcal{C}(\eta)\Pr[s=0]\Pr[s=1] \end{array}\right] &> 0
\end{aligned} \tag{41}$$

$\Pr[s=0]$ and $\Pr[s=1]$ are linear in $\xi\eta$, and the term

$$\mathcal{C}(\eta)\Pr[s=0]\Pr[s=1] \propto \left[\begin{array}{c} \frac{(1-\omega)(1-\theta)}{\omega+(1-\omega)\theta} \frac{\Pr[s=0]\Pr[s=1]}{2\tau-1} \\ +\left(\tau\Pr[s=0]-(1-\tau)\Pr[s=1]\right)\xi\eta \end{array} \right] \tag{42}$$

is quadratic in $\xi\eta$. Hence $F(\eta)>0$ is equivalent to a polynomial in η of degree at most 3 being positive. $F(\eta)$ is negative at $\eta=0$ and $\eta=1/\xi$ (using $\mathcal{C}(0)>0$ under $\omega<1$).[18] Two disjoint positive sub-intervals of $[0,1/\xi]$ would require at least four zeros, impossible for a degree-3 polynomial; so $\{F(\eta)>0\}\cap[0,1/\xi]$ is empty or a single interval, and therefore so is $\{F(\eta)>0\}\cap(\eta_{\max},1)$.

Write $\{F(\eta)>0\}\cap(\eta_{\max},1)=(\underline{\eta}^*,\overline{\eta}^*)$ when non-empty, where $\underline{\eta}^*=\eta_{\max}$ if $\{F(\eta)>0\}$ extends to $\eta_{\max}$ (otherwise the smaller root of F), and $\overline{\eta}^*=1$ as a limit if $\{F(\eta)>0\}$ extends to 1 (otherwise the larger root). Since φ_{int} is strictly increasing on $[\eta_{\max},1]$ (φ_{int} is U-shaped with minimum at $\eta_{\max}$), each $\varphi\in\Phi_c$ corresponds to exactly one $\eta^*\in(\underline{\eta}^*,\overline{\eta}^*)$ with $\varphi_{\text{int}}(\eta^*)=\varphi$, so $\Phi_c=\left(\varphi_{\text{int}}(\underline{\eta}^*),\varphi_{\text{int}}(\overline{\eta}^*)\right)$.

Now combine Φ_a, Φ_b, and Φ_c. By the discussion above, Φ_a and Φ_b are both non-empty iff the peak value $\mathcal{B}(1;\varphi_l(1))>0$, which is exactly $F(1)>0$; when non-empty they meet at $\varphi_l(1)$. Φ_c requires $F(\eta)>0$ somewhere in $(\eta_{\max},1)$, which is automatic when $F(1)>0$ and $\eta_{\max}<1$ (continuity at $\eta=1$).

Case $F(1)>0$. Then $\Phi_a\cup\Phi_b=(\varphi_{\text{int}}(1),\varphi_{\text{tilt}})$. If $\eta_{\max}<1$, continuity forces $\overline{\eta}^*=1$ as a limit, so Φ_c meets Φ_b at $\varphi_{\text{int}}(1)$ and the window is $(\varphi_{\text{int}}(\underline{\eta}^*),\varphi_{\text{tilt}})$.[19] If $\eta_{\max}=1$ (i.e., $\xi\le 1/2$), the domain $(\eta_{\max},1)$ is empty, so Φ_c is empty and the window is $(\varphi_{\text{int}}(1),\varphi_{\text{tilt}})$.

Case $F(1)\le 0$. Φ_a and Φ_b are empty. The window is Φ_c when non-empty, equal to $(\varphi_{\text{int}}(\underline{\eta}^*),\varphi_{\text{int}}(\overline{\eta}^*))$; otherwise empty.

In all cases, the window is a single interval $(\underline{\varphi},\overline{\varphi})$ with

$$\underline{\varphi}=\begin{cases}\varphi_{\text{int}}(\underline{\eta}^*) & \text{if } \Phi_c \text{ is non-empty,}\\ \varphi_{\text{int}}(1) & \text{if } \Phi_c \text{ is empty,}\end{cases} \qquad \overline{\varphi}=\begin{cases}\varphi_{\text{tilt}} & \text{if } \Phi_a \text{ is non-empty,}\\ \varphi_{\text{int}}(\overline{\eta}^*) & \text{if } \Phi_a \text{ is empty.}\end{cases} \tag{43}$$

[18]The interval $[0,1/\xi]$ is convenient for the polynomial-sign argument; under $\xi\in(0,1)$, $1/\xi>1$, so this interval extends past the economic range $\eta\in[0,1]$. The relevant range $\eta\in(\eta_{\max},1)$ lies strictly inside $[0,1/\xi]$.

[19]At $\varphi=\varphi_{\text{int}}(1)$ the strict inequalities defining Φ_b and Φ_c both fail, but $\mathcal{B}_{\text{int}}(1;\varphi_{\text{int}}(1))=0$ and $\eta_{\max}<1$, so by Step 2 $\eta^*=1$ is a stable equilibrium there. Hence $\varphi_{\text{int}}(1)$ belongs to the window.

Within Φ_c, η^* is strictly increasing in φ: at the stable root, $\partial\mathcal{B}_{\text{int}}/\partial\eta < 0$ and $\partial\mathcal{B}_{\text{int}}/\partial\varphi > 0$, so by the implicit function theorem, raising φ shifts the zero to the right. Combined with $\eta^* = 1$ on $\Phi_a \cup \Phi_b$, η^* is weakly increasing in φ on $(\underline{\varphi}, \overline{\varphi})$.

Finally, we show the φ-window is non-empty for small c, vanishes as $c \to \infty$, and shrinks monotonically in between — defining a threshold $\hat{c}$.

At $c = 0$, $\mathcal{C} \equiv 0$, so for any $\varphi \in (0, \varphi_l(1))$, $\mathcal{B}_{\text{int}}(1; \varphi) > 0$ (since $\alpha(1; 1) > \alpha(0; 1)$), and $\eta^* = 1$ is a stable pure equilibrium. By continuity of $\mathcal{B}$ in c, the window remains non-empty for all sufficiently small $c > 0$. As $c \to \infty$, $x_R(1; \eta) - x_R(0; \eta)$ does not depend on c, while $\mathcal{C}(\eta)$ is linear and strictly increasing in c uniformly in η (the $(1 - \omega)$ term of (16) contributes a positive η-independent slope under $\omega < 1$). Hence $\mathcal{B} \to -\infty$ uniformly, so for c large enough no equilibrium with $\eta^* > 0$ exists.

Monotonicity. If an equilibrium with $\eta^* > 0$ exists at (φ, c''), then one also exists at (φ, c') for any $c' < c''$.

Case 1: pure-strategy equilibrium at c''. $\mathcal{B}(1; \varphi, c'') > 0$ and $\mathcal{B}$ strictly decreasing in c give $\mathcal{B}(1; \varphi, c') > 0$, so $\eta^* = 1$ is a pure-strategy equilibrium at c'.

Case 2: mixed-strategy equilibrium $\eta''^* \in (\eta_{\max}, 1)$ at c''. $\mathcal{B}_{\text{int}}(\eta''^*; \varphi, c'') = 0$ and $\mathcal{B}_{\text{int}}$ strictly decreasing in c give $\mathcal{B}_{\text{int}}(\eta''^*; \varphi, c') > 0$. If $\mathcal{B}_{\text{int}}(1; \varphi, c') \geq 0$, $\eta^* = 1$ is a pure-strategy equilibrium at c'. Otherwise, strict concavity of $\mathcal{B}_{\text{int}}$ with $\mathcal{B}_{\text{int}}(\eta''^*; \varphi, c') > 0$ and $\mathcal{B}_{\text{int}}(1; \varphi, c') < 0$ gives a unique downward-crossing root $\eta'^* \in (\eta''^*, 1)$, which stays in the int regime since $\eta'^* > \eta''^* > \eta_l(\varphi)$.

Conclusion. The three observations together give a threshold $\hat{c} \in (0, \infty)$ such that the window is non-empty if and only if $c < \hat{c}$.[20]

Boundary case $\omega = 1$. Note that $\mathcal{C}(\eta) = c(\alpha(1; \eta) - \alpha(0; \eta))/(\kappa_H(\overline{q} + q))$, so

$$\mathcal{B}_{\text{int}}(\eta; \varphi) = \left[\frac{\varphi q r \lambda \bar{e}}{(\kappa_R/\kappa_H + 2)(1 - \theta)} - \frac{c}{\kappa_H(\overline{q} + q)}\right] (\alpha(1; \eta) - \alpha(0; \eta)). \tag{44}$$

Since $\alpha(1; \eta) - \alpha(0; \eta) > 0$ on $(0, 1]$, the sign of $\mathcal{B}_{\text{int}}$ on $(0, 1]$ equals the sign of the bracketed coefficient. This rules out any interior zero of $\mathcal{B}_{\text{int}}$. Note that $\mathcal{B}_{\text{tilt}}$ is strictly increasing in η when

[20] Note that at the knife-edge case $c = \hat{c}$, the window is empty. From the combined step, the window is non-empty if and only if $F(\eta) > 0$ for some $\eta \in [\eta_{\max}, 1]$. Since $F(\eta; c)$ is continuous and strictly decreasing in c for each fixed η, any η_0 with $F(\eta_0; \hat{c}) > 0$ would still satisfy $F(\eta_0; c) > 0$ for c slightly above $\hat{c}$, so the window would be non-empty there, contradicting the definition of $\hat{c}$.

the bracketed coefficient is non-negative and convex when it is negative, and $\mathcal{B}_{\text{tilt}}(0;\varphi) \leq 0$ in the tilt regime (now with equality possible, since $\mathcal{C}(0) = 0$ at $\omega = 1$), so any interior zero is an ascending crossing. Hence no stable mixed equilibrium exists. The pure-strategy case $\eta^* = 1$ proceeds as in the main argument, except that the large-c step now follows directly from the bracketed coefficient being strictly decreasing in c: for c large enough, the bracket is negative at any admissible φ, so $\mathcal{B}_{\text{int}}(1;\varphi,c) < 0$ and the window vanishes. Step 1 also changes at $\omega = 1$: since $\mathcal{C}(0) = 0$, the sign of $\mathcal{B}(\eta;\varphi)$ on $(0,1]$ equals the sign of the bracketed coefficient, so when the coefficient is strictly positive, $\mathcal{B}(\epsilon;\varphi) > 0$ for small $\epsilon > 0$ and the no-action equilibrium fails the stability condition; $\eta^* = 1$ is then the unique stable equilibrium. ■

Proof of Corollary 1. Based on the proof of Proposition 1, an equilibrium with $\eta^* = 1$ exists if and only if $c < \hat{c}$ and $\varphi \in (\varphi_{\text{int}}(1), \varphi_{\text{tilt}})$,[21] where

$$\begin{aligned}
\varphi_{\text{tilt}} &= \varphi_u(1)\left(1 - \mathcal{C}(1)\frac{\kappa_R/\kappa_H + 2}{q}\right) \\
&= \frac{\frac{1}{r} - \frac{q}{\overline{q}+q}}{\lambda\left(1 - \alpha(1;1)\overline{e}\right)}\left[1 - \frac{c}{(\overline{q}+q)q}\left(\frac{\kappa_R}{\kappa_H^2} + \frac{2}{\kappa_H}\right)\left(\begin{array}{c}\frac{1-\theta}{\frac{\omega}{1-\omega}+\theta}\frac{1}{2\tau-1} \\ +\alpha(1;1) - \alpha(0;1)\end{array}\right)\right]
\end{aligned} \tag{45}$$

and

$$\begin{aligned}
\varphi_{\text{int}}(1) &= \frac{1}{\frac{r\lambda}{1-\theta}[\alpha(1;1) - \alpha(0;1)]\overline{e}}\mathcal{C}(1)\frac{\kappa_R/\kappa_H + 2}{q} \\
&= \frac{1-\theta}{r\lambda\overline{e}}\frac{c}{(\overline{q}+q)q}\left(\frac{\kappa_R}{\kappa_H^2} + \frac{2}{\kappa_H}\right)\left(\frac{1-\theta}{\frac{\omega}{1-\omega}+\theta}\frac{1}{2\tau-1}\frac{1}{\alpha(1;1) - \alpha(0;1)} + 1\right).
\end{aligned} \tag{46}$$

It is straightforward to see that φ_{tilt} is increasing in $\overline{e}$, decreasing in c, increasing in ω, increasing in κ_H, and decreasing in κ_R. Similarly, that $\varphi_{\text{int}}(1)$ has exactly the opposite comparative statics.

Next, consider an equilibrium with $\eta^* \in (0,1)$. Then, η^* is defined by $\mathcal{B}_{\text{int}}(\eta^*;\varphi) = 0$. By the implicit function theorem,

$$\frac{\partial \eta^*}{\partial z} = -\frac{\frac{\partial \mathcal{B}_{\text{int}}(\eta^*;\varphi)}{\partial z}}{\frac{\partial \mathcal{B}_{\text{int}}(\eta^*;\varphi)}{\partial \eta}}, \tag{47}$$

[21] At the knife-edge endpoint $\varphi = \varphi_{\text{int}}(1)$, an equilibrium with $\eta^* = 1$ also exists when $\eta_{\max} < 1$ (see Step 2 of the proof of Proposition 1). This endpoint is immaterial for the comparative statics below.

where z is a parameter of the model. Since the equilibrium is stable, it must be $\frac{\partial \mathcal{B}_{\text{int}}(\eta^*;\varphi)}{\partial \eta} < 0$. Therefore,

$$\text{sign}\left(\frac{\partial \eta^*}{\partial z}\right) = \text{sign}\left(\frac{\partial \mathcal{B}_{\text{int}}(\eta^*;\varphi)}{\partial z}\right). \tag{48}$$

Since

$$\begin{aligned}\mathcal{B}_{\text{int}}(\eta;\varphi) &= \varphi\frac{q}{\kappa_R/\kappa_H+2}\frac{r\lambda}{1-\theta}[\alpha(1;\eta)-\alpha(0;\eta)]\overline{e}-\mathcal{C}(\eta) \\ &= \varphi\frac{q}{\frac{\kappa_R}{\kappa_H}+2}\frac{r\lambda}{1-\theta}[\alpha(1;\eta)-\alpha(0;\eta)]\overline{e} \\ &\quad -\frac{c}{\kappa_H(\overline{q}+q)}\left(\frac{1-\theta}{\frac{\omega}{1-\omega}+\theta}\frac{1}{2\tau-1}+\alpha(1;\eta)-\alpha(0;\eta)\right),\end{aligned} \tag{49}$$

$\mathcal{B}_{\text{int}}(\eta;\varphi)$ (and hence η^*) is increasing in $\overline{e}$, decreasing in c, increasing in ω, increasing in κ_H, and decreasing in κ_R. ■

Proof of Proposition 2. **Part (i)**. Consider $\mathcal{B}(\eta;\tau)$, the manager's net benefit from choosing $a=1$ when the action is effective, parameterized by τ, and recall that an equilibrium with $\eta^*>0$ exists only if $\mathcal{B}(\eta^*;\tau)\geq 0$. The trade differential is bounded, $x_R(1;\eta)-x_R(0;\eta)\leq \frac{q}{\kappa_R/\kappa_H+2}$, whereas the cost diverges, $\lim_{\tau\searrow\frac{1}{2}}\mathcal{C}(\eta)=\infty$ for every $\eta\in[0,1]$. Consequently, $\lim_{\tau\searrow\frac{1}{2}}\mathcal{B}(\eta;\tau)=-\infty$ for every $\eta\in[0,1]$. It follows that there exists a threshold $\underline{\tau}\in(\frac{1}{2},1]$ such that $\max_{\eta\in[0,1]}\mathcal{B}(\eta;\tau)\geq 0$ only if $\tau>\underline{\tau}$. Hence, an equilibrium with $\eta^*>0$ can exist only if $\tau>\underline{\tau}$; that is, $\mathcal{T}_A\subseteq(\underline{\tau},1]$ and $\underline{\tau}>\frac{1}{2}$.

From Proposition 1, for all $\tau\in[0.5,1]$ there exists $\overline{c}_\tau\geq 0$ such that $\tau\in\mathcal{T}_A\Leftrightarrow c<\overline{c}_\tau$, with $\overline{c}_\tau=0$ implying no action equilibrium exists at precision τ. Next, consider the net benefit at $\tau=1$:

$$\mathcal{B}(\eta;1)=x_R(1;\eta)-x_R(0;\eta)-\frac{c}{\kappa_H(\overline{q}+q)\,(\omega+(1-\omega)\theta)} \tag{50}$$

Since $\alpha(s;\eta)$ does not depend on η when $\tau=1$, the difference $x_R(1;\eta)-x_R(0;\eta)$ likewise does not depend on η, and hence neither does $\mathcal{B}(\eta;1)$. It follows that

$$\overline{c}_1=[x_R(1;\eta)-x_R(0;\eta)]\;\kappa_H(\overline{q}+q)\,(\omega+(1-\omega)\theta)\,. \tag{51}$$

Indeed, if $c<\overline{c}_1$, then $\mathcal{B}(\eta;1)>0$ for every η, so $\eta^*=1$ is a stable equilibrium at $\tau=1$. If

$c > \bar{c}_1$, then $\mathcal{B}(\eta;1) < 0$ for every η, so no action equilibrium exists at $\tau = 1$. If $c = \bar{c}_1$, then $\mathcal{B}(\cdot;1) \equiv 0$: every $\eta^* > 0$ satisfies the equilibrium condition, but none satisfies the stability condition, which requires $\mathcal{B}(\eta^* - \epsilon) > 0$. Therefore, $1 \in \mathcal{T}_A$ if and only if $c < \bar{c}_1$.

We provide sufficient and necessary conditions for $\bar{c}_1 > 0$ and $\frac{\partial \bar{c}_\tau}{\partial \tau}\big|_{\tau=1} < 0$. When these hold, a cost $c = \bar{c}_1$ admits no action equilibrium at $\tau = 1$, yet one exists for τ sufficiently close to but below one.

We let $\mathcal{B}(\eta;\tau,c)$ denote the benefit function in (15) parametrized by (τ, c). We first establish that the short-sale constraint must bind. To this end, we prove $\frac{\partial \mathcal{B}_{\text{int}}}{\partial \tau}|_{c=\bar{c}_1,\tau=1} > 0$. Let $\eta_{\max}(\tau, c) = \arg\max_{\eta\in[0,1]} \mathcal{B}_{\text{int}}(\eta;\tau,c)$. Then, $\bar{c}_\tau$ satisfies

$$\mathcal{B}_{\text{int}}(\eta_{\max}(\tau,\bar{c}_\tau), \tau; \bar{c}_\tau) = 0, \tag{52}$$

implying

$$\bar{c}_1 = \frac{q\left(\kappa_H(\bar{q}+q)\right)}{\kappa_R/\kappa_H + 2} \frac{\frac{\varphi r \lambda}{1-\theta}\bar{e}}{\frac{(1-\omega)(1-\theta)}{\omega+(1-\omega)\theta} + 1} > 0. \tag{53}$$

By the implicit function theorem,

$$\frac{\partial \bar{c}_\tau}{\partial \tau} = -\frac{\frac{\partial \mathcal{B}_{\text{int}}}{\partial \tau} + \frac{\partial \mathcal{B}_{\text{int}}}{\partial \eta}\frac{\partial \eta_{\max}}{\partial \tau}}{\frac{\partial \mathcal{B}_{\text{int}}}{\partial c} + \frac{\partial \mathcal{B}_{\text{int}}}{\partial \eta}\frac{\partial \eta_{\max}}{\partial c}}. \tag{54}$$

Thus, if $\frac{\partial \mathcal{B}_{\text{int}}}{\partial \eta} \neq 0$ then it must be $\eta_{\max}(\tau,\bar{c}_\tau) \in \{0,1\}$. Either way, $\frac{\partial \mathcal{B}_{\text{int}}}{\partial \eta} = 0$ and $\frac{\partial \bar{c}_\tau}{\partial \tau} = -\frac{\frac{\partial \mathcal{B}_{\text{int}}}{\partial \tau}}{\frac{\partial \mathcal{B}_{\text{int}}}{\partial c}}$, implying $sign(\frac{\partial \bar{c}_\tau}{\partial \tau}) = sign(\frac{\partial \mathcal{B}_{\text{int}}}{\partial \tau}|_{c=\bar{c}_\tau})$. Notice

$$\begin{aligned}\frac{\partial \mathcal{B}_{\text{int}}(\eta,\tau;c)}{\partial \tau} \quad &= \quad \frac{q}{\kappa_R/\kappa_H + 2}\frac{\varphi r \lambda}{1-\theta}\bar{e}\frac{\partial\left[\alpha(1;\eta) - \alpha(0;\eta)\right]}{\partial \tau} \\ &\quad + \frac{c}{\kappa_H(\bar{q}+q)}\left(\frac{(1-\omega)(1-\theta)}{\omega+(1-\omega)\theta}\frac{2}{(2\tau-1)^2} - \frac{\partial\left[\alpha(1;\eta)-\alpha(0;\eta)\right]}{\partial \tau}\right)\end{aligned} \tag{55}$$

where $\frac{\partial[\alpha(1;\eta)-\alpha(0;\eta)]}{\partial\tau} > 0$. Substituting in $\overline{c}_1$, we get

$$\begin{aligned} \frac{\partial\mathcal{B}_{\text{int}}}{\partial\tau}|_{c=\overline{c}_1,\tau=1} &= \frac{q}{\kappa_R/\kappa_H+2}\frac{\varphi r\lambda}{1-\theta}\overline{e}\frac{\partial\left[\alpha(1;\eta)-\alpha(0;\eta)\right]}{\partial\tau}|_{\tau=1} \\ &\quad +\frac{\overline{c}_1}{\kappa_H(\overline{q}+q)}\left(2\frac{(1-\omega)(1-\theta)}{\omega+(1-\omega)\theta}-\frac{\partial\left[\alpha(1;\eta)-\alpha(0;\eta)\right]}{\partial\tau}|_{\tau=1}\right) \\ &= \frac{q}{\kappa_R/\kappa_H+2}\frac{\varphi r\lambda}{1-\theta}\overline{e}\left(\frac{\partial\left[\alpha(1;\eta)-\alpha(0;\eta)\right]}{\partial\tau}|_{\tau=1}+2\right)(1-\omega)(1-\theta)>0, \end{aligned} \tag{56}$$

implying that the short-sale constraint must bind, as required.

If the short-sale constraint binds, then the relevant region is tilt, and it must be $\eta^*=1$. Thus, we require $x_R(0,1)\leq 0$ for every $\tau\in\left[\frac{1}{2},1\right]$, implying $\frac{\varphi r\lambda}{1-\theta}>1$. In this case, $\overline{c}_\tau$ is defined by

$$\mathcal{B}_{\text{tilt}}(\tau;\overline{c}_\tau)=0, \tag{57}$$

implying

$$\overline{c}_1=\frac{q\left(\kappa_H(\overline{q}+q)\right)}{\kappa_R/\kappa_H+2}\frac{1-\dfrac{\varphi r\lambda}{1-\theta}(1-\overline{e})}{\frac{(1-\omega)(1-\theta)}{\omega+(1-\omega)\theta}+1}, \tag{58}$$

where $\overline{c}_1>0\Leftrightarrow\frac{\varphi r\lambda}{1-\theta}<\frac{1}{1-\overline{e}}$. Notice $\frac{\partial\overline{c}_\tau}{\partial\tau}=-\frac{\frac{\partial\mathcal{B}_{\text{tilt}}}{\partial\tau}}{\frac{\partial\mathcal{B}_{\text{tilt}}}{\partial c}}$, implying $sign(\frac{\partial\overline{c}_\tau}{\partial\tau})=sign(\frac{\partial\mathcal{B}_{\text{tilt}}}{\partial\tau}|_{c=\overline{c}_\tau})$. Also notice

$$\begin{aligned} \frac{\partial\mathcal{B}_{\text{tilt}}}{\partial\tau} &= \frac{q}{\kappa_R/\kappa_H+2}\frac{\varphi r\lambda}{1-\theta}\overline{e}\frac{\partial\alpha(1;1)}{\partial\tau} \\ &\quad +\frac{c}{\kappa_H(\overline{q}+q)}\left(\frac{(1-\omega)(1-\theta)}{\omega+(1-\omega)\theta}\frac{2}{(2\tau-1)^2}-\frac{\partial\left[\alpha(1;1)-\alpha(0;1)\right]}{\partial\tau}\right). \end{aligned} \tag{59}$$

Substituting in $\frac{\partial\alpha(1;1)}{\partial\tau}|_{\tau=1}=\frac{1-\xi}{\xi}$, $\frac{\partial\alpha(0;1)}{\partial\tau}|_{\tau=1}=-\frac{\xi}{1-\xi}$, and $\overline{c}_1$, we get

$$\frac{\partial\mathcal{B}_{\text{tilt}}}{\partial\tau}|_{c=\overline{c}_1,\tau=1}=\frac{q}{\kappa_R/\kappa_H+2}\left[\frac{\varphi r\lambda}{1-\theta}\overline{e}\frac{1-\xi}{\xi}-\left(1-\frac{\varphi r\lambda}{1-\theta}(1-\overline{e})\right)G(\xi)\right] \tag{60}$$

where

$$G(\xi)\equiv\frac{\omega+(1-\omega)\theta}{\xi(1-\xi)}-2. \tag{61}$$

Thus, $\frac{\partial\mathcal{B}_{\text{tilt}}}{\partial\tau}|_{c=\overline{c}_1,\tau=1}<0$ if and only if $G(\xi)>0$ and $\frac{\varphi r\lambda}{1-\theta}<\frac{1}{\overline{e}\frac{1-\xi}{\xi}\frac{1}{G(\xi)}+1-\overline{e}}$. Combined with the requirement $1 < \frac{\varphi r\lambda}{1-\theta} < \frac{1}{1-\overline{e}}$, we get that $\overline{c}_1 > 0$ and $\frac{\partial \overline{c}_\tau}{\partial \tau}|_{\tau=1} < 0$ if and only if $G(\xi) > 0$ and

$$1 < \frac{\varphi r\lambda}{1-\theta} < \frac{1}{\overline{e}\frac{1-\xi}{\xi}\frac{1}{G(\xi)} + 1 - \overline{e}}. \tag{62}$$

Since $\frac{1}{\overline{e}\frac{1-\xi}{\xi}\frac{1}{G(\xi)}+1-\overline{e}} \to \frac{1}{1-\overline{e}}$ as $\xi \to 1$, if the short-sale constraint binds in the action equilibrium (i.e., $1 < \frac{\varphi r\lambda}{1-\theta}$) and $\overline{c}_1 > 0$ in the tilt region (i.e., $\frac{\varphi r\lambda}{1-\theta} < \frac{1}{1-\overline{e}}$), then there exists $\overline{\xi} \in (0,1)$ such that if $\xi > \overline{\xi}$ then $\overline{c}_\tau$ is decreasing function at $\tau = 1$, as required.

Part (ii). If $\eta^* = 1$, then η^* is constant in τ and the claim is immediate. If $\eta^* \in (0,1)$, the equilibrium lies in the interior regime and satisfies $\mathcal{B}_{\text{int}}(\eta^*;\tau) = 0$ with $\frac{\partial \mathcal{B}_{\text{int}}}{\partial \eta}|_{\eta^*} < 0$ (due to the stability condition). Thus, by the implicit function theorem it's sufficient to prove that $\frac{\partial \mathcal{B}_{\text{int}}(\eta;\tau)}{\partial \tau} > 0$. From (22) and (16)

$$\begin{aligned}\mathcal{B}_{\text{int}}(\eta;\tau) &= \left(\frac{q}{\kappa_R/\kappa_H + 2}\frac{\varphi r\lambda}{1-\theta}\overline{e} - \frac{c}{\kappa_H(\overline{q}+q)}\right)[\alpha(1;\eta) - \alpha(0;\eta)] \\ &\quad - \frac{c}{\kappa_H(\overline{q}+q)}\frac{(1-\omega)(1-\theta)}{\omega + (1-\omega)\theta}\frac{1}{2\tau - 1}.\end{aligned} \tag{63}$$

The condition $\mathcal{B}_{\text{int}}(\eta^*;\tau) = 0$ implies that the first term is positive. Since $\alpha(1;\eta) - \alpha(0;\eta)$ increases in τ and $\frac{1}{2\tau-1}$ decreases in τ, we have $\frac{\partial \mathcal{B}_{\text{int}}(\eta;\tau)}{\partial \tau} > 0$, as required. ■

Proof of Proposition 3. **Part (i).** If $\eta^* = 0$, then the signal s is uninformative, the posteriors in (14) are zero after both realizations, and x_R is independent of s and τ. Therefore, $\mathbb{E}[U_M]$ does not depend on τ.

If $\eta^* > 0$, using $\mathbb{E}[v] = \frac{\mu + rq\,\mathbb{E}[p] - c\xi\eta^*}{\overline{q}+q}$ and $\mathbb{E}[a] = \xi\eta^*$, we have

$$\begin{aligned}\mathbb{E}[U_M] &= (\omega + (1-\omega)\theta)\,\mathbb{E}[p] + (1-\omega)\frac{\mu - c\xi\eta^*}{\overline{q}+q}. \\ &= (\omega + (1-\omega)\theta)\frac{\mu - c\xi\eta^* - (q - \mathbb{E}[x_R])(\overline{q}+q)\kappa_H}{\overline{q}+q-rq} + (1-\omega)\frac{\mu - c\xi\eta^*}{\overline{q}+q} \\ &= \frac{1}{1-\theta}\frac{1}{\overline{q}+q}(\mu - c\xi\eta^*) - \left(\omega + \frac{\theta}{1-\theta}\right)\kappa_H(q - \mathbb{E}[x_R])\end{aligned} \tag{64}$$

and

$$\frac{\partial \mathbb{E}[U_M]}{\partial \tau} = -\frac{1}{1-\theta}\frac{c\xi}{\overline{q}+q}\frac{\partial \eta^*}{\partial \tau} + \left(\omega + \frac{\theta}{1-\theta}\right)\kappa_H\frac{\partial \mathbb{E}[x_R]}{\partial \tau}. \tag{65}$$

There are three cases to consider. First, suppose $\eta^* = 1$ and $x_R(1) > x_R(0) > 0$. In this case, $\frac{\partial \eta^*}{\partial \tau} = 0$ and

$$\begin{aligned} \mathbb{E}[x_R] &= \Pr[s=1]\, x_R(1) + \Pr[s=0]\, x_R(0) \\ &= \frac{q}{\kappa_R/\kappa_H + 2}\left(1 - \frac{\varphi r \lambda}{1-\theta}(1 - \xi \overline{e})\right), \end{aligned} \tag{66}$$

which does not depend on τ, and thus, $\frac{\partial \mathbb{E}[U_M]}{\partial \tau} = 0$.

Second, suppose $\eta^* = 1$ and the short-sale constraint binds after $s = 0$: the expression inside the max operator in (12) is non-positive at $s = 0$ and strictly positive at $s = 1$ (with slight abuse of notation, $x_R(0) \leq 0 < x_R(1)$). Then $\mathbb{E}[x_R] = \Pr[s=1]\, x_R(1)$ and

$$\frac{\partial \mathbb{E}[x_R]}{\partial \tau} = \frac{q}{\kappa_R/\kappa_H + 2}\left(\left(1 - \frac{\varphi r \lambda}{1-\theta}\right)(2\xi - 1) + \frac{\varphi r \lambda}{1-\theta}\xi \overline{e}\right). \tag{67}$$

Notice that $x_R(0) \leq 0 < x_R(1)$ implies

$$\frac{1}{1 - \alpha(0,1)\,\overline{e}} \leq \frac{\varphi r \lambda}{1-\theta} < \frac{1}{1 - \alpha(1,1)\,\overline{e}}, \tag{68}$$

and hence, evaluating $\frac{\partial \mathbb{E}[x_R]}{\partial \tau}$ at $\frac{\varphi r \lambda}{1-\theta} = \frac{1}{1-\alpha(s,1)\overline{e}}$ gives

$$\frac{\partial \mathbb{E}[x_R]}{\partial \tau} = \frac{q}{\kappa_R/\kappa_H + 2}\frac{\alpha(s,1)(1-\xi) + \xi(1 - \alpha(s,1))}{1 - \alpha(s,1)\,\overline{e}}\overline{e} > 0. \tag{69}$$

Since $\frac{\partial \mathbb{E}[x_R]}{\partial \tau}$ is affine in $\frac{\varphi r \lambda}{1-\theta}$, and strictly positive at the end-points $\frac{\varphi r \lambda}{1-\theta} \in \left\{\frac{1}{1-\alpha(0,1)\overline{e}}, \frac{1}{1-\alpha(1,1)\overline{e}}\right\}$, it must also be strictly positive for every $\frac{\varphi r \lambda}{1-\theta} \in \left[\frac{1}{1-\alpha(0,1)\overline{e}}, \frac{1}{1-\alpha(1,1)\overline{e}}\right]$. Since $\frac{\partial \mathbb{E}[x_R]}{\partial \tau} > 0$, $\frac{\partial \mathbb{E}[U_M]}{\partial \tau} > 0$ as well.

Third, if $\eta^* \in (0,1)$ then we must have $x_R(1) > x_R(0) > 0$. Therefore,

$$\mathbb{E}[x_R] = \frac{q}{\kappa_R/\kappa_H + 2}\left(1 - \frac{\varphi r \lambda}{1-\theta}(1 - \eta^* \xi \overline{e})\right), \tag{70}$$

and

$$\frac{\partial \mathbb{E}[x_R]}{\partial \tau} = \frac{q}{\kappa_R/\kappa_H + 2}\frac{\varphi r \lambda}{1-\theta}\frac{\partial \eta^*}{\partial \tau}\xi \overline{e}. \tag{71}$$

Thus,

$$\frac{\partial \mathbb{E}[U_M]}{\partial \tau} = \frac{\kappa_H}{1-\theta}\xi\frac{\partial \eta^*}{\partial \tau}\left[(\omega+(1-\omega)\theta)\frac{q}{\kappa_R/\kappa_H+2}\frac{\varphi r\lambda}{1-\theta}\overline{e} - \frac{c}{(\overline{q}+q)\,\kappa_H}\right]. \tag{72}$$

The indifference condition $\mathcal{B}_{\text{int}}(\eta^*;\varphi)=0$ implies that the bracketed term is strictly positive, and the proof of Proposition 2(ii) gives $\frac{\partial\eta^*}{\partial\tau}>0$; hence $\frac{\partial\mathbb{E}[U_M]}{\partial\tau}>0$, as required.

Part (ii). As shown above, if $c<\overline{c}_1$ then $1\in\mathcal{T}_A$. We first show that in this case, the manager prefers the equilibrium with $\eta^*=1$ to the equilibrium with $\eta^*=0$.

Consider a manager who observes $e=\overline{e}$. Existence of $\eta^*=1$ requires that this manager does not want to deviate from taking the action ($a=1$) to not taking the action ($a=0$). The deviation payoff, however, is identical to the payoff in the no-action equilibrium with $\eta^*=0$ because $s=0$ with probability one. As a result, the responsible investor's trades, the stock price, and the firm value are identical in the deviation and the no-action equilibrium. When $c<\overline{c}_1$, this incentive constraint holds strictly ($\mathcal{B}(\eta;1)>0$), so the manager strictly prefers the action equilibrium at $\tau=1$ to the no-action equilibrium.

Recall that, according to part (i), the manager weakly prefers the action equilibrium at $\tau=1$ to any action equilibrium at $\tau<1$. Therefore, if $c<\overline{c}_1$, the manager chooses some $\tau^*\in\mathcal{T}_A$ attaining the payoff he would receive under $\tau=1$. If $c>\overline{c}_1$, then the manager strictly prefers the no-action equilibrium to the action equilibrium at $\tau=1$, if the latter were to exist; and since that payoff is weakly higher than that of any action equilibrium with $\tau<1$, the manager strictly prefers the no-action equilibrium. Since the manager's expected payoff is invariant to τ in the no-action equilibrium, by the tie-breaking assumption mentioned in the text, he then chooses $\tau^*=\frac{1}{2}$. In the knife-edge case $c=\overline{c}_1$, we have $\mathcal{B}(\cdot;1)\equiv 0$, so the payoff the manager would obtain in an action equilibrium at $\tau=1$ equals the no-action payoff; by part (i), any action equilibrium with $\tau<1$ then yields a weakly lower payoff than the no-action equilibrium. The manager thus weakly prefers the no-action equilibrium, and the tie-breaking assumption again gives $\tau^*=\frac{1}{2}$. ■

Proof of Proposition 4. Part (i). Let $f(\eta^*)$ and $p(\eta^*)$ denote, respectively, the expected externalities and the stock price in the equilibrium where the action is taken with probability η^*. Note that

$$\mathbb{E}[a^*e] = \overline{e}\xi\eta^*. \tag{73}$$

Thus, $f(\eta^*) > f(0)$ if and only if

$$\begin{aligned} \mathbb{E}\left[(\mu + rqp(\eta^*))\,(1 - a^*e)\right] &> \mu + rqp(0) \\ \mu &< rq\frac{\mathbb{E}\left[p(\eta^*; s)\,(1 - a^*e)\right] - p(0)}{\mathbb{E}\left[a^*e\right]}. \end{aligned} \tag{74}$$

Since $B(\eta)$ does not depend on μ, it follows that η^* or a^* do not depend on μ either. Notice that the derivative of the right-hand side of (74) with respect to μ is $-\frac{rq}{\overline{q}+q-rq} < 0$, and hence, there exists $\overline{\mu} > 0$ such that $f(\eta^*) > f(0) \Leftrightarrow \mu < \overline{\mu}$, as required.

Finally, suppose that $\eta^* = 1$ and $x_R(0; 0) \leq 0$. Since the right-hand side of (74) is decreasing in μ, we have $sign(\frac{\partial \overline{\mu}}{\partial \varphi}) = sign(\frac{\partial \mathbb{E}[p(\eta^*;s)(1-a^*e)]}{\partial \varphi})$. Since $\eta^* = 1$,

$$\begin{aligned} \frac{\partial \mathbb{E}\left[p(\eta^*; s)\,(1 - a^*e)\right]}{\partial \varphi} &\propto \mathbb{E}\left[\frac{\partial\left(x_R\left(s; 1\right)\left(1 - \left(a^*e\right)\right)\right)}{\partial \varphi}\right] \\ &= \Pr\left[s = 1\right]\frac{\partial x_R\left(1; 1\right)}{\partial \varphi}\left(1 - \alpha(1; 1)\overline{e}\right) + \Pr\left[s = 0\right]\frac{\partial x_R\left(0; 1\right)}{\partial \varphi}\left(1 - \alpha(0; 1)\overline{e}\right), \end{aligned} \tag{75}$$

and since $x_R(s; \eta)$ is weakly decreasing in φ for every realization of s, and $\alpha(s; 1)\overline{e} < 1$, it follows that $\frac{\partial \mathbb{E}[p(\eta^*;s)(1-a^*e)]}{\partial \varphi} < 0$. Therefore, $\frac{\partial \overline{\mu}}{\partial \varphi} < 0$ in those cases.

Part (ii). The left panel of Figure 5 provides a numerical example in which expected firm value is higher in the action equilibrium for intermediate φ and lower for low and high φ, establishing the first statement.

For the second statement, expected firm value in the equilibrium with action probability η^* is

$$\mathbb{E}[v|\eta^*] = \frac{\mu - c\xi\eta^* - \left(q - \mathbb{E}[x_R(s; \eta^*)]\right) rq\kappa_H}{\overline{q} + q - rq}. \tag{76}$$

Differencing between the action and no-action equilibria gives

$$\mathbb{E}[v|\eta^*] > \mathbb{E}[v|0] \iff \mathbb{E}[x_R(s; \eta^*)] - x_R(s; 0) > \frac{c\,\xi\eta^*}{rq\kappa_H}. \tag{77}$$

Recall from the proof of Proposition 1 that, holding $\varphi \in (\underline{\varphi}, \overline{\varphi})$ fixed, the action equilibrium continues to exist as c falls and satisfies $\eta^* = 1$ for all c below some $c_0 > 0$, since $\mathcal{B}(1; \varphi) > 0$ at $c = 0$ and $\mathcal{B}(\eta, \varphi; c)$ is continuous and strictly decreasing in c. Evaluated at $\eta^* = 1$, the left-hand side of (77) does not depend on c and is strictly positive. Indeed, $\alpha(s; 1) > 0$ for

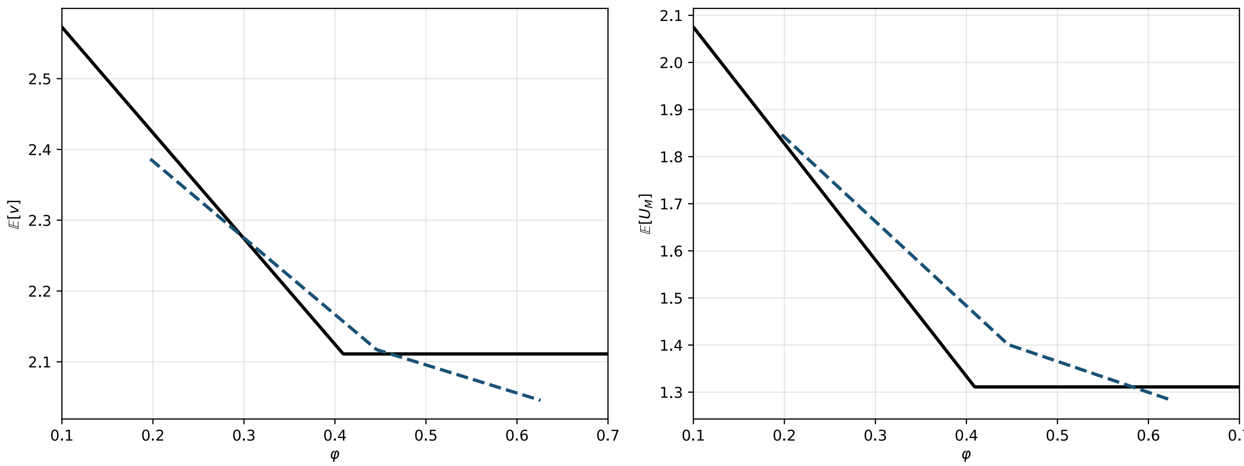


Figure 5: Expected per-share firm value $\mathbb{E}[v]$ and manager utility $\mathbb{E}[U_M]$ as a function of the responsible investor's social concern φ. The solid line is the equilibrium with $\eta^* = 0$ and the dashed line is the equilibrium with $\eta^* > 0$. Parameters: $\xi = 0.5$, $\tau = 0.85$, $\omega = 0.8$, $\bar{e} = 0.55$, $c = 0.22$, $\mu = 3$, $r = 1.1$, $q = \bar{q} = 1$, $\kappa_H = 1$, $\kappa_R = 0$, $\lambda = 1$.

$s \in \{0, 1\}$, so by Lemma 1, $x_R(1; 1) > x_R(1; 0)$ and $x_R(0; 1) \geq x_R(0; 0)$. Hence, evaluated at $\eta^* = 1$, the left-hand side of (77) is independent of c and positive, while the right-hand side increases in c and equals zero at $c = 0$; therefore (77) holds for $c > 0$ sufficiently small. Thus, there exists $\underline{c} > 0$ such that for all $c < \underline{c}$ we have $\eta^* = 1$ and expected firm value strictly higher in the action equilibrium.

Part (iii). Let $\mathbb{E}[U_R(x, \eta)|s]$, $\mathbb{E}[p(x, \eta)|s]$, and $\mathbb{E}[f(x, \eta)|s]$ denote, respectively, the responsible investor's expected utility, expected price, and expected externalities conditional on signal s and trade x, when the action is taken with probability η conditional on being effective. Notice that

$$\mathbb{E}[p(x, \eta)|s] = \frac{\mu - c\,\mathbb{E}[a|s] - (q - x)(q + \bar{q})\kappa_H}{q + \bar{q} - rq}. \tag{78}$$

Fix $x \geq 0$. Then (78) implies that the responsible investor's trading profits (gross of trading costs) are $x(v - p) = x\kappa_H(q - x)$, which is independent of η. Moreover, since households price in the costly action, it must be that $\mathbb{E}[p(x, 0)|s] > \mathbb{E}[p(x, \eta)|s]$. Since both the corrective action and the lower price reduce the firm's externalities, it must also be that $\mathbb{E}[f(x, 0)|s] >$

$\mathbb{E}[f(x,\eta)|s]$.[22] Therefore, for every $x \geq 0$ and every signal realization s,

$$\mathbb{E}[U_R(x,0)|s] < \mathbb{E}[U_R(x,\eta)|s] \leq \max_{x\geq 0} \mathbb{E}[U_R(x,\eta)|s]. \tag{79}$$

Evaluating (79) at $x = x_R(s;0)$, the responsible investor's equilibrium trade when $\eta^* = 0$, and taking expectations over s shows that her expected utility is strictly higher in the action equilibrium.

Part (iv). Recall $x_i(s) = \frac{\mathbb{E}[v|s]-p}{\kappa_H}$ and $p = \mathbb{E}[v|s] - \kappa_H (q - x_R(s))$. Thus, in equilibrium, the households' expected net trading profits conditional on signal s are

$$\begin{aligned} & \frac{\mathbb{E}[v|s]-p}{\kappa_H}(\mathbb{E}[v|s]-p) - \frac{\kappa_H}{2}\left(\frac{\mathbb{E}[v|s]-p}{\kappa_H}\right)^2 \qquad (80) \\ = \; & \frac{1}{2}\frac{1}{\kappa_H}(\mathbb{E}[v|s]-p)^2 \\ = \; & \frac{\kappa_H}{2}(q - x_R(s))^2, \end{aligned}$$

as required. The households prefer the action equilibrium if and only if,

$$\frac{\kappa_H}{2}\mathbb{E}[(q - x_R(s;\eta^*))^2] + \bar{q}\mathbb{E}[v|\eta^*] > \frac{\kappa_H}{2}\mathbb{E}[(q - x_R(s;0))^2] + \bar{q}\mathbb{E}[v|0]. \tag{81}$$

Given (76), this expression can be rewritten as

$$-\frac{1}{\kappa_H}\frac{\bar{q}}{q}\frac{1}{\bar{q}+q-rq}c\xi\eta^* > \mathbb{E}\left[(x_R(s;\eta^*) - x_R(s;0))\left(\frac{\bar{q}+q}{\bar{q}+q-rq}(1-r) - \frac{x_R(s;0)+x_R(s;\eta^*)}{2q}\right)\right]. \tag{82}$$

Notice $r > 1$ and $x_R(s;\eta^*) > x_R(s;0)$, thus the right-hand side of (82) is negative. Recall from Corollary 1 that an action equilibrium with $\eta^* = 1$ is more likely to exist as κ_H increases. Moreover, as $\kappa_H \to \infty$, the left-hand side of (82) converges to zero, whereas, given (12), the right-hand side of (82) remains strictly negative at this limit (recall the action equilibrium with $\eta^* = 1$ continues to exists). Therefore, for κ_H sufficiently large, households prefer the action equilibrium in spite of lower trading profits. ■

[22] At the knife-edge case $\tau = 1$ and $s = 0$, the unfavorable signal perfectly reveals that no effective action was taken, so $\alpha(0;\eta) = 0$ and these inequalities, as well as the first inequality in (79), hold with equality. This does not affect the result: at $s = 1$ the inequalities are strict for every τ, since $\alpha(1;\eta) > 0$, and $\Pr[s = 1] > 0$, so the expectation over s below remains strict.

The following Lemma establishes properties of the posteriors $\alpha(1;\eta)$ and $\alpha(0;\eta)$ that are used in the proofs above.

Lemma 3 *Let* $\tau \in (\frac{1}{2}, 1)$.

(i) Both posteriors satisfy $\alpha(s;0) = 0$ *and are strictly increasing in* η. *For all* $\eta > 0$, $\alpha(1;\eta) > \xi\eta > \alpha(0;\eta)$. *Moreover,* $\alpha(1;\eta)$ *is concave in* η *and* $\alpha(0;\eta)$ *is convex in* η.

(ii) The difference $\alpha(1;\eta) - \alpha(0;\eta)$ *is globally concave in* η*, and its derivative satisfies*

$$\operatorname{sign}\left(\frac{\partial[\alpha(1;\eta) - \alpha(0;\eta)]}{\partial\eta}\right) = \operatorname{sign}(1 - 2\xi\eta)\,. \tag{83}$$

Proof of Lemma 3. Write $z \equiv \xi\eta \in [0, \xi]$ for the unconditional probability that the action is both taken and effective. It follows that

$$\Pr(s{=}1) = z\tau + (1{-}z)(1{-}\tau), \qquad Pr(s{=}0) = z(1{-}\tau) + (1{-}z)\tau \tag{84}$$

denote the signal probabilities. Then $\alpha(1) = z\tau/\Pr(s{=}1)$ and $\alpha(0) = z(1{-}\tau)/\Pr(s{=}0)$.

Part (i). At $z = 0$ (i.e. $\eta = 0$), both numerators vanish, giving $\alpha(1;0) = \alpha(0;0) = 0$. Differentiating with respect to η (using $dz/d\eta = \xi$):

$$\frac{\partial\alpha(s = \hat{s};\eta)}{\partial\eta} = \frac{\xi\tau(1{-}\tau)}{\Pr(s{=}\hat{s})^2} > 0, \tag{85}$$

so both posteriors are strictly increasing in η for all $\tau \in (\frac{1}{2}, 1)$.

To verify the ordering, direct computation gives:

$$\alpha(1;\eta) - \alpha(0;\eta) = z(1{-}z)(2\tau{-}1)/(\Pr(s{=}1)\ \Pr(s{=}0)) > 0 \tag{86}$$

for $\eta > 0$, since $z = \xi\eta \in (0, 1)$ and $\tau > \frac{1}{2}$.

To verify $\alpha(1;\eta) > \xi\eta > \alpha(0;\eta)$, note that $\alpha(1;\eta) = z\tau/\Pr(s{=}1)$ and the unconditional mean is $\xi\eta \cdot 1 + (1{-}\xi\eta) \cdot 0 = z$. Since $\alpha(1;\eta)$ is the posterior after the more informative signal, $\alpha(1;\eta) > z > \alpha(0;\eta)$ by the law of iterated expectations together with the strict ordering $\alpha(1;\eta) > \alpha(0;\eta)$.

Differentiating the first derivatives again:

$$\frac{\partial^2\alpha(1;\eta)}{\partial\eta^2} = -\frac{2\xi^2\tau(1-\tau)(2\tau-1)}{\Pr(s{=}1)^3} < 0, \qquad \frac{\partial^2\alpha(0;\eta)}{\partial\eta^2} = \frac{2\xi^2\tau(1-\tau)(2\tau-1)}{\Pr(s{=}0)^3} > 0, \tag{87}$$

where the signs follow from $\tau \in (\frac{1}{2}, 1)$ implying $2\tau-1 > 0$, $\Pr(s{=}1) > 0$, and $\Pr(s{=}0) > 0$. Hence $\alpha(1;\eta)$ is strictly concave and $\alpha(0;\eta)$ is strictly convex in η.

Part (ii). Let the difference in posteriors be $D(\eta) \equiv \alpha(1;\eta) - \alpha(0;\eta)$. Since both posteriors are zero at $\eta = 0$, $D(0) = 0$. The derivative of $D(\eta)$ with respect to η is

$$\frac{\partial D(\eta)}{\partial\eta} = \xi\tau(1-\tau)\left(\frac{1}{\Pr(s{=}1)^2} - \frac{1}{\Pr(s{=}0)^2}\right) = \xi\tau(1-\tau)\,\frac{\Pr(s{=}0)^2 - \Pr(s{=}1)^2}{\Pr(s{=}1)^2\,\Pr(s{=}0)^2}. \tag{88}$$

Since $\Pr(s{=}1) + \Pr(s{=}0) = 1$ and $\Pr(s{=}1) - \Pr(s{=}0) = (2z-1)(2\tau-1)$, we have

$$\Pr(s{=}0)^2 - \Pr(s{=}1)^2 = (\Pr(s{=}0) - \Pr(s{=}1))(\Pr(s{=}0) + \Pr(s{=}1)) = (2\tau-1)(1-2z)\cdot 1, \tag{89}$$

giving

$$\frac{\partial D(\eta)}{\partial\eta} = \frac{\xi\tau(1-\tau)(2\tau-1)(1-2\xi\eta)}{\Pr(s{=}1)^2\,\Pr(s{=}0)^2}. \tag{90}$$

Since $\xi\tau(1-\tau)(2\tau-1) > 0$ and $\Pr(s{=}1)^2\,\Pr(s{=}0)^2 > 0$, the sign of $\partial D/\partial\eta$ is determined by $1 - 2\xi\eta$. Three cases follow:

Case $\xi > 1/2$. The derivative changes sign at $\eta^\dagger \equiv 1/(2\xi) \in (0,1)$: positive for $\eta < \eta^\dagger$ and negative for $\eta > \eta^\dagger$. Combined with $D(0) = 0$ and continuity, D first increases then decreases, with a unique maximum at $\eta^\dagger$.

Case $\xi < 1/2$. Then $1 - 2\xi\eta \geq 1 - 2\xi > 0$ for all $\eta \in [0,1]$, so D is strictly increasing on $[0,1]$ and peaks at $\eta = 1$.

Case $\xi = 1/2$. Then $1 - 2\xi\eta = 1 - \eta \geq 0$ with equality only at $\eta = 1$, so D is strictly increasing on $[0,1)$ with $D'(1) = 0$; it peaks at $\eta = 1$.

Finally, global concavity of D follows from $D''(\eta) = \alpha''(1;\eta) - \alpha''(0;\eta)$: part (i) shows $\alpha''(1;\eta) < 0$ and $\alpha''(0;\eta) > 0$, so $D'' < 0$ for all $\eta \in (0,1]$ and $\tau \in (\frac{1}{2}, 1)$. ∎

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