

# Market Liquidity, Investor Participation and Managerial Autonomy: Why do Firms go Private?

Finance Working Paper N° 119/2006

October 2005

Arnoud Boot

Washington University in St.Louis.

Radhakrishnan Gopalan

Washington University in St.Louis.

Anjan Thakor

Washington University in St.Louis.

© Arnoud Boot, Radhakrishnan Gopalan and Anjan Thakor 2005. All rights reserved. Short sections of text, not to exceed two paragraphs, may be quoted without explicit permission provided that full credit, including © notice, is given to the source.

This paper can be downloaded without charge from:  
<https://www.ecgi.global/publications/working-papers>

ECGI Working Paper Series in Finance

---

# Market Liquidity, Investor Participation and Managerial Autonomy: Why do Firms go Private?

---

Working Paper N° 119/2006  
October 2005

**Arnoud Boot**  
**Radhakrishnan Gopalan**  
**Anjan Thakor**

---

We thank Yakov Amihud, Patrick Bolton, Kose John, Rafael Repullo, seminar participants at the 2005 ACLE/JFI Conference on "The Ownership of the Modern Corporation: Economic and Legal Perspectives on Private versus Publicly-listed Corporations" in Amsterdam, and seminar participants at Amsterdam, BI Oslo, Frankfurt, Georgetown, and NYU for useful co?

© Arnoud Boot, Radhakrishnan Gopalan and Anjan Thakor 2005. All rights reserved. Short sections of text, not to exceed two paragraphs, may be quoted without explicit permission provided that full credit, including © notice, is given to the source.

# Market Liquidity, Investor Participation and Managerial Autonomy: Why do Firms Go Private?

by

*Arnoud W. A. Boot, Radhakrishnan Gopalan and Anjan V. Thakor \**

---

\*Boot is from the University of Amsterdam and CEPR, Gopalan and Thakor are from the Olin School of Business, Washington University in St.Louis. We thank Yakov Amihud, Patrick Bolton, Kose John, Rafael Repullo, seminar participants at the 2005 ACLE/JFI Conference on “The Ownership of the Modern Corporation: Economic and Legal Perspectives on Private versus Publicly-listed Corporations” in Amsterdam, and seminar participants at Amsterdam, BI Oslo, Frankfurt, Georgetown, and NYU for useful comments.

# Market Liquidity, Investor Participation and Managerial Autonomy: Why do Firms Go Private?

## ABSTRACT

We analyze a publicly-traded firm's decision to stay public or go private, focusing on the stochastic nature of investor participation in the public market. The liquidity of public ownership is both a blessing and a curse: it facilitates trading and lowers the cost of capital, but it also introduces volatility in a firm's shareholder base. This exposes management to uncertainty regarding the identity of future shareholders and their intervention in management decisions, consequently affecting the manager's perceived decision-making autonomy and curtailing managerial inputs. We extract predictions about how investor participation affects stock price level and volatility and the public firm's incentives to go private, thereby providing a link between *investor participation* and *firm participation* in public markets.

“Hotshot executives are fleeing the scrutiny of public companies  
for the mad money of the private-equity boom”,

*Business Week*, February 27<sup>th</sup> 2006, p. 52

When should a publicly-traded firm decide to go private? This question, which we address in this paper, is of central importance in the theory of the firm, and has been brought into sharper focus by recent events. During the 1990s, scores of companies went public, many quite young. However, since the precipitous decline of the stock market, the “going-public” wave appears to have been replaced by a delisting surge. The number of U.S. companies delisting was 83 in 1999, 86 in 2000, 262 in 2003 and 188 in 2004.<sup>1</sup> Many have conjectured that the decline in stock prices after 2000 has induced firms to go private, a sort of flip side of the observation that IPOs are largely a bull-market phenomenon (e.g. Ritter and Welch (2002)). But why should privately-held firms go public when stock prices are high and publicly-traded firms go private when stock prices are low?

An additional development that has been suggested as a factor is the recent set of changes in the corporate governance of publicly-traded firms. In the U.S., this involved the passage of the Sarbanes-Oxley Act (SOX). This Act, passed in the wake of accounting scandals and other corporate abuses at highly-visible publicly-traded firms, is intended in part to restore investor confidence in the public stock market and ensure continued investor participation. But it has been suggested that this may increase the costs of being public and discourage public ownership.<sup>2</sup> There is, however, no formal theory that provides any link between a firm’s decision to go private, investor participation in public capital markets, the level of its stock price, and the stringency of its corporate governance.

---

<sup>1</sup>A similar trend has been observed in Europe. Although delisting includes both “going dark” and going-private (wherein one or more private investors buy out the company’s shares) transactions, the delisting data are nonetheless indicative of firms eschewing public ownership. Hill (2007) reports that the number of companies going private since 2001 has doubled, and includes many large transactions such as Hertz, Nieman Marcus, Metro-Goldwyn-Mayer, Toys “R” Us, and the \$ 32 billion buyout of energy company TXU in 2007.

<sup>2</sup>For example, Deutsch (2005) reports in the *New York Times*: “The shares of Fidelity Bancorp have always been thinly traded, and its executives wondered why it bothered to be a public company at all. Still, they never really considered delisting the stock until Congress passed the Sarbanes-Oxley Act, with its myriad new reporting requirements, in 2002.

In November, the bank announced that it was “going dark” - delisting its stock from the Nasdaq market.”

In addition to inducing public U.S. firms to go private, it has also been alleged that the higher costs of being public post-SOX have induced public firms to be listed on exchanges other than the U.S. For example, according to the *Financial Services Forum*, the U.S. accounted for 20% of all IPO sales worldwide in 2006, down from 35% in 2001. Zingales (2007) notes that U.S. equity market share has dropped dramatically during 2000-2005 and attributes this partly to the increased compliance costs for public U.S. companies. This conclusion has been challenged by Doidge, Karolyi and Stultz (2007). They note that cross-listings in both New York and London declined during 1990-2005 and that SOX has not resulted in any decline in New York cross-listings.

Also unexplained is the stylized fact that the age of firms going public has been diminishing in the U.S., stock prices and corporate governance issues notwithstanding (see Fink et al., (2004)). Perhaps this has something to do with the development of the U.S. capital markets and enhanced investor participation. We do not really know. But it seems important for any theory of the choice of ownership structure to address this phenomenon as well.

We develop a theory that assigns center stage to investor participation and explores the potential interaction between investor participation and firm participation in the context of a publicly-traded firm's choice of whether to stay public or go private. We model a publicly-traded firm whose objective is to maximize the wealth of its initial shareholders. The firm has assets in place and also access to a growth opportunity. When the growth opportunity arrives, it is possible that the manager's posterior belief about its value will diverge from the shareholders' (investors') posterior belief. This can lead to disagreement about the optimal course of action. That is, disagreement can arise because beliefs are heterogeneous but rational in the sense of Kurz (1994a,b).<sup>3</sup> We show that investors optimally give the manager some decision-making discretion or autonomy to overcome such disagreement and pursue the project-choice decision he thinks is best. In determining optimal managerial autonomy through the stringency of corporate governance, investors face the following tradeoff. On the one hand, investors want to make governance as stringent as possible to minimize the likelihood of the manager making a project-choice decision they don't like. On the other hand, greater stringency induces lower managerial effort in uncovering a good growth opportunity. We examine how this endogenously-determined managerial autonomy varies across public and private ownership.

A key aspect of our analysis is investor heterogeneity. Different investors may have different propensities to agree with the manager. This implies that the optimal degree of managerial autonomy as well as the valuation of the firm may differ depending on the investor base. With private ownership, the ownership base is stable because the relative illiquidity of this ownership discourages investors from frequent trading. This leads to stability in the alignment between the manager and the investors and hence

---

<sup>3</sup>Heterogeneous priors have been used in various settings previously, e.g. Abel and Mailath (1994), Allen and Gale (1999), Boot, Gopalan and Thakor (2006), Garmaise (2001), Morris (1995), Song and Thakor (forthcoming), and Van den Steen (2004, 2006). Recently, Acemoglu, Chernozhukov and Yildiz (2006) have shown that convergence of heterogeneous prior beliefs to a common posterior belief may *not* occur when individuals are uncertain about the interpretation of signals. Disagreement can also arise due to a variety of reasons other than the one we focus on, such as overconfidence on the part of either management or investors (Bernardo and Welch (2001), and Daniel, Hirshleifer and Subramanyam (1998)), and excessive pessimism (Coval and Thakor (2005)) or optimism (Manove and Padilla (1999)).

in the autonomy given to the manager. In the public capital market, the composition of shareholders is not fixed; it changes as investors trade in and out of the firm’s shares because of market liquidity. Consequently, there is uncertainty about the level of alignment between the manager and investors.

For public firms, trading-induced uncertainty about the future ownership base means that managerial autonomy is based on an *expectation* of what future investors will consider optimal. By contrast, in a private firm, corporate governance is determined by private investors, who will *not* trade away their (illiquid) ownership, so managerial autonomy is chosen to be precisely optimal for those investors. In deciding whether to take a publicly-held firm private, shareholders trade off the higher liquidity of public ownership against the more precisely-determined corporate governance of private ownership. This tradeoff is affected by the cross-sectional heterogeneity among investors and the degree of investor participation in the public market. The “butter and knife” in the tradeoff that determines ownership choice are essentially one and the same, namely the greater liquidity of public ownership relative to private ownership. On the one hand this greater liquidity generates a relative benefit for public ownership by lowering the firm’s cost of capital, and on the other hand it generates a relative disadvantage for public ownership by creating greater uncertainty about the firm’s ownership base.

We also modify our basic set-up to explore extensions of the model. In one extension, we discuss the role of lock-up agreements as a way to increase investor stability in the public market. In other extensions, we analyze the impact of alternative incentive contracts for the manager and examine how the level of investor participation in the public market affects the age of the firm choosing to go private.

Our main results are as follows. First, the likelihood of a publicly-traded firm going private is decreasing in the level of its stock price, and increasing in the volatility of its stock price. Second, when a public firm goes private, it experiences an increase in value. Third, an increase in investor participation in the public equity market leads to an increase in the attractiveness of public ownership by elevating the firm’s stock price, decreasing its price volatility, and increasing the autonomy of its manager. Fourth, public firms will go private only at substantial premia above the pre-transaction stock prices.<sup>4</sup> Fifth, a decrease in public-market investor participation encourages younger firms to go private.

---

<sup>4</sup>We argue that the likelihood that a private investor will show up who is willing to pay the premium is increasing in the number of potential private investors, and that this number has been enlarged by the development of private equity firms.

Our work is related to the research on the determinants of public versus private ownership (e.g. Zingales (1995)), and particularly the role of the tension between liquidity and control in this determination. Some believe that public ownership comes with liquidity benefits but is too dispersed in ownership to offer the effective control that private ownership offers to investors (Bhide (1993) and Coffee (1991)).<sup>5</sup> Bebchuk and Jolls (1999) argue that an initial owner in a public company may defeat investors' attempts to control him by maintaining control through his ownership in order to protect his private control benefits. However, others have challenged this view. Bolton and von Thadden (1998) show that a limited degree of ownership concentration in a public firm captures both the benefits of liquidity and control. Faure-Grimaud and Gromb (2004) also highlight the role of a liquid and informative public market in increasing the activism incentives of a large shareholder and hence the attractiveness of public ownership. Kahn and Winton (1998) further illuminate this issue by showing that large shareholders are more likely to play an active role when the stock price is low, when stock liquidity is low, and when their holdings are large. Maug (1998) presents a viewpoint similar to Bolton and von Thadden (1998), but goes even further to suggest that investors in public firms could potentially exercise even better control over management than possible in a private firm.<sup>6</sup> Burkart, Gromb and Panunzi (1997) question even the desirability of investor control over the manager, since such control may adversely affect ex ante managerial incentives.

While there are numerous differences between these papers and ours, the key distinction is that the existing literature has not addressed what constitutes the primary focus of our analysis, namely the manner in which a publicly-traded firm's choice of whether to go private is influenced by investor participation in public equity markets and particularly the difference in the stability of the firm's investor base across private and public ownership.<sup>7</sup> This focus and our assumption of universal risk neutrality distinguish our paper naturally from papers that focus on the diversification and risk-sharing benefits of

---

<sup>5</sup>This improved investor control may permit investors to limit managerial expropriation of shareholder wealth or other forms of abuses such as those discussed by Jensen (1986). The point that private ownership offers better control has also been made by Black and Gilson (1998). While this literature has focussed on private versus public equity, others have also examined the firm's choice between private and public borrowing (e.g., Detragiache (1994)).

<sup>6</sup>Although not addressing the issue of the choice between private and public ownership, Bennedsen and Wolfenzon (2000) show how the owner of the firm can choose an ownership structure with several—instead of one—large shareholders in order to form a *coalition* to acquire control. The coalition takes more efficient actions than any individual member would.

<sup>7</sup>The disclosure literature has also examined the link between investor participation and disclosure (see Bhattacharya and Nicodono (2001), for example). Maksimovic and Pichler (2001) argue that public ownership is accompanied by less flexible disclosure requirements than private ownership. Chemmanur and Fulghieri (1999) argue that the public capital markets involve duplicated monitoring costs.

public markets, such as Admati, Pfleiderer and Zechner (1994), Pagano (1993), and Shah and Thakor (1988).<sup>8</sup>

Our analysis assigns a pivotal role to control in the choice between private and public ownership, something that has been empirically documented (see Brau and Fawcett (2006)). This relates our paper to the literature on private control benefits. Aghion and Bolton (1992) examine both efficient and inefficient exogenous private control benefits, with the manager having a hard-wired preference for control; by contrast, in our model the manager has *no* innate preference for control, and his desire for autonomy emerges endogenously via the potential for disagreement.<sup>9</sup>

Pagano and Roell (1998) model an entrepreneur who has full control and can engage in socially-inefficient private benefits extraction. The entrepreneur would benefit from a credible precommitment not to extract private benefits, but investors need to monitor to limit entrepreneurial diversion. Private companies may experience excessive monitoring due to ownership concentration, thereby inducing the firm to go public. In discussing model extensions, Pagano and Roell (1998) note, as we do, that one additional disadvantage of public ownership is the entrepreneur's possible loss of control over the ownership base. In their model, this arises from the arrival of a new shareholder who can create value with a superior monitoring technology and who buys out the previous monitoring shareholder. In contrast to public ownership, private ownership allows the entrepreneur to have veto power over such a transfer of ownership and hence extract a greater share of the surplus created.

There are several key differences between Pagano and Roell's (1998) analysis and ours. First, the effect of investor participation in the public market on the firm's choice between private and public ownership is the focal point of our analysis, and absent in theirs. Second, the possible arrival of a shareholder with a superior monitoring technology unambiguously increases firm value even with public ownership in Pagano and Roell (1998).<sup>10</sup> By contrast, shareholder volatility with public ownership is an *ex ante* cost in our model and the entrepreneur is better off when this volatility is diminished via greater

---

<sup>8</sup>See, also Jensen (1986), Black and Gilson (1998), and Kaplan and Stromberg (2003).

<sup>9</sup>In Aghion and Bolton (1992), the objectives of the manager and investors diverge, unlike in our model. Moreover, (exogenously-fixed) security benefits are identically valued by the manager and investors, whereas in our model the very reason why autonomy matters is that endogenously-determined security benefits are valued differently by the manager and investors.

<sup>10</sup>That is, if we were to account for the impact of the possible loss of control over the shareholder base on *ex ante* firm value, we would find that this value would be higher and all shareholders would be better off due to the future loss of control because it would strengthen the entrepreneur's precommitment not to divert. In other words, this loss of control with public ownership in Pagano and Roell (1998) is a disadvantage for the owner only *ex post*. He benefits *ex ante*.

investor participation. Third, we differ significantly in the empirical predictions. Pagano and Roell's (1998) main prediction is that public ownership is preferred when large amounts of external financing are needed and/or private benefit consumption is relatively efficient. Our analysis produces a host of different predictions that depend critically on investor participation and the alignment between the firm and investors. Finally, the public-to-private transactions we consider are quite different from private-to-public transactions that Pagano and Roell (1998) consider. Since our public-to-private transaction involves bidding for the shares of small investors and only the highest bidder is likely to succeed, it cannot be explained on the basis of managers taking firms private to preserve inefficient private benefit consumption or to avoid hostile takeovers.<sup>11</sup>

The paper closest to ours is Boot, Gopalan and Thakor (2006), which also examines the entrepreneur's choice between private and public ownership in a setting in which the manager has an endogenous control preference due to potential disagreement with investors.<sup>12</sup> There are numerous important differences, however, between that paper and our work here. First, in Boot, Gopalan and Thakor (2006), the stability of the investor base and hence investor participation and the stringency of public market governance are held exogenously fixed. By contrast, our focus is on investor participation and its effect on the firm's future ownership base and the endogenously-determined stringency of public market governance. More specifically, a novel aspect of this paper is its examination of the role of market liquidity in public capital markets simultaneously creating *both* the principal advantage and disadvantage of public ownership relative to private ownership. Second, in contrast to earlier research, we are able to show why public firms will tend to go private when their stock prices are low and exhibit high volatility. Third, unlike previous research, we also examine the link between firm age and investor participation. This allows us to analyze the key role that investor participation plays in the age of a

---

<sup>11</sup>On the other hand, *hostile* going-private transactions *can* be explained on the basis of inefficient managerial benefits consumption. This explanation was offered for the hostile takeovers of the 1980s (Jensen (1986)). But as we discuss after Proposition 3, there are numerous key differences between our predictions and those of a private-benefits model. Furthermore, these models of hostile takeovers equate the choice between public and private ownership with one between dispersed and concentrated ownership.

<sup>12</sup>In that paper we provide a discussion of the motivation for a heterogeneous-priors based approach like the one adopted here, rather than the usual asymmetric information or agency approach, as well as the differences in key insights emerging from the different approaches. In addition to the discussion there, we note that asymmetric information models provide little help in understanding why a public firm would go private when its stock price is low and/or the volatility of this price is high. To explain a firm's decision to go private based on asymmetric information would require hypothesizing that asymmetric information problems with public ownership are more severe at low stock prices. However, existing models may well suggest the opposite. High future growth opportunities should lead to high market-to-book ratios and a greater fraction of the firm value coming from future opportunities rather than assets currently in place, thereby implying greater information asymmetries. Moreover, managers appear to issue equity when stock prices are high because of perceived wealth transfer gains, suggesting that informational asymmetries are greater at higher stock prices.

public firm going private. Finally, we highlight key new differences between our disagreement-based approach and private control benefits. In particular, we show formally that while giving the manager sufficiently high stock ownership eliminates project-choice distortion in a private control benefits setting, it has *no* such effect in our model.

The rest is organized as follows. Section I contains a description of the model. Section II has the analysis. Model extensions and empirical predictions of our analysis appear in Section III. Section IV concludes. All proofs are in the Appendix.

## I. Model Description

Our analysis starts out with an all-equity public firm that may go private or stay public in anticipation of the arrival of a growth opportunity.

### A. Growth Opportunity and Disagreement

The economy has universal risk neutrality and a riskfree interest rate of zero. Consider a publicly-listed firm in which the manager owns  $\alpha \in (0, 1)$  of the firm, and public investors own the remaining  $1 - \alpha$ . The manager's ownership in the firm is his only wealth and his goal is to maximize the value of his shareholding. There are five dates 0, 1, 2, 3 and 4. At  $t = 0$ , the firm has assets in place that will yield a non-stochastic cash flow of  $S > 0$  at  $t = 4$ . At  $t = 0$ , a coalition of investors may try to take the public firm private by buying all the outstanding public shares and delisting the company. If the coalition is unsuccessful in taking the firm private, it remains public. Once the private/public choice is made, the firm's governance structure is chosen. This choice is made by the private investors in the case of the private firm and by the outside shareholders in the case of a public firm. The governance structure in turn determines the degree of autonomy for the manager in the event of disagreement with investors. All this happens at  $t = 0$ . At  $t = 1$ , the manager can expend privately costly *search effort*  $e$  to find a growth opportunity ("project" from now) that will become available with probability  $e$  at  $t = 3$ . Trading by investors occurs at  $t = 2$ ; this will be described later. At  $t = 3$ , the manager and investors learn about project availability. The manager's private cost of search effort  $e$  is  $\frac{\beta e^2}{2}$ , where  $\beta > 0$  is a constant. The project will generate cash flows in addition to those from existing assets at

$t = 4$ .

Conditional on project availability at  $t = 3$ , the manager and investors draw prior beliefs about its quality. Project quality can be one of two types: Good ( $G$ ) or Bad ( $B$ ). The cash flow from a  $G$  project depends on a second managerial effort, which is the preparation effort,  $\varepsilon$ , which includes activities like feasibility studies. If the manager expends  $\varepsilon$  at  $t = 3$ , he incurs a personal cost  $\Psi > 0$ , a cost that can be avoided by not expending effort,  $\varepsilon$ . With preparation effort, a  $G$  project increases the firm's cash flow from  $S$  to  $X_G$ . Without preparation effort, the  $G$  project does not affect the cash flows. Irrespective of preparation effort, a  $B$  project reduces the cash flow from  $S$  to 0.

There are thus two managerial effort choices. The first is the search effort  $e$ , chosen at  $t = 1$ , that affects the probability of project availability at  $t = 3$ . Conditional on the project being available at  $t = 3$ , there is a preparation effort,  $\varepsilon$ , that affects project profitability. These two efforts serve different purposes in the model. The subsequent analysis shows that the presence of search effort ensures that investors find it optimal to give the manager some autonomy, and the presence of preparation effort ensures that investors do not attempt to “force” the manager to undertake a project he believes is bad.

We let  $\theta_m$  represent the manager's prior belief about the probability that the project is of type- $G$ , and assume that  $\theta_m \in \{\theta_l, \theta_h\}$ , with  $\theta_h > \theta_l$ . Let  $\delta \in (0, 1)$  be the probability of the manager drawing  $\theta_h$  and  $1 - \delta$  the probability of the manager drawing  $\theta_l$ . The manager's perception of project value depends on his prior belief  $\theta_m$  and his preparation effort choice. Define  $X_h \equiv \theta_h X_G + [1 - \theta_h]0 = \theta_h X_G$  and  $X_l \equiv \theta_l X_G + [1 - \theta_l]0 = \theta_l X_G$  as the perceived values of the project when the manager expends preparation effort and his priors are  $\theta_h$  and  $\theta_l$  respectively. We assume that  $X_l < S < X_h$ , so that the project is valuable (with preparation effort) when the prior belief is  $\theta_h$  and it destroys value (regardless of effort) when the prior belief is  $\theta_l$ . Consequently, the manager will choose not to expend preparation effort whenever his prior about project quality is  $\theta_l$ , giving rise to a project value of  $\theta_l S + [1 - \theta_l]0 = \theta_l S < S$ , which means that the manager will reject the project in this case.

The manager and investors may have different priors about the nature of the firm's project. Let  $\theta_i$  represent the investors' prior probability that the project is of type- $G$ , where,  $\theta_i \in \{\theta_l, \theta_h\}$ . When  $\theta_m \neq \theta_i$ , the manager and the investors disagree about the desirability of the project. We permit  $\theta_m$  and  $\theta_i$  to be correlated as follows:  $Pr(\theta_i = \theta_m) = \rho \in [0, 1]$ , and  $Pr(\theta_i \neq \theta_m) = 1 - \rho$ . Thus,  $\rho$

represents the probability the manager and investors have a common prior belief about project quality. Each prior is privately observed and can neither be verified nor contracted upon. We assume that  $\rho$  is distributed according to the distribution function  $G(\rho)$  and a density function  $g(\rho)$ , with support  $[\rho_l, \rho_h] \subset (0, 1)$ . Define  $E(\rho) \equiv \int_{\rho_l}^{\rho_h} \rho g(\rho) d\rho$ .

If both the manager and the investors draw a prior belief of  $\theta_l$ , the manager does not expend preparation effort, and they agree to eschew the project since they both perceive a negative net present value (NPV) of  $\theta_l S - S$ . Similarly, if both the manager and the investors draw a prior of  $\theta_h$ , the manager expends preparation effort and there is mutual agreement to undertake the project with a positive NPV of  $X_h - S$ . Disagreement *may* arise when prior beliefs differ. If the manager draws  $\theta_l$ , then he will not expend preparation effort,  $\varepsilon$ , and will perceive project value as  $\theta_l S$ . If in this case, investors draw a prior of  $\theta_h$ , then anticipating that the manager will not expend preparation effort, investors will perceive project value as  $\theta_h S$ , and will also want to avoid the project. However, when the manager's prior is  $\theta_h$ , he will expend preparation effort and will wish to implement the project, provided that the cost  $\Psi$  is not prohibitive,<sup>13</sup> but investors will wish to reject the project if their prior belief is  $\theta_l$ . It is this disagreement that we focus on. Since the disagreement arises due to different *prior* beliefs about project quality, there is no incentive for either the manager or the investors to change these beliefs based on what the other believes. Beliefs will be revised only in the face of new information, and *not* due to the different prior belief of someone with no more information.<sup>14</sup>

Even though agents have heterogeneous priors, we assume that these prior beliefs are rational in the sense of Kurz (1994a,b). The essential aspect of the theory of rational beliefs for our analysis is that the observables in the economy that agents form beliefs about have the technical property of “stability” but not stationarity.<sup>15</sup> That is, for beliefs to be rational, agents cannot have beliefs that are precluded by historical data. However, since a stable but non-stationary process is not generally uniquely identified even with countably infinite data points, there can be multiple rational beliefs that are consistent with the historical data. Not all these beliefs will conform to rational expectations; with non-stationarity,

<sup>13</sup>A sufficient condition for the manager to invest  $\Psi$  is given in Section II A in expression (3). For now we assume that this condition is satisfied.

<sup>14</sup>See Kreps (1990) who views prior beliefs as part of the primitives of any model, and asserts that heterogeneous priors represent a more general specification than homogenous priors.

<sup>15</sup>Kurz (1994a) shows that every stable process is associated with a specific stationary measure, and that multiple stable processes can give rise to the same associated stationary measure. While historical data can be used to construct the stationary measure, they cannot generally be used to distinguish between multiple stable processes associated with the same stationary measure.

rational expectations would require agents to have information that cannot be derived from historical data. Even in the case where the observables in the economy are stationary, projects from one period to the next may be unique, so that there may simply be insufficient data on each project to permit convergence to common posterior beliefs if agents start out with heterogeneous priors.<sup>16</sup>

The greater is the value of  $\rho$ , the higher is the likelihood of agreement between the manager and the investors;  $\rho = 1$  indicates perfect agreement while  $\rho = 0$  indicates perfect disagreement. The agreement parameter  $\rho$  can be thought of as being affected by the attributes of the project (i.e. the nature of the firm's business), the length of the time series of relevant historical data (e.g. the age of the firm), the manager's previous track record in managing similar projects of that type, and possibly also the general level of investor confidence in the corporate sector. If the project is one that the manager has previously dealt with successfully for a long time and investors are familiar with it as well,  $\rho$  will tend to be high. Since project familiarity and confidence in the manager's prior track record can be expected to vary in the cross-section of investors, there can be heterogeneity among investors in the extent of their agreement with the manager, even when all investors and the manager have rational beliefs.

### *B. Investor Participation in the Market*

We model heterogeneity of  $\rho$  across investors by assuming that the public market is comprised of  $N$  investors and that the  $\rho$  for each investor at any point in time is an independent draw from a continuous probability distribution  $G(\rho)$ , with the associated density function  $g(\rho)$  and support  $[\rho_l, \rho_h]$ . Whenever a project decision is to be made, the investors holding the firm's shares in the public market are the ones who have the highest level of agreement with the manager among the cross-section of investors; this is because they assign the highest value to the firm and can thus outbid all other investors in a competitive capital market. We assume that the investors with the highest  $\rho$  have collectively sufficient wealth to own all the shares of the firm i.e., all the investors holding a firm's shares have the same  $\rho$ . Thus, although there is heterogeneity among public-market investors, for a particular firm, there is homogeneity in the identity of its actual investors, in terms of their agreement with the manager.

### *C. Liquidity Cost*

---

<sup>16</sup>If we assume that the manager and investors are uncertain about the precision of additional signals, then Acemoglu, Chernozhukov and Yildiz (2006) show that convergence may never occur.

We assume investors experience interim liquidity needs. In particular, with probability 1, investors require cash at  $t = 2$  in order to satisfy a liquidity need at that time.<sup>17</sup> Investors have two alternate ways of meeting their liquidity need. They can either sell some illiquid asset in their portfolio, other than their shares in the firm, or they can sell their shares in the firm. If the investor chooses to sell some other illiquid asset, a liquidity cost of  $L > 0$  is incurred by the seller.<sup>18</sup> If the investor chooses instead to sell his shares in the firm, the cost depends on whether the firm is public or private.

Consider first an investor in a public firm. If the investor decides to sell his shares at  $t = 2$  in response to a liquidity need, then public listing of the firm's shares ensures that the sales transaction occurs in a well-defined market that has active, low-cost trading, with the market price at any point reflecting the valuation of the "maximal" investors (those with the highest agreement with the manager). At  $t = 2$ , the investor thus assesses the valuation of the firm at  $t = 3$  as being dependent on  $\bar{\rho}$ , the expected value, assessed at  $t = 2$ , of the  $\rho$  of the maximal investor at  $t = 3$ ; we will state the expression for  $\bar{\rho}$  later and prove formally that the share sale price at  $t = 2$  will be increasing in  $\bar{\rho}$ .

Moreover, selling at  $t = 2$  in a liquid public market will involve no search costs in finding a buyer, and the investor can thus sell his shares without delay or significant cost. The only possible cost such an investor may perceive is that his own  $\rho$  may be higher than  $\bar{\rho}$ , in which case he is selling his shares at a price lower than his own valuation. The investor will nonetheless choose to sell his shares if the perceived cost of doing so is less than  $L$ , the liquidity cost of selling some other illiquid asset. We later provide a sufficient condition that guarantees this.

The high liquidity of public ownership thus manifests itself in the form of enabling investors to satisfy their liquidity need by selling the firm's shares at virtually no cost. An important consequence of such trading is that it creates uncertainty about the identity of future shareholders and hence about  $\rho$ . Since the project selection decision is taken at  $t = 3$ , it is only the  $\rho$  at that point in time that matters. The  $\rho$  at  $t = 3$  is the  $\rho$  of the "maximal investor" who buys shares at  $t = 2$ , because there is no further liquidity trading after  $t = 2$ . At  $t = 0$ , the manager, is uncertain about this  $\rho$  because at

---

<sup>17</sup>The assumption that investors face an interim liquidity need with probability 1 is to highlight the cost of market liquidity. A stochastic liquidity shock that occurs with probability less than 1 can be easily accommodated, and could introduce an additional motive for trade. Investors may wish to sell shares even in the absence of a liquidity need, if there is an investor with a higher valuation. Even in this case, however, if the probability of liquidity-motivated trade is sufficiently large, all our conclusions will be preserved.

<sup>18</sup>An alternative assumption is that the investor borrows from a bank using his ownership in the firm as collateral, and incurs a dissipative cost  $L$  in doing so.

$t = 2$ , the existing investors in the firm’s shares exit due to liquidity-need-based trading.

By contrast, private ownership lacks an analogous market transaction mechanism. Hence, when private investors wish to sell shares, finding a buyer involves costly and time-consuming search, one that we assume involves a cost  $c > 0$ . Such a search results in finding an investor who represents only a random draw from the cross-section of potential buyers, with an expected agreement with the manager that equals the mean of the distribution of all  $\rho$ ’s in the cross-section, i.e.  $E(\rho)$ . This contrasts with the competitive market-clearing mechanism that continuously reflects the valuation of the *marginal* investor in the case of the public market. Private investors will retain the firm’s shares and incur the liquidity cost  $L$  in selling some other illiquid asset if the sum of the search cost  $c$  and the loss (gain) from selling to a buyer with an “average valuation” is greater than  $L$ . We assume this to be the case for all private investors and later state the precise restrictions on exogenous parameters for this to be satisfied. The private investors thus retain their ownership in the firm, implying that the relative illiquidity of private ownership results in the identity of the initially-identified investors remaining unchanged.<sup>19</sup>

In modeling the search in the private market, we implicitly assume that the private investors sell the shares to the first buyer that they are able to identify. We make this assumption for simplicity. A more extensive search model would involve the private investors sequentially searching optimally for the “best” buyer. We have formally analyzed such a mechanism and verified that all our results are preserved in such a setting. Details are available upon request.

#### *D. Managerial Autonomy*

Disagreement over project choice only matters when the manager wants to invest but the outside shareholders don’t. There must be a rule to resolve this disagreement. The structure of our model is that in the case of disagreement both the manager and the outside shareholders are simultaneously in control. Disagreement is resolved by determining the probability with which the manager gets to decide and the probability with which the outside shareholders decide. This can be thought of as a bargaining game in which there is some probability with which the manager moves first and makes a take-it-or-leave-it offer to the outside shareholders and some probability with which the outside shareholders get

---

<sup>19</sup>An aspect of this liquidity that we have not emphasized is that the shareholder base can also become misaligned because the firm’s business changes, say from paint to plastics. The liquidity of the public market permits shareholders to relatively easily sell their shares to another group that may be more aligned with the new strategy.

to move first. If the take-it-or-leave-it offer is rejected, the firm comes to a standstill and the manager as well as investors get a payoff of 0. The probability with which the manager moves first is called the *managerial autonomy*  $\eta \in [0, 1]$ . Because the first mover in this game has all the bargaining power,  $\eta$  is the probability that the manager will invest in the project when the outside shareholders disagree with him;  $1 - \eta$  is the probability that outside shareholders will stop the manager from investing in this state. We interpret  $\eta$  as a corporate governance parameter that specifies the degree of control given to the manager, with a lower  $\eta$  representing more stringent governance.<sup>20</sup>

*Ceteris paribus* the manager prefers more autonomy to less. This is *not* because he has an innate preference for control, but because greater autonomy gives him greater ability to maximize his assessment of *security benefits*.<sup>21</sup> The managerial autonomy  $\eta$  is endogenously determined (following the ownership choice) at  $t = 0$ , after which the manager simply takes  $\eta$  as fixed for his subsequent decisions. With either ownership mode, the optimal  $\eta$  is set by investors to maximize the share price.

In our analysis, while we endogenize the governance arrangement with either ownership mode, we take the managerial share ownership  $\alpha$  as exogenously fixed. Observe that any optimal contract, designed to provide effort incentives for the manager, will involve a non-zero managerial ownership stake. Given a positive managerial ownership stake, autonomy becomes relevant to the manager because he perceives the value of his ownership to be increasing in managerial autonomy. Allowing  $\alpha$  to be endogenously determined as part of an optimal contract will not qualitatively affect our analysis.<sup>22</sup> In Section III B we analyze the implications of incentive contracts other than straight equity for the manager. An important assumption throughout is that the manager does not have sufficient liquid assets to buy out the investors. With potential disagreement between the manager and the investors and the manager being indispensable for project implementation, the manager would want to buy out the investors. Insufficient managerial wealth precludes this.

### *E. Summary of Sequence of Events*

The sequence of events is as follows. At  $t = 0$ , the ownership mode (become private or continue to

---

<sup>20</sup>Since we consider an existing public firm, it is possible that its governance structure as a public firm has already been determined. However, rather than take this as exogenous, we allow it to be endogenously determined at  $t = 0$ .

<sup>21</sup>This is an important distinguishing feature of our approach from that of private benefits models, wherein control is preferred to preserve private benefits.

<sup>22</sup>See Boot, Gopalan and Thakor (2006) for the derivation of an optimal financing contract involving a positive managerial ownership and a non-zero level of managerial autonomy.

be public) is determined in the best interest of the investors. The firm's investors then determine the managerial autonomy  $\eta$  at  $t = 0$ . The manager invests in privately-costly search effort  $e$  at  $t = 1$ . This search effort affects the probability of the availability of the project at  $t = 3$ . At  $t = 2$ , investors have a need for liquidity, and if the firm is publicly traded, investors can satisfy this need by selling their shares in the market. If the firm is private, the private investors do not sell their shares and incur a liquidity cost  $L$  in satisfying their liquidity need. At  $t = 3$ , both the manager and the investors learn about project availability. If the manager learns that a project is available, he draws his private prior belief about project quality at  $t = 3$ . Subsequently, he proposes the project to the outside shareholders, who then draw their own prior belief about project quality at  $t = 3$ . After this, the decision about whether to go ahead with the project is made at  $t = 3$ . If the project is accepted, the manager chooses his preparation effort,  $\varepsilon$ , at  $t = 3$ . This effort choice affects the cash flow enhancement possible with the project. Cash flows are realized at  $t = 4$ . *Figure 1* summarizes the sequence of events.

————— FIGURE 1 GOES HERE —————

## II. Analysis

We begin the analysis by consolidating all our key assumptions related to restrictions on the exogenous parameters. We then proceed with the analysis using backward induction. Since no decisions are made at  $t = 4$ , we begin with events at  $t = 3$ , and examine them for both public and private ownership. This is when the manager and investors learn whether the growth opportunity is available and also whether they agree on pursuing it. This is based on the expected agreement between the manager and investors, which is  $\rho_{pr}$  for the private firm and  $\bar{\rho}$  for the public firm. We examine how these are determined, and the manager's choice of preparation effort,  $\varepsilon$ . We then move to  $t = 2$ . This is when investors experience a liquidity need; we examine the behavior of both public and private investors in response to this need. Next we examine the manager's choice of search effort,  $e$ , at  $t = 1$ , with both public and private ownership. Finally, we examine how managerial autonomy is determined with public and private ownership at  $t = 0$  and how the firm makes its ownership choice at that time.

### A. Assumptions

We make two types of assumptions. First, we need to ensure that the greater liquidity of the publicly-traded shares relative to privately-owned shares results in the investors selling their shares with public ownership and retaining them with private ownership (Assumption 1). Second, we impose restrictions on project cash flows to ensure that the manager exerts preparation and search efforts and that investors will not wish to grant the manager complete autonomy (Assumption 2).

**Assumption 1**

$$c > L > Q \quad (1)$$

$$\text{where } Q \equiv [\rho_h - E(\rho)][1 - \alpha]\delta \left[ \frac{\delta E(\rho)[X_h - X_l][\alpha[X_h - S] - \Psi]}{2\beta[S - X_l]} \right] \left[ \frac{[X_h - S] - E(\rho)[X_h - X_l]}{2[1 - E(\rho)]} \right] \quad (2)$$

The condition  $L > Q$  guarantees that with public ownership investors will indeed sell their shares to satisfy a liquidity need. It is transparent that a sufficiently large  $L$  will induce investors to sell their shares.  $Q$  is a measure of the discount at which investors expect to sell their shares relative to their own valuation.<sup>23</sup> As long as  $L$  is large enough, public-market investors are willing to sell the shares even at a discount. Private ownership is different. In that case, a search cost  $c$  is incurred in addition to the price concession associated with selling.<sup>24</sup> For  $c$  large enough, investors choose not to sell. Specifically,  $c > L$  guarantees that investors will prefer to hold on to their shares in the firm.

**Assumption 2**

$$A : \quad \alpha[X_h - S] \geq \Psi \quad (3)$$

$$B : \quad E(\rho)[X_h - S] + [1 - E(\rho)][X_l - S] > 0 \quad (4)$$

$$C : \quad X_h + X_l > 2S \quad (5)$$

$$\rho_h < \frac{2[S - X_l]}{X_h - X_l} \quad (6)$$

Assumption 2A guarantees that the manager will choose to expend preparation effort if he is permitted to undertake a project he believes will enhance value i.e., when his prior belief is  $\theta_h$ . Assumption 2B guarantees that investors attach a positive value to the manager exerting (some) search effort.<sup>25</sup> Finally, Assumption 2C guarantees that investors do not wish to give the manager either complete or

<sup>23</sup>Although a seller expects to sell to an investor with  $\rho = \bar{\rho}$ ,  $Q$  is not expressed in terms of  $\bar{\rho}$ , because  $\bar{\rho}$  is an endogenous variable. This explains why  $\rho_h$  and  $E(\rho)$  show up in (2).

<sup>24</sup>We will show later that  $\rho_{pr}$ , the agreement of an investor who wishes to take the public firm private, will exceed  $E(\rho)$ , the expected agreement of a randomly-chosen investor. Hence, when an investor who took the firm private at  $t = 0$  sells his shares at  $t = 1$ , he expects to make a loss.

<sup>25</sup>This assumption is sufficient for investors to attach a positive expected value to the project, since (4) represents the expected value of the project as assessed by an investor with an average agreement with the manager. Since both  $\bar{\rho}$  and  $\rho_{pr}$  will be shown to be higher than  $E(\rho)$ , this assumption ensures that investors attach a positive expected value to the project with both public and private ownership.

no autonomy over project choice. Essentially (5) ensures that the project is sufficiently valuable relative to its opportunity cost so that the investors wish to undertake it, whereas (6) ensures that the cost of disagreement  $S - X_t$  is sufficiently large relative to the benefit of higher managerial effort so that investors do not wish to give complete autonomy to the manager.

### *B. Determination of Level of Agreement and Project Preparation Effort at $t = 3$*

Given Assumption 2, we know that the manager will expend preparation effort  $\varepsilon$  after he draws a prior belief  $\theta_h$  that the project is type-G and has been accepted. For any other prior belief, the manager rejects the project, so the issue of investing in preparation effort is moot.

**Level of Agreement With Public Ownership:** We now examine how  $\bar{\rho}$ , the expected level of agreement of the investors with the manager at  $t = 3$ , is determined with public ownership. When the project implementation decision is made at  $t = 3$ , the firm's shares will be held by the investors with the highest agreement with the manager. This is because, for a given level of managerial autonomy in the public market, such investors will have the maximum valuation for the firm's shares and should be able to outbid all others. Thus,  $\bar{\rho}$  is the expected agreement of the “maximal investor” with the manager.<sup>26</sup> When  $\rho$  varies cross-sectionally and the  $\rho$  for any investor is a random draw from a probability distribution, the highest value of  $\rho$  among the  $N$  investors – the  $\rho$  of the “maximal investor” – is the  $N^{th}$  order statistic of  $\rho$ , say  $\rho_N$ , where

$$\rho_N = \max_{1 \leq i \leq N} \{\rho_i\}.$$

Note that  $\bar{\rho}$  is the expected value of the  $N^{th}$  order statistic, i.e.  $E(\rho_N) \equiv \bar{\rho}$ . Since  $\rho$  is distributed according to the distribution function  $G(\rho)$  and a density function  $g(\rho)$ , with support,  $[\rho_l, \rho_h]$ , we know from the standard properties of the  $N^{th}$  order statistic (see Galton and Pearson (1902)) that:

$$\bar{\rho} = \frac{N!}{(N-1)!} \int_{\rho_l}^{\rho_h} G(\rho_N)^{N-1} g(\rho_N) \rho_N d\rho_N \quad (7)$$

We now have:

**LEMMA 1:** *The expected value of the  $N^{th}$  order statistic,  $\rho_N$ , is increasing in  $N$ , i.e.,  $\frac{\partial \bar{\rho}}{\partial N} > 0$ .*

---

<sup>26</sup>This  $\bar{\rho}$  will generally be different from the  $\rho$  of the original investor at  $t = 0$  since that investor will sell out at  $t = 2$  to satisfy his interim liquidity need.

The variance of  $\rho_N$ ,  $\sigma_N^2$ , is decreasing in  $N$  as long as  $T(N) \equiv \left\{ \frac{g[G^{-1}(\frac{1}{N+1})]}{N} \right\}^{-1}$  is non-increasing in  $N$ .

The intuition is that the greater the number of investors in the market, the higher will be the agreement parameter of the maximal investor since there are more investors to choose from. Thus,  $\bar{\rho}$  is increasing in  $N$ . Similarly, the greater the number of participating investors, the smaller will be the variance in the agreement of the maximal investor,  $\sigma_N^2$ . The condition in Lemma 1 restricts the shape of the density function of  $\rho$ . Intuitively, the condition states that the density  $g$  should not increase too rapidly with  $\rho$ . The uniform distribution, for which  $T(N)$  decreases with  $N$ , is an example of an admissible distribution. Violation of this condition implies that a disproportionately large fraction of the population has a very high agreement with the manager. In this case, disagreement would not matter. To preclude this, we assume henceforth that the condition on  $T(N)$  in Lemma 1 is satisfied.

Lemma 1 highlights two important effects of higher investor participation. The first is an increase in the expected value ( $\bar{\rho}$ ) of the agreement parameter ( $\rho_N$ ), and the second is a reduction in its variance. Because there is a one-to-one relationship between the expected value of  $\rho_N$  and the level of the stock price with public ownership and also between the volatility of  $\rho_N$  and the volatility of the stock price, this result implies that the level of the stock price will increase and its volatility will diminish as investor participation increases. This implies a negative correlation between stock returns and volatility that has been empirically documented (e.g. Black (1976)). The usual explanation for this phenomenon is that the firm's financial leverage decreases with rising stock prices, and this reduces stock volatility. Our model shows that an increase in investor participation readily explains this inverse relationship between stock returns and volatilities, without relying on financial leverage.

**Level of Agreement with Private Ownership:** If the firm went private at  $t = 0$ , its private investors have an agreement parameter of  $\rho_{pr}$  with the manager. For now we take  $\rho_{pr}$  as a given. We will derive it as an endogenous variable when we examine the firm's choice of whether to continue with public ownership or go private.

### *C. Events at $t = 2$ : Investors' Liquidity Need*

**Public Ownership:** Given Assumption 1, the investors sell their shares at  $t = 2$  when faced with

a liquidity need.

**Private Ownership:** Given Assumption 1, the private investors hold on to their shares and sell some other illiquid asset, incurring a cost  $L$ . In making this assumption, we adopt the framework that the investors sell their shares to the first buyer they encounter. See Section I C.

#### D. Events at $t = 1$ : Manager's Optimal Choice of Search Effort

**Public Ownership:** The manager's choice of search effort  $e$  at  $t = 1$  takes as a given the autonomy  $\eta_{pub}$  determined at  $t = 0$  and maximizes  $V_{pub}^M$ , his expected payoff, which is given by:

$$\begin{aligned} V_{pub}^M &= E(\alpha\{e\delta\rho[X_h - S] + e\delta[1 - \rho]\eta_{pub}[X_h - S] + S\} - e\delta\{\rho + [1 - \rho]\eta_{pub}\}\Psi - \frac{\beta e^2}{2}) \\ &= \alpha W_{pub}^M - e\delta[\bar{\rho} + \{1 - \bar{\rho}\}\eta_{pub}]\Psi - \frac{\beta e^2}{2} \end{aligned} \quad (8)$$

where  $W_{pub}^M \equiv e\delta\bar{\rho}[X_h - S] + e\delta[1 - \bar{\rho}]\eta_{pub}[X_h - S] + S$  is the manager's assessment of the firm value at  $t = 1$ . Note that the first term in  $W_{pub}^M$  is the increase in the firm's cash flow when there is a project (occurs with a probability  $e$ ) and the investors and the manager both have prior beliefs  $\theta_h$  (occurs with a probability  $\delta\bar{\rho}$ ) and hence agree on implementing the project. The second term in  $W_{pub}^M$  is the increase in cash flow when there is a project, the manager draws a prior  $\theta_h$  but the outside shareholders prior belief is  $\theta_l$  (occurs with a probability  $\delta[1 - \bar{\rho}]$ ). In this case the manager is able to undertake the project only with probability  $\eta_{pub}$ . The last term in  $W_{pub}^M$  is the cash flow  $S$  from assets in place, since in all states other than those reflected in the first two terms in  $W_{pub}^M$ , the project is not accepted and the cash flow remains  $S$ . The two terms not involving  $W_{pub}^M$  in (8) are the private effort costs of the manager. The term  $e\delta\{\bar{\rho} + [1 - \bar{\rho}]\eta_{pub}\}\Psi$  represents the preparation effort cost  $\Psi$  multiplied with the probability  $e$  of having a project, the conditional probability  $\delta$  of the manager drawing a prior belief  $\theta_h$  and the probability that investors will accept the manager's project choice,  $\bar{\rho} + [1 - \bar{\rho}]\eta_{pub}$ . The last term in (8) is  $\frac{\beta e^2}{2}$ , which is the cost of the manager's search effort. We assume that the manager's shareholding is sufficient to satisfy his individual rationality (IR) constraint. We now have:

**LEMMA 2:** *The manager's uniquely optimal choice of effort level with public ownership is:*

$$e_{pub}^* = \frac{\delta}{\beta} [\{\bar{\rho} + [1 - \bar{\rho}]\eta_{pub}\} \{\alpha[X_h - S] - \Psi\}] \quad (9)$$

where  $e_{pub}^*$  is strictly increasing in the expected level of agreement,  $\bar{\rho}$ , at  $t = 3$  between the manager and

public shareholders, the managerial ownership fraction,  $\alpha$ , and the managerial autonomy,  $\eta_{pub}$ , and is strictly decreasing in the effort disutility parameter,  $\beta$ .

Managerial effort,  $e_{pub}^*$ , is increasing in  $\bar{p}$  because the probability that the project will actually be implemented is increasing in  $\bar{p}$ ; thus, the manager's return to effort is greater when  $\bar{p}$  is higher. Similar reasoning holds for the impact of the autonomy parameter  $\eta_{pub}$ . The effects of managerial ownership  $\alpha$  and the effort disutility parameter  $\beta$  on effort are transparent.

Substituting (9) in (8) gives the value of the manager's expected payoff at the optimum,  $e_{pub}^*$ , as perceived by the manager:

$$V_{pub}^{M*}(\eta_{pub}) = \frac{\beta e_{pub}^{*2}}{2} + \alpha S \quad (10)$$

**Private Ownership:** As in the case of public ownership, the manager takes as a given the autonomy  $\eta_{pr}$  determined at  $t = 0$  and chooses his search effort  $e_{pr}$  at  $t = 1$  to maximize  $V_{pr}^M$ , his expected payoff, which is given by:

$$\begin{aligned} V_{pr}^M &= \alpha \{e\delta\rho_{pr}[X_h - S] + e\delta[1 - \rho_{pr}]\eta_{pr}[X_h - S] + S\} - e\delta\{\rho_{pr} + [1 - \rho_{pr}]\eta_{pr}\}\Psi - \frac{\beta e^2}{2} \\ &= \alpha W_{pr}^M - e\delta[\rho_{pr} + \{1 - \rho_{pr}\}\eta_{pr}]\Psi - \frac{\beta e^2}{2} \end{aligned} \quad (11)$$

where  $W_{pr}^M \equiv e\delta\rho_{pr}[X_h - S] + e\delta[1 - \rho_{pr}]\eta_{pr}[X_h - S] + S$  is the manager's assessment of the firm value at  $t = 1$ .  $W_{pr}^M$  as well as (11) are similar to their public-ownership counterparts (see (8)). This now yields:

**LEMMA 3:** *The manager's uniquely optimal choice of effort level with private ownership is:*

$$e_{pr}^* = \frac{\delta}{\beta} [\{\rho_{pr} + [1 - \rho_{pr}]\eta_{pr}\} \{\alpha[X_h - S] - \Psi\}] \quad (12)$$

where  $e_{pr}^*$  is strictly increasing in the agreement,  $\rho_{pr}$ , the managerial ownership fraction,  $\alpha$ , and the managerial autonomy,  $\eta_{pr}$ , and is strictly decreasing in the effort disutility parameter,  $\beta$ .

#### *E. Events at $t = 0$ : Choice of Managerial Autonomy with Public and Private Ownership*

We first determine the optimal managerial autonomy with public ownership,  $\eta_{pub}^*$ , and then the optimal managerial autonomy with private ownership,  $\eta_{pr}^*$ .

**Public Ownership:** Investors determine  $\eta_{pub}^*$  at  $t = 0$  to maximize their expected payoff taking into account the impact of this choice on the manager's subsequent choice of optimal search effort,  $e_{pub}^*$ , at  $t = 0$ . That is, at  $t = 1$ , investors choose  $\eta_{pub}^*$  to maximize:

$$\begin{aligned} V_{pub}^I &= E([1 - \alpha]\{e_{pub}^*\delta\rho[X_h - S] + e_{pub}^*\delta[1 - \rho]\eta_{pub}[X_l - S] + S\}) \\ &= [1 - \alpha]W_{pub}^I \end{aligned} \quad (13)$$

where  $W_{pub}^I \equiv e_{pub}^*\delta\bar{\rho}[X_h - S] + e_{pub}^*\delta[1 - \bar{\rho}]\eta_{pub}[X_l - S] + S$  is the investors' assessment of the value of the firm (the stock price) at  $t = 0$ . Comparing  $W_{pub}^M$  and  $W_{pub}^I$  we see that a key difference between these two expressions arises due to the disagreement state when the manager draws a prior of  $\theta_h$  and the investors draw a prior of  $\theta_l$  (this state occurs with a probability  $\delta[1 - \bar{\rho}]$ ). In that state, investors perceive the NPV of the project as  $X_l - S < 0$ , while the manager perceives it as  $X_h - S > 0$ . We can see that  $W_{pub}^I$  is increasing in  $\bar{\rho}$ . Thus, a higher agreement connotes a higher stock price.

**LEMMA 4:** *The optimal managerial autonomy with public ownership,  $\eta_{pub}^*$ , is given by:*

$$\eta_{pub}^* = \frac{\bar{\rho}[X_h + X_l - 2S]}{2[1 - \bar{\rho}][S - X_l]} \in (0, 1) \quad (14)$$

where  $\eta_{pub}^*$  is increasing in  $\bar{\rho}$ , and the growth opportunity cash flow ( $X_h$  or  $X_l$ ).

Lemma 4 characterizes the properties of the public market governance regime. Managerial autonomy is increasing in  $\bar{\rho}$ . Intuitively, the greater the propensity of investors to agree with the manager, the lower is the “cost” of granting autonomy to the manager. Managerial autonomy  $\eta_{pub}^*$  is also increasing in the attractiveness of the growth opportunity ( $X_h$  or  $X_l$ ) – the greater is  $X_h$  or  $X_l$ , the lower is the cost of managerial autonomy.

This generalizes Boot, Gopalan and Thakor (2006) where  $\eta_{pub}$  is viewed as exogenous. In that paper, the assumption of an exogenous  $\eta_{pub}$  is used as a convenient approximation, rather than a precise representation of how public-market governance actually works.<sup>27</sup> We endogenize  $\eta_{pub}$  because we want to analyze the impact of investor participation on public governance arrangements, something that Boot, Gopalan and Thakor (2006) do *not* address. In any case, the common element in both papers is that public-market corporate governance is more rigid than private corporate governance.<sup>28</sup>

<sup>27</sup>Boot, Gopalan and Thakor (2006) state on pp. 825-826, “In practice, public firms have had some latitude in choosing corporate governance stringency, through the choice of the number of outside versus inside directors, which antitakeover provisions to adopt and so on.”

<sup>28</sup>This is because  $\eta_{pub}^*$  is set based on the expected value,  $\bar{\rho}$ , whereas the private-ownership autonomy parameter,  $\eta_{pr}$ ,

In the event of disagreement, following the specification in Section 2.4,  $\eta_{pub}^*$  is the endogenously-determined probability with which the manager has control. If  $\eta_{pub}^*$  were either 0 or 1, we would have unilateral control (exclusive investor control if  $\eta_{pub}^* = 0$  and exclusive managerial control if  $\eta_{pub}^* = 1$ ). However, the fact that  $\eta_{pub}^* \in (0, 1)$  means that “joint control” is optimal.

This should be contrasted with Aghion and Bolton (1992) who consider three control regimes: exclusive entrepreneurial control, exclusive investor control, and state-contingent control wherein control transfers *exclusively* to one party *ex post* based on an observable signal. They find that when security benefits are comonotonic with total benefits, investor control is optimal and the first-best is achieved. When neither security benefits nor private benefits are comonotonic with total benefits in all states of the world, unilateral control allocations are dominated by state-contingent allocations. Aghion and Bolton (1992) define joint control as a situation in which the probability with which the manager is in control and the probability with which investors are in control add up to more than 1. Using this definition, they show that joint control is never optimal.<sup>29</sup> Our definition of joint control is clearly different from that in Aghion and Bolton (1992) since the managerial and investor control probabilities add up to 1. Tirole (2006) refers to the joint control we characterize as “stochastic control” (see pp. 390-391). What we have also differs from state-contingent control in Aghion and Bolton (1992), where one party gains exclusive control *ex post* based on the realization of a contractible signal, and this is *ex post* efficient. There is no commonly-observed signal of this sort that agents can contract upon in our model. Moreover, when disagreement occurs, *neither* party views it as being *ex post* efficient to transfer control to the other party, so state-contingent exclusive control is never optimal. From Lemmas 1 and 4, we have the following proposition.

**PROPOSITION 1:** *Higher levels of investor participation in the public capital market,  $N$ , lead to*

is based on the realized  $\rho$  (see the subsequent discussion of private ownership).

<sup>29</sup>See Proposition 5 in Aghion and Bolton (1992). We quote from Aghion and Bolton (1992): “In our models, joint ownership is always (weakly) dominated by either unilateral or contingent control”. (p.486). The key assumptions in Aghion and Bolton (1992) that make joint control inefficient are that *ex post* both agents agree on the first-best action, and that in the event that joint control is granted, *either* party can bring the firm to a standstill by vetoing the action the other party wishes to take. In our model, we endogenously derive the probability with which each party has veto power. That is, extending our framework to Aghion and Bolton’s (1992) model, the manager and investors would agree *ex ante* that if they find themselves in an *ex post* state in which they are both in control, the manager would have veto power with probability  $\eta$  and investors would have veto power with probability  $1 - \eta$ . While the interaction between disagreement and the cost of capital makes such a randomized allocation of veto power *ex ante* efficient and renegotiation proof in our model, it is not clear it would be efficient in Aghion and Bolton (1992). In any case, if the definition of joint control is viewed strictly as a situation in which both parties must simultaneously have control (so that control probabilities add up to more than 1) with *no* tie breaker, then joint control is *not* efficient in our model as well. See Halonen (2000) for a depiction of joint control using Aghion and Bolton’s (1992) definition.

higher levels of  $\bar{\rho}$ , the expected value of the agreement parameter,  $\rho_N$ , lower values of  $\sigma_N^2$ , the variance of  $\rho_N$ , and higher values of the optimal public-market managerial autonomy,  $\eta_{pub}^*$ .

This proposition states that greater investor participation in the equity market results in corporate governance that grants more decision-making autonomy to management. The intuition is that greater investor participation leads, on average, to higher agreement between the manager and investors (Lemma 1), paving the way for greater autonomy for the manager (Lemma 4). If firms with more analysts following them (i.e. the larger and better-known firms) can be viewed as those with greater investor participation, then this result says that the larger and better-known firms will optimally have more lax corporate governance as represented by greater managerial autonomy. Moreover, these firms with greater autonomy will also have lower stock price volatility (due to lower  $\sigma_N^2$ ).

With this interpretation, Proposition 1 provides a testable prediction. If one views greater protection for managers against being replaced via takeovers as an empirical proxy for greater autonomy, then evidence supporting Proposition 1 appears in Gompers, Ishi and Metrick (2003). They infer from the values of their governance index that managers in larger firms enjoy greater takeover protection.

This result seems uniquely linked to our disagreement-based autonomy framework. If we had only asymmetric information between investors and the manager along with private control benefits, then it would follow that greater investor participation would reduce this informational asymmetry<sup>30</sup> and investors would exercise greater control to limit the manager's private benefit extraction. Thus, an asymmetric-information setup with private control benefits is likely to produce a prediction *opposite* to ours. Our next result addresses the attractiveness of public ownership for the manager.

**PROPOSITION 2:** *The manager's assessment of his security benefits with public ownership,  $V_{pub}^M$ , is increasing in the level of investor participation in the public equity market.*

This proposition implies that greater investor participation in the public equity market makes public ownership more attractive. This is intuitive. Greater investor participation increases the probability of finding investors with higher levels of agreement with the manager and hence induces higher valuations. The flip side of this is that low levels of investor participation will reduce the valuations of public firms,

---

<sup>30</sup>Large firms, which typically have more analyst following and greater investor participation, also exhibit lower adverse-selection components in their bid-ask spreads (see Stoll (2000)). The finding that large firms face lower informational asymmetries can be found in a number of papers (e.g. Hong, Stein and Lim (2000)).

and this may provide a possible explanation for why relatively small firms, that complain about having received inadequate attention from security analysts and investors, have recently begun to go private.<sup>31</sup>

Models with exogenous private control benefits are unlikely to yield this result. For example, in Aghion and Bolton (1992), security benefits are exogenous and thus unaffected by investor participation. In our model, security benefits are *endogenized* via a managerial-autonomy-contingent “investor approval” process that affects the expected value of security benefits for the manager.

**Private Ownership:** If the firm goes private, then at  $t = 0$ , managerial autonomy  $\eta_{pr}^*$  is set to maximize the expected payoff of private investors given by:

$$\begin{aligned} V_{pr}^I &= [1 - \alpha]\{e_{pr}^*[\delta\rho_{pr}[X_h - S] + \delta[1 - \rho_{pr}]\eta_{pr}[X_l - S]] + S\} - L \\ &= [1 - \alpha]W_{pr}^I - L \end{aligned} \tag{15}$$

where  $W_{pr}^I \equiv e_{pr}^*[\delta\rho_{pr}[X_h - S] + \delta[1 - \rho_{pr}]\eta_{pr}[X_l - S]] + S$  is the investors’ valuation of the firm at  $t = 0$ . This expression is similar to that for  $W_{pub}^I$  except that the private investors’ level of agreement  $\rho_{pr}$  may differ from the expected level of agreement  $\bar{\rho}$ . Since the investors experience a liquidity shock before project availability is revealed, they incur the liquidity penalty  $L$  with probability 1. Because  $W_{pr}^I$  is similar to  $W_{pub}^I$ , the optimal managerial autonomy with private ownership takes a form similar to the optimal autonomy with public ownership ( $\eta_{pub}^*$  (14)) with  $\rho_{pr}$  substituted in place of  $\bar{\rho}$ .

**LEMMA 5:** *The optimal managerial autonomy with private ownership,  $\eta_{pr}^*$ , is given by:*

$$\eta_{pr}^* = \frac{\delta\rho_{pr}[X_h + X_l - 2S]}{2\delta[1 - \rho_{pr}][S - X_l]} \in (0, 1) \tag{16}$$

where  $\eta_{pr}^*$  is increasing in  $\rho_{pr}$ , and the project cash flow ( $X_h$  or  $X_l$ ).

#### F. Renegotiation at the Time of Project Choice

The corporate governance structure stipulates that in case of disagreement, the manager gets to decide with probability  $\eta$  and the investors get to decide with a probability  $1 - \eta$ . Since  $\eta$  is chosen to be *ex ante* efficient, it is natural to ask whether there are any *ex post* incentives to renegotiate the original control structure. This is potentially important in the disagreement state in which the manager wants to go ahead with the project and the outside shareholders want to reject it. We can

---

<sup>31</sup>See Thornton (2004), for example.

show that when the managerial ownership  $\alpha$  is neither too high nor too low, the ex ante optimal governance structure is renegotiation proof. The following lemma formalizes the sufficient condition for renegotiation-proofness.

**LEMMA 6:** *The optimal managerial autonomy is renegotiation proof if and only if*

$$\frac{S - X_l}{X_h - X_l} < \alpha < \frac{X_h[S - X_l]}{X_h - X_l} \quad (17)$$

The intuition for this condition is as follows. When the managerial ownership,  $\alpha$ , is sufficiently high, foregoing the project is too costly for the manager and hence he optimally buys back full control from the investors by surrendering some cash flow rights. Similarly, when the investors' ownership is very high, going ahead with the project is too costly for the investors and hence they optimally buy back control from the manager. We will assume henceforth that  $\alpha$  satisfies (17).

### *G. Choice Between Public and Private Ownership*

We now analyze the investors' choice between the ownership modes. At  $t = 0$ , the coalition of investors with the maximal valuation for the firm's shares will prefer to take the firm private if their valuation of the private firm exceeds the share price with continued public ownership. This is also a necessary condition for a going-private transaction to succeed. We now have:

**PROPOSITION 3:** *There exists a cutoff value of the agreement parameter  $\rho_{pr}$ , call it  $\hat{\rho}$ , with  $\hat{\rho} > \bar{\rho}$ , such that private investors with  $\rho_{pr} > \hat{\rho}$  will strictly prefer to take the firm private. The probability of the firm going private is thus decreasing in  $\hat{\rho}$ . The cutoff  $\hat{\rho}$  is increasing in: the expected agreement in the public market,  $\bar{\rho}$ , the liquidity cost of private ownership,  $L$ , the effort disutility parameter,  $\beta$ , the cash flow without the project,  $S$ , the level of investor participation in the public market,  $N$ , and is decreasing in: the variance of the agreement parameter,  $\sigma_N^2$ , and in the cash flow with the project,  $X_h$  or  $X_l$ .*

Proposition 3 highlights the factors that influence the investors' choice of taking the firm private. The investors' perception of firm value is increasing in the agreement with the manager  $\rho_{pr}$ . Thus, only private investors with a relatively high level of agreement with the manager will be able to pay a sufficiently high price to take the firm private. In fact, since  $\rho_{pr} > \hat{\rho} > \bar{\rho}$  and firm value is strictly

increasing in  $\rho$ , we have the prediction that going-private transactions will occur only at significant premia over the pre-announcement stock price. The reason why  $\hat{\rho}$ , the cutoff value of agreement above which private ownership is preferred, is strictly greater than  $\bar{\rho}$  is because  $L$ , the liquidity cost with private ownership, decreases the private investors' valuation.  $\hat{\rho} > \bar{\rho}$  also ensures that  $\hat{\rho} > E(\rho)$ . Thus, whenever the private investors try to sell the shares, they can do so only at a loss, which means that as long as (1) is satisfied, they will retain the firm's shares and incur the liquidity cost  $L$ . The cutoff  $\hat{\rho}$  is increasing in the expected public-market agreement parameter,  $\bar{\rho}$ , because the stock price with public ownership is increasing in  $\bar{\rho}$ .  $\hat{\rho}$  is increasing in the effort disutility parameter  $\beta$ , the cash flow with no project  $S$ , and decreasing in the cash flow with the project  $X_h$  or  $X_l$ , because an increase in  $\beta$ ,  $S$ , or a decrease in  $X_h$  or  $X_l$ , makes the project and hence private ownership less attractive. Note that the benefit of private ownership is the greater agreement between the manager and the investors ( $\hat{\rho} > \bar{\rho}$ ), and consequently a higher probability of implementing the project. Finally,  $\hat{\rho}$  is increasing in the level of investor participation,  $N$ , because greater investor participation leads to a higher  $\bar{\rho}$ , and a lower  $\sigma_N^2$ .

The prediction that the probability with which private ownership is preferred is decreasing in public-market investor participation distinguishes our approach from that involving exogenous private benefits of control. Further, we have the result that going-private transactions are more likely when the cash flow without the growth project,  $S$ , is *low* and the cash flow with the project,  $X_h$  or  $X_l$ , is *high*; this is the opposite of what a private-benefits-of-control model would predict (e.g. Jensen (1986)), namely that firms with significant assets in place and free cash flows but lacking in growth options are more likely to benefit from the increased monitoring of private ownership and hence more likely to go private.

Our analysis also permits us to say something about the attractiveness of public ownership and the level and volatility of the firm's stock price. Because the cut-off  $\hat{\rho}$  is increasing in  $N$ , lower investor participation will lead to a greater propensity for firms to go private. By Lemma 1, we know that a lower  $N$  means a lower  $\bar{\rho}$  (lower stock price). This implies that we will have public firms going private more often when stock prices are low than when stock prices are high. If we assume that low and high valuations are correlated with general market conditions (e.g. due to investor sentiment), then this result is consonant with the empirical evidence that IPOs are largely a bull-market phenomenon (see, for example, Ritter and Welch (2002) as well as the anecdotal evidence mentioned in the Introduction). Moreover, we know from Lemma 1 that a decrease in  $N$  increases the volatility of the public firm's

stock price. Hence, the prediction is that public firms tend to go private when the stock price volatility is high.

While our result that public firms will go private when their stock prices are low could perhaps be obtained with alternative (perhaps simpler) models, the result that high price *volatility* will also induce such behavior seems special to our model since it depends on the simultaneous effect of investor participation on price volatility and manager-investor agreement. None of the existing models that employ asymmetric information or private control benefits are likely to generate this result.

Having highlighted the investors' preference between the two ownership modes, the following proposition highlights the manager's preference between the ownership modes.

**PROPOSITION 4:** *Whenever  $\rho_{pr}$ , the agreement parameter of the private investors, is higher than  $\bar{\rho}$ , the agreement parameter reflected in the firm's stock price with continued public ownership, the manager strictly prefers private ownership.*

Contrast this with Proposition 3, which says that a private investor will be willing to take the firm private only if  $\rho_{pr} > \hat{\rho} > \bar{\rho}$ . Since the manager is willing to go private whenever  $\rho_{pr} > \bar{\rho}$ , this implies that the manager has a stronger preference for private ownership than the private investor and thus will always be willing to go private when the investor wishes to do so. The intuition is as follows. The manager's total payoff with either ownership mode is increasing in managerial search effort,  $e$ . This can be seen from (10) and the analogous expression with private ownership.<sup>32</sup> Moreover, since managerial search effort with either ownership mode is increasing in  $\rho$  (Lemmas 2 and 3), it follows that the manager's total payoff with either ownership mode is also increasing in  $\rho$ . That is, from the manager's standpoint, the choice between private and public ownership comes down simply to choosing the mode that offers the higher  $\rho$ . Thus, the manager prefers private ownership whenever  $\rho_{pr} > \bar{\rho}$ . But, as we saw in Proposition 3, unlike the manager the private investor faces a liquidity cost,  $L$ , which pushes the investor's cut-off agreement parameter for private ownership,  $\hat{\rho}$ , above  $\bar{\rho}$ .

An interesting implication emerges from Proposition 3 and 4. Since  $\rho_{pr} > \hat{\rho} > \bar{\rho}$ , it follows that managerial search effort is also higher if the firm is taken private than if it continues to be public. This

---

<sup>32</sup>The expression is  $V_{pr}^{M*} = \frac{\beta e_{pr}^{*2}}{2} + \alpha S$ .

implies a higher firm value with private ownership, generating the prediction that a public firm that goes private is worth more *after* going private. Evidence consistent with this prediction appears in Smith (1990).

Two additional points are noteworthy. First, since there is agreement between the manager and the private investors regarding the preferred ownership mode, whenever the private investors bid for the firm, they will always succeed in taking the firm private. Second, since the private investors are buying out all the outstanding publicly-traded shares, there will not be free riding by individual shareholders as in Grossman and Hart (1980). But the price at which the private investors acquire these shares needs to be pinned down. The fact that the private investors' valuation is greater than the stock price with public ownership ensures that the going-private transaction generates a surplus. We assume that the public shareholders get a fraction  $\gamma$  of the surplus while the private investors get  $1 - \gamma$ . Thus, the price at which the private investors can acquire the shares is  $P = W_{pub}^I + \gamma[W_{pr}^I - W_{pub}^I]$ .<sup>33</sup>

Our analysis implies that a firm can be taken private only if a private investor with a sufficiently high  $\rho_{pr}$  shows up. The probability of this occurrence can be expected to be increasing in the number of potential private investors. The emergence of private equity as a major force in taking firms private can be viewed as an enlargement of the pool of potential private investors, and hence an increase in the probability that any public firm will encounter a private investor with a sufficiently high  $\rho_{pr}$  to take it private. Recent events involving private equity are thus consistent with our analysis.

**PROPOSITION 5:** *The set of exogenous parameter values that support either private or public ownership is nonempty.*

We establish this through a numerical example that shows a wide range of exogenous parameter values for which Assumptions 1 and 2 are satisfied and the optimal choice can be either private or public ownership.

### III. Model Extensions and Empirical Predictions

In this section, we first discuss whether mechanisms could be designed for greater investor stability

---

<sup>33</sup>We can generate such a surplus-sharing rule via an explicit Nash bargaining game.

in the public capital market. Subsequently we examine the robustness of our analysis to alternative managerial ownership levels and incentive contracts, and also extend the model to analyze how firm age could affect the agreement between the manager and the investors and hence a firm's optimal ownership choice. We close the section with a discussion of the main empirical predictions of the analysis.

#### *A. Lock-up Agreements and Investor Stability in the Public Market*

In our model, the private investors hold on to the firm's shares until  $t = 3$ . This provides stability to the agreement between the manager and the investors, which facilitates better matching between  $\eta_{pr}$  and  $\rho_{pr}$  and ensures a higher level of managerial effort. Can this ever be achieved with public ownership? That is, can investors with  $\rho = \rho_{pr}$  buy all the public shares at  $t = 0$ , fix the level of managerial autonomy to maximize their valuation and commit not to sell the shares until  $t = 3$  even with public ownership? The answer is no. This is because the investors suffer a liquidity need at  $t = 2$ , after the manager has expended search effort at  $t = 1$ . This makes any commitment by them not to sell their shares time-inconsistent. Anticipating such a sale, the manager will not expend the optimal search effort. Consequently, public ownership will not result in the same search effort as private ownership. Thus, somewhat ironically, it is the very *liquidity* of the public equity market that keeps a publicly-traded firm from being able to elicit the same high level of search effort from the manager as a privately-held firm. In our model, the role of private ownership is to prevent any sale of shares by the private investors, which ensures investor stability and relatively high managerial search effort. This role can not be duplicated by a *liquid* public market.

While public-market liquidity creates volatility in the firm's ownership base, there are mechanisms that could, at least temporarily, dampen this volatility. For example, venture capitalists opt for a partial exit via an IPO, but often retain a sizeable ownership stake even after the IPO. This facilitates stability in the ownership base, at least for some time. Similarly, lock-up agreements compel managers and investors to hold on to their shares following an IPO, which again improves ownership stability.

IPO transactions sometimes also include financial inducements for investors to hold on to their shares. For example, in the 1998 Deutsche Telecom IPO, retail (non-institutional) investors were offered an 8% reward (in terms of a lower IPO price) if they held on to their shares for at least 24 months.

The existence of dual-class shares may also enhance investor stability. The voting shares, which are typically held by management and related parties, are typically less liquid and are also accompanied by transfer restrictions and hence contribute to greater investor stability.

While these examples help to see how our theory illuminates the popularity of practices that increase public ownership stability, the fact remains that the liquidity of public ownership is an undeniable force in making public ownership less stable than private ownership. The practices we have discussed may reduce the shareholder-instability disadvantage of public ownership, but will not eliminate it.

### *B. The Impact of Managerial Ownership and Alternative Incentive Contracts*

We have two goals in this subsection. The first is to contrast the impact of managerial ownership on the autonomy given to the manager in our model with that in a model in which the friction is (exogenous) private control benefits. The second is to examine optimal incentive contracting in our model.

#### **Impact of Managerial Ownership With Disagreement vs Private Control Benefits**

With heterogeneous-prior-induced disagreement, managerial autonomy with either public or private ownership is unaffected by  $\alpha$ , the level of managerial shareholding (see (14) and (16)). This is unlikely to be the case with exogenous private benefits of control. For illustration, consider a model with exogenous private benefits of control, such that the manager enjoys a private benefit of  $B > 0$  if the status quo is preserved and no project is implemented. Investors and the manager have the same prior beliefs, so there is no disagreement due to differences in beliefs. With the project, firm cash flows are  $X$  and the manager gets  $\alpha$  fraction of this. We assume that  $X > B$  so the first-best choice is to implement the project. The manager will agree to implement the project if  $\alpha X \geq B$  or if  $\alpha \geq \hat{\alpha} = \frac{B}{X}$ . For  $\alpha \leq \hat{\alpha}$ , the manager would not wish to implement the project. Given this we now have:

**PROPOSITION 6:** *With heterogeneous-priors-induced disagreement, managerial autonomy is unaffected by managerial ownership  $\alpha$ . In contrast,  $\alpha$  affects disagreement and managerial autonomy with exogenous private benefits. For  $\alpha < \hat{\alpha}$  there is disagreement about the optimal project choice and full investor control is optimal, whereas for  $\alpha \geq \hat{\alpha}$  there is no disagreement about the optimal project choice and full managerial autonomy is (weakly) optimal.*

This proposition exposes an important difference between our model and those with exogenous private benefits of control. While greater managerial ownership can diminish disagreement when the friction is private benefits of control, it can provide no amelioration when the friction is disagreement caused by heterogeneous priors. That is, the principal will perceive the manager’s project-choice distortion as being eliminated by sufficiently high managerial ownership in a private-control-benefits setting, but *not* in a heterogeneous prior beliefs setting.

### Alternative Incentive Contracts

In our analysis, the manager’s incentive contract comprises of equity ownership in the firm. In practice, incentive contracts take many forms, including bonus, stock options etc. In this section we consider a more general incentive contract than the one we have used thus far and show that our results are preserved. In our model, firm value at  $t = 4$  can take on three possible values, namely  $S$ ,  $X_h$  and  $X_l$ . Let  $w_0, w_1$  and  $w_2$  be the wages in these three states of the world. We restrict the wages to be non-negative i.e. we assume limited liability for the manager. We now have:

**PROPOSITION 7:** *The optimal contract will have  $w_0 = w_2 = 0$  and  $w_1 > 0$ .*

The intuition for the proposition is as follows. Since firm value is fixed at  $S$  when the manager does not find a project, giving a non-zero wage in this state could reduce the managerial effort incentives. Hence, the shareholders will optimally set  $w_0 = 0$ . According to the manager, the firm value of  $X_l$  is a zero-probability event. This is because, the project is implemented only when the manager thinks it is good. Hence, setting a positive managerial wage when firm value equals  $X_l$  will not impact either the manager’s effort incentives or participation. Given these two results, it is obvious that we need  $w_1 > 0$ , both to ensure managerial participation and to provide effort incentives. With our simple technology, this general contract resembles an equity contract.<sup>34</sup> We can also show that all of our earlier results will continue to hold even in this general case.

### C. Firm Age, Investor Participation and Optimal Ownership Choice

Our analysis can be used to gain some insight into how firm age impacts the agreement between the

---

<sup>34</sup>To see this, normalize  $S = 0$  in the manager’s objective (8) and let  $w_i \equiv \alpha X_h$ . This result is derived within the context of our model. A more general treatment of optimal incentive contracts in the presence of differences of opinion can be found in Bolton, Scheinkman and Xiong (2006).

manager and investors and hence a firm's ownership choice. In general, older firms are likely to have a longer history of project implementation and a higher correlation between their new projects and their past successful projects. Such intertemporal linkages across projects enable the manager and investors to learn about project returns, and potentially reduce the heterogeneity of beliefs among them. Such opportunities for beliefs convergence are greater in firms with longer project implementation histories. In this section, we explore learning by the manager and investors and analyze how it impacts the manager-investor agreement for old and young firms and hence their ownership choices. To allow for learning, we modify the way we specify prior beliefs. Apart from this, our set-up remains unchanged.

Firms have access to incremental projects every period. Project payoffs are uncertain and the manager and the investors have heterogenous prior beliefs about project payoffs. As before, the investors who hold the firm's shares are those with beliefs closest to that of the manager, or equivalently the highest agreement with the manager. Every period, the manager and the investors observe a common signal about the project, but may disagree about the optimal implementation decision. Conditional on this disagreement, the manager has control with probability  $\eta$ .

Let us define  $r$  as the probability that an old firm's incremental project is a repeat project that the firm has successfully implemented in the past; with probability  $1 - r$ , the incremental project is a completely novel project without any history. For a young firm, the incremental project is completely novel with probability 1. Hence, prior to the nature of the project being revealed, the expected agreement between the manager of an old firm and investors is equal to  $\rho_{old} = r\rho_r + [1 - r]\rho_n$ , where  $\rho_r$  is the agreement for a repeat project and  $\rho_n$  is the agreement for a novel project. For a young firm, the agreement is equal to  $\rho_{young} = \rho_n$ . The agreement for an old firm,  $\rho_{old}$ , will be greater than the agreement for a young firm,  $\rho_{young}$ , if

$$r\rho_r + [1 - r]\rho_n > \rho_n \quad \text{or} \quad \rho_r > \rho_n \quad (18)$$

What (18) says is that the expected agreement will be greater for an old firm if the agreement for a repeat project,  $\rho_r$ , is greater than that for a novel project,  $\rho_n$ . To identify the conditions for this to happen, we need to specify the prior beliefs of the manager and the investors. To illustrate the main ideas, we use normally distributed priors.

The manager's prior belief about the expected project payoffs is  $N(\mu^M, \sigma^2)$  and investor  $i$ 's prior

belief is  $N(\mu^i, \sigma^2)$ . For simplicity, we assume that the manager and the investors have the same belief about the variance of the expected payoffs. In the cross-section of investors, the  $\mu^i$ 's are random draws from a continuous distribution with support  $[0, \mu^M]$ . Thus, the manager's prior mean represents the maximum possible prior mean of the investors. This ensures that the manager always has a "higher" prior about the expected project payoffs than the investors<sup>35</sup> and that the agreement between the manager and the investors is increasing in the mean of the investors' prior beliefs. Since the investors' prior means are random draws from a continuous distribution, from Lemma 1 we know that the prior belief of the maximal investors and hence the agreement between the manager and the firm's shareholders is increasing in the level of investor participation,  $N$ .

Every period, the manager and the investors observe a common signal  $Y$  about the expected project payoff.  $Y$  is a random draw from a normal distribution with a mean equal to the expected project payoff and a variance of  $s^2$ ; the manager and the investors agree on the variance of the signal. Since the manager and the investors differ in their prior beliefs about the expected project payoff, they will also differ in their interpretation of the common signal.<sup>36</sup> Given risk neutrality and a zero riskfree rate, the manager and the investors will agree to invest in the project if their posterior means are positive. Since the manager and the investors have different prior beliefs, there can be potential disagreement between them on project implementation. From standard results in Bayesian statistics (DeGroot (1970)), we know that after a signal realization of  $Y = y$ , the manager's posterior about the expected project payoff will be normally distributed with a mean of  $\frac{\mu^M s^2 + y \sigma^2}{s^2 + \sigma^2}$ , and a variance of  $\frac{s^2 \sigma^2}{s^2 + \sigma^2}$ . The manager will wish to invest in the project if  $\mu^M s^2 + y \sigma^2 \geq 0$ , or if  $y \geq -\frac{\mu^M s^2}{\sigma^2}$ . Similarly, an investor with prior  $N(\mu^i, \sigma^2)$  will agree to invest in the project only if  $\mu^i s^2 + y \sigma^2 \geq 0$  or if  $y \geq -\frac{\mu^i s^2}{\sigma^2}$ . Thus, the probability that the investor will agree with the manager on the project implementation decision, conditional on the manager wishing to invest, is:

$$\rho = \frac{Pr(y \geq -\frac{\mu^i s^2}{\sigma^2})}{Pr(y \geq -\frac{\mu^M s^2}{\sigma^2})}. \quad (19)$$

---

<sup>35</sup>The justification is that the manager is likely to pick the projects he is most bullish about.

<sup>36</sup>In this set-up, this difference in the interpretation of the common signal would vanish asymptotically if a sufficiently long history of signals were available. Such convergence of beliefs is natural to expect for repeat projects. However, for novel projects, our assumption is that a different novel project is drawn in each period, so that there simply is not a long enough sequence of signals available on any given novel project to permit beliefs convergence. An alternative specification would have been to start with heterogeneous prior beliefs about project payoffs as we do and assume that there is uncertainty about the precision of the subsequent common signals the manager and investors observe for novel projects but no such uncertainty exists for repeat projects. Then the recent results of Acemoglu, Chernozhukov and Yildiz (2006) imply that asymptotic agreement will then be reached for repeat projects and may never be attained for novel projects. All of our results will obtain in this setting.

The following lemma compares the agreement for novel and repeat projects.

**LEMMA 7:** *The agreement for a successful repeat project,  $\rho_r$ , is higher than that for a novel project,  $\rho_n$ .*

The intuition is as follows. A project success, namely a high realized payoff, increases both the manager's and the investors' posterior mean of the project payoff. But since the investors' prior mean  $\mu^i$  is below the manager's prior mean  $\mu^M$ , the investors' posterior belief exhibits a bigger response to a project success. This reduces the gap between the beliefs of the manager and the investors and increases agreement, leading to  $\rho_r > \rho_n$ . From (18), it then follows that an old firm with a successful repeat project will have a higher agreement with investors than will a young firm.

We have thus far taken the extent of investor participation,  $N$ , as fixed. However,  $N$  may change over time for various reasons, including changes in transactions costs for trading, improved regulation etc. Our next result explores the effect of a change in  $N$ .

**PROPOSITION 8:** *A change in the level of investor participation  $N$  will have a greater impact on the agreement for a young firm than for an old firm i.e.  $|\frac{\partial \rho_{young}}{\partial N}| > |\frac{\partial \rho_{old}}{\partial N}|$ .*

The intuition is as follows. A change in  $N$  affects agreement by changing the “maximal” investors' prior mean about the project payoff. An increase in  $N$  increases the prior beliefs of the investors with the maximal mean (Lemma 1) and hence the agreement between the manager and the investors. The impact of this increase in the investors' prior belief on their posterior beliefs will be lower for a successful repeat project than for a novel project, because the realized payoff on the repeat project dampens the impact of the prior. Since the incremental projects of young firms are more likely to be novel, a change in  $N$  will have a greater impact on agreement for young firms.

The elevated sensitivity of young firms with respect to investor participation also implies that these firms will more often switch between private and public ownership (Proposition 3). That is, an increase in  $N$  will have a bigger impact on the preference of young firms for either remaining or going public. Similarly, a decrease in  $N$  pushes young public firms more strongly towards private ownership.

This prediction has some empirical support in the recent work of Fink, Fink, Grullon and Weston (2004), who show that the average age of firms going public declined from forty years in the 1960s

to five years in the 1990s. This was also a period during which public-market investor participation increased significantly. Proposition 8 suggests that disproportionately many young firms went public during this period. Further evidence can be found in international comparisons. For example, a typical Initial Public Offering (IPO) in the US, where investor participation is higher than in Italy, involves firms that are much younger than the firms doing IPOs in Italy.<sup>37</sup> Our result also agrees well with anecdotal evidence that the recent going-private wave is disproportionately concentrated among the younger firms that went public during the Internet boom of the late 1990s.

#### *D. Empirical Predictions*

For the most part, the empirical predictions that we discuss below are novel and are yet to be tested. The key to testing them will be to come up with reasonable empirical proxies for agreement,  $\rho$ , and managerial autonomy,  $\eta$ . Proxies for  $\rho$  may be linked to proxy fights (the fewer of them, the higher the inferred  $\rho$ ), shareholder reactions to major investments in R&D, and acquisitions (the higher the announcement effect, the greater the implied shareholder endorsement of management decisions and hence the higher the inferred  $\rho$ ), and the dual-class control premium (the higher this premium, the higher the value investors attach to having control in states of disagreement with the manager and hence the lower the inferred  $\rho$ ).<sup>38</sup> Proxies for  $\eta$  may be provisions in IPO charters, and relevant variables in the Gompers, Ishi and Metrick (GIM (2003)) governance index.

1. The greater the investor participation for a publicly-traded firm, the lower should be the stringency of corporate governance for the firm (Proposition 1). If we view better-known and larger firms as having greater investor participation, then this prediction is consistent with GIM (2003) that managers in larger firms have greater takeover protection, assuming that greater protection signifies greater autonomy. However, this evidence may be a bit tangential, and one would need to more directly test this prediction by using other proxies for investor participation, such as depth, and the number of analysts following the firm.
2. A public firm will go private when its stock price is sufficiently low and/or the volatility of this price is sufficiently high (Proposition 3). This prediction is consistent with the anecdotal evidence

---

<sup>37</sup>Pagano, Panetta and Zingales (1998) report that a typical firm doing an IPO in Italy is six times older than a typical firm doing an IPO in the US.

<sup>38</sup>Dittmar and Thakor (2007) use many of these proxies to empirically document that disagreement has incremental power in explaining the firm's security issuance decision.

presented in the Introduction, but we are not aware of any large-sample empirical evidence.

3. The probability of a going-private transaction declines as the liquidity cost of private ownership increases, as the assets in place increase in value, and as the growth opportunity becomes less valuable (Proposition 3). One way to test this would be to compare going-private transactions across markets or countries that have private equity markets with varying levels of liquidity.
4. A going-private transaction will occur only at a price that represents a substantial premium above the firm's pre-transaction stock price (Proposition 3).
5. A public firm that goes private will exhibit improved performance after going private and be worth more. This result follows from Proposition 4, which shows that whenever the firm goes private, managerial effort is greater with private ownership.
6. *Ceteris paribus*, the higher the price paid to take the firm private the less stringent is the corporate governance put in place *after* the firm is taken private. This follows from Lemma 5, which shows that managerial autonomy with private ownership is increasing in  $\rho_{pr}$ , and from the fact that the higher is  $\rho_{pr}$  the higher is the private investors' valuation of the firm.
7. A reduction in public-market investor participation will result in younger public firms going private, whereas an increase leads to younger firms going public (Proposition 8).

## IV. Conclusion

The development of liquid capital markets for publicly-traded firms is one of the most noteworthy features of industrialized economies. This liquidity has an obvious advantage for public firms in that it lowers the firm's cost of capital. While this advantage also appears in our analysis, we show that public-market liquidity also has a surprising dark side which manifests itself in volatility in the firm's ownership base and hence uncertainty about the degree of alignment between the manager and shareholders. That is, the liquidity of public ownership is both its blessing and its curse.

A central feature of our analysis is that the firm's ownership mode determines the three key factors – corporate governance, liquidity and the stability of the firm's shareholder base – that influence the firm's decision of whether to be public or private. As part of our analysis, we provide an approach for

*endogenously* determining both security benefits and the allocation of control between the manager and investors, in a setting in which the manager has *no* innate preference for control via exogenous private benefits of control. We show how this determination differs across private and public ownership due to the inherently greater liquidity of the latter.

We also show that greater public-market investor participation strengthens the incentives for firms to stay public. This works via two channels. First, higher investor participation leads to a higher valuation for the marginal investor holding the stock. And second, the (cross-sectional) variance of the valuations of investors holding the firm's stock is smaller when there are more investors. Both effects work in concert and raise the firm's stock price by increasing the expected manager-investor agreement. Thus, investor participation and firm participation in the stock market go hand-in-hand.

Our major results are unlikely to obtain with asymmetric information or exogenous private control benefits, and we highlight some key differences between our approach and the usual approaches. For example, with private control benefits, giving the manager greater ownership in the firm resolves the problem of inefficient project choice, and the manager can be given complete project choice autonomy if managerial ownership is sufficiently high. In contrast, with heterogeneous prior beliefs, managerial ownership has no effect on either manager-investor disagreement or managerial autonomy. We believe our approach can be fruitfully used to address other issues in the theory of the firm. For example, the alignment between the firm and its investors will be affected by the boundaries of the firm, thereby producing an interaction between firm boundaries and its choice of ownership mode.

## APPENDIX

*Proof of Lemma 1:* The distribution function of  $\rho_N$ ,  $Q(\rho_N)$  can be written as  $Q(\rho_N) = G^N(\rho)$  where  $G(\rho)$  is the distribution function of  $\rho$ . Since  $G(\rho) \leq 1$ , it is clear that for any two values of  $N$ , say  $N_1$  and  $N_2$  with  $N_1 < N_2$ ,  $Q(\rho_{N_2})$  first-order stochastically dominates  $Q(\rho_{N_1})$ . This in turn implies  $\bar{\rho}_{N_2} < \bar{\rho}_{N_1}$ . Thus, we have proved the first part of the lemma. To prove the second part of the lemma, we first note that  $\rho_N$  has the following asymptotically degenerate distribution with 0 variance:

$$F(\rho_N) = \begin{cases} 1 & \rho_N = \rho_h \\ 0 & \rho_N \neq \rho_h \end{cases}$$

To show that for finite  $N$  the variance of  $\rho_N$  is decreasing in  $N$ , we can use an approximate inverse Taylor series expansion of the expression of the variance of  $\rho_N$ . That is, the variance of  $\rho_N$  is given as  $Var(\rho_N) = E(\sigma_N^2) - E(\rho_N)^2$ , where

$$E(\rho_N) = \int_0^1 G(\rho)^{N-1} g(\rho) \rho d\rho \quad (A1)$$

We know from the probability integral transformation that  $G(\rho_N) = u_N$ , where  $u_N$  is the  $N^{th}$  order statistic from a uniform distribution. Thus, we can express any function of the  $N^{th}$  order statistic of any continuous distribution as a function of the  $N^{th}$  order statistic of the uniform distribution. Using this and performing a Taylor series expansion of the variance of  $\rho_N$ , a good approximation for  $Var(\rho_N)$  can be written as follows:

$$\sigma_N^2 \equiv Var(\rho_N) \approx \frac{N}{[N+1]^2[N+2]} \{g[G^{-1}(\frac{1}{N+1})]\}^{-2} \quad (A2)$$

For a detailed derivation of (A2), see Gibbons (1971). (A2) shows when  $Var(\rho_N)$  is decreasing in  $N$ . We note that  $G^{-1}(\frac{1}{N+1})$  is decreasing in  $N$ . It is difficult to say much about  $\{g[G^{-1}(\frac{1}{N+1})]\}^{-2}$ , unless we know the shape of the distribution. The condition required for  $Var(\rho_N)$  to decrease in  $N$  is that  $T(N) \equiv \{g[G^{-1}(\frac{1}{N+1})]\}^{-2}$  should not increase with  $N$  at an order of magnitude greater than  $N^2$ , which is the rate at which  $\frac{N}{[N+1]^2[N+2]}$  decreases with  $N$ . Thus, the condition required is that  $\{g[G^{-1}(\frac{1}{N+1})]\}^{-2}$  should not increase at an order of magnitude greater than  $N^2$ . This is clearly true when  $T(N)$  is non-increasing in  $N$ . Q.E.D.

*Proof of Lemma 2:* Since  $V_{pub}^M$  in (8) is concave in  $e$ , the first-order condition (FOC) is necessary and sufficient for the unique maximum. The optimal effort level  $e_{pub}^*$  is determined through the FOC

as:

$$\begin{aligned} \delta[\bar{\rho} + [1 - \bar{\rho}]\eta_{pub}][\alpha[X_h - S] - \Psi] - \beta e &= 0 \quad \text{which yields} \\ e_{pub}^* &= \frac{\delta}{\beta}[\bar{\rho} + [1 - \bar{\rho}]\eta_{pub}][\alpha[X_h - S] - \Psi] \end{aligned} \quad (\text{A3})$$

It is now clear that  $\frac{\partial e_{pub}^*}{\partial \bar{\rho}} > 0$ ,  $\frac{\partial e_{pub}^*}{\partial \alpha} > 0$ ,  $\frac{\partial e_{pub}^*}{\partial \eta_{pub}} > 0$  and  $\frac{\partial e_{pub}^*}{\partial \beta} < 0$ . Q.E.D.

*Proof of Lemma 3:* Since  $V_{pr}^M$  in (11) is concave in  $e$ , the FOC is necessary and sufficient for the unique maximum. The optimal effort level  $e_{pr}^*$  is determined through the FOC as:

$$\begin{aligned} \delta[\rho_{pr} + [1 - \rho_{pr}]\eta_{pr}][\alpha[X_h - S] - \Psi] - \beta e &= 0 \quad \text{which yields} \\ e_{pr}^* &= \frac{\delta}{\beta}[\rho_{pr} + [1 - \rho_{pr}]\eta_{pr}][\alpha[X_h - S] - \Psi] \end{aligned} \quad (\text{A4})$$

It is now clear that  $\frac{\partial e_{pr}^*}{\partial \rho_{pr}} > 0$ ,  $\frac{\partial e_{pr}^*}{\partial \alpha} > 0$ ,  $\frac{\partial e_{pr}^*}{\partial \eta_{pr}} > 0$  and  $\frac{\partial e_{pr}^*}{\partial \beta} < 0$ . Q.E.D.

*Proof of Lemma 4:* The optimal managerial autonomy with public ownership is fixed so as to maximize the  $t = 0$  stock price of the firm. The stock price at  $t = 0$  is given by (13). Since  $V_{pub}^I$  is concave in  $\eta_{pub}$ , the FOC is necessary and sufficient for a unique maximum. We can write  $V_{pub}^I$  as:

$$\begin{aligned} V_{pub}^I &= [1 - \alpha][e_{pub}^*\delta[\bar{\rho}\{X_h - S\} + [1 - \bar{\rho}]\eta_{pub}\{X_l - S\}] + S] \\ &= [1 - \alpha]\left[\frac{\delta}{\beta}[\bar{\rho} + \{1 - \bar{\rho}\}\eta_{pub}][\alpha[X_h - S] - \Psi]\delta[\bar{\rho}\{X_h - S\} + [1 - \bar{\rho}]\eta_{pub}\{X_l - S\}] + S\right] \end{aligned}$$

Differentiating with respect to  $\eta_{pub}$  we get the FOC:

$$\begin{aligned} \frac{\partial V_{pub}^I}{\partial \eta_{pub}} &= [1 - \alpha]\frac{\delta}{\beta}[1 - \bar{\rho}][\alpha\{X_h - S\} - \Psi]\delta[\bar{\rho}\{X_h - S\} + \{1 - \bar{\rho}\}\eta_{pub}^*\{X_l - S\}] \\ &\quad + [1 - \alpha]\left[\frac{\delta}{\beta}[\bar{\rho} + \{1 - \bar{\rho}\}\eta_{pub}^*][\alpha[X_h - S] - \Psi]\delta[1 - \bar{\rho}][X_l - S]\right] = 0 \end{aligned} \quad (\text{A5})$$

Simplifying we have  $\bar{\rho}[X_h - S] + [1 - \bar{\rho}]\eta_{pub}^*[X_l - S] + [\bar{\rho} + \{1 - \bar{\rho}\}\eta_{pub}^*][X_l - S] = 0$

$$\text{or} \quad \eta_{pub}^* = \frac{\bar{\rho}[X_h + X_l - 2S]}{2[1 - \bar{\rho}][S - X_l]} \quad (\text{A6})$$

To verify the second-order condition is satisfied we can differentiate  $\frac{\partial V_{pub}^I}{\partial \eta_{pub}}$  with respect to  $\eta_{pub}$  to get

$$\begin{aligned} \frac{\partial^2 V_{pub}^I}{\partial \eta_{pub}^2} &= [1 - \alpha]\left[\frac{\delta}{\beta}[1 - \bar{\rho}][\alpha\{X_h - S\} - \Psi]\delta\{1 - \bar{\rho}\}\{X_l - S\}\right] \\ &\quad + [1 - \alpha]\left[\frac{\delta}{\beta}\{1 - \bar{\rho}\}[\alpha[X_h - S] - \Psi]\delta[1 - \bar{\rho}][X_l - S]\right] \end{aligned}$$

Since  $X_l < S$ , it is obvious that the right hand side of the above expression is negative. From (A6) it is also clear that  $\frac{\partial \eta_{pub}^*}{\partial \bar{\rho}} > 0$ ,  $\frac{\partial \eta_{pub}^*}{\partial X_h} > 0$  and  $\frac{\partial \eta_{pub}^*}{\partial X_l} > 0$ . Q.E.D.

*Proof of Proposition 1:* Follows readily from Lemma 1 and Lemma 4. Q.E.D.

*Proof of Proposition 2:* Note that  $\frac{\partial V_{pub}^M}{\partial N} = [\frac{\partial V_{pub}^M}{\partial \bar{\rho}} \cdot \frac{\partial \bar{\rho}}{\partial N}] + [\frac{\partial V_{pub}^M}{\partial \eta_{pub}^*} \cdot \frac{\partial \eta_{pub}^*}{\partial N}]$ . Clearly  $\frac{\partial V_{pub}^M}{\partial \bar{\rho}} > 0$ . From Lemma 1 we know  $\frac{\partial \bar{\rho}}{\partial N} > 0$ . Moreover  $\frac{\partial V_{pub}^M}{\partial \eta_{pub}^*} > 0$  (see (8)) and  $\frac{\partial \eta_{pub}^*}{\partial N} > 0$  (see Proposition 1). Q.E.D.

*Proof of Lemma 5:* The optimal managerial autonomy with private ownership is fixed so as to maximize the valuation of the private investors. The valuation of the private investors at  $t = 0$  is given by (15). Since  $V_{pr}^I$  is concave in  $\eta$ , the FOC is necessary and sufficient for a unique maximum. We can write  $V_{pr}^I$  as:

$$\begin{aligned} V_{pr}^I &= [1 - \alpha]\{e_{pr}^*[\delta\rho_{pr}[X_h - S] + \delta[1 - \rho_{pr}]\eta_{pr}[X_l - S]] + S\} - L \\ &= [1 - \alpha]\left\{\frac{\delta}{\beta}\{\rho_{pr} + [1 - \rho_{pr}]\eta_{pr}^*\}[\alpha\{X_h - S\} - \Psi][\delta\rho_{pr}[X_h - S] + \delta[1 - \rho_{pr}]\eta_{pr}[X_l - S]] + S\right\} - L \end{aligned}$$

Differentiating with respect to  $\eta_{pr}$  we get

$$\begin{aligned} \frac{\partial V_{pr}^I}{\partial \eta_{pr}} &= [1 - \alpha]\left\{\frac{\delta}{\beta}[1 - \rho_{pr}][\alpha\{X_h - S\} - \Psi][\delta\rho_{pr}[X_h - S] + \delta[1 - \rho_{pr}]\eta_{pr}^*[X_l - S]] + S\right\} \\ &\quad + [1 - \alpha]\left[\frac{\delta}{\beta}\{\rho_{pr} + [1 - \rho_{pr}]\eta_{pr}^*\}[\alpha\{X_h - S\} - \Psi]\delta[1 - \rho_{pr}][X_l - S]\right] = 0 \end{aligned}$$

Simplifying we have  $\delta\rho_{pr}[X_h - S] + \delta[1 - \rho_{pr}]\eta_{pr}^*[X_l - S] + \delta[\rho_{pr} + [1 - \rho_{pr}]\eta_{pr}^*][X_l - S] = 0$

$$\text{which implies } \eta_{pr}^* = \frac{\delta\rho_{pr}[X_h + X_l - 2S]}{2\delta[1 - \rho_{pr}][S - X_l]} \quad (\text{A7})$$

Verification of the second-order condition,  $\frac{\partial^2 V_{pr}^I}{\partial \eta_{pr}^2} < 0$ , is along the same lines as in the proof of Lemma 4. Q.E.D.

*Proof of Lemma 6:* The optimal control allocation divides control between the manager and the investors. Hence, two kinds of renegotiation can occur: the investors can acquire additional control from the manager, or the manager can acquire additional control from the investors, with the acquisition of additional control in each case involving an offer to increase the other party's ownership share; the party acquiring control ends up with total control. We consider each kind of renegotiation. It is convenient to define  $\alpha_1 \equiv \frac{S - X_l}{X_h - X_l}$  and  $\alpha_2 \equiv \frac{X_h[S - X_l]}{X_h - X_l}$ .

**Investors Acquiring Control from Manager:** Starting with  $\eta_{pr}^*$ , the expected loss suffered by investors in the event of disagreement is  $Loss = \eta_{pr}^*[1 - \alpha][S - X_l]$ , and the expected benefit for the

manager is  $Benefit = \alpha\eta_{pr}^*[X_h - S]$ . For renegotiation to fail, the loss suffered by the investors should be less than the benefit perceived by the manager:

$$\begin{aligned} \eta_{pr}^*[1 - \alpha][S - X_l] &< \alpha\eta_{pr}^*[X_h - S], \\ \text{which implies } \alpha &> \frac{S - X_l}{X_h - X_l} \end{aligned} \quad (A8)$$

**Manager Acquiring Control from the Investors:** The manager can acquire total control by agreeing to reduce his ownership stake from  $\alpha$  to some  $\alpha_0 < \alpha$ . The manager's expected utility with the original contract is  $\alpha\{\eta_{pr}^*X_h + [1 - \eta_{pr}^*]S\}$ . If he acquires total control, he can guarantee investment with probability 1, and his utility is  $\alpha_0X_h$ . So a necessary condition for the manager to be willing to renegotiate is:

$$\begin{aligned} \alpha_0X_h &\geq \alpha\{\eta_{pr}^*X_h + [1 - \eta_{pr}^*]S\}, \quad \text{which implies} \\ \alpha_0 &\geq \frac{\alpha[\eta_{pr}^*X_h + [1 - \eta_{pr}^*]S]}{X_h} \equiv \hat{\alpha} \end{aligned} \quad (A9)$$

The investors' expected utility is  $[1 - \alpha][\eta_{pr}^*X_l + \{1 - \eta_{pr}^*\}S]$  without renegotiation, and it is  $[1 - \alpha_0]X_l$  with renegotiation. Thus, for the investors to be willing to renegotiate we need

$$\begin{aligned} [1 - \alpha_0]X_l &\geq [1 - \alpha][\eta_{pr}^*X_l + \{1 - \eta_{pr}^*\}S], \quad \text{which implies} \\ \alpha_0 &\leq 1 - \frac{[1 - \alpha][\eta_{pr}^*X_l + \{1 - \eta_{pr}^*\}S]}{X_l} \equiv \hat{\alpha} \end{aligned} \quad (A10)$$

For renegotiation to be feasible, we need  $[\hat{\alpha}, \hat{\alpha}]$  to be a nonempty set, i.e.  $\hat{\alpha} > \hat{\alpha}$ . Thus, renegotiation proofness is guaranteed if

$$\hat{\alpha} < \hat{\alpha} \quad (A11)$$

Substituting for  $\hat{\alpha}$  and  $\hat{\alpha}$  in (A11) means that the following is sufficient for renegotiation-proofness:

$$\begin{aligned} 1 - \frac{[1 - \alpha][\eta_{pr}^*X_l + \{1 - \eta_{pr}^*\}S]}{X_l} &< \frac{\alpha[\eta_{pr}^*X_h + [1 - \eta_{pr}^*]S]}{X_h}, \quad \text{which implies} \\ \alpha\left\{\frac{\eta_{pr}^*X_h + [1 - \eta_{pr}^*]S}{X_h} - \frac{\eta_{pr}^*X_l + [1 - \eta_{pr}^*]S}{X_l}\right\} &> 1 - \frac{\eta_{pr}^*X_l + [1 - \eta_{pr}^*]S}{X_l} \end{aligned}$$

The above inequality can be further simplified as:

$$\alpha \frac{S\{X_l - X_h\}\{1 - \eta_{pr}^*\}}{X_lX_h} > \frac{\{1 - \eta_{pr}^*\}\{X_l - S\}}{X_l} \quad (A12)$$

Noting that  $X_l < S < X_h$ , the following suffices for (A12) to hold:

$$\alpha < \frac{X_h[S - X_l]}{X_h - X_l} \quad (\text{A13})$$

Combining (A8) and (A13), we see that renegotiation-proofness obtains if

$$\alpha \in \left( \frac{S - X_l}{X_h - X_l}, \frac{X_h[S - X_l]}{X_h - X_l} \right)$$

Q.E.D.

*Proof of Proposition 3:* The stock price at  $t = 0$ , when the firm has public ownership, is given by (13), while the private investors' valuation is given by (15). Comparing (13) and (15), private investors will be able to purchase all the outstanding shares iff:

$$W_{pr}^I \geq W_{pub}^I$$

$$\text{or } [1 - \alpha][e_{pr}^*[\delta\rho_{pr}[X_h - S] + \delta[1 - \rho_{pr}]\eta_{pr}[X_l - S]] + S] - L \geq [1 - \alpha]e_{pub}^*\delta[\bar{\rho}\{X_h - S\} + [1 - \bar{\rho}]\eta_{pub}^*\{X_l - S\}] + S$$

Substituting for  $e_{pr}^*$ ,  $\eta_{pr}^*$ ,  $e_{pub}^*$  and  $\eta_{pub}^*$  (see (12), (14), (3) and (11)) and simplifying we have:

$$\begin{aligned} & \{1 - \alpha\} \left\{ \left[ \frac{\delta\rho_{pr}[X_h - X_l][\alpha[X_h - S] - \Psi]}{2\beta[S - X_l]} \right] \left[ \frac{\delta\rho_{pr}[X_h - X_l]}{2} \right] + S \right\} - L \\ & \geq \{1 - \alpha\} \left\{ \left[ \frac{\delta\bar{\rho}[X_h - X_l][\alpha[X_h - S] - \Psi]}{2\beta[S - X_l]} \right] \left[ \frac{\delta\bar{\rho}[X_h - X_l]}{2} \right] + S \right\} \\ \text{simplifying we have} \quad & \{1 - \alpha\} \left\{ \left[ \frac{\delta^2[X_h - X_l]^2[\alpha[X_h - S] - \Psi]}{4\beta[S - X_l]} \right] [\rho_{pr}^2 - \bar{\rho}^2] \right\} \geq L \\ \text{which implies} \quad & \rho_{pr} \geq \bar{\rho} + \sqrt{\frac{4L\beta[S - X_l]}{[1 - \alpha][\delta^2[X_h - X_l]^2[\alpha[X_h - S] - \Psi]]}} \equiv \hat{\rho} \end{aligned} \quad (\text{A14})$$

It is clear that  $\hat{\rho}$  is increasing in  $\bar{\rho}$ ,  $L$ ,  $\beta$  and  $S$ . That  $\hat{\rho}$  is decreasing in  $X_h$  and  $X_l$  can be seen by differentiating the expression for  $\hat{\rho}$  with respect to  $X_h$  and  $X_l$  and noting that Assumption 2 implies  $X_h + X_l > 2S$ . Further,  $\frac{\partial \hat{\rho}}{\partial N} > 0$  follows from  $\bar{\rho}$  increasing in  $N$  and  $\frac{\partial \hat{\rho}}{\partial \bar{\rho}} > 0$ .  $\frac{\partial \hat{\rho}}{\partial \sigma_N^2} < 0$  follows from  $\sigma_N^2$  decreasing in  $N$ . (Proposition 1)

Q.E.D.

*Proof of Proposition 4:* The manager will strictly prefer private ownership whenever  $V_{pr}^{M*} > V_{pub}^{M*}$ , where  $V_{pr}^{M*}$  is given by (11) after replacing  $e$  with  $e_{pr}^*$  from (12), and  $V_{pub}^{M*}$  is given by (10) after substituting for  $e_{pub}^*$  from (9). Thus, the manager prefers private ownership when:

$$\begin{aligned} \frac{\beta}{2} \left\{ \frac{[\delta\rho_{pr}[X_h - X_l][\alpha[X_h - S] - \Psi]]^2}{2\beta[S - X_l]} \right\} + \alpha S & > \frac{\beta}{2} \left\{ \frac{[\delta\bar{\rho}[X_h - X_l][\alpha[X_h - S] - \Psi]]^2}{2\beta[S - X_l]} \right\} + \alpha S \\ \text{or} \quad \rho_{pr} & > \bar{\rho} \end{aligned} \quad (\text{A15})$$

Comparing (A14) to (A15) we see that (A15) is satisfied whenever (A14) is satisfied.

Q.E.D.

*Proof of Proposition 5:* Define the set of exogenous parameter values  $\Lambda \equiv \{\rho_h, \rho_l, N, L, c, \alpha, \delta, \beta, \Psi, X_h, X_l, S\}$ .

We prove that  $\Lambda$  is nonempty and for this we replace  $E(\rho)$  in Assumption 1 with  $\bar{\rho}$  (see footnote 23).

We provide a numerical example in which we first vary the level of investor participation  $N$ , then the expected liquidity cost  $L$ , the cash flows with the project  $X_h$ , cash flows without the project  $S$ , and managerial search cost parameter  $\beta$ . For this numerical example, we assume that the agreement in the cross-section of investors is uniformly distributed between  $[\rho_l, \rho_h]$ .

| A. Exogenous Parameters |       |       |       |       |       |       |
|-------------------------|-------|-------|-------|-------|-------|-------|
| $\rho_h$                | .7    | .7    | .7    | .7    | .7    | .7    |
| $\rho_l$                | .2    | .2    | .2    | .2    | .2    | .2    |
| $E(\rho)$               | .45   | .45   | .45   | .45   | .45   | .45   |
| $c$                     | .5    | .5    | .5    | .5    | .5    | .5    |
| $\delta$                | 1     | 1     | 1     | 1     | 1     | 1     |
| $\Psi$                  | .0001 | .0001 | .0001 | .0001 | .0001 | .0001 |
| $\alpha$                | .2    | .2    | .2    | .2    | .2    | .2    |
| $X_l$                   | .4    | .4    | .4    | .4    | .4    | .4    |
| $N$                     | 2     | 3     | 2     | 2     | 2     | 2     |
| $L$                     | .10   | .10   | .12   | .10   | .10   | .10   |
| $X_h$                   | 1.75  | 1.75  | 1.75  | 1.80  | 1.75  | 1.75  |
| $S$                     | .9    | .9    | .9    | .9    | 1     | .9    |
| $\beta$                 | .17   | .17   | .17   | .17   | .17   | .18   |
| B. Endogenous Variables |       |       |       |       |       |       |
| $\bar{\rho}$            | .533  | .58   | .533  | .533  | .533  | .533  |
| $\eta_{pub}^*$          | .40   | .47   | .40   | .46   | .14   | .40   |
| $e_{pub}^*$             | .72   | .78   | .72   | .79   | .53   | .68   |
| $V_{pub}^I$             | .93   | .96   | .93   | .96   | .95   | .92   |
| $\hat{\rho}$            | .65   | .68   | .67   | .63   | .69   | .66   |

Panel A gives the values of the exogenous parameters while Panel B gives the corresponding values of the endogenous variables. In Columns 2-6 we sequentially vary the level of investor participation  $N$ , the expected liquidity cost  $L$ , cash flows with the project  $X_h$ , cash flows without the project  $S$ , and managerial search cost parameter  $\beta$ . As can be seen from the above numerical analysis in all cases, there is an interior value of  $\hat{\rho} \in (\rho_l, \rho_h)$  such that for all  $\rho_{pr} \geq \hat{\rho}$  the firm goes private and otherwise the firm remains public. When we increase  $N$  with all other exogenous parameters held fixed, we see that  $\hat{\rho}$  increases, implying that public ownership becomes more attractive.  $\hat{\rho}$  increases and public ownership becomes more attractive if  $L$  increases (Column 3), if  $S$  increases (Column 5) and if  $\beta$  increases (Column 6), public ownership becomes less attractive if  $X_h$  increases (Column 4). We have thus shown that  $\Lambda$  is nonempty.

Q.E.D.

*Proof of Proposition 6:* This is clear from the discussion preceding Proposition 6. Q.E.D.

*Proof of Proposition 7:* We prove the proposition for public ownership. A similar proof can be written for private ownership. The investors' problem with public ownership can be written as:

$$\max_{w_0, w_1, w_2, \eta_{pub}} \quad e\delta\bar{\rho}[X_h - S - w_1 - w_0] + e\delta[1 - \bar{\rho}]\eta_{pub}[X_l - S - w_2 - w_0] + S + w_0 \quad (\text{A16})$$

$$\begin{aligned} \text{s.t} \quad & e\delta\bar{\rho}[w_1 - w_0] + e\delta[1 - \bar{\rho}]\eta_{pub}[w_1 - w_0] + S + w_0 - e\delta\{\bar{\rho} + [1 - \bar{\rho}]\eta_{pub}\}\Psi \\ & -\frac{\beta e^2}{2} \geq 0 \end{aligned} \quad (\text{A17})$$

$$\begin{aligned} \text{s.t} \quad & e^* \in \operatorname{argmax}\{e\delta\bar{\rho}[w_1 - w_0] + e\delta[1 - \bar{\rho}]\eta_{pub}[w_1 - w_0] + S + w_0 - e\delta\{\bar{\rho} + [1 - \bar{\rho}]\eta_{pub}\}\Psi \\ & -\frac{\beta e^2}{2}\} \end{aligned} \quad (\text{A18})$$

$$w_0 \geq 0 \quad w_1 \geq 0 \quad w_2 \geq 0$$

From the above problem, we can see that  $w_2$  does not enter either the manager's IR constraint (A17) or the IC constraint (A18). Since (A16) is decreasing in  $w_2$ , it is optimal to set  $w_2 = 0$ . To prove that it is optimal to set  $w_0 = 0$ , we substitute the manager's IC constraint, with the corresponding FOC. The manager's FOC for effort choice is:

$$\begin{aligned} & \delta\bar{\rho}[w_1 - w_0] + \delta[1 - \bar{\rho}]\eta_{pub}[w_1 - w_0] - \delta\{\bar{\rho} + [1 - \bar{\rho}]\eta_{pub}\}\Psi - \beta e = 0 \\ \text{or} \quad e^* &= \frac{1}{\beta} \{\delta\bar{\rho}[w_1 - w_0] + \delta[1 - \bar{\rho}]\eta_{pub}[w_1 - w_0] - \delta\{\bar{\rho} + [1 - \bar{\rho}]\eta_{pub}\}\Psi \end{aligned} \quad (\text{A19})$$

From the FOC, it is clear that the optimal managerial effort is decreasing in  $w_0$ . Since (A16) is also decreasing in  $w_0$ , it is optimal to set  $w_0 = 0$ . Substituting the manager's optimal effort as a function of  $w_1$  and  $\eta_{pub}$  from the FOC into the investors' objective function, we can solve for the optimal  $w_1$  and  $\eta_{pub}$  and show that it is optimal to set  $w_1 > 0$ . Q.E.D.

*Proof of Lemma 7:* To prove that  $\rho_r > \rho_n$  for a successful repeat project, we begin by noting that the agreement  $\rho$  on a novel project, as perceived by the investor with a prior of  $N(\mu^i, \sigma)$  is:

$$\rho_n^i = \frac{Pr(y \geq -\frac{\mu^i s^2}{\sigma^2})}{Pr(y \geq -\frac{\mu^M s^2}{\sigma^2})} = \frac{\int_{-\frac{\mu^i s^2}{\sigma^2}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{[x - \mu^i]^2}{2s^2}} dx}{\int_{-\frac{\mu^M s^2}{\sigma^2}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{[x - \mu^i]^2}{2s^2}} dx} = \frac{\int_{-\frac{\mu^i s^2}{\sigma^2}}^{\infty} e^{-\frac{[x - \mu^i]^2}{2s^2}} dx}{\int_{-\frac{\mu^M s^2}{\sigma^2}}^{\infty} e^{-\frac{[x - \mu^i]^2}{2s^2}} dx}$$

Throughout the proof, we estimate the agreement for an investor with  $\mu = \mu^i$  who we assume to be the maximal investor.

Let  $R$  denote the past realized payoff for the repeat project and let  $s^2$  be the variance of the project payoff. For a repeat project with a realized return of  $R$ , the manager's posterior about project return will be normally distributed with a mean of  $\frac{\mu^M s^2 + R\sigma^2}{s^2 + \sigma^2}$  and a variance of  $\frac{s^2\sigma^2}{s^2 + \sigma^2}$ . Similarly, the investor's posterior will be normally distributed with a mean of  $\frac{\mu^i s^2 + R\sigma^2}{s^2 + \sigma^2}$  and a variance of  $\frac{s^2\sigma^2}{s^2 + \sigma^2}$ . If the firm gets this project a second time and the manager and the investor observe a signal  $Y = y$  about the project payoff, the manager's posterior about the project payoff will now be normally distributed with a mean of  $\frac{\mu^M s^2 + R\sigma^2 + y\sigma^2}{2s^2 + \sigma^2}$ . The manager will be willing to implement the project a second time if the signal  $y \geq -\frac{\mu^M s^2 + R\sigma^2}{\sigma^2}$ . Similarly, it can be shown that the investor will be willing to implement the project if  $y \geq -\frac{\mu^i s^2 + R\sigma^2}{\sigma^2}$ . Hence, before observing the signal  $Y$ , the probability of agreement between the manager and the investor on a repeat project is:

$$\rho_r = \frac{\int_{-\frac{\mu^i s^2 + R\sigma^2}{\sigma^2}}^{\infty} e^{-\frac{[x - \frac{\mu^i s^2 + R\sigma^2}{s^2 + \sigma^2}]^2}{\frac{s^2\sigma^2}{s^2 + \sigma^2}}} dx}{\int_{-\frac{\mu^M s^2 + R\sigma^2}{\sigma^2}}^{\infty} e^{-\frac{[x - \frac{\mu^M s^2 + R\sigma^2}{s^2 + \sigma^2}]^2}{\frac{s^2\sigma^2}{s^2 + \sigma^2}}} dx}$$

Thus, the difference in the agreement between a repeat project and a novel project is:

$$\rho_r - \rho_n = \frac{\int_{-\frac{\mu^i s^2 + R\sigma^2}{\sigma^2}}^{\infty} e^{-\frac{[x - \frac{\mu^i s^2 + R\sigma^2}{s^2 + \sigma^2}]^2}{\frac{s^2\sigma^2}{s^2 + \sigma^2}}} dx}{\int_{-\frac{\mu^M s^2 + R\sigma^2}{\sigma^2}}^{\infty} e^{-\frac{[x - \frac{\mu^M s^2 + R\sigma^2}{s^2 + \sigma^2}]^2}{\frac{s^2\sigma^2}{s^2 + \sigma^2}}} dx} - \frac{\int_{-\frac{\mu^i s^2}{\sigma^2}}^{\infty} e^{-\frac{[x - \frac{\mu^i}{s}]^2}{\frac{s^2\sigma^2}{s^2 + \sigma^2}}} dx}{\int_{-\frac{\mu^M s^2}{\sigma^2}}^{\infty} e^{-\frac{[x - \frac{\mu^M}{s}]^2}{\frac{s^2\sigma^2}{s^2 + \sigma^2}}} dx}$$

Converting into standard normal variables and letting  $x_n^i \equiv \frac{-\frac{\mu^i s^2}{\sigma^2} - \mu^i}{s}$ ,  $x_n^M \equiv \frac{-\frac{\mu^M s^2}{\sigma^2} - \mu^M}{s}$ ,  $x_r^i \equiv \frac{-\frac{\mu^i s^2 + R\sigma^2}{\sigma^2} - \frac{\mu^i s^2 + R\sigma^2}{s^2 + \sigma^2}}{s}$  and  $x_r^M \equiv \frac{-\frac{\mu^M s^2 + R\sigma^2}{\sigma^2} - \frac{\mu^M s^2 + R\sigma^2}{s^2 + \sigma^2}}{s}$ , we have:

$$\begin{aligned} \rho_r - \rho_n &= \frac{\int_{x_r^i}^{\infty} e^{-\frac{z^2}{2}} dx}{\int_{x_r^M}^{\infty} e^{-\frac{z^2}{2}} dx} - \frac{\int_{x_n^i}^{\infty} e^{-\frac{z^2}{2}} dx}{\int_{x_n^M}^{\infty} e^{-\frac{z^2}{2}} dx} \\ &= \frac{\int_{x_r^i}^{\infty} e^{-\frac{z^2}{2}} dx \int_{x_n^M}^{\infty} e^{-\frac{z^2}{2}} dx - \int_{x_n^i}^{\infty} e^{-\frac{z^2}{2}} dx \int_{x_r^M}^{\infty} e^{-\frac{z^2}{2}} dx}{\int_{x_n^M}^{\infty} e^{-\frac{z^2}{2}} dx \int_{x_r^M}^{\infty} e^{-\frac{z^2}{2}} dx} \end{aligned} \quad (A20)$$

Since  $x_n^M$  and  $x_n^i$  are independent of  $R$  while  $x_r^M$  and  $x_r^i$  are decreasing in  $R$ , for some  $R \geq \hat{R}$  we have  $x_n^M \geq x_r^M$  and  $x_n^i \geq x_r^i$ . When  $R \geq \hat{R}$ , we can write  $\rho_r - \rho_n$  as:

$$\rho_r - \rho_n = \frac{\int_{x_r^i}^{\infty} e^{-\frac{z^2}{2}} dx \int_{x_n^M}^{\infty} e^{-\frac{z^2}{2}} dx - \int_{x_n^i}^{\infty} e^{-\frac{z^2}{2}} dx \int_{x_r^M}^{\infty} e^{-\frac{z^2}{2}} dx}{\int_{x_n^M}^{\infty} e^{-\frac{z^2}{2}} dx \int_{x_r^M}^{\infty} e^{-\frac{z^2}{2}} dx} \quad (A21)$$

Thus,  $\rho_r - \rho_n > 0$  if:

$$\int_{x_r^i}^{\infty} e^{-\frac{z^2}{2}} dx \int_{x_n^M}^{\infty} e^{-\frac{z^2}{2}} dx \geq \int_{x_n^i}^{\infty} e^{-\frac{z^2}{2}} dx \int_{x_r^M}^{\infty} e^{-\frac{z^2}{2}} dx \quad (A22)$$

Since  $\mu^M \geq \mu^i$ , we have  $x_n^M \leq x_n^i$ . Hence the second integral on the left-hand side is greater than the second integral on the right-hand side. Further, since  $\max\{x_r^M, x_r^i, x_n^M, x_n^i\} < 0$ ,  $x_r^M \leq x_r^i$  and  $x_r^i - x_n^i = x_r^M - x_n^M$ , we see that the first integral on the left-hand side is also greater than the first integral on the right-hand side. From these two results it is clear that the above inequality holds. Thus we have shown that for all  $R \geq \hat{R}$ ,  $\rho_r \geq \rho_n$ . If successful projects are those for which we have  $R \geq \hat{R}$ , then this proves the lemma. Q.E.D.

*Proof of Proposition 8:* To prove the proposition, it is sufficient to show that  $\frac{\partial \rho_r}{\partial \mu^i} < \frac{\partial \rho_n}{\partial \mu^i}$ . This is because, from Lemma 1 and the discussion in Section III C we know that  $\frac{\partial \mu^i}{\partial N} > 0$ , and it is easy to show that  $\frac{\partial \rho_r}{\partial \mu^i} > 0$  and  $\frac{\partial \rho_n}{\partial \mu^i} > 0$ . Differentiating  $\rho_r$  and  $\rho_n$  from their expressions in (A20) with respect to  $\mu^i$  using Leibnitz's rule, we need to show:

$$\frac{\int_{x_r^M}^{\infty} e^{-\frac{z^2}{2}} dx [-e^{-\frac{(x_r^i)^2}{2}} \frac{\partial x_r^i}{\partial \mu^i}] - \int_{x_r^i}^{\infty} e^{-\frac{z^2}{2}} dx [-e^{-\frac{(x_r^M)^2}{2}} \frac{\partial x_r^M}{\partial \mu^i}]}{[\int_{x_r^M}^{\infty} e^{-\frac{z^2}{2}} dx]^2} \leq \frac{\int_{x_n^M}^{\infty} e^{-\frac{z^2}{2}} dx [-e^{-\frac{(x_n^i)^2}{2}} \frac{\partial x_n^i}{\partial \mu^i}] - \int_{x_n^i}^{\infty} e^{-\frac{z^2}{2}} dx [-e^{-\frac{(x_n^M)^2}{2}} \frac{\partial x_n^M}{\partial \mu^i}]}{[\int_{x_n^M}^{\infty} e^{-\frac{z^2}{2}} dx]^2} \quad (\text{A23})$$

Before we prove (A23), we first show that

$$-e^{-\frac{(x_r^i)^2}{2}} \frac{\partial x_r^i}{\partial \mu^i} + e^{-\frac{(x_r^M)^2}{2}} \frac{\partial x_r^M}{\partial \mu^i} \leq -e^{-\frac{(x_n^i)^2}{2}} \frac{\partial x_n^i}{\partial \mu^i} + e^{-\frac{(x_n^M)^2}{2}} \frac{\partial x_n^M}{\partial \mu^i} \quad (\text{A24})$$

Differentiating the expressions for  $x_r^i$ ,  $x_r^M$ ,  $x_n^i$  and  $x_n^M$  with respect to  $\mu_i$  and substituting for  $\frac{\partial x_r^i}{\partial \mu^i}$ ,  $\frac{\partial x_r^M}{\partial \mu^i}$ ,  $\frac{\partial x_n^i}{\partial \mu^i}$  and  $\frac{\partial x_n^M}{\partial \mu^i}$ , we have:

$$e^{-\frac{(x_r^i)^2}{2}} [\frac{s^2}{\sigma^2} + \frac{s^2}{s^2 + \sigma^2}] - e^{-\frac{(x_r^M)^2}{2}} [\frac{s^2}{s^2 + \sigma^2}] \leq e^{-\frac{(x_n^i)^2}{2}} [\frac{s^2}{\sigma^2} + 1] - e^{-\frac{(x_n^M)^2}{2}} \quad (\text{A25})$$

When  $R \geq \hat{R}$ , we know that  $x_r^i \leq x_n^i$  and  $x_r^M \leq x_n^M$ . Furthermore, we can show that  $x_r^i - x_r^M = x_n^i - x_n^M$ . From these results and since  $e^{-\frac{z^2}{2}}$  is a convex function, we can show that:

$$e^{-\frac{(x_r^i)^2}{2}} - e^{-\frac{(x_r^M)^2}{2}} \leq e^{-\frac{(x_n^i)^2}{2}} - e^{-\frac{(x_n^M)^2}{2}} \quad \text{or} \quad e^{-\frac{(x_r^M)^2}{2}} \geq e^{-\frac{(x_r^i)^2}{2}} - [e^{-\frac{(x_n^i)^2}{2}} - e^{-\frac{(x_n^M)^2}{2}}] \quad (\text{A26})$$

To prove (A25), it is sufficient to show that the following is true:

$$e^{-\frac{(x_r^i)^2}{2}} [\frac{s^2}{\sigma^2} + \frac{s^2}{s^2 + \sigma^2}] - [e^{-\frac{(x_r^i)^2}{2}} - [e^{-\frac{(x_n^i)^2}{2}} - e^{-\frac{(x_n^M)^2}{2}}]] [\frac{s^2}{s^2 + \sigma^2}] \leq e^{-\frac{(x_n^i)^2}{2}} [\frac{s^2}{\sigma^2} + 1] - e^{-\frac{(x_n^M)^2}{2}}$$

Simplifying  $e^{-\frac{(x_r^i)^2}{2}} \frac{s^2}{\sigma^2} \leq e^{-\frac{(x_n^i)^2}{2}} \frac{s^2}{\sigma^2} + [e^{-\frac{(x_n^i)^2}{2}} - e^{-\frac{(x_n^M)^2}{2}}] \frac{\sigma^2}{s^2 + \sigma^2}$

Since  $x_n^i \geq x_r^i$  and  $x_n^i \geq x_n^M$ , the above inequality is true. This proves (A25). We can rewrite (A25)

as:

$$e^{\frac{-(x_r^M)^2}{2}} \left[ \frac{s^2}{[s^2 + \sigma^2]} \right] \leq e^{\frac{-(x_r^i)^2}{2}} \left[ \frac{s^2}{\sigma^2} + \frac{s^2}{s^2 + \sigma^2} \right] - \left[ e^{\frac{-(x_n^i)^2}{2}} \left[ \frac{s^2}{\sigma^2} + 1 \right] - e^{\frac{-(x_n^M)^2}{2}} \right] \quad (\text{A27})$$

To prove (A23) it is sufficient to show that it is true when we substitute for  $e^{\frac{-(x_r^M)^2}{2}} \left[ \frac{s^2}{[s^2 + \sigma^2]} \right]$  with the left-hand side of (A27). Substituting for  $e^{\frac{-(x_r^M)^2}{2}} \left[ \frac{s^2}{[s^2 + \sigma^2]} \right]$  in (A23) and simplifying we have to show

$$\begin{aligned} & \frac{e^{\frac{-(x_r^i)^2}{2}} \left\{ \frac{s^2}{\sigma^2} + \frac{s^2}{s^2 + \sigma^2} \right\} \left[ \int_{x_r^M}^{\infty} e^{\frac{-z^2}{2}} dx - \int_{x_r^i}^{\infty} e^{\frac{-z^2}{2}} dx \right] + K \int_{x_r^i}^{\infty} e^{\frac{-z^2}{2}} dx}{\left[ \int_{x_r^M}^{\infty} e^{\frac{-z^2}{2}} dx \right]^2} \\ & - \frac{e^{\frac{-(x_n^i)^2}{2}} \left[ \frac{s^2}{\sigma^2} + 1 \right] \left[ \int_{x_n^M}^{\infty} e^{\frac{-z^2}{2}} dx - \int_{x_n^i}^{\infty} e^{\frac{-z^2}{2}} dx \right] + K \int_{x_n^i}^{\infty} e^{\frac{-z^2}{2}} dx}{\left[ \int_{x_n^M}^{\infty} e^{\frac{-z^2}{2}} dx \right]^2} \leq 0 \end{aligned}$$

where  $K \equiv e^{\frac{-(x_n^i)^2}{2}} \left[ \frac{s^2}{\sigma^2} + 1 \right] - e^{\frac{-(x_n^M)^2}{2}}$ . Given that  $x_n^M \geq x_r^M$  and  $x_n^i \geq x_r^i$  and  $\rho_r \geq \rho_n$ , the above inequality holds. Q.E.D.

## REFERENCES

- Abel, Andrew and George Mailath, 1994, Financing losers in competitive markets, *Journal of Financial Intermediation* 3-2, 139-165.
- Acemoglu, Daron, Victor Chernozhukov, and Muhamet Yildiz, 2006, Learning and disagreement in an uncertain world, NBER Working Paper No. W12648.
- Admati, Anat, Paul Pfleiderer, and Josef Zechner, 1994, Large shareholder activism, risk-sharing, and financial market equilibrium, *Journal of Political Economy* 102, 1097-1130.
- Aghion, Phillipe, and Patrick Bolton, 1992, An incomplete contracts approach to financial contracting, *Review of Economic Studies* 59, 473-494.
- Allen, Franklin, and Douglas Gale, 1999, Diversity of opinion and financing of new technologies, *Journal of Financial Intermediation* 8, 68-89.
- Bebchuk, Lucian A., and Christine Jolls, 1999, Managerial value diversion and shareholder wealth, *Journal of Law Economics and Organization*, 15, 487-502.
- Bennedsen, Morten, and Daniel Wolfenzon, 2000, The balance of power in closely held corporations, *Journal of Financial Economics* 58, 113-140.
- Bernardo, Antonio, and Ivo Welch, 2001, On the evolution of overconfidence and entrepreneurs, *Journal of Economics and Management Strategy* 10, 301-330.
- Bhattacharya, Sudipto, and Giovanna Nicodano, 2001, Insider trading, investment and liquidity: A welfare analysis, *Journal of Finance* 56, 1141-1156.
- Bhide, Amar, 1993, The hidden costs of stock market liquidity, *Journal of Financial Economics* 34, 31-51.
- Black, Bernard S., and Ronald J. Gilson, 1998, Venture capital and the structure of capital markets: Banks versus stock markets, *Journal of Financial Economics* 47, 243-277.
- Black, Fisher, 1976, Studies of stock price volatility changes, *Proceedings of the 1976 meetings of the American Statistical Association, Business and Economics Statistics Section*, 177-181.

- Bolton, Patrick, Jose Scheinkman, and Wei Xiong, 2006, Executive compensation and short-termist behavior in speculative markets, *Review of Economic Studies* 73, 577-610.
- Bolton, Patrick, and Ernst-Ludwig von Thadden, 1998, Blocks, liquidity and corporate control, *Journal of Finance* 53, 1-25.
- Boot, Arnoud, W. A., Radhakrishnan Gopalan, and Anjan V. Thakor, 2006, The entrepreneur's choice between private and public ownership, *Journal of Finance* 61, 803-837.
- Brau, James C., and Stanley E. Fawcett, 2004, Initial public offerings: An analysis of theory and practice, *Journal of Finance* 61, 399-436.
- Burkart, Mike, Denis Gromb, and Fausto Panunzi, 1997, Large shareholders, monitoring, and the value of the firm, *Quarterly Journal of Economics* 112, 693-728.
- Chemmanur, Thomas J., and Paolo Fulghieri, 1999, A theory of the going-public decision, *Review of Financial Studies* 12, 249-279.
- Coffee, John C., 1991, Liquidity versus control: The institutional investor as corporate monitor, *Columbia Law Review* 91, 1277-1368.
- Coval, Joshua and Anjan V. Thakor, 2005, Financial intermediation as a beliefs-bridge between optimists and pessimists, *Journal of Financial Economics* 75, 535-570.
- Daniel, Kent, David Hirshleifer, and Avanidhar Subramaniam, 1998, Investor psychology and security market under and overreactions, *Journal of Finance* 53, 1839-1885.
- Degroot, Morris L., 1970, *Optimal Statistical Decisions* (McGraw Hill.)
- Detragiache, Enrica, 1994, Private versus public borrowing: A theory with implications for bankruptcy reform, *Journal of Financial Intermediation* 3, 327-354.
- Deutsch, Claudia, The higher price of staying public, *The New York Times* January 23, 2005.
- Dittmar, Amy, and Anjan V. Thakor, 2007, Why do firms issue equity?, *Journal of Finance* 62, 1-54.
- Doidge, Craige, Andrew Karolyi, and Rene Stultz, 2007, Has New York become less competitive in global markets? Evaluating foreign listing choices over time, NBER Working Paper No. 13079.

- Faure-Grimaud, Antoine, and Denis Gromb, 2004, Public trading and private incentives, *Review of Financial Studies* 17, 985-1014.
- Fink, Jason, Kristin Fink, Gustavo Grullon, and James P. Weston, 2004, IPO vintage and the rise of idiosyncratic risk, Rice University Working Paper.
- Galton, Francis F.R.S., and Karl Pearson, 1902, The most suitable proportion between the value of first and second prices, *Biometrika* 1, 385-399.
- Garmaise, Mark J., 2001, Rational beliefs and security design, *Review of Financial Studies* 14, 1183-1213.
- Gibbons, Jean Dickinson, 1971, *Nonparametric Statistical Inference*, McGraw-Hill Series in Probability and Statistics, McGraw Hill.
- Gompers, Paul A., Joy L. Ishii, and Andrew Metrick, 2003, Corporate governance and equity prices, *Quarterly Journal of Economics* 118, 107-155.
- Grossman, Sanford, and Oliver D. Hart, 1980, Take-over bids, the free rider problem and the theory of the corporation, *Bell Journal of Economics and Management Science* 11, 42-64.
- Halonen, Maija, 2002, Reputation and the allocation of ownership, *Economic Journal* 112, 539-558.
- Hill, Patricia, Red tape drives firms overseas, *The Washington Times*, March 12, 2007.
- Hong, Harrison, Terence Lim, and Jeremy C. Stein, 2000, Bad news travels slowly: size, analyst coverage and the profitability of momentum strategies, *Journal of Finance* 55, 265-296.
- Jensen, Michael, 1986, Agency costs of free cash flows, corporate finance and takeovers, *American Economic Review* 76, 323-329.
- Kahn, Charles, and Andrew Winton, 1998, Ownership structure, speculation and shareholder intervention, *Journal of Finance* 53, 99-129.
- Kaplan, Steven N., and Per Stromberg, 2003, Financial contracting theory meets the real world: An empirical analysis of venture capital contracts, *Review of Economic Studies* 70, 281-315.
- Kreps, David M., 1990, *A course in microeconomic theory*, (F.T. Prentice Hall).

- Kurz, Mordecai, 1994a, On rational belief equilibria, *Economic Theory* 4, 859-876.
- Kurz, Mordecai, 1994b, On the structure and diversity of rational beliefs, *Economic Theory* 4, 877-900.
- Maksimovic, Vojislav, and Pegaret Pichler, 2001, Technological innovation and IPOs, *Review of Financial Studies* 14, 459-494.
- Manove, Michael, and Jorge A. Padilla, 1999, Banking (conservatively) with optimists, *RAND Journal of Economics* 30, 324-350.
- Maug, Ernst, 1998, Large shareholders as monitors: Is there a trade-off between liquidity and control?, *Journal of Finance* 53, 65-84.
- Morris, Stephen E., 1995, The common prior assumption in economic theory, *Economics and Philosophy* 11, 227-253.
- Pagano, Marco, 1993, The floatation of companies on the stock market: A co-ordination failure model, *European Economic Review* 37, 1101-1125.
- Pagano, Marco, Fabio Panetta, and Luigi Zingales, 1998, Why do companies go public ? An empirical analysis, *Journal of Finance* 53, 27-64.
- Pagano, Marco, and Ailsa Roell, 1998, The choice of stock ownership structure: Agency costs, monitoring and the decision to go public, *Quarterly Journal of Economics* 113, 187-225.
- Ritter, Jay, and Ivo Welch, 2002, A review of IPO activity, pricing and allocations, *The Journal of Finance* 57, 1795-1828.
- Shah, Salman, and Anjan V. Thakor, 1988, Private versus public ownership: Investment, ownership distribution, and optimality, *Journal of Finance* 43, 41-59.
- Smith, Abbie J., 1990, Corporate ownership structure and performance: The case of management buyouts, *Journal of Financial Economics* 27, 143-164.
- Song, Fenghua and Anjan V. Thakor, 2007, Relationship banking, fragility and the asset-liability matching problem, *Review of Financial Studies* forthcoming.
- Stoll, Hans R., 2000, Friction, *Journal of Finance* 55, 1479-1514.

Thorton, Emily, 2004, A little privacy please. More small outfits are deciding that being a public company isn't worth the hassle, *Business Week*, May 2.

Tirole, Jean, 2006, *The theory of corporate finance*, (Princeton University Press).

Van den Steen, Eric, 2004, Rational overoptimism (and other biases), *American Economic Review* 94, 1141-1151.

Van den Steen, Eric, 2006, Disagreement and the allocation of control, *MIT Sloan Working Paper # 4610-06*, June.

Zingales, Luigi, 1995, Insider ownership and the decision to go public, *Review of Economic Studies* 62, 425-448.

Zingales, Luigi, 2007, Is the U.S capital market losing its competitive edge?, University of Chicago Working Paper.

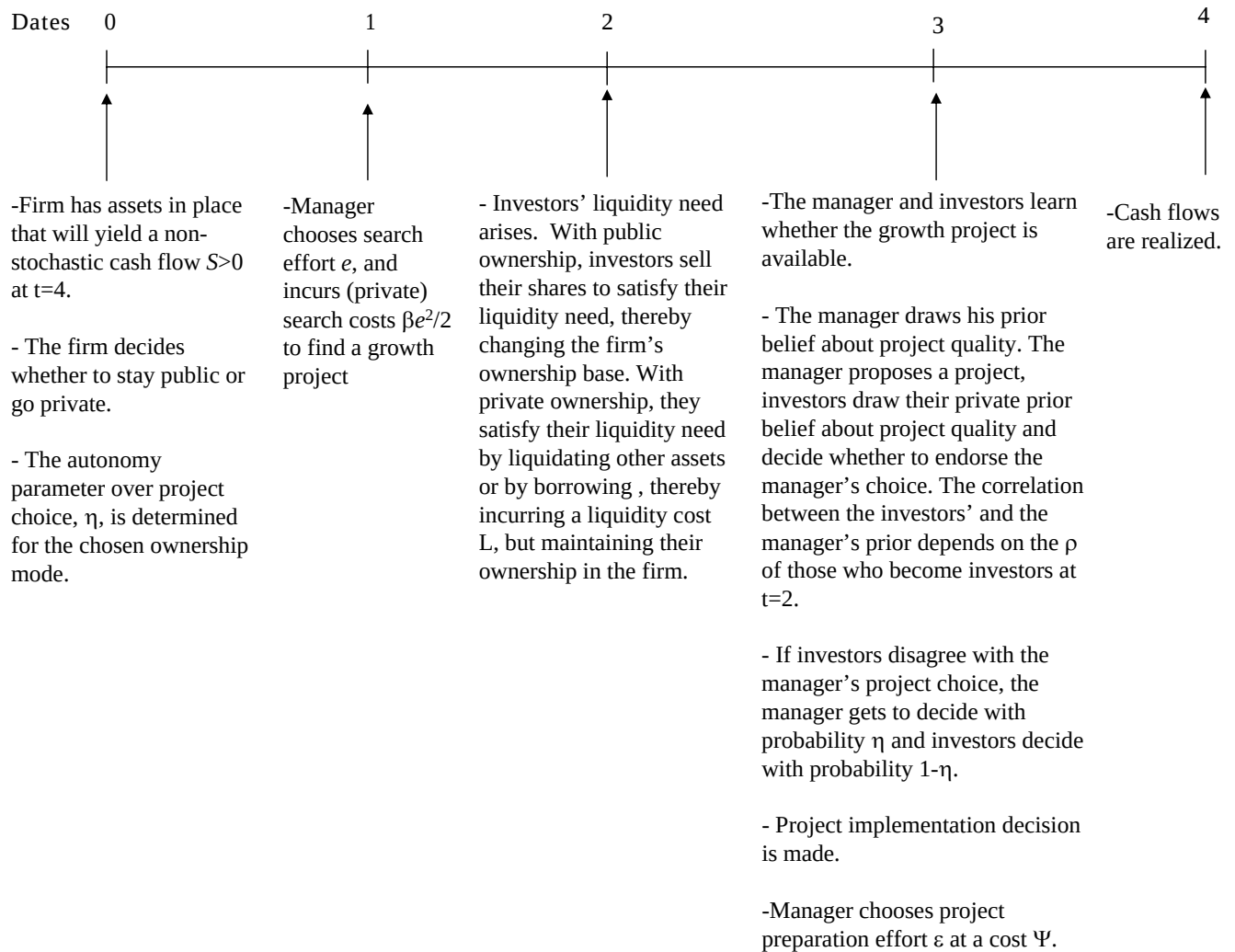


Figure 1: SEQUENCE OF EVENTS

## About ECGI

ECGI is an international non-profit membership organisation that provides a platform for debate and dialogue among academics, policymakers, and business leaders, with a focus on major corporate governance, ESG, and stewardship issues.

The views expressed in this working paper are those of the authors, not those of the ECGI or its members.

**[www.ecgi.global](http://www.ecgi.global)**

## ECGI Working Paper Series in Finance

### Editorial Board

#### Editor

**Nadya Malenko**, Professor of Finance, Boston College

#### Consulting Editors

**Renée Adams**, Professor of Finance, University of Oxford

**Franklin Allen**, Professor of Finance and Economics, Imperial London

**Julian Franks**, Professor of Finance, London Business School

**Mireia Giné**, Professor of Finance, IESE Business School

**Marco Pagano**, Professor of Economics, Facoltà di Economia Università di Napoli  
Federico II

#### Series Manager

**Asif Malik**, ECGI Working Paper Series Manager

<https://www.ecgi.global/publications/working-papers>