Complex Organizations, Leverage, and Interest Rates

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September 2026

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Complex Organizations, Leverage, and Interest Rates⋆

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Disclosure Statement

Giovanna Nicodano. In accordance with the AFA Conflict of Interest Disclosure Policy, I declare the following:

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(2) *[DA CONFERMARE]* I have not received, in the past three years, financial support of $10,000 or more (consulting fees, retainers, grants, research support, equity, or in-kind support) from any interested party with a stake related to this article.

(3) I hold no paid or unpaid position as an officer, director, advisor, or board member of any organization whose policy positions, goals, or financial interests relate to this article.

(4) The disclosures above also apply to my close relatives and partner.

(5) No party had the right to review this paper prior to its circulation, including supervisors or employers.

(6) No support received by me is subject to any non-disclosure obligation.

(7) Not applicable: this article does not involve the collection of data on human subjects.

Luca Regis. In accordance with the AFA Conflict of Interest Disclosure Policy, I declare the following:

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1. Introduction

Trade-off models predict that lower interest rates reduce optimal leverage because they compress the tax advantage of debt without a comparable reduction in associated costs (Fischer et al., 1989; Goldstein et al., 2001; Ju and Ou-Yang, 2006). Yet borrowing by highly leveraged sponsored entities often expands when financing conditions ease, despite their sensitivity to tax and distress considerations.[2]

This paper reconciles this tension. We study a trade-off model in which a sponsor chooses leverage across entities and provides contingent support to a backed unit at the time of debt repayment.[3] The anticipated sponsor support relaxes the backed unit's future cash-flow constraint, allowing leverage to be reallocated within the organization. Lower interest rates reduce sponsor leverage incentives, but also increase the value of preserving financial flexibility to support the backed unit. As a result, leverage may shift toward backed entities as rates fall.

Leverage responds heterogeneously not only within but also across organizations. We show that stand-alone firms (modeled as in Leland, 2007) optimally delever when interest rates fall, while backed units may increase borrowing at increasing spreads, default probabilities, and losses given default. This occurs when rates become sufficiently low, so that the sponsor may optimally choose zero leverage in order to maximize support capacity. These results imply that organizational form shapes the transmission of monetary conditions to leverage and credit risk. In complex organizations, lower rates do not uniformly affect anticipated cash-flow constraints. Instead, they induce a tax-driven internal reallocation of leverage toward supported entities.

The interaction between tax shields and contingent support within the organization is key. Lower interest rates reduce the tax advantage of debt for both stand-alone firms and sponsors, weakening their incentives to borrow. In contrast, sponsor contingent support alters the backed unit's tax–default trade-off for a given debt level by reducing both expected default costs and lenders' recovery, thereby increasing spreads and tax shield. As a result, leverage can shift toward the backed unit. The standard trade-off prediction of deleveraging may therefore reverse within complex organizations, potentially increasing total organizational debt as interest rates fall.

These results matter for policy reasons, as well. The high leverage of complex organizations, in

2. See Gompers et al. (2016), Jiangli et al. (2007), Hanlon and Heitzman (2022) and Brok (2024) for evidence that complex entities respond to tax incentives, and Axelson et al. (2013), Aramonte and Avalos (2019) for examples where leverage increases when aggregate credit conditions ease.

3. Support arrangements are prevalent in private equity (Bernstein et al., 2019; Hotchkiss et al., 2021; Haque et al., 2023), in groups (Bodie and Merton, 1992; Beaver et al., 2019; Shi et al., 2023), securitization (Gorton and Souleles, 2006), banks (Segura and Zeng, 2020) and shadow banks (Allen et al., 2023).

low-interest-rate environments, raises concerns about their contribution to financial instability when interest rates eventually return to higher levels.[4] Our model implies that default costs peak when interest rates fall to sufficiently low levels, rather than when they subsequently rebound. Moreover, heterogeneity across organizational forms may dampen aggregate default risk, as stand-alone firms delever.

We illustrate the model using two benchmark parameterizations based on Leland (2007) and Nicodano and Regis (2019). In the first, resembling securitization structures, the sponsor delevers as interest rates fall while the backed unit expands borrowing. In the second, resembling an LBO structure, the sponsor is optimally zero-leverage and the backed unit carries substantially more debt than an equivalent stand-alone firm. Across both cases, lower interest rates reallocate leverage toward backed units and increase their default risk. These effects are amplified when sponsor and backed-unit cash flows are more correlated, increasing the value of contingent support. Importantly, borrowing expands despite higher lenders' losses given default and therefore higher spreads. The mechanism is therefore not cheaper financing, but the preservation of tax benefits on risky debt within the organization.

Our paper contributes to the trade-off theory of capital structure. Seminal models characterize firms' optimal leverage as the outcome of a trade-off between tax benefits and distress costs, with lower interest rates generally reducing optimal debt by compressing the tax advantage of leverage (Leland and Toft, 1996; Goldstein et al., 2001; Ju and Ou-Yang, 2006). We show that this comparative static need not hold once financing decisions are jointly determined within a multi-entity organization.[5]

The paper also connects to the literature on contingent support arrangements. Contingent guarantees can expand borrowing capacity by reallocating resources across affiliated entities and by reducing expected distress costs in supported units (Bodie and Merton, 1992; Luciano and Nicodano, 2014; Segura and Zeng, 2020) while unconditional support in conglomerates may reduce value (Leland, 2007). Previous work studies the valuation and incentive effects of such arrangements, but does not analyze how they modify the response of optimal leverage to changes in interest rates. We show that contingent support can overturn the standard trade-off prediction that lower interest rates induce deleveraging.

4. See Andersson et al. (2025), Aramonte and Avalos (2021), European Systemic Risk Board (2024), for private equity; European Securities and Markets Authority (2019), Powell (2019), and Rosengren (2019), for securitization.

5. Abel (2018) focuses instead on profitability, showing that it can reduce optimal leverage. While consistent with the data, this is the opposite relationship between profitability and debt in most trade-off settings.

The paper also relates to the literature on internal capital markets and organizational design. Existing work studies resource allocation across conglomerate divisions and business-group affiliates (Stein, 1997; Rajan et al., 2000; Inderst and Müller, 2003; Cestone and Fumagalli, 2005; Almeida and Wolfenzon, 2006; Shi et al., 2023), but largely abstracts from capital structure choices and their sensitivity to interest rates. Related research on private equity, structured finance, and sponsor-backed firms features layered financial structures and contingent support arrangements (Axelson et al., 2009; DeMarzo, 2005; Nicodano and Regis, 2019), but does not analyze how the organizational structure modifies the response of optimal leverage to changes in interest rates.

More broadly, the paper contributes to the literature on monetary policy transmission and corporate risk-taking. Existing mechanisms emphasize either improved debt-service capacity when rates fall due to cheaper borrowing (Greenwald, 2019; Lian and Ma, 2021) or increased risk-taking induced by search-for-yield incentives (Martinez-Miera and Repullo, 2017; Becker and Ivashina, 2015). Our mechanism is different. In our setting, lower interest rates can increase borrowing even though lenders correctly price risk and spreads rise. The response arises from the interaction between taxes and contingent support within complex organizations, which preserve the value of risky debt despite lower interest rates.

In summary, we show that organizational structure alters how leverage responds to changes in interest rates, generating comparative statics absent from standard trade-off models. The results therefore highlight a corporate-governance dimension of monetary conditions in multi-entity firms.

The paper unfolds as follows. Section 2 studies both the stand-alone and the complex organization's leverage choice. In Sections 3 and 4, via numerical analyses, we provide insights on leverage, default probability and lenders' losses-upon-default adjustments following a change in interest rates. While Section 3 studies the case of a zero-leverage sponsor, Section 4 shifts attention to a leveraged sponsor. Section 5 concludes. All proofs are gathered in the Appendix.

2. The Model

This section introduces the model in two steps. We first characterize the benchmark stand-alone firm, highlighting the standard trade-off between tax benefits and default costs. We then turn to a complex organization with a sponsor providing contingent support to a backed unit. We show how this organizational structure alters the response of optimal leverage to interest rates.

At time 0, a controlling entity owns two units, $i = S, B$. Each unit has a random exogenous operating cash flow, X_i, which is realized at time T. The symbol $G(\cdot)$ denotes the cumulative distribution function and $f(\cdot)$ the density of X_i at time T, which we assume to be identical for the two units; $g(\cdot,\cdot)$ is the joint distribution of X_S and X_B and ρ their correlation coefficient. At time 0, the controlling entity issues a zero-coupon debt with maturity T. [6]

The face value F_i of the debt issued by each unit i maximizes the total arbitrage-free value (ν_{SB}) of equity, E_i, and debt, D_i:

$$\nu_{SB} = \max_{F_S,F_B} \sum_{i=S,B} (E_i + D_i)\,. \tag{1}$$

Each unit pays a flat proportional income tax at an effective rate $0 < \tau_i < 1$ and loses a proportion, $0 < \alpha_i < 1$, of its cash flows in default.

As with coupon debt, interest on debt is entirely deductible from taxable income. Tax authorities indeed treat the accretion of a zero-coupon bond's value as tax-deductible imputed interest. This tax advantage of debt generates a trade-off. On the one hand, increasing leverage generates tax savings, while on the other it increases expected default costs because – everything else equal – higher leverage increases default likelihood.

At time T cash flows are realized and distributed in the following order. First, corporate income taxes are paid. Then, the debt obligations are fulfilled, if possible. When a unit cannot meet its debt obligations, its income, net of both taxes and default costs, is distributed to the lenders. Shareholders will receive net residual cash flows, if any, after debt is fully reimbursed.

The expressions for the values of equity and debt, E_i and D_i, are in the Appendix.

Because cash flows are exogenous and markets are frictionless except for taxes and default costs, maximizing firm value is equivalent to minimizing the sum of expected tax burden (T_i) and expected default costs (C_i):

$$\nu_{SB} = \min_{F_S,F_B} \sum_{i=S,B} T_i + C_i. \tag{2}$$

The expected tax burden of each unit is proportional to its expected operational cash flow X_i, net of the tax shield X_i^Z, which captures the interest deductibility benefit of debt. Since debt is modeled as a zero-coupon bond, interests – and hence the tax shield – are equal to the difference between the face value of debt, F_i, and its market value D_i: $X_i^Z = F_i - D_i$.

Default costs are proportional to operating cash flows.

Below, we model two different ways of owning the two units. In the first case, they are separately

6. The choice of zero-coupon debt follows Leland (2007) for analytical convenience, since the tax shield equals the difference between the face value and the market value of debt. Importantly, most tax codes allow to deduct yearly changes in this difference exactly as coupon interest would be, even though no cash interest is paid until maturity.

owned and they constitute two stand-alone entities. In the other case, units are connected through a conditional support guarantee. In such a complex organization leverage choices are interdependent, since borrowing in one unit affects the marginal value of borrowing in the other.

2.1. The Stand-Alone Units

We begin with the case of unconnected, stand-alone units, which isolates the standard tax–default trade-off and serves as a benchmark for the effects of organizational structure. By risk neutrality, the value of the tax burden in each stand-alone (SA) unit is equal to:

$$T_i^{SA}(F_i) = \tau_i \phi \mathbb{E}[(X_i - X_i^Z)^+], \tag{3}$$

where $\phi = \frac{1}{(1+r)^T}$ is the discount factor for the time-T horizon at which the cash flows are realized. The superscripts and subscripts, i, indicate whether the stand-alone unit is endowed with the sponsor ($i = S$) or backed unit ($i = B$) parameters.

Each stand-alone unit defaults at T when its realized net cash flow is less than the face value of debt. In other words, default occurs when cash flows are less than a default threshold, X_i^d, defined as the minimum cash flow required to fully repay debt: $X_i^d = F_i + \frac{\tau_i}{1-\tau_i} D_i$.[7] Expected default costs, that are a dead-weight loss, are equal to:

$$C_i^{SA}(F_i) = \alpha_i \phi \mathbb{E}\left[X_i 1_{\{0 < X_i < X_i^d\}}\right]. \tag{4}$$

They are proportional to the default cost parameter, α_i, and increase in realized cash flows, when the unit goes bankrupt. A rise in the nominal value of debt, F_i, increases the default threshold, X_i^d, thereby increasing the expected default costs.

When units are owned separately, the value of the objective function (2) is simply the sum of the values of the taxes and default costs in each unconnected unit. Notice that the optimal value of each unit is equal to:

$$V_i(F_i^*(\phi); \phi) = V_i(0; \phi) + TS_i(F_i^*(\phi); \phi) - C_i(F_i^*(\phi); \phi), \tag{5}$$

where $F_i^*(\phi)$ is the optimal face value of debt in unit i, $V_i(0; \phi)$ is the value of a zero-leverage unit, and $TS_i(F_i; \phi) = T_i^{SA}(0; \phi) - T_i^{SA}(F_i; \phi)$ is the present value of the tax savings from leverage, equal to the difference between the taxes paid by a zero-leverage unit and a unit that issues debt

7. Indeed, the firm defaults at time T whenever the realized cash flow net of taxes is insufficient to repay the principal, i.e. when $(1 - \tau_i)X_i + \tau_i X_i^Z < F_i$. Using $X_i^Z = F_i - D_i$ we get the expression in the text.

F_i. Equation (5) highlights that, among other parameters, the optimal solution depends on the discount factor ϕ, and thus on the risk-free rate.
It is worth recalling some properties of the components of the stand-alone unit's value. The tax shield is a convex function of F_i. In fact, increasing the face value of debt increases the tax shield, thereby reducing the tax burden. This occurs because the market value of debt, D_i, increases with F_i at a decreasing rate (due to higher risk). The default threshold, X_i^d, is instead concave in the face value of debt, F_i. Luciano and Nicodano (2014) prove that a stand-alone unit has positive optimal debt if the sum of tax burden and default costs is convex in the face value of debt. Nicodano and Regis (2019) further show that it raises positive debt even with a zero risk-free rate because the endogenous spread generates a positive tax shield.
We can now analyze the optimal response to a reduction in the risk-free rate that increases the discount factor, ϕ. [8] Since the solution of problem (1) is not available in closed-form, even in the case of stand-alone units, we can not directly study the sensitivity of optimal debt to interest-rate changes. However, we obtain interesting results at fixed debt levels. We first prove a lemma on the deductible cost of debt, i.e. the yield to maturity $y_i = \left(\frac{F_i}{D_i}\right)^{\frac{1}{T}} - 1$. [9]

Lemma 1. *The yield to maturity, y_i, falls as the interest rate, r, declines holding fixed the face value of debt, F_i. As a consequence, the deductible cost of debt decreases when the interest rate falls.*

The lemma establishes that a decline in the risk-free rate increases the market value of debt, for fixed face value of debt, thereby reducing the yield to maturity. An implication is that the incentive to borrow for tax deductions falls with the risk-free rate. This is the signature feature of trade-off capital structure models.[10]

We now study how a decline in interest rates, hence an increase in the discount factor, affects the tax shield and default costs at a given level of debt. Two opposing forces are at play (see equations (3) and (4)). First, the higher discount factor directly reduces the expected discounted value of both tax deductions and default costs. Second, it indirectly affects both the tax shield,

8. To keep the cash-flow distribution fixed, we let the expected present value of cash flows vary with ϕ assuming valuation is performed under the risk-neutral measure.

9. See the Appendix for all the proofs.

10. It extends to other types of debt, such as perpetual coupon debt (Leland and Toft, 1996), dynamically refinanced debt (Fischer et al., 1989; Goldstein et al., 2001), stochastic-rate coupon debt with endogenous maturity (Ju and Ou-Yang, 2006), beyond the zero-coupon debt (Leland, 2007) which we analyze.

X_i^Z, and the default threshold, X_i^d, by changing the market value of debt, D_i. The proposition below summarizes these effects.

Proposition 1. *Assume the interest rate falls. Then, in a stand-alone company with fixed face value of debt F_i, (a) the tax shield falls and the no-default threshold increases; (b) if $\tau < 0.5$, the former effect dominates; (c) the expected values of both taxes and default costs increase.*

While results concerning taxes (in Part (a) and (c) of the proposition) are straightforward, because the tax shield falls together with the interest rate, the one concerning default costs is subtler. It stems from a reduction in net after-tax cash flows available to repay debt, due to the reduction in the tax shield. This indeed leads to an increase in the probability of default. Part (b) establishes that, for reasonable values of the tax rate ($\tau < \frac{1}{2}$), the tax shield decreases more than the no-default threshold.

We are now able to determine the value changes in both debt and equity due to the interest-rate reduction. First, a lower discount factor raises the value of both debt and equity. Second, the variations in both the tax shield and no-default thresholds captured by Proposition 1 increase the default probability, reducing both market values. The first effect prevails, as the next Proposition proves.

Proposition 2. *Assume the interest rate falls. Then, in a stand-alone company, the market value of the firm increases, holding fixed the face value of debt.*

As the interest rate falls, the equity value may fall for very high levels of debt face value. However, the increase in the debt value due to the interest-rate reduction prevails and, hence, the market value of a stand-alone company increases for any F_i.

We now turn to the analysis of the optimal level of debt for the stand-alone firm with the following proposition:

Proposition 3. *Assume convexity of the objective (2) and $\frac{\partial^2 D}{\partial F \partial \phi} > 0$, so that a decline in the interest rate raises the marginal contribution of leverage to the debt value. Then the optimal face value of debt in a stand-alone unit decreases as the interest rate falls if Condition (19) in the Appendix holds. This condition is satisfied for every τ when $\alpha \to 1$.*

Proposition 3 highlights that, even in the stand-alone case, a decline in interest rates does not mechanically reduce optimal leverage. Leverage decreases when, at the optimum, the increase in marginal default costs (MDC) outweighs the change in marginal tax savings (MTS). In normal circumstances, this is the relevant case. Lower interest rates compress the tax advantage of debt by reducing the tax-deductible cost of debt (see Lemma 1), while simultaneously increasing default risk since the lower tax shield shrinks cash flows available for the service of debt. As a result, MTS fall and MDC rise, leading firms to delever. The conditions in the theorem and the Appendix guarantee that this is the case.

These sufficient conditions may fail in extreme cases. First, marginal default costs can decrease if the firm is already highly leveraged, so that the default threshold lies in a sufficiently downward-sloping region of the cash-flow density ($|f'(X^d)| < 0$). Second, marginal tax benefits may increase if the decline in interest rates raises the sensitivity of debt value to face value sufficiently strongly — an effect that becomes relevant only when the tax shield is already large. Both cases require leverage levels well above those typically observed at the optimum.

In sum, the familiar implication of trade-off models holds unless the stand-alone unit is initially very highly levered. But for extreme parameter values, lower interest rates reduce marginal tax incentives while increasing marginal default costs, leading to a decline in optimal debt.

2.2. The Sponsor and its Backed Unit

We now consider two affiliated units linked by a contingent guarantee, following Luciano and Nicodano (2014). This sponsor–backed (SB) structure introduces an additional governance margin: the sponsor endogenously allocates leverage across units by trading off its own tax benefits against the borrowing capacity of the backed unit. The guarantee allows the sponsor to transfer resources to a distressed but viable backed unit whenever such a transfer prevents default. Formally, the sponsor covers the shortfall $F_B - X_B^n$, where X_i^n denotes cash flows net of taxes, provided its own net resources suffice to meet both its obligations and the transfer, i.e., ($X_S^n - F_S \geq F_B - X_B^n$). This implies that the sponsor retains limited liability with respect to the backed unit's debt outside these contingencies.

The value of the guarantee is given by the expected default costs saved in the backed unit thanks to the transfer. For given F_S and F_B, its value is

$$\Gamma(F_S, F_B) = \phi\alpha_B E\left[X_B 1_{\{0 \leq X_B \leq X_B^d,\, X_S \geq h(X_B)\}}\right], \tag{6}$$

where $h(\cdot)$ is defined in the Appendix. The Appendix also reports the values of equity and debt for both the sponsor and the backed unit. The guarantee directly affects the sponsor's equity value and the backed unit's debt value through the transfer of resources across units. The change in the backed unit's debt value, D_B, also has an indirect effect because both the tax shield, X_B^Z, and the default threshold, X_B^d, depend on it. For a given debt level F_S, the sponsor's taxes and default costs remain identical to those of a stand-alone firm. Those of the backed unit are instead:

$$T_B = \phi\tau_B E\left[(X_B - X_B^Z)1_{\{X_B \geq X_B^Z\}}\right],$$

$$C_B^{SB} = \phi\alpha_B E\left[X_B 1_{\{0 \leq X_B \leq X_B^d\}}\right] - \Gamma(F_S, F_B).$$

where X_B^Z and X_B^d differ from their stand-alone counterparts. The guarantee therefore modifies the backed unit's tax–default trade-off. Expected default costs fall because the guarantee covers some default states, while the tax burden increases because the higher market value of debt reduces the tax shield. The reduction in expected default costs dominates, making the guarantee value-enhancing for a given interest rate. [11]

The ex-post transfer from the sponsor's shareholders to the backed unit's lenders affects both ex-ante values and leverage choices. As a result, financing decisions become interdependent across units. Reducing sponsor leverage increases the likelihood that support can be provided, thereby relaxing the backed unit's borrowing constraint. This mechanism is illustrated in Figure 1. A reduction in sponsor debt has two effects. First, it increases the value of the backed unit's debt, shifting its tax-shield threshold leftward and its no-default threshold rightward, from X_B^d to $X_B^{d'}$. Second, it lowers the level of sponsor cash flows required to service the sponsor's own debt. The latter effect implies that, for every realization of the backed unit's cash flow below the default threshold X_B^d, the guarantee becomes effective in a larger set of states. It also becomes effective between X_B^d and $X_B^{d'}$ whenever sponsor cash flows are sufficiently high.

Although the higher default threshold slightly expands the set of states in which the backed unit defaults, the improvement in guarantee effectiveness more than compensates for this effect. It may therefore be optimal to forego the sponsor's tax shield in order to expand borrowing—and thus tax benefits—in the backed unit, when the two entities share the same tax and bankruptcy parameters (see Luciano and Nicodano, 2014). The effects of a drop in interest rates on the guarantee are illustrated in figure 2. A decline in the interest rate increases the market value of the backed unit's debt, shifting the tax-shield threshold leftward and the no-default threshold rightward. Since the

11. In contrast, unconditional guarantees embedded in conglomerate mergers can be value-reducing (see Leland, 2007). The conditional support can be also self-enforcing in a repeated setting Luciano and Nicodano (2014).

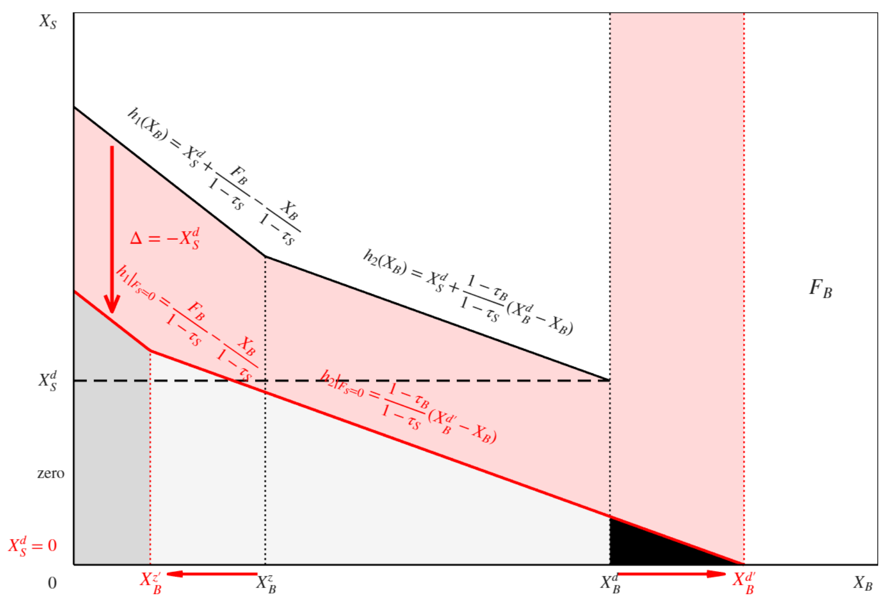

Figure 1: Reduction in sponsor leverage and guarantee's effectiveness. On the horizontal and the vertical axes are the backed unit's and sponsor's cash flows, respectively. The curves represent the sponsor's support schedules and the arrow marks the downward shift from eliminating sponsor debt. The guarantee is effective whenever sponsor cash flow lies above the relevant curve. Coloring shows how guarantee status changes: *white*: effective under both regimes; *red*: effective only once the sponsor is zero-leverage; *black*: the backed unit defaults only when the sponsor is zero-leverage due to the rightward shift in the no-default threshold. The red region dominates the black one, so lower sponsor leverage expands guarantee effectiveness.

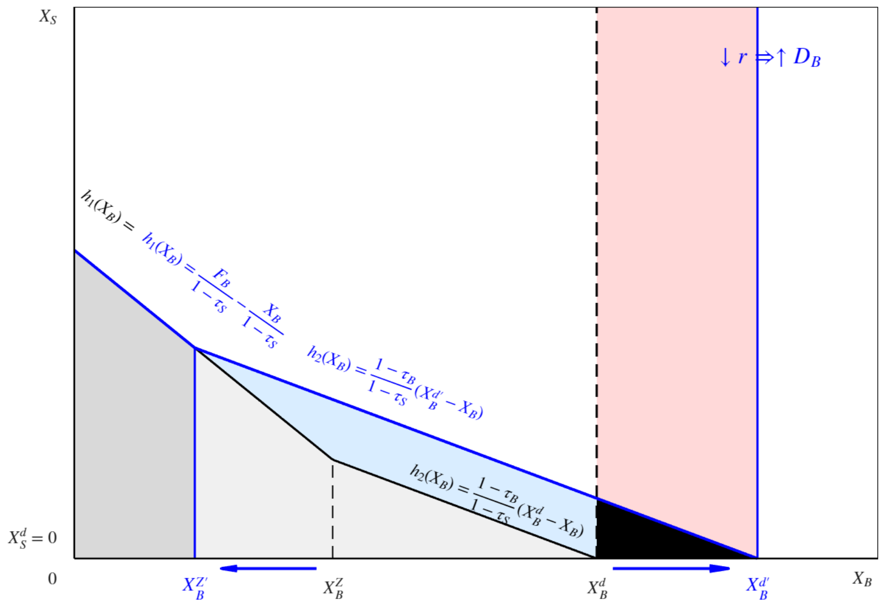

Figure 2: Reduction in the interest rate and the guarantee's effectiveness. On the horizontal and vertical axes are the backed unit's and sponsor's cash flows, respectively. The curves represent the sponsor's support schedules, before and after the interest-rate decline. The guarantee is effective whenever sponsor cash flow lies above the relevant curve. Coloring shows how guarantee status changes: *white*: effective both before and after the rate drop; *red*: effective only after the drop; *blue*: effective before the drop but not after; *black*: the backed unit defaults only after the rate drop, due to the rightward shift in the no-default threshold. A lower interest rate can both expand and contract guarantee effectiveness, depending on the size of the regions.

sponsor's cash flows have to cover a larger area – between $X_B^{Z'}$ and $X_B^{d'}$ – the guarantee becomes less effective in some states between these new thresholds. However, the guarantee operates in a new region where the gains from preventing default are particularly large.

Figure 3 shows that, following a decline in interest rates, reducing sponsor leverage enhances support capacity and may more than offset the reduction in guarantee effectiveness that arises in some regions after the interest-rate decline. Below, we show that a zero-leverage sponsor is optimal, for unrestricted parameter values, when interest rates are sufficiently low.

Proposition 4. *Assume a convex objective function in (2). Then, there exists a threshold interest-rate level $\bar{r}(\tau_S, \tau_B, \alpha_B, G, g)$ such that, for all $r \leq \bar{r}$, the optimal sponsor debt is zero, $F_S^* = 0$. Holding all other parameters fixed, $\bar{r}$ decreases with τ_S for $\tau_S < 0.5$.*

As in the stand-alone case, lower interest rates reduce the value of the sponsor's tax shield. In a multi-entity structure, however, they also increase the value of preserving financial flexibility

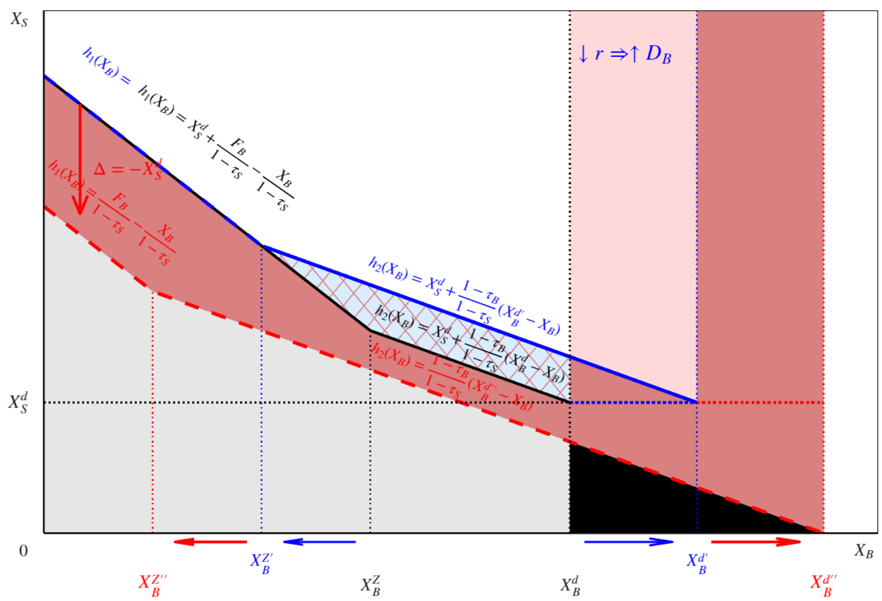

Figure 3: Combined effect of lower sponsor leverage and a lower interest rate on the guarantee's effectiveness. Axes and support schedules are as in Figures 1&2, now shown for a change in both sponsor leverage and in the interest rate. Coloring shows how guarantee status changes relative to the initial (levered sponsor, higher rate) case: *white*: always effective; *light red*: effective once the interest rate falls, holding sponsor debt fixed; *blue*: ineffective once the interest rate falls; *dark red*: effective once the sponsor becomes zero-leverage, on top of the rate decline; *black*: the backed unit defaults only under the combined shift, due to the rightward move in the no-default threshold.

to support the backed unit. When interest rates become sufficiently low, the value of preserving support capacity outweighs the tax benefit of sponsor borrowing, inducing the sponsor to specialize in providing support rather than issuing debt. As a result, leverage is reallocated toward the backed unit, as illustrated in the numerical analysis performed in the next section. This reallocation increases both the organization's tax shield and the backed unit's exposure to default risk. [12] At low interest rates, the conditional guarantee from the sponsor to the backed unit is also more likely to be self-enforcing, as shown in Luciano and Nicodano (2014). The threshold interest rate $\bar{r}$ is lower when the sponsor's own tax incentives are stronger. Numerical analysis further reveals that the threshold rate increases when borrowing incentives in the backed unit become stronger, i.e., when α_B is lower and τ_B is higher. Also, it increases with the probability that sponsor's cash flow is higher when the backed unit's default cost would be higher absent support, that is with cash-flow correlation ρ. In sum, while stand-alone firms adjust leverage through a standard tax–default trade-off, complex organizations introduce an additional margin through the internal allocation of debt. This margin is key to understanding how leverage responds to changes in interest rates.

We can also prove the following corollary to Proposition 4:

Corollary 1. *Let (F_S^*, F_B^*) denote the optimal debt levels of the complex organization. Assume:*

1. *a convex objective function in (2);*
2. *that, as stand-alone firms, both units satisfy the sufficient conditions of Proposition 3 and therefore optimally delever following a decline in interest rates.*
3. *the conditions of Proposition 4 hold, so that $F_S^* = 0$.*

Then, following a decline in interest rates, the backed unit's optimal debt can increase, i.e. $\frac{dF_B^*}{d\phi} > 0$*, even though the same unit would delever as a stand-alone firm.*

Proof. See the Appendix. ■

The corollary highlights the central distinction between stand-alone firms and backed units. A decline in interest rates weakens borrowing incentives in a stand-alone firm through the mechanism described in Proposition 3. By contrast, sponsor support introduces an additional margin: lower interest rates may increase the value of preserving support capacity and thereby relax the backed

12. An implication, which reinforces the results in Luciano and Nicodano (2014), is that lower interest rates increase the value of unilateral guarantees relative to mutual guarantees, i.e. where both units guarantee each other's debt, that would limit such deleveraging.

unit's effective borrowing constraint. Consequently, a backed unit may increase leverage even when the same entity would optimally delever in isolation.

This may raise the question of why a stand-alone would not purchase default protection from a third party instead of relying on a sponsor. Three unmodeled features distinguish the two arrangements. First, a market-purchased guarantee is a contractual claim, typically priced at an actuarially fair premium by the protection seller. That premium would include the value that instead accrues to the complex organization. Second, the sponsor's support is discretionary and protected by limited liability. It is honored when privately optimal and sustainable through repeated interaction (as in Luciano and Nicodano, 2014) or legal enforcement specific to affiliated entities, whereas a market hedge is a binding obligation regardless of both the seller's and interest-rate circumstances. Third, an external guarantor faces information frictions, absent between units in the complex organization, which a competitive market would price into the premium. Last, and most importantly, the model shows that much of the guarantee value stems from changes in the distribution of debt between sponsor and backed unit.

We further explore the differences between stand-alone units and complex organizations, as parameters vary, through the numerical analysis presented in the next section.

3. Optimal Leverage and Credit Risk Sensitivity to the Risk-Free Rate

In this section, we numerically analyze the changes in the optimal capital structure associated with variation in the risk-free rate. We also analyze the changes in the endogenous default probability, spread and loss given default. We compare stand-alone units to the connected units belonging to a complex organization, assuming they display the same parameters.

Table 1 displays the base-case calibration parameters, as in Leland (2007), which refer to a typical BBB company. We set the tax rate and the proportional bankruptcy costs to $\tau_i = 20\%$ and $\alpha_i = 23\%$, $i = S, B$ respectively, and then proceed to parametric changes. We fix the marginal distributions of cash flows at maturity (5 years) to a normal distribution with mean $100 * (1.05)^5$ and standard deviation $\sigma = 22 * \sqrt{5}$ and we maintain a joint normality assumption for connected units, letting correlation vary.

We start by assessing the capital structure changes in the stand-alone unit, when the risk-free rate falls from 5% to 1%.

3.1. The Stand-Alone Company

Our first result is that lower interest rates reduce incentives to lever in the stand-alone case (see Table 2). Condition 24, when checked numerically, holds for our parametric case and, thus,

Table 1: Base-case parameters		
Symbol	**Parameter**	**Value**
τ	Tax rate	20%
α	Default costs rate	23%
r	Annual risk-free rate	5%
$\phi = \frac{1}{(1+r)^5}$	Discount factor	0.78
$X(0)$	Cash-flow present value	100
V_U	Zero-leverage unit after-tax present value	80.05
T	Time horizon	5
σ	Cash flow volatility at T	$22\sqrt{5}$
ρ	Cash flow correlation	0.2

Table 1: This table displays the base-case parameters, following Leland (2007). The zero-leverage value, (V_U), is the present value of positive cash flows net of taxes. The annual cash-flow volatility, 22, is scaled to the 5-year horizon.

the optimal face value of debt falls by about 40%, from 57.1 to 34.3, and the market value of debt declines from 42.2 to 31.8 when the interest rate lowers from 5% to 1%.[13]

While the value of the hypothetical zero-leverage firm increases sharply due to lower discounting (from 80.05 to 97.20), the value of leverage -the difference between the optimally leveraged and the zero-leverage unit value- collapses (from 1.42 to 0.25). This reflects a compression in the tax shield, which falls from 14.89 to 2.46, and in tax savings (from 2.32 to 0.47).

Lower leverage reduces the default threshold (from 67.65 to 42.26), leading to a decline in expected default costs (from 0.89 to 0.22) and in default probability (from 11.14% to 4.13%). As a result, spreads fall from 1.23% to 0.50%. Losses given default decline in absolute terms due to the lower face value of debt, although they increase as a fraction of the principal conditional on default. As the default threshold moves towards zero with lower borrowing, recovery upon default falls from 50% to 41% of the principal, since defaults occur when after-tax cash flows are smaller.

These patterns are robust across (unreported) parameter variations. The decline in leverage is milder when incentives to lever are stronger —- namely, when volatility or tax rates are higher, or

13. It is well-known that observed leverage falls short of trade-off predictions, a gap that can be explained by the opportunity cost of borrowing now rather than preserving the option to borrow later (DeAngelo et al., 2011), tax avoidance (Brok, 2026), risk-neutral rather than physical default probabilities resulting in higher risk premia (Almeida and Philippon, 2007), and agency considerations such as covenant constraints (Nini et al., 2012).

Table 2: Optimal Stand-Alone Organization			
Variable	**Formula**	Interest rate 5%	1%
Optimal zero-coupon bond principal	F^*	57.10	34.30
Optimal levered value	$V^* = D^* + E^*$	81.47	97.45
Value of optimal debt	D^*	42.21	31.84
Value of optimal equity	E^*	39.26	65.61
Tax shield	$X_Z^* = F - D$	14.89	2.46
No-default threshold	$X^d = F + \frac{\tau}{1-\tau} D$	67.65	42.26
Zero-leverage unit value	V_U	80.05	97.20
Value of optimal leverage	$V^* - V_U$	1.42	0.25
Debt yield	$y^* = (F^*/D^*)^{\frac{1}{5}} - 1$	6.23%	1.50%
Spread	$s^* = \left(\frac{F}{D}\right)^{\frac{1}{5}} - \left(\frac{1}{\phi}\right)^{\frac{1}{5}}$	1.23%	0.50%
Taxes	$T^* = \tau\phi\mathbb{E}[(X - X^Z)^+]$	17.70	23.84
Tax savings	$TS^* = T(0) - T^*$	2.32	0.47
Default costs	$C^* = \alpha\phi\mathbb{E}\left[X\mathbf{1}_{\{0<X<X^d\}}\right]$	0.89	0.22
5-year default probability	$DP^* = \int_{-\infty}^{X^d} f(x)\,dx$	11.14%	4.13%
Loss given default	$LGD^* = \frac{F^* - D^* \frac{1}{\phi}}{DP^*}$	28.96	20.16

Table 2: This table displays the optimal structure of a stand-alone unit with parameters as in Table 1, for two levels of interest rates, 5% and 1%. Yields and spreads are annualized. The default probability is defined as the probability that the unit is not able to repay its lenders after $T = 5$ years, when the cash flows are realized. The cash-flow distribution is fixed at T: $X \sim N(127.63, 49.19)$.

proportional default costs are lower.

We can summarize these results as follows, assuming that everything else is unchanged including the distribution of future cash flows:

Observation 1. *When the risk-free interest rate decreases, the optimal leverage of a stand-alone company falls together with its expected default costs, default probability and losses given default.*

3.2. A Zero-Leverage sponsor: the LBO Case

We now consider a sponsor–backed unit. We begin from the base-case calibration in Table 1 and focus on a low cash-flow correlation (0.2) benchmark, illustrated in Figures 4, 5 and Table 3.

At $r = 5\%$, the optimal allocation of leverage within the organization is highly asymmetric: the sponsor is zero-leverage, while the backed unit issues more debt than two comparable stand-alone firms combined (220 vs. 114). This configuration resembles LBO structures (Cohn et al., 2014). The key mechanism is governance-based: by forgoing its own tax shield, the sponsor preserves financial slack and thereby maximizes its capacity to provide contingent support. This support relaxes the backed unit's default constraint ex-post, enabling it to borrow more ex-ante and extract a substantially larger tax shield (103 vs. 14.9 in the stand-alone benchmark), albeit at the cost of higher expected default costs.

When the risk-free rate declines to 1%, this internal reallocation margin becomes even more pronounced. The sponsor remains optimally zero-leverage, while the backed unit further expands debt (to 247). In contrast to the stand-alone benchmark, lower interest rates induce more, not less, leverage in the backed unit.

Quantitatively, tax savings in the backed unit increase (from 14.6 to 19.1), but this comes with a sharp rise in default costs (from 8.1 to 14.5) and default probability (from 47% to 63%). Spreads and losses given default rise accordingly. Thus, the expansion in borrowing is not driven by cheaper financing, but by a reallocation of debt toward an entity with looser cash-flow constraint. Importantly, sponsor support does not eliminate default risk. While support reduces expected default costs for a given debt level, equilibrium borrowing shifts distress toward states with lower recovery and higher losses given default. As a result, debt in the backed unit remains risky and spreads may increase even as interest rates fall. The mechanism is therefore distinct from standard guarantee or cheap-credit stories: borrowing expands because risky debt continues to generate valuable tax shields within the organization.

Note in fact that further borrowing does not operate through additional sponsor deleveraging, since the sponsor *remains* zero-leverage (see Proposition 4). Instead, the backed unit adjusts along its own tax–default margin. To understand this adjustment, let us compare the stand-alone unit and the backed unit for fixed debt. In the stand-alone benchmark a decline in the interest rate increases the value of debt thereby reducing the tax shield and the spread (see Lemma 1), thereby raising expected default costs (see Proposition 1), leading—under standard conditions—to a decline in optimal leverage (see Proposition 3).

This logic changes in the backed unit because support alters the distribution of default states rather than eliminating distress risk. Lower rates shift probability mass toward states in which the backed unit pays taxes but still defaults, since taxes increase the cash-flow shortfall for given face value of debt. In these states, lenders receive only partial recovery. Thus the market value of debt rises due to lower discounting thereby reducing the tax shield, but not enough to compress the spread.[14] When debt is then optimized, the backed unit exploits this persistent spread, since increasing debt face value raises the tax shield despite lower risk-free rates. The mechanism leading to higher debt in the backed unit is therefore unrelated to cheaper funding; it is the interaction of support, risky debt pricing, and interest deductibility. The comparison between the leverage choices with different interest-rate levels is evident in Figure 4, whose upper left panel shows a numerical example of the behaviour described by Corollary 1, where the different organizational arrangement leads to divergent optimal responses.

This pattern is robust across interest-rate levels and strengthens with cash-flow correlation (see Table 3). Higher correlation increases the value of support by raising the likelihood that the sponsor can intervene in distress states. The backed-unit optimal face value of debt and the default probability top 255 and 63.5%, respectively, when cash-flow correlation and the interest rate are equal to 0.8 and 1%. The table also shows that, irrespective of correlation, the joint default probability of the complex organization is negligible because of the zero-default-probability of the zero-leverage sponsor.

The statement below summarizes some of these patterns, assuming that only the risk-free rate varies:

Observation 2. *When the risk-free rate falls, a backed unit supported by a zero-leverage sponsor increases borrowing, spreads, default probability, and losses given default. These effects are amplified by higher cash-flow correlation.*

Our results bear implications for empirical work. They imply that information on the structure of organizations is essential to understand leverage choices, as the response to interest rates may be opposite for a stand-alone and a backed unit with the same characteristics. In this respect, our findings are broadly consistent with the observed divergent response, by public companies and comparable LBO targets, to lower interest rates. Axelson et al. (2013) find that the ratio of debt to EBITDA is higher in LBO targets but not in matched public companies when interest rates fall. The spreads they find depend on the type of debt. The median spread is equal to 262bp and 937bp for senior and junior bank loans respectively, reaching up to 916bp and 1048bp for senior

14. An unreported quantitative assessment of lower rates, holding debt at the optimum level when r=5%, is revealing. For the stand-alone firm, the probability of default while paying taxes increases only slightly (from 10% to 11.3%), the tax shield more than halves (from 14.89 to 6.15), the spread remains low (roughly at 1.23%). In the backed unit, for $\rho = 0.2$, the probability of default while paying taxes markedly increases (from 47% to 63%), the tax shield falls by only 16% (from 103 to 86.5), while the spread increases substantially (from 8.5% to 12%).

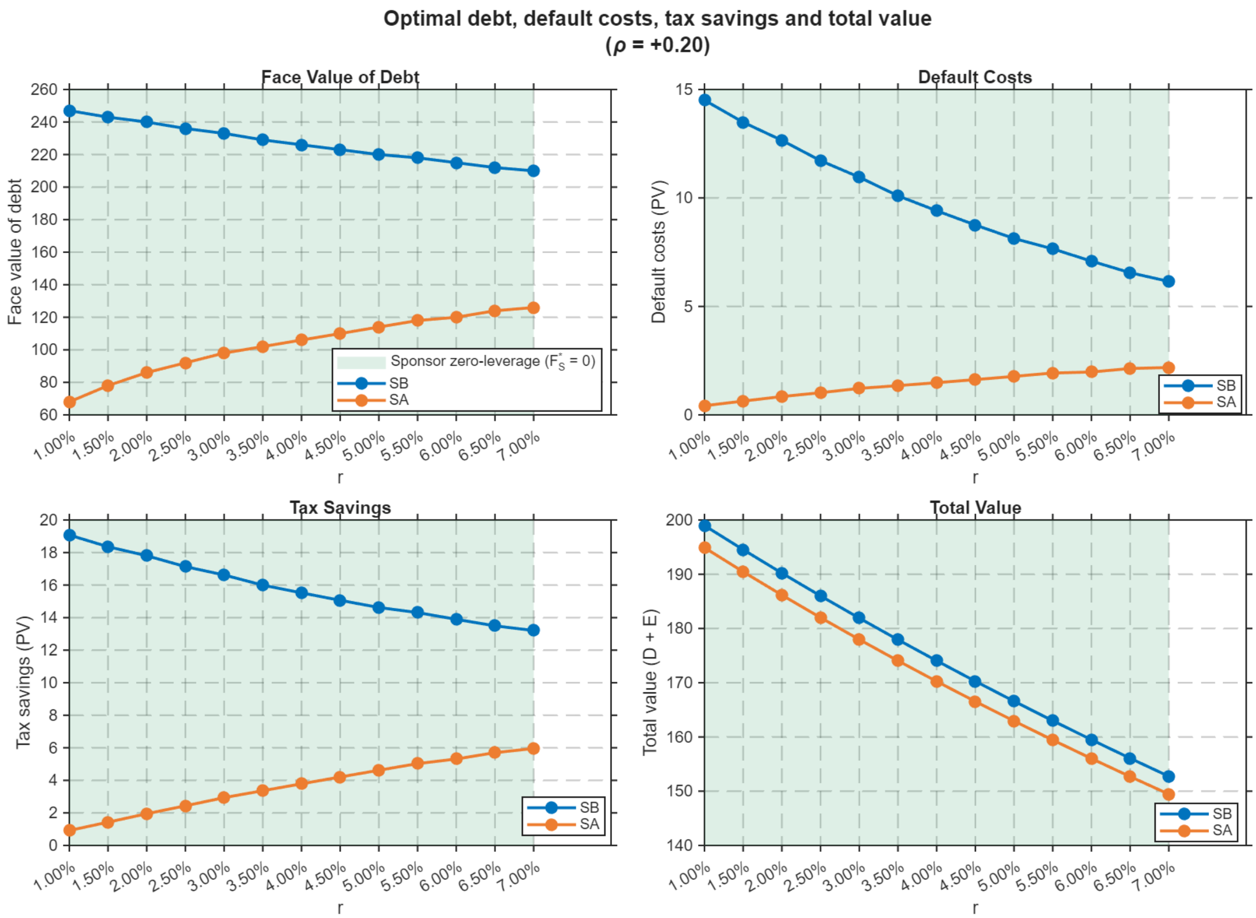

Figure 4: The four panels report the optimal face value of debt, default costs, tax savings, and total value for the backed unit supported by a zero-leverage sponsor (in blue) and the equivalent stand-alone (in orange), as the risk-free rate varies from 1% to 7%. Parameters are as in Table 1. Cash flows are jointly normal, with 0.2 correlation.

and subordinated bonds respectively. The spread implied by our model, when 1% (5%) is the level of interest rates, is equal to 528bp (197bp) when cash-flow correlation between the fund and the LBO target is -0.8, reaching 1210bp (845bp) when cash-flow correlation is 0.2. Our numerical exercise also shows that these spreads allow to reduce the tax burden of the LBO target to up to one fifth of the taxes paid by a similar zero-leverage company. Furthermore, while we know that information about the complex structure improves on default prediction (Beaver et al., 2019), our model suggests a flip in the sign of the prediction depending on the presence of the sponsor.

Our analysis also sheds light on regulators' concerns of potential financial instability observing the rise – from below 4 to above 5 - in the multiple of average total debt to EBITDA for leveraged

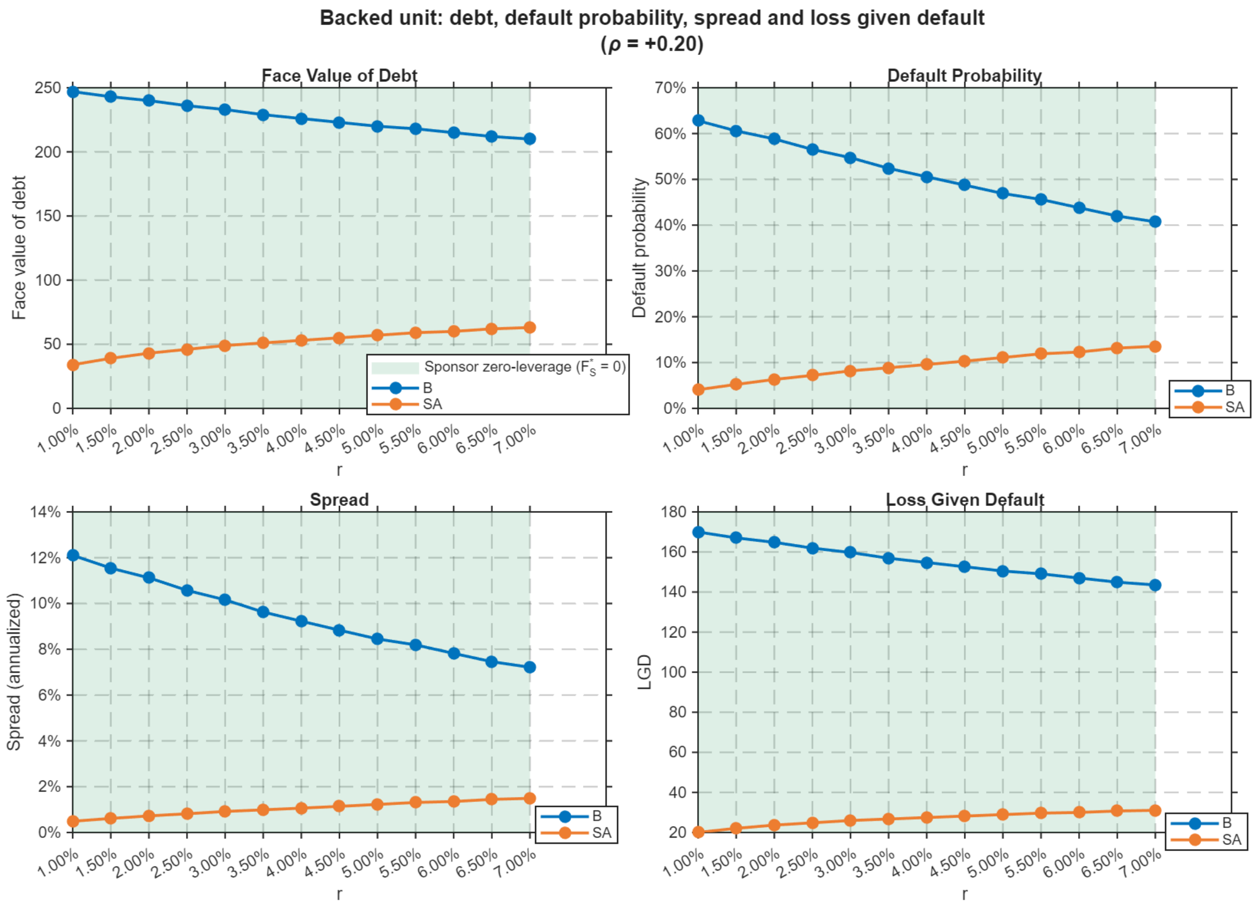

Figure 5: The four panels report the optimal face value of debt, 5-year default probability, annualized spread, and loss given default for the backed unit supported by a zero-leverage sponsor (in blue) and the equivalent stand-alone (in orange), as the risk-free rate varies from 1% to 7%. Parameters are as in Table 1. Cash flows are jointly normal, with 0.2 correlation.

transactions priced at or above LIBOR + 225bp, as interest rates were falling (Rosengren, 2019). Our results indicate that when leverage increases in response to lower interest rates, both default probabilities and, to a lesser extent, lenders' losses given default also rise with respect to the stand-alone benchmark.

However, a fuller quantitative assessment would require allowing for differences in parameters across organizational forms. For instance, private equity research suggests that leveraged buyout targets may display higher expected cash flows, lower cash-flow volatility, shorter and matched debt maturity, than stand-alone counterparts. The comparison should also account for the presence of sponsors that rarely default relative to independent units. Moreover, default probabilities need not move uniformly across firm types: distress risk in sponsored structures may increase precisely when other firms become safer and cash flows improve under easier monetary conditions. These last considerations suggest that organizational heterogeneity may smooth aggregate default risk across interest-rate scenarios.

The numerical exercise should therefore be interpreted as isolating how organizational structure alone affects the relationship between interest rates and credit risk.

4. A Leveraged Sponsor: the Parent–Subsidiary and Securitization Cases

We now turn to the case in which the sponsor is initially leveraged, which is empirically relevant for multinational groups and securitization arrangements.[15] We therefore allow for a higher tax rate in the sponsor than in the backed unit ($\tau_S = 24\%$; $\tau_B = 16\%$), and also increase cash-flow volatility ($\sigma_S = \sigma_B = \sigma = 44 * \sqrt{5}$) to ensure stronger incentives to borrow. Table 4 collects the relevant parameters.

We begin with the baseline correlation ($\rho = 0.2$) and refer to Figure 6, Figure 7 and Table 5.

At the initial level of interest rates ($r = 5\%$), the complex organization raises more total debt than two equivalent stand-alone firms (190 vs. 171), but allocates it asymmetrically: the backed unit issues more debt than the sponsor (111 vs. 79), despite the sponsor's stronger tax incentives. This reflects a governance trade-off. The sponsor internalizes the value of contingent support and therefore limits its own leverage relative to a stand-alone benchmark (79 vs. 103), preserving financial slack. The backed unit, in turn, borrows more than its stand-alone counterpart (111 vs. 68), benefiting from relaxed effective borrowing constraints. This reallocation does not impact much total tax savings (10.14 vs. 10.32) but greatly reduces aggregate default costs (5.25 vs. 6.09) relative to stand-alone firms, increasing total value (170.54 vs. 169.88).

When the interest rate declines, two forces interact. First, as in the stand-alone case, lower rates compress the sponsor's tax advantage. Second, they increase the value of preserving financial flexibility to support the backed unit (Proposition 4). As a result, total leverage displays a non-monotonic response: it initially declines as rates fall from 5% to 3%, and then increases. This contrasts sharply with the monotonic deleveraging of stand-alone firms.

The reallocation of leverage has first-order implications for credit risk. As borrowing concentrates in the backed unit, both default probabilities and losses given default increase sharply. For instance, at $r = 1\%$, backed-unit default costs are 11 times those of a stand-alone unit (8.85 vs. 0.79). The (5-year) default probability reaches almost 44%, up from 24.6% in the baseline 5%

15. See respectively Anantavrasilp et al. (2020), Bianco and Nicodano (2006), and Brok (2024); and Allen et al. (2023), Jiangli et al. (2007), and Gorton and Souleles (2006).

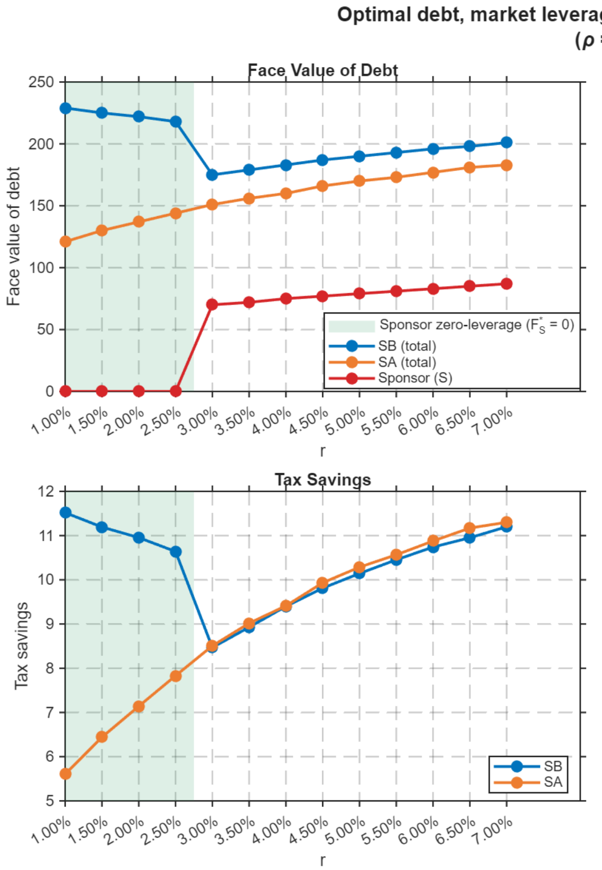

Figure 6: This figure portrays the optimal debt (face value and market value), default costs and tax savings of the sponsor/backed unit arrangement (in blue) when interest rate ranges from 1% to 7% and compares the figures with those of equivalent stand-alone units (orange). In the upper left panel, the red line depicts the optimal debt of the sponsor.

interest rate. Losses given default also deteriorate, rising to 206.98 from 109.7 (75.61% vs. 71.31% in percentage terms), and spreads rise from 6.01% to 10.64%.

An additional insight emerges from comparing Table 3 and Table 5. In Table 3, where the sponsor is optimally zero-leverage, the value of the guarantee declines slightly as interest rates fall. Lower rates increase the market value of the backed unit's debt, reducing its tax shield and expanding the set of default states. Since further sponsor's deleveraging is not possible, the guarantee becomes less effective on net. By contrast, in Table 5, the value of the guarantee increases sharply when interest rates fall. In this case, lower rates induce the sponsor to delever, thereby expanding the set of states in which support can be provided. The resulting increase in support capacity more than offsets the reduction in guarantee effectiveness associated with the larger default

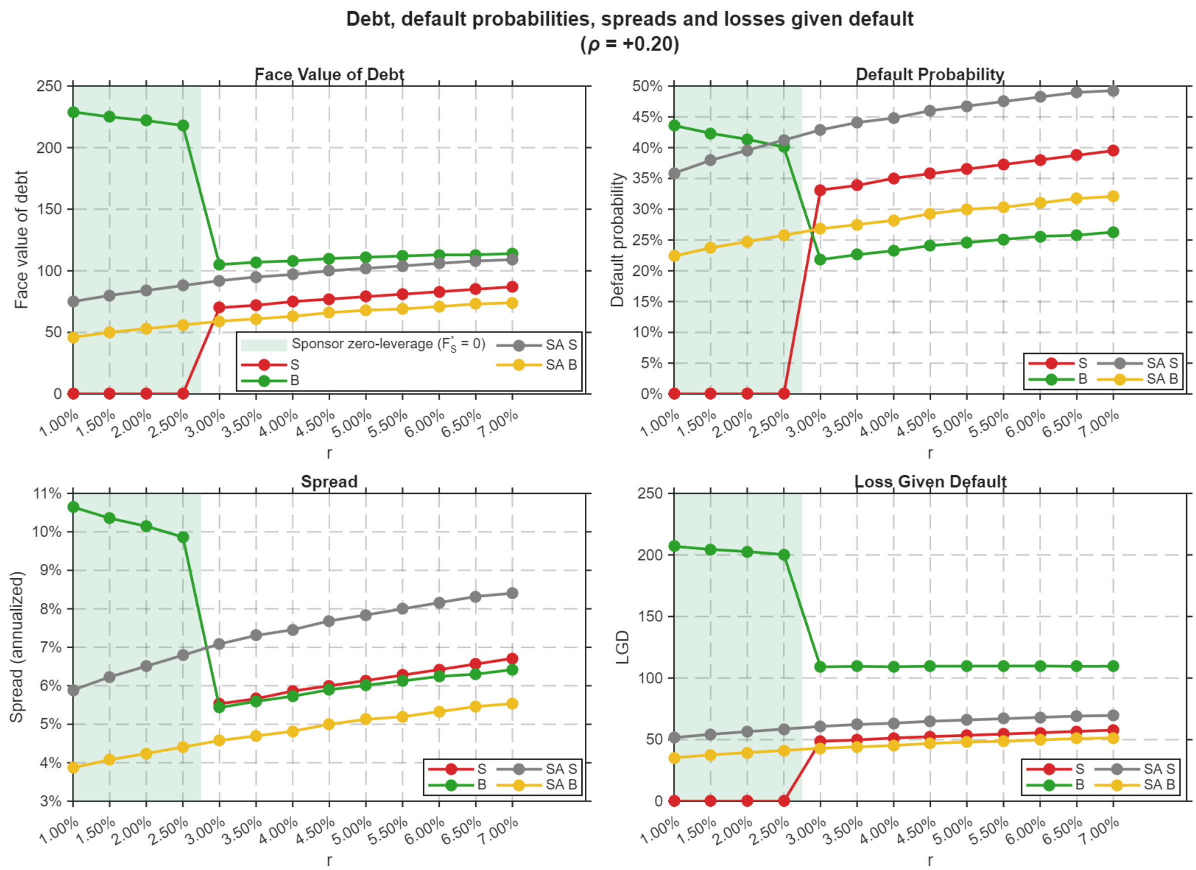

Figure 7: This figure portrays the debt (face value), default probabilities, spreads and loss given defaults of the sponsor (red), the backed unit (green) and compares their figures with those of equivalent stand-alone units (in grey and yellow, respectively) when interest rate ranges from 1% to 7%. Spreads are annualized, the default probabilities are the probabilities that lenders are not repaid in full when the cash flows are realized at $T = 5$ years.

region of the backed unit. This comparison quantifies the value of sponsor deleveraging as an adjustment margin within complex organizations.

Importantly, the increase in borrowing is not driven by cheaper funding. Instead, it reflects the persistence—and in some cases amplification—of spreads. As leverage rises, a larger fraction of states falls into tax-paying default, where lenders receive only partial recovery. This keeps debt risky preventing the market value of debt from converging to its face value. The resulting wedge between the two sustains the tax advantage of debt even as interest rates decline. Thus, the key mechanism is governance-based: lower interest rates shift leverage toward backed entities where debt remains risky. Their tax shields are preserved through endogenous spreads.

Thus, for our selected parameters, we observe a transformation of the levered parent–subsidiary

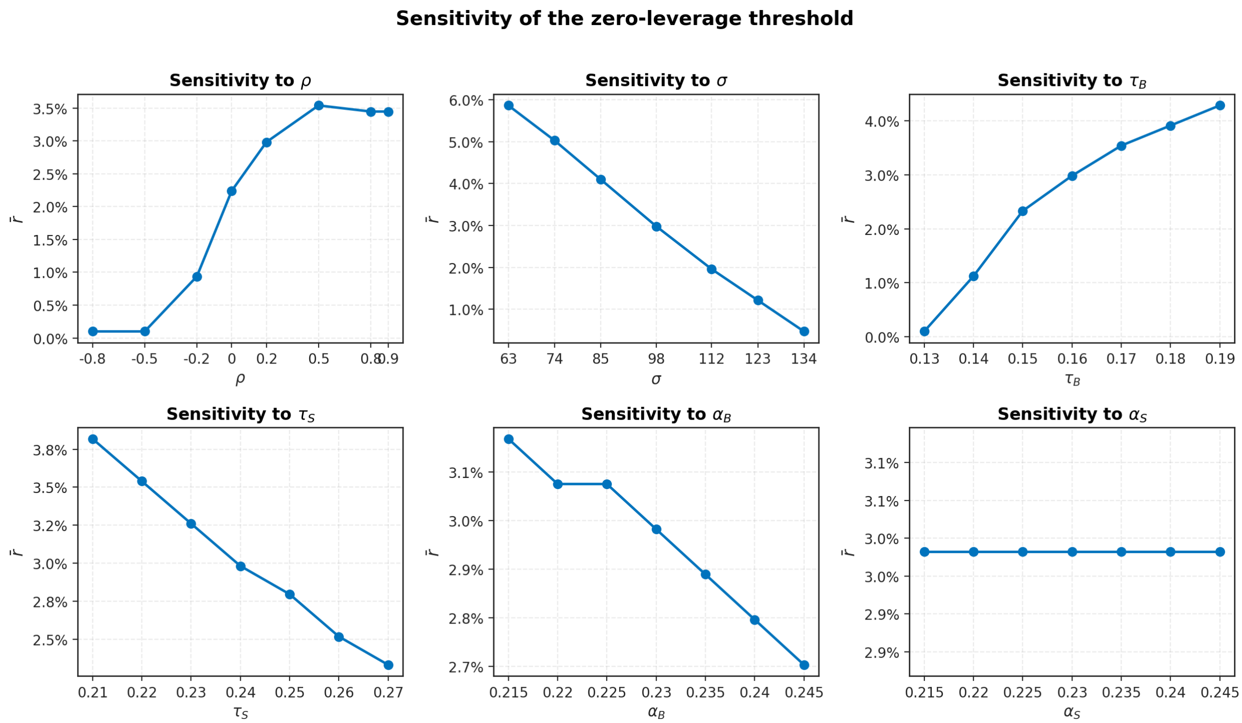

Figure 8: The six panels report the sensitivity of the threshold $\bar{r}$ to changes in the parameters. Parameters change one at a time, while the others are fixed as in Table 4. We set a floor to $\bar{r}$ equal to zero.

arrangement to a zero-leverage LBO fund and its highly leveraged portfolio company. The likelihood of this transformation depends on cash-flow correlation (see Table 5). Higher correlation increases the probability that the sponsor can intervene in distress states. Consequently, the zero-leverage sponsor outcome is more likely when correlation is high, i.e., when diversification is limited.

Indeed, Figure 8 shows a positive relation between $\bar{r}$, the threshold below which the sponsor is optimally zero-leverage and ρ, that flattens out at high correlation levels. As correlation approaches its upper limit, the benefits from support fade away because the states in which the sponsor can intervene are likely those in which the backed unit is already solvent. The figure also analyses sensitivity of the threshold rate $\bar{r}$ to other relevant parameter changes, relative to their values in Table 4. The results show that a zero-leverage sponsor is optimal below a higher threshold rate the lower τ_S (as predicted by Proposition 4) and α_B, the higher τ_B, thus the weaker the incentives to issue debt in the sponsor as opposed to the backed unit. The default cost rate of the sponsor, α_S, does not affect the threshold rate because, at zero leverage, the marginal effect of debt on the default costs of the sponsor is null. Finally, $\bar{r}$ decreases with σ, which makes the tax shield more valuable.

The analysis of our results generates the following prediction:

Observation 3. *Lower interest rates can induce discrete organizational changes, reallocating leverage toward backed units and transforming parent–subsidiary structures into LBO-like arrangements.*

These results speak to the empirical literature. They are consistent with evidence that leveraged private firms increase borrowing at higher spreads during monetary expansions, while comparable public firms delever (Caglio et al., 2022; Axelson et al., 2013). They also provide a structural rationale for the expansion of securitization activity in low-rate environments (Powell, 2019). A caveat regarding implications for groups has to do with unmodeled, but frequently observed, dividend payments from subsidiaries,[16] which enhance the sponsor's incentive to borrow (see Nicodano and Regis, 2019). We expect the threshold interest rate, $\bar{r}$, triggering the sale of a subsidiary to a private equity fund, to be lower for sponsors receiving larger dividend distributions.

From a policy perspective, the model highlights a composition effect in risk-taking. As interest rates fall, risk is reallocated within organizations rather than uniformly reduced. Backed units become more leveraged and riskier, while sponsors reduce leverage and default risk. This implies that aggregate default risk may evolve differently from firm-level risk, and that monitoring organizational structure is essential for financial stability assessments.

At the same time, the analysis points to a potential divergence between privately optimal and socially optimal structures. At low interest rates, complex organizations maximize value by concentrating leverage in backed units, but this may increase total default costs relative to stand-alone firms. Such divergence does not occur at the higher interest-rate level, as complex organizations are both privately and socially optimal.

16. See e.g. Almeida and Wolfenzon (2006); Anantavrasilp et al. (2020); Desai et al. (2004).

Table 3: Optimal Value and Debt — Sponsor / Backed Unit						
	Correlation					
	-0.8		0.2		0.8	
Variable	Interest Rate		Interest Rate		Interest Rate	
	5%	1%	5%	1%	5%	1%
Face Value of Debt	187	205	220	247	229	255
	(0;187)	(0;205)	(0;220)	(0;247)	(0;229)	(0;255)
Market Debt Value	133.52	151.17	117.06	133.45	115.44	133.90
	(0.00;133.52)	(0.00;151.17)	(0.00;117.06)	(0.00;133.45)	(0.00;115.44)	(0.00;133.90)
Equity Value	32.34	44.87	49.52	65.53	51.92	66.37
	(31.98;0.35)	(44.75;0.12)	(49.46;0.07)	(65.52;0.01)	(51.88;0.04)	(66.36;0.01)
Total Value	165.86	196.04	166.58	198.98	167.36	200.27
	(31.98;133.87)	(44.75;151.29)	(49.46;117.13)	(65.52;133.46)	(51.88;115.48)	(66.36;133.91)
Value of Leverage	5.76	1.64	6.49	4.57	7.27	5.86
Tax Savings	8.17	9.98	14.62	19.07	15.71	19.91
	(0.00;8.17)	(0.00;9.98)	(0.00;14.62)	(0.00;19.07)	(0.00;15.71)	(0.00;19.91)
Taxes	31.85	38.62	25.40	29.53	24.31	28.69
	(20.01;11.84)	(24.30;14.32)	(20.01;5.39)	(24.30;5.23)	(20.01;4.30)	(24.30;4.39)
Default Costs	2.41	8.34	8.13	14.50	8.44	14.05
	(0.00;2.41)	(0.00;8.34)	(0.00;8.13)	(0.00;14.50)	(0.00;8.44)	(0.00;14.05)
Guarantee value	19.33	19.06	14.56	13.38	14.38	13.86
Yield	(N/A;6.97%)	(N/A;6.28%)	(N/A;13.45%)	(N/A;13.10%)	(N/A;14.68%)	(N/A;13.75%)
Spread	(N/A;1.97%)	(N/A;5.28%)	(N/A;8.45%)	(N/A;12.10%)	(N/A;9.68%)	(N/A;12.75%)
Default Probability	(0.00%;13.18%)	(0.00%;34.35%)	(0.00%;46.91%)	(0.00%;62.80%)	(0.00%;50.77%)	(0.00%;63.47%)
Joint Default Prob.	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Loss Given Default	(0.00;125.93)	(0.00;134.24)	(0.00;150.49)	(0.00;169.97)	(0.00;160.86)	(0.00;180.03)

Table 3: This table reports the optimal capital structure and credit-risk measures of a sponsor–backed unit organization. Both units have the parameters in Table 1. Sponsor and backed-unit values are reported in parentheses; totals are reported outside parentheses. Cash flows are jointly normal, with marginal distributions as in Table 1 and correlation $\rho \in \{-0.8, 0.2, 0.8\}$. Yields and spreads are annualized. Default probabilities are 5-year probabilities.

Table 4: Leveraged Sponsor parameters		
Symbol	**Parameter**	**Value**
τ_S	Sponsor Tax rate	24%
τ_B	Backed unit Tax rate	16%
$\alpha_S = \alpha_B$	Default costs rate	23%
$\sigma_S = \sigma_B$	Cash flow volatility at T	$44\sqrt{5}$
ρ	Cash flow correlation	0.2

Table 4: This table displays the baseline parameters in the SB case with a leveraged sponsor, following Nicodano and Regis (2019).

Table 5: Optimal Value and Debt — Transformation of a Complex Organization

Variable	Correlation −0.8		−0.2		0		0.2		0.8	
	Interest Rate		Interest Rate		Interest Rate		Interest Rate		Interest Rate	
	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%
Face Value of Debt	183	158	188	151	189	150	190	229	241	256
	(74;109)	(57;101)	(81;107)	(61;90)	(81;108)	(60;90)	(79;111)	(0;229)	(25;216)	(0;256)
Market Value of Debt	115.53	126.68	113.11	116.46	112.73	114.97	112.44	132.03	117.04	133.62
	(44.32;71.21)	(43.16;83.52)	(47.48;65.64)	(45.65;70.81)	(47.48;65.25)	(45.04;69.93)	(46.60;65.84)	(0.00;132.03)	(16.94;100.10)	(0.00;133.62)
Equity Value	55.49	77.14	57.54	87.05	57.87	88.50	58.10	71.79	54.02	71.91
	(27.96;27.53)	(41.30;35.84)	(28.94;28.60)	(45.09;41.96)	(29.60;28.27)	(46.46;42.04)	(30.90;27.20)	(68.10;3.69)	(49.51;4.51)	(69.91;1.99)
Total Value	171.02	203.82	170.65	203.51	170.59	203.47	170.54	203.82	171.07	205.53
	(72.28;98.74)	(84.46;119.36)	(76.42;94.23)	(90.74;112.77)	(77.07;93.52)	(91.50;111.97)	(77.50;93.04)	(68.10;135.72)	(66.45;104.62)	(69.91;135.62)
Value of Leverage	5.37	2.67	5.01	2.36	4.95	2.31	4.89	2.67	5.42	4.38
Guarantee value	3.37	4.27	2.27	2.41	2.15	2.25	2.15	13.53	10.13	15.38
Taxes	32.44	45.12	31.48	44.59	31.34	44.55	31.27	38.77	29.18	36.56
	(19.97;12.47)	(27.36;17.76)	(19.37;12.11)	(27.06;17.53)	(19.37;11.97)	(27.14;17.42)	(19.54;11.72)	(30.17;8.60)	(23.49;5.69)	(30.17;6.38)
Tax Savings	8.97	5.17	9.94	5.70	10.08	5.74	10.14	11.52	12.23	13.73
	(4.87;4.09)	(2.81;2.36)	(5.48;4.46)	(3.11;2.58)	(5.48;4.60)	(3.04;2.70)	(5.30;4.84)	(0.00;11.52)	(1.36;10.87)	(0.00;13.73)
Default Costs	3.59	2.50	4.93	3.34	5.13	3.42	5.25	8.85	6.81	9.36
	(2.19;1.40)	(1.58;0.93)	(2.69;2.24)	(1.84;1.50)	(2.69;2.44)	(1.77;1.65)	(2.55;2.71)	(0.00;8.85)	(0.19;6.63)	(0.00;9.36)
Yield	(10.80%;8.89%)	(5.72%;3.87%)	(11.28%;10.27%)	(5.97%;4.91%)	(11.28%;10.60%)	(5.91%;5.18%)	(11.14%;11.01%)	(N/A;11.64%)	(8.09%;16.63%)	(N/A;13.89%)
Spread	(5.80%;3.89%)	(4.72%;2.87%)	(6.28%;5.27%)	(4.97%;3.91%)	(6.28%;5.60%)	(4.91%;4.18%)	(6.14%;6.01%)	(N/A;10.64%)	(3.09%;11.63%)	(N/A;12.89%)
Default Probability	(34.35%;12.57%)	(28.12%;7.27%)	(37.39%;20.59%)	(29.78%;13.34%)	(37.39%;22.42%)	(29.36%;14.84%)	(36.52%;24.58%)	(0.00%;43.60%)	(16.14%;44.77%)	(0.00%;48.93%)
Joint Default Probability	3.54%	1.78%	14.44%	9.35%	17.60%	11.82%	20.67%	0.00%	16.14%	0.00%
Loss Given Default	(50.75;144.16)	(41.40;181.78)	(54.58;112.79)	(43.71;116.82)	(54.58;110.26)	(43.14;111.21)	(53.49;109.70)	(0.00;206.98)	(20.94;197.10)	(0.00;236.17)

Table 5: This table displays the optimal figures of a complex organization when the parameters for the two units are those in Table 1, apart from $\tau_S = 24\%$, $\tau_B = 16\%$, $\sigma_S = \sigma_B = \sigma = 44\sqrt{5}$, for two levels of interest rates, 5% and 1%. Sponsor and backed-unit figures are reported in parentheses, respectively, while the total figure is outside the parenthesis. Cash flows are jointly normally distributed, with marginal distributions as in Table 1 and correlation parameter ranging from −0.8 to 0.8.

5. Concluding Remarks

This paper studies how organizational structure shapes the response of optimal leverage to changes in interest rates. In a multi-entity organization with contingent support arrangements, the standard trade-off prediction of deleveraging need not hold uniformly across affiliated entities. Instead, financing choices reflect both the tax benefits of debt and the value of preserving internal support capacity. As a result, changes in interest rates may alter not only the level of borrowing, but also its allocation within the organization and the associated distribution of credit risk.

The analysis uncovers a new channel of monetary transmission to leverage. Higher debt arises from the interaction between tax shields and internal support arrangements rather than from cheaper borrowing, moral hazard, or investors' search for yield. Importantly, greater borrowing may coexist with higher spreads and expected losses upon default, implying that financing conditions need not become safer even when interest rates decline.

These insights apply to a broad range of organizational forms, including leveraged buyout structures, securitization vehicles, and other entities connected through internal guarantees or support arrangements. More generally, the results imply that organizational structure is essential for understanding how interest rates affect leverage choices and credit risk between firms with otherwise similar characteristics. Our tractable, static organization design with zero-coupon debt allows to uncover the general mechanism driving the relation between interest rates and leverage, namely that contingent sponsor support alters the marginal tax-bankruptcy trade-off, moving debt towards the guaranteed unit especially below a critical interest rate. We conjecture this mechanism carries over to multi-period complex organizations with alternate coupon debt, but leave these demanding extensions for future research.

The paper also carries implications for financial stability. In the model, changes in interest rates can increase leverage concentration and expected distress costs within particular organizational structures even in the absence of agency distortions. At the same time, risk can become redistributed rather than uniformly amplified, implying that aggregate outcomes depend on the composition of organizational forms within the economy. A fuller assessment of these equilibrium interactions remains an important avenue for future research. More broadly, the paper highlights how internal organizational arrangements shape the transmission of monetary conditions to leverage and credit risk.

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6. Appendix

6.1. Definition of the $h(\cdot)$ function

The function $h(X_B)$ defines the set of states of the world in which the sponsor has enough funds to intervene and save its affiliate from default, while at the same time remaining solvent. The rescue occurs if the cash flows of the sponsor X_S are enough to cover both its own debt obligations and the remaining part of those of the affiliate. The function $h(X_B)$, which defines the level of parent cash flows above which the rescue occurs, is defined as:

$$h(X_B) = \begin{cases} X_S^d + \frac{F_B}{1-\tau_S} - \frac{X_B}{1-\tau_S} = h_1(X_B) & X_B < X_B^Z, \\ X_S^d + \frac{1-\tau_B}{1-\tau_S}\left[X_B^d - X_B\right] = h_2(X_B) & X_B \geq X_B^Z. \end{cases}$$

6.2. Expressions for $D_i^{SA}, E_i^{SA}, D_S, D_B, E_S, E_B$

This subsection collects the formal definitions of debt and equity values for stand-alone companies, sponsors, and backed units.

$$D_i^{SA} = \phi\left[(1-\alpha_i)\int_0^{X_i^Z} xf(x)dx + (1-\alpha_i-\tau_i)\int_{X_i^Z}^{X_i^d} xf(x)dx + \tau_i X_i^Z\int_{X_i^Z}^{X_i^d} f(x)dx + F_i\int_{X_i^d}^{+\infty} f(x)dx\right]. \tag{7}$$

$$E_i^{SA} = \phi\left[(1-\tau_i)\int_{X_i^d}^{+\infty} xf(x)dx + (\tau_i X_i^Z - F_i)\int_{X_i^d}^{+\infty} f(x)dx\right]. \tag{8}$$

$$D_S = \phi\left[(1-\alpha_S)\int_0^{X_S^Z} xf(x)dx + (1-\alpha_S-\tau_S)\int_{X_S^Z}^{X_S^d} xf(x)dx + \tau_S X_S^Z\int_{X_S^Z}^{X_S^d} f(x)dx + F_S\int_{X_S^d}^{+\infty} f(x)dx\right]. \tag{9}$$

$$E_S = \phi\left[(1-\tau_S)\int_{X_S^d}^{+\infty} xf(x)dx + (\tau_S X_S^Z - F_S)\int_{X_S^d}^{+\infty} f(x)dx\right] - \Psi(F_S, F_B). \tag{10}$$

$$D_B = \phi \Bigg[(1-\alpha_B) \int_0^{X_B^Z} \int_{-\infty}^{h_1(x)} x f(x,y) dx dy +$$

$$+ (1-\alpha_B-\tau_B) \int_{X_B^Z}^{X_B^d} \int_{-\infty}^{h_2(x)} x f(x,y)\, dx\, dy + \tau_B X_B^Z \int_{X_B^Z}^{X_B^d} \int_{-\infty}^{h_2(x)} f(x,y) dx dy +$$

$$+ F_B \left[\int_{X_B^d}^{+\infty} f(x)\, dx + \int_0^{X_B^Z} \int_{h_1(x)}^{+\infty} f(x,y)\, dx\, dy + \int_{X_B^Z}^{X_B^d} \int_{h_2(x)}^{+\infty} f(x,y)\, dx\, dy \right] \Bigg] + \Psi(F_S, F_B). \tag{11}$$

$$E_B = \phi \left[(1-\tau_B) \int_{X_B^d}^{+\infty} x f(x) dx + (\tau_B X_B^Z - F_B) \int_{X_B^d}^{+\infty} f(x) dx \right]. \tag{12}$$

$$\Psi(F_S, F_B) = \phi \left[\int_0^{X_B^Z} \int_{h_1(x)}^{+\infty} (F_B - x) f(x,y)\, dy\, dx + \int_{X_B^Z}^{X_B^d} \int_{h_2(x)}^{+\infty} \Big(F_B - \big[(1-\tau_S) x + \tau_S X_B^Z \big] \Big) f(x,y)\, dy\, dx \right] \tag{13}$$

Notice that

$$D_B = D_B^{SA}(F_B; D_B) + \Psi(F_S, F_B) + \Gamma(F_S, F_B), \tag{14}$$

where $D_B^{SA}(F_B; D_B)$ indicates the debt value of a stand-alone whose face value is F_B and whose thresholds X_B^Z and X_B^d are evaluated at the debt value D_B.

6.3. Proof of Lemma 1

We suppress the subscript i for simplicity. The derivative of the yield to maturity with respect to ϕ is

$$\frac{\partial y}{d\phi} = \frac{1}{T} \left(\frac{F}{D} \right)^{\frac{1}{T}-1} \left(-\frac{F}{D^2} \right) \frac{\partial D}{\partial \phi} = -\frac{1}{T} \left(\frac{F}{D} \right)^{1/T} \frac{1}{D} \frac{\partial D}{\partial \phi}.$$

In Proposition 1, we prove that $\frac{\partial D}{\partial \phi} = \frac{D}{\phi k}$, where $k \geq 1$. Substituting and using the definition of y, we get:

$$\frac{\partial y}{d\phi} = -\frac{1}{T} \left(\frac{F}{D} \right)^{1/T} \frac{1}{D} \frac{D}{\phi \kappa} = -\frac{(F/D)^{1/T}}{T \phi \kappa} = -\frac{1+y}{T \phi \kappa} \; < \; 0.$$

This condition holds unconditionally, because $T, \phi, \kappa > 0$.

6.4. Proof of Proposition 1

The derivatives of the expected discounted values of taxes and default costs are respectively (we suppress dependence on F_i and the subscript i for notational convenience):

$$\frac{\partial T}{d\phi} = \frac{T}{\phi} - \frac{\partial X^Z}{d\phi}(1 - G(X^Z))\phi\tau \tag{15}$$

$$\frac{\partial C}{d\phi} = \alpha\phi\frac{\partial X^d}{d\phi}X^d f(X^d) + \frac{C}{\phi}. \tag{16}$$

Recalling that $X^Z = F - D$, $X^d = F + \frac{\tau}{1-\tau}D$, indeed we have:

$$\frac{\partial X^Z}{d\phi} = -\frac{\partial D}{d\phi}, \tag{17}$$

$$\frac{\partial X^d}{d\phi} = \frac{\tau}{1-\tau}\frac{\partial D}{d\phi}. \tag{18}$$

Hence, we need to focus on the derivative of market debt value with respect to ϕ, for fixed F (dependence of D on F is suppressed for notational convenience):

$$\begin{aligned}
\frac{\partial D}{d\phi} &= \frac{D}{\phi} + \phi\left[(1-\alpha)\frac{\partial X^d}{d\phi}X^d f(X^d) - \tau\frac{\partial X^d}{d\phi}\left(X^d - X^Z\right)f(X^d) + \tau\frac{\partial X^Z}{d\phi}\left[G(X^d) - G(X^Z)\right] + \right.\\
&- \left. F\frac{\partial X^d}{d\phi}f(X^d)\right]\\
\frac{\partial D}{d\phi} &= \frac{D}{\phi} + \phi\left[(1-\alpha)\frac{\partial X^d}{d\phi}\left(\frac{\tau}{1-\tau}D\right)f(X^d) - \alpha F\frac{\partial X^d}{d\phi}f(X^d) - \tau\frac{\partial X^d}{d\phi}\left(X^d - X^Z\right)f(X^d) + \right.\\
&+ \left. \tau\frac{\partial X^Z}{d\phi}\left[G(X^d) - G(X^Z)\right]\right]\\
\frac{D}{\phi} &= \frac{\partial D}{d\phi}\left[1 - \phi(1-\alpha)\left(\frac{\tau}{1-\tau}\right)\left(\frac{\tau}{1-\tau}D\right)f(X^d) + \phi\alpha F\frac{\tau}{1-\tau}f(X^d) + \right.\\
&+ \left. \phi\tau\frac{\tau}{1-\tau}\left(X^d - X^Z\right)f(X^d) + \phi\tau\left[G(X^d) - G(X^Z)\right]\right] \implies \frac{\partial D}{d\phi} = \frac{D}{\phi}\frac{1}{\kappa}
\end{aligned}$$

Since $\frac{D}{\phi} \geq 0$, the sign of $\frac{\partial D}{d\phi}$ depends on the sign of κ. Rearranging it, we have:

$$\begin{aligned}
\kappa &= 1-\phi(1-\alpha)\left(\frac{\tau}{1-\tau}\right)\left(\frac{\tau}{1-\tau}D\right)f(X^d)+\phi\alpha F\frac{\tau}{1-\tau}f(X^d)+ \\
&+ \phi\tau\frac{\tau}{1-\tau}\frac{D}{1-\tau}f(X^d)+\phi\tau\left[G(X^d)-G(X^Z)\right]= \\
&= 1+\phi\alpha\left(\frac{\tau}{1-\tau}\right)\left(\frac{\tau}{1-\tau}D\right)f(X^d)+\phi\alpha F\frac{\tau}{1-\tau}f(X^d)+\phi\tau\left[G(X^d)-G(X^Z)\right]\geq 1.
\end{aligned}$$

As a consequence, using (17) and (18), we have $\frac{\partial X^Z}{d\phi} \leq 0$ and $\frac{\partial X^d}{d\phi} \geq 0$, i.e. the tax shield decreases (increases) and the no-default threshold increases (decreases) with the discount factor ϕ (the interest rate r). This proves part (a) of the proposition.

Also, we have that

$$\left|\frac{\partial X^Z}{d\phi}\right| > \left|\frac{\partial X^d}{d\phi}\right| \implies 1 > \frac{\tau}{1-\tau}$$
$$i.e.\ \tau < \frac{1}{2}.$$

which proves part (b) of the proposition.

Using (15) and (16), it follows directly from part (a) that $\frac{\partial T}{d\phi} \geq 0$ and $\frac{\partial C}{d\phi} \geq 0$, which proves part (c).

6.5. Proof of Proposition 2

When proving Proposition 1, we proved that $\frac{\partial D}{d\phi} \geq 0$ for fixed F. To prove that the value of the SA is increasing with ϕ, we have to prove now the sum of equity and debt is increasing in ϕ, for fixed F:

$$\begin{aligned}
\frac{\partial E}{\partial\phi} &= \frac{E}{\phi}+\phi\left[-(1-\tau)X^d f(X^d)\frac{\partial X^d}{\partial\phi}+\tau\left(1-G(X^d)\right)\frac{\partial X^Z}{\partial\phi}-(\tau X^Z-F)f(X^d)\frac{\partial X^d}{\partial\phi}\right]= \\
&= \frac{E}{\phi}+\phi f(X^d)\frac{\partial X^d}{\partial\phi}\underbrace{\left[-(1-\tau)X^d-\tau X^Z+F\right]}_{=0}+\phi\tau\left(1-G(X^d)\right)\frac{\partial X^Z}{\partial\phi}= \\
&= \frac{E}{\phi}+\phi\tau\left(1-G(X^d)\right)\frac{\partial X^Z}{\partial\phi}=\frac{E}{\phi}-\phi\tau\left(1-G(X^d)\right)\frac{\partial D}{\partial\phi}.
\end{aligned}$$

The above expression is not always strictly greater than zero, and can in fact become negative for very high values of F. However, if we consider firm value $V = E + D$:

$$\frac{\partial V}{\partial \phi} = \frac{\partial D}{\partial \phi}+\frac{\partial E}{\partial \phi}=\frac{\partial D}{\partial \phi}+\frac{E}{\phi}-\phi\tau\left(1-G(X^d)\right)\frac{\partial D}{\partial \phi}=\frac{E}{\phi}+\frac{\partial D}{\partial \phi}\left[1-\phi\tau\left(1-G(X^d)\right)\right].$$

Since the last term in parenthesis is always positive and we proved in Proposition 1 that $\frac{\partial D}{\partial \phi}\geq 0$, then the Proposition is proved.

6.6. Conditions and proof of Proposition 3.

Proposition 3 holds for every $\alpha>0,\tau>0$ if

$$f'(X^d)\geq 0;\ and\ \frac{\partial^2 D}{\partial F\partial \phi}(1-G(X^Z))-(1-\frac{\partial D}{\partial F})f(X^Z)\frac{\partial D}{\partial \phi}\geq 0. \tag{19}$$

When $\alpha\to 1$, the condition is verified for every $0<\tau<1$. If $\tau\to 0$ optimal (zero) debt is unaffected by a change in the interest rate.

Proof. Let us derive the condition. First, we turn to the F.O.C. for the stand-alone firm. Define $\Psi(F,\phi)$ as the sum of the marginal tax burden and default costs. It reads

$$\Psi(F,\phi)=-\tau\frac{dX^Z}{dF}\left(1-G(X^Z)\right)+\alpha X^d f(X^d)\frac{dX^d}{dF}. \tag{20}$$

We know that

$$\Psi(F^*,\phi)=0$$

where F^* is the optimum. Since F^* is $F^*(\phi)$, we have

$$\Psi(F^*(\phi),\phi)=0$$

Let us differentiate both sides wrt ϕ now:

$$\frac{\partial\Psi}{\partial F}\cdot\frac{dF^*}{d\phi}+\frac{\partial\Psi}{\partial\phi}=0$$

Then, we have :

$$\frac{dF^*}{d\phi}=-\frac{\frac{\partial\Psi}{\partial\phi}|_{F=F^*}}{\frac{\partial\Psi}{\partial F}|_{F=F^*}}.$$

For simplicity, from here onwards we omit dependence of the derivatives on the r.h.s. and all of the following quantities on F^*, which is the value of F at which they are evaluated. $\frac{\partial\Psi}{\partial F}>0$, because F^* is a minimum. As a consequence, the sign of $\frac{dF^*}{d\phi}$ is opposite to the sign of $\frac{\partial\Psi}{\partial\phi}$. Then, to ensure

$\frac{dF^*}{d\phi} < 0$ (F^* decreasing with ϕ,i.e. increasing with r), we obtain the following condition:

$$\underbrace{-\tau\frac{\partial^2 X^Z}{\partial F \partial \phi}[1 - G(X^Z)] + \tau\frac{\partial X^Z}{\partial F}f(X^Z)\frac{\partial X^Z}{\partial \phi}}_{\text{Marginal tax burden change}} + \tag{21}$$

$$+\underbrace{\alpha\left[\frac{\partial^2 X^d}{\partial F \partial \phi}X^d f(X^d) + \frac{\partial X^d}{\partial F}\frac{\partial X^d}{\partial \phi}f(X^d) + \frac{\partial X^d}{\partial F}X^d f'(X^d)\frac{\partial X^d}{\partial \phi}\right]}_{\text{Marginal default cost change}} > 0. \tag{22}$$

It follows that

$$\frac{\alpha}{\tau} > \frac{\frac{\partial^2 X^Z}{\partial F \partial \phi}[1 - G(X^Z)] - \frac{\partial X^Z}{\partial F}f(X^Z)\frac{\partial X^Z}{\partial \phi}}{\frac{\partial^2 X^d}{\partial F \partial \phi}X^d f(X^d) + \frac{\partial X^d}{\partial F}\frac{\partial X^d}{\partial \phi}f(X^d) + \frac{\partial X^d}{\partial F}X^d f'(X^d)\frac{\partial X^d}{\partial \phi}}, \tag{23}$$

where the numerator of the r.h.s. is the marginal tax benefit change when ϕ changes, for a unitary tax rate, and the denominator is the marginal default cost change when ϕ changes, for a unitary default cost rate.

The assumption $\frac{\partial^2 D}{\partial F \partial \phi} > 0$ implies that $\frac{\partial^2 X^Z}{\partial F \partial \phi} < 0$ and $\frac{\partial^2 X^d}{\partial F \partial \phi} > 0$.

Hence:

- $-\tau\frac{\partial^2 X^Z}{\partial F \partial \phi}[1 - G(X^Z)] > 0$.
- $\tau\frac{\partial X^Z}{\partial F}f(X^Z)\frac{\partial X^Z}{\partial \phi} < 0$.
- $\frac{\partial^2 X^d}{\partial F \partial \phi}X^d f(X^d) > 0$.
- $\frac{\partial X^d}{\partial F}\frac{\partial X^d}{\partial \phi}f(X^d) > 0$.
- $\frac{\partial X^d}{\partial F}X^d f'(X^d)\frac{\partial X^d}{\partial \phi}$: its sign depends on whether the slope of the density is positive or negative.

Since by assumption $\frac{\partial^2 D}{\partial F \partial \phi} > 0$, the denominator of the r.h.s. of (23) is surely positive as soon as $f'(X^d) \geq 0$. Assume this is the case. Since $\frac{\partial X^Z}{\partial \phi} = -\frac{\partial D}{\partial \phi}$, $\frac{\partial X^d}{\partial \phi} = \frac{\tau}{1-\tau}\frac{\partial D}{\partial \phi}$, we can rewrite the numerator in (23) as:

$$(1 - \frac{\partial D}{\partial F})f(X^Z)\frac{\partial D}{\partial \phi} - \frac{\partial^2 D}{\partial F \partial \phi}(1 - G(X^Z)). \tag{24}$$

When this numerator is negative, tax benefits are decreasing in ϕ and F^* is decreasing in ϕ for any α and τ. This happens if

$$\frac{\partial^2 D}{\partial F \partial \phi}(1 - G(X^Z)) > (1 - \frac{\partial D}{\partial F})f(X^Z)\frac{\partial D}{\partial \phi}.$$

To complete the proof, let us notice that, when $\alpha \to 1$ and $\tau > 0$, then $F^* \to 0$. Then, looking at equation (22), it is easy to see that the l.h.s. is always positive. When $\tau \to 0$, instead, the l.h.s. is always zero and the F.O.C. is unaffected by a change in ϕ. ■

6.7. Proof of Proposition 4

Following Nicodano and Regis (2019), a necessary and sufficient condition for the sponsor to be zero-leverage is:

$$
\begin{aligned}
\frac{\tau_S(1-G(0))(1-\phi(1-G(0)))}{\alpha_B\left[1+\frac{\tau_S}{1-\tau_S}\phi(1-G(0))\right]} \quad &\leq \quad \int_0^{X_B^{Z,*}} xg\left(x, \frac{F_B^*}{1-\tau_S} - \frac{x}{1-\tau_S}\right)dx + \\
&+ \quad \int_{X_B^{Z,*}}^{X_B^{d,*}} xg\left(x, \frac{1-\tau_B}{1-\tau_S}(X_B^{d,*}-x)\right)dx, \qquad (25)
\end{aligned}
$$

where $X_B^{Z,*}, X_B^{d,*}$ indicate the tax shield and the no-default threshold of an optimally leveraged backed unit. The condition ensures that, when the sponsor's debt marginally increases, the increase in the sponsor's tax savings (on the lhs) is no larger than the increase in the backed unit's default costs (on the rhs) which are saved through the guarantee.

The left hand side of the above inequality is decreasing in ϕ (increasing with r), because its derivative with respect to ϕ is negative and it reaches a finite lower bound when $\phi \to \infty$. The right hand side is continuous in ϕ and always non-negative. Then, a $\bar{\phi}$ (or, equivalently, $\bar{r}$) such that (25) is satisfied as an equality exists and, for any $\phi \geq \bar{\phi}$ or, alternatively, for $r \leq \bar{r}$, the condition will be satisfied and the sponsor will be zero-leverage. Since the right hand side is increasing in F_B, it is sufficient to evaluate the condition at a (fixed) level of $F_B \geq F_B^*$ to obtain a necessary condition for the sponsor to be zero-leverage. Let us now prove the last part of the proposition. The l.h.s. of (25) increases in τ_S as soon as $\tau_S < 0.5$. The r.h.s. decreases in τ_S because the support region shrinks as τ_S increases everything else equal (there are less net cash flows available to rescue the backed unit). Hence, $\bar{r}$ decreases with τ_S for $\tau_S < 0.5$. Notice that this is a sufficient condition (evaluated when $\phi = 1$).

6.8. Proof of Corollary 1

Let us consider the first-order conditions for an optimum in SB, where $F_S^* = 0$. Hence, we have a F.O.C. that involves F_B, and reads:

$$\Lambda_B(F_S,F_B,\phi)=\frac{\partial T_B}{\partial F_B}+\frac{\partial C_B}{\partial F_B}-\frac{\partial\Gamma}{\partial F_B}=-\tau_B\phi\frac{\partial X_B^Z}{\partial F_B}(1-G(X_B^Z))+\alpha_B\phi X_B^d f_B(X_B^d)\frac{\partial X_B^d}{\partial F_B}-\frac{\partial\Gamma}{\partial F_B}=\lambda_1, \tag{26}$$

while the F.O.C. involving F_S is met with $F_S^*=0$:

$$\Lambda_S(F_S^*,F_B,\phi)=\frac{\partial T_S}{\partial F_S}+\frac{\partial C_S}{\partial F_S}+\frac{\partial T_B}{\partial F_S}+\frac{\partial C_B}{\partial F_S}-\frac{\partial\Gamma}{\partial F_S}= \tag{27}$$

$$=\left[-\tau_B\phi\frac{\partial X_B^Z}{\partial F_S}(1-G(X_B^Z))+\alpha_B\phi X_B^d f_B(X_B^d)\frac{\partial X_B^d}{\partial F_S}\right]-\frac{\partial\Gamma}{\partial F_S}=\lambda_2 \tag{28}$$

Deriving with respect to ϕ, we get:

$$\frac{\partial^2\Lambda}{\partial F_S^2}\frac{dF_S^*}{d\phi}+\frac{\partial^2\Lambda}{\partial F_S\partial F_B}\frac{dF_B^*}{d\phi}+\frac{\partial^2\Lambda}{\partial F_S\partial\phi}=0$$

$$\frac{\partial^2\Lambda}{\partial F_B\partial F_S}\frac{dF_S^*}{d\phi}+\frac{\partial^2\Lambda}{\partial F_B^2}\frac{dF_B^*}{d\phi}+\frac{\partial^2\Lambda}{\partial F_B\partial\phi}=0,$$

which can be expressed in a more compact form as a system of equations:

$$\underbrace{\begin{pmatrix}\Lambda_{SS} & \Lambda_{SB}\\ \Lambda_{BS} & \Lambda_{BB}\end{pmatrix}}_{H}\begin{pmatrix}\frac{dF_S^*}{d\phi}\\ \frac{dF_B^*}{d\phi}\end{pmatrix}=\begin{pmatrix}-\Lambda_{S\phi}\\ -\Lambda_{B\phi}\end{pmatrix},$$

where Λ_{yx}, $y=S,B$ $x=S,B,\phi$ represents the derivative of Λ_y with respect to x. The solution to the system is

$$\frac{dF_S^*}{d\phi}=\frac{-\Lambda_{S\phi}\Lambda_{BB}+\Lambda_{B\phi}\Lambda_{SB}}{|H|}$$

$$\frac{dF_B^*}{d\phi}=\frac{-\Lambda_{B\phi}\Lambda_{SS}+\Lambda_{S\phi}\Lambda_{SB}}{|H|},$$

where H, the Hessian, is positive definite because (F_S^*,F_B^*) is a minimum; $\Lambda_{SS}>0$,$\Lambda_{BB}>0$ because of convexity. When $F_S^*=0$, which happens by Proposition 4 when $r\leq\bar{r}$, since locally $\frac{dF_S^*}{d\phi}=0$, we have that

$$\frac{dF_B^*}{d\phi}=-\frac{\Lambda_{B\phi}}{\Lambda_{BB}}.$$

Since $\Lambda_{BB} > 0$, the optimal debt of B increases as the interest rate falls if $\Lambda_{B\phi} < 0$. $\Lambda_{B\phi}$ captures the effect of a change in ϕ on the marginal net costs of increasing F_B, as a result of the change in the tax burden and the default costs. Let us now compute $\Lambda_{B\phi}$ explicitly:

$$\Lambda_{B\phi} = \frac{\partial^2(T_B + C_B)}{\partial F_B \partial \phi} - \frac{\partial \Omega}{\partial F_B} + \phi \frac{\partial D_B}{\partial \phi} \left[\frac{\partial^2(T_B + C_B)}{\partial F_B \partial D_B} - \frac{\partial^2 \Omega}{\partial F_B \partial D_B} \right] \tag{29}$$

where $\Omega = \frac{\Gamma}{\phi}$. Notice that, since the sponsor is zero-leverage, there is no effect of the interest-rate change on sponsor's debt value: $-\phi \frac{\partial D_S}{\partial \phi} \left[\frac{\partial^2 \Omega}{\partial F_B \partial D_S} \right] = 0$. B increases its debt optimally when r decreases if its marginal costs decrease ($\Lambda_{B\phi} < 0$). The condition for which this happens is thus:

$$\frac{\partial^2(T_B + C_B)}{\partial F_B \partial \phi} - \frac{\partial \Omega}{\partial F_B} + \phi \frac{\partial D_B}{\partial \phi} \left[\frac{\partial^2(T_B + C_B)}{\partial F_B \partial D_B} - \frac{\partial^2 \Omega}{\partial F_B \partial D_B} \right] < 0$$

$$\frac{\partial \Omega}{\partial F_B} + \phi \frac{\partial D_B}{\partial \phi} \frac{\partial^2 \Omega}{\partial F_B \partial D_B} > \frac{\partial^2(T_B + C_B)}{\partial F_B \partial \phi} + \phi \frac{\partial D_B}{\partial \phi} \frac{\partial^2(T_B + C_B)}{\partial F_B \partial D_B}$$

This inequality compares the effects on the marginal value of the guarantee with those on the marginal costs of B as a stand-alone firm. It states that the sum of the direct increase in the marginal value of debt thanks to the guarantee and the indirect expansion effect (via B's debt valuation) must outweigh the sum of the direct and indirect effects of an interest-rate change on B's stand-alone marginal costs. Notice that this inequality may hold even if the conditions for B to delever as a stand-alone are verified. This completes the proof of our statement.

To understand better when this may occur, let us now analyze the four terms that appear in (29). The first term, under our assumptions, is positive, because we assume that the backed unit behaves as a "normal" stand-alone. The second term depends on the sign of $\frac{\partial \Omega}{\partial F_B}$. Indeed, this is a pure valuation effect, which captures how the marginal value of the guarantee moves when the interest rate changes:

$$
\begin{aligned}
\frac{\partial\Omega}{\partial F_B} = \alpha_B \Bigg[& \underbrace{\int_{\frac{1-\tau_B}{1-\tau_S}\left[X_B^d - X_B\right]}^{+\infty} X_B^d g(X_B^d, X_S)\, dX_S \cdot \frac{\partial X_B^d}{\partial F_B}}_{\text{Increase in guarantee effectiveness (1)}} + \\
& - \underbrace{\int_0^{X_B^Z} X_B g(X_B, \frac{F_B}{1-\tau_S} - \frac{X_B}{1-\tau_S}) \left[\frac{1}{1-\tau_S}\right] dX_B}_{\text{Loss of effectiveness below } X_B^Z \text{ (2)}} + \\
& - \underbrace{\int_{X_B^Z}^{X_B^d} X_B g(X_B, \frac{1-\tau_B}{1-\tau_S}\left[X_B^d - X_B\right]) \left[\frac{1-\tau_B}{1-\tau_S}\frac{\partial X_B^d}{\partial F_B}\right] dX_B}_{\text{Loss of effectiveness above } X_B^Z \text{ (3)}} \Bigg],
\end{aligned}
$$

where we used that $\frac{\partial X_S^d}{\partial F_B} = 0$. Indeed, there are two opposing effects: 1) the increase in X_B^d makes the rescue more valuable, when it is possible (term (1)); 2) the increase in X_B^d makes the rescue more difficult, because the amount of S cash flows required for providing support reduces (terms (2) and (3)). The third term (29) captures the change in B's costs due to the change in the market values of D_B following a change in interest rates. It affects both the marginal costs as a "stand-alone" and the guarantee effectiveness. Indeed, the effect on the guarantee effectiveness is:

$$
\begin{aligned}
\frac{\partial^2\Omega}{\partial F_B \partial D_B} = \alpha_B \Bigg\{ & \left(\frac{\tau_B}{1-\tau_B}\right) \int_0^{+\infty} \frac{\partial}{\partial X_B^d}\left(X_B^d g(X_B^d, 0)\right) dX_S \\
& - \left(\frac{\tau_B}{1-\tau_S}\right) X_B^d g(X_B^d, 0) \\
& + \left(\frac{\tau_B}{1-\tau_S}\right) X_B^Z g(X_B^Z, \frac{D_B}{1-\tau_S}) \\
& - \frac{\tau_B(1-\tau_B)}{(1-\tau_S)^2} \int_{X_B^Z}^{X_B^d} X_B \frac{\partial g(X_B, h_2)}{\partial h_2}\, dX_B \Bigg\}.
\end{aligned}
$$

This term can be positive or negative, depending on the levels of the thresholds X_B^d and X_B^Z and on the joint density f.

About ECGI

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