

When do proxy advisors improve corporate decisions?

Finance Working Paper N° 832/2022

April 23, 2026

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Abstract

This paper studies the impact of proxy advisors on shareholder decision-making. We posit two assumptions: (i) the board is at least as well informed as any individual shareholder; (ii) shareholders can condition their information acquisition on the proxy advisor's recommendation. If only (i) holds, shareholders optimally rubber-stamp the board's proposal and do not invest in private research. If (ii) additionally holds, disagreement between the board and the proxy advisor can trigger private information acquisition by some shareholders. We identify the conditions under which this mechanism restores value-of-information logic and improves the informational quality of shareholder voting outcomes.

Keywords: Collective decision-making, information acquisition, proxy advisors, strategic voting

JEL Classification: G23, D72, D83

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Abstract

This paper studies the impact of proxy advisors on shareholder decision-making. We posit two assumptions: (i) the board is at least as well informed as any individual shareholder; (ii) shareholders can condition their information acquisition on the proxy advisor’s recommendation. If only (i) holds, shareholders optimally rubber-stamp the board’s proposal and do not invest in private research. If (ii) additionally holds, disagreement between the board and the proxy advisor can trigger private information acquisition by some shareholders. We identify the conditions under which this mechanism restores value-of-information logic and improves the informational quality of shareholder voting outcomes.

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1 Introduction

Shareholders vote on a variety of important issues, including director elections, executive compensation, and certain aspects of mergers and acquisitions. During the past two decades, shareholders’ decision making has changed due to the rise of a new business: Proxy advisory firms (such as ISS and Glass Lewis) provide voting recommendations to shareholders. These recommendations have substantial impact on shareholder voting outcomes.¹ There is an ongoing public and scientific debate about the effects of proxy advisors (PAs in what follows) on the quality of decision making in shareholder meetings.² The regulation of PAs is highly contentious, as evidenced, for example, by frequent and substantial changes of rules applied by the U.S. Securities and Exchange Commission (SEC) within the last few years. A key point of contention is that PAs may crowd out shareholders’ incentives to invest in own research. Shareholders who rely on a PA’s recommendation as a substitute for own research save costs individually, but negatively affect the collective by not contributing new information into the decision-making process (Malenko and Malenko, 2019).

In this paper, we show that under two practically relevant assumptions, the presence of a PA actually leads to either more, or, at least, not fewer shareholders investing into private information acquisition, and hence improves corporate decision quality. The basic intuition is that shareholders who otherwise, without a PA, “rubber-stamp” any board proposal, use the PA as a filter to identify controversial issues that deserve further investigation. Proving that this intuition holds generally, even when voters are strategic, and bringing to light its hidden assumptions, necessitates a formal model. By solving the model, we provide the conditions under which disagreement of public signals endogenously restores the acquisition of private information in voting. From a conceptual perspective, the mechanism we study can be understood as a form of sequential value-of-information logic: shareholders optimally acquire a costly private signal only when earlier public signals leave posterior beliefs sufficiently close to indifference.

In our model, there are three types of agents: Shareholders, the firm’s board of directors, and a PA. Shareholders and the board both care about firm value, whereas the PA cares for its profit. There is a proposal on an issue (e.g., a director election). Agents are imperfectly informed about the correct decision on the issue, i.e., about which decision will increase firm

¹See, for example, Alexander et al. (2010); Choi, Fisch, and Kahan (2010); Ertimur, Ferri, and Oesch (2013); Iliev and Lowry (2015); Larcker, McCall, and Ormazabal (2013, 2015); Li (2018); Malenko and Shen (2016); McCahery, Sautner, and Starks (2016); Matsusaka and Shu (2021). PAs can also have an impact by increasing shareholder engagement with firms (Dey, Starkweather, and White, 2022).

²Spatt (2021) provides a survey of the literature with a focus on regulatory issues.

value most. The board and the PA receive a private imperfect and independently distributed signal about the correct decision. The board recommends a decision based on its own signal. For brevity, we refer to this as the “board’s proposal.” Then, each shareholder individually decides whether to buy the PA’s vote recommendation, i.e., the PA’s signal, and whether to invest in own research, i.e., to obtain a private signal. Finally, shareholders vote, and the simple majority rule determines the outcome.

Two key assumptions set this model apart from the existing literature. First, we posit that the board is better-informed than any single shareholder alone. This assumption, “BIB” (for better-informed board), is in line with a long tradition of studies in corporate governance arguing that insiders (the board and management) have information about the company that may be superior to that of shareholders (Jensen and Meckling, 1976). The second key assumption is “PAF” (proxy advice first): *after* receiving proxy advice, a shareholder can decide upon additional research about the issue at hand. This assumption is more likely to hold in regulatory settings that provide shareholders with sufficient time to conduct research.

We solve our game-theoretic model for pure Perfect Bayesian Nash equilibria, mainly focusing on equilibria that are not Pareto-dominated by other equilibria.³ We find that the presence of a PA increases the shareholders’ incentives to invest in own research or leaves these incentives unchanged relative to the case without a PA.

The underlying logic builds on standard value-of-information reasoning, but its application in a voting environment with strategic agents is non-trivial. Assume first that there is no PA. Each shareholder only needs to consider the case in which her vote decides whether the proposal passes or not, i.e., where she is pivotal. If all shareholders vote on the board’s proposal based on their own information, then a shareholder being pivotal must mean that the affirmative and dissenting signals of other shareholders are equally frequent. In this situation, only the information of the pivotal shareholder and the board’s information are crucial. Even if the pivotal shareholder has invested in her own research and this signal is not in favor of the board’s proposal, she is still better off voting in favor of the proposal since the board, by Assumption BIB, is better informed than the shareholder. Therefore, the shareholder would not invest in her own research in the first place and would prefer to always endorse the proposal. This behavior is typically inefficient. Substituting the own research with the informativeness of the board’s proposal is individually rational but collectively harmful, as it leads to correlated mistakes.⁴ Thus, although sequential value-of-information logic would

³Pareto-dominated equilibria are based on coordination failure.

⁴This result holds as long as shareholders do not anticipate the board’s proposal to be so conflicted as to be less informative than even a single shareholder’s own research and, thus, close to uninformative, a model

prescribe private research in controversial cases, it breaks down in the benchmark voting environment due to free-riding and pivotality considerations.

Compared to this benchmark, the presence of a PA leads to higher decision quality in our model. In this sense, the presence of an early second public signal restores the conditions under which standard sequential value-of-information logic becomes operative despite the voting externality. Intuitively, for a shareholder it pays off to invest in own research when there is sufficient controversy about whether the proposal should be accepted. Thus, a PA does not just contribute an additional signal into the decision-making process, but triggers shareholders to conditionally generate their own signals. This complementarity effect is strongest when the board and the PA are similarly well informed such that their contradicting signals indicate strong controversy. We show these results for the whole parameter space and address the cases where each of the two baseline assumptions is violated—providing the exact conditions for endogenous signal acquisition. We also discuss how the core mechanism extends to settings with non-trivial proxy advisor fees or conflicts of interest, clarifying which aspects of the results are robust to monopolistic pricing or biased advice and which require stronger assumptions.

We further extend the analysis to asymmetric equilibria. Asymmetric equilibria allow shareholders to specialize in different strategies, even though they are *ex ante* identical.⁵ It turns out that the crucial strategy of conditionally investing in research is also pervasive in asymmetric equilibria and is, in fact, used in a much larger area of the parameter space than in symmetric equilibria. In addition, depending on the parameters, there are shareholders who always invest in an own signal without subscribing to the PA, shareholders who rubber-stamp the board’s proposals or shareholders who “robo-vote,” i.e., always vote according to the PA’s recommendations. With asymmetric equilibria, too, PAs make the voting outcome more informed.

Although our model is necessarily stylized, it provides important perspectives on some recent regulatory developments, which we discuss in Section 6. For example, some recent proposals would arguably lead to a failure of Assumption PAF, because timelines around shareholder meetings would be adjusted such that investors simply would not have time to do research *contingent on* PA advice. Such interventions would, therefore, inadvertently lead to lower quality corporate decision-making.

variation violating BIB that we discuss in Section 5.

⁵Of course, another reason for shareholders to behave heterogeneously in practice is that they differ *ex ante*. Studies that feature shareholder heterogeneity include Levit, Malenko, and Maug (2021) and Levit, Malenko, and Maug (2024).

Our paper contributes to the literature on common interest voting and on voting in corporations. The literature on strategic voting in a setting of common interest started with Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1996, 1997, 1998).⁶ Informational inefficiencies of voting occur when the symmetry assumptions of the standard Condorcet model are violated, either with regard to the signal technology (Austen-Smith and Banks, 1996) or with regard to the information-transmission process (Gerardi and Yariy, 2007; Iaryczower, Shi, and Shum, 2018; Buechel and Mechtenberg, 2019). Inefficiencies are also generated when private information becomes costly (Persico, 2004; Gershkov and Szentes, 2009), or when public information is provided that is not of sufficiently high quality (Kawamura and Vlaseros, 2017; Jeong, 2019; Liu, 2019; Malenko and Malenko, 2019). Generally speaking, inefficiencies can be generated by correlation of private beliefs across voters, either through public information or through information-transmission processes between voters that are not optimally tailored to the signal (quality) distribution.⁷

A related strand is the literature on voting in corporations. Seminal works in this literature that specifically elucidate the role of strategic voting include Maug (1999), Maug and Yilmaz (2002), and Maug and Rydqvist (2009). A relatively recent innovation has been to enhance the analysis by considering the presence of proxy advisors (Malenko and Malenko, 2019; Malenko, Malenko, and Spatt, 2025). Other work on (strategic) voting in the corporate finance context includes Matvos and Ostrovsky (2006), Brav and Mathews (2011), Levit and Malenko (2011), Van Wesep (2014), Bar-Isaac and Shapiro (2020), Ma and Xiong (2021), Meirowitz and Pi (2022), and Parlasca and Voss (2023), among others.

We contribute to both strands of literature: First, for common-interest voting environments, we characterize when standard sequential value-of-information logic fails and when it is restored once an additional early public signal is introduced. Addressing the whole parameter space, we find the boundaries of signal precision where information acquisition and information aggregation breaks down. Second, we show that proxy advice given early enough can foster shareholders' own investments in research. This has the policy implication that shareholders need to have sufficient time to conduct such research *after* receiving the advice from PAs. Under these conditions, PAs are likely to improve decision quality and social welfare.

⁶For studies of informational efficiency in games with strategic complements and strategic substitutes, see Angeletos and Pavan (2007) and Hellwig and Veldkamp (2009).

⁷Accordingly, Levy and Razin (2015) show that correlation neglect can enhance informational efficiency.

2 Model Setup

2.1 Basic Ingredients

We model voting on corporate decisions as strategic voting under uncertainty (Austen-Smith and Banks, 1996; Feddersen and Pesendorfer, 1996, 1997, 1998) and follow frameworks such as Kawamura and Vlaseros (2017), Malenko and Malenko (2019), Bar-Isaac and Shapiro (2020) and Ma and Xiong (2021). A firm is owned by $N > 1$ shareholders, where N is odd. The firm faces uncertainty with respect to a binary decision. Examples vary by jurisdiction and include director elections, dividends, shareholder proposals, compensation-related matters, etc. Making the ex post correct decision will increase firm value by an amount normalized to 1, while the wrong decision leaves it unchanged.

More formally, there are two states of the world $\theta \in \{A, B\}$ with equal prior probability. Slightly abusing notation, we assume that the firm has to decide on a binary issue $\{A, B\}$ that yields value 1 if and only if the decision matches the true state.⁸

The board of directors receives a binary signal s_B regarding the issue to be voted on. The signal takes on values a or b . The signal quality is $q_B \in (\frac{1}{2}, 1)$, i.e., $Pr[s_B = a | \theta = A] = Pr[s_B = b | \theta = B] = q_B$. We assume that the board then recommends either action A or B . We call this the “board’s proposal.”

A profit-maximizing proxy advisor (PA) offers advice to shareholders at fee $f > 0$. The PA receives a signal about the true state as well. The quality of that signal is $q_P \in (\frac{1}{2}, 1)$. The PA provides a vote recommendation *for* or *against* the board’s proposal to subscribing shareholders.

Before that, shareholders decide whether to subscribe to the PA’s offer. If a shareholder subscribes, she receives the PA’s recommendation.⁹ A shareholder *then* decides whether to invest $c > 0$ in own research about the issue at hand. If a shareholder expends own research costs, this leads to a private signal of quality $q_S \in (\frac{1}{2}, 1)$. Hence, a shareholder knows the board’s signal, can subscribe to get the PA’s signal, and then can invest into an own private signal. When the shareholder meeting is held, each shareholder votes *yes* or *no*. Abstentions are excluded.¹⁰ For simplicity, each shareholder holds one share of the firm and each share provides one vote. The decision that receives a majority of votes is implemented.

⁸As long as the vote we consider remains a decision between two different actions, with one Pareto-dominating the other conditional on the unobservable state of the world, the main argument of the paper would still apply when introducing a continuous state space or a continuous signal technology.

⁹Hence, the PA sells information directly in the sense of Admati and Pfleiderer (1990).

¹⁰In practice, shareholders may also abstain. However, according to most institutional settings abstentions are counted (either as *yes* or *no*) and hence shareholders’ voting action is essentially binary.

Conditional on state θ , all signals are independent, and precision levels q_B , q_P , and q_S are common knowledge.

Starting from this standard framework, our first key assumption is that the board knows better than any single shareholder what is good for the company.

Assumption 1 (BIB). *The board is at least as well informed as a single shareholder, i.e.,*

$$q_S \leq q_B.$$

“BIB” stands for better-informed board. One interpretation is that the proposal itself already contains valuable information that would be hard to match for a single shareholder alone. For the quality of the PA q_P we do not make any analogous assumption that restricts it to be above or below the other agents’ qualities.

In the course of the analysis it will come in handy to transform signal qualities $q \in (0.5, 1)$ into log-odds $\log(\frac{q}{1-q}) \in (0, \infty)$. We denote the log-odds of the board being correct as $\ell_B := \log(\frac{q_B}{1-q_B})$ and likewise $\ell_S := \log(\frac{q_S}{1-q_S})$ for the shareholders and $\ell_P := \log(\frac{q_P}{1-q_P})$ for the PA. Then Assumption BIB reads $\ell_S \leq \ell_B$.¹¹ This notation is convenient since it allows us to aggregate signal qualities by summation. To see this, consider the board’s signal b and assume, for instance, that both the PA and one shareholder have received signals a and there is no further information. Then, the board’s signal is rather correct than not if and only if $q_B(1 - q_P)(1 - q_S) \geq (1 - q_B)q_Pq_S$, which is equivalent to $\ell_B \geq \ell_P + \ell_S$.

Our second key assumption is that shareholders can condition their research investment on the PA’s recommendation.

Assumption 2 (PAF). *Subscribing shareholders decide upon own research investment after they have received the PA’s recommendation.*

“PAF” stands for “proxy advice first” and means that the proxy advisor’s recommendation arrives early enough to still acquire private information on the specific issue.¹²

¹¹Nitzan and Paroush (1982) show that among voters with idiosyncratic signal precision the optimal voting weights would be according to these log-odds.

¹²Relaxing this assumption would change the timing of our model such that shareholders have to decide simultaneously about subscribing to the PA and about investing in own research. We discuss the consequences of making this assumption in our model in Section 6.2 and provide the corresponding results in Supplementary Online Material (SOM) Section 1.3.

2.2 Simplification and Timeline

It turns out that we can substantially simplify the exposition by fixing the signal and behavior of the board and the behavior of the PA. The board receives a signal and then makes a recommendation. We let the board's signal be always b (for board).¹³ We fix the board's behavior by assuming it makes the proposal according to its signal, i.e., it has received signal b and now proposes action B .¹⁴ Likewise, we fix the PA's behavior to set fee $f > 0$ and recommend according to its signal, i.e., it recommends *for* if it has received signal b (for board) and it recommends *against* if it has received signal a (against board).

These simplifications do not affect the substance of our analysis and results. Since the board, like the shareholders, aims at maximizing firm value, revealing its signal to help shareholders decide for the optimal policy is in its own interest. The proxy advisor, in turn, can generate profits only from helping individual shareholders to make a decision that is even more informed than it would be without the PA. The reason is that the shareholders' willingness to pay for proxy advice depends only on the PA's contribution to the informedness of the vote.¹⁵ In this sense, the fee f should be interpreted as reflecting shareholders' willingness to pay for an incremental public signal, rather than as the outcome of strategic pricing, market power, or contractual frictions on the part of the proxy advisor. Hence, the optimal strategy of the PA is to reveal its true signal whenever asked to do so, and to set the fee f equal to the shareholder's willingness to pay for this revelation. Hence, explicitly including a strategic PA and an endogenous fee does not add interesting results for the informational mechanism we study, such that we can focus on the main implications derived from the simplified setup. The role of non-trivial fees and research costs for equilibrium behavior is discussed explicitly in Section 5.3.

The timeline, which is illustrated by Figure 1, summarizes the simplified setup. At $t = 0$ nature draws a state of the world and signals for all potential recipients of signals. The players learn the board's proposal and hence its signal. At $t = 1$ each shareholder decides whether to pay the fee for the PA's report. Those who pay the fee receive the truthful vote recommendation which is equivalent to learning the PA's signal. At $t = 2$ each shareholder decides whether to invest costs c to receive an own independently and identically distributed

¹³This will exclude strategies that depend on the label of the alternative, such as always voting *yes* for alternative A and *no* for alternative B independent of which alternative the board has proposed.

¹⁴In Section 5.4, we discuss to which extent, and with which implications, a re-interpretation of our model covers conflicts of interest between board and shareholders.

¹⁵In Malenko, Malenko, and Spatt (2021), the PA biases a costless public recommendation to enhance shareholders' perceived pivotality since their willingness to pay for the PA's costly signal increases in pivotality. Heterogeneity in voters, which allows these manipulations to be effective, is created by subjective values.

signal of quality q_S . At $t = 3$ shareholders vote. At $t = 4$ the proposal passes if a majority approves it and payoffs are realized.

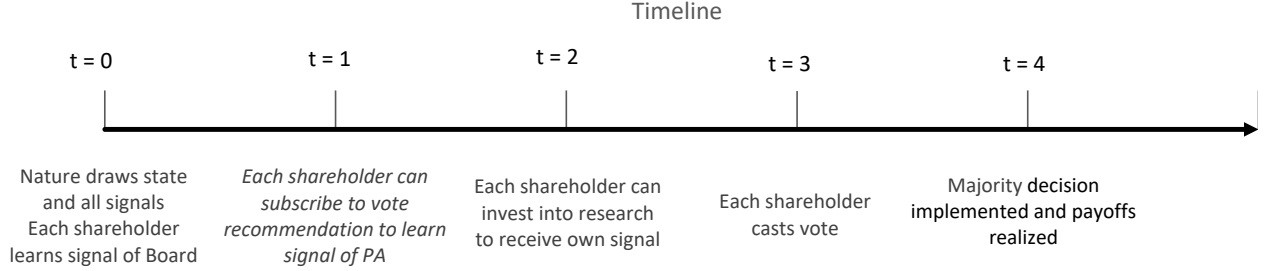


Figure 1: Timeline. For simplicity, the board’s and PA’s behavior is fixed. (Actions in italics only apply if there is a PA.)

2.3 Strategies

The most important strategic aspects concern the shareholders. They have several strategies both on the information acquisition stages ($t = 1$ and $t = 2$, respectively) and on the voting stage ($t = 3$). On the information acquisition stages, there are six strategies: A shareholder who does not subscribe may invest in own research (NotSubscribe-Invest) or not (NotSubscribe-NotInvest); a shareholder who does subscribe may unconditionally invest in research (Subscribe-Invest) or not (Subscribe-NotInvest) or, else, may invest in research only if the recommendation is *for* (Subscribe-InvestIFF*for*) or only if the recommendation is *against* (Subscribe-InvestIFF*against*).

In the voting stage, any shareholder chooses *yes* or *no*. The set of voting strategies depends on the acquired information which may include the PA’s signal and the own signal. For instance, for a shareholder who acquired both kinds of information (e.g., with Subscribe-Invest), a voting strategy is a mapping $v_i : \{for, against\} \times \{a, b\} \rightarrow \{yes, no\}$. Slightly abusing notation, we write σ_i for the information acquisition *and* voting strategy of a shareholder i , and we use $\sigma = (\sigma_1, \dots, \sigma_N)$ to denote a strategy profile of shareholders.

We study Perfect Bayesian Nash equilibria, i.e., players best respond to their beliefs and update their beliefs according to Bayes’ rule wherever possible. We focus on pure strategies, but analyze both symmetric and asymmetric strategy profiles.

Asymmetric strategy profiles capture heterogeneous behavior of shareholders better than symmetric mixed strategies would; and they are more efficient in common interest voting games (see, e.g., Kawamura and Vlaseros, 2024). Existence of equilibria is not an issue, as a symmetric equilibrium always exists, e.g., every shareholder *rubber-stamps* (i.e., votes *yes*

without subscribing to the PA nor investing into an own signal). Multiplicity of equilibria we handle by selecting those that are Pareto-efficient among them. This eliminates equilibria that are due to miscoordination.

To analyze the model we take the perspective of a regulator who compares a market with a PA, as in the game defined above, with a market in which no PA is admitted (i.e., the game above without actions at $t = 1$). The regulator maximizes welfare which requires maximizing decision quality in our setup. We will assume that costs of information acquisition, be it fee f or costs c , are relatively small compared to benefits of decision quality. Hence, when shareholders have to trade off costs of information acquisition with benefits of higher firm value, we will assume that the latter dominates.¹⁶ When there are two strategies with the same decision quality, then shareholders strictly prefer the one with lower costs, as we assume that costs are strictly positive. The quality of corporate decisions is measured by $\Pi(\sigma)$, the ex ante probability that the decision will match the true state.¹⁷ In what follows, all propositions of the main text are proven in Appendix A in this document. Some further arguments used in the discussion in the main text are formally shown in the Supplementary Online Material (SOM), which is available here: <https://bit.ly/proxy-SOM>.

3 Main Results

Benchmark: Without Proxy Advisor. Consider first the benchmark situation that no PA is admitted. Thus, posit that in the timeline of Figure 1 actions at $t = 1$ are suppressed. Then a shareholder’s information acquisition decision reduces to whether to acquire an own signal or not in $t = 2$. Suppose for a moment that all shareholders do acquire such a signal and vote according to it. We call this strategy profile *UNIS*, for “UNconditional Investment in own Signal,” where the term “unconditional” will be justified later, when shareholders could potentially condition their investment in own research on the PA’s vote recommendation.¹⁸ In this strategy profile the decision quality amounts to $\Pi(\sigma^{UNIS}) = \pi(N)$, where $\pi(N) := \sum_{i=\frac{N+1}{2}}^N \binom{N}{i} q_S^i (1 - q_S)^{N-i}$ is the probability that a majority decision of N shareholders is correct.

¹⁶We discuss the relaxation of this assumption in Section 5.3.

¹⁷This is also called *informational efficiency*, which can be distinguished from *economic efficiency* (see, e.g., Buechel and Mechtenberg, 2019). Economic efficiency means welfare, which here can be defined as $\Pi(\sigma)$ net of the investment costs in own research since the prices paid to the PA are transfers. When investment costs c become arbitrarily small, the two concepts coincide.

¹⁸For simplicity, we use the same labels for strategies as for the symmetric strategy *profiles* composed of these strategies.

While the decision quality of such voting behavior is usually very high (De Caritat, 1785), it is unfortunately not an equilibrium under Assumption BIB. The intuition is straightforward once spelled out. A single shareholder i can improve by deviating to not acquire a signal and vote *yes*. When this shareholder i is pivotal, the signals of all $N - 1$ other shareholders are split: there are as many a -signals as there are b -signals among them. Now, even if i 's signal points against the board's proposal, Assumption BIB, i.e., the assumption that the board is at least as well informed as i , makes it beneficial to vote *yes*, i.e., for the board's proposal, and not to acquire own information in the first place. We call this latter strategy and its corresponding strategy profile “*Rubber-stamping*”.¹⁹ This insight generalizes.

Proposition 1 (SYM without PA). *Suppose no PA is admitted. If Assumption BIB holds, then there does not exist a symmetric equilibrium in which shareholders invest in own research. Hence, decision quality in symmetric equilibria is bounded by: $\Pi(\sigma) \leq q_B$. The Pareto-efficient²⁰ symmetric equilibrium is Rubber-stamping and leads to decision quality $\Pi(\sigma^{Rubber}) = q_B$.*

Proposition 1 shows that without a PA the quality of decision making is bounded by the quality of the board whose proposal is rubber-stamped by the shareholders. This result is related to the first theorem in Austen-Smith and Banks (1996), which shows that it is not an equilibrium to vote according to the private signal when a proposal is ex ante more likely to be correct. The result is also related to the substitution effect of Malenko and Malenko (2019), but occurs on a different level: For the scenario that the board's proposals are uninformative, Malenko and Malenko (2019) obtain over-reliance of shareholders on the PA's recommendations. For the scenario that the board's proposals are informative, we obtain over-reliance on this proposal without any PA, as a new benchmark.²¹

Including Proxy Advice. The presence of a proxy advisor substantially increases a shareholder's set of information-acquisition strategies. One of them, *Subscribe-InvestIFFagainst*, gives rise to the following symmetric strategy profile, which we denote by $\hat{\sigma}$ and call “*CAIS* (Conditional on Advice Invest in Signal):” All shareholders subscribe to proxy advice; if the recommendation is *for*, they vote *yes*; if the recommendation is *against*, they invest in own

¹⁹Assumption BIB, $q_S \leq q_B$, is in fact necessary and sufficient for the conclusion of Proposition 1. Since we have Assumption BIB as a leading assumption, we only show sufficiency in the proof of Proposition 1.

²⁰Recall that the criterion of Pareto-efficiency is applied within the set of symmetric equilibria.

²¹The result that no shareholder invests in own research is stylized. When studying asymmetric equilibria without PA, we find the same message in a less stylized form: Without PA, there always exist shareholders who do not invest in own research, while the number of shareholders who do is bounded.

research and vote according to their own signal, i.e., vote *yes* if the signal is *b* and *no* if it is *a*. CAIS is illustrated in Table 1.

In this strategy profile shareholders use the PA's recommendation as a filter: *for* recommendations are followed without being challenged; *against* recommendations trigger further investigation of the issue.

		Own Signal	
		<i>b</i> (for board)	<i>a</i> (against board)
PA's recommendation	<i>for</i>	yes	
	<i>against</i>	yes	no

Table 1: Strategy CAIS: Invest in own research if and only if vote recommendation is *against*; and after *for* recommendation vote *yes*, after *against* recommendation vote *yes* if and only if signal is for board.

It turns out that based on this strategy profile the negative result of Proposition 1 can be mitigated by the presence of a PA.

Proposition 2 (SYM with PA). *Let Assumptions BIB and PAF hold. Let costs c be arbitrarily small and let fee f be sufficiently smaller. Suppose there is a PA with $\ell_P \in (\ell_B - \ell_S, \ell_B + \ell_S)$. Then there exists a symmetric equilibrium in which shareholders conditionally invest in own research. The Pareto-efficient equilibrium is CAIS and leads to decision quality $\Pi(\hat{\sigma}) > q_B$. Otherwise (i.e., if $\ell_P \notin (\ell_B - \ell_S, \ell_B + \ell_S)$), the Pareto-efficient equilibrium is Rubber-stamping with $\Pi(\sigma^{Rubber}) = q_B$.*

In the proof of Proposition 2 (Appendix A.2) we proceed as in the proof of Proposition 1 (Appendix A.1), by providing first the full characterization of all equilibria as a lemma and then selecting those that are Pareto-efficient among them. Comparing Proposition 2 with Proposition 1, we conclude that the presence of a PA either strictly improves decision quality or leaves it unchanged, compared to the setting without a PA. The condition for the strict improvement can be rewritten as $|\ell_B - \ell_P| < \ell_S$, which has the following interpretation: the difference in quality of board and PA is smaller than the information quality of one shareholder. If this condition is satisfied, we have a new equilibrium (CAIS) with a PA in which all shareholders conditionally invest in own research.

The first intuition for the conditions of the PA being beneficial as stated in Proposition 2 can be seen from their violations. Consider the symmetric strategy profile CAIS. If $\ell_P \leq \ell_B - \ell_S$, we have $\ell_S + \ell_P \leq \ell_B$, i.e., the board is better informed than the PA and one shareholder together. Then there is a deviation from CAIS to Rubber-stamping. Intuitively,

the board is sufficiently well informed that it does not individually pay off to acquire any information, even if it were costless. If $\ell_P \geq \ell_B + \ell_S$, i.e., the PA is better informed than the board and one shareholder together, then there is a deviation from CAIS, e.g., to not investing and to voting against the board's proposal. Indeed, the deviating shareholder's vote is only pivotal if board and PA disagree and voting *no* improves decision quality, given that the PA is so well informed. If costs c or f are not small enough, there is again a beneficial deviation, e.g., to Rubber-stamping, which saves costs. Finally, if the PA's fee f is not sufficiently smaller than the costs c , then deviating to UNIS saves costs without affecting the outcome.²²

In Section 1 of the Supplementary Online Material (SOM), we extend the analysis to the complete characterization of all symmetric equilibria in pure strategies, with and without Assumptions BIB and PAF. Most importantly, that analysis shows that the two key assumptions Assumption BIB and PAF are not only sufficient but also necessary for the conclusion, as we will discuss below.

4 Illustration and Discussion

Numerical Example. Example 1 numerically illustrates Propositions 1 and 2.

Example 1 (Symmetric Equilibria). *Let $q_B = 0.75$, $q_P = 0.7$, and $q_S = 0.6$. Then $\ell_B = 0.477$, $\ell_P = 0.368$, and $\ell_S = 0.176$ such that the condition $\ell_P \in (\ell_B - \ell_S, \ell_B + \ell_S)$ of Proposition 2 is satisfied, as $0.368 \in (0.477 - 0.176, 0.477 + 0.176)$. Table 2 illustrates the implications of Propositions 1 and 2 for decision quality. First, not admitting a PA leads to Rubber-stamping and hence to a decision quality of $q_B = 0.75$, independent of the number of shareholders N (Proposition 1). Second, when a PA is admitted, CAIS is the Pareto-efficient symmetric equilibrium, which delivers a strictly higher decision quality (by Proposition 2). Its decision quality is further increasing in the number of shareholders N and approaching $0.925 < 1$ for large N . The decision quality differs from one by the probability that the board's and the PA's signal agree on the wrong state. Finally, Table 2 shows the hypothetical case in which all shareholders play UNIS, i.e., invest in own research. This is not an equilibrium but a classic benchmark capturing the quality of majority decisions by N independently informed*

²²The assumption c small enough assures that shareholders who can improve decision quality by investing in own research would not shy away due to the high costs. The assumption that the costs are larger than zero matters when deviations that do not affect decision quality are considered. The assumption that fees f are sufficiently smaller than c means that the results answer the question whether there is a fee f such that a PA can profitably be active in the market.

voters, as already pointed out by the Marquis de Condorcet (*De Caritat*, 1785). In this benchmark case, decision quality may start low, but becomes larger than in equilibrium for a sufficiently large number of voters.

Setting	Decision quality	$N = 3$	$N = 5$	$N = 21$	$N = 101$	$N = 1,001$
No PA	$\Pi(\sigma^{Rubber}) = q_B$	0.75	0.75	0.75	0.75	0.75
With PA	$\Pi(\hat{\sigma}) = q_B q_P + p^{dis} \pi(N)$	0.784	0.798	0.855	0.917	0.925
Hypothetical	$\Pi(\sigma^{UNIS}) = \pi(N)$	0.648	0.683	0.826	0.979	1.0

Table 2: Decision quality in Example 1, illustrating Propositions 1 and 2 for $q_B = 0.75$, $q_P = 0.7$, and $q_S = 0.6$. The table considers the two Pareto-efficient symmetric equilibria, Rubber-stamping and CAIS, and strategy profile UNIS, which is not an equilibrium. $p^{dis} := (1 - q_B)q_P + q_B(1 - q_P)$ is the probability that the board's and the PA's signal differ.

To illustrate how information-acquisition costs affect welfare comparisons, consider the individual costs $C(\sigma)$ of the three symmetric strategy profiles: $C(\sigma^{Rubber}) = 0$, $C(\hat{\sigma}) = f + p^{dis}c$, and $C(\sigma^{UNIS}) = c$ for each shareholder. While f is a welfare-neutral transfer from shareholders to the proxy advisor, non-trivial costs c matter for the welfare comparison, as they represent real resource expenditures. Admitting non-trivial costs, CAIS yields higher welfare than Rubber-stamping if and only if $\Pi(\hat{\sigma}) - \Pi(\sigma^{Rubber}) > Np^{dis}c$, which is equivalent to $c < \frac{1}{N}(\pi(N) - \frac{q_B(1-q_P)}{p^{dis}})$, where $p^{dis} := (1 - q_B)q_P + q_B(1 - q_P)$. It measures when the costs of the more informative equilibrium are justified by higher decision quality. For instance, in Example 1 with $N = 21$, that is $c < 0.0125$, which means that the individual private costs of information acquisition must not exceed 1.25% of the normalized value of the decision for the firm.²³

We now turn to illustrating Propositions 1 and 2 graphically, while at the same time extending our analysis to the entire parameter space.

Graphical Illustration. Figure 2 illustrates the full parameter space. An entry (x, y) in this coordinate system has the simple interpretation that the board is equally well informed as x shareholders, while the PA is equally well informed as y shareholders.²⁴

In the upper panel of Figure 2, no PA is admitted. By Proposition 1, Rubber-stamping is the Pareto-efficient equilibrium under Assumption BIB, i.e., for $q_S \leq q_B$. This is illustrated

²³We will return to non-trivial costs and study their implications for deviations and equilibria in Subsection 5.3.

²⁴“Equally well informed” means that if x shareholders have received a signal a (against the board) then both states A and B are equally likely. Hence, if more than x shareholders have received a signal a and there is no other information, then the board should be overruled.

in the area $\frac{\ell_B}{\ell_S} \geq 1$. Assumption BIB is necessary and sufficient for this conclusion as UNIS is the Pareto-efficient equilibrium for $\frac{\ell_B}{\ell_S} < 1$. Hence, when there is no PA, information acquisition occurs if and only if the board is less well informed than a single shareholder, i.e., when Assumption BIB is violated.²⁵

In the lower panel of Figure 2, there is a PA and Assumption PAF is satisfied. Proposition 2 has shown that under Assumption BIB, i.e., for $\frac{\ell_B}{\ell_S} \geq 1$, we have either CAIS or Rubber-stamping as Pareto-efficient symmetric equilibrium. Moreover, by Proposition 2 the parameter space in which CAIS is an equilibrium is given by the condition $\ell_P \in (\ell_B - \ell_S, \ell_B + \ell_S)$, which defines a corridor around the 45-degree line.²⁶ On the 45-degree line the board and the PA are exactly equally well informed, i.e., $\frac{\ell_P}{\ell_S} = \frac{\ell_B}{\ell_S}$ (or $q_B = q_P$). Note that this corridor is not bounded from the upper right. Hence, for arbitrarily well-informed board and PA, there is still an equilibrium with conditional information acquisition of all shareholders, as long as the board and the PA are roughly equally-well informed. The intuition is that whenever their signals contradict each other, there is sufficient controversy to invest in own research. Below this corridor, the board's signal is more informative than the PA's and a single shareholder's signal together such that any shareholder can deviate to Rubber-stamping. Similarly, above the corridor the PAs' signal is more informative than the board's and a single shareholder's signal together such that any shareholder has an incentive to deviate, e.g., to subscribe to the PA and follow its advice. A strategy profile consisting of only this strategy, call it *Follow* (or "robo-voting"), is however not an equilibrium, as any agent can save costs by not subscribing to the PA without affecting the outcome.²⁷ That is why Rubber-stamping is also the Pareto-efficient symmetric equilibrium in this area of the parameter space.

We now assess the comparative statics of changing information quality. Assume $\frac{\ell_B}{\ell_S} > 1$ and start with an uninformed PA: $q_P \approx 0.5$, i.e., $\frac{\ell_P}{\ell_S} \approx 0$. Decision quality remains unaffected by the PA's information quality q_P (or $\frac{\ell_P}{\ell_S}$) at first, then discontinuously increases from q_B to $\Pi(\hat{\sigma})$. Within the region where CAIS is an equilibrium, decision quality further improves as $\Pi(\hat{\sigma})$ is continuously increasing in q_P . Finally, it returns to the level q_B when Rubber-stamping is played again. Hence, there is a non-monotonic effect of a PA's information quality on the corporate decision quality with the latter being highest for a PA that is slightly better informed than the board.²⁸ Comparative-static effects of the board's information quality

²⁵The additional results, due to the violation of Assumption BIB, are provided in the Supplementary Online Material (SOM) Section 1.

²⁶When studying asymmetric equilibria, we show that CAIS can be played by a majority of shareholders far beyond this corridor. The corridor only restricts the area in which *all* shareholders play CAIS.

²⁷We will revisit this question when studying asymmetric equilibria. It turns out that Follow can be part of an equilibrium strategy profile.

²⁸A non-monotonic effect of the PA's recommendation quality on the corporate decision quality is also

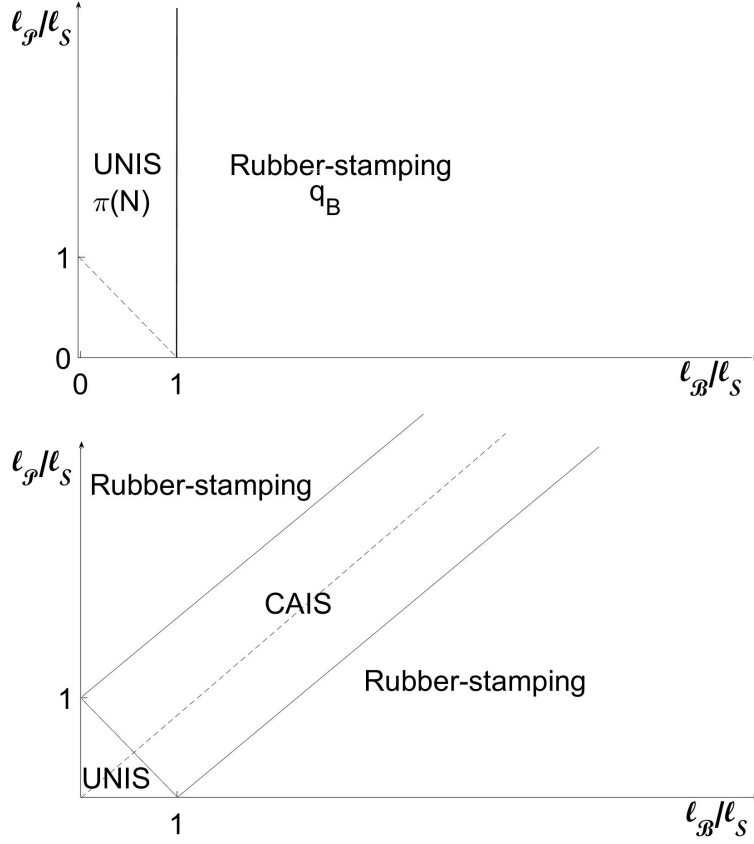


Figure 2: Parameter space with Pareto-efficient symmetric equilibria. Upper panel: without a PA; lower panel: with a PA. Both panels: The areas to the left of $\frac{\ell_B}{\ell_S} = 1$ are precluded by Assumption BIB; they are still depicted here to show the effect of the assumption.

are analogous if $\frac{\ell_P}{\ell_S} > 1$, i.e., the PA is better informed than a single shareholder. Finally, increasing signal quality of the shareholders, q_S , reduces $\frac{\ell_B}{\ell_S}$ and $\frac{\ell_P}{\ell_S}$, which means graphically moving towards the origin. This improves decision quality of CAIS because shareholders base their decision on their own information when the PA's recommendation is *against*.

Assumption BIB, i.e., $\frac{\ell_B}{\ell_S} \geq 1$, rules out UNIS, the strategy profile in which all shareholders acquire information. Violating BIB while satisfying PAF, UNIS is the Pareto-efficient symmetric equilibrium in the lower left corner of the parameter space (in the lower panel of Figure 2), which is defined by the condition $\ell_S > \ell_B + \ell_P$. Hence, in the presence of a PA, UNIS requires that one single shareholder must be better informed than board and PA

predicted by the analysis of Malenko and Malenko (2019). In their model, higher PA quality always weakly reduces the shareholders' investment in private signals such that maximal research incentives are obtained for the lowest PA quality. In our model, maximal research incentives are obtained for intermediate PA quality, namely when it equals the board's quality. If proxy advice does not arrive sufficiently early or if the PA is already better informed than the board, a competence-increasing regulation of the PA may undermine shareholders' research incentives and affect decision quality negatively.

together. Interestingly, this is an even stronger condition than the condition for UNIS when no PA is admitted ($\ell_S > \ell_B$).

Let us now compare the upper panel with the lower panel. Under Assumption BIB, i.e., for $\frac{\ell_B}{\ell_S} \geq 1$, the presence of a PA weakly improves decision quality, as it replaces Rubber-stamping with CAIS if anything. When Assumption BIB is violated, there can be a different effect. Suppose that the quality of the board is not much better than a coin flip, i.e., $q_B \approx 0.5$. Then $\frac{\ell_B}{\ell_S} \approx 0$ and there is the equilibrium with full information acquisition (UNIS) and high decision quality, as long as no PA is admitted. The presence of a PA who is better informed than a single shareholder ($\frac{\ell_P}{\ell_S} > 1$) destroys this equilibrium and reduces decision quality from $\pi(N)$ to $q_B \approx 0.5$. The reason is that conditional on pivotality a shareholder prefers to follow the PA's recommendation over acquiring and using the own signal. Hence, Assumption BIB dramatically changes how admission of a PA affects decision quality when studying symmetric equilibria.

5 Extensions and Robustness

5.1 Asymmetric Equilibria

We have characterized the (Pareto-efficient) symmetric equilibria. Since the characterization of all (Pareto-efficient) asymmetric equilibria is quite cumbersome and involves many case distinctions, we relegate it to Supplementary Online Material (SOM) Section 2 and provide only the main result and the essence of the other findings here. Interestingly, although we model shareholders as ex ante homogeneous, there is specialization on different strategies in the Pareto-efficient equilibria, e.g., in one typical equilibrium, some shareholders play CAIS, some shareholders play UNIS, while others play either Rubber-stamping, or always follow the PA's recommendation, depending on whether the board or the PA is better informed.

An overview of the Pareto-efficient asymmetric equilibria is presented in Figure 3 which is proven in SOM Lemma 2.3. *Protest* is defined as the strategy in which a shareholder does not invest in research and votes *no*. *Follow* refers to the strategy where a player buys the PA's recommendation and follows it when voting (without investing in own research).

Our basic results remain similar. In particular, we can first show that without PA, the number of shareholders who invest in own research is bounded from above. That is, in the equilibrium without a PA there are always some shareholders not investing in research, given that Assumption BIB is satisfied. Second, when admitting a PA whose signal quality is not too far from the board's, the number of shareholders who invest or conditionally

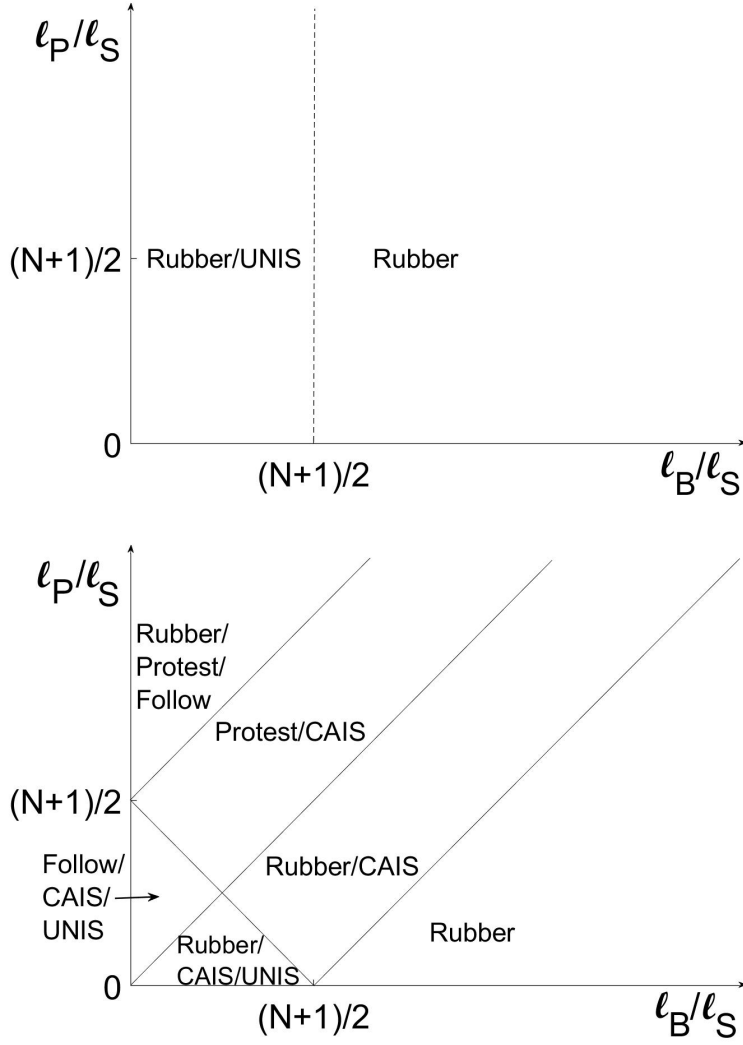


Figure 3: Parameter space with Pareto-efficient (potentially asymmetric) equilibria. Upper panel without a PA based on SOM Proposition 2.1, lower panel with a PA based on SOM Lemma 2.3. CAIS is part of the equilibrium strategy profile in large areas of the parameter space.

invest weakly increases. Again, the crucial innovation is that the PA's recommendation is used as a condition to invest in own research in the strategy CAIS. While this was true for all shareholders in Proposition 2 in a certain parameter range, we now find this in much larger areas of the parameter space, while no longer all shareholders use this strategy (see again Figure 3). More precisely, if we have $\frac{|\ell_B - \ell_P|}{\ell_S} < \frac{N+1}{2}$, then $N - \frac{|\ell_B - \ell_P|}{\ell_S}$, i.e., more than half of all shareholders, invest in own information, either conditionally as in CAIS or even unconditionally as in UNIS. The above condition means that the difference between the information quality of the PA and the information quality of the board must not exceed the aggregated information quality of about half of all shareholders together, which graphically widens the corridor in the lower panel of Figure 2 from starting at 1 to starting at $\frac{N+1}{2}$ in the lower panel of Figure 3 on both axes.²⁹ Moreover, the number of investing shareholders, $N - \frac{|\ell_B - \ell_P|}{\ell_S}$, is decreasing in the difference of information quality between the board and the PA. Hence, we find the strongest research incentives for shareholders when the PA is as well informed as the board.³⁰

Finally, the question remains how the effects of a PA on equilibrium behavior translates into decision quality. Proposition 3 provides the answer.

Proposition 3 (ASYM). *Let costs c and f be sufficiently small. For any setting of signal qualities $q_B, q_P, q_S \in (\frac{1}{2}, 1)$, decision quality in any Pareto-efficient equilibrium with a PA under Assumption PAF, $\Pi(\sigma^*)$, is weakly higher than the decision quality in any strategy profile without a PA (including their Pareto-efficient equilibria), i.e., $\Pi(\sigma^*) \geq \bar{\Pi}^{no-PA}$, where $\bar{\Pi}^{no-PA}$ is the maximal decision quality for any strategy profile in the game without a PA.*

The proof of Proposition 3 considers all Pareto-efficient strategy profiles and shows that each of them is an equilibrium with maximal decision quality. It then exploits the insight that any decision quality attainable without a PA can be replicated in the presence of a PA that is simply ignored. Hence, decision quality cannot be reduced by admitting a PA. There are some technical differences between Proposition 3 on asymmetric equilibria and the propositions on symmetric equilibria. First, the simple line of argument used in this proof does not apply to symmetric equilibria. Accordingly, the inefficiency issue that arises there does not carry over to asymmetric equilibria. Second, the assumptions differ: Proposition 3 does not require the fee f to be sufficiently smaller than the cost of private research c . This makes the analysis consistent with the interpretation in Section 2.2, where the fee f reflects

²⁹Observe that the larger the number of shareholders N , the less demanding this assumption is.

³⁰Other comparative-static effects might be different for asymmetric equilibria than for symmetric, see Supplementary Online Material (SOM) Section 2.

shareholders' willingness to pay for the informational value of proxy advice, rather than strategic pricing or market power on the part of the proxy advisor.

Despite these technical differences, the main insight that PAs weakly improve decision quality carries over. This result holds throughout the entire parameter space when considering asymmetric strategy profiles. Moreover, the novel type of equilibrium behavior identified in this paper (CAIS) exists in a broad range of the parameter space.

5.2 One Dominant Shareholder

We have thus far assumed that $N > 1$. Let us now consider the case of only one shareholder $N = 1$, which applies to any company with a shareholder who holds a decisive majority of shares. We can show that both main results carry over to this case. First, without a PA, there is no incentive to invest in research under Assumption BIB, i.e., for $q_S \leq q_B$. Second, the presence of a PA with appropriate information quality improves decision quality, as it leads to a Pareto-efficient equilibrium in which the shareholder conditionally invests in research.

Does this imply that the voting context is unnecessary for our analysis? Certainly not. The single-shareholder case masks the importance of the timing. Since a single shareholder is always pivotal, the Assumption PAF is not necessary for research investment in that special case. That is, even when the subscription decision and the information acquisition decision are made simultaneously, there is an equilibrium with investment in own research for $N = 1$. In this equilibrium strategy the shareholder subscribes to the vote recommendation and invests in own research (Subscribe-Invest) and votes *yes* if and only if at least one of the two supports the board's proposal. Hence, for the case of only one shareholder, there is a complementarity between proxy advice and own research, independently of the timing of the two decisions; while our analysis of voting brings to light that complementarity relies on the timing and addresses the free-rider problem.

5.3 Non-trivial Costs

We assumed that costs of private information acquisition c are small and, for Proposition 2, that the fee for subscribing to proxy advice f is sufficiently smaller. This subsection examines how restrictive these assumptions are and clarifies how behavior changes when fees or research costs are non-trivial, starting with the role of the proxy advisor's fee.

If the PA's fee f is not sufficiently smaller than the costs c , then deviating from symmetric CAIS to UNIS saves costs without affecting the outcome. Indeed, decision quality would not change because in the case of pivotality, the potentially deviating shareholder votes according to the own signal in both strategy profiles. However, this shareholder's costs would change from $C(\hat{\sigma}) = f + p^{dis}c$, with $p^{dis} = q_B(1 - q_P) + (1 - q_B)q_P$, to $C(\sigma^{UNIS}) = c$. Hence, the symmetric equilibrium CAIS requires $f \leq \bar{f} := (1 - p^{dis})c$, which we have formulated as f sufficiently smaller than c . As $p^{dis} \in (0, \frac{1}{2})$, a sufficient condition is $f \leq \frac{c}{2}$. In words, the PA's fee must not exceed half of the individual costs of private research, an assumption that seems realistic. If the fee exceeds this bound, shareholders optimally refrain from subscribing to the proxy advisor, and the conditional information acquisition mechanism no longer operates.

Consider now a monopolistic PA that chooses the subscription fee to maximize profits. As long as shareholders are homogeneous and we focus on symmetric strategy profiles, this extension does not alter the equilibrium logic of the model. If the fee set by the PA exceeds shareholders' willingness to pay for proxy advice, demand collapses and the proxy advisor earns zero profits. Hence, a profit-maximizing monopolistic PA optimally sets the fee up to the shareholders' willingness to pay for the subscription, which we determined here as $\bar{f}$. Accordingly, treating the fee f as exogenous and sufficiently small can be interpreted as a reduced-form representation of monopolistic pricing under homogeneous demand.

Let us now turn to the costs of private information acquisition c . If they are not sufficiently small, a shareholder has a profitable deviation from CAIS to Rubber-stamping. This move saves private costs for the deviating shareholder but reduces expected decision quality. Computing the expected change in costs and benefits yields the following *small cost condition*, which is sufficient for CAIS to be an equilibrium, given $f \leq \frac{c}{2}$.³¹

$$c < \frac{1}{N} \binom{N-1}{\frac{N-1}{2}} (1 - q_S)^{\frac{N-1}{2}} q_S^{\frac{N-1}{2}} [(1 - q_B) q_P q_S - q_B(1 - q_P)(1 - q_S)].$$

The first factor on the right-hand side stems from the fact that any voter owns only the share $\frac{1}{N}$ of the firm. This factor disappears when costs are expressed relative to the

³¹The derivation and some examples are provided in Supplementary Online Material (SOM) Section 1.4.

voter's per-share stake, i.e., as $c^{rel} := \frac{c}{1/N}$. The last factor on the right-hand side, in square brackets, is the conditional change in decision quality and depends on the signal qualities. The middle factors capture the probability of being pivotal, which decreases with N and, for given symmetry, also declines as q_S moves away from 0.5. More precisely, they capture the probability that all $N - 1$ other agents' signals are split, which decreases quickly with N when q_S is not close to 0.5. How restrictive this small cost condition is depends on the signal qualities and on the number of voters N .

In Example 1 in the main text, the condition requires private costs to be below 0.18% of the per-share stake for $N = 21$ and below 0.52% for $N = 5$. These values are around 1% for $N = 21$ and 3% for $N = 5$ in some other examples that also satisfy all conditions of Proposition 2.

This illustrates that the symmetric equilibrium with conditional private research by all shareholders is most plausible as a benchmark for settings with a limited number of actively engaged voters or with very low marginal research costs, rather than as a literal description of broad shareholder participation in large firms. Thus, the small-cost assumption is restrictive and corresponds to situations in which much is at stake for the firm in the decision or when shareholders indeed face very low marginal research costs. Otherwise, private research equilibria, despite being potentially efficient, unravel due to a free-rider problem that emerges with non-trivial costs.

In reality, costs of private research are heterogeneous across shareholders. There are large investors with economies of scale and small costs per issue, while private costs for other shareholders may be prohibitively high. Our analysis therefore highlights that proxy advice is most likely to improve decision quality when at least a subset of shareholders faces sufficiently low research costs.

5.4 Conflicts of Interest

In a re-interpretation of our model the board has a partial conflict of interest with the shareholders. Suppose the effect of conflicted interests is that it reduces the likelihood that the board's proposal is correct from q_B to some $\tilde{q}_B < q_B$. More technically, suppose the board's bias is a random variable that is drawn and private information. Shareholders know the distribution of the bias, but not its realization. The distribution of the bias is such that either the board's proposal is determined by the bias or that it is determined by the signal. Moreover, suppose that the board's bias is symmetrically distributed around zero.³² This

³²Asymmetry of the bias distribution would make one type of proposal more informative than the other.

introduces noise into the informativeness of the board’s proposal as shareholders put positive probability on the case that the proposal is independent of the signal. Then the assumption of a high quality board $\tilde{q}_B \geq q_S$ thus means that the board is not only better informed, but also that the board’s agency problem is limited. Conversely, a low $\tilde{q}_B$ means either that the board has a low expertise or that it has a high agency problem such that its proposal is not very informative.

Reconsidering the comparative-statics on q_B (cf., e.g., Figure 2), we can thus also address how the agency problem affects the decision quality. Start with a very well informed board and a small agency problem: $q_B > \tilde{q}_B > q_P > q_S$. Reducing $\tilde{q}_B$ first fosters the shareholders’ research incentives up to the point $\tilde{q}_B \approx q_P$, then reduces them. This non-monotonicity makes it possible that agency problems may even increase the quality of corporate decision making, as boards whose proposals are less informative may incentivize shareholders to (conditionally) invest in own research.

If we further increase the agency problem up to a point where the board’s proposal becomes close to uninformative (i.e., $\tilde{q}_B \approx \frac{1}{2}$), a proxy advisor is indeed detrimental for the reasons elaborated in the literature and illustrated at the end of Section 4. However, a second PA might now challenge the first PA’s recommendation. This could induce the controversy needed to make shareholders invest in own research.

Now, consider instead a biased PA. The implications are analogous to the discussion above. If the bias is symmetric across states and private information to the PA, it affects the model as a reduction in the effective precision of the PA’s signal q_P . We discussed those comparative-static effects already in Section 4. If, instead, the PA or the board is biased in a non-symmetric way, one type of recommendation becomes more informative than the other, which leads to a non-trivial extension beyond the scope of the present analysis.

6 Practical and Regulatory Implications

6.1 Shareholder Behavior in Practice

Our theoretical findings are consistent with observed heterogeneity in how institutional investors use proxy advisors. Proxy advisory reports typically include both a vote recommendation and detailed background analysis. Many large asset managers state that they use these materials as an input into their own decision process rather than mechanically following the recommendation (Swipra, 2018), and bespoke voting policies are increasingly common. This reflects the optionality that our model identifies when shareholders decide whether to

acquire private information in addition to public signals from the board and a proxy advisor.

At the same time, empirical studies find that a nontrivial share of investors often vote in line with proxy advisor recommendations without additional discernible research, especially for routine proposals (Shu, 2021). In the context of our model, such “robo-voting” can be interpreted in two distinct ways. First, even conditionally informed shareholders often vote with advice on routine items. Second, when private information acquisition costs vary across investors, some investors optimally choose strategies that resemble rubber-stamping or following advice without further research, as in the asymmetric equilibria we characterize. Related evidence at the issuer level is provided by Ertimur, Kim, and Pawliczek (2024), who document that firms sometimes respond to negative proxy advisor recommendations by issuing detailed supplemental proxy filings. These filings are effective only when they lead to a revision of the proxy advisor’s recommendation, highlighting the role of proxy advisors as informational gatekeepers and as focal points in the voting process.

6.2 Regulation of Timing of Proxy Advice

Consider the situation when proxy advice arrives *after* the shareholders’ decision to invest in own research, i.e., when Assumption PAF is violated. All actions occur as illustrated in the timeline (Figure 1), but proxy advice arrives at the end of period $t = 2$. We consider the cases where Assumption BIB holds and where it does not hold. If Assumption BIB holds, the Pareto-efficient symmetric equilibrium is Rubber-stamping and hence decision quality is bounded by q_B . Hence, there is no positive effect of having a PA, as decision quality with or without a PA is bounded by the quality of the board. If Assumption BIB does not hold, i.e., if the board does not have the best information regarding what is good for the company, we find that UNIS is an equilibrium and Pareto-efficient if and only if $\ell_S \geq \ell_B + \ell_P$; otherwise, Rubber-stamping is the Pareto-efficient equilibrium. This condition is the same as in our model with early proxy advice (see bottom right corner of the lower panel of Figure 2). It is more demanding than the condition in the setting without a PA, which was $\ell_S > \ell_B$. Specifically, the condition $\ell_S \geq \ell_B + \ell_P$ means that a single shareholder has to be better informed, not only than the board, but than both the board and the PA together. Hence, the introduction of a PA whose information arrives late, if anything, weakens the shareholders’ research incentives.

Overall, a key insight from our model is that the informational value of proxy advice critically depends on its timing relative to shareholders’ decisions to acquire private information. In the model, “sufficiently early” has a precise meaning: proxy advice must arrive

before shareholders decide whether to incur the cost c of private research. Only in this case can shareholders condition their information acquisition on the advice, which is the margin through which proxy advice affects equilibrium behavior.

To interpret this condition in practice, it is useful to think in terms of an *effective research window*, that is, the time between the arrival of proposal-specific information and investors' internal voting cutoffs. Proxy advice is “early” in the sense of the model if it expands this window and thereby allows investors to further update their information. By contrast, advice that arrives only after investors have effectively committed to their information acquisition strategy can influence final votes but has very different consequences for voting strategies.

Recent empirical evidence supports this interpretation. Hirst and Robertson (2022) show that investors often do not know the content of proposals sufficiently in advance, implying that the relevant window for informed voting may open relatively late in the process. Iliev, Kalodimos, and Lowry (2021) document that investor attention to governance is limited and varies systematically over time, consistent with constraints on processing large volumes of information. Complementing this, Calluzzo and Kedia (2025) show that the concentration of shareholder meetings during the proxy season leads to substantial workload compression for proxy advisors and is associated with lower-quality recommendations.

These findings suggest that the effectiveness of proxy advice depends not only on its informational content but also on whether it arrives at a point in time when investors have the capacity to process it and adjust their behavior. This interpretation aligns with Hu, Malenko, and Zytnick (2024), who show that institutional voting follows a two-stage structure in which investors decide, after observing proposals, whether to conduct additional research and deviate from default voting policies.

Viewed through this lens, recent SEC interventions can be understood as attempts to ensure that proxy advice is provided within the effective research window. The relevant issue is not merely whether recommendations are distributed before the shareholder meeting, but whether they are disseminated with sufficient lead time for investors to process them, compare them with issuer responses, and, if warranted, conduct additional research. In our model, this is exactly the condition under which proxy advice improves decision quality.

For example, in August 2019, the U.S. Securities and Exchange Commission (SEC) issued guidance stating that investment advisers fulfill their fiduciary duties of care and loyalty only if they exercise oversight over proxy advisory firms, including scrutiny of methodologies, engagement practices, and error-correction procedures.³³ Subsequent SEC

³³<https://www.sec.gov/rules/interp/2019/ia-5325.pdf>

proposals and rule changes focused explicitly on the timing of proxy advice and issuer responses. A proposed rule from November 2019 raised concerns that investors might receive proxy advisor recommendations too late to meaningfully conduct independent research.³⁴ While the rule adopted in July 2020 partially addressed this issue by requiring mechanisms for timely awareness of issuer responses,³⁵ a further proposal in November 2021 emphasized the importance of delivering independent proxy voting advice to investors in a timely manner.³⁶ This approach was finalized in July 2022.³⁷

Taken together, these developments highlight that the regulatory relevance of timing is more specific than simply “earlier is better”: rules matter insofar as they ensure that proxy advice arrives at a stage where investors can still revise their information acquisition decisions, rather than merely their final votes.

Recent developments in the proxy advisory industry further underscore the relevance of our analysis. Major proxy advisors have adjusted their service offerings, not by abandoning voting recommendations altogether, but by complementing them with additional analytical tools and greater customization. For the 2026 proxy season, ISS continues to issue standardized benchmark voting recommendations, while at the same time expanding governance research platforms that provide analytical content without explicit vote recommendations.³⁸ Similarly, Glass Lewis has moved away from a single uniform recommendation style and now offers multiple voting policy frameworks, allowing institutional clients to choose among different recommendation styles that better align with their own governance preferences.³⁹

Importantly, these changes do not eliminate the presence of public signals in the form of proxy advisor recommendations. Rather, they modify how such signals are generated, interpreted, and integrated into investors’ decision-making processes. From the perspective of our model, these developments can be understood as affecting the precision, framing, and salience of the proxy advisor’s signal, while preserving its role as an early public input into shareholder voting. As long as recommendations (or clearly interpretable voting perspectives) remain available sufficiently early, the core mechanism we study---namely that disagreement between public signals can trigger conditional private information acquisition---continues to apply.

³⁴<https://www.sec.gov/rules/proposed/2019/34-87457.pdf>

³⁵<https://www.sec.gov/rules/final/2020/34-89372.pdf>

³⁶<https://www.sec.gov/rules/proposed/2021/34-93595.pdf>

³⁷<https://www.sec.gov/rules/final/2022/34-95266.pdf>

³⁸<https://insights.issgovernance.com/posts/iss-stoxx-introduces-new-research-services-to-support-investors-proprietary-stewardship-programs/>

³⁹<https://www.glasslewis.com/article/personal-commitment-change-proxy-voting-practices>

More broadly, the coexistence of standardized recommendations and optional analytical tools highlights that proxy advice increasingly serves as a screening and prioritization device rather than a mechanical voting instruction. This interpretation is consistent with our theoretical framework, in which proxy advice helps shareholders identify controversial proposals that warrant further investigation, while routine proposals are more likely to be decided without additional private research.

6.3 Regulation of Proxy Advisor Competence

Regulators continue to focus on disclosure and oversight of proxy advisory methodologies and conflicts of interest. For example, U.S. regulatory actions have stressed compliance with antifraud provisions and enhanced transparency around methodologies and conflicts, while in the EU the Shareholder Rights Directive mandates disclosure of methodology and conflicts to institutional investor clients.

These competence-enhancing measures interact with the model’s insight on signal quality. In our model, a proxy advisor with somewhat better information than the board provides incentives for private information acquisition, and yields the highest collective decision quality. At the same time, if the proxy advisor is sufficiently better informed than the board, or if improvements in signal precision make one public expert dominant, the incentives for shareholders to acquire private information may weaken, potentially lowering decision quality. Taken together, these observations suggest that regulatory efforts to improve proxy advisor competence and transparency should be evaluated not only in terms of signal quality per se, but in terms of how they affect both the timing of information and relative precision vis-à-vis board information.

7 Conclusion

In this paper we analyze the effects of proxy advisory firms (PAs) on corporate decisions. As a benchmark PAs are not admitted. When the board’s proposals are sufficiently informative, shareholders do not have incentives to conduct their own research and simply rubber-stamp the board’s proposals. Hence in the absence of PAs, decision quality is bounded by the quality of the board (Proposition 1). Introducing a PA whose information level is not too far from the board’s alters this result and leads to a higher decision quality (Proposition 2). From a conceptual perspective, this effect reflects the restoration of standard sequential value-of-information logic in a voting environment: shareholders optimally acquire a costly private

signal only when earlier public signals generate sufficient disagreement. This result hinges on the timing of proxy advice: only when the PA’s recommendation arrives sufficiently early can shareholders condition their private research decisions on it. Extending the analysis from symmetric equilibria to asymmetric equilibria, we find that many but not all shareholders play this conditional investment strategy. Importantly, we arrive at the same overall conclusion: PAs improve corporate decision quality (Proposition 3).

Importantly, our results do not imply that adding more public signals is always beneficial. The effect depends on whether additional signals increase disagreement and thereby trigger private information acquisition. While two (at best equally precise) signals create more controversy than one, adding another one reverts the effect: while two signals are offsetting each other, the third is provided as if there was only a single one. Hence, multiple public signals induce more information acquisition if and only if they create more controversy, e.g., if there is an even number of equally precise signals, or if signal qualities differ in a way that increases controversy. Having an informative proposal and one similarly informed advisor is a cheap and simple way of doing so.⁴⁰

While our analysis is tailored to shareholder voting, related concerns arise in other environments in which public signals may crowd out or stimulate private information acquisition. For example, Sangiorgi and Spatt (2017) argue that credit rating agencies can crowd out independent information production by investors. Our results highlight the mechanism through which public signals can either suppress or restore private research incentives in collective decision-making problems.

More broadly, the paper shows that in voting environments with endogenous information acquisition, standard value-of-information logic may fail due to free-riding and pivotality considerations, and may be restored only under specific timing and information conditions. In the context of shareholder voting, these conditions help explain when proxy advice improves decision quality and when it does not.

⁴⁰The larger the (even) number of conditionally independent informative signals of equal quality, the smaller is the probability of a tie in the signal realizations.

A Appendix: Proofs

A.1 Proof of Proposition 1

To prove Proposition 1 Lemma A.1 is helpful.

Lemma A.1 (SYM without PA: All Equilibria). *Let Assumption BIB hold. Suppose no PA is admitted.*

- i. Define Protest as the strategy profile in which no shareholder invests in research and all shareholders vote no. Protest is a symmetric equilibrium for any $q_B, q_S \in (0.5, 1)$. Its decision quality is $1 - q_B$.*
- ii. Rubber-stamping (i.e., no shareholder invests in research and all shareholders vote yes) is a symmetric equilibrium for any $q_B, q_S \in (0.5, 1)$. Its decision quality is q_B .*
- iii. There are no other symmetric equilibria.*

Proof. We address each part separately.

- i. We have $N \geq 3$ shareholders (because $N > 1$ and odd). When all shareholders vote *no*, a single shareholder is never pivotal. Hence, there is no way to increase decision quality. Deviations can thus only affect costs. Since no information is acquired in this information-acquisition strategy (NotSubscribe-NotInvest), costs are minimal. Hence, there is no unilateral improvement.

Decisions always implement the opposite of the board's proposal. By assumption of the simplified model, the board's proposal corresponds to its signal (B). Hence, the ex ante probability that the true state matches the decision equals the probability that the board's signal does not match the true state, which is $1 - q_B$.

- ii. The proof that Rubber-stamping is an equilibrium is fully analogous to part i. of Lemma A.1. With Rubber-stamping, the decision quality equals the ex ante probability that the board's signal matches the true state, which is q_B .
- iii. There are only two information-acquisition strategies. For not investing in an own signal both strategies are symmetric equilibria (see part i. and ii.). Consider now investment in an own signal: Since shareholders pay c they must condition on their own signal. Otherwise, they could improve their utility by voting the same and not investing c . Conditioning on their signal leaves two pure strategies: vote *yes* if b and *no* if a (i.e., UNIS) or the opposite (vote *yes* if a and *no* if b). If voting *yes* after a (against) was optimal, then voting *no* after a would also be so. Hence, shareholders could profitably deviate to unconditionally voting A .

We finally show that UNIS is not an equilibrium under Assumption BIB, i.e., $q_S \leq q_B$.⁴¹ Consider shareholder i deviates to Rubber-stamping. The deviation changes the outcome only if i is pivotal and the own signal is a : Under UNIS i would vote *no*, under Rubber-stamping i would vote *yes*. Pivotality implies that among the $N - 1$ other shareholders

⁴¹In fact, UNIS is a symmetric equilibrium if and only if $q_S > q_B$.

the signals are split in $\frac{N-1}{2}$ a -signals and $\frac{N-1}{2}$ b -signals. Conditional on that case, B is more likely to be true than A (such that Rubber-stamping weakly improves decision quality) if and only if

$$\begin{aligned} q_B(1 - q_S) \binom{N-1}{\frac{N-1}{2}} q_S^{\frac{N-1}{2}} (1 - q_S)^{\frac{N-1}{2}} &\geq (1 - q_B) q_S \binom{N-1}{\frac{N-1}{2}} (1 - q_S)^{\frac{N-1}{2}} q_S^{\frac{N-1}{2}} \\ q_B(1 - q_S) &\geq (1 - q_B) q_S \\ \frac{q_B}{1 - q_B} &\geq \frac{q_S}{1 - q_S} \\ \ell_B &\geq \ell_S. \end{aligned}$$

Hence, Rubber-stamping weakly improves decision quality for $q_S \leq q_B$, which is Assumption BIB. Moreover, Rubber-stamping saves costs c . Therefore, it strictly improves utility of the deviating shareholder i .

□

Now, we use Lemma A.1 to prove Proposition 1. Under Assumption BIB there are only two equilibria. Equilibrium Rubber-stamping leads to the same costs as the Protest equilibrium. Rubber-stamping Pareto-dominates Protest because it leads to higher decision quality $\Pi(\sigma^{Rubber}) = q_B > 0.5 > 1 - q_B = \Pi(\sigma^{Protest})$.

A.2 Proof of Proposition 2

To prove Proposition 2 Lemma A.2 is helpful.

Lemma A.2 (SYM with PA: All Equilibria). *Let Assumptions BIB and PAF hold. Let costs c be arbitrarily small and let fee f be sufficiently smaller.*

- i. Protest (i.e., no shareholder invests in research and all shareholders vote no) is a symmetric equilibrium for any $\ell_B, \ell_S \in (0, \infty)$. Its decision quality is $1 - q_B$.*
- ii. Rubber-stamping (i.e., no shareholder invests in research and all shareholders vote yes) is a symmetric equilibrium for any $\ell_B, \ell_S \in (0, \infty)$. Its decision quality is q_B .*
- iii. CAIS is a symmetric equilibrium if and only if $\ell_P \in (\ell_B - \ell_S, \ell_B + \ell_S)$. Its decision quality is: $\Pi(\hat{\sigma}) = q_B q_P + [(1 - q_B) q_P + q_B(1 - q_P)] \pi(N)$, with $\pi(N) := \sum_{i=\frac{N+1}{2}}^N \binom{N}{i} q_S^i (1 - q_S)^{N-i}$.*
- iv. CAIS-2 is a symmetric equilibrium if and only if $\ell_P \in (\ell_B - \ell_S, \ell_B + \ell_S)$. Its decision quality is: $\Pi(\sigma^{CAIS-2}) = (1 - q_B)(1 - q_P) + [(1 - q_B) q_P + q_B(1 - q_P)] \pi(N)$.*
- v. There are no other symmetric equilibria. In particular, there is no equilibrium in which all shareholders subscribe to proxy advice and unconditionally invest in own signal (Subscribe-Invest).*

Proof. We address each part of Lemma A.2 separately.

- i. The proof is identical to the proof of Lemma A.1, part i.

ii. The proof is identical to the proof of Lemma A.1, part ii.

iii. CAIS is illustrated in Table 1. We show that CAIS is an equilibrium if and only if $\ell_P \in (\ell_B - \ell_S, \ell_B + \ell_S)$.

Suppose first that $\ell_P \notin (\ell_B - \ell_S, \ell_B + \ell_S)$, i.e., either $\ell_P \leq \ell_B - \ell_S$ or $\ell_P \geq \ell_B + \ell_S$. We show that CAIS cannot be an equilibrium. In CAIS pivotality implies that the vote recommendation is *against* and that among the $N - 1$ other shareholders the signals are split in $\frac{N-1}{2}$ *a*-signals and $\frac{N-1}{2}$ *b*-signals. (Indeed, after recommendation *for* no shareholder is pivotal.)

Let $\ell_P \leq \ell_B - \ell_S$. Consider a shareholder i who deviates to Rubber-stamping. This deviation alters the decision in comparison to CAIS if the vote recommendation is *against*, all other shareholder's signals are split, and i 's signal is *a*: In CAIS, i would vote *no*, in the deviation i would vote *yes*. This deviation weakly improves decision quality if $\ell_B \geq \ell_P + \ell_S$, which holds by assumption. Since, the deviation saves costs c , it increases i 's expected utility.

Let $\ell_P \geq \ell_B + \ell_S$. Consider a shareholder i who deviates to voting *no* without information acquisition (as in Protest). This deviation alters the decision in comparison to CAIS if the vote recommendation is *against*, all other shareholders' signals are split, and i 's signal is *b*: In CAIS, i would vote *yes*, in the deviation i would vote *no*. This deviation weakly improves decision quality if $\ell_P \geq \ell_B + \ell_S$, which holds by assumption. Since the deviation saves costs c , it increases i 's expected utility.

Hence, if $\ell_P \notin (\ell_B - \ell_S, \ell_B + \ell_S)$, CAIS is not an equilibrium.

Now suppose that $\ell_P \in (\ell_B - \ell_S, \ell_B + \ell_S)$. In order to show that CAIS is an equilibrium, we demonstrate that there is no individual deviation that improves utility. We use the following principle: if a deviation is more attractive than an other deviation in terms of utility, then excluding the former is sufficient to exclude the latter. We organize the potential deviations by information-acquisition strategy. There are six information-acquisition strategies to consider. Pivotality always implies that the vote recommendation is *against* and that among the $N - 1$ other shareholders the signals are split in $\frac{N-1}{2}$ *a*-signals and $\frac{N-1}{2}$ *b*-signals.

(1) NotSubscribe-NotInvest. Deviating to NotSubscribe-NotInvest and voting *yes* as in Rubber-stamping is not an improvement for low enough costs given $\ell_S + \ell_P > \ell_B$. This deviation only changes the outcome if the PA has recommended *against*, i has received signal *a* (against), and all other shareholders' signals are split. It would weakly improve decision quality iff

$$\begin{aligned} q_B(1 - q_P)(1 - q_S) \binom{N-1}{\frac{N-1}{2}} q_S^{\frac{N-1}{2}} (1 - q_S)^{\frac{N-1}{2}} &\geq (1 - q_B) q_P q_S \binom{N-1}{\frac{N-1}{2}} (1 - q_S)^{\frac{N-1}{2}} q_S^{\frac{N-1}{2}} \\ q_B(1 - q_P)(1 - q_S) &\geq (1 - q_B) q_P q_S \\ \frac{q_B}{1 - q_B} &\geq \frac{q_P}{1 - q_P} \cdot \frac{q_S}{1 - q_S} \\ \ell_B &\geq \ell_P + \ell_S. \end{aligned}$$

By assumption $\ell_P > \ell_B - \ell_S$, this deviation strictly decreases decision quality. It does save costs f always and c with probability $q_B(1 - q_P) + (1 - q_B)q_P$. For low enough costs f and c , Rubber-stamping does not increase utility because of its lower decision quality.

Deviation to vote *no* without information acquisition (as in Protest) is not an improvement for low enough costs given $\ell_P < \ell_B + \ell_S$.

- (2) NotSubscribe-Invest. Deviation NotSubscribe-Invest and voting according to the own signal as in UNIS does not change the outcome. Indeed, after a *for* recommendation i is not pivotal, after an *against* recommendation i votes under her deviation as she does under CAIS. Hence, this deviation is an improvement only if it saves costs. It is not an improvement if $f \leq c[q_B q_P + (1 - q_B)(1 - q_P)]$, which is satisfied if f is sufficiently lower than c .
- (3) Subscribe-NotInvest. The deviation to buying the PA's recommendation and following it is not an improvement given $\ell_P < \ell_B + \ell_S$ and low enough c .
- (4) Subscribe-Invest. Deviation to buy both recommendation and signal. Case 1, illustrated in Table A.2, is outcome equivalent, but more costly. Case 2, illustrated in Table A.3, is not an improvement given $\ell_P < \ell_B + \ell_S$.
- (5) Subscribe-InvestIFF*for*. Consider the deviation to buying the PA's recommendation and investing in an own signal iff the recommendation is *for*. The case illustrated in Table A.4 is not an improvement given $\ell_P < \ell_B + \ell_S$. The alternative case, which differs by voting *yes* after the *against* recommendation, is not an improvement given $\ell_S + \ell_P > \ell_B$.
- (6) Subscribe-InvestIFF*against*. Consider the deviation to applying the same information-acquisition strategy as in CAIS, but a different voting strategy. The most attractive deviation is to vote *no* after the *for* recommendation. This is outcome equivalent and equally costly and, hence, not an improvement.

Hence, under the conditions assumed in part iii. of the Lemma CAIS is an equilibrium.

Finally, concerning decision quality, notice that if board and PA receive the same signal, this signal determines the decision, and if they receive a different signal, the signal that is received by a majority of shareholders determines the decision. Therefore, decision quality in CAIS is $(q_B q_P) * 1 + (1 - q_B)(1 - q_P) * 0 + [(1 - q_B)q_P + q_B(1 - q_P)]\pi(N) * 1$, as $q_B q_P$ is the probability that the board and the PA both receive the same and correct signal, and $[(1 - q_B)q_P + q_B(1 - q_P)]$ is the probability that the two receive signals that are different from each other.

iv. CAIS-2 is illustrated in Table A.1.

		Own Signal	
		b (for board)	a (against)
PA	for	no	
	$against$	yes	no

Table A.1: CAIS-2: Invest in research iff vote recommendation is *against*; after *for* recommendation vote *no*, after *against* recommendation vote *yes* iff the own signal is b .

The proof that CAIS-2 is an equilibrium if and only if $\ell_P \in (\ell_B - \ell_S, \ell_B + \ell_S)$ is identical to the proof that CAIS is an equilibrium under these conditions (cf. Proof of Lemma A.2, part iii.).⁴²

Concerning decision quality, notice that if board and proxy advisor receive the same signal, the decision is contrary to this signal, and if they receive different signals, the signal that is received by a majority of shareholders determines the decision. Therefore, decision quality in CAIS-2 is $(q_B q_P) * 0 + (1 - q_B)(1 - q_P) * 1 + [(1 - q_B)q_P + q_B(1 - q_P)]\pi(N) * 1$.

v. To show that there are no additional equilibria, we exhaustively discuss all pure strategies. Again, we organize the discussion by information-acquisition strategy.

- (1) NotSubscribe-NotInvest. There are only voting strategies *yes* or *no*. Both lead to equilibria as shown in parts i and ii.
- (2) NotSubscribe-Invest. Since shareholders pay c they must condition on their own signal. Otherwise, they could improve their utility by voting the same and not investing c . Conditioning on the own signal leaves two pure strategies: vote *yes* if the signal is b and *no* if the signal is a (i.e., as in UNIS) or the opposite (vote *yes* if the signal is a and *no* if the signal is b). If voting *yes* after a (*against*) was optimal, then voting *no* after a would also be optimal. Hence, shareholders could improve their utility by unconditionally voting A . Only UNIS remains. Under Assumption BIB, NotSubscribe-NotInvest and voting *yes* as in Rubber-stamping is a profitable deviation from UNIS.

- (3) Subscribe-NotInvest. Since shareholders pay f they must condition on the PA's recommendation. For instance, they vote *yes* after *for* and *no* after *against*; or they do the opposite. In either case, no shareholder is pivotal since all vote for the same, given a particular recommendation.

A shareholder can improve her utility by not paying f and voting, e.g., *yes*. Hence, there is no symmetric equilibrium with this information-acquisition strategy.

- (4) Subscribe-Invest. Since shareholders pay both f and c they must condition their voting strategy on both the PA's vote recommendation and the own signal. Otherwise, they could improve their utility with the same voting behavior, but saving costs. This means that in fact only two voting strategies remain.

Case 1: vote *yes* except if the PA's recommendation is *against* and the own signal is a , as in Table A.2. In this case no shareholder is pivotal if the PA recommends *for*.

⁴²This is not surprising, as both strategies have the same information-acquisition strategy, Subscribe-InvestIFF*against*, and they only differ in a voting action, where no player is pivotal.

Hence, shareholder i can only be pivotal if the recommendation is *against*. If so, i would vote according to her signal. Hence, deviating to unconditionally investing in an own signal and voting accordingly, as in UNIS, would not change the outcome because either i is not pivotal or i would also vote according to the signal. However, acting as in UNIS saves fee f . Thus, this is a profitable deviation, and the strategy profile of case 1, illustrated in Table A.2, cannot be an equilibrium.

		Own Signal	
		b (for board)	a (against)
PA	for	yes	yes
	$against$	yes	no

Table A.2: A strategy profile based on acquiring both proxy advice and own signal, case 1: Subscribe-Invest and vote *yes*, except if PA's recommendation is *against* and the own signal is a .

Case 2: vote *no* except if both the PA's recommendation is *for* and the own signal is b , as in Table A.3. The analogous argument as above for case 1 applies, as follows: In this case no shareholder is pivotal if the PA recommends *against*. Hence, shareholder i can only be pivotal if the recommendation is *for*. If so, i would vote according to signal. Hence, deviating to unconditionally investing in an own signal as in UNIS would not change the outcome because either i is not pivotal or i would also vote according to the signal. Acting as in UNIS, however, saves fee f . Thus, strategy profile of case 2 cannot be an equilibrium.

		Own Signal	
		b (for board)	a (against)
PA	for	yes	no
	$against$	no	no

Table A.3: A strategy profile based on acquiring both proxy advice and own signal, case 2: Subscribe-Invest and vote *no*, except if PA's recommendation is *for* and the own signal is b .

Therefore, there cannot be a symmetric equilibrium with this information-acquisition strategy (Subscribe-Invest), in which shareholders unconditionally buy both the PA's recommendation and an own signal.

- (5) Subscribe-Invest IFF *for*. Since shareholders pay f and sometimes c they must condition their voting strategy on the recommendation and the own signal when they acquire them. In particular, after having bought the own signal on top of the recommendation *for*, shareholders must vote according to their signal in equilibrium. Voting the opposite is dominated, and not conditioning as well. This leaves two cases, which we address as Cand. 5a and Cand. 5b. We show that none of them is an equilibrium under Assumption 1.⁴³ Consider first Cand. 5a: shareholders vote

⁴³In fact, each of these strategy profiles is a symmetric equilibrium if and only if $\ell_S > \ell_B + \ell_P$.

yes except if the PA's vote recommendation is *for* and the own signal is *a* (against) as in Table A.4.

		Own Signal	
		<i>b</i> (for board)	<i>a</i> (against)
PA	<i>for</i>	yes	no
	<i>against</i>	yes	

Table A.4: Cand. 5a. A strategy profile based on acquiring an own signal iff the recommendation is *for*: Subscribe-InvestIFF*for* and vote *yes*, except if PA's recommendation is *for* and the own signal is *b*.

Consider shareholder *i* who deviates to NotSubscribe-NotInvest and voting *yes*, as in Rubber-stamping. This deviation only alters the outcome when the vote recommendation is *for*, all other shareholders' signals are split, and *i*'s signal is *a* (*against*): Under Cand. 5a, *i* would vote *no*, but under her deviation she votes *yes*. Decision quality improves by this deviation if $\ell_B + \ell_P > \ell_S$. This condition is satisfied by Assumption 1. Moreover, the costs are lower under this deviation than under Cand. 5a. Hence, Cand. 5a cannot be an equilibrium.

Now consider Cand. 5b. Shareholders vote *no* except if the PA's vote recommendation is *for* and the own signal is *b*. Again, no shareholder is pivotal after recommendation *against*. Hence, deviating to NotSubscribe-NotInvest and voting *yes* as in Rubber-stamping is an improvement, identical to the case of Cand. 5a above.

- (6) Subscribe-InvestIFF*against*. Since shareholders pay *f* and sometimes *c* they must condition their voting strategy on the recommendation and the own signal when they acquire them. In particular, after having bought the own signal on top of the recommendation *against*, shareholders must vote according to their signal in equilibrium. Voting the opposite is dominated, and not conditioning as well. This leaves two cases: CAIS and CAIS-2, which we have addressed. Hence, there are no further equilibria.

□

Now we can turn to the proof of Proposition 2. Suppose there is a PA with $\ell_P \in (\ell_B - \ell_S, \ell_B + \ell_S)$. To show that CAIS is an equilibrium and Pareto-efficient, we use Lemma A.2, which shows that besides CAIS there are three further equilibria in this parameter space: Rubber-stamping, Protest, and CAIS-2. It remains to show that CAIS Pareto-dominates in this area.

First, CAIS has the same costs as CAIS-2 and decision qualities are: $\Pi(\hat{\sigma}) = q_B q_P + [(1 - q_B)q_P + q_B(1 - q_P)]\pi(N)$. $\Pi(\sigma^{CAIS-2}) = (1 - q_B)(1 - q_P) + [(1 - q_B)q_P + q_B(1 - q_P)]\pi(N)$. CAIS has higher decision quality iff $q_B q_P > (1 - q_B)(1 - q_P)$, which always holds as $q_B, q_P > 0.5$. Hence, CAIS Pareto-dominates CAIS-2.

Second, decision quality of Rubber-stamping is q_B and decision quality of Protest is

$1 - q_B < q_B$. CAIS has strictly higher decision quality than both iff

$$\begin{aligned}
\Pi(\hat{\sigma}) &> q_B \\
q_B q_P + [q_B(1 - q_P) + (1 - q_B)q_P]\pi(N) &> q_B \\
q_B(1 - q_P)\pi(N) + (1 - q_B)q_P\pi(N) &> q_B(1 - q_P) \\
(1 - q_B)q_P\pi(N) &> q_B(1 - q_P)[1 - \pi(N)] \\
\frac{\pi(N)}{1 - \pi(N)} \cdot \frac{q_P}{1 - q_P} &> \frac{q_B}{1 - q_B} \\
\log\left(\frac{\pi(N)}{1 - \pi(N)}\right) + \log\left(\frac{q_P}{1 - q_P}\right) &> \log\left(\frac{q_B}{1 - q_B}\right) \\
\ell_N + \ell_P &> \ell_B, \\
\text{where } \ell_N &:= \log\left(\frac{\pi(N)}{1 - \pi(N)}\right).
\end{aligned} \tag{A.1}$$

Since $\ell_N > \ell_S$ and by assumption $\ell_P > \ell_B - \ell_S$, we have $\ell_N + \ell_P > \ell_S + \ell_P > \ell_B$. Hence, CAIS leads to strictly higher decision quality than both Rubber-stamping and Protest. It induces higher costs f and c . Thus, for low enough costs, CAIS Pareto-dominates them.

Now, suppose that $\ell_P \notin (\ell_B - \ell_S, \ell_B + \ell_S)$. To show that the Pareto-efficient equilibrium is Rubber-stamping, we use again Lemma A.2. Under Assumption BIB and for $\ell_P \notin (\ell_B - \ell_S, \ell_B + \ell_S)$, only two equilibria remain: Rubber-stamping and Protest. Rubber-stamping Pareto-dominates because it leads to higher decision quality $\Pi(\sigma^{Rubber}) = q_B > 0.5 > 1 - q_B = \Pi(\sigma^{Protest})$, while it induces the same costs.

A.3 Proof of Proposition 3

Suppose there is a PA and Assumption PAF holds.⁴⁴ Let S be the set of all pure strategy profiles.⁴⁵ Let $\Pi : S \rightarrow [0, 1]$ be the decision quality. Let $S^{max} \subset S$ be the set of all strategy profiles that maximize Π . As S is finite, S^{max} is non-empty.

A player's strategy is called *minimal* if the voting strategy conditions on all pieces of information that are acquired. For instance, consider information-acquisition strategy *Subscribe-InvestIFFagainst*: When an own signal has been acquired after vote recommendation *against*, the voting behavior must differ between signal realization a and signal realization b to be part of a minimal strategy. Observe that for any strategy that is not minimal, the voting behavior can be mimicked by a strategy with lower costs, saving c or f or both. A strategy profile(!) is called *minimal* if all players' strategies are minimal *and* if any player's reduction of information acquisition (not subscribing and/or not acquiring an

⁴⁴We are particularly thankful to Maximilian Janisch and Thomas Lehéricy who suggested this proof idea for this proposition. Interestingly, it can be applied to asymmetric equilibria, but not to symmetric equilibria. The reason is that when studying symmetric equilibria, the space under consideration changes because deviations to strategies that thus form an asymmetric strategy profile are admitted. An alternative proof approach would consider our game as a game with a potential function in the sense of (Monderer and Shapley, 1996).

⁴⁵Then there are 16 strategies for each shareholder. We restrict attention to pure strategies.

own signal) changes the decision quality.⁴⁶ Now let $S^* \subset S^{max}$ be the strategy profiles that are minimal and lead to maximal decision quality.

We first show, as Claim 1, that all $\sigma^* \in S^*$ are equilibria. A shareholder can only achieve higher utility than in σ^* by higher decision quality or lower costs. Higher decision quality is impossible per definition. Lower costs reduce decision quality because σ^* is minimal. As costs are by assumption sufficiently small, lower costs that reduce decision quality are not an improvement.

Second, we show, as Claim 2, that any Pareto-efficient strategy profile must belong to S^* . Suppose first that σ' is Pareto-efficient, but not in S^* . Then it is either not maximizing decision quality or not minimal. If it does not maximize decision quality, take another strategy profile, say σ^* , that does and every shareholder is better off. The reason is that any difference in decision quality is always larger than the difference in costs, which are by assumption sufficiently small; formally: $\Pi(\sigma^*) - \Pi(\sigma') > c + f \implies u_i(\sigma^*) > u_i(\sigma')$ for all i . If σ' is not minimal, there is a player who can save costs without affecting decision quality and utility of other shareholders.

By Claim 1 and 2 together, each Pareto-efficient strategy profile is an equilibrium with maximal decision quality. Hence, there clearly exists an equilibrium, say σ^* , with maximal decision quality. Now consider any Pareto-efficient equilibrium σ , i.e., an equilibrium that is not Pareto-dominated by any other equilibrium. This equilibrium must also maximize decision quality, i.e., $\sigma \in S^{max}$. Otherwise, it would be dominated by σ^* , as higher decision quality means strictly higher utility for every shareholder (again due to the small cost assumption). Therefore, *every Pareto-efficient equilibrium must maximize decision quality*.

Let us now turn to the model without a PA.⁴⁷ Let T be the set of all pure strategy profiles (without a PA). In full analogy to above, we define T^* as the set of strategy profiles that are maximizing decision quality and that are minimal. Now observe that any strategy profile in T (without a PA) corresponds to a strategy profile in S (with a PA) where simply no player subscribes to the PA's vote recommendation. Consequently, any decision quality obtained with a strategy profile in T can also be obtained with a strategy profile in S . Let $\tilde{\sigma} \in S$ be a strategy profile that mimicks the maximal decision quality obtainable without a PA. Let $\bar{\Pi}^{with-PA}$, respectively $\bar{\Pi}^{no-PA}$, denote the maximal decision quality in the framework with a PA, respectively without a PA (for any strategy profile in the corresponding games). Since in the Pareto-efficient Nash equilibria with a PA, decision quality is maximal, we have $\Pi(\sigma^*) = \bar{\Pi}^{with-PA} \geq \Pi(\tilde{\sigma}) = \bar{\Pi}^{no-PA}$ for any Pareto-efficient equilibrium σ^* in the game with a PA.

⁴⁶Indeed, there are strategy profiles where all strategies are minimal, but still some players can reduce their information-acquisition without affecting the decision. For instance, when $N - 1$ players play NotSubscribe-NotInvest and vote *yes* as if in Rubber-stamping and one player does not subscribe to the PA but unconditionally invests into an own signal, as in UNIS. If the latter player stops acquiring an own signal, decisions are unaffected because she is never pivotal. By definition, such strategy profiles are not minimal.

⁴⁷There are only four strategies for each player: as in Rubber-stamping, as in Protest, as in UNIS, and voting contrary to the own signal.

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Supplementary Online Material (SOM) for “When do proxy advisors improve corporate decisions?”

1 All Symmetric Equilibria

- 1.1 Remaining Symmetric Equilibria in Benchmark Setting without a Proxy Advisor
- 1.2 Remaining Symmetric Equilibria with an Early Proxy Advisor
- 1.3 All Symmetric Equilibria with a Late Proxy Advisor

2 Asymmetric Equilibria

- 2.1 Asymmetric Equilibria in Benchmark Setting without a Proxy Advisor
- 2.2 Lemmata for Asymmetric Equilibria with a Proxy Advisor
- 2.3 Research Incentives Increase with a Proxy Advisor
- 2.4 Decision Quality Improves with a Proxy Advisor

These Supplementary Online Materials are available at <https://bit.ly/proxy-SOM>.

1 All Symmetric Equilibria

In the main text, the equilibrium analysis was restricted to the Assumptions BIB and PAF, i.e., that the board is better informed than a single shareholder and that proxy advice arrives before shareholders decide on own research. In this section we characterize all symmetric equilibria with and without these two assumptions. An overview of all symmetric equilibria is provided in SOM Table 1.1.

Information-acquisition strategy	No PA Benchmark	With early PA Under PAF	With late PA Violating PAF
NotSubscribe-NotInvest	Rubber*, Protest	Rubber*, Protest	Rubber*, Protest
NotSubscribe-Invest	(UNIS*)	(UNIS*)	(UNIS*)
Subscribe-NotInvest		--	--
Subscribe-Invest		--	--
Subscribe-InvestIFF <i>for</i>		(Cand. 5a), (Cand. 5b)	
Subscribe-InvestIFF <i>against</i>		CAIS*, CAIS-2	

Table 1.1: All symmetric equilibria arranged by information-acquisition strategy. Equilibria in brackets are precluded by Assumption BIB. Equilibria marked by “*” are Pareto-efficient in some area of the parameter space. The “--” indicates that there are symmetric strategy profiles with this information-acquisition strategy, but none of them is an equilibrium; in contrast to the empty cells which indicate that these information-acquisition strategies cannot be played due to the setting.

When there is no PA there are three symmetric equilibria. UNIS, in which all shareholders invest in research, is restricted to an area of the parameter space where Assumption BIB is violated. Shareholders who do not invest in research can play Rubber-stamping or do the opposite: vote *no* unconditionally, which we call Protest. Both these symmetric strategy profiles are trivial equilibria, as no shareholder is ever pivotal and they incur no costs. Protest induces a decision quality $\Pi(\sigma) = 1 - q_B$ because it leads to the correct decision whenever the board’s proposal is wrong. Clearly, it is Pareto-dominated by Rubber-stamping, as $q_B > 0.5 > 1 - q_B$ and both induce the same costs (none).

These three equilibria also exist when there is a PA (last two columns of SOM Table 1.1) and their discussion is analogous. With information-acquisition strategy Subscribe-InvestIFF *for*, there are two additional symmetric equilibria, labelled Cand. 5a and Cand. 5b. However, they are both precluded by Assumption BIB and moreover Pareto-dominated (in fact by UNIS). Moreover, there are two additional equilibria, based on information-acquisition strategy Subscribe-InvestIFF *against*. One of them is CAIS. The other equilibrium, CAIS-2, only differs from CAIS in the voting behavior when the vote recommendation is *for*. In CAIS shareholders vote *yes*, while shareholders in CAIS-2 vote *no*, i.e., they do not approve the board’s proposal when the PA recommends to. CAIS-2 is Pareto-dominated by CAIS since it induces the same costs, but a lower decision quality than CAIS.

More striking than the additional equilibria which are Pareto-dominated is the observation that there are no equilibria with information-acquisition strategies Subscribe-NotInvest and Subscribe-Invest (independent of Assumptions BIB and PAF). To see why not, note that buying only the vote recommendation, i.e., information-acquisition strategy Subscribe-NotInvest, is only worthwhile when using this information in an instance of pivotality. However, if all shareholders symmetrically use the vote recommendation, then no shareholder is ever pivotal. Similarly, acquiring both signals,

i.e., information-acquisition strategy Subscribe-Invest, is worthwhile only if shareholders condition their vote on both PA advice and own signal such that none is superfluous, e.g., by voting *yes* if and only if one of the latter is in favor of the proposal. When all shareholders adopt this strategy, pivotality already implies that the recommendation was *against*. Hence, saving the subscription fee by not subscribing to the PA is a profitable deviation.

Remember that Lemmas A.1 and A.2 in the Appendix of the main text provide all symmetric strategy profiles that can be equilibria, together with their parameter conditions and decision quality under the Assumptions BIB and PAF. We now provide the corresponding results for all remaining equilibria that were excluded by Assumptions BIB and PAF -- first, for the benchmark case that there is no PA, then for a PA whose advice arrives early, i.e., under Assumption PAF, finally for the case that there is a PA whose advice arrives later, i.e., violating Assumption PAF. In SOM Table 1.1, this means that the main text has already addressed the first two columns with the equilibria that are not in brackets and we now complete the analysis by addressing the equilibria in brackets of the first column in SOM Section 1.1, then the equilibria in brackets of the second column in SOM Section 1.2; and finally all equilibria of the last column in SOM Section 1.3.

1.1 Remaining Symmetric Equilibria in Benchmark Setting without a Proxy Advisor

Lemma 1.1 (SYM without PA: All Remaining Equilibria). *Suppose no PA is admitted. In addition to the equilibria provided in Lemma A.1, there is the following symmetric equilibrium when Assumption BIB is relaxed:*

- i. UNIS, i.e., all shareholders invest in own research and vote according to their signal, is a symmetric equilibrium if and only if $q_S > q_B$. Its decision quality is $\Pi(\sigma^{UNIS}) = \pi(N)$.*

Proof. Suppose first that $q_S \leq q_B$ (i.e., Assumption BIB holds). Then UNIS is not an equilibrium as shown by Lemma A.1 (in the Appendix A of the main text).

Now, suppose $q_S > q_B$. In order to show that UNIS is an equilibrium, we show that there is no individual deviation that improves utility. We use the following principle: if a deviation is more attractive than an other deviation in terms of utility, then excluding the former is sufficient to exclude the latter. We organize the potential deviations by information-acquisition strategy. As we consider the setting without a PA, there are only two information-acquisition strategies: acquiring an own signal or not. Pivotality always implies that among the $N - 1$ other shareholders the signals are split in $\frac{N-1}{2}$ *a*-signals and $\frac{N-1}{2}$ *b*-signals, which occurs with positive probability.

1. When no own signal is acquired, strategies are Rubber-stamping and Protest.

Consider first the deviation from UNIS to Rubber-stamping. When pivotal, voting *yes* would weakly increase decision quality if $q_B \geq q_S$. Rubber-stamping decreases decision quality as $q_S > q_B$ holds by assumption. Rubber-stamping saves costs c . For small enough c (as it is assumed in the lemma), this deviation does not increase utility.

The deviation to always vote *no* without information acquisition (Protest) changes the outcome to *no* in case of pivotality. It would then induce *A* despite the fact that, conditional on pivotality, all other shareholder's signals balance each other and that the board's signal is *b*. Hence, decision quality is affected in a worse way than with Rubber-stamping, while cost savings are equal. Therefore this latter deviation is not as attractive as the deviation to Rubber-stamping.

2. When a signal is acquired, the information-acquisition strategy is unchanged. Deviation to a different voting strategy is not an improvement. First, not conditioning on the acquired signal is less attractive than the deviation to Rubber-stamping or to unconditionally voting *no*, as it involves higher costs. Conditioning on the own signal leaves one deviation on the voting stage: vote *yes* if *a* and *no* if *b*, which is the opposite of UNIS. However, if voting *yes* after *a* (against) was optimal in case of pivotality, then voting *no* after *a* would also be. Hence, shareholders could improve by unconditionally voting *no*.

We have established that Rubber-stamping is the most attractive deviation. For $q_B > q_S$, the deviation to Rubber-stamping strictly decreases decision quality, and hence does not increase utility for low enough costs c .

Finally, the decision quality of UNIS equals $\pi(N)$ because the signal that has been received by a majority of the N (odd) voters determines the decision. Hence, the ex ante probability that the decision matches the true state equals the probability that among N independent signals of quality q_S the majority is correct, which is the definition of $\pi(N)$.

Let us now show that there are no further symmetric equilibria. There are only two information-acquisition strategies. For not investing in an own signal both strategy profiles are symmetric equilibria, as already addressed in parts i. and ii. of Lemma A.1). Consider now investment in an own signal. Since shareholders pay c they must condition their vote on their own signal. Otherwise, they could improve by voting in the same way and not investing c . Conditioning on their signal leaves two pure strategies: vote *yes* if *b* and *no* if *a* (i.e., UNIS) or the opposite (vote *yes* if *a* and *no* if *b*). If voting *yes* after *a* (against) was optimal, then voting *no* after *a* would also be. Hence, shareholders could improve by unconditionally voting *A*. Only UNIS remains when shareholders acquire an own signal. □

1.2 Remaining Symmetric Equilibria with an Early Proxy Advisor

Lemma 1.2 (SYM with PA under Assumption PAF: All Remaining Equilibria). *Let Assumption PAF hold. Let costs c be arbitrarily small and let fee f be sufficiently smaller. In addition to the equilibria provided in Lemma A.2, there are the following symmetric equilibria when Assumption BIB is relaxed:*

- i. UNIS is a symmetric equilibrium if and only if $\ell_S > \ell_B + \ell_P$. Its decision quality is $\Pi(\sigma^{UNIS}) = \pi(N)$.
- ii. Cand. 5a (i.e., shareholders subscribe to PA and invest in own research iff the vote recommendation is for, i.e., *Subscribe-InvestIFFfor*; when the recommendation is for, they vote *yes* iff their own signal is *b*; when the recommendation is against, they vote *yes*), as illustrated in Table A.4, is a symmetric equilibrium if and only if $\ell_S > \ell_B + \ell_P$. Its decision quality: $\Pi(\sigma^{Cand. 5a}) = q_B(1 - q_P) + [q_B q_P + (1 - q_B)(1 - q_P)](\pi(N))$.
- iii. Cand. 5b (i.e., shareholders subscribe to PA and invest in own research iff the vote recommendation is for, i.e., *Subscribe-InvestIFFfor*; when the recommendation is for, they vote *yes* iff their own signal is *b*; when the recommendation is against, they vote *no*), as illustrated in SOM Table 1.2, is a symmetric equilibrium if and only if $\ell_S > \ell_B + \ell_P$. Its decision quality is $\Pi(\sigma^{Cand. 5b}) = (1 - q_B)q_P + [q_B q_P + (1 - q_B)(1 - q_P)](\pi(N))$.

Proof. We address each part of SOM Lemma 1.2 separately.

- i. Suppose first that $\ell_S \leq \ell_B + \ell_P$. We show that there is a utility improving deviation.

Consider shareholder i who deviates to strategy CAIS. (CAIS is illustrated in Table 1.) A deviation to CAIS differs from UNIS only when the vote recommendation is *for* and the own signal is a : with UNIS she would vote *no*, with CAIS she votes *yes*. It weakly improves decision quality iff $\ell_B + \ell_P \geq \ell_S$, which we assumed at the beginning of this argument. It saves costs if $f < c[q_B q_P + (1 - q_B)(1 - q_P)]$, which is satisfied by assumption that f is sufficiently lower than c .⁴⁸ Hence, i improves utility by deviating to CAIS.

Suppose now that $\ell_S > \ell_B + \ell_P$. In order to show that UNIS is an equilibrium, we show that there is no individual deviation that improves utility. We use the following principle: if a deviation is more attractive than an other deviation in terms of utility, then excluding the former is sufficient to exclude the latter. We organize the potential deviations by information-acquisition strategy. Pivotality always implies that among the $N - 1$ other shareholders the signals are split in $\frac{N-1}{2}$ a -signals and $\frac{N-1}{2}$ b -signals, which occurs with positive probability.

- (1) NotSubscribe-NotInvest. Deviating to Rubber-stamping decreases decision quality, as $\ell_S > \ell_B + \ell_P > \ell_B$ implies that $q_S > q_B$. It saves costs c . Hence, the deviation to Rubber-stamping is not beneficial for low enough c . Deviation to unconditionally voting *no* is even less attractive than deviating to Rubber-stamping (see also Proof of SOM Lemma 1.1).
- (2) NotSubscribe-Invest. A deviation to using the same information-acquisition strategy as in UNIS (NotSubscribe-Invest) but a different voting strategy is not an improvement (see Proof SOM Lemma 1.1).
- (3) Subscribe-NotInvest. Deviating to subscribe to the PA without investing into an own signal (Subscribe-NotInvest) is most attractive when voting according to the PA's recommendation. (When not conditioning on the vote recommendation, the shareholder could better deviate to no information acquisition, i.e., NotSubscribe-NotInvest.) Voting according to the PA's recommendation weakly improves decision quality iff $q_P \geq q_S$, which is precluded by $\ell_S > \ell_B + \ell_P$. Hence, this deviation strictly decreases decision quality. Consider now the costs. The deviation costs an additional f but saves c . If costs c are sufficiently low (as it is assumed in SOM Lemma 1.2), the decision quality difference dominates any cost difference, and hence, this deviation does not increase i 's utility.
- (4) Subscribe-Invest. Deviating to subscribe to the PA and invest into an own signal requires conditioning on both the vote recommendation and the own signal. Otherwise, there are other deviations which are more attractive because of the costs. Two cases are possible.

Case 1 is the deviation illustrated in Table A.2, i.e., voting *yes* except when both the own signal and the PA's recommendation contradict the board's proposal, then the deviating shareholder votes *no*. This deviation cannot alter the outcome after recommendation *against* because voting according to one's signal is like in UNIS. The difference to UNIS occurs if the vote recommendation is *for* and i 's signal is a (in UNIS i would vote *no*, in this deviation she would vote *yes*). Conditional on that case and on pivotality, the deviation weakly improves decision quality iff $\ell_B + \ell_P \geq \ell_S$, which is precluded by assumption $\ell_S > \ell_B + \ell_P$. Considering that the deviation is more costly than UNIS, it is not beneficial. The second case is illustrated in Table A.3, i.e., voting *no* except when both the own signal and the PA's recommendation are aligned with the board's proposal. This deviation cannot alter the outcome after recommendation *for* because voting according to one's

⁴⁸We use this assumption here only for the special case $\ell_B + \ell_P = \ell_S$.

signal is like in UNIS. The difference to UNIS hence only occurs if the vote recommendation is *against* and the signal is *b*: in UNIS *i* would vote *yes*; in this deviation she would vote *no*. Conditional on that case and conditional on pivotality, the deviation weakly improves decision quality iff $\ell_P \geq \ell_S + \ell_B$, which is precluded by assumption $\ell_S > \ell_B + \ell_P$. Hence, this deviation is not beneficial, considering that it is also more costly than UNIS.

- (5) *Subscribe-InvestIFFfor*. Consider the deviation to subscribing to the PA and investing into an own signal iff the PA's recommendation is *for*.

Since shareholders pay f and sometimes c they must condition their voting strategy on the information they have acquired. (Otherwise, there would be a more attractive deviation without these costs.) In particular, they must vote according to their signal after the *for* recommendation. There are two cases. They correspond to the strategies Cand. 5a and Cand. 5b.

Cand. 5a: shareholders vote *yes* except if the vote recommendation is *for* and the own signal is *a* (against) as in Table A.4. The deviation to Cand. 5a cannot alter the outcome after recommendation *for* because voting according to one's signal is like in UNIS. The difference to UNIS hence only occurs if the vote recommendation is *against* and the signal is *a* (in UNIS *i* would vote *no*, in this deviation she would vote *yes*). Conditional on that case and on pivotality, the deviation would weakly improve decision quality iff $\ell_B \geq \ell_S + \ell_P$. Since $\ell_S > \ell_B + \ell_P$ by assumption, we have $\ell_B < \ell_S + \ell_P$. Hence, this deviation decreases decision quality. The deviation costs f with certainty and c with probability $q_B q_P + (1 - q_B)(1 - q_P)$. UNIS costs c with certainty. The cost difference (costs of UNIS minus costs of Cand. 5b) is hence $c[q_B(1 - q_P) + (1 - q_B)q_P] - f$ which is small for c arbitrarily small (as it is assumed in the lemma). Hence, the deviation does not increase utility of a deviating shareholder.

Cand. 5b: shareholders vote *no* except if the vote recommendation is *for* and the own signal is *b* (for board) as in SOM Table 1.2.

		Own Signal	
		<i>b</i> (for board)	<i>a</i> (against)
PA	<i>for</i>	yes	no
	<i>against</i>	no	

Table 1.2: Cand. 5b: A strategy based on acquiring an own signal iff the PA's recommendation is *for*: *Subscribe-InvestIFFfor* and vote *no*, except if the PA's recommendation is *for* and the own signal is *b*.

The deviation to Cand. 5b cannot alter the outcome after recommendation *for* because voting according to one's signal is like in UNIS. The difference to UNIS hence only occurs if the vote recommendation is *against* and the signal is *b* (in UNIS *i* would vote *yes*, in this deviation she would vote *no*). Conditional on that case and on pivotality, the deviation would weakly improve decision quality iff $\ell_P \geq \ell_S + \ell_B$. By assumption $\ell_S > \ell_B + \ell_P$, we have $\ell_P < \ell_S + \ell_B$. Hence, this deviation reduces decision quality. The cost difference (costs of UNIS minus costs of Cand. 5b) is again $c[q_B(1 - q_P) + (1 - q_B)q_P] - f$, which is small for c arbitrarily small (as it is assumed in the lemma). Hence, the deviation does not increase utility of a deviating shareholder.

- (6) *Subscribe-InvestIFFagainst*. Consider a deviation to subscribing to the PA and investing into an own signal iff the vote recommendation is *against*.

Since shareholders pay f and sometimes c they must condition their voting strategy on the information they have acquired. (Otherwise, there would be a more attractive deviation, saving the costs.) In particular, they must vote according to their signal after the *against* recommendation. Two cases remain: CAIS as illustrated in Table 1 and CAIS-2 as illustrated in Table A.1. A deviation to CAIS differs from UNIS only when the vote recommendation is *for* and the own signal is a (with UNIS she would vote *no*, with CAIS she votes *yes*). It would weakly improve decision quality iff $\ell_B + \ell_P \geq \ell_S$. Since $\ell_S > \ell_B + \ell_P$, it strictly decreases decision quality. The cost difference (costs of UNIS minus costs of CAIS) is $c[q_B q_P + (1 - q_B)(1 - q_P)] - f$ and decreases in c . Hence, the difference in decision quality dominates, and the deviation to CAIS lowers the deviating shareholder's utility lower.

Consider now a deviation to strategy CAIS-2 (which only differs from CAIS by voting *no* when the vote recommendation is *for*). It differs from UNIS only when the vote recommendation is *for* and the own signal is b (indeed, with UNIS she votes *yes*, with CAIS-2 she would vote *no*). The deviation to CAIS-2 would weakly improve decision quality iff $\ell_B + \ell_S + \ell_P \leq 0$, which is never satisfied. It decreases decision quality. Since we assume sufficiently low costs, this deviation is not profitable.

Now, we have covered all deviations that can occur. If $\ell_S > \ell_B + \ell_P$, UNIS is an equilibrium for low enough costs c .

- ii. We have to show that Cand. 5a is a symmetric equilibrium if and only if $\ell_S > \ell_B + \ell_P$. The voting strategy is illustrated in Table A.4. A shareholder is not pivotal after the *against* recommendation, but after the *for* recommendation when all the other $N - 1$ shareholder's signals are exactly split.

Suppose first that $\ell_S \leq \ell_B + \ell_P$. We show that there is a beneficial deviation. Consider shareholder i who deviates to Rubber-stamping. This deviation only alters the outcome when the vote recommendation is *for*, all other shareholders' signals are split, and i 's signal is a (against): Under Cand. 5a, i would vote *no*, but under Rubber-stamping she votes *yes*. Decision quality weakly improves by this deviation given $\ell_B + \ell_P \geq \ell_S$. Moreover, costs are lower. Hence, Cand. 5a cannot be an equilibrium.

Suppose now that $\ell_S > \ell_B + \ell_P$. In order to show that Cand. 5a is an equilibrium, we show that there is no individual deviation that improves utility. We use the following principle: if a deviation is more attractive than an other deviation in terms of utility, then excluding the former is sufficient to exclude the latter. We organize the potential deviations by information-acquisition strategy. Pivotality always implies that the vote recommendation is *for* and among the $N - 1$ other shareholders the signals are split in $\frac{N-1}{2}$ a -signals and $\frac{N-1}{2}$ b -signals, which occurs with positive probability.

- (1) NotSubscribe-NotInvest. Deviating to Rubber-stamping, while saving costs, decreases decision quality as $\ell_B + \ell_P < \ell_S$. Hence, the deviation to Rubber-stamping is not beneficial for low enough c and f .

Deviating to unconditionally voting *no* (Protest) lowers decision quality in any case and is therefore less attractive than deviating to Rubber-stamping.

- (2) NotSubscribe-Invest. Consider deviating to information-acquisition strategy NotSubscribe-Invest. The most attractive combination of this strategy with a voting strategy is UNIS:

UNIS conditions the vote on the acquired signal in a way that maximizes decision quality. However, a deviation to UNIS does not change the outcome, since in case of pivotality (when the vote recommendation is *for*) the voting strategies are identical. Cand. 5a is less expensive than UNIS if $f + [q_B q_P + (1 - q_B)(1 - q_P)]c \leq c$, which is $f \leq c(1 - [q_B q_P + (1 - q_B)(1 - q_P)])$. This is satisfied for f sufficiently lower than c . Hence, UNIS is not a profitable deviation.

- (3) *Subscribe-NotInvest*. A deviation to subscribing to the PA is most attractive when voting according to the recommendation. (Voting the opposite of the recommendation reduces decision quality. When not conditioning on the vote recommendation, the shareholder could save costs and deviate to no information acquisition, *NotSubscribe-NotInvest*.) Voting according to the recommendation alters the outcome if vote recommendation is *for*, other shareholder's signals are split, and i 's signal is a (against): With Cand. 5a, i would vote *no*, with this deviation i would vote *yes*. The deviation weakly improves decision quality iff $\ell_B + \ell_P \geq \ell_S$, which is precluded by $\ell_S > \ell_B + \ell_P$. It decreases decision quality. The deviation saves costs c , but for low enough c it is not beneficial to deviate.
- (4) *Subscribe-Invest*. A deviation to subscribing to the PA and investing into an own signal requires to conditioning on both vote recommendation and own signal. Otherwise, there are other deviations which are more attractive, producing the same decision quality and saving the costs. Two cases are possible.

Case 1 is the deviation illustrated in Table A.2. The difference to UNIS only occurs if the vote recommendation is *for* and the own signal is a (in UNIS i would vote *no*, in this deviation she would vote *yes*). Conditional on that case and conditional on pivotality, the deviation weakly improves decision quality iff $\ell_B + \ell_P \geq \ell_S$, which is precluded by assumption. Considering that the deviation is more costly than Cand. 5a, this deviation is never beneficial.

The second case is illustrated in Table A.3. This deviation does not alter the outcome (after vote recommendation *for*, voting is according to signal like in Cand. 5a, and after recommendation *against*, the deviating shareholder is not pivotal). Considering that the deviation is also more costly than Cand. 5a, this deviation is never beneficial.

- (5) *Subscribe-InvestIFFfor*. Consider a deviation that keeps information-acquisition strategy *Subscribe-InvestIFFfor* but changes the voting strategy. Changes after the recommendation *against* are ineffective, as no shareholder is pivotal. Since shareholders pay c after recommendation *for* they must condition their voting strategy on the information they have acquired. (Otherwise, there would be a more attractive deviation that produces the same decision quality but saves c .) In particular, they must condition their vote on their signal after the *for* recommendation. This means that the deviation is voting *yes* when the own signal is a (*against*) and voting *no* when the own signal is b (for board). This deviation leads to the same costs as Cand. 5a but clearly reduces decision quality.
- (6) *Subscribe-InvestIFFagainst*. Consider a deviating shareholder who subscribes to the PA and invests into an own signal iff the PA's recommendation is *against*. Since the shareholder pays f and sometimes c , she must condition her voting strategy on the information she has acquired. (Otherwise, there would be a more attractive deviation that produces the same decision quality but saves costs.) In particular, she must vote according to her signal after the *against* recommendation. Two cases remain.

Case 1 is CAIS as illustrated in Table 1. It alters the outcome of Cand. 5a only when the vote recommendation is *for* and the own signal is a : Indeed, with Cand. 5a she would vote *no*,

with CAIS she votes *yes*. The deviation weakly improves decision quality iff $\ell_B + \ell_P \geq \ell_S$, which is precluded by assumption. It reduces decision quality. It does save some cost: Cand. 5a costs $f + [q_B q_P + (1 - q_B)(1 - q_P)]c$, CAIS costs $f + [q_B(1 - q_P) + (1 - q_B)q_P]c$. For low enough costs c , CAIS is not an improvement.

Case 2 is called CAIS-2 and illustrated in Table A.1 (it only differs from CAIS by prescribing to vote *no* when the vote recommendation is *for*). CAIS-2 differs from Cand. 5a only when the vote recommendation is *for* and the own signal is b (with Cand. 5a the shareholder would vote *yes*, with CAIS-2 she votes *no*). The deviation would weakly improve decision quality iff $\ell_B + \ell_S + \ell_P \leq 0$, which is precluded by assumption. It reduces decision quality. Hence, even for small enough costs c , this is not a beneficial deviation.

Now, we have covered all deviations that can occur. If $\ell_S > \ell_B + \ell_P$, Cand. 5a is an equilibrium for low enough costs c and sufficiently lower f .

The decision quality of the equilibrium Cand. 5a amounts to $q_B q_P * \pi(N) + q_B(1 - q_P) * 1 + (1 - q_B)q_P * 0 + (1 - q_B)(1 - q_P) * \pi(N)$.

- iii. The proof that Cand. 5b is an equilibrium under the same conditions as Cand. 5a is identical to the proof for Cand. 5a (immediately above).

The decision quality of the equilibrium Cand. 5b amounts to $q_B q_P * \pi(N) + q_B(1 - q_P) * 0 + (1 - q_B)q_P * 1 + (1 - q_B)(1 - q_P) * \pi(N)$.

To show that there are no additional equilibria, we exhaustively discuss all pure strategies. Again, we organize the discussion by information-acquisition strategy.

- (1) NotSubscribe-NotInvest. There are only voting strategies *yes* or *no*. Both lead to equilibria as shown in parts i. and ii. of Lemma A.1. (Assumption BIB does not matter for these results.)
- (2) NotSubscribe-Invest. Since shareholders pay c they must condition on their own signal. Otherwise, they could improve by voting in the same way and not investing c . Conditioning on the own signal leaves two pure strategies: voting *yes* if the signal is b and voting *no* if the signal is a (i.e., UNIS) or the opposite (voting *yes* if the signal is a and voting *no* if the signal is b). If voting *yes* after the own signal is a (against) was optimal, then voting *no* after signal a would also be so. Hence, shareholders could improve by unconditionally voting *no*. Only UNIS remains. When UNIS is an equilibrium has been already addressed in this lemma, SOM Lemma 1.2, part i.
- (3) Subscribe-NotInvest. Since shareholders pay f , they must condition on the PA's recommendation. Hence, they either vote *yes* after *for* and *no* after *against*, or they do the opposite. In either case, no shareholder is pivotal since all votes are the same given one particular vote recommendation. A shareholder can improve by not paying f and voting unconditionally, e.g., *yes*. Hence, there is no symmetric equilibrium with this information-acquisition strategy.
- (4) Subscribe-Invest. Since shareholders pay both f and c , they must condition their voting strategy on both the vote recommendation and the own signal. Otherwise, they could improve their utility by exhibiting the same voting behavior, but saving costs. This means that only two voting strategies remain.

Case 1: Consider the strategy to vote *yes* except if the vote recommendation is *against* and the signal is a , as in Table A.2. In this case no shareholder is pivotal if the PA recommends *for* (as the recommendation is common for all shareholders). Hence, shareholder i can only be

pivotal if the vote recommendation is *against*. If so, i votes according to her own signal. Hence, deviating to UNIS would not change the outcome because either i is not pivotal or i would also vote according to the own signal. However, UNIS saves fee f and is hence a profitable deviation. Thus, the strategy profile of case 1, illustrated in Table A.2, cannot be a symmetric equilibrium.

Case 2: Consider the strategy to vote *no* except if the vote recommendation is *for* and the own signal is b (for board), as in Table A.3. The analogous argument as above for case 1 applies, as follows: In this case no shareholder is pivotal if the PA recommends *against* (as the recommendation is common for all shareholders). Hence, shareholder i can only be pivotal if the recommendation is *for*. If so, i votes according to the own signal. Hence, deviating to UNIS would not change the outcome because either i is not pivotal or i would also vote according to the own signal. However, UNIS saves fee f and is hence a profitable deviation. Thus, the strategy profile of case 2 cannot be a symmetric equilibrium.

Therefore, there cannot be a symmetric equilibrium with this information-acquisition strategy (Subscribe-Invest), in which shareholders unconditionally buy both PA's recommendation and own signal.

- (5) Subscribe-InvestIFF*for*. Since shareholders pay f and sometimes c , they must condition their voting strategy on the vote recommendation and the own signal when they acquire them. In particular, after having bought the own signal on top of the recommendation *for*, shareholders must vote according to their signal in equilibrium. Voting the opposite is dominated and not conditioning as well. This leaves two cases, which we have already addressed as Cand. 5a and Cand. 5b in this lemma (SOM Lemma 1.2) in parts ii. and iii.
- (6) Subscribe-InvestIFF*against*. Since shareholders pay f and sometimes c , they must condition their voting strategy on the vote recommendation and the own signal when they acquire them. In particular, after having bought the own signal on top of the recommendation *against*, shareholders must vote according to their signal in equilibrium. Voting the opposite is dominated and not conditioning as well. This leaves two cases: CAIS and CAIS-2, which we have already addressed in Lemma A.2. (Assumption 1 does not affect these equilibria.)

Hence, there are no further symmetric equilibria. □

1.3 All Symmetric Equilibria with a Late Proxy Advisor

Consider now the situation when a PA is admitted and proxy advice arrives after the shareholders' decision to invest in own research. That is, all actions occur as illustrated in the timeline (Figure 1), but proxy advice arrives at the end of period $t = 2$. This timeline admits information-acquisition strategies NotSubscribe-NotInvest, NotSubscribe-Invest, Subscribe-NotInvest, and Subscribe-Invest, which are all also playable when Assumption PAF is satisfied. However, the violation of Assumption PAF precludes the two information-acquisition strategies Subscribe-InvestIFF*for* and Subscribe-InvestIFF*against*, the latter of which was crucial for our main results. SOM Table 1.1 illustrates this and already indicates that in this setting, the equilibria are very similar to the setting where there is no PA.

Comparing these two settings (the benchmark setting of no PA with a PA whose recommendation arrives late) we note that the presence of a PA who violates Assumption PAF admits two additional strategies: Subscribe-NotInvest, and Subscribe-Invest. It turns out that none of these two additional

information-acquisition strategies is part of an equilibrium. Intuitively, buying only the vote recommendation, i.e., information-acquisition strategy Subscribe-NotInvest, is only worthwhile when using this information in an instance of pivotality. However, if all shareholders symmetrically use the vote recommendation, then no shareholder is ever pivotal. If they do not use it, they could save costs by not paying the fee f . Similarly, and more importantly, acquiring both signals, i.e., information-acquisition strategy Subscribe-Invest, is never part of an equilibrium. The reason is that shareholders who acquire both the PA's signal and an own signal must condition their vote on both such that none is superfluous, e.g., by voting *yes* if and only if one of them is in favor of the proposal. When all shareholders adopt such a strategy, pivotality already implies the content of the vote recommendation. For instance, if shareholders vote *yes* if and only if either the vote recommendation or the own signal is in favor of the proposal, pivotality implies that the recommendation was *against*. Hence, there is a deviation to not paying the subscription fee. Analogous arguments exist for every other possible voting strategy that conditions on combinations of PA advice and the own signal. Therefore, admitting a late PA does not lead to additional equilibria. In contrast, it may destroy equilibria that existed without a PA because it offers additional deviation possibilities. This in fact occurs for some parameter range, as illustrated in SOM Figure 1.1 in the left area above the triangle, where UNIS ceases to be an equilibrium, while Rubber-stamping becomes the Pareto-efficient equilibrium. SOM Proposition 1.1 provides the general result.

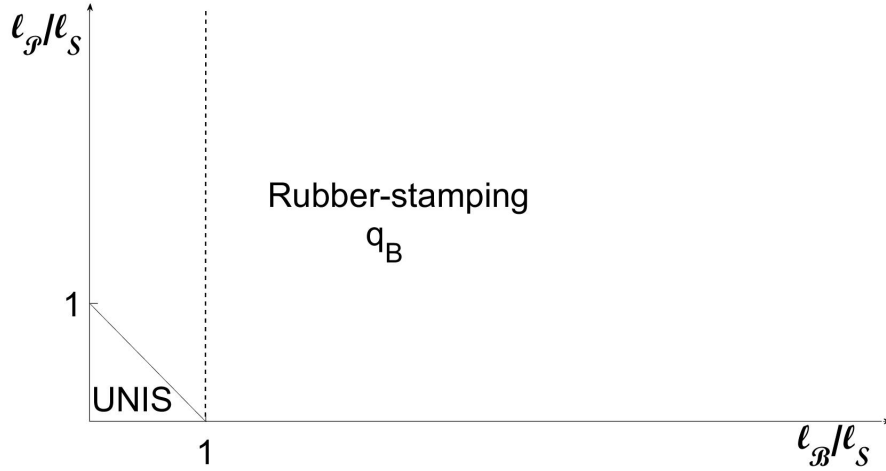


Figure 1.1: Pareto-efficient symmetric equilibria with late PA, i.e., when Assumption PAF is violated.

Proposition 1.1 (SYM with PA violating Assumption PAF). *Let costs c be arbitrarily small. Suppose there is a PA whose vote recommendation arrives after shareholders decided upon own research, i.e., Assumption PAF is violated.*

- i. *If $\ell_S \geq \ell_B + \ell_P$, then there exists a symmetric equilibrium in which shareholders invest in own research. The Pareto-efficient equilibrium is UNIS and leads to decision quality $\Pi(\sigma^{UNIS}) = \pi(N)$.*
- ii. *Otherwise, there does not exist a symmetric equilibrium in which shareholders invest in own research. The Pareto-efficient equilibrium is Rubber-stamping and leads to decision quality is $\Pi(\sigma^{rubber}) = q_B$.*

Proof. To show part i. of SOM Proposition 1.1, we use SOM Lemma 1.3 below, which shows that UNIS is an equilibrium (part iii.) and that there are two further equilibria: Rubber-stamping and Protest. It remains to show that UNIS Pareto-dominates rubber-stamping and the other trivial equilibrium (Protest) for $\ell_S \geq \ell_B + \ell_P$. We have

$$\Pi(\sigma^{UNIS}) = \pi(N) > q_S > q_B = \Pi(\sigma^{Rubber}) > 1 - q_B = \Pi(\sigma^{Protest}),$$

where $\sigma^{Protest}$ stands for the equilibrium in which shareholders acquire no information (NotSubscribe-NotInvest) and vote *no* unconditionally. Now, for costs c low enough (as it is assumed in the proposition), UNIS Pareto-dominates because of its higher decision quality.

To show part ii. of SOM Proposition 1.1, we use again SOM Lemma 1.3. For $\ell_S < \ell_B + \ell_P$, UNIS is not an equilibrium (by SOM Lemma 1.3) and only two equilibria remain, Rubber-stamping and Protest. Rubber-stamping Pareto-dominates because it leads to higher decision quality $\Pi(\sigma^{Rubber}) = q_B > 0.5 > 1 - q_B = \Pi(\sigma^{Protest})$. $\square$

It is important to observe that Assumption BIB, $q_S \leq q_B$, implies $\ell_S < \ell_B + \ell_P$, which precludes UNIS here. Hence, we can summarize.

Remark. Under Assumption BIB, unconditional information acquisition neither occurs with late PA (SOM Proposition 1.1), nor with an early PA (SOM Proposition 1.2). Conditional information acquisition occurs with an early PA (Proposition 2), but not with a late PA (SOM Proposition 1.1).

Considering the benchmark setting without a PA, SOM Proposition 1.1 is very similar to Proposition 1, but differs in the condition for the two equilibria. In fact, the condition for an equilibrium with information acquisition becomes more demanding when a PA is admitted: $\ell_S \geq \ell_B + \ell_P$ means that a single shareholder has to be better informed, not only than the board, but than both the board and the PA together. The reason is that there is an additional deviation possibility, compared to the setting without a PA: A shareholder could invest in an own signal and buy the vote recommendation and then vote *no* only if both the vote recommendation and the own signal are against the board.⁴⁹ This deviation only changes the voting outcome, compared to the UNIS strategy profile, if i is pivotal and the PA's recommendation is *for* and i 's signal is *a*: in UNIS i votes *no*, in this deviation she would vote *yes*. Conditional on that case, the deviation improves decision quality iff

$$\begin{aligned} q_B q_P (1 - q_S) \binom{N-1}{\frac{N-1}{2}} q_S^{\frac{N-1}{2}} (1 - q_S)^{\frac{N-1}{2}} &> (1 - q_B)(1 - q_P) q_S \binom{N-1}{\frac{N-1}{2}} (1 - q_S)^{\frac{N-1}{2}} q_S^{\frac{N-1}{2}} \\ \frac{q_B q_P (1 - q_S)}{1 - q_B} &> \frac{(1 - q_B)(1 - q_P) q_S}{1 - q_S} \\ \frac{q_P}{1 - q_P} &> \frac{q_S}{1 - q_S} \\ \ell_B + \ell_P &> \ell_S. \end{aligned}$$

When this condition is satisfied, then for a sufficiently small fee f , the deviation is an improvement for i since it increases decision quality. Hence, $\ell_S \geq \ell_B + \ell_P$ is a necessary condition for UNIS to be an equilibrium when costs c and fee f are arbitrarily small.

The foundation for SOM Proposition 1.1 is SOM Lemma 1.3 that we establish next.

Lemma 1.3 (SYM with PA violating Assumption PAF: All Equilibria). *Let costs c be arbitrarily small and let fee f be sufficiently smaller. Suppose there is a PA whose vote recommendation*

⁴⁹This strategy is illustrated in Table A.2.

arrives after shareholders decided upon own research, i.e., Assumption PAF is violated. Then in contrast to Lemma A.2 and SOM Lemma 1.2, we have:

- i. Protest (i.e., no shareholder invests in own research and all shareholders vote no) is a symmetric equilibrium for any $\ell_B, \ell_S \in (0, \infty)$. Its decision quality is $1 - q_B$.
- ii. Rubber-stamping (i.e., no shareholder invests in research and all shareholders vote yes) is a symmetric equilibrium for any $\ell_B, \ell_S \in (0, \infty)$. Its decision quality is q_B .
- iii. UNIS is a symmetric equilibrium if and only if $\ell_S \geq \ell_B + \ell_P$. Its decision quality is $\Pi(\sigma^{UNIS}) = \pi(N)$.
- iv. There are no other symmetric equilibria. In particular, there is no equilibrium in which all shareholders subscribe to proxy advice and invest in own signal (Subscribe-Invest).

Proof. We address each part of SOM Lemma 1.3 separately.

- i. The proof is identical to the proof of Lemma A.1, part i.
- ii. The proof is identical to the proof of Lemma A.1, part ii.
- iii. Suppose first that $\ell_S < \ell_B + \ell_P$. We show that there is a beneficial deviation for f low enough.

Consider shareholder i deviates to the following strategy σ'_i : subscribe and invest (Subscribe-Invest) and vote *yes*, except if the PA's recommendation is *against* and the own signal is a . This strategy is illustrated Table A.2.

Pivotality in UNIS implies that among the $N - 1$ other shareholders the signals are split in $\frac{N-1}{2}$ a -signals and $\frac{N-1}{2}$ b -signals. This deviation σ'_i does not alter the outcome after recommendation *against* because voting according to signal is like in UNIS. The difference to UNIS only occurs if the recommendation is *for* and i 's signal is a : in UNIS i would vote *no*, in this deviation she would vote *yes*. Conditional on that case and conditional on pivotality, the deviation improves decision quality if and only if $\ell_B + \ell_P > \ell_S$, which holds by the assumption made at the beginning of this argument. Note that the deviation has the same research costs c but additional costs f . Hence, for f small enough (as it is assumed in this lemma) it strictly improves utility of i .

Suppose now that $\ell_S \geq \ell_B + \ell_P$. In order to show that UNIS is an equilibrium, we have to show that there is no individual deviation that improves utility. In the proof of SOM Lemma 1.2, part i., we consider all possible deviations from UNIS and show that there is no individual deviation that improves utility for $\ell_S > \ell_B + \ell_P$. Here, there are less deviation possibilities to consider, as Subscribe-InvestIFF*for* and Subscribe-InvestIFF*against* cannot be played. For the other four information-acquisition strategies, all arguments from the proof of SOM Lemma 1.2 still apply under the assumption $\ell_S \geq \ell_B + \ell_P$ when simply using that $\ell_S \geq \ell_B + \ell_P$ implies $q_S > q_B$ and $q_S > q_P$.

- iv. There are four information-acquisition strategies to consider.

- (1) For NotSubscribe-NotInvest, i.e., shareholders neither subscribe to the PA nor invest in an own signal, both strategy profiles are symmetric equilibria, as already addressed in parts i. and ii. of this Lemma, SOM Lemma 1.3.

- (2) For NotSubscribe-Invest, i.e., shareholders do not subscribe to the PA but invest in an own signal, UNIS is an equilibrium as established in SOM Lemma 1.3, part i. The other strategy with NotSubscribe-Invest, voting the opposite of the signal, is not an equilibrium. The proof is the same as in SOM Lemma 1.1.
- (3) Consider Subscribe-NotInvest, i.e., shareholders subscribe to the PA but do not invest in an own signal. Since shareholders pay f , they must condition their voting strategy on the recommendation. (Otherwise, they could improve by using the same voting strategy, but saving fee f .) To condition the voting strategy on the recommendation means either to vote *yes* after *for* and *no* after *against*, or the opposite voting strategy. In either case, no shareholder is pivotal since all vote in the same way after a given vote recommendation. A shareholder can improve by not paying f and voting, e.g. *yes*. Hence, there is no symmetric equilibrium with this information-acquisition strategy.
- (4) Consider now Subscribe-Invest, i.e., shareholders subscribe to the PA and invest in an own signal. Since shareholders pay both f and c , they must condition their voting strategy on both the vote recommendation and their own signal. Otherwise, they could improve exhibiting the same voting behavior, but saving costs. This means that in fact only two voting strategies remain. Case 1: The strategy to vote *yes* except if the vote recommendation is *against* and the own signal is a , when i votes according to the own signal, as illustrated in Table A.2. In this case 1, no shareholder is pivotal if the PA recommends *for* (as the recommendation is common for all shareholders). Hence, shareholder i can only be pivotal if the recommendation is *against*. If so, i votes according to the own signal. Hence, deviating to strategy UNIS would not change the outcome because either i is not pivotal or i would also vote according to the own signal. UNIS, however, saves fee f . Thus, the strategy profile of case 1 (Table A.2) cannot be a symmetric equilibrium.
- Case 2: The strategy to vote *no* except if both the vote recommendation is *for* and the own signal is b (for board), when i votes according to the own signal, as in Table A.3. The analogous argument as above for case 1 applies, as follows: In this case 2, no shareholder is pivotal if the PA recommends *against* (as the recommendation is common for all shareholders). Hence, shareholder i can only be pivotal if the vote recommendation is *for*. If so, i votes according to the own signal. Hence, deviating to strategy UNIS would not change the outcome because either i is not pivotal or i would also vote according to the own signal. UNIS, however, saves fee f . Thus, the strategy profile of case 2 (Table A.3) cannot be a symmetric equilibrium.
- Therefore, there cannot be a symmetric equilibrium with information-acquisition strategy Subscribe-Invest, in which shareholders unconditionally buy both the PA's recommendation and an own signal.

□

1.4 Derivation of Small Cost Condition

We derive the *small cost condition* discussed in Section 5.3, which is sufficient for the symmetric CAIS to be an equilibrium. If the costs c are not small, a shareholder has a profitable deviation from CAIS to Rubber-stamping, which does not involve any information acquisition costs. In contrast, CAIS has expected costs $C(\hat{\sigma}) = f + p^{dis}c$ for any shareholder. To compute the expected loss of benefits, focus on a potentially deviating shareholder i . Denote by $p^{split}(N-1, q_S)$ the probability

that the other $N - 1$ voters have their signals exactly split into $\frac{N-1}{2}$ a -signals and the same number of b signals:

$$p^{split}(N - 1, q_S) := \binom{N - 1}{\frac{N-1}{2}} (1 - q_S)^{\frac{N-1}{2}} q_S^{\frac{N-1}{2}}.$$

Denote by $\Delta(\hat{\sigma}, \sigma')$ the difference in expected decision quality between the symmetric strategy profile CAIS, and the strategy profile σ' in which one voter has deviated from CAIS to Rubber-stamping, conditioning on the case that all other shareholders' signals are split:

$$\Delta(\hat{\sigma}, \sigma') := (1 - q_B) q_P q_S - q_B (1 - q_P) (1 - q_S).$$

As any voter owns only the share $\frac{1}{N}$ of the firm, the symmetric CAIS equilibrium requires: $f + p^{dis} c < \frac{1}{N} p^{split}(N - 1, q_S) \cdot \Delta(\hat{\sigma}, \sigma')$. With $p^{dis} \in (0, \frac{1}{2})$ and $f \leq \frac{c}{2}$, we have $f + p^{dis} c < c$. Hence,

$$c < \frac{1}{N} p^{split}(N - 1, q_S) \cdot \Delta(\hat{\sigma}, \sigma') = \frac{1}{N} \binom{N - 1}{\frac{N-1}{2}} (1 - q_S)^{\frac{N-1}{2}} q_S^{\frac{N-1}{2}} [(1 - q_B) q_P q_S - q_B (1 - q_P) (1 - q_S)]$$

is a sufficient condition for CAIS to be an equilibrium. Translated into costs per-share stake, $c^{rel} := \frac{c}{1/N}$, this *small cost condition* becomes

$$c^{rel} < p^{split}(N - 1, q_S) \cdot \Delta(\hat{\sigma}, \sigma').$$

How restrictive this assumption is depends on the signal qualities and on the number of voters N . In Example 1 of the main text, we have $\Delta(\hat{\sigma}, \sigma') = 0.015$; and $p^{split} \approx .1171$ (.3456) when setting $N = 21$ ($N = 5$). Hence, the private costs of one shareholder had to be smaller than 0.18% (0.52%) of the per-share stake in this example. These values are 1.07% (3.15%) in another example that also satisfies all conditions of Proposition 2, where we set $q_B = q_S = .6$ and $q_P = .69$. These values are 0.39% (3.33%) in a third example, where we set $q_B = q_S = .7$ and $q_P = .8$.

2 Asymmetric Equilibria

The main text provides one key result for asymmetric equilibria (Proposition 3), while it summarizes the others. This section of the Supplementary Online Material (SOM) provides the detailed analysis for what happens when we drop the symmetry assumption. We first show that without a PA, the number of shareholders who invest in own research is bounded from above (SOM Section 2.1). We then show how admitting a PA alters this result. More specifically, we analyze asymmetric equilibria with a PA first by establishing three lemmata (SOM Section 2.2), then by comparing the shareholders' research incentives with and without a PA (SOM Section 2.3), and finally by comparing the resulting decision quality (SOM Section 2.3). As we show, the number of shareholders who invest or conditionally invest, as well as the decision quality, weakly increase due to the presence of a PA, confirming the conclusions from the analysis of symmetric equilibria.

2.1 Asymmetric Equilibria in Benchmark Setting without a Proxy Advisor

Consider again the benchmark setting in which no PA is admitted. While Proposition 1 stated that under Assumption BIB, there is no symmetric equilibrium in which *every* shareholder invests in own research, the next result extends this to asymmetric equilibria in some parameter range. It also states that generally, in equilibrium without a PA there are always some shareholders not investing in research.

Proposition 2.1 (ASYM without PA). *Suppose no PA is admitted.*

- i. *If $\frac{\ell_B}{\ell_S} \geq \frac{N+1}{2}$, there does not exist an equilibrium in which any shareholder invests in own research. In any Pareto-efficient equilibrium $N' \in \{\frac{N+1}{2}, \dots, N\}$ shareholders (i.e., a majority) play Rubber-stamping and $N - N'$ play Protest, which leads to decision quality $\Pi(\sigma^{Rubber}) = q_B$.*
- ii. *Otherwise, i.e., if $\frac{\ell_B}{\ell_S} < \frac{N+1}{2}$, in equilibrium the number of shareholders who invest in own research is at most $z_1 := N - \lfloor \frac{\ell_B}{\ell_S} \rfloor$.⁵⁰ In the Pareto-efficient equilibrium under sufficiently small costs c , $N - \lfloor \frac{\ell_B}{\ell_S} \rfloor = z_1$ shareholders play UNIS and $\lfloor \frac{\ell_B}{\ell_S} \rfloor$ shareholders play Rubber-stamping.*

Proof. The proof is organized in four paragraphs.

All Strategies. Consider all pure strategies. First, those who do not buy the signal have the following pure strategies: voting *yes* (Rubber-stamping) and voting *no* (Protest).

Second, a shareholder that invests into the own signal must condition his voting behavior on the signal and be pivotal in at least one draw of nature. Otherwise, i.e., when voting unconditionally or never being pivotal, there is an improvement by keeping the voting strategy and not investing into the signal, saving costs c . A shareholder that invests and conditions on the signal can either vote in line with his signal (i.e., vote *yes* if b (for board) and *no* if a (against board)), which we call UNIS, or do the opposite (*yes* iff a). The opposite (*yes* iff a) cannot be part of an equilibrium strategy. Indeed, if voting *no* after receiving signal b (for board) is a best response, then, conditional on signal b , state A , which does not match the board's proposal, must be more likely than B , the state that matches the board's proposal. Since the information technology of signals is monotonic, receiving signal a (against board) makes state A even more likely such that this voter also prefers to vote *no*

⁵⁰The mathematical expression $\lfloor z \rfloor$ is defined as the largest integer that is lower or equal to z .

if the signal is a (against). Hence, the opposite voting strategy can be ruled out and only UNIS remains for those who buy the signal.

Conditions for Investing in Own Signal. For a given strategy profile σ , a given realization of signals, and a shareholder i , let us define two numbers d and δ_{-i} . For the voters who have not invested, let $\boxed{d}$ be the number of unconditional *yes*-votes minus the unconditional *no* votes; for the voters who have invested, let $\boxed{\delta_{-i}}$ be the number of a (against) signals received minus the number of b (for board) signals received when excluding the focal shareholder i . (Recall that in equilibrium those who received signal b will vote *yes* and those who received signal a will vote *no*).

Suppose shareholder i invested in research and is pivotal. Pivotality implies that the number of total votes of others, $N - 1$, is fifty-fifty split in *yes* and *no* votes. This implies that $d = \delta_{-i}$.

The two necessary and jointly sufficient conditions for optimality of voting according to the own signal are: $\ell_B + 1\ell_S > \delta_{-i}\ell_S$, which is required for (and implies) optimality of voting *yes* when the own signal is b , and $\delta_{-i}\ell_S + 1\ell_S < \ell_B$, which is required for (and implies) optimality of voting *no* after the own signal a . This yields $d = \delta_{-i} \in (\frac{\ell_B}{\ell_S} - 1, \frac{\ell_B}{\ell_S} + 1)$. In fact, the interval is open. Suppose to the contrary, that $d = \frac{\ell_B}{\ell_S} - 1$. Then $\delta_{-i} = d$ implies $\delta_{-i}\ell_S + 1\ell_S = \ell_B$, i.e., an informed shareholder i is indifferent between voting *yes* and voting *no* after receiving signal a . After receiving signal b , this shareholder prefers to vote *yes*. Hence, if this shareholder would unconditionally vote *yes*, she would induce the same decision quality. Therefore, this shareholder could unilaterally improve (upon her strategy UNIS) by Rubber-stamping, i.e., not investing into an own signal and voting *yes* unconditionally, which saves costs $c > 0$. Analogously, it would be beneficial to switch to unconditionally voting *no* in case of $d = \frac{\ell_B}{\ell_S} + 1$.

Part i. For part i. of the proposition, we use in particular that we have $d = \delta_{-i} > \frac{\ell_B}{\ell_S} - 1$ when a shareholder invests, while $\frac{\ell_B}{\ell_S} \geq \frac{N+1}{2}$ by assumption. Thus, $d > \frac{N+1}{2} - 1$ and hence $d \geq \frac{N+1}{2}$ (as d is a natural number). Hence, the assumption that a shareholder invests in own research leads to the implication that at least $d \geq \frac{N+1}{2}$ more shareholders unconditionally vote *yes* than unconditionally vote *no*. The latter, however, implies that no voter is ever pivotal (since there is always a majority voting *yes*). This, in turn, contradicts the assumption that a shareholder invests in research. Thus, there cannot be an informed voter in equilibrium.

Without informed voter, each shareholder votes either unconditionally *yes* (Rubber-stamping) or *no* (Protest). Information quality is q_B if a majority votes unconditionally *yes* and $1 - q_B$ otherwise. Hence, in any Pareto-efficient equilibrium $N' \in \{\frac{N+1}{2}, \dots, N\}$ play the strategy of Rubber-stamping and $N - N'$ play the strategy of Protest such that decision quality is $\Pi^* = q_B$.

Part ii. In order to prove the upper bound for the number of informed shareholders, we use again that we have $d > \frac{\ell_B}{\ell_S} - 1$ when a shareholder invests. This implies $d \geq \lfloor \frac{\ell_B}{\ell_S} \rfloor$, as d is a natural number. To create a vote difference of d among those who did not invest, it takes at least d voters, e.g., when d vote *yes* while zero vote *no*. Hence, the number of informed voters is at most $N - \lfloor \frac{\ell_B}{\ell_S} \rfloor = z_1$.

We now prove that $\lfloor \frac{\ell_B}{\ell_S} \rfloor$ players voting always *yes* (strategy Rubber-stamping) and $N - \lfloor \frac{\ell_B}{\ell_S} \rfloor$ players voting according to their private signals (strategy UNIS) is indeed an equilibrium. To this end, we show that no shareholder in these two groups has an incentive to deviate from her strategy.

- Consider a shareholder who invests in an own signal and votes according to it (strategy UNIS). She is pivotal if the others' votes are split, which happens if and only if there are $d = \lfloor \frac{\ell_B}{\ell_S} \rfloor = \delta_{-i}$ more a -signals than b -signals among the other informed shareholders. In that occasion, deviating to voting *no* after signal b is suboptimal because it would reject

the board's proposal although $\ell_B + 1\ell_S \not\geq \delta_{-i}\ell_S$; and deviating to voting *yes* after signal a is suboptimal because it would accept the board's proposal although $\ell_B \not\geq \delta_{-i}\ell_S + 1\ell_S$. As pivotality occurs with positive probability, deviations certainly reduce decision quality, while they can only save costs c that are by assumption arbitrarily small.

- Consider now a shareholder who always votes *yes*. Even if she knew her signal in case of being pivotal, she would never want to deviate to voting *no*, as she is pivotal if and only if there are $\frac{N-1}{2} - (\lfloor \frac{\ell_B}{\ell_S} \rfloor - 1)$ players who obtained positive signals and $\frac{N-1}{2}$ players with negative ones: A negative signal would make her prefer to vote *no* if and only if $\ell_B + (\frac{N-1}{2} - \lfloor \frac{\ell_B}{\ell_S} \rfloor + 1) \cdot \ell_S < \frac{N-1}{2} \cdot \ell_S + \ell_S$, which is equivalent to $\ell_B - \lfloor \frac{\ell_B}{\ell_S} \rfloor \cdot \ell_S < 0$ and thus to $\frac{\ell_B}{\ell_S} < \lfloor \frac{\ell_B}{\ell_S} \rfloor$, which is a contradiction.

We will now prove that the equilibrium is Pareto-efficient. To this end, we first prove that for any equilibrium in which a positive number of players invests in own research, the difference d between players always voting *yes* and those always voting *no* must equal $\lfloor \frac{\ell_B}{\ell_S} \rfloor$. Above, we have already derived that $d \in (\frac{\ell_B}{\ell_S} - 1, \frac{\ell_B}{\ell_S} + 1)$, which can only happen for $d = \lfloor \frac{\ell_B}{\ell_S} \rfloor$ or $d = \lfloor \frac{\ell_B}{\ell_S} \rfloor + 1$. For any fixed strategy profile and signal distribution, let $\boxed{\delta}$ denote the difference between the number of informed shareholders voting *no* and informed shareholders voting *yes*. A shareholder always voting *yes* will then be pivotal if and only if $d - 1 = \delta$ or, equivalently, $d = \delta + 1$. Now, let us assume that this shareholder still had the opportunity for own research and obtained a signal which happens to contradict the board's proposal. This shareholder would like to vote *no* if $\ell_B < \delta\ell_S + \ell_S$, which is equivalent to $\ell_B < d\ell_S$ or $\frac{\ell_B}{\ell_S} < d$. Thus, whenever $d > \frac{\ell_B}{\ell_S}$ and costs are sufficiently low, we cannot have a rubber-stamping player in equilibrium because such a player would benefit from deviating to UNIS. As indeed $\lfloor \frac{\ell_B}{\ell_S} \rfloor + 1 > \frac{\ell_B}{\ell_S}$, $d = \lfloor \frac{\ell_B}{\ell_S} \rfloor + 1$ cannot result in an equilibrium. Thus, we overall must have $d = \lfloor \frac{\ell_B}{\ell_S} \rfloor$ in any equilibrium in which some shareholders invest in own research.

Overall, we now have established that apart from trivial equilibria in which no shareholder invests in private research, there can only be equilibria in which the difference between those always voting *yes* and those always voting *no* is exactly equal to $\lfloor \frac{\ell_B}{\ell_S} \rfloor$. Possible non-trivial equilibria are thus characterized by $\lfloor \frac{\ell_B}{\ell_S} \rfloor + \alpha$ shareholders always voting *yes* and α shareholders always voting *no*, with α being a non-negative integer. Define $\pi(l, k) := \sum_{i=k}^l \binom{l}{i} q_S^i (1 - q_S)^{l-i}$, i.e., the probability that among l realizations of signals with precision q_S at least k are correct. Decision quality is then

$$\Pi(\sigma) = q_B \cdot \pi \left(N - (d + \alpha), \frac{N+1}{2} - d \right) + (1 - q_B) \cdot \pi \left(N - (d + \alpha), \frac{N+1}{2} \right)$$

for $d = \lfloor \frac{\ell_B}{\ell_S} \rfloor$, as $N - (d + \alpha)$ shareholders play strategy UNIS, $d + \alpha$ play strategy Rubber-stamping, and α play Protest. Since the function π is increasing in its first argument, decision quality is maximized for $\alpha = 0$. Hence, in the equilibrium of this type that provides the highest decision quality, $N - \lfloor \frac{\ell_B}{\ell_S} \rfloor$ play strategy UNIS, while $\lfloor \frac{\ell_B}{\ell_S} \rfloor$ play strategy Rubber-stamping. The corresponding decision quality is larger than the highest decision quality of all equilibria in which no shareholder invests in own research, which is q_B . The reason is the following: whenever the proposal of the board is accepted, the decisions coincide, but when a proposal is rejected, we have at least $d + 1$ more signals against the proposal than for the proposal, which makes it more likely that the proposal is wrong: $\ell_B \leq (d + 1)\ell_S$ for $d = \lfloor \frac{\ell_B}{\ell_S} \rfloor$. $\square$

To understand SOM Proposition 2.1, consider a shareholder who invested in own research. This investment can only be part of an equilibrium if this shareholder conditions on her own

signal in some instance in which she is pivotal. In particular, this shareholder must vote *no* if the signal is *a* (against). This is a best response if pivotality implies that a sufficient number of other informed shareholders also have received information against the board's proposal. This, in turn, is possible in strategy profiles in which several uninformed shareholders Rubber-stamp the board's proposal. When the number of shareholders who Rubber-stamp is by roughly $\frac{\ell_B}{\ell_S}$ larger than the number of shareholders who vote unconditionally *no* (i.e., play Protest), then there might indeed be incentives to invest in own research and vote according to one's signal. If this difference, however, exceeds half of all shareholders, as considered in part i., then it is impossible to be pivotal in the first place. Otherwise, i.e., in the case addressed in part ii., it is possible to have informed shareholders, but their number is bounded from above by $N - \lfloor \frac{\ell_B}{\ell_S} \rfloor$. It turns out that the strategy profile with the highest decision quality is then $\sigma^{\mu, \nu}$, with $\mu = N - \lfloor \frac{\ell_B}{\ell_S} \rfloor$ shareholders investing in own research and voting according to signal (strategy UNIS), and $\nu = \lfloor \frac{\ell_B}{\ell_S} \rfloor$ shareholders playing strategy Rubber-stamping. This strategy profile is Pareto-efficient and yields the upper bound for the decision quality. For the description of decision quality it is helpful to define $\pi(l, k)$ as the probability that among l realizations of signals with precision q_S at least k are correct, i.e., $\pi(l, k) := \sum_{i=k}^l \binom{l}{i} q_S^i (1 - q_S)^{l-i}$. Then the decision quality in case of part ii. of SOM Proposition 2.1 is $\Pi(\sigma) = q_B \cdot \pi(z_1, z_1 - \frac{N-1}{2}) + (1 - q_B) \cdot \pi(z_1, \frac{N+1}{2})$, where still $z_1 := N - \lfloor \frac{\ell_B}{\ell_S} \rfloor$.

Comparative statics imply that the maximal number of shareholders who invest is decreasing in the board's relative information quality $\frac{\ell_B}{\ell_S}$, starting with $N - 1$ for $\lfloor \frac{\ell_B}{\ell_S} \rfloor = 1$, decreasing down to $\frac{N+1}{2}$ for $\lfloor \frac{\ell_B}{\ell_S} \rfloor = \frac{N-1}{2}$, and then discontinuously jumping to 0.⁵¹ This validates the insight we gained from the symmetric equilibria: Without a PA, well informed boards reduce shareholders' research incentives.⁵²

2.2 Lemmata for Asymmetric Equilibria with a Proxy Advisor

Before analyzing the effects of introducing a PA on shareholders' research incentives (SOM Section 2.3) and on decision quality (SOM Section 2.4), we establish three lemmata about asymmetric equilibria when a PA is admitted. SOM Lemma 2.1 shows two properties of equilibrium behavior, SOM Lemma 2.2 provides bounds on the number of informed shareholders, and SOM Lemma 2.3 characterizes the Pareto-efficient strategy profiles.

Lemma 2.1 (ASYM with PA: Equilibrium Behavior). *Let Assumption PAF hold. Let costs $c > 0$ be arbitrarily small and let fee $f > 0$ be sufficiently smaller. In equilibrium the following statements hold:*

- i. *There is no shareholder who buys the vote recommendation and unconditionally invests in own research, i.e., uses Subscribe-Invest.*
- ii. *Every shareholder who acquires an own signal votes according to this signal, i.e., votes yes if the signal is *b* (board) and vote no if the signal is *a* (against).*

Proof. We first show part i. and then use part i. to show part ii.

- i. Suppose shareholder i uses Subscribe-Invest. Since i pays both f and c she must condition her voting strategy on both the PA's vote recommendation and the own signal. This excludes

⁵¹Only if $\frac{\ell_B}{\ell_S} < 1$, which is the negation of Assumption BIB, all N shareholders could be informed.

⁵²Their overall effect on decision quality might however still be positive -- we will discuss comparative-static effects on decision quality further below in SOM Section 2.4.

unconditional voting (such as always *yes*) and voting conditional only on one type of information (such as only voting according to the vote recommendation, or only voting according to the signal). Indeed, compared to these strategies, the shareholder could improve by exhibiting the same voting behavior, but saving costs. This means that only one type of voting strategy remains, namely conditioning on both the vote recommendation and the own signal. Consider one such strategy, namely voting *yes* except if the vote recommendation is *against* and the signal is *a*, then the shareholder votes *no* (as in Table A.2). In this case, the shareholder votes *yes* after the *for* recommendation, independently of the own signal. She could improve by keeping the same voting behavior, but changing the information-acquisition strategy, switching to *Subscribe-InvestIFFagainst*, which saves costs c with positive probability. Likewise, there is an improvement for all strategies of this type. In particular, consider voting *no* except if the vote recommendation is *for* and the own signal is *b* (for board), as in Table A.3. Here, there is an improvement to *Subscribe-InvestIFFfor*.

- ii. There are four information-acquisition strategies that involve investing in an own signal: *NotSubscribe-Invest*, *Subscribe-Invest*, *Subscribe-InvestIFFfor*, and *Subscribe-InvestIFFagainst*.

Suppose there is a shareholder i who plays *NotSubscribe-Invest* and violates the assertion. This shareholder either votes independently of the own signal or votes for the opposite of what the signal indicates. In the former case, i could improve by using the same voting strategy, but not acquiring a signal. That would lead to the same decision quality but reduce her costs. In the latter case, the shareholder is either never pivotal and could again improve by not acquiring the signal, or she is pivotal with positive probability. If pivotal with positive probability, i would only find it optimal to vote for the opposite of what the signal indicates if, conditional on signal b and pivotality, state A that does not match the board's proposal is at least as likely as state B that does (and conditional on signal a and pivotality, state B is at least as likely as A). However, as the signal is drawn independently of the occurrence of pivotality and is informative as $q_S > 0.5$, this is impossible (conditional on signal b , state B is strictly more likely than it would be when conditioning on signal a). Therefore, a shareholder who uses *NotSubscribe-Invest* in equilibrium plays strategy UNIS.

There is no shareholder who plays *Subscribe-Invest*, as shown in part i. of this proof.

Suppose there is a shareholder i who plays *Subscribe-InvestIFFfor* and violates the assertion. After vote recommendation *for*, this shareholder either votes independently of the signal or for the opposite of what the signal indicates. There are improvements by switching to not acquiring the signal or to voting according to the signal, in complete analogy to the information-acquisition strategy *NotSubscribe-Invest* above. Likewise, this holds for *Subscribe-InvestIFFagainst*.

□

Lemma 2.2 (ASYM with PA: Bounds). *Let Assumption PAF hold. Let costs $c > 0$ be arbitrarily small and let fee $f > 0$ be sufficiently smaller. In equilibrium the following holds.*

- i. If $\frac{\ell_B + \ell_P}{\ell_S} < \frac{N+1}{2}$, then the number of shareholders who invest in own research is at most $N - \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor$ after vote recommendation *for* and at most $N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ after vote recommendation *against*.
- ii. If $\frac{|\ell_B - \ell_P|}{\ell_S} < \frac{N+1}{2} \leq \frac{\ell_B + \ell_P}{\ell_S}$, then no shareholder invests in own research after vote recommendation *for* and at most $N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ shareholders invest after vote recommendation *against*.

iii. Otherwise, i.e., if $\frac{N+1}{2} \leq \frac{|\ell_B - \ell_P|}{\ell_S}$, no shareholder invests in own research.

Proof. We first establish two statements and then proceed with the three parts of SOM Lemma 2.2.

- (B1) In equilibrium, the number of shareholders who invest in own research is at most $\max\{N - \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor, 0\}$ after vote recommendation *for*.
- (B2) In equilibrium, the number of shareholders who invest in own research is at most $\max\{N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor, 0\}$ after vote recommendation *against*. This holds for both (i) $\ell_B \geq \ell_P$ and (ii) $\ell_B < \ell_P$.

Ad B1 Suppose statement (B1) is violated. Then there is an equilibrium where the number of informed shareholders after vote recommendation *for*, x_1 , satisfies $x_1 > N - \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor$ and $x_1 > 0$.

Since the information-acquisition strategy Subscribe-Invest is never part of an equilibrium (by part i. of SOM Lemma 2.1), $x_1 > 0$ implies that every informed player after vote recommendation *for* either plays NotSubscribe-Invest or Subscribe-InvestIFF*for*.

Suppose first that there is a shareholder i who plays Subscribe-InvestIFF*for* in equilibrium. This player must be pivotal after vote recommendation *for* with positive probability, otherwise she could improve by saving costs c without affecting decision quality. Pivotality implies that $\frac{N-1}{2}$ of the $N - 1$ other shareholders vote *yes* and $\frac{N-1}{2}$ vote *no*. Let $\boxed{d}$ be the difference between the *yes*-votes and *no*-votes of uninformed players (i.e., those who have not acquired a signal) after vote recommendation *for*. By part ii. of SOM Lemma 2.1, all shareholders who have acquired a signal after vote recommendation *for* must vote according to it. Therefore, conditional on having received recommendation *for* and on pivotality, shareholder i knows that among the other informed shareholders, exactly $\delta_{-i} = d$ more have received signal a than b . (Again $\boxed{\delta_{-i}}$ designates the number of a (against) signals received minus the number of b (for board) signals received when excluding the focal shareholder i .) However, as the number of uninformed players $y_1 := N - x_1$ satisfies $y_1 < \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor$, the signal difference that makes player i pivotal satisfies $\delta_{-i} = d \leq y_1 < \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor$. As δ_{-i} is a natural number, $\delta_{-i} < \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor$ implies $\delta_{-i} \leq \frac{\ell_B + \ell_P}{\ell_S} - 1$, which is equivalent to $\ell_B + \ell_P \geq \delta_{-i}\ell_S + 1\ell_S$. The final inequality means that state B , which matches the board's proposal, is more likely than state A , which does not match the proposal, when i is pivotal after vote recommendation *for*, even if i has received signal a . Hence, shareholder i could improve by voting *yes* after vote recommendation *for* and not investing in the own signal, which would weakly increase decision quality and strictly decrease costs, contradicting the assumption that the strategy profile with i playing Subscribe-InvestIFF*for* was an equilibrium.

Suppose now that there is no shareholder who plays Subscribe-InvestIFF*for* in equilibrium. Then all informed shareholders play NotSubscribe-Invest. By part ii. of SOM Lemma 2.1, all of them play UNIS. Consider the unilateral deviation of a shareholder i from UNIS to strategy CAIS. By the assumption on costs that f is sufficiently lower than c , this saves costs. Moreover, this would weakly improve decision quality: after the *against* recommendation the decision is the same, but after the *for* recommendation the decision quality weakly improves. The argument for the latter claim is exactly as before when considering a player using Subscribe-InvestIFF*for*. When shareholder i is pivotal after vote recommendation *for*, we have $\delta_{-i} = d \leq y_1 = N - x_1 < \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor$, which implies $\ell_B + \ell_P \geq \delta_{-i}\ell_S + 1\ell_S$,

meaning that state B is more likely than state A even if i has received signal a . Hence, the deviation to strategy CAIS is an improvement for shareholder i , contradicting the assumption that the strategy profile with no shareholder playing Subscribe-InvestIFF *for* was an equilibrium.

Hence, if (B1) is violated, we have that no shareholder plays Subscribe-InvestIFF *for* in equilibrium and that a strategy profile without a shareholder playing Subscribe-InvestIFF *for* cannot be an equilibrium -- a contradiction.

Ad B2(i) Suppose statement (B2) is violated and $\ell_B \geq \ell_P$ holds. Then there is an equilibrium where the number of informed shareholders after the *against* recommendation, x_2 , satisfies $x_2 > N - \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$ and $x_2 > 0$.

Since the information-acquisition strategy Subscribe-Invest is never part of an equilibrium (by part i. of SOM Lemma 2.1), $x_2 > 0$ implies that every informed player after the *against* recommendation either plays NotSubscribe-Invest or Subscribe-InvestIFF *against*.

Suppose first that there is a shareholder i who plays Subscribe-InvestIFF *against* (e.g., strategy CAIS) in equilibrium. This shareholder must be pivotal with positive probability, otherwise she could improve by saving costs without affecting the decision quality. By part ii. of SOM Lemma 2.1, all shareholders who are informed after vote recommendation *against* vote according to their signal. In particular, shareholder i must vote *no* after signal a , which requires: $\ell_B < \ell_P + \delta_{-i}\ell_S + \ell_S$, where δ_{-i} is again the vote difference of the other informed shareholders. (A weak inequality cannot be part of an equilibrium strategy as the deviation to voting always *yes* after vote recommendation *against* and saving costs c would be an improvement.) That is, $\delta_{-i} > \frac{\ell_B - \ell_P}{\ell_S} - 1$. Since all informed shareholders vote according to their signal, the condition above and pivotality imply that at least $\frac{\ell_B - \ell_P}{\ell_S}$ shareholders are uninformed, which is in contradiction to the assumption $x_2 > N - \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$ as this inequality implies that strictly less than $\lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$ are uninformed.⁵³ Hence, in equilibrium no shareholder can play Subscribe-InvestIFF *against*.

Suppose now that there is no player who plays Subscribe-InvestIFF *against* in equilibrium. Then all informed shareholders after the *against* recommendation play strategy UNIS (by part ii. of SOM Lemma 2.1). Consider the unilateral deviation of a shareholder i from UNIS to Subscribe-InvestIFF *for*, voting unconditionally *yes* after vote recommendation *against*. By the assumption on costs that f is sufficiently lower than c , this saves costs. Moreover, this would weakly improve decision quality because after the *for* recommendation, the decision is the same, but after the *against* recommendation, the decision weakly improves, as we show now. For the given strategy profile in the assumed equilibrium, we define the following numbers for the case that the vote recommendation is *against*: Let $y_2 = N - x_2$ be the number of uninformed shareholders, let d be the difference of *yes*-votes and *no*-votes among the y_2 uninformed shareholders, and let δ_{-i} be the difference of a -signals and b -signals of the x_2 informed shareholders. When shareholder i is pivotal after vote recommendation *against*, we have $\delta_{-i} = d \leq y_2 = N - x_2 < \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$. For $\ell_B \geq \ell_P$, and since δ_{-i} is a natural number, this implies $\delta_{-i} \leq \frac{\ell_B - \ell_P}{\ell_S} - 1$, which is equivalent to $\ell_B \geq \ell_P + \delta_{-i}\ell_S + 1\ell_S$. The final inequality means that state B , which matches the board's proposal, is more likely than state A , which does not match the proposal, when shareholder i is pivotal after vote recommendation *against*, even if i has received signal a . Hence, the deviation weakly

⁵³For an integer x , $x \geq \lfloor \tau \rfloor$ is equivalent to $x > \tau - 1$.

improves decision quality, while it strictly saves costs, contradicting the assumption that the strategy profile with no shareholder playing *Subscribe-InvestIFFagainst* was an equilibrium. Hence, if B2(i) is violated, we have that no shareholder plays *Subscribe-InvestIFFagainst* in equilibrium and that a strategy profile without a shareholder playing *Subscribe-InvestIFFagainst* cannot be an equilibrium -- a contradiction.

Ad B2(ii) Suppose statement (B2) is violated and $\ell_B < \ell_P$ holds. Then there is an equilibrium where the number of informed shareholders after the *against* recommendation, x_2 , satisfies $x_2 > N - \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$ and $x_2 > 0$.

Since the information-acquisition strategy *Subscribe-Invest* is never part of an equilibrium (by part i. of SOM Lemma 2.1), $x_2 > 0$ implies that every informed player after the *against* recommendation either plays *NotSubscribe-Invest* or *Subscribe-InvestIFFagainst*.

Suppose first that there is a shareholder i who plays *Subscribe-InvestIFFagainst* (e.g., strategy CAIS) in equilibrium. For the given strategy profile in the assumed equilibrium, we define the following numbers for the case that the vote recommendation is *against*: Let $y_2 = N - x_2$ be the number of uninformed shareholders, let $\tilde{d} = -d$ be the difference of *no*-votes and *yes*-votes among the y_2 uninformed shareholders, and let $\tilde{\delta}_{-i} = -\delta_{-i}$ be the difference of *b*-signals and *a*-signals (in favor of the proposal) of the x_2 informed shareholders who are not i . When shareholder i is pivotal after vote recommendation *against*, we have $\tilde{\delta}_{-i} = \tilde{d} \leq y_2 = N - x_2 < \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$. For $\ell_B < \ell_P$ and since $\tilde{\delta}_{-i}$ is a natural number, this implies $\tilde{\delta}_{-i} \leq \frac{\ell_P - \ell_B}{\ell_S} - 1$, which is equivalent to $\ell_B + \tilde{\delta}_{-i}\ell_S + 1\ell_S \leq \ell_P$. The final inequality means that state B , which matches the board's proposal, is less likely than state A , which does not match the proposal, when shareholder i is pivotal after vote recommendation *against*, even if i has received signal b . Hence, i could improve by deviating to unconditionally voting *no* after vote recommendation *against* and not investing in an own signal. This deviation weakly improves decision quality, while it strictly saves costs, contradicting the assumption that the strategy profile with i playing *Subscribe-InvestIFFagainst* was an equilibrium.

Suppose now that there is no player who plays *Subscribe-InvestIFFagainst*. Then all informed shareholders after the *against* recommendation play UNIS. Consider the unilateral deviation of a shareholder i from UNIS to *Subscribe-InvestIFFfor*, unconditionally voting *no* after vote recommendation *against*. By the assumption on costs that f is sufficiently lower than c , this saves costs. Moreover, this would weakly improve decision quality by the same argument used immediately above: When shareholder i is pivotal after vote recommendation *against*, we have $\tilde{\delta}_{-i} = \tilde{d} \leq y_2 = N - x_2 < \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$, which implies $\ell_B + \tilde{\delta}_{-i}\ell_S + 1\ell_S \leq \ell_P$. The final inequality means that state B is less likely than state A when shareholder i is pivotal after vote recommendation *against* even if i has received signal b .

Hence, if B2(ii) is violated, we have that no shareholder plays *Subscribe-InvestIFFagainst* in equilibrium and that a strategy profile without a shareholder playing *Subscribe-InvestIFFagainst* cannot be an equilibrium -- a contradiction.

Now, we can use (B1) and (B2) to address the three parts of SOM Lemma 2.2.

- i. By the assumption $\frac{\ell_B + \ell_P}{\ell_S} < \frac{N+1}{2}$, we have $\max\{N - \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor, 0\} = N - \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor$ and the result after vote recommendation *for* follows directly from statement (B1). Likewise, we have $\frac{\ell_B - \ell_P}{\ell_S} \leq \frac{\ell_B + \ell_P}{\ell_S} < \frac{N+1}{2}$, implying $\max\{N - \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor, 0\} = N - \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$ and the result after vote recommendation *against* follows directly from statement (B2).

- ii. By the assumption $\frac{|\ell_B - \ell_P|}{\ell_S} < \frac{N+1}{2}$, we have $\max\{N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor, 0\} = N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ and the result after vote recommendation *against* follows directly from statement (B2).

It remains to show that after vote recommendation *for*, no shareholder invests in own research. Suppose, in contrast, that there is an equilibrium where the number x_1 of informed shareholders after vote recommendation *for* satisfies $x_1 > 0$.

Statement (B1) implies $x_1 \leq N - \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor$. Hence, the number of uninformed shareholders after vote recommendation *for* satisfies $y_1 = N - x_1 \geq \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor$, which implies $y_1 > \frac{\ell_B + \ell_P}{\ell_S} - 1$. The assumption $\frac{N+1}{2} \leq \frac{\ell_B + \ell_P}{\ell_S}$ now implies $\frac{N+1}{2} \leq \frac{\ell_B + \ell_P}{\ell_S} < y_1 + 1$. Hence, there are at least $\frac{N+1}{2}$ uninformed shareholders after vote recommendation *for*. Therefore, no shareholder can be pivotal in that case. Any of the $x_1 > 0$ shareholders who invest in an own signal can beneficially deviate to (still) buying the vote recommendation, but not investing in an own signal after vote recommendation *for*, as this saves costs and does not affect decision quality.

- iii. Suppose, in contrast, that there is an equilibrium where either (1) the number of informed shareholders after the *for* recommendation, x_1 , satisfies $x_1 > 0$, or (2) the number of informed shareholders after the *against* recommendation, x_2 , satisfies $x_2 > 0$, or both. We show that there is a contradiction, first for statement (1) and then for statement (2).

The first case (1) is excluded by statement B1, exactly as shown immediately above.

In the second case (2), statement B2 implies $x_2 \leq N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$. Hence, the number of uninformed shareholders after vote recommendation *against* satisfies $y_2 = N - x_2 \geq \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$, which implies $y_2 > \frac{|\ell_B - \ell_P|}{\ell_S} - 1$. The assumption $\frac{N+1}{2} \leq \frac{|\ell_B - \ell_P|}{\ell_S}$ now implies $\frac{N+1}{2} \leq \frac{|\ell_B - \ell_P|}{\ell_S} < y_2 + 1$. Hence, there are at least $\frac{N+1}{2}$ uninformed shareholders after vote recommendation *against*. Therefore, no shareholder can be pivotal in that case. Any of the $x_2 > 0$ shareholders who invest in an own signal can beneficially deviate to (still) buying the vote recommendation, but not investing in an own signal after vote recommendation *against*, as this saves costs and does not affect decision quality.

□

Lemma 2.3 (ASYM with PA: Pareto-efficient Equilibria). *Let Assumption PAF hold. Let costs $c > 0$ be arbitrarily small and let fee $f > 0$ be sufficiently smaller. Then the following strategy profiles are Pareto-efficient equilibria.*⁵⁴

- i. If $\frac{\ell_B + \ell_P}{\ell_S} < \frac{N+1}{2}$ and $\ell_B \geq \ell_P$, then $\lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ shareholders play strategy Rubber-stamping, $\lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ play strategy CAIS, and $N - \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor$ play strategy UNIS. Hence, decision quality is $\Pi^* = q_B q_P \pi(N - \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor, \frac{N+1}{2} - \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor) + (1 - q_B)(1 - q_P) \pi(N - \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor, \frac{N+1}{2}) + q_B(1 - q_P) \pi(N - \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor, \frac{N+1}{2} - \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor) + (1 - q_B) q_P \pi(N - \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor, \frac{N+1}{2})$.
- ii. If $\frac{\ell_B + \ell_P}{\ell_S} < \frac{N+1}{2}$ and $\ell_B < \ell_P$, then $\lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ shareholders play Follow (buy recommendation and follow it), $\lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ play strategy CAIS, and $N - \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor$ play strategy UNIS. Hence, decision quality is $\Pi^* = q_B q_P \pi(N - \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor, \frac{N+1}{2} - \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor) + (1 - q_B)(1 - q_P) \pi(N - \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor, \frac{N+1}{2}) + q_B(1 - q_P) \pi(N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor, \frac{N+1}{2}) + (1 - q_B) q_P \pi(N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor, \frac{N+1}{2} - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor)$.

⁵⁴We are particularly thankful to Stefan Kloessner who foresaw this result during a walk in the forest.

- iii. If $\frac{|\ell_B - \ell_P|}{\ell_S} < \frac{N+1}{2} \leq \frac{\ell_B + \ell_P}{\ell_S}$ and $\ell_B \geq \ell_P$, then $\lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ shareholders play strategy Rubber-stamping and $N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ play strategy CAIS. Hence, decision quality is $\Pi^* = q_B q_P + q_B(1 - q_P)\pi(N - \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor, \frac{N+1}{2} - \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor) + (1 - q_B)q_P\pi(N - \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor, \frac{N+1}{2})$.
- iv. If $\frac{|\ell_B - \ell_P|}{\ell_S} < \frac{N+1}{2} \leq \frac{\ell_B + \ell_P}{\ell_S}$ and $\ell_B < \ell_P$, then $\lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ shareholders play strategy Protest and $N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ play strategy CAIS. Hence, decision quality is $\Pi^* = q_B q_P + q_B(1 - q_P)\pi(N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor, \frac{N+1}{2}) + (1 - q_B)q_P\pi(N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor, \frac{N+1}{2} - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor)$.
- v. If $\frac{|\ell_B - \ell_P|}{\ell_S} \geq \frac{N+1}{2}$ and $\ell_B \geq \ell_P$, then $N' \in \{\frac{N+1}{2}, \dots, N\}$ shareholders play strategy Rubber-stamping and $N - N'$ play strategy Protest. Hence, decision quality is $\Pi^* = q_B$.
- vi. If $\frac{|\ell_B - \ell_P|}{\ell_S} \geq \frac{N+1}{2}$ and $\ell_B < \ell_P$, then $\frac{N-1}{2}$ shareholder play strategy Rubber-stamping, $\frac{N+1}{2}$ play Protest, and 1 plays Follow. Hence, decision quality is $\Pi^* = q_P$.

SOM Lemma 2.3 is illustrated in the lower panel of Figure 3 of the main text, where any region of the figure corresponds to one case of the lemma. In each region a Pareto-efficient equilibrium is indicated. The comparison with the Pareto-efficient equilibria in the benchmark setting without proxy advisor, illustrated in the upper panel of Figure 3, is discussed in the next two subsections. The remainder of this subsection is dedicated to prove SOM Lemma 2.3.

Proof. We first determine the theoretically maximal decision quality. Then we show that in every area of the parameter space the claimed strategy profile reaches this maximum, is an equilibrium, and is Pareto-efficient.

Generally, decision quality is maximal for a given number of signals if for any realization of these signals, the alternative that is more likely to match the true state is implemented. Moreover, additional signals increase the maximal decision quality. In our model, we always have the signal of the board of quality q_B and the signal of the PA of quality q_P . Additionally, we can have a number of signals of quality q_S . This number is restricted by SOM Lemma 2.2 and depends on whether the PA's signal agrees with the board (i.e., the vote recommendation is *for*) or not (i.e., the vote recommendation is *against*). Let $\bar{n}_1$ denote the maximal number of signals of quality q_S when the PA's signal agrees with the board's (vote recommendation *for*) and let $\bar{n}_2$ denote the maximal number of signals of quality q_S when it disagrees (vote recommendation *against*). For any realization of signals of quality q_S let δ be the number of *a*-signals minus the number of *b*-signals, i.e., the difference against the board, and define $\delta = 0$ if there is no such signal.

Then maximal decision quality under the constraints given by the upper bounds $\bar{n}_1$ and $\bar{n}_2$ is reached if and only if the following two conditions are satisfied:

- (C1) When the PA's signal is aligned with the board's, then $\bar{n}_1$ signals of quality q_S are generated and the proposal is accepted if $\delta < \frac{\ell_B + \ell_P}{\ell_S}$ and rejected if $\delta > \frac{\ell_B + \ell_P}{\ell_S}$ (and either accepted or rejected if $\delta = \frac{\ell_B + \ell_P}{\ell_S}$).
- (C2) When the PA's signal is *against* board, then $\bar{n}_2$ additional signals are generated and the proposal is accepted if $\delta < \frac{\ell_B - \ell_P}{\ell_S}$ and rejected if $\delta > \frac{\ell_B - \ell_P}{\ell_S}$ (and either accepted or rejected if $\delta = \frac{\ell_B - \ell_P}{\ell_S}$).

Indeed, when the PA's signal agrees with the board's and $\bar{n}_1 = 0$, the proposal of the board must be accepted. When the PA's signal disagrees with the board's and $\bar{n}_2 = 0$, the proposal must be

accepted if $q_B > q_P$ and rejected if $q_B < q_P$. When the PA's signal agrees with the board's and $\bar{n}_1 > 0$, then the condition for acceptance in C1 is equivalent to the following.

$$\begin{aligned}\delta &< \frac{\ell_B + \ell_P}{\ell_S} \\ \delta \ell_S &< \ell_B + \ell_P \\ \delta \log\left(\frac{q_S}{1 - q_S}\right) &< \log\left(\frac{q_B}{1 - q_B}\right) + \log\left(\frac{q_P}{1 - q_P}\right) \\ \left(\frac{q_S}{1 - q_S}\right)^\delta &< \frac{q_B}{1 - q_B} \cdot \frac{q_P}{1 - q_P},\end{aligned}$$

which is indeed the condition for state B being more likely than state A , conditional on the signal realizations. The analogous arguments hold for $\delta > \frac{\ell_B + \ell_P}{\ell_S}$ and Condition C2. Observe that in C2, $\delta < \frac{\ell_B - \ell_P}{\ell_S}$ is negative when $q_B < q_P$, requiring more b -signals than a -signals to accept the proposal, as it should be. In the knife-edge case where $\delta = \frac{\ell_B + \ell_P}{\ell_S}$, respectively $\delta = \frac{\ell_B - \ell_P}{\ell_S}$, both states are equally likely, conditional on the signal realizations, and hence it does not matter for decision quality which decision is made. It is an important part of the Conditions C1 and C2 that the maximal possible number of signals of quality q_S is generated, as any lower number of signals would lead to a strictly lower decision quality even if the decisions, given the realizations of signals, were according to the inequalities in C1 and C2.

We now set out to show for each case of SOM Lemma 2.3 that the claimed strategy satisfies the two conditions C1 and C2, with upper bounds $\bar{n}_1$ and $\bar{n}_2$ derived from SOM Lemma 2.2, and hence the maximum decision quality is attained. Moreover, we show that no shareholder has an incentive to deviate.

- i. For $\frac{\ell_B + \ell_P}{\ell_S} < \frac{N+1}{2}$ and $\ell_B \geq \ell_P$, part i. of SOM Lemma 2.2 applies. Hence, $\bar{n}_1 = N - \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor$ and $\bar{n}_2 = N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$. With the proposed strategy profile, we have $N - \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor$ shareholders who play strategy UNIS, which makes $\bar{n}_1$ informed shareholders after the *for* recommendation. After the *against* recommendation, we have additional $\lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ informed shareholders who play strategy CAIS. This makes $N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor = \bar{n}_2$ informed shareholders after the *against* recommendation.

After the *against* recommendation, the $\lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ Rubber-stamping shareholders vote *yes* and all other shareholders (those who play strategy CAIS and those who play strategy UNIS) invest in an own signal and vote according to it. Thus, the proposal is accepted if and only if $\delta < \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$. If $\lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ is even, then $\bar{n}_2$ is odd and thus δ is odd. If $\lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ is odd, then $\bar{n}_2$ is even and thus δ is even. Thus, for any realization of signals we have $|\delta - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor| \geq 1$ because exactly one of these integers is even and one of them is odd.⁵⁵ Taken together, if the vote recommendation is *against* and $\delta < \frac{\ell_B - \ell_P}{\ell_S}$, then $\delta < \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ and the proposal is accepted, as required by C2. If the vote recommendation is *against* and $\delta > \frac{\ell_B - \ell_P}{\ell_S}$, then $\delta > \frac{\ell_B - \ell_P}{\ell_S} \geq \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ and the proposal is rejected, as required by C2. Hence, Condition C2 is satisfied.

Suppose the recommendation is *for*. Then, $\lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor$ shareholders vote *yes* (those playing strategy Rubber-stamping plus those playing strategy CAIS), while $N - \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor = \bar{n}_1$ shareholders invest in a signal of quality q_S and vote according to it. Thus, the proposal is accepted if and only

⁵⁵Hence, $\delta = \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ is not possible.

if $\delta < \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor$. Of the integers $\lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor$ and δ exactly one is even and one is odd.⁵⁶ (δ is even iff $\bar{n}_1$ is even, while $\bar{n}_1 = N - \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor$, whereas N is always odd.) If $\delta < \frac{\ell_B + \ell_P}{\ell_S}$, then $\delta < \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor$ and the proposal is accepted, as required by C1. If $\delta > \frac{\ell_B + \ell_P}{\ell_S}$, then $\delta > \frac{\ell_B + \ell_P}{\ell_S} \geq \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor$ and the proposal is rejected, as required by C1. Hence, Condition C1 is satisfied.

Hence, the proposed strategy profile satisfies Conditions C1 and C2. Thus, decision quality is maximized under the constraints that the maximal numbers of informed shareholders after recommendations *for* and *against* must not exceed upper bounds $\bar{n}_1$ and $\bar{n}_2$, respectively. Deviating to a strategy that lowers decision quality is not an improvement as costs are assumed to be sufficiently small and hence cannot compensate any loss in decision quality. Potentially beneficial deviations must therefore either reduce costs without affecting decision quality, or increase decision quality (which requires relaxing at least one of the constraints that impose upper bounds $\bar{n}_1$ and $\bar{n}_2$).⁵⁷

The shareholders who Rubber-stamp cannot reduce costs, as their costs are zero. Suppose shareholder i deviates from strategy Rubber-stamping and improves decision quality. She is pivotal after the *for* recommendation if the others' votes are split (there are $\frac{N+1}{2}$ *yes*-votes and $\frac{N+1}{2}$ *no*-votes). As $\lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor - 1$ others play strategy Rubber-stamping and $\lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor - \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$ play strategy CAIS, there are $\lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor - 1$ unconditional *yes*-votes after recommendation *for*. The remaining $N - \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor$ shareholders play UNIS and are thus informed. Hence, i is pivotal after *for* if and only if $\delta = \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor - 1$ (where δ is again the signal difference against the board among the informed). With Rubber-stamping, i would vote *yes*. To change the decision with a deviation, she must deviate to voting *no*. This deviation would be most likely to increase decision quality if she had received signal a . Decision quality would improve in that case iff $\ell_B + \ell_P < (\lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor - 1)\ell_S + 1\ell_S$, which is equivalent to $\frac{\ell_B + \ell_P}{\ell_S} < \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor$, but clearly not true. Hence, the Rubber-stamping shareholder i cannot improve decision quality after vote recommendation *for*.

The Rubber-stamping shareholder i is pivotal after the *against* recommendation if the others' votes are split. As $\lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor - 1$ others play strategy Rubber-stamping, there are $\lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor - 1$ unconditional *yes*-votes after recommendation *against*. The remaining $N - \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$ shareholders play strategy UNIS or strategy CAIS and are thus informed. Hence, i is pivotal after recommendation *against* if and only if $\delta = \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor - 1$. With Rubber-stamping, i would vote *yes*. To change the decision with a deviation she must deviate to voting *no*. This deviation would be most likely to increase decision quality if she had received signal a . Decision quality would improve in that case iff $\ell_B < \ell_P + (\lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor - 1)\ell_S + 1\ell_S$, which is equivalent to $\frac{\ell_B - \ell_P}{\ell_S} < \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$ and clearly not true, as $|\ell_B - \ell_P| = \ell_B - \ell_P$ for $q_B \geq q_P$ (which holds by assumption in Case i. of SOM Lemma 2.3). Therefore, a shareholder who Rubber-stamps cannot beneficially deviate.

The shareholders who play strategy UNIS bear the costs c . The following information-acquisition strategies have lower costs (than NotSubscribe-Invest): NotSubscribe-NotInvest, Subscribe-NotInvest, Subscribe-InvestIFF*for*, and Subscribe-InvestIFF*against*, by the assumption that

⁵⁶Hence, $\delta = \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor$ is not possible.

⁵⁷We have to check for deviations that might potentially increase decision quality, although we have already established that decision quality in equilibrium is maximal under the constraints given by SOM Lemma 2.2. While the bounds of SOM Lemma 2.2 hold in any equilibrium, decision quality after deviations from a strategy profile is not *per se* restricted by the respective upper bound in SOM Lemma 2.2.

f is sufficiently lower than c . Suppose one shareholder deviates to a strategy that involves one of these information-acquisition strategies. If this shareholder uses NotSubscribe-NotInvest, Subscribe-NotInvest, or Subscribe-InvestIFF*against*, then the number of informed shareholders after *for* is smaller than $\bar{n}_1$, namely $\bar{n}_1 - 1$, and hence C1 is violated. If this shareholder uses Subscribe-InvestIFF*for*, then the number of informed shareholders after recommendation *against* is smaller than $\bar{n}_2$, namely $\bar{n}_2 - 1$, and hence C2 is violated. Thus, any cost-saving deviation of a shareholder who plays strategy UNIS decreases decision quality. To improve decision quality, while at most $\bar{n}_1$ or $\bar{n}_2$ shareholders are informed after recommendation *for* or *against*, respectively, is not possible because decision quality is already maximal (by satisfying C1 and C2). To improve decision quality by deviating from strategy UNIS and increasing the number of informed shareholders after *for* or *against* is also not possible, as a shareholder who plays strategy UNIS is already informed in both cases. Thus, there is no beneficial deviation for a shareholder who plays strategy UNIS.

The shareholders who play strategy CAIS and hence use Subscribe-InvestIFF*against* could save costs by deviating and using the information-acquisition strategies NotSubscribe-NotInvest or Subscribe-NotInvest. Indeed, both would save costs by the assumption that f is sufficiently smaller than c . Deviating to the other information-acquisition strategies would increase costs by the same assumption and by the fact that the recommendation *for* is more likely than *against*: $q_B q_P > q_B(1 - q_P) + (1 - q_B)q_P$. If a shareholder deviates to a strategy involving NotSubscribe-NotInvest or Subscribe-NotInvest, this reduces decision quality because the number of signals after recommendation *against* is smaller than $\bar{n}_2$ and thus, Condition C2 is violated.

Hence, it remains to check whether a shareholder i who plays strategy CAIS can improve decision quality with some deviation. After recommendation *for*, she votes *yes* according to CAIS and hence has to vote *no* and be pivotal to affect the decision with her deviation. After recommendation *for*, $\lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor - 1$ shareholders vote *yes* without being informed when excluding i (summing up those who play strategy Rubber-stamping and those who play strategy CAIS, minus one player). Pivotality after recommendation *for* requires that the votes of the $N - 1$ others are split, which occurs if and only if the signal difference of the $\bar{n}_1$ informed shareholders, δ_{-i} , satisfies $\delta_{-i} = \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor - 1$ because this is the number of shareholders who vote *yes* without being informed. Conditioning on pivotality, voting *no* would be most likely to increase decision quality if the own signal was a . Decision quality would improve in that case iff $\ell_B + \ell_P < (\lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor - 1)\ell_S + 1\ell_S$, which is equivalent to $\frac{\ell_B + \ell_P}{\ell_S} < \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor$ and clearly not true. After the *against* recommendation, a shareholder i who plays strategy CAIS votes according to the own signal, as do all others who play strategy CAIS or strategy UNIS. The $\lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$ Rubber-stamping shareholders vote *yes*. In order to affect the decision with her deviation, shareholder i has to vote *yes* after signal a or *no* after signal b and be pivotal. Pivotality after recommendation *against* requires that the votes of the $N - 1$ others are split, which occurs if and only if $\delta_{-i} = \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$, where δ_{-i} is again the signal difference against the proposal among the informed shareholders, excluding i . Conditional on pivotality after recommendation *against*, deviating to voting *yes* after signal a would improve decision quality iff $\ell_B > \ell_P + \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor \ell_S + 1\ell_S$, which is equivalent to $\frac{\ell_B - \ell_P}{\ell_S} > \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor + 1$, which is clearly not true. Deviating to *no* after signal b would improve decision quality in that case iff $\ell_B + 1\ell_S < \ell_P + \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor \ell_S$, which is equivalent to $\frac{\ell_B - \ell_P}{\ell_S} + 1 < \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$, which is clearly not true either. Thus, there is no beneficial deviation for a shareholder who plays strategy CAIS.

Taken together, above arguments imply that the proposed strategy profile is an equilibrium as no shareholder can profitably deviate. This equilibrium maximizes decision quality under the

constraints of SOM Lemma 2.2 as it satisfies the Conditions C1 and C2 for these constraints. By SOM Lemma 2.2 there cannot be an equilibrium with looser constraints. Hence, among all equilibria it attains maximal decision quality. A Pareto-superior equilibrium would require lower costs for some shareholders without increasing the costs of any other shareholder. The total costs after vote recommendation *for* are $\bar{n}_1 c + \left(\lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor - \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor \right) f$; and after vote recommendation *against*, total costs are $\bar{n}_2 c + \left(\lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor - \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor \right) f$. Reducing the number of informed shareholders in either case to save costs c would lead to a violation of the Conditions C1 or C2. Reducing the number of shareholders who subscribe to the PA (by an integer $t \in \{0, 1, \dots, \left(\lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor - \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor \right)\}$) to decrease costs related to fee f would either lead to a decrease in the number of informed shareholders or would require a “compensation” by having t more shareholders who unconditionally invest in an own signal (Subscribe-NotInvest). The former violates the Conditions C1 or C2, the latter is even more costly as $tc > tf$ holds by assumption. Thus, the proposed equilibrium is Pareto-efficient.

Finally, decision quality for this equilibrium can be constructed by using the probability that under the equilibrium strategy profile the decision is correct, considering the four combinations of whether the board’s signal is correct or wrong and whether the PA’s signal is correct or wrong, together with the probability $\pi(l, k)$ that among l realizations with precision q_S at least k are correct: $\Pi^* = q_B q_P \pi(N - \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor, \frac{N+1}{2} - \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor) + (1 - q_B)(1 - q_P) \pi(N - \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor, \frac{N+1}{2}) + q_B(1 - q_P) \pi(N - \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor, \frac{N+1}{2} - \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor) + (1 - q_B) q_P \pi(N - \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor, \frac{N+1}{2})$.

- ii. For $\frac{\ell_B + \ell_P}{\ell_S} < \frac{N+1}{2}$ and $\ell_B < \ell_P$, part i. of SOM Lemma 2.2 applies. Hence, $\bar{n}_1 = N - \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor$ and $\bar{n}_2 = N - \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$. With the proposed strategy profile, we have $N - \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor$ shareholders who play strategy UNIS, which makes $\bar{n}_1$ informed shareholders after the *for* recommendation. After the *against* recommendation, we have additional $\lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor - \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$ informed shareholders who play strategy CAIS. This makes $N - \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor = \bar{n}_2$ informed shareholders after the *against* recommendation. The remaining $\lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$ shareholders play Follow, i.e., they are uninformed and always vote according to the vote recommendation.

Suppose the vote recommendation is *for*. Then, $\lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor$ shareholders vote *yes* (those playing Follow plus those playing strategy CAIS), while $N - \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor = \bar{n}_1$ shareholders invest into a signal of quality q_S and vote according to it. Thus, the proposal is accepted if and only if $\delta < \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor$, where δ is again the signal difference against the board. Of the integers $\lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor$ and δ , exactly one is even and one is odd (see above). If $\delta < \frac{\ell_B + \ell_P}{\ell_S}$, then $\delta < \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor$ and the proposal is accepted, as required by C1. If $\delta > \frac{\ell_B + \ell_P}{\ell_S}$, then $\delta > \frac{\ell_B + \ell_P}{\ell_S} \geq \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor$ and the proposal is rejected, as required by C1. Hence, Condition C1 is satisfied.

After the *against* recommendation, the $\lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$ shareholders who play Follow vote *no* and all other shareholders (who play strategy CAIS or strategy UNIS) invest into an own signal and vote according to it. Thus, the proposal is accepted if and only if $\tilde{\delta} > \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$, where $\tilde{\delta} = -\delta$ is the number of b -signals minus the number of a -signals (that is the vote difference in favor of the board). Of the integers $\lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$ and $\tilde{\delta}$ exactly one is even and one is odd. ($\tilde{\delta}$ is even iff $\bar{n}_2$ is even, while $\bar{n}_2 = N - \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$, whereas N is always odd.) Thus, for any realization of signals we have $|\tilde{\delta} - \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor| \geq 1$. The condition $\tilde{\delta} > \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$ is hence equivalent to $\tilde{\delta} > \frac{\ell_B - \ell_P}{\ell_S}$ and (since $\ell_B < \ell_P$) to $-\delta > \frac{\ell_P - \ell_B}{\ell_S}$ and finally to $\delta < \frac{\ell_B - \ell_P}{\ell_S}$, as required by C2.

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