The Design and Business Models of Financial Ratings

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Abstract

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Keywords: corporate governance, crowding-out effect, information acquisition, ESG ratings, short-termism, socially responsible investment

JEL Classifications: G14, G24

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Abstract

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Keywords: information aggregation, motivational ratings, rating agencies, short-termism.
JEL classifications: D82, D83, G14, G24.

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Financial intermediaries such as credit rating agencies and ESG certifiers produce much of the information used by investors, and are typically paid by the firms they assess. This firm-paid model has persisted despite long-standing concerns about conflicts of interest and the resulting distortions, such as rating inflation (Skreta and Veldkamp, 2009; Bolton, Freixas, and Shapiro, 2012). Regulators have responded to these concerns, most notably through credit rating agency reforms in Title IX of the Dodd-Frank Act. At the same time, other segments of the market for financial information rely on investor-paid intermediaries, including independent equity research and ESG ratings sold directly to asset managers.

The persistence of the firm-paid model despite its well-documented costs suggests that it delivers economic benefits. Yet its coexistence with investor-paid alternatives suggests that these benefits do not always outweigh those of the latter. What are these benefits, why do different segments adopt different funding models, and what are the consequences for firm behavior?

To address these questions, we develop a framework for the design and funding of financial information. A profit-maximizing intermediary observes two signals: one about the action taken by a firm insider, and one about the firm's fundamentals. It relies on these signals to produce information about firm value (a "rating"), and chooses whether to sell it to the firm or to investors. Investors and firm insiders value information for different reasons. Investors want to better assess firm value, in order to increase their trading profits in a financial market in the spirit of Kyle (1985). By contrast, the firm insider values a rating that reflects his action, because it affects the price at which he may have to sell his stake before the firm's output is realized, as in Faure-Grimaud and Gromb (2004). This captures exit by founders, managers, or blockholders such as venture capitalists. These two distinct objectives lead to different rating designs, and the intermediary's funding model determines which one obtains in equilibrium.

Our first set of results characterizes equilibrium rating design under each funding model. An investor-paid intermediary produces an informational rating that puts all weight on the signal about fundamentals, so as to maximize the precision of investors' value assessments. A firm-paid intermediary instead produces a motivational rating by combining both signals, as in Hörner and Lambert (2021). The motivational rating makes the asset price sensitive to the insider's action. Bundling signals is essential: a rating that were uninformative about fundamentals would be ignored by the market, would have no price impact, and would therefore provide no effort incentives.[1]

An important implication is that the rating design that is most informative about firm value induces the lowest effort. This contrasts with the classic view that more informative

[1] That effort is correctly anticipated is not crucial. What matters is the distinction between information that improves incentives and information that resolves uncertainty. See Online Appendix B for an extension of the model in which the insider's effort is uncertain.

market prices strengthen managerial incentives (Holmström and Tirole, 1993). In our model, the effect arises from the endogenous design of information: the intermediary's choice of rating weights determines the sensitivity of the stock price to effort.

Our main result shows that, when firms pay for ratings and fundamental uncertainty is sufficiently high, endogenously designed ratings completely eliminate short-termism. In this case, the equilibrium rating provides implicit incentives such that the insider's effort equals the level attainable when he can commit to effort, regardless of how likely he is to exit early. High uncertainty makes the signal about firm fundamentals more informative, which increases the sensitivity of the market price to any rating which depends on this signal. When the intermediary chooses the rating weight that maximizes the insider's willingness to pay, the market price is then sufficiently sensitive to actual effort, so even an insider with a short horizon fully internalizes long-run value creation via the sale price. This time horizon irrelevance result contrasts with standard models of exit and moral hazard (Faure-Grimaud and Gromb, 2004; Chemla and Hennessy, 2014), where exit always weakens incentives relative to the commitment benchmark.

The choice between the two business models depends on three factors. The intermediary sells informational ratings to investors when liquidity is sufficiently high, insiders' horizons are sufficiently long, or their stakes are sufficiently low, and sells motivational ratings to firms otherwise. In the investor-paid regime, "bad information", which maximizes trading profits but not output, crowds out "good information". This highlights a new cost of stock market liquidity.[2]

We extend the baseline model to allow for competition between intermediaries. The equilibrium falls into three regimes, determined jointly by market liquidity and the entrant's information acquisition cost. When markets are illiquid or entry costs are high, entry is blockaded and the incumbent designs a motivational rating. For intermediate values of these parameters, the incumbent distorts its rating weight (rather than prices or quantities, as in standard entry-deterrence models) to deter entry. Thus, the threat of entry lowers equilibrium effort, even when no entry occurs. When markets are sufficiently liquid, the incumbent accommodates entry by selling a private informational rating to investors, ceding the motivational role to the entrant. Both aforementioned business models coexist in this regime. This extension reinforces the "bad information drives out good information" mechanism: the informational rating gives investors an informational advantage that they exploit with strategic trading, which reduces the sensitivity of the market price to effort and weakens the incentive effec-

[2]This is in contrast to the classic result that public disclosure crowds out private information acquisition (Verrecchia, 1982). The literature has identified other costs of liquidity, including inadequate internal monitoring (Bhide, 1993) and liquidations that contribute to financial contagion (Allen and Gale, 2000). Our result also contrasts with Edmans (2009), where liquidity facilitates governance by enabling blockholders to trade on private information. In our model, liquidity instead shifts the business model toward investor-paid ratings, at the expense of implicit incentives.

tiveness of the motivational rating. These results suggest that regulatory efforts to promote competition among rating agencies or to make ratings more informative may have unintended consequences.

The framework applies beyond the rating agency context to any setting in which an intermediary chooses how to aggregate multidimensional information about a firm, including investment banks in IPOs, auditors of private companies,[3] and ESG intermediaries.

ESG ratings sold to investors may emphasize exposure to sustainability risks rather than motivating real social or environmental improvement, consistent with the stated methodologies of major ESG rating providers (e.g., MSCI, Sustainalytics). Firm-oriented certification can instead function as a commitment device for real improvement. Our model predicts that both business models coexist when information production is sufficiently cheap and stock markets are liquid. We also show that "impact funds" can generate social impact not because of their investment decisions or their engagement, but because of their influence on the design of financial information. This is a new channel in the literature on socially responsible investment (Broccardo, Hart, and Zingales, 2022; Edmans, Levit, and Schneemeier, 2024; Oehmke and Opp, 2025; Chen, Gupta, and Starmans, 2026).

In a variant of the model adapted to the specificities of debt and credit ratings,[4] we show that issuer-paid ratings have motivational value that investor-paid ratings do not: they make refinancing terms sensitive to managerial actions and thereby induce effort. This offers a new rationale for the issuer-paid model in credit rating markets, and suggests that mandating investor-paid ratings would have an unintended cost absent from existing analyses of credit rating regulation.

The time horizon irrelevance result sheds new light on venture capital (VC). When uncertainty is high, blockholders with a short horizon can still be involved in long-term projects because the design of financial information delivers strong implicit incentives. This helps rationalize how VC funds with 7 to 10 year horizons finance firms whose value unfolds over longer horizons, and why venture capitalists concentrate on high-uncertainty early-stage ventures (Gompers and Lerner, 2001; Janeway, Nanda, and Rhodes-Kropf, 2021), a narrow focus noted with concern by Lerner and Nanda (2020). Our framework also provides a new rationale for what they describe as a puzzling fact: the limited variation in fund horizons across industries despite great diversity in typical project lengths.

An additional implication is that, when uncertainty is high enough that the commitment

[3]Audit reports for private companies are flexible relative to public companies: disclosure can combine GAAP and non-GAAP earnings (Leung and Veenman (2018)), risk assessment procedures vary (Knechel et al. (2013)), and so do the levels of complexity and disaggregation (Chen, Miao, and Shevlin, 2015; Hoitash and Hoitash, 2018). These degrees of freedom can shift the weight of different signals in the report.

[4]We embed our information aggregation mechanism in the framework of Bolton, Freixas, and Shapiro (2012). The firm's default probability depends on both an exogenous credit type and the insider's effort to improve creditworthiness, and the insider benefits from a higher rating because of its effect on the price at which the firm's debt is refinanced. See Section 5.2 for a discussion of this application.

outcome is achievable with rating design, there is no additional role for debt as a commitment device, in contrast to the standard moral hazard model of Innes (1990).[5]

1 Related literature

Our paper contributes to two strands of literature on information production in financial markets. The first, initiated by Admati and Pfleiderer (1986, 1988), studies how informed agents sell exogenously given information, with subsequent work analyzing how much information to produce (Admati, Pfleiderer, and Zechner, 1994; Peress, 2010; Ferreira, Manso, and Silva, 2014; Dasgupta and Mathews, 2024), and interactions between public and private information (Goldstein and Yang, 2017). The second strand studies information design problems where the designer cares about actions or outcomes (Bergemann and Morris, 2019). Most closely related, Hörner and Lambert (2021) analyze optimal ratings designed to maximize social surplus, which is equivalent to maximizing effort in their setting.

In our model, a profit-maximizing intermediary chooses both rating design and its funding model, in a setting with insider exit. This delivers two results absent from this prior work: time horizon irrelevance and the crowding out of motivational by informational ratings in liquid markets. Equilibrium rating weights generally differ from those that induce first-best effort; when uncertainty is large enough, the rating weight can induce effort above the first-best level, although this does not happen in equilibrium.

Our model also differs from two recent papers on information intermediaries. Pollrich and Strausz (2024) show that the fee structure is irrelevant in a setting with one seller and one buyer where transparency does not affect efficiency. Saeedi and Shourideh (2024) study optimal design when information is sold to a given third party but the information provider only observes one signal.

The distinction between motivational and informational uses of ratings is related to certification models where buyers and sellers use information for different purposes, such as inspection versus signaling (Stahl and Strausz, 2017). Closer to our setup, Lizzeri (1999) and Bouvard and Levy (2018) study profit-maximizing certifiers. Ratings depend on type or quality in these models, whereas the rating can depend on both effort and type in our model. Several papers study ratings in contracting problems without insider effort (Faure-Grimaud, Peyrache, and Quesada, 2009; Daley, Green, and Vanasco, 2020; Kashyap and Kovrijnykh, 2016), and many analyze optimal weights on performance signals in principal-agent settings (e.g. Banker and Datar, 1989; Paul, 1992). By contrast, we show that the optimal aggregation of signals depends on who pays for the information.

[5]This is broadly consistent with the evidence that firms with high market-to-book ratios and R&D intensity are more likely to be unlevered (Strebulaev and Yang, 2013).

A large literature studies credit rating agencies and conflicts of interest arising from issuer payment. Bolton, Freixas, and Shapiro (2012) show that issuer fees can lead to inflated ratings and that competition enables rating shopping. Related work examines shopping behavior (Sangiorgi and Spatt, 2017) and competition effects (Doherty, Kartasheva, and Phillips, 2012). Reputational concerns and disciplining mechanisms are studied in Mathis, McAndrews, and Rochet (2009), Bar-Isaac and Shapiro (2013), Fulghieri, Strobl, and Xia (2014), and Piccolo and Shapiro (2022), with empirical evidence of catering to issuers in Griffin, Nickerson, and Tang (2013) and Kraft (2015). By contrast, we study the effect of the funding model on rating design when ratings affect implicit effort incentives from market prices.

A parallel literature studies ESG ratings, where both issuer-paid and investor-paid models coexist. ESG ratings exhibit substantial variations across providers (Berg, Kölbel, and Rigobon, 2022). Lovo and Olivier (2026) analyze who should pay for ESG ratings when investors have heterogeneous social responsibility preferences, while Azarmsa and Shapiro (2025) study whether competing ESG agencies generalize or specialize. An important distinction is that these papers do not model effort choices by rated firms. In the broader literature on socially responsible investment, Oehmke and Opp (2025) show that impact arises through engagement, while Edmans, Levit, and Schneemeier (2024) study shareholder divestment. These papers do not model the design of ESG information. We contribute to this literature by identifying a new benefit of firm-paid information, and by showing that when impact investors purchase ESG information, rating agencies respond by producing ratings that motivate ESG improvements.

A large literature studies the real effects of financial information. Our result that investor-paid information can depress output while maximizing market efficiency contributes to the literature on the real effects of financial markets (Bond, Edmans, and Goldstein, 2012) and specifically to papers on information disclosure and real efficiency (Goldstein and Yang, 2017; Edmans, Heinle, and Huang, 2016). It is reminiscent of the bad equilibrium in Dow and Gorton (1997) and the detrimental effect of disclosure on real efficiency in Goldstein and Yang (2019), although the mechanism differs: in these papers, inefficiency arises from feedback effects when managers learn from prices, whereas in ours, prices matter for effort incentives.

In the standard framework of Holmström and Tirole (1993), more informative stock prices improve incentive provision by providing better signals of managerial actions. Banerjee, Davis, and Gondhi (2026) show that when prices become more informative about investment opportunities, they become noisier signals of effort, which reduces the optimal sensitivity of the manager's pay to the stock price. In their model, the sensitivity of the stock price to effort is constant, and the mechanism operates through the optimal contract.[6] In our model, the result instead arises from the endogenous rating design, which determines the sensitivity of

[6] Lin, Liu, and Sun (2019) study how stock price informativeness affects optimal contracting and show that a more informative stock price reduces the incentives necessary to elicit good investment decisions.

the stock price to effort, which in turn affects the insider's return to effort.

The literature on short-termism shows that a short horizon or potential exit weakens incentives relative to the outcome with commitment (Faure-Grimaud and Gromb, 2004; Chemla and Hennessy, 2014), and examines how to mitigate short-termism through contracting (Marinovic and Varas, 2019) or monitoring (Von Thadden, 1995). Our contribution is to show that with proper rating design, short horizons need not reduce effort: when uncertainty is sufficiently high and ratings are funded by firms, equilibrium effort equals the commitment level regardless of the exit probability. This time horizon irrelevance result provides a new perspective on when and why short-termism emerges, highlighting the design of financial information as a mechanism that can restore efficient incentives even with short horizons.

2 Model

2.1 The firm

A firm is partially owned by a risk neutral "insider" who exerts effort to improve its output. The insider initially owns a fraction $1-\delta$ of the shares, such that he has effective control of the firm. As in Faure-Grimaud and Gromb (2004), the insider can be viewed as an entrepreneur who founded or purchased the firm, or as an involved shareholder such as a venture capitalist. The output of the firm, which is realized at $t = 2$, is given by:

$$\tilde{X} \quad = \quad A + \tilde{\theta}. \tag{1}$$

where A is the insider's effort, and θ is the firm's "type". The latter should be interpreted broadly as a source of uncertainty about firm output.

Effort A is unobservable and optimally chosen by the insider at $t = 0$. The firm type θ, which is unknown at $t = 0$, is drawn from a normal distribution with mean $\bar{\theta}$ and variance σ_θ^2. Information about θ is symmetric: the realization θ of the random variable $\tilde{\theta}$ is unobservable.[7] Effort is privately costly for the insider, with a monetary cost of $C(A)$, with $C' > 0$, $C'' > 0$, $C(0) = 0$, and $C'(0) = 0$.

2.2 Financial market

Trading occurs at $t = 1$. The financial market is in the spirit of Kyle (1985). In addition to the insider, who may or may not trade as specified below, it involves three types of economic agents. First, noise traders submit a random market order $\tilde{u}$, which is normally distributed with mean 0 and variance σ_u^2 and is independent of the firm type $\tilde{\theta}$. Second, a risk-neutral

[7] If θ is instead interpreted as the insider's ability, this is consistent with a standard model of career concerns, where the agent's ability is unknown ex-ante.

speculator obtains information about the firm by observing a rating R, then she submits a market order $q(R)$. Third, a competitive and risk-neutral market maker observes whether the insider is hit by a liquidity shock as well as the total order flow. He may observe the rating or not, depending on the business model chosen by the intermediary (the "rating agency"). He sets the price of the asset to its expected value given his information.

As in Faure-Grimaud and Gromb (2004), with probability $\pi \in (0,1]$, the insider is hit by a publicly observable liquidity shock at $t = 1$ and sells his shares in the firm. The insider's trade is observable and cannot be anonymous.[8] The model allows for $\pi = 1$, in which case the insider has a fixed horizon of $t = 1$.

The game is sequential: the insider and the informed speculator submit market orders, then the market maker sets the stock price p.

2.3 Rating agency

A rating agency observes two signals at $t = 1$, one about effort and the other about the firm type:

$$\tilde{S}_1 = A + \tilde{\varepsilon}_1 \tag{2}$$

$$\tilde{S}_2 = \theta + \tilde{\varepsilon}_2 \tag{3}$$

where θ is the realization of $\tilde{\theta}$; for $i \in \{1,2\}$, $\tilde{\varepsilon}_i$ follows a standard normal distribution and is independent from other random variables. The unit variance assumption simplifies expressions and could be relaxed.

The rating agency chooses its rating methodology to maximize its expected profits. It chooses the rating design at $t = 0$ and reveals the rating R at $t = 1$. Commitment to an information structure is standard in the information design literature (Bergemann and Morris, 2019; Hörner and Lambert, 2021) and in models of certification intermediaries (Lizzeri 1999; Bouvard and Levy, 2018).[9] In practice, major credit rating and ESG rating agencies publish detailed methodologies in advance and are required by regulation to apply them consistently.[10] Below, we consider different business models for the rating agency, depending on whether it sells its rating to the firm or to the speculator. When the rating is purchased by the firm, the

[8] As in Faure-Grimaud and Gromb (2004), this assumption allows to abstract from adverse selection issues that could lead to a market breakdown when initial owners have private information about firm value.

[9] It is also the natural outcome when the rating agency has reputational concerns: deviating from a pre-announced methodology would undermine its credibility and future revenues. Mathis, McAndrews, and Rochet (2009) and Bar-Isaac and Shapiro (2013) formalize how reputation sustains commitment by rating agencies.

[10] SEC Rule 17g-2 requires Nationally Recognized Statistical Rating Organizations to document their rating procedures, and Rule 17g-8 requires them to apply those procedures consistently. The EU ESG Ratings Regulation (Regulation (EU) 2024/3005) requires ESG rating providers to disclose and consistently apply their methodologies.

fee is paid from corporate funds. The insider, who owns a fraction $1-\delta$ of the firm, bears the same fraction of the cost, and makes the decision to maximize his objective function.

Note that a market participant who observes two linearly independent ratings can invert them to recover both underlying signals (see Lemma 3 for a formal proof). Observing two ratings is therefore equivalent to observing both signals. Moreover, since the insider's effort is anticipated in equilibrium, only information about S_2 affects the stock price. A rating that depended only on S_1 would have no value, but combining S_1 and S_2 in a single rating can make the stock price sensitive to effort. Accordingly, as in Hörner and Lambert (2021), we assume that the rating agency designs a single rating such that:

$$\tilde{R} \;\; = \;\; k\tilde{S}_1 + (1-k)\,\tilde{S}_2. \tag{4}$$

where $k \in [0,1]$ is chosen by the rating agency and is public information (we discuss the restriction to linear ratings in Online Appendix C). The parameter k is the rating weight: a higher k makes the rating more sensitive to effort, whereas a lower k makes it more informative about the firm type. For a given k, the variance of the rating is:

$$\sigma_R^2 \equiv (1-k)^2\,\sigma_\theta^2 + k^2 + (1-k)^2\,. \tag{5}$$

The timeline of events is summarized in Figure 1.

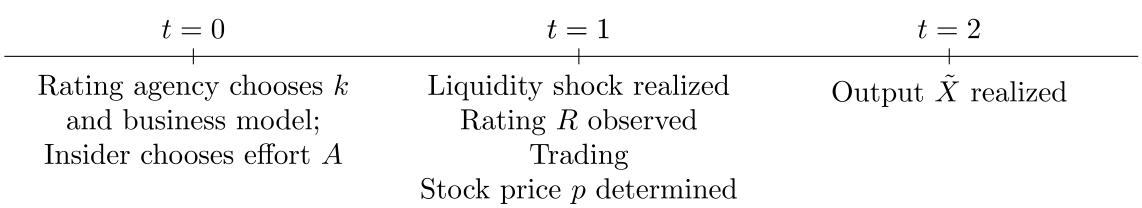

Figure 1: Timeline of the model.

3 Single rating agency

3.1 Market equilibrium

We characterize the market equilibrium for two information structures: when the rating is privately observed by the speculator, and when it is publicly disclosed to all market participants.

Since the market maker observes the market order of the corporate insider and the total order flow, he can infer $y \equiv q + u$, which is the part of the order flow emanating from the speculator and the noise trader. The insider's order is driven only by an exogenous shock whose probability is independent of firm value, so that it does not affect beliefs about firm value. Let I denote the market maker's information at $t = 1$. When the market maker observes the rating, his information includes the net order flow $q + u$ and the rating R, otherwise it just includes the net order flow $q + u$.

We derive the linear rational expectations equilibrium as in Kyle (1985).

Definition 1 *An equilibrium is a market order $q(R) = \alpha + \beta R$ from the speculator and an equilibrium price $p = \mu + \lambda y$ such that $q(R) = \arg\max_q \mathbb{E}\left[\left(\tilde{X} - p(q+u)\right)q \middle| R\right]$ and $p = \mathbb{E}\left[\tilde{X}\middle|I\right]$.*

Proposition 1 derives the market equilibrium when the rating is privately observed by the speculator. Let the insider's actual effort be A and his anticipated effort be $\hat{A}$. Even though $A = \hat{A}$ in equilibrium, implicit effort incentives from the firm's market valuation depend on the relation between actual effort A and firm value.

Proposition 1 (private rating equilibrium) *When the rating R is privately observed by the speculator, the equilibrium is such that:*
(i) Speculator's strategy:

$$q(R) \quad = \quad \alpha + \beta R \quad \text{where} \quad \alpha = -\frac{\sigma_u}{\sigma_R}\left(k\hat{A} + (1-k)\,\bar{\theta}\right) \quad \text{and} \quad \beta = \frac{\sigma_u}{\sigma_R} \tag{6}$$

(ii) Market maker's pricing rule:

$$p \quad = \quad \mu + \lambda y \quad \text{where} \quad \mu = \hat{A} + \bar{\theta} \quad \text{and} \quad \lambda = \frac{(1-k)\,\sigma_\theta^2}{2\sigma_u \sigma_R} \tag{7}$$

(iii) Speculator's expected profit:

$$\mathbb{E}[\Pi] \quad = \quad \frac{\sigma_u\,(1-k)\,\sigma_\theta^2}{2\sigma_R} \tag{8}$$

(iv) Expected stock price conditional on effort:

$$\mathbb{E}\left[p\middle|A, \hat{A}\right] \quad = \quad \hat{A} + \bar{\theta} + \frac{1}{2}\frac{k(1-k)\sigma_\theta^2}{\sigma_R^2}(A - \hat{A}) \tag{9}$$

Parameter λ, the sensitivity of the stock price to the order flow, is a measure of market illiquidity. As in the Kyle (1985) model, it is increasing in $(1-k)\sigma_\theta^2$, which is a measure of the speculator's informational advantage, and decreasing in σ_u, which reflects the random part of the order flow. In addition, λ is decreasing in σ_R. Intuitively, holding σ_θ constant, a higher σ_R makes the speculator's signal less precise, which makes her trade less informative, reducing the price sensitivity to the order flow.

Parameter β represents the trading aggressiveness of the speculator. As in Kyle (1985), it is increasing in the magnitude σ_u of noise trading. It is also decreasing in the standard deviation σ_R of the rating.[11]

Proposition 2 characterizes the market equilibrium when the rating is publicly observable. As the speculator has no informational advantage, she does not trade; the order flow is pure noise and is not taken into account by the market maker.

Proposition 2 (public rating equilibrium) *When the rating R is observed by all market participants, the equilibrium is such that:*
(i) The speculator does not trade: $q(R) = 0$ for all R;
(ii) Market maker's pricing rule:

$$p \;=\; \mathbb{E}[X|R] = \hat{A} + \bar{\theta} + \frac{(1-k)\sigma_\theta^2}{\sigma_R^2}\left(R - k\hat{A} - (1-k)\bar{\theta}\right) \tag{10}$$

(iii) Speculator's expected profit: $\mathbb{E}[\Pi] = 0$.
(iv) Expected stock price conditional on effort:

$$\mathbb{E}\left[p|A,\hat{A}\right] \;=\; \hat{A} + \bar{\theta} + \frac{k(1-k)\sigma_\theta^2}{\sigma_R^2}\left(A - \hat{A}\right) \tag{11}$$

Comparing Propositions 1 and 2, the sensitivity of the expected stock price to actual effort is twice as large when the rating is public ($\frac{k(1-k)\sigma_\theta^2}{\sigma_R^2}$) versus private ($\frac{k(1-k)\sigma_\theta^2}{2\sigma_R^2}$). Intuitively, the speculator's strategic trading reduces the transmission of information into the stock price when the rating is private.

3.2 Effort and implicit incentives

We now analyze the insider's effort incentives, focusing on the implicit incentives that arise from the rating's impact on the stock price when the insider might need to sell his stake at $t = 1$ (i.e. $\pi > 0$). Implicit incentives arise from the sensitivity of the expected stock price $\mathbb{E}[p|A,\hat{A}]$ to actual effort A, as described in Propositions 1(iv) and 2(iv): the insider internalizes that higher effort shifts the distribution of the rating, which in turn shifts the stock price at which he might exit.[12]

[11] This is because σ_R is increasing in prior fundamental uncertainty σ_θ and in the noise in the signals $\tilde{S}_1$ and $\tilde{S}_2$ used in the rating. As in Kyle (1985), a higher σ_θ decreases market liquidity and therefore trading aggressiveness. In addition, the noise in signals $\tilde{S}_1$ and $\tilde{S}_2$ reduces the reliability of the speculator's information, and consequently her trading aggressiveness.

[12] This is the standard mechanism in models of career concerns (Holmström, 1999). As we show in Online Appendix B, the mechanism has a natural counterpart in a model where the insider privately observes a parameter $\overline{A}$ of his effort cost function. The rating remains informative about an uncertain variable (now $\overline{A}$ rather than θ), and the two models are mathematically equivalent, but the equilibrium effort now depends on the privately observed $\overline{A}$ and is unknown to market participants.

As a benchmark, we consider the effort that would be optimally chosen by the insider if he could commit to a level of effort at $t = 0$. We denote this effort by A^{LT}. It solves:

$$\max_{A}\,(1-\delta)\left(\pi\mathbb{E}\left[p\middle|A,A\right]+(1-\pi)\left(A+\bar{\theta}\right)\right)-C(A) \quad \Leftrightarrow \quad \max_{A}\,\{(1-\delta)A-C(A)\} \tag{12}$$

This gives:

$$C'(A^{LT}) = 1-\delta \tag{13}$$

We call this level of effort A^{LT} because it is also the level of effort that would be achieved at the second-best if the insider were never hit by a liquidity shock, i.e. if he had a long-term ("LT") horizon with $\pi = 0$. Online Appendix D derives the first-best effort and the rating weights that elicit this level of effort.

The insider's effort incentives depend in part on how effort affects the expected market value of the firm. These implicit incentives only exist when $k \in (0,1)$. For $k = 0$, the rating is independent of effort and therefore does not provide effort incentives. For $k = 1$, the rating only depends on effort, not on firm type $\tilde{\theta}$, so that it does not change beliefs and does not affect the stock price.

Lemma 1 characterizes the insider's equilibrium effort when the rating is privately observed by the speculator and when it is publicly disclosed. For now, we take the rating weight k as given.

Lemma 1 (effort given rating weight)
(i) When the rating is privately observed by the speculator, the insider's effort satisfies:

$$C'(A) = (1-\delta)\left(1-\pi+\pi\frac{k(1-k)\sigma_\theta^2}{2\sigma_R^2}\right) \tag{14}$$

The insider effort is equal to the level under commitment A^{LT} if and only if $\sigma_\theta \geq \sigma_\theta^* \equiv 2\sqrt{2+\sqrt{5}}$ and $k = k^{LT} \equiv \frac{4+5\sigma_\theta^2 \pm \sqrt{\sigma_\theta^4-16-16\sigma_\theta^2}}{2(4+3\sigma_\theta^2)}$.
(ii) When the rating is publicly disclosed, the insider's effort satisfies:

$$C'(A) = (1-\delta)\left(1-\pi+\pi\frac{k(1-k)\sigma_\theta^2}{\sigma_R^2}\right) \tag{15}$$

The insider effort is equal to the level under commitment A^{LT} if and only if $\sigma_\theta \geq \sigma_\theta^d \equiv \sqrt{2(1+\sqrt{2})}$ and $k = k_d^{LT} \equiv \frac{3\sigma_\theta^2+2 \pm \sqrt{\sigma_\theta^4-4\sigma_\theta^2-4}}{4\sigma_\theta^2+4}$.
(iii) The threshold level of fundamental uncertainty is lower with a public rating: $\sigma_\theta^d < \sigma_\theta^*$.

Implicit incentives are stronger when prior uncertainty about firm value (σ_θ) is higher, as the rating is then more informative and has a greater impact on firm valuation (for $k \in (0,1)$).

When fundamental uncertainty σ_θ is above a threshold, the insider can achieve the effort level under commitment A^{LT} – the effort he would choose if he never faced early exit ($\pi = 0$). This requires an appropriately designed rating weight k. When σ_θ is above the threshold, there are two rating weights k^{LT} that achieve A^{LT}. Intuitively, in this case, the rating weight that maximizes effort induces effort above A^{LT}. To achieve exactly A^{LT}, the rating weight can be either increased (making the rating more sensitive to effort but less informative about θ, reducing price impact) or decreased (making the rating less sensitive to effort). Both adjustments achieve the same effort level through different information structures.[13]

Implicit incentives depend on the business model of the rating agency. As already noted, the sensitivity of the expected stock price to the insider's actual effort is twice as large when the rating is public. As a result, the minimum level of uncertainty needed to achieve the level of effort under commitment is lower when the rating is public: $\sigma_\theta^d < \sigma_\theta^*$. The doubled sensitivity of firm value to effort makes it easier to provide adequate implicit incentives.

Proposition 3 (time horizon irrelevance) *When uncertainty about firm value is sufficiently high ($\sigma_\theta \geq \sigma_\theta^*$ for private ratings, $\sigma_\theta \geq \sigma_\theta^d$ for public ratings), there exists a rating weight k such that the insider achieves the level of effort under commitment A^{LT} regardless of the probability π of early exit.*

Proposition 3 shows that the insider's investment horizon becomes irrelevant for effort provision when information is appropriately designed and uncertainty is high enough. To see why, note that the insider's return to effort comes from two channels: with probability $1 - \pi$, he receives his share of the output at $t = 2$ (a unit return per unit of effort); with probability π, he sells at a stock price that partially reflects his effort, with a sensitivity given by the fraction in equations (14) and (15). The commitment effort A^{LT} is achieved when these two channels deliver the same return, i.e., when this sensitivity equals 1. This sensitivity depends on the weight k that the rating puts on the signal about effort, but not on π: when both channels yield the same return, shifting probability between them does not change the insider's incentives. Moreover, for $k \in (0, 1)$, this sensitivity is increasing in σ_θ, because higher prior uncertainty about firm fundamentals amplifies the stock price impact of any rating that partially resolves it. When fundamental uncertainty is large enough, properly aggregated information creates stock price-based implicit incentives that replicate those of permanent ownership.

Remark 1 *As σ_θ increases, one of the rating weights k_d^{LT} that induces the level of effort under commitment with a public rating tends asymptotically to $\frac{1}{2}$.*

[13] We provide graphical illustrations in Online Appendix E. Note that the agency is indifferent between the two k^{LT} values because the insider's willingness to pay depends only on the effort level induced, which is the same for both. This willingness to pay, when compared against the willingness to pay of the speculator for a $k = 0$ rating, drives the equilibrium outcome in section 3.4.

The optimal rating corresponding to the lowest value of k_d^{LT} in Lemma 1(ii) converges asymptotically to $k = \frac{1}{2}$ as uncertainty about firm value increases. Intuitively, when uncertainty about firm value is very high, there is a lot of scope for learning, so that the rating has a very high price impact. To not provide excessive effort incentives, the rating weight converges to either $\frac{1}{2}$ or to 1. For $k = \frac{1}{2}$, the rating is simply a linear transformation of a noisy version of output. Thus, as uncertainty about firm value is sufficiently high, a rating which is simply informative about output will approximately induce the level of effort under commitment.

The time horizon irrelevance result has important theoretical and practical implications. At a theoretical level, it shows that the tension between short investment horizons and long-term value creation can be completely resolved through equilibrium rating design when uncertainty is sufficiently high. This contrasts with the standard view in corporate finance that shorter horizons reduce effort and efficiency (Stein, 1989; Faure-Grimaud and Gromb, 2004).

Practically, this result helps explain empirical patterns. Most notably, it provides a foundation for understanding how venture capitalists can be successful despite an apparent investment horizon mismatch, as further discussed in section 5.1. More broadly, the result suggests that concerns about short-termism depend on the uncertainty environment: they are most acute in mature, stable industries where σ_θ is low, but may be largely irrelevant in high-growth, high-uncertainty sectors. The condition for time horizon irrelevance is independent of δ and π, so it applies across ownership structures and investment horizons. This suggests that short-termism might be more effectively addressed by the design of financial information, which we study in section 3.3, than by contractual restrictions on exit.

3.3 Rating design

Having established the effort implications of different rating designs, we now determine the equilibrium rating design for different business models (the optimal business model will be determined in section 3.4 below). Start with the case when the agency sells the rating to the speculator by making a take-it-or-leave-it offer.

Proposition 4 (investor-paid rating) *When the agency sells the rating to the speculator, it designs a rating which is as informative about firm fundamentals as possible:*

$$k = k^{**} \equiv 0 \tag{16}$$

When the rating agency sells the rating to the speculator, it sets k to maximize her willingness to pay, which is equal to her expected trading profit (Proposition 1(iii)). This profit is maximized by a rating that is as informative as possible about firm fundamentals θ, since information about effort has no trading value when equilibrium effort is correctly anticipated. The rating weight that maximizes the speculator's willingness to pay is therefore $k = k^{**} = 0$, which puts all weight on the signal about firm fundamentals.[14]

Proposition 4 corresponds to the business model of ESG ratings sold to investors, and some equity research conducted by investment banks and brokerage houses that is privately communicated to their investor clientele.

We now consider the business model where the agency sells the rating to the firm, by distinguishing between the cases when it privately communicates the rating to the speculator and when it provides a publicly observable rating. In any case, the agency sets the rating weight and makes a take-it-or-leave-it offer to the insider, who decides whether or not to use the firm's money to purchase the rating.

Proposition 5 (firm-paid rating) *The effort-maximizing rating weight is:*

$$k^* \equiv \frac{1+\sigma_\theta^2 - \sqrt{1+\sigma_\theta^2}}{\sigma_\theta^2}. \tag{17}$$

When the agency sells the rating to the firm:
(i) When it privately communicates the rating to the speculator, it sets:

$$k = \begin{cases} k^* & \text{if } \sigma_\theta < \sigma_\theta^* \\ k^{LT} & \text{if } \sigma_\theta \geq \sigma_\theta^* \end{cases} \tag{18}$$

(ii) When it publicly discloses the rating, it sets:

$$k = \begin{cases} k^* & \text{if } \sigma_\theta < \sigma_\theta^d \\ k_d^{LT} & \text{if } \sigma_\theta \geq \sigma_\theta^d \end{cases} \tag{19}$$

In both cases, the rating agency chooses k to maximize the insider's willingness to pay, which depends on the level of effort that the information environment will elicit. A rating weight that induces the same level of effort A^{LT} as when the insider can commit to effort is the most valuable rating design for the insider. It has weight k^{LT} in part (i), and k_d^{LT} in part (ii). However, it only exists when fundamental uncertainty is above a threshold: when $\sigma_\theta \geq \sigma_\theta^*$ in part (i), and when $\sigma_\theta \geq \sigma_\theta^d$ in part (ii). When σ_θ is below the threshold, the most valuable rating design simply maximizes effort, with weight k^* as defined in equation (17). The information mix that maximizes effort incentives does not change when the rating is communicated to the speculator or when it is publicly disclosed. It is noteworthy that none of these optimal rating weights depend on π: the rating agency does not need to know the insider's exit probability to design ratings.

[14] In Online Appendix F, we show that when the speculator is endowed with shares in the firm, the rating weight that maximizes her willingness to pay is strictly positive.

In part (i), the agency is paid by the firm but provides the rating directly to investors. In part (ii), the firm pays for a public rating. This is the business model of credit rating agencies and of investment banks that produce publicly available equity research.

Figure 2 depicts the rating weights corresponding to the two business models considered in Proposition 5. In both panels, the optimal rating weight k^* is increasing in σ_θ: the greater the prior uncertainty about firm value, the more weight the rating puts on the effort signal S_1. Intuitively, higher fundamental uncertainty amplifies the price impact of any rating informative about firm value, so the agency can put more weight on S_1 while still maintaining a sufficiently high price impact of the rating. Above the threshold value of σ_θ for commitment effort (σ_θ^* in panel (i), σ_θ^d in panel (ii)), two weights yield $A = A^{LT}$: the lower weight makes the rating more informative about θ but less sensitive to effort, while the higher weight does the opposite.

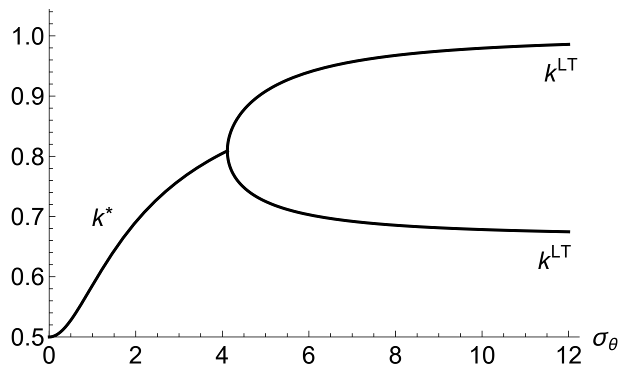

Figure 2: Rating weight from Proposition 5(i) on the left panel, and from Proposition 5(ii) on the right panel, as a function of σ_θ.

3.4 Business model

We now determine the profit-maximizing business model for the agency (with associated rating design as derived in section 3.3). We start by establishing that one of the three business models considered is always weakly dominated.

Lemma 2 *When the agency sells the rating to the firm, public disclosure of the rating weakly dominates private communication of the rating to the speculator.*

Intuitively, a public rating has a higher price impact compared to a rating privately observed by a strategic speculator, so that the insider's effort has a stronger effect on the stock price when the rating is public. When uncertainty is sufficiently high that the level of effort under commitment can be elicited whether the rating is publicly disclosed or privately communicated, the same outcome can be achieved either way, so the insider has the same willingness to pay for these two options, and the agency is indifferent.

Proposition 6 shows how the equilibrium business model depends on model parameters.

Proposition 6 *The rating agency sells the rating to the speculator rather than the firm if the volume of noise trading σ_u is sufficiently high, or if the insider's stake $1-\delta$ or probability of exit π is sufficiently low.*

In an illiquid stock market with a low σ_u, the expected profit of the informed speculator is very low, and so is her willingness to pay for a rating. By contrast, if the stock market is sufficiently liquid, the rating agency can sell its rating at a higher price to the speculator than to the firm. Thus, when market liquidity is sufficiently high that selling to the speculator is more profitable, the agency produces an informational rating rather than a motivational rating: information useful for trading displaces information that improves incentives and real outcomes. This highlights a new cost of financial market liquidity.

Now consider the insider's liquidity shock, which proxies for his time horizon. If we had $\pi = 0$, the insider would not value the rating: he would not sell his stake in the firm and therefore would not care about its market value. More generally, when π is low, the insider has a low willingness to pay for the rating since he already largely internalizes the benefits of his effort. Therefore, the rating agency will rather sell a rating to the speculator.

Similarly, a lower insider stake $1-\delta$ favors the speculator-paid business model: as δ increases, the insider's willingness to pay for the rating declines while the speculator's is unaffected, so the rating agency prefers to sell to the speculator.

4 Competition between rating agencies

We now extend the analysis to consider competition between rating agencies. This extension addresses an important practical consideration: in many markets, multiple rating agencies are competing. Does competition improve or worsen the information environment? Our analysis shows that competition can reduce effort by creating strategic distortions in rating design.

4.1 Model setup

We modify the baseline model to allow for potential entry by a second rating agency. The timing is as follows. As in the baseline model, the first rating agency (incumbent) observes the signals $S_1 = A + \varepsilon_1$ and $S_2 = \theta + \varepsilon_2$, designs a rating $R_1 = k_1 S_1 + (1-k_1) S_2$, and decides whether to sell it to the firm or to the speculator. In addition, a second rating agency (potential entrant) observes the first rating $R_1(k_1)$ and decides whether to acquire the same signals S_1 and S_2 at cost K.[15] If the second agency enters, it designs a rating $R_2 = k_2 S_1 + (1-k_2) S_2$

[15] This allows us to focus on the effects of competition between agencies holding their information constant, as opposed to the well-understood effects of disseminating differential information.

and decides whether to sell it publicly or privately. Trading and price determination are as in the baseline model, but market participants may observe multiple ratings.

Lemma 3 (signal recovery) *If an agent observes $R_1 = k_1 S_1 + (1-k_1)S_2$ and $R_2 = k_2 S_1 + (1-k_2)S_2$ with $k_1 \neq k_2$, then she can recover $S_1 = \frac{R_1(1-k_2)-R_2(1-k_1)}{k_1(1-k_2)-k_2(1-k_1)}$ and $S_2 = \frac{R_1 k_2 - R_2 k_1}{k_2(1-k_1)-k_1(1-k_2)}$.*

This follows from solving a system of two linear equations in two unknowns. This result has implications for equilibrium configurations. If two public ratings were issued, the market maker would observe both and recover the underlying signals, making one rating redundant. Likewise, if the speculator purchased two private ratings, she would recover the signals, making one rating redundant.

Corollary 1 *In equilibrium, at most one public rating and one private rating can coexist.*

4.2 Equilibrium with public and private ratings

We focus on the configuration where the first agency provides a public rating R_1 and the second agency, if it enters, provides a private rating R_2 to the speculator. When a public rating R_1 and a private rating R_2 coexist, the market microstructure differs from the baseline model. The market maker observes R_1 and the order flow y, while the speculator observes both R_1 and R_2. As the speculator can recover the signals from observing the two ratings, the equilibrium with a public and a private rating does not depend on the specific design of the second rating as its informational value for the speculator is the same as that of a fully informational rating ($k_2 = 0$).

Proposition 7 (market equilibrium with two ratings) *The equilibrium is such that:*
(i) Speculator's strategy:

$$q(R_1, R_2) = \beta(R_2 - \mathbb{E}[R_2]) + \gamma(R_1 - \mathbb{E}[R_1]), \tag{20}$$

where $\beta = \frac{\sigma_{R_1}\sigma_u}{k_1 \sigma_{R_2}}$ and $\gamma = -\frac{(1-k_1)\sigma_{R_2}\sigma_u}{k_1 \sigma_{R_1}}$.
(ii) Market maker's pricing:

$$p = \mu + \lambda(y - \mathbb{E}[y]) + \nu(R_1 - \mathbb{E}[R_1]), \tag{21}$$

where $\mu = \hat{A} + \bar{\theta}, \lambda = \frac{\sigma_\theta^2 k_1}{2\sigma_u \sigma_{R_1}\sigma_{R_2}}$, and $\nu = \frac{\sigma_\theta^2(1-k_1)}{\sigma_{R_1}^2}$.
(iii) Expected price conditional on effort:

$$\mathbb{E}\left[p \middle| A, \hat{A}\right] = \hat{A} + \bar{\theta} + \frac{1}{2}\frac{\sigma_\theta^2(1-k_1)k_1}{\sigma_{R_1}^2}(A - \hat{A}). \tag{22}$$

(iv) Speculator's expected profit:

$$\mathbb{E}[\Pi] = \lambda\sigma_u^2 = \frac{k_1\sigma_\theta^2\sigma_u}{2\sigma_{R_1}\sigma_{R_2}}. \tag{23}$$

The variances of the ratings are: $\sigma_{R_1}^2 = k_1^2 + (1-k_1)^2(1+\sigma_\theta^2)$ *and* $\sigma_{R_2}^2 = 1+\sigma_\theta^2$ *(with* $k_2 = 0$*).*

Proposition 7 extends the baseline equilibrium to the setting where a public rating R_1 and a private rating R_2 coexist. In particular, part (iii) shows that the sensitivity of the expected price to effort is $\frac{1}{2}\frac{(1-k_1)k_1\sigma_\theta^2}{\sigma_{R_1}^2}$, which is half the sensitivity obtained when the same rating R_1 is the only source of information and is publicly disclosed (see Proposition 2). The public rating, which is sensitive to the insider's effort, affects the market price in two ways. First, the market maker prices R_1 directly, which is captured by the coefficient ν in equation (21). Second, the speculator, who has recovered the fundamentals signal S_2 and holds an informational advantage, trades against the public rating ($\gamma < 0$); this affects the price through the order flow with coefficient $\lambda\gamma$. The second channel offsets half of the first ($\lambda\gamma = -\nu/2$), so the net sensitivity of the expected price to effort is half of what it would be if R_1 were the only source of information and were publicly disclosed. In sum, the coexistence of informational and motivational ratings reduces the incentive effect of the motivational rating.

In the two-rating equilibrium, the speculator's profit increases with the motivational content of the public rating.

Lemma 4 (monotonicity of speculator profit) *The speculator's expected profit,* $\frac{k_1\sigma_\theta^2\sigma_u}{2\sigma_{R_1}\sigma_{R_2}}$, *is strictly increasing in* k_1 *on* $[0, 1]$.

When the first agency makes its public rating more motivational (higher k_1), it makes the rating less informative about the firm's type θ. This increases the value of the second agency's private rating, which can reveal information about θ. Thus, paradoxically, a better rating from the insider's perspective makes entry by a competing agency more attractive.

4.3 Competition regimes

In this subsection, we assume $\sigma_\theta \le \sigma_\theta^d$. By Proposition 5(ii), in this region the weight of the firm-paid public rating is k^*. Thus, this rating weight is uniquely determined, making the competition analysis well-defined.[16]

[16] The restriction $\sigma_\theta \le \sigma_\theta^d$ keeps the analysis tractable by ensuring that the rating weight that maximizes the insider's willingness to pay is unique. The weak inequality includes the boundary case $\sigma_\theta = \sigma_\theta^d$ in which the unique weight induces the full commitment effort A^{LT}. Online Appendix G extends the analysis to $\sigma_\theta > \sigma_\theta^d$, where two weights k_d^{LT} both implement A^{LT}: the three-regime structure of Proposition 8 and the accommodation outcome are unchanged, but the blockaded region expands while the deterrence region weakly shrinks because the incumbent can now minimize the entry threat by selecting the lower of the two weights.

The second agency enters only if selling an additional rating is profitable. Define

$$\Delta(k, \sigma_u) \equiv \frac{k\, \sigma_\theta^2\, \sigma_u}{2\, \sigma_R(k)\, \sqrt{1 + \sigma_\theta^2}}.$$

By Proposition 7(iv), $\Delta(k, \sigma_u)$ is the speculator's expected trading profit when a public rating with weight k coexists with a private informational rating with zero weight. It is the entrant's payoff when the incumbent sells a public rating with weight k and the entrant sells a private rating, and the incumbent's payoff when it sells a private rating and the entrant sells a public rating with weight k. By Lemma 4, $\Delta(0, \sigma_u) = 0$ and $\Delta(k, \sigma_u)$ is strictly increasing in k on $[0, 1]$.

Let $V(k)$ be the insider's willingness to pay for a public rating in the single-public-rating environment, and $V^T(k)$ the corresponding willingness to pay in the two-rating environment. We have $V^T(k) \leq V(k)$ for all k, and, with $\sigma_\theta \leq \sigma_\theta^d$, both functions are maximized at k^*, with $V(\cdot)$ strictly increasing on $[0, k^*]$. Define:

$$\hat{k}_1(\sigma_u, K) \equiv \begin{cases} k^* & \text{if } K \geq \Delta(k^*, \sigma_u) \\ \hat{k}_1 \in (0, k^*) \text{ s.t. } \Delta(\hat{k}_1, \sigma_u) = K & \text{if } K < \Delta(k^*, \sigma_u) \end{cases}$$

where uniqueness follows from the strict monotonicity of $\Delta(\cdot, \sigma_u)$. Let $W(\sigma_u) \equiv \frac{\sigma_u \sigma_\theta^2}{2\sqrt{1+\sigma_\theta^2}}$ denote the speculator's willingness to pay for a purely informational private rating with $k = 0$. Define the incumbent's payoff when it sells a private rating to the speculator:

$$\Pi^P(\sigma_u, K) \equiv \begin{cases} \Delta(k^*, \sigma_u), & \text{if } K < V^T(k^*) \\ W(\sigma_u), & \text{if } K \geq V^T(k^*) \end{cases}$$

Indeed, in this case, the entrant sells a public rating if and only if $V^T(k^*) > K$. We maintain the following conventions: an entrant that makes zero profits stays out; first-stage payoff ties lead to a public rating choice; if $V(\hat{k}_1) = V^T(k^*)$, the incumbent chooses deterrence.

The incumbent's optimal strategy is to select the element of $\{V(k^*), V(\hat{k}_1), V^T(k^*), \Pi^P(\sigma_u, K)\}$, that maximizes its payoff. This corresponds respectively to selling a public rating with $k_1 = k^*$ when this does not trigger entry, deterring entry with $k_1 = \hat{k}_1$, accommodating entry with $k_1 = k^*$, and selling a private rating with $k_1 = 0$. Proposition 8 determines which strategy is optimal.

Proposition 8 (competition regimes) *There are three possible equilibrium regimes:*
(i) Blockaded entry when either

$$K \geq \Delta(k^*, \sigma_u) \quad \textit{and} \quad V(k^*) \geq \Pi^P(\sigma_u, K),$$

in which case the incumbent sells a public rating with $k_1 = k^*$, *or*

$$\Pi^P(\sigma_u, K) > \max\left\{V(\hat{k}_1(\sigma_u, K)),\ V^T(k^*)\right\} \quad \text{and} \quad K \geq V^T(k^*),$$

in which case the incumbent sells a private rating with $k_1 = 0$.
(ii) Deterred entry when

$$K < \Delta(k^*, \sigma_u) \quad \text{and} \quad V(\hat{k}_1(\sigma_u, K)) \geq \max\left\{V^T(k^*),\ \Pi^P(\sigma_u, K)\right\}.$$

Then the incumbent sells a public rating with $k_1 = \hat{k}_1(\sigma_u, K) < k^*$.
(iii) Accommodated entry when either

$$K < \Delta(k^*, \sigma_u) \quad \text{and} \quad V^T(k^*) \geq \max\left\{V(\hat{k}_1(\sigma_u, K)),\ \Pi^P(\sigma_u, K)\right\},$$

in which case the incumbent sells a public rating with $k_1 = k^*$ *to the firm and the entrant sells a private rating with* $k_2 = 0$ *to the speculator, or*

$$\Pi^P(\sigma_u, K) > \max\left\{V(\hat{k}_1(\sigma_u, K)),\ V^T(k^*)\right\} \quad \text{and} \quad K < V^T(k^*),$$

in which case the incumbent sells a private rating with $k_1 = 0$ *to the speculator and the entrant sells a public rating with* $k_2 = k^*$ *to the firm.*

The three regimes are jointly determined by two parameters, the measure of market liquidity σ_u and the entrant's information-acquisition cost K (see Figure 3). Liquidity raises the trading value of informational ratings, while the cost K determines whether entry is profitable. The incumbent's optimal strategy balances three considerations: maximizing the insider's willingness to pay for a public rating, extracting value from the speculator's trading activity, and managing the entry threat.

An important question is whether the entry cost K exceeds the speculator's trading profit $\Delta(k^*, \sigma_u)$ at the insider-optimal public rating. When $\Delta(k^*, \sigma_u) \leq K$, entry is unprofitable for any $k_1 \in [0, k^*]$ chosen by the incumbent, who is then unconstrained by the entry threat. When $K < \Delta(k^*, \sigma_u)$, entry is now a credible threat when $k_1 = k^*$, and the incumbent compares three strategies: deterring entry by reducing k_1 below k^* (with payoff $V(\hat{k}_1)$), accommodating entry while maintaining $k_1 = k^*$ (with payoff $V^T(k^*)$), and switching to serve the speculator with a private rating (with payoff $\Pi^P(\sigma_u, K)$). The equilibrium regime depends on which strategy yields the highest payoff.

In regime (i) with blockaded entry, the incumbent operates as a monopolist. There are two cases. When market liquidity is low, the speculator's profit from trading on a private rating is small, so $\Delta(k^*, \sigma_u)$ is low, and if $\Delta(k^*, \sigma_u) \leq K$ the entrant cannot profitably sell a private rating to the speculator. In this case, the incumbent provides the insider-optimal public rating

$k_1 = k^*$ and earns $V(k^*)$, so that the rating design is undistorted. When market liquidity is high enough that $W(\sigma_u) > V(k^*)$ and the entry cost is also high enough ($K \geq V^T(k^*)$), the speculator is the most profitable client and the entrant cannot profitably enter with a public rating. In this case, the incumbent sells a private informational rating ($k_1 = 0$) and earns the speculator's willingness to pay $W(\sigma_u)$.

In regime (ii) with deterred entry, when entry is a credible threat at the undistorted rating weight ($K < \Delta(k^*, \sigma_u)$), the incumbent distorts its rating weight to $k_1 = \hat{k}_1 < k^*$ to prevent entry. Intuitively, a less motivational and more informative public rating leaves less residual uncertainty about fundamentals, which reduces the value of a competing private rating to the speculator (since $\Delta(k_1, \sigma_u)$ is increasing in k_1). Deterrence is the incumbent's preferred strategy when it yields a higher payoff than accommodating entry or selling a private rating to the speculator, i.e. $V(\hat{k}_1) \geq \max\{V^T(k^*), \Pi^P(\sigma_u, K)\}$.

In regime (iii) with accommodated entry, entry occurs and the two business models coexist. The incumbent's strategy depends on which side of the market is more profitable. When liquidity is moderate, the incumbent sells a rating with $k_1 = k^*$ to the firm and the entrant sells a private rating to the speculator; the insider pays only $V^T(k^*) < V(k^*)$ for the public rating because the coexisting informational rating reduces the sensitivity of the price to effort (Proposition 7(iii)). When liquidity is high, the incumbent instead sells a private informational rating ($k_1 = 0$) and the entrant sells a public rating with $k_2 = k^*$ to the insider, so that the incumbent earns $\Delta(k^*, \sigma_u)$ from the speculator.

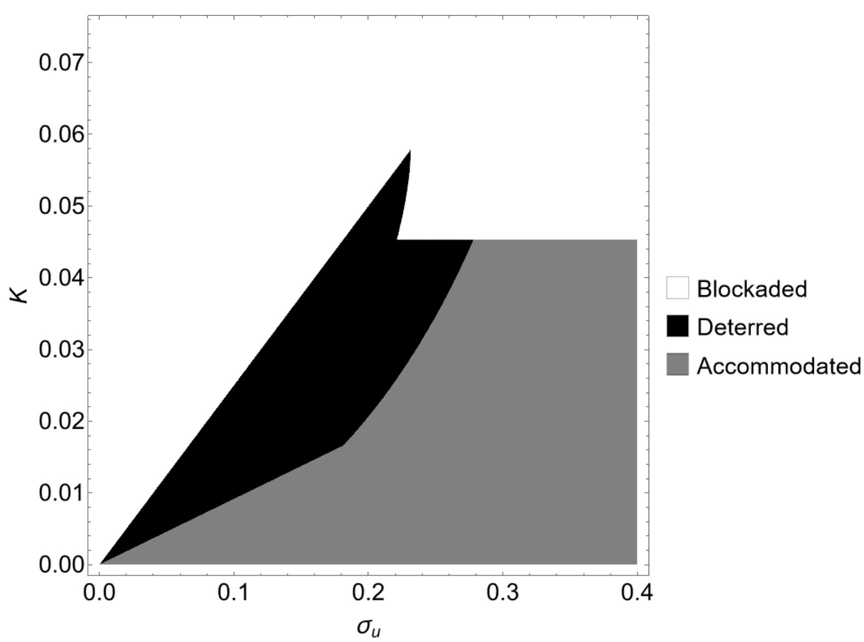

Figure 3: The three competition regimes in Proposition 8 as a function of the measure σ_u of market liquidity and the cost K faced by the potential entrant. Parameter values: $\sigma_\theta = 1$, $\delta = 0.5$, $\pi = 0.75$, and insider cost of effort $C(A) = \frac{A^2}{2}$.

Two effects are especially noteworthy. First, regime (ii) features entry deterrence via information distortion: the incumbent sets $k_1 < k^*$ to forestall entry, so the threat of entry reduces effort and expected output even though entry does not occur. Second, in regime (iii), the two business models coexist but the existence of an informational rating reduces the incentive effectiveness of the motivational rating (see Proposition 7(iii)).

Although welfare is not generally well-defined in models with noise traders (whose utility is unspecified), in our setting welfare comparisons reduce to rankings of expected firm output net of effort cost and the entrant's information-acquisition cost (when entry occurs). Noise traders' orders are exogenous and unaffected by effort or rating design, and their expected losses are offset by the speculator's expected gains (the market maker breaks even by construction and rating fees are pure transfers between risk-neutral agents). Total surplus (in monetary terms) thus reduces to expected firm output net of effort cost, $A + \bar{\theta} - C(A)$, in the blockaded and deterred regimes, and $A + \bar{\theta} - C(A) - K$ in the accommodated regime, where $A + \bar{\theta} - C(A)$ is strictly increasing in equilibrium effort for $A \in [0, A^{LT}]$.

Thus, the results have policy implications. Reducing barriers to entry for rating agencies enlarges the regions with deterred or accommodated entry, where ratings are either distorted or less motivationally effective. Mandating ratings of different types is not a panacea: the coexistence of motivational and informational ratings reduces effort incentives.

These results are also related to an empirical literature. Competition improves the informativeness of ratings but worsens their real effects. This can help explain the mixed evidence on competition and rating quality (Becker and Milbourn, 2011; Xia, 2014), depending on whether quality is measured by informational accuracy or real outcomes.

5 Applications

The theoretical results developed have implications for several important financial markets. In this section, we examine three applications that illustrate how optimal information aggregation affects real-world rating and certification systems.

5.1 Venture capital

Venture capitalists are shareholders with a short investment horizon who are actively involved in the firms in their portfolio and whose engagement can increase firm value (Hellmann and Puri, 2002; Bernstein, Giroud, and Townsend, 2016). Yet venture capital funds typically have investment horizons of 7 to 10 years, while the companies they finance often require 15 to 20 years to reach their full potential. Standard theories of short-termism (e.g. Stein (1989)) suggest that this mismatch should lead to poor outcomes.

Our time-horizon irrelevance result (Proposition 3) provides an explanation. Venture-backed companies typically face enormous uncertainty about their prospects because they rely on new technologies and unproven business models. Indeed, venture capitalists tend to finance "technological revolutions" (Lerner and Nanda, 2020), in which firm-level uncertainty is compounded with uncertainty about the space that each firm is trying to enter. This high fundamental uncertainty (σ_θ) is precisely the condition under which rating design can eliminate short-termism: when σ_θ exceeds the threshold σ_θ^*, effort can reach the commitment level A^{LT} regardless of exit probability π. This is consistent with the finding that VC funding is especially beneficial for firms facing high uncertainty (Park and Steensma, 2012).

An important condition is that financial information about VC-backed firms be designed to cater to these firms rather than to outside investors. VC contracts often include contractible milestones, such as product launches or clinical-trial endpoints (Kaplan and Strömberg, 2003), which play the role of the effort-sensitive signal S_1 in the model: their verification creates an information environment where signals about effort are reflected in firm valuation. Likewise, when exit takes the form of an IPO, disclosure is designed by an investment bank paid by the issuing firm. In both cases, the information generated reflects the value-improving actions taken by venture capitalists, which provides implicit incentives.

5.2 Credit ratings

Credit ratings present a well-known puzzle. Despite extensive evidence of conflicts of interest in the issuer-paid business model (Bolton, Freixas, and Shapiro, 2012; Griffin, Nickerson, and Tang, 2013), and despite periodic regulatory proposals to mandate investor-paid ratings, the issuer-paid model has persisted for decades. Our framework provides an explanation based on the motivational value of issuer-paid credit ratings.

To formalize this application, we develop a dedicated model in section H of the Online Appendix. It adapts our baseline framework to the specificities of debt and credit ratings by embedding our information aggregation mechanism in the framework of Bolton, Freixas, and Shapiro (2012, henceforth BFS).

We extend their model so that the firm's default probability depends on an exogenous credit type θ and on the insider's effort to improve creditworthiness. The credit rating agency (henceforth CRA) chooses a rating weight k that determines how the rating combines information about effort and information about the firm type. When $k = 0$, the rating depends only on the exogenous type, i.e. it is a purely informational rating as in BFS. When $k > 0$, the rating also reflects effort, which introduces a motivational component that is not considered in BFS.

The main difference with respect to our baseline model is the reason why the insider cares about the rating. In the credit rating application, the insider holds equity and cares about the price at which the firm's outstanding debt is refinanced at $t = 1$. A higher refinancing price reduces borrowing costs, which benefits shareholders. Effort at $t = 0$ improves the expected value of the firm's debt, but this improvement is reflected in the refinancing price only if the credit rating incorporates information about effort. When the credit rating is purely informational ($k = 0$), the refinancing price is insensitive to the insider's (actual) effort. The insider then exerts zero effort, and the debt is refinanced at unfavorable terms, reflecting only the expected credit type.

Some results from the Online Appendix are similar to those of the baseline model, even though the setup differs. An investor-paid CRA sets $k = 0$, since the speculator values information about the firm's type to generate trading profits, not information about effort which is correctly anticipated in equilibrium. An issuer-paid CRA, by contrast, sets $k > 0$, producing a rating that makes refinancing terms sensitive to managerial actions and thereby motivates effort. When uncertainty about the firm's credit type is sufficiently large, this rating design induces the effort level under commitment – the binary analogue of the condition $\sigma_\theta \geq \sigma_\theta^d$ in the baseline model. This provides a rationale for the dominance of the issuer-paid business model: issuer-paid ratings generate a motivational effect that investor-paid ratings do not replicate.

Since the setting in Online Appendix H assumes binary firm types and binary signals rather than normal distributions, it also confirms that the motivational-versus-informational trade-off of the baseline model, as well as the achievability of commitment-level effort under high enough fundamental uncertainty, do not crucially rely on distributional assumptions.

This result also generates the policy implication that regulatory interventions in credit rating markets should account for both the informational and motivational roles of ratings. Policies that shift the industry toward investor-paid models (as considered after the 2008 financial crisis) may improve informativeness about exogenous credit risk, but they would eliminate the motivational benefits that operate through the refinancing channel, resulting in worse refinancing terms for borrowers.

5.3 ESG and impact investing

The ESG information market features two coexisting business models. ESG ratings (provided by agencies such as MSCI and Sustainalytics) are sold to institutional investors for portfolio screening and focus on financially material ESG risks. Certifications (such as B-Corp and LEED) are purchased by companies themselves; firms pursue them to "give visibility to commitment" and "encourage self-assessment for continuous improvement" (Diez-Busto, Sanchez-Ruiz, and Fernandez-Laviada, 2022), which can be viewed as motivational rather than purely informational objectives.

In the ESG context, the insider's effort A represents costly actions to improve environmental and social outcomes that matter for firm valuation, such as reducing carbon emissions or improving labor practices. The firm type θ captures its ESG profile, which depends on factors such as its technology and its exposure to regulation. An intermediary can obtain information ("signals") about the firm's effort and its ESG profile. Uncertainty about the firm's ESG profile is measured by σ_θ.

These two business models correspond to different types of firms: in practice, investor-paid ratings predominantly cover large public firms, while firm-paid certifications are more common among smaller or private firms. Our framework explains this coexistence. Investor-paid ESG ratings, which are informative about the ESG profile of the firm ($k^{**} = 0$), emerge in firms with dispersed or passive ownership (low $1 - \delta$ or disengaged shareholders who do not add value to the firm) or in firms with a liquid market for their stock. Investors who purchase these ratings are looking for information about ESG risks for trading and investment purposes. On the contrary, the model predicts that firms with concentrated ownership and active shareholder(s) (high $1 - \delta$) with an illiquid stock will pursue firm-paid ESG assessments such as ESG certifications, which will involve $k > 0$. Examples include early-stage clean technology firms and family businesses undergoing sustainability transitions. These certifications allow firms to credibly commit to high ESG effort by ensuring that ESG improvements that only materialize in the long run are reflected earlier in firm valuation. They are especially valuable when firm insiders have relatively short horizons and ESG uncertainty is high.

The model also sheds new light on impact investing. Now suppose that the insider's effort A generates social impact. In addition, following Oehmke and Opp (2025), suppose that the speculator maximizes expected wealth plus social impact.

Corollary 2 *When the insider's effort improves social outcomes and the speculator is socially responsible, the rating design that maximizes her willingness to pay is such that* $k > 0$.

Socially responsible speculators value information about effort A because they intrinsically value social outcomes. Their willingness to pay is maximized by a rating that increases their informational advantage about $\tilde{\theta}$ (for trading) and is sensitive to effort A (for impact). Importantly, impact arises not through the speculator's trading activity per se, but through her influence on the design of financial information: when impact investors purchase ESG information, rating agencies respond by producing ESG ratings with $k > 0$ that create incentives for ESG improvements by firm insiders. This contrasts with existing theories where impact arises through divestment (Edmans, Levit, and Schneemeier, 2024), engagement (Oehmke and Opp, 2025), portfolio concentration (Bisceglia, Piccolo, and Schneemeier, 2026), or the agency costs of reduced price informativeness from sustainable investing (Chen, Gupta, and Starmans, 2026).

Finally, this analysis has regulatory implications. The SEC's proposed climate disclosure rules and the EU's Corporate Sustainability Reporting Directive (CSRD) emphasize standardized disclosure of financially material ESG risks (effectively mandating information with $k = 0$). While this improves comparability and is aimed at reducing greenwashing, it may unintentionally compromise impact-oriented information (with $k > 0$) that motivates ESG improvements. More generally, our results suggest that the coexistence of both types of ESG information is an equilibrium feature that reflects the distinct needs of different market participants.

6 Conclusion

We have shown that the funding structure of financial intermediaries has real effects that go beyond the well-studied problem of rating bias. First, who pays for financial information determines which type of information is produced: firm-paid intermediaries produce motivational ratings that strengthen effort incentives, while investor-paid intermediaries produce informational ratings that do not. Second, properly designed ratings can fully resolve the tension between short-term horizons and long-term value creation. This time horizon irrelevance result contrasts with standard models of moral hazard with exit. Third, market liquidity and competition among intermediaries can undermine the provision of motivational ratings.

These results have implications for several ongoing debates. In the regulation of credit rating agencies, they identify an overlooked cost of shifting toward investor-paid models – weaker incentive provision arising from the endogenous rating design. In venture capital, they help explain why short-horizon funds can successfully finance long-horizon projects when fundamental uncertainty is sufficiently high. In ESG markets, they rationalize the coexistence of investor-paid ratings and firm-paid certifications as an equilibrium outcome. For regulators, a general principle is that information markets cannot be assessed based only on accuracy: a reform that improves rating quality may reduce output if it replaces motivational ratings with informational ratings.

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A Proofs

Proof of Proposition 1: The speculator solves:

$$\max_q \mathbb{E}\left[\left(\tilde{X}-p\right)q\middle|R\right] \quad \Leftrightarrow \quad \max_q \left(\mathbb{E}[\tilde{X}|R]-(\mu+\lambda q)\right)q \tag{24}$$

The second-order condition for a maximum is satisfied, so the optimum $q^*(R)$ is described by the first-order necessary condition:

$$q^*(R) \quad = \quad \frac{\mathbb{E}\left[\tilde{X}\middle|R\right]-\mu}{2\lambda} \tag{25}$$

We have:

$$\begin{aligned}\mathbb{E}[\tilde{X}|R] \quad &= \quad \hat{A}+\bar{\theta}+\frac{cov(\tilde{X},\tilde{R})}{var(\tilde{R})}\left(R-k\hat{A}-(1-k)\,\bar{\theta}\right)\\ &= \quad \hat{A}+\bar{\theta}+\frac{(1-k)\,\sigma_\theta^2}{\sigma_R^2}\left(R-k\hat{A}-(1-k)\,\bar{\theta}\right)\end{aligned} \tag{26}$$

Denoting $b_I \equiv \frac{(1-k)\sigma_\theta^2}{\sigma_R^2}$, we have:

$$q^*\left(R\right)=\frac{\hat{A}+\bar{\theta}-\mu}{2\lambda}+\frac{b_I\left(R-k\hat{A}-(1-k)\,\bar{\theta}\right)}{2\lambda} \tag{27}$$

So we get

$$\beta=\frac{b_I}{2\lambda} \qquad \text{and} \qquad \alpha=\frac{\hat{A}+\bar{\theta}-\mu}{2\lambda}-\frac{b_I\left(k\hat{A}+(1-k)\,\bar{\theta}\right)}{2\lambda} \tag{28}$$

The price set by the market maker corresponds to the expected value of the firm given y. As already explained, the net order flow y includes an order $\alpha+\beta R$ from the speculator and an order u from noise traders, but excludes the corporate insider's order since it is observable and uninformative.

$$p \;=\; \mathbb{E}\left[\tilde{X}\middle|y\right] \;=\; \mathbb{E}\left[\tilde{X}\right]+\frac{cov(\tilde{X},\tilde{y})}{Var(\tilde{y})}\left(y-\mathbb{E}[\tilde{y}]\right) \;=\; \hat{A}+\bar{\theta}+\frac{\beta\,(1-k)\,\sigma_\theta^2}{\beta^2(\sigma_R^2)+\sigma_u^2}\left(y-\mathbb{E}[\tilde{y}]\right) \tag{29}$$

In particular, when $y=\mathbb{E}[\tilde{y}]$:

$$p=\hat{A}+\bar{\theta}=\mu+\lambda\left(\alpha+\beta\left(k\hat{A}+(1-k)\,\bar{\theta}\right)\right) \tag{30}$$

We get:

$$\lambda=\frac{\beta\,(1-k)\,\sigma_\theta^2}{\beta^2(\sigma_R^2)+\sigma_u^2} \qquad \text{and} \qquad \mu=\hat{A}+\bar{\theta}-\lambda\left(\alpha+\beta\left(k\hat{A}+(1-k)\,\bar{\theta}\right)\right)$$

Solving for α, β, λ and μ yields:

$$\beta = \frac{\sigma_u}{\sigma_R} \tag{31}$$

$$\lambda = \frac{(1-k)\,\sigma_\theta^2}{2\sigma_u \sigma_R} \tag{32}$$

$$\alpha = -\beta\mathbb{E}[\tilde{R}] = -\beta\left(k\hat{A} + (1-k)\,\bar{\theta}\right) \tag{33}$$

$$\mu = \mathbb{E}[\tilde{X}] = \hat{A} + \bar{\theta} \tag{34}$$

The expected stock price as a function of the expected effort of the insider, $\hat{A}$, and his actual effort, A, is:

$$\begin{aligned}
\mathbb{E}\left[p\middle|A, \hat{A}\right] &= \mu + \lambda\mathbb{E}\left[y|A\right] \\
&= \mu + \lambda\left(\alpha + \beta\mathbb{E}\left[R|A\right]\right) \\
&= \hat{A} + \bar{\theta} + \lambda\beta k\left(A - \hat{A}\right) \\
&= \hat{A} + \bar{\theta} + \frac{1}{2}\left(1-k\right)k\frac{\sigma_\theta^2}{\sigma_R^2}\left(A - \hat{A}\right)
\end{aligned}$$

The expected (ex-ante) profit of the speculator who is trading after privately observing rating R is:

$$\begin{aligned}
\mathbb{E}\left[\left(\tilde{X} - p(\tilde{y})\right)q\right] &= \mathbb{E}\left[(\tilde{X} - \hat{A} - \bar{\theta} - \lambda\beta(R - \mathbb{E}[\tilde{R}]) - \lambda u)\beta(R - \mathbb{E}[\tilde{R}])\right] \\
&= \beta\mathbb{E}\left[(\tilde{X} - \hat{A} - \bar{\theta})(R - \mathbb{E}[\tilde{R}])\right] - \lambda\beta^2\mathbb{E}\left[\left(R - \mathbb{E}[\tilde{R}]\right)^2\right] \\
&= \frac{\sigma_u}{\sigma_R}\left(1-k\right)\sigma_\theta^2 - \lambda\beta^2\sigma_R^2 \\
&= \frac{\sigma_u\left(1-k\right)\sigma_\theta^2}{2\sigma_R} \qquad (35)
\end{aligned}$$

Proof of Proposition 2: With a public rating, the market maker sets the market value of the firm equal to its expected output given the rating. The order flows are irrelevant as they do not include any additional information about firm value. Given rating R, the market value of the firm is:

$$p = \hat{A} + \mathbb{E}\left[\tilde{\theta}\middle|R\right] = \hat{A} + \mathbb{E}\left[\tilde{\theta}\right] + \frac{cov(\tilde{\theta}, \tilde{R})}{var(\tilde{R})}\left(R - \mathbb{E}\left[\tilde{R}\right]\right) = \hat{A} + \bar{\theta} + \frac{(1-k)\sigma_\theta^2}{\sigma_R^2}\left(R - \mathbb{E}\left[\tilde{R}\right]\right)$$

The expected market value conditional on actual effort A and expected effort $\hat{A}$ is:

$$\mathbb{E}\left[p\middle|A,\hat{A}\right] \quad = \quad \hat{A}+\bar{\theta}+k(1-k)\frac{\sigma_\theta^2}{\sigma_R^2}(A-\hat{A}) \tag{36}$$

Proof of Lemma 1: Parts (i) and (ii). The insider maximizes:

$$\max_A\,(1-\delta)\left((1-\pi)\left(A+\bar{\theta}\right)+\pi\left(\left(\hat{A}+\bar{\theta}\right)+S(k)\left(A-\hat{A}\right)\right)\right)-C(A) \tag{37}$$

where $S(k)=\lambda\beta k=k\,(1-k)\,\frac{\sigma_\theta^2}{2\sigma_R^2}$ with a private rating, and $S(k)=k(1-k)\frac{\sigma_\theta^2}{\sigma_R^2}$ with a public rating. The second-order condition for a maximization problem is satisfied, so that the optimal effort is given by the first-order necessary condition:

$$C'(A)=(1-\delta)\,(1-\pi+\pi S(k)) \tag{38}$$

From the solution to the insider's maximization problem in equation (38) and the definition of the effort A^{LT} preferred by the insider in equation (13), the rating induces A^{LT} if and only if:

$$(1-\delta)\,(1-\pi+\pi S(k))=1-\delta \quad\Leftrightarrow\quad S(k)=1 \tag{39}$$

There are two cases. First, with a private rating, $S(k)=1$ rewrites as:

$$k\,(1-k)\,\frac{\sigma_\theta^2}{2\sigma_R^2}=1 \qquad\Leftrightarrow\qquad k\,(1-k)\,\sigma_\theta^2=2\left((1-k)^2\left(\sigma_\theta^2+1\right)+k^2\right) \tag{40}$$

Second, with a public rating, $S(k)=1$ rewrites as:

$$k(1-k)\frac{\sigma_\theta^2}{\sigma_R^2}=1 \qquad\Leftrightarrow\qquad k\,(1-k)\,\sigma_\theta^2=\left((1-k)^2\left(\sigma_\theta^2+1\right)+k^2\right) \tag{41}$$

In any of these two cases, solving for k gives the optimal weights.

Part (iii). For any $k\in(0,1)$, the incentive coefficient $\frac{k(1-k)\sigma_\theta^2}{\sigma_R^2}$ is strictly increasing in σ_θ^2. Indeed:

$$\frac{k(1-k)\sigma_\theta^2}{\sigma_R^2}=\frac{k(1-k)\sigma_\theta^2}{k^2+(1-k)^2+(1-k)^2\sigma_\theta^2}, \tag{42}$$

and differentiating with respect to σ_θ^2 yields a numerator proportional to $k^2+(1-k)^2>0$. Since the private-rating coefficient $\frac{k(1-k)\sigma_\theta^2}{2\sigma_R^2}$ is exactly half the public-rating coefficient at every k, it is also strictly increasing in σ_θ^2 for each k.

From parts (i) and (ii), $A=A^{LT}$ requires $S(k)=1$, with the relevant value for the coefficient $S(k)$ depending on whether the rating is private or public. By definition of σ_θ^d, at $\sigma_\theta = \sigma_\theta^d$ there exists k_0 such that the public-rating coefficient equals 1; at this same k_0, the private-rating coefficient equals $\frac{1}{2}$. Since the private-rating coefficient is equal to the public-rating coefficient multiplied by $\frac{1}{2}$, its maximum over k at σ_θ^d is also $\frac{1}{2} < 1$. Strict monotonicity of the private-rating coefficient in σ_θ at each k therefore implies that its maximum over k is strictly increasing in σ_θ, so the lowest σ_θ at which this maximum reaches 1 is such that $\sigma_\theta^* > \sigma_\theta^d$.

Proof of Remark 1: Dividing the numerator and the denominator by σ_θ^2:

$$\frac{3\sigma_\theta^2 - \sqrt{\sigma_\theta^4 - 4\sigma_\theta^2 - 4} + 2}{4\sigma_\theta^2 + 4} = \frac{3 - \sqrt{1 - \frac{4}{\sigma_\theta^2} - \frac{4}{\sigma_\theta^4}} + \frac{2}{\sigma_\theta^2}}{4 + \frac{4}{\sigma_\theta^2}} \xrightarrow{\sigma_\theta \to \infty} \frac{3-1}{4} = \frac{1}{2}. \tag{43}$$

Proof of Proposition 3: Consider the private rating case. From Lemma 1(i), $A = A^{LT}$ requires $C'(A^{LT}) = 1 - \delta$, i.e.:

$$(1-\delta)\left(1 - \pi + \pi \frac{k(1-k)\sigma_\theta^2}{2\sigma_R^2}\right) = 1 - \delta \quad \Leftrightarrow \quad \frac{k(1-k)\sigma_\theta^2}{2\sigma_R^2} = 1. \tag{44}$$

This condition is independent of π. By Lemma 1(i), a solution $k = k^{LT}$ exists if and only if $\sigma_\theta \geq \sigma_\theta^*$. Since the same k^{LT} satisfies the condition for every $\pi \in [0, 1]$, effort equals A^{LT} regardless of the insider's time horizon.

The argument for the public rating case is identical, with $\frac{k(1-k)\sigma_\theta^2}{\sigma_R^2} = 1$ (from Lemma 1(ii)) replacing the condition above, and σ_θ^d replacing σ_θ^*.

Proof of Proposition 4: The rating weight that maximizes the speculator's willingness to pay is given by:

$$\max_k \left(\frac{\sigma_u (1-k)\, \sigma_\theta^2}{2\sigma_R}\right) \tag{45}$$

Since σ_u and σ_θ are constants, and substituting for σ_R, this is equivalent to:

$$\max_k \left(\frac{1-k}{\sqrt{(1-k)^2\, (\sigma_\theta^2 + 1) + k^2}}\right) \tag{46}$$

The first derivative with respect to k is:

$$-\frac{k}{(k^2 + (k-1)^2(1+\sigma_\theta^2))^{\frac{3}{2}}} \tag{47}$$

which is negative for any $\sigma_\theta > 0$ and $k \in (0, 1)$. The optimal weight is given by: $k = k^{**} \equiv 0$.

Proof of Proposition 5: Let $S(k)$ denote the coefficient defined in the proof of Lemma 1, which is equal to $\frac{k(1-k)\sigma_\theta^2}{2\sigma_R^2}$ with a private rating (as in part (i) of the Proposition) and $\frac{k(1-k)\sigma_\theta^2}{\sigma_R^2}$ with a public rating (as in part (ii) of the Proposition). Given equilibrium effort $\hat{A}$, the agency chooses the rating weight to maximize the insider's willingness to pay using the firm's funds (such that he only bears a fraction $1-\delta$ of the cost), which is:

$$W_I \equiv \frac{1}{1-\delta}\left((1-\delta)\hat{A} - C(\hat{A}) - (1-\delta)\hat{A}_0 + C(\hat{A}_0)\right). \tag{48}$$

where $\hat{A}_0$ is the equilibrium effort without a rating, which solves $C'(\hat{A}_0) = (1-\pi)(1-\delta)$. The first-order condition (FOC) is:

$$\left(1-\delta - C'(\hat{A})\right)\frac{d\hat{A}}{dk} = 0. \tag{49}$$

From equation (38) in the proof of Lemma 1, $\hat{A}$ satisfies $C'(\hat{A}) = (1-\delta)(1-\pi+\pi S(k))$, so $\hat{A}$ is strictly increasing in $S(k)$, and the insider's willingness to pay in equation (48) is strictly increasing in $\hat{A}$ for $\hat{A} < A^{LT}$.

First, if there exists k such that $S(k) = 1$, then $C'(\hat{A}) = 1-\delta$, so $\hat{A} = A^{LT}$ and the insider's willingness to pay reaches its maximum. By Lemma 1, such a rating weight k exists if and only if $\sigma_\theta \geq \sigma_\theta^*$ with a private rating (part (i) of the Proposition) and $\sigma_\theta \geq \sigma_\theta^d$ with a public rating (part (ii) of the Proposition). Then, the corresponding rating weight is k^{LT} in part (i) and k_d^{LT} in part (ii).

Second, if there does not exist k such that $S(k) = 1$, then $S(k) < 1$ and $\hat{A} < A^{LT}$ for every $k \in [0,1]$. Then, the insider's willingness to pay in equation (48) is strictly increasing in $\hat{A}$, and therefore it is increasing in $S(k)$. Thus, the optimal rating weight maximizes $S(k)$. In parts (i) and (ii), $S(k)$ is proportional to:

$$\frac{k(1-k)}{\sigma_R^2} = \frac{k(1-k)}{(1-k)^2(\sigma_\theta^2+1)+k^2}, \tag{50}$$

so the effort-maximizing rating weight is the same in parts (i) and (ii). The FOC for (50) can be written as:

$$k^2\sigma_\theta^2 - 2(\sigma_\theta^2+1)k + (\sigma_\theta^2+1) = 0, \tag{51}$$

whose smaller root, corresponding to the maximum, is:

$$k^* \equiv \frac{\sigma_\theta^2 + 1 - \sqrt{1+\sigma_\theta^2}}{\sigma_\theta^2}. \tag{52}$$

Proof of Lemma 2: When the insider pays the rating agency and the rating is communicated privately to the speculator, the insider's equilibrium payoff is $(1-\delta)\hat{A}^p$+constant$-C(\hat{A}^p)$, where $\hat{A}^p$ solves $C'(\hat{A}^p) = (1-\delta)(1-\pi+\pi\frac{k(1-k)\sigma_\theta^2}{2\sigma_R^2})$. When the rating is publicly disclosed, the insider's equilibrium payoff is $(1-\delta)\hat{A}^d$ + constant $-$ $C(\hat{A}^d)$, where $\hat{A}^d$ solves $C'(\hat{A}^d) = (1-\delta)\left(1-\pi+\pi\frac{k(1-k)\sigma_\theta^2}{\sigma_R^2}\right)$.

For any $k \in (0,1)$, the implicit incentive coefficient is strictly larger under public disclosure. By the proof of Proposition 5, the insider's willingness to pay is monotonic in $(1-\delta)\hat{A}-C(\hat{A})$, and is increasing in effort $\hat{A}$ up to A^{LT}, and the effort-maximizing weight k^* is the same in both cases. Therefore, for any σ_θ, the insider's willingness to pay is weakly higher under public disclosure. The inequality is strict when $\sigma_\theta < \sigma_\theta^*$, because the optimal effort under public disclosure is strictly higher. When $\sigma_\theta \geq \sigma_\theta^*$, both business models achieve A^{LT} (with the appropriate rating weight), so the insider's willingness to pay is the same.

Proof of Proposition 6: The rating agency makes a take-it-or-leave-it offer and therefore extracts the full surplus from its client. We compare the willingness to pay of the speculator and of the insider under each business model.

First, consider the effect of σ_u. From Lemma 2, the relevant comparison is between the speculator's willingness to pay and the insider's willingness to pay under public disclosure. The speculator's willingness to pay with $k = k^{**} = 0$ is, from Proposition 1(iii): $W_S = \frac{\sigma_u\sigma_\theta^2}{2\sqrt{\sigma_\theta^2+1}}$, which is linearly increasing in σ_u. The insider's willingness to pay with the firm's funds, W_I as defined in equation (48), is independent of σ_u because neither the insider's objective function nor his equilibrium effort depends on σ_u. Therefore, there exists a threshold $\overline{\sigma}_u$ such that $W_S \geq W_I$ if and only if $\sigma_u \geq \overline{\sigma}_u$.

Second, consider the effect of π. The speculator's willingness to pay W_S is independent of π. The insider's willingness to pay (W_I as defined above) depends on π via the equilibrium effort. At $\pi = 0$, the equilibrium effort satisfies $C'(\hat{A}^d) = (1-\delta)$ regardless of k, so $W_I = 0$ when $\pi = 0$. The insider's effort $\hat{A}^d(k,\pi)$ is continuous in (k,π) by the implicit function theorem applied to the insider's FOC. Since the agency chooses k from the compact set $[0,1]$ to maximize $W_I(k,\pi)$, the Theorem of the Maximum implies that W_I is continuous in π. Since $W_S > 0$ is independent of π, there exists a threshold $\underline{\pi} > 0$ such that $W_S > W_I$ for all $\pi < \underline{\pi}$, in which case the rating agency sells the rating to the speculator.

Third, consider the effect of δ. The speculator's willingness to pay W_S is independent of δ. The insider's willingness to pay W_I defined above depends on δ through the factor $(1-\delta)$ in his objective function and the factor $1/(1-\delta)$ that accounts for the fact that he uses the firm's funds to purchase the rating. As $\delta \to 1$, the insider's effort incentive coefficient $(1-\delta)\big(1-\pi+\pi k(1-k)\frac{\sigma_\theta^2}{\sigma_R^2}\big) \to 0$, so equilibrium effort $\hat{A}^d$ tends to $C'^{-1}(0)$. The effort without a rating, $\hat{A}_0$, also tends to $C'^{-1}(0)$. A Taylor expansion of $C(\cdot)$ around $C'^{-1}(0)$ shows that the numerator of W_I is of order $(1-\delta)^2$, while the denominator is of order $1-\delta$, so $W_I \to 0$ as $\delta \to 1$. Since $W_S > 0$, there exists a threshold $\overline{\delta}$ such that $W_S \geq W_I$ for all $\delta \geq \overline{\delta}$.

Proof of Proposition 7: The proof is similar to the proofs of Propositions 1 and 2. See Online Appendix I.

Proof of Lemma 4: See Online Appendix I.

Proof of Proposition 8: We characterize the incumbent's optimal strategy in three steps: deriving the entrant's best response to each incumbent choice; determining the incumbent's optimal public rating weight given entry incentives; comparing payoffs across all strategies.

If the incumbent provides a public rating, the entrant cannot profitably provide a second public rating. The entrant's relevant option is providing a private informational rating to the speculator, earning $\Delta(k_1, \sigma_u) - K$. Entry occurs if and only if $\Delta(k_1, \sigma_u) > K$. If the incumbent sells a private informational rating ($k_1 = 0$), this reveals S_2 to the speculator. A second private rating would allow the speculator to recover S_1 using Lemma 3. However, $S_1 = A + \varepsilon_1$ contains only anticipated effort A (known in equilibrium) and noise ε_1 orthogonal to fundamentals θ. Since S_1 reveals no information about the firm's tradeable value θ, a second private rating earns zero profit. The entrant's relevant option is providing a public rating to the firm. With the incumbent's private rating already revealing S_2, Proposition 7 characterizes the two-rating equilibrium where the market maker observes the public rating R_2 and the speculator observes both. This yields an insider willingness to pay of $V^T(k_2)$, maximized at $k_2 = k^*$. Entry occurs if and only if $V^T(k^*) > K$.

Any public rating with $k_1 > k^*$ is weakly dominated by k^*: this replacement weakly raises the incumbent's payoff both with entry (V^T single-peaked at k^*) and without entry (V single-peaked at k^*), while weakly lowering entry pressure (since $\Delta(\cdot, \sigma_u)$ increases in k_1 by Lemma 4). Thus, we restrict attention to $k_1 \in [0, k^*]$.

For $K \geq \Delta(k^*, \sigma_u)$, entry is unprofitable for all $k_1 \in [0, k^*]$ since $\Delta(\cdot, \sigma_u)$ is maximized at k^*. The incumbent's optimal public rating is $k_1 = k^*$, earning $V(k^*)$.

For $K < \Delta(k^*, \sigma_u)$, the incumbent chooses between deterring and accommodating entry. Setting $k_1 \leq \hat{k}_1(\sigma_u, K)$ deters entry. Since $V(\cdot)$ increases on $[0, k^*]$, the optimal deterrence choice is $k_1 = \hat{k}_1$, yielding payoff $V(\hat{k}_1)$. Setting $k_1 > \hat{k}_1$ allows entry. Since $V^T(\cdot)$ is maximized at k^*, the optimal accommodation choice is $k_1 = k^*$, yielding payoff $V^T(k^*)$.

If the incumbent serves the speculator with $k_1 = 0$, it extracts the speculator's reservation value, which depends on the entrant's response. If $K < V^T(k^*)$, entry occurs, reducing the speculator's reservation value to $\Delta(k^*, \sigma_u)$. If $K \geq V^T(k^*)$, entry is blockaded, the speculator's reservation value is $W(\sigma_u)$. Thus the incumbent's payoff with a private rating is $\Pi^P(\sigma_u, K)$ as defined above.

The incumbent's optimal strategy is determined by comparing its payoff with the following rating and strategy:

(a) Public rating, blockaded entry: $V(k^*)$ (only possible when $K \geq \Delta(k^*, \sigma_u)$);

(b) Public rating, deterred entry: $V(\hat{k}_1)$ (only possible when $K < \Delta(k^*, \sigma_u)$);

(c) Public rating, accommodated entry: $V^T(k^*)$ (only possible when $K < \Delta(k^*, \sigma_u)$);
(d) Private rating: $\Pi^P(\sigma_u, K)$ (always available).

The three regimes correspond to which strategy yields the highest payoff among the four available. Blockaded (regime (i)) arises when (a) dominates (incumbent sells public $k_1 = k^*$, no entry) or when (d) dominates with $K \geq V^T(k^*)$ (incumbent sells private $k_1 = 0$, no entry). Deterred (regime (ii)) arises when (b) dominates (incumbent sells public $k_1 = \hat{k}_1 < k^*$, no entry). Accommodated (regime (iii)) arises when (c) dominates (incumbent sells public $k_1 = k^*$, entrant sells private) or when (d) dominates with $K < V^T(k^*)$ (incumbent sells private $k_1 = 0$, entrant sells public $k_2 = k^*$). Ties are broken per the Proposition statement.

Proof of Corollary 2: See Online Appendix I.

Online Appendix

B Uncertain effort

In our model, the agent's equilibrium effort is known, as is common in moral hazard models (Bolton and Dewatripont (2004)). This is not a crucial assumption.

Indeed, the model can be reinterpreted when there is uncertainty about the insider's action (i.e. the level of effort) rather than about "firm fundamentals". Suppose that firm output is simply:

$$X = A, \tag{A.1}$$

and the cost of effort is now:

$$C(A) = \frac{c}{2}\left(A - \overline{A}\right)^2 \tag{A.2}$$

where c is common knowledge and $\overline{A}$, which is drawn from $\overline{A} \sim \mathcal{N}(\bar{\theta}, \sigma_{\theta}^2)$, is privately learned by the insider at the effort choice stage.[17] There is uncertainty about the parameter $\overline{A}$, which enters the effort cost function and affects the equilibrium effort. This is similar to the specification in Akerlof and Kranton (2005) and Fischer and Huddart (2008), where $\overline{A}$ would be interpreted as the "effort norm". This implies:

$$C'(A) = c\left(A - \overline{A}\right) \tag{A.3}$$

The FOC with respect to the insider's effort is now, given a first derivative of the insider's wealth with respect to effort ξ:[18]

$$\xi = c\left(A - \overline{A}\right) \quad \Leftrightarrow \quad A = \frac{\xi}{c} + \overline{A} \tag{A.4}$$

Thus, output, as defined in equation (A.1), can be decomposed as:

$$X = \underbrace{A - \overline{A}}_{\text{correctly anticipated}} + \underbrace{\overline{A}}_{\text{uncertain}} = \frac{\xi}{c} + \overline{A} \tag{A.5}$$

In this setting, the two signals which can be combined into a rating are $\tilde{S}_1 = A - \overline{A} + \tilde{\varepsilon}_1$, and $\tilde{S}_2 = \overline{A} + \tilde{\varepsilon}_2$. As in the main model, the first signal is sensitive to the level of effort A

[17] This is for tractability and to avoid issues that arise with asymmetric information at the contracting stage (Lizzeri (1999)).

[18] There are several possible expressions for ξ, depending on the business model used by the rating agency.

chosen by the insider, and the second signal reduces uncertainty about firm output. Note that the first signal is not informative in equilibrium: it is known that $A - \overline{A} = \frac{\xi}{c}$ (see equation (A.4)). Thus, as in the main model, S_1 is not informative in equilibrium but it can still be used to provide effort incentives since it is sensitive to the level of effort A actually chosen by the insider. Note that since $\overline{A}$ is a random variable privately observed by the insider, the equilibrium effort $A = \frac{\xi}{c} + \overline{A}$ is itself a random variable from outsiders' perspective, i.e. effort is uncertain.

For comparison purposes, the FOC with respect to effort in the main model, which relies on the more standard specification $C(A) = \frac{c}{2}A^2$, would be:

$$\xi = cA \quad \Leftrightarrow \quad A = \frac{\xi}{c} \quad \Rightarrow \quad X = \frac{\xi}{c} + \theta \tag{A.6}$$

Comparing the equalities on the right of equations (A.4) and (A.6) shows that the firm output X is additive in a constant $(\frac{\xi}{c})$ and either $\overline{A}$ (in the alternative setting) or $\tilde{\theta}$ (in the main model). In both cases, this latter variable is unknown, normally distributed, and the signal S_2 reduces uncertainty about it: $\tilde{S}_2 = \theta + \tilde{\varepsilon}_2$ in the main model, and $\tilde{S}_2 = \overline{A} + \tilde{\varepsilon}_2$ in this alternative setting.

In sum, the tradeoff remains essentially the same in this alternative setting: shall information motivate effort or resolve uncertainty about firm value? The model in the main paper and the one sketched in this Appendix generate the same joint distribution of (X, S_1, S_2), with $\overline{A}$ taking the role of θ: in both models, $X = \frac{\xi}{c} + Z$, $S_1 = \frac{\xi}{c} + \tilde{\varepsilon}_1$, and $S_2 = Z + \tilde{\varepsilon}_2$, where $Z \sim \mathcal{N}(\bar{\theta}, \sigma_\theta^2)$ denotes either θ (main model) or $\overline{A}$ (uncertain-effort model). The market maker's pricing rule, the speculator's strategy, and the insider's incentives are therefore identical. The difference is interpretative: uncertainty is about the firm type θ in the main model, and it is about the baseline level of effort $\overline{A}$ in this Online Appendix.

C Linear rating

We consider ratings R such that $R = kS_1 + (1-k)S_2$ with $k \in [0, 1]$, which restricts the rating to be linear in the signals. First of all, our main result about time horizon irrelevance, which shows that there exists a rating with some properties, would still hold if we were to consider a more general class of ratings.

In our model, all primitive random variables ($\tilde{\theta}$, $\tilde{\varepsilon}_1$, $\tilde{\varepsilon}_2$, $\tilde{u}$) are Gaussian. In the market equilibria we characterize, the speculator's market order is linear in the rating and the market maker's pricing rule is linear in observable variables, as is standard in the Kyle (1985) framework. In such Gaussian-linear environments, the sufficient statistic for any signal's informational content is its covariance structure with the relevant variables. Two signals with the same covariance structure with $(\tilde{X}, \tilde{\theta}, \tilde{u})$ are informationally equivalent for all agents in the model. Since any function $h(S_1, S_2)$ that preserves joint normality with the other random variables must be an affine function of (S_1, S_2), the restriction to linear ratings is without loss of generality within the class of Gaussian information structures.

In principle, a rating which is a nonlinear transformation $h(S_1, S_2)$ could generate a non-Gaussian signal, but characterizing market equilibria with such a signal would require a departure from the linear equilibrium framework. Models based on Kyle (1985) usually rely on the linearity assumption for tractability. Hörner and Lambert (2021) impose the same restriction for similar reasons.

D First-best outcome

The first-best insider effort A^{FB} maximizes expected output A minus the effort cost $C(A)$, so that it is defined by:

$$C'(A^{FB}) = 1 \tag{A.7}$$

The insider's optimally chosen effort is first-best if he owns the whole firm and is never hit by a liquidity shock, i.e. $\delta = 0$ and $\pi = 0$. In this case, he knows ex-ante that he will own the whole firm ex-post, and therefore fully internalizes the effect of his effort on firm value.

We now show that under some conditions a rating can lead to the insider to exert the first-best effort. Define:

$$c \equiv 1 + \frac{1}{\pi}\frac{\delta}{1-\delta}. \tag{A.8}$$

Lemma 5 shows that, when uncertainty about firm value is sufficiently high, there are rating weights that induce the first-best effort defined in equation (A.7).

Lemma 5 *(i) When the speculator privately observes the rating, The insider effort is A^{FB} if and only if $\sigma_\theta^4 \geq 16c^2(\sigma_\theta^2 + 1)$ and*

$$k = k^{FB} \equiv \frac{4c(\sigma_\theta^2+1) + \sigma_\theta^2 \pm \sqrt{\sigma_\theta^4 - 16c^2(\sigma_\theta^2+1)}}{2\left(2c(\sigma_\theta^2+2) + \sigma_\theta^2\right)}. \tag{A.9}$$

(ii) When the rating is publicly disclosed, the insider effort is A^{FB} if $\sigma_\theta \geq \Sigma(\delta, \pi)$ and $k = k_d^{FB}$, where $\Sigma(\delta, \pi)$ and k_d^{FB} are defined in the proof of Lemma 5.

The insider's effort is first-best when the implicit incentives from the resale value of the firm are sufficiently high to outweigh the insider's partial ownership of the firm (when $\delta > 0$). This is only possible for a subset of parameter values. First, as in the baseline model, prior uncertainty about firm value, as captured by σ_θ, must be sufficiently high. Second, insider ownership $1-\delta$ must be high enough. Otherwise, there is no rating design such that market pricing based on the rating delivers first-best incentives. Intuitively, the insider's incentives can be decomposed into two components. First, the explicit incentives from his (expected) long-term ownership. These incentives are proportional to $(1-\delta)(1-\pi)$. Second, the implicit incentives from the firm's market valuation when he needs to liquidate his stake. These incentives are proportional to $(1-\delta)\pi$. By contrast, first-best effort incentives are independent of the insider's ownership $1-\delta$. As a result, when the insider does not own the whole firm, i.e. $1-\delta<1$, a necessary condition for effort to reach the first-best level is that the implicit incentives from market valuation be sufficiently higher than the explicit incentives from the insider's long-term ownership, which is a stronger condition than in Lemma 1.

Proof of Lemma 5:

Part (i). The insider maximizes:

$$\max_{A}\,(1-\delta)\left((1-\pi)\left(A+\bar{\theta}\right)+\pi\left(\left(\hat{A}+\bar{\theta}\right)+\lambda\beta k\left(A-\hat{A}\right)\right)\right)-C(A) \tag{A.10}$$

The second-order condition for a maximization problem is satisfied, so that the optimal effort is given by the first-order necessary condition:

$$C'(A)=(1-\delta)\left(1-\pi+\pi\lambda\beta k\right) \tag{A.11}$$

From the solution to the insider's maximization problem in equation (A.11) and the definition of the first-best effort, the rating induces the first-best effort if and only if:

$$(1-\delta)\left(1-\pi+\pi\lambda\beta k\right)=1 \quad\Leftrightarrow\quad \lambda\beta k=1+\frac{1}{\pi}\frac{\delta}{1-\delta} \tag{A.12}$$

Substituting for λ and β, the condition is: $\frac{k(1-k)\sigma_\theta^2}{2\sigma_R^2}=1+\frac{1}{\pi}\frac{\delta}{1-\delta}$. Solving for k gives the optimal weights.

Part (ii). The insider maximizes:

$$\max_{A}\,(1-\delta)\left((1-\pi)\left(A+\bar{\theta}\right)+\pi\left(\hat{A}+\bar{\theta}+k(1-k)\frac{\sigma_\theta^2}{\sigma_R^2}(A-\hat{A})\right)\right)-C(A) \tag{A.13}$$

The second-order condition for a maximization problem is satisfied, so that the optimal effort is given by the first-order necessary condition:

$$C'(A)=(1-\delta)\left(1-\pi+\pi k(1-k)\frac{\sigma_\theta^2}{\sigma_R^2}\right) \tag{A.14}$$

From the solution to the insider's maximization problem in equation (A.14) and the definition of the first-best effort in equation (A.7), the rating induces the first-best effort if and only if:

$$(1-\delta)\left(1-\pi+\pi k(1-k)\frac{\sigma_\theta^2}{\sigma_R^2}\right)=1 \quad \Leftrightarrow \quad k(1-k)\frac{\sigma_\theta^2}{\sigma_R^2}=\frac{1}{\pi}\frac{\delta}{1-\delta}+1 \tag{A.15}$$

Solving for k gives the optimal weights $k=k_d^{FB}$, which exist for $\sigma_\theta \geq \Sigma(\delta,\pi)$, where k_d^{FB} and $\Sigma(\delta,\pi)$ are defined as:

$$\begin{aligned}
k_d^{FB} &\equiv \frac{\frac{2\delta\sigma_\theta^2}{(1-\delta)\pi}+\frac{2\delta}{(1-\delta)\pi}+3\sigma_\theta^2+2\pm\sqrt{-\frac{4\delta^2\sigma_\theta^2}{(1-\delta)^2\pi^2}-\frac{4\delta^2}{(1-\delta)^2\pi^2}-\frac{8\delta\sigma_\theta^2}{(1-\delta)\pi}-\frac{8\delta}{(1-\delta)\pi}+\sigma_\theta^4-4\sigma_\theta^2-4}}{2\left(\frac{\delta\sigma_\theta^2}{(1-\delta)\pi}+\frac{2\delta}{(1-\delta)\pi}+2\sigma_\theta^2+2\right)} \\
\Sigma(\delta,\pi) &\equiv \left(\frac{2\left(\delta^2\pi^2-2\delta^2\pi+\delta^2-2\delta\pi^2+2\delta\pi+\pi^2\right)}{(\delta-1)^2\pi^2}\right. \\
&\quad \left. +2\sqrt{\frac{2\delta^4\pi^4-6\delta^4\pi^3+7\delta^4\pi^2-4\delta^4\pi+\delta^4-8\delta^3\pi^4+18\delta^3\pi^3-14\delta^3\pi^2+4\delta^3\pi+12\delta^2\pi^4-18\delta^2\pi^3+7\delta^2\pi^2-8\delta\pi^4+6\delta\pi^3+2\pi^4}{(\delta-1)^4\pi^4}}\right)^{\frac{1}{2}}
\end{aligned}$$

E Iso-effort curves

This Appendix displays iso-effort curves. Figures 4 and 5 depict on the y-axis the rating weights that induce a given effort for two business models considered. In particular, they show that the minimum level of fundamental uncertainty σ_θ required for a given effort is lower when the rating is publicly disclosed.

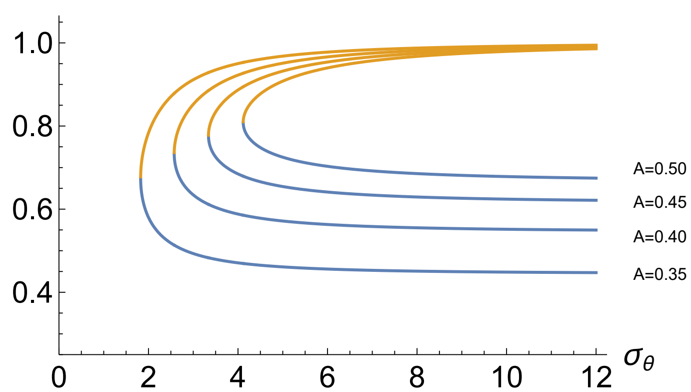

Figure 4: Iso-effort curves in the $\{\sigma_\theta, k\}$ plane that induce effort levels of $A = 0.35, 0.40, 0.45, 0.50$, for $\delta = 0.5$, $\pi = 0.5$, and $C(A) = \frac{A^2}{2}$, when the rating is privately communicated to the speculator. The rating weight k^{LT}, as defined in Lemma 1(i), induces $A = 0.5$. For each effort level depicted, the two parts of the iso-effort curve with different colors correspond to the two rating weights that induce this effort.

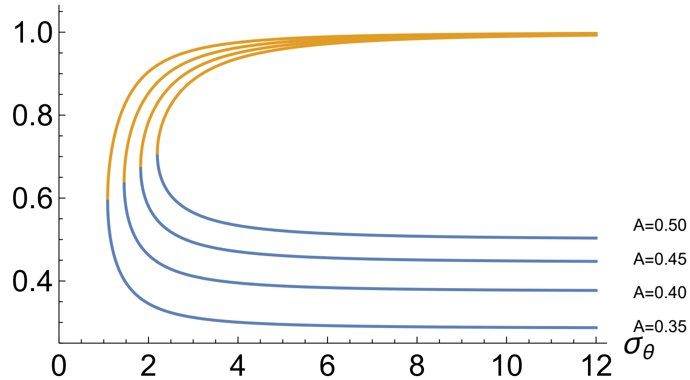

Figure 5: Iso-effort curves in the $\{\sigma_\theta, k\}$ plane that induce effort levels of $A = 0.35, 0.40, 0.45, 0.50$, for $\delta = 0.5$, $\pi = 0.5$, and $C(A) = \frac{A^2}{2}$, when the rating is publicly disclosed. The rating weight k_d^{LT}, as defined in Lemma 1(ii), induces $A = 0.5$. For each effort level depicted, the two parts of the iso-effort curve with different colors correspond to the two rating weights that induce this effort.

F Speculator endowed with shares

We assumed that the speculator did not initially own any shares in the firm. We now relax this assumption to allow the speculator to also be an initial shareholder.

Corollary 3 *When the speculator owns some shares in the firm at $t = 0$, the rating that maximizes her willingness to pay involves a strictly positive weight k.*

A speculator who initially owns shares in the firm does not only care about maximizing her informational advantage. She also cares, to an extent that depends on her initial stake, about maximizing the expected value of the firm. The latter is achieved by increasing effort incentives, which in turn involves increasing the rating weight above the level that maximizes the rating's informativeness, i.e. above $k^{**} = 0$.

This result has ambiguous implications for economic efficiency. On the one hand, if the rating agency sells the rating to the speculator, it will now sell a rating that is not purely informational but instead partly motivational. On the other hand, a speculator with a stake in the firm has a higher willingness to pay for the rating, which makes it more profitable for the rating agency to sell the rating to her.

Proof of Corollary 3: The equilibrium of the financial market follows the one in Proposition 1. When the speculator initially owns q_0 shares in the firm, her expected payoff from a

$t = 0$ perspective is now equal to:

$$\frac{\sigma_u (1-k) \sigma_\theta^2}{2\sigma_R} + q_0(\hat{A} + \bar{\theta}) \tag{A.16}$$

where $\hat{A}$ denotes the equilibrium effort, which depends on k. The speculator's objective is the sum of two terms in equation (A.16). Denote the first term by $f(k)$ and the second by $g(k)$.

Both f and g are continuous and differentiable on $(0,1)$. We will show that $f'(k)+g'(k) > 0$ at $k = 0$, which implies that the optimum is at some $k > 0$. We have:

$$f'(k) = -\frac{\sigma_u \sigma_\theta^2}{2} \frac{k}{(k^2 + (1-k)^2(1+\sigma_\theta^2))^{\frac{3}{2}}} \tag{A.17}$$

At $k = 0$, we have $f'(0) = 0$. Moreover, $g'(k) = q_0 \frac{d\hat{A}}{dk}$. From the insider's FOC, equilibrium effort is implicitly defined by $C'(\hat{A}) = (1-\delta)\left(1 - \pi + \pi \frac{k(1-k)\sigma_\theta^2}{2\sigma_R^2}\right)$. By the implicit function theorem:

$$\frac{d\hat{A}}{dk} = \frac{(1-\delta)\pi}{C''(\hat{A})} \frac{d}{dk}\left[\frac{k(1-k)\sigma_\theta^2}{2\sigma_R^2}\right] \tag{A.18}$$

The derivative of the expression $\frac{k(1-k)}{\sigma_R^2} = \frac{k(1-k)}{(1-k)^2(1+\sigma_\theta^2)+k^2}$ is equal to $\frac{1}{1+\sigma_\theta^2} > 0$ at $k = 0$, so that $\frac{d\hat{A}}{dk} > 0$ at $k = 0$. Thus, for $q_0 > 0$, we have $g'(0) = q_0 \frac{d\hat{A}}{dk} > 0$ at $k = 0$. In sum, at $k = 0$ we have:

$$f'(0) + g'(0) = 0 + q_0 \frac{d\hat{A}}{dk} > 0 \tag{A.19}$$

Since the objective function in equation (A.16) is continuous on $[0,1]$ and its derivative at $k = 0$ is strictly positive, the maximum cannot be at $k = 0$, i.e. the optimal rating weight is strictly positive.

G Competition with high uncertainty

We extend section 4.3 to $\sigma_\theta \in (\sigma_\theta^d, \sigma_\theta^*)$, where $\sigma_\theta^* = 2\sqrt{2+\sqrt{5}}$ is the threshold from Lemma 1(i). The argument for the case $\sigma_\theta \geq \sigma_\theta^*$ is similar, as sketched in footnote 19 below.

By Lemma 1(ii), two weights $k_d^{LT,-} < k^* < k_d^{LT,+}$ both induce A^{LT}, so $V(k_d^{LT,-}) = V(k_d^{LT,+}) \equiv V(A^{LT})$, the global maximum of V. Since $S(k^*) > 1$ for $\sigma_\theta > \sigma_\theta^d$ (where $S(k)$ is the public rating coefficient defined in the proof of Lemma 1), effort is higher than A^{LT} at k^*, and $V(k^*) < V(A^{LT})$. With two ratings, the coefficient $S(k_1)/2$ is lower than 1 for $k_1 \in [0,1]$ for $\sigma_\theta < \sigma_\theta^*$, so V^T remains single-peaked at k^* by the same argument as in the proof of Proposition 8.

The single-public-rating problem is therefore bimodal at k_d^{LT} while V^T is unchanged. Lemma 4 (Δ strictly increasing in k_1) selects $k_d^{LT,-}$ over $k_d^{LT,+}$ whenever there is any entry threat. There are two cases: (a) if $K \geq \Delta(k_d^{LT,-}, \sigma_u)$, entry is unprofitable when the incumbent sets $k_1 = k_d^{LT,-}$ so entry is blockaded; (b) if $K < \Delta(k_d^{LT,-}, \sigma_u)$, the incumbent either distorts the rating weight to $\hat{k}_1 < k_d^{LT,-}$ to deter entry as in section 4.3, accommodates entry at $k_1 = k^*$, or sells a private rating with $k_1 = 0$ to the speculator, depending on whether $V(\hat{k}_1)$, $V^T(k^*)$, or Π^P is largest. In sum:

Corollary 4 *For $\sigma_\theta \in (\sigma_\theta^d, \sigma_\theta^*)$, the regimes of Proposition 8 apply with $V(k^*)$ replaced by $V(A^{LT})$ in every comparison involving the single-public-rating payoff, and with k^* replaced by $k_d^{LT,-}$ as the incumbent's optimal weight for a public rating. In the accommodation regime, we still have $k_1 = k^*$ with payoff $V^T(k^*)$. The blockaded region expands and the deterrence region weakly shrinks: $K \geq \Delta(k_d^{LT,-}, \sigma_u)$ is now enough for entry to be blockaded at $k_1 = k_d^{LT,-}$.*

Thus, there is a complementarity between two important results of the paper: when fundamental uncertainty is high enough for time horizon irrelevance, the incumbent's two effort-equivalent designs k_d^{LT} provide a free instrument to manage entry, which weakens the motivation-deterrence trade-off described in section 4.3.[19]

H Credit rating application

This appendix adapts the baseline model of the paper to a credit rating setting with binary firm types and binary signals, building on the framework of Bolton, Freixas, and Shapiro (2012, henceforth BFS). We show how the information aggregation mechanism of the main model applies when an insider cares about the price at which the firm's debt is refinanced, and a credit rating agency (CRA) produces ratings that affect refinancing terms. Specifically, effort to improve creditworthiness is only reflected in the refinancing price when the credit rating incorporates information about effort.

H.1 Model setup

A firm has issued debt with face value F, which must be refinanced at $t = 1$. The firm's credit type is $\theta \in \{\theta_H, \theta_L\}$ such that $\theta_H > \theta_L > 0$, with $\Pr(\tilde{\theta} = \theta_H) = \frac{1}{2}$. We denote $\bar{\theta} \equiv \frac{1}{2}(\theta_H + \theta_L)$ and $\Delta\theta \equiv \theta_H - \theta_L > 0$.

[19] For $\sigma_\theta \geq \sigma_\theta^*$, V^T also becomes bimodal, peaking at the private-rating roots k^{LT} from Lemma 1(i), and the same selection logic applies: in the accommodation regime, the incumbent switches from k^* to $k^{LT,-}$, which induces A^{LT} with entry. The blockaded regime continues to induce A^{LT} at $k_d^{LT,-}$ (the relevant root differs because, in the accommodated regime, the price impact of the public rating is halved by the speculator's strategic trading on her private rating).

The probability that the firm does not default is $e+\theta$, where $e \geq 0$ is the insider's effort (denoted A in the main model). With binary types, the no-default probability is well-defined and lies in $(0,1)$ provided $e+\theta_H \leq 1$ and $e+\theta_L \geq 0$, which we assume.[20]

Effort e represents managerial actions that improve creditworthiness, such as improving operations, managing cash flows, reducing risk exposures, or maintaining asset quality. The firm type θ captures exogenous factors that affect default risk, including industry conditions and the firm's competitive position.

At $t=1$, the firm refinances the debt: it issues new debt with face value F to replace the maturing debt. The refinancing price, i.e. the amount investors are willing to pay for the new debt, depends on the market's assessment of the firm's creditworthiness, which in turn depends on the credit rating. The new debt pays F if the firm does not default and 0 otherwise. The value of the debt conditional on type θ is:

$$V(\theta) \quad = \quad (e+\theta)\,F \tag{A.20}$$

We denote $V_H \equiv (e+\theta_H)F$ and $V_L \equiv (e+\theta_L)F$, with $\Delta V \equiv V_H - V_L = (\Delta\theta)\,F$.

An insider holds a fraction $1-\delta$ of the firm's equity. Effort is privately costly, with a monetary cost $C(e)$ satisfying $C' > 0$, $C'' > 0$, $C(0)=0$, and $C'(0)=0$, as in the main model. To ensure existence of a solution, we also assume $\lim_{e\to\infty} C'(e) = \infty$.

The insider's payoff depends on the price p at which the firm's debt is refinanced at $t=1$. A higher refinancing price reduces the firm's borrowing costs, which makes equity holders better off. Specifically, the insider's expected payoff is $U = (1-\delta)\,p - C(e)$, where p is the refinancing price determined by the market at $t=1$.[21]

The insider's effort incentives emanate from the refinancing channel. Effort improves the expected value of the firm's debt, but this improvement is only reflected in the refinancing price if the credit rating incorporates information about effort. For example, with a purely informational credit rating that only depends on the exogenous type θ, the refinancing price does not reflect the insider's effort choice.

A credit rating agency (CRA) observes a binary signal $s \in \{G, B\}$ about the firm at $t=1$, similarly to BFS. The rating design choice of the CRA boils down to choosing parameter $k \in [0,1]$ of its information acquisition technology, which determines how its signal depends

[20] A sufficient condition is $e \in [0, 1-\theta_H]$, which is guaranteed when $\theta_H < 1$ and the cost of effort is sufficiently steep.

[21] This payoff can be microfounded as follows. When the firm refinances debt with face value F at price $p \leq F$, the difference $F-p$ represents an additional cost borne by equity holders. The insider, holding a fraction $1-\delta$ of equity, bears $(1-\delta)(F-p)$ of this cost. Since F is fixed, maximizing the insider's payoff is equivalent to maximizing $(1-\delta)p - C(e)$.

on the insider's effort e and the firm type θ. Specifically, the signal probabilities are:

$$\begin{aligned} \Pr(\tilde{s}=G|\theta_H,e) &\equiv \gamma_H(k)=(1-k)\gamma+k\,e \\ \Pr(\tilde{s}=G|\theta_L,e) &\equiv \gamma_L(k)=(1-k)(1-\gamma)+k\,e \end{aligned}$$

where the exogenous parameter $\gamma \in (\frac{1}{2},1)$ is the quality of the CRA's information about the firm type θ, and k is the endogenously determined weight on effort.[22]

The CRA can commit to truthfully report its signal, and it publicly discloses a credit report or "rating" $m \in \{G,B\}$. The aggregation weight k plays the same role as in the main model. When $k=0$, the signal depends only on θ with precision γ, i.e. $\Pr(\tilde{s}=G|\theta_H)=\gamma$ and $\Pr(\tilde{s}=G|\theta_L)=1-\gamma$; this is a purely informational rating, identical to the CRA signal in BFS. When $k=1$, the signal depends only on effort, with $\Pr(\tilde{s}=G)=e$ regardless of θ; this is a purely motivational rating. When $k\in(0,1)$, the signal depends on both effort and type; this is a mixed rating, analogous to the rating $R=kS_1+(1-k)S_2$ in the main model.

We define the unconditional probability of a G signal as: $\Sigma \equiv \gamma_H+\gamma_L=(1-k)+2ke$, so that $\Pr(\tilde{s}=G)=\Sigma/2$ and $\Pr(\tilde{s}=B)=(2-\Sigma)/2$.

The informativeness of the rating is:

$$\gamma_H-\gamma_L \;=\; (1-k)(2\gamma-1) \tag{A.21}$$

which is decreasing in k: a higher weight on effort reduces the rating's informativeness about the firm type. The "effort sensitivity" of the rating is:

$$\frac{\partial \Pr(\tilde{s}=G|\theta,e)}{\partial e}=k \tag{A.22}$$

The market for the firm's debt at $t=1$, where the refinancing price is determined, operates in the spirit of Kyle (1985), as in the main model. It involves three types of agents: noise traders, who submit a random market order $\tilde{u}$, normally distributed with mean 0 and variance σ_u^2, independent of $\tilde{\theta}$; a risk-neutral speculator, who observes the credit rating m (privately or publicly, depending on the business model) and submits a market order $q(m)$; a competitive, risk-neutral market maker who observes the total order flow and (depending on the business model) possibly the public rating, and who sets the refinancing price of the debt equal to its expected value given his information.

[22] In BFS, e denotes the CRA's signal precision. We use γ for signal precision to avoid confusion with the insider's effort e. We assume parameters are such that $\gamma_H,\gamma_L\in[0,1]$.

H.2 Equilibrium refinancing prices

We first characterize the equilibrium when the credit rating m is publicly disclosed, which is the standard business model for major CRAs. In this case, the speculator has no informational advantage, so she does not trade. The market maker sets the refinancing price equal to the expected value of debt given the public rating.

Lemma 6 (belief updating) *Bayesian updating gives:*

$$\Pr(\theta_H|m=G)=\frac{\gamma_H}{\Sigma},\qquad \Pr(\theta_H|m=B)=\frac{1-\gamma_H}{2-\Sigma} \tag{A.23}$$

The posterior expected type conditional on the rating is:

$$\mathbb{E}[\tilde{\theta}|m=G] = \frac{\gamma_H\theta_H+\gamma_L\theta_L}{\Sigma} \tag{A.24}$$

$$\mathbb{E}[\tilde{\theta}|m=B] = \frac{(1-\gamma_H)\theta_H+(1-\gamma_L)\theta_L}{2-\Sigma} \tag{A.25}$$

The equilibrium refinancing prices under a public rating are:

$$p(G) = \left(\hat{e}+\mathbb{E}[\tilde{\theta}|m=G]\right)F=\left(\hat{e}+\frac{\gamma_H\theta_H+\gamma_L\theta_L}{\Sigma}\right)F \tag{A.26}$$

$$p(B) = \left(\hat{e}+\mathbb{E}[\tilde{\theta}|m=B]\right)F=\left(\hat{e}+\frac{(1-\gamma_H)\theta_H+(1-\gamma_L)\theta_L}{2-\Sigma}\right)F \tag{A.27}$$

where $\hat{e}$ denotes the market maker's belief about the insider's effort.

Lemma 7 (price spread) *The refinancing price spread between a "G" and a "B" rating is:*

$$p(G)-p(B) = \frac{(1-k)(2\gamma-1)(\Delta\theta)F}{\Sigma(2-\Sigma)} \tag{A.28}$$

The price spread is increasing in the rating's informativeness $(1-k)(2\gamma-1)$, the type spread $\Delta\theta$, and the face value F. It is decreasing in the aggregation weight k: a more motivational rating leads to a smaller price difference between G and B reports, because the rating is less informative about the firm type.

Lemma 8 (refinancing terms, public rating) *When the rating is publicly disclosed and the insider's actual effort is e (while the market anticipates $\hat{e}$), the expected refinancing price is:*

$$\mathbb{E}\left[p|e,\hat{e}\right]=(\hat{e}+\bar{\theta})F+\frac{k(1-k)(2\gamma-1)(\Delta\theta)F}{\Sigma(2-\Sigma)}(e-\hat{e}) \tag{A.29}$$

The coefficient $\frac{k(1-k)(2\gamma-1)(\Delta\theta)F}{\Sigma(2-\Sigma)}$ is the sensitivity of the expected refinancing price to the insider's actual effort. It is the product of the effort sensitivity of the rating (k), the informativeness of the rating ($(1-k)(2\gamma-1)$), the type spread ($\Delta\theta$), and the face value (F), scaled by $\Sigma(2-\Sigma)$. This coefficient is the binary analogue of $\frac{k(1-k)\sigma_\theta^2}{\sigma_R^2}$ in the main model. When $k=0$, this coefficient is zero: the refinancing price does not depend on effort when the rating is purely informational.

We now characterize the equilibrium when the rating is privately communicated to the speculator, as in a model in the spirit of Kyle (1985) with binary private information. The speculator observes $m \in \{G, B\}$ and submits a market order. The market maker observes the total order flow $y = q + u$ (where $u \sim \mathcal{N}(0, \sigma_u^2)$) and sets the refinancing price.

Lemma 9 (private rating equilibrium) *When the credit rating m is privately observed by the speculator, and with $\Sigma \equiv 1 + k(2\hat{e}-1)$ evaluated at the equilibrium effort $\hat{e}$, the linear equilibrium is characterized by:*
(i) Speculator's strategy: $q_G = \sqrt{(2-\Sigma)/\Sigma}\,\sigma_u$ when $m = G$ and $q_B = -\sqrt{\Sigma/(2-\Sigma)}\,\sigma_u$ when $m = B$;
(ii) Market maker's pricing rule: $p(y) = (\hat{e}+\bar{\theta})F + \lambda\, y$, with:

$$\lambda \;=\; \frac{\sqrt{\Sigma(2-\Sigma)}}{4\sigma_u}(V_G - V_B) \;=\; \frac{(1-k)(2\gamma-1)(\Delta\theta)F}{4\sigma_u\sqrt{\Sigma(2-\Sigma)}}, \tag{A.30}$$

where $V_G - V_B = p(G) - p(B)$ as in Lemma 7;
(iii) Speculator's expected profit:

$$\mathbb{E}[\Pi] \;=\; \lambda\sigma_u^2 \;=\; \frac{(1-k)(2\gamma-1)(\Delta\theta)F\,\sigma_u}{4\sqrt{\Sigma(2-\Sigma)}}; \tag{A.31}$$

(iv) Expected refinancing price as a function of effort:

$$\mathbb{E}[p|e,\hat{e}] = (\hat{e}+\bar{\theta})F + \frac{k(1-k)(2\gamma-1)(\Delta\theta)F}{2\,\Sigma(2-\Sigma)}(e-\hat{e}). \tag{A.32}$$

As in the main model, the sensitivity of the expected refinancing price to effort is twice as large when the rating is public as opposed to private, because of the speculator's strategic trading.

H.3 Effort incentives and rating design

If the refinancing market could directly observe the insider's effort, the refinancing price would be $(e+\theta)F$ conditional on type θ, giving an expected price of $(e+\bar{\theta})F$. The insider would then solve:

$$\max_{e} \, (1-\delta)(e+\bar{\theta})F - C(e), \tag{A.33}$$

which would give $C'(e^{FI}) = (1-\delta)F$. We call e^{FI} the full-information effort level, i.e. the level of effort that the insider would like to commit to. It represents the effort the insider would choose if the refinancing price fully reflected his effort. A higher face value F increases effort incentives by making the refinancing channel more important.

Lemma 10 (effort with public rating) *When the credit rating is publicly disclosed, the insider's equilibrium effort e is implicitly defined by:*

$$C'(e) \;=\; (1-\delta)\,\frac{k(1-k)(2\gamma-1)(\Delta\theta)F}{\Sigma(e)\big(2-\Sigma(e)\big)}, \tag{A.34}$$

In particular, when $k=0$, the RHS is zero and $e=0$.

Lemma 11 (effort with private rating) *When the rating is privately observed by the speculator, the insider's equilibrium effort e is implicitly defined by:*

$$C'(e) \;=\; (1-\delta)\,\frac{k(1-k)(2\gamma-1)(\Delta\theta)F}{2\,\Sigma(e)\big(2-\Sigma(e)\big)} \tag{A.35}$$

In particular, when $k=0$, the RHS is zero and $e=0$.

Unlike in the Gaussian model of the main model, where the effort coefficient $k(1-k)\frac{\sigma_\theta^2}{\sigma_R^2}$ depends on k alone, the coefficient on the RHS of (A.34)-(A.35) depends on equilibrium effort e through $\Sigma(e)$. Equations (A.34) and (A.35) are therefore fixed-point equations rather than closed-form expressions for e. A solution exists by the intermediate value theorem: at $e=0$, the RHS is strictly positive (for $k \in (0,1)$) while $C'(0)=0$; at the upper bound $e = 1-\theta_H$, the assumption that effort cost is sufficiently steep (so that $e^{FI} < 1-\theta_H$) ensures that $C'(1-\theta_H)$ is higher than the bounded RHS, so the two continuous functions must cross in $(0, 1-\theta_H)$. The solution is unique if:

$$C''(e) \;>\; (1-\delta)\,\frac{\partial}{\partial e}\left[\frac{k(1-k)(2\gamma-1)(\Delta\theta)F}{\Sigma(e)(2-\Sigma(e))}\right] \tag{A.36}$$

at every fixed point, which is satisfied with a sufficiently convex effort cost.

Comparing equations (A.34) and (A.35), implicit effort incentives are twice as high under the public credit rating, as in the main model. Equations (A.34) and (A.35) are the binary analogues of the effort equations in Lemma 1.

A purely informational rating ($k = 0$) depends only on the exogenous type θ and conveys no information about effort, so that a change in effort does not affect the refinancing price. With no marginal return to effort through the refinancing channel, the insider optimally exerts zero effort. This leads to the worst possible refinancing terms: the debt is refinanced at price $\bar{\theta}F$, reflecting only the prior expected type and zero effort.[23] By contrast, when $k > 0$, the rating is sensitive to effort, which affects the refinancing price, thereby generating positive effort incentives and better refinancing terms.

Next, we establish under which condition the refinancing channel provides high effort incentives.

Proposition 9 (full incentive provision) *The insider achieves the full-information effort level e^{FI} if and only if:*

$$\frac{k(1-k)(2\gamma-1)(\Delta\theta)}{\Sigma(2-\Sigma)} = 1 \quad (\textit{public rating}) \tag{A.37}$$

$$\frac{k(1-k)(2\gamma-1)(\Delta\theta)}{2\Sigma(2-\Sigma)} = 1 \quad (\textit{private rating}) \tag{A.38}$$

Each of these conditions can be satisfied by setting the weight k optimally when $(2\gamma - 1)\Delta\theta$ is sufficiently large (i.e., the precision of the CRA information γ and uncertainty about the firm type $\Delta\theta$ are sufficiently high).

Condition (A.37) is the binary analogue of the condition $k(1-k)\frac{\sigma_\theta^2}{\sigma_R^2} = 1$ in the main model. The type spread $\Delta\theta$ plays the same role as σ_θ: when $\Delta\theta$ is large, the credit rating is highly informative, so that refinancing prices are highly sensitive to the rating (when $k < 1$), which results in strong implicit effort incentives (when $k > 0$).

Propositions 10 and 11 study optimal rating design.

Proposition 10 (investor-paid credit rating) *When the CRA sells the credit rating to the speculator, it designs a purely informational rating about the firm's credit type θ by setting $k = 0$.*

An investor-paid credit rating focuses entirely on the exogenous component of default risk (θ). Intuitively, the speculator's trading profit depends on her informational advantage about θ, not about effort, which is correctly anticipated in equilibrium. With $k = 0$, the rating does not provide any implicit effort incentives: from equation (A.34), the insider exerts zero effort, and the debt is refinanced at price $\bar{\theta}F$, which is the worst possible refinancing terms.

[23] This sharp outcome of zero effort when $k = 0$ is specific to the credit rating setting, where the insider is only concerned with the refinancing price. In the baseline model of the main text, the insider also receives a direct payoff proportional to $(1 - \pi)$ from the long-horizon channel. As a result, a rating with $k = 0$ still yields positive effort as long as as $\pi < 1$.

Proposition 11 (issuer-paid credit rating) *When the insider pays the CRA to produce a public credit rating, the CRA optimally sets $k > 0$. Moreover, if there exists $k \in (0,1)$ that satisfies equation (A.37), then the CRA sets k to achieve the full-information effort level e^{FI}. Otherwise, the CRA sets k to maximize effort e, which is the value k^* that maximizes:*

$$\frac{k(1-k)(2\gamma-1)(\Delta\theta)}{\Sigma(2-\Sigma)}$$

This result implies that issuer-paid CRAs (including Moody's, S&P, and Fitch) create value not only through information provision but also through their motivational effect on firm behavior. The optimal credit rating aggregates signals about managerial actions (the effort dimension, with weight $k > 0$) with signals about the firm's fundamental creditworthiness (the type dimension, with weight $1-k$). This type of rating provides implicit incentives that purely informational investor-paid ratings ($k = 0$) cannot replicate: the latter lead to zero effort and the worst refinancing terms.

H.4 Proofs

Proof of Lemma 6: By Bayes' rule: $\Pr(\theta_H|m = G) = \frac{\frac{1}{2}\Pr(\tilde{m}=G|\theta_H)}{\Pr(\tilde{m}=G)} = \frac{\gamma_H/2}{\Sigma/2} = \frac{\gamma_H}{\Sigma}$. The second expression follows from the same reasoning.

Proof of Lemma 7: We have:

$$\begin{aligned}
p(G) - p(B) &= F\left(\mathbb{E}[\theta|m=G] - \mathbb{E}[\theta|m=B]\right) \\
&= F\frac{(2-\Sigma)(\gamma_H\theta_H + \gamma_L\theta_L) - \Sigma((1-\gamma_H)\theta_H + (1-\gamma_L)\theta_L)}{\Sigma(2-\Sigma)} \\
&= F\frac{2(\gamma_H\theta_H + \gamma_L\theta_L) - \Sigma(\theta_H + \theta_L)}{\Sigma(2-\Sigma)}
\end{aligned}$$

The numerator simplifies to:

$$\begin{aligned}
2\gamma_H\theta_H + 2\gamma_L\theta_L - (\gamma_H + \gamma_L)(\theta_H + \theta_L) &= (\gamma_H - \gamma_L)(\theta_H - \theta_L) \\
&= (1-k)(2\gamma-1)(\Delta\theta)
\end{aligned}$$

where we used equation (A.21).

Proof of Lemma 8: The expected price conditional on actual effort e is:

$$\begin{aligned}
\mathbb{E}[p|e,\hat{e}] &= \Pr(\tilde{m} = G|e)\cdot p(G) + \Pr(\tilde{m} = B|e)\cdot p(B) \\
&= p(B) + \Pr(m = G|e)(p(G) - p(B))
\end{aligned}$$

In equilibrium ($e = \hat{e}$): $\mathbb{E}[p|\hat{e}, \hat{e}] = (\hat{e} + \bar{\theta})F$. For a deviation $e \neq \hat{e}$:

$$\mathbb{E}[p|e, \hat{e}] - \mathbb{E}[p|\hat{e}, \hat{e}] \;=\; (\Pr(\tilde{m} = G|e) - \Pr(\tilde{m} = G|\hat{e}))\,(p(G) - p(B))$$

Since $\Pr(\tilde{m} = G|e) = \frac{1}{2}(\gamma_H(e) + \gamma_L(e)) = \frac{1}{2}((1-k) + 2ke)$, we have:

$$\Pr(\tilde{m} = G|e) - \Pr(\tilde{m} = G|\hat{e}) = k(e - \hat{e})$$

Substituting from Lemma 7:

$$\mathbb{E}[p|e, \hat{e}] - (\hat{e} + \bar{\theta})F = \frac{k(1-k)(2\gamma - 1)(\Delta\theta)F}{\Sigma(2 - \Sigma)}(e - \hat{e})$$

Proof of Lemma 9: The speculator observes $m \in \{G, B\}$ with probabilities $\Pr(\tilde{m} = G) = \Sigma/2$ and $\Pr(\tilde{m} = B) = (2-\Sigma)/2$, and conditional expected debt values $V_G = (\hat{e}+\mathbb{E}[\tilde{\theta}|m = G])F$ and $V_B = (\hat{e} + \mathbb{E}[\theta|m = B])F$. By Lemma 7, $V_G - V_B = (1-k)(2\gamma - 1)(\Delta\theta)F/(\Sigma(2 - \Sigma))$.

Conjecture a linear equilibrium $p(y) = \mu + \lambda y$ with $\mu = (\hat{e} + \bar{\theta})F$. The speculator's FOC gives $q_m = (\mathbb{E}[V|m] - \mu)/(2\lambda)$, so $q_G - q_B = (V_G - V_B)/(2\lambda)$. Setting $p = \Sigma/2$, the market maker's zero-profit condition $\lambda = \mathrm{Cov}(V, y)/\mathrm{Var}(y)$ becomes

$$\lambda = \frac{p(1-p)(V_G - V_B)(q_G - q_B)}{p(1-p)(q_G - q_B)^2 + \sigma_u^2}.$$

Substituting $q_G - q_B = (V_G - V_B)/(2\lambda)$ and solving yields $(q_G - q_B)^2 = \sigma_u^2/(p(1-p)) = 4\sigma_u^2/(\Sigma(2 - \Sigma))$ so that:

$$\lambda = \frac{(V_G - V_B)\sqrt{\Sigma(2 - \Sigma)}}{4\sigma_u}.$$

Using $V_G - \mu = (1-p)(V_G - V_B)$ and $V_B - \mu = -p(V_G - V_B)$ in the FOC gives $q_G = \sqrt{(2 - \Sigma)/\Sigma}\,\sigma_u$ and $q_B = -\sqrt{\Sigma/(2 - \Sigma)}\,\sigma_u$, establishing (i). Substituting in λ gives (A.30). As in the standard Kyle model, $\mathbb{E}[\Pi] = \lambda\sigma_u^2$, giving (A.31).

For part (iv), note that in any linear equilibrium, $\lambda(q_G - q_B) = (V_G - V_B)/2$ regardless of the specific values of λ, q_G, and q_B. Since a deviation $e \neq \hat{e}$ shifts $\Pr(\tilde{m} = G|e)$ by $k(e - \hat{e})$, the expected order flow shifts by $k(e - \hat{e})(q_G - q_B)$, so

$$\mathbb{E}[p|e, \hat{e}] - \mathbb{E}[p|\hat{e}, \hat{e}] \;=\; \lambda k(e - \hat{e})(q_G - q_B) \;=\; \frac{k(V_G - V_B)}{2}(e - \hat{e}).$$

Substituting $V_G - V_B$ from Lemma 7 gives equation (A.32). Part (iv) holds in any linear equilibrium (symmetric or asymmetric): it relies only on the identity $\lambda(q_G - q_B) = (V_G - V_B)/2$, which follows from the speculator's FOC independently of Σ.

Proof of Lemma 10: The insider maximizes:

$$\max_e \,(1-\delta)\,\mathbb{E}[p|e,\hat{e}] - C(e)$$

Substituting (A.29) and taking the FOC with respect to e, evaluated at $e=\hat{e}$ in equilibrium, gives (A.34). When $k=0$, the RHS is zero, and $e=0$ follows from $C'(0)=0$.

Proof of Proposition 9: Setting $C'(e)=(1-\delta)F$ in (A.34) and evaluating Σ at $e=e^{FI}$ gives the requirement $\frac{k(1-k)(2\gamma-1)(\Delta\theta)}{\Sigma(e^{FI})(2-\Sigma(e^{FI}))}=1$, which is (A.37). F does not appear explicitly in (A.37) but it enters implicitly through e^{FI} (defined by $C'(e^{FI})=(1-\delta)F$) and thus through $\Sigma(e^{FI})=1+k(2e^{FI}-1)$. The same argument applies to (A.35), which gives (A.38).

Proof of Proposition 10: From (A.31), the speculator's expected profit, evaluated at the equilibrium effort $\hat{e}(k)$ induced by the rating weight k, is

$$\mathbb{E}[\Pi(k)] \;=\; \frac{(1-k)(2\gamma-1)(\Delta\theta)F\,\sigma_u}{4\sqrt{1-k^2(2\hat{e}(k)-1)^2}}.$$

At $k=0$, equation (A.35) implies $\hat{e}(0)=0$, so $\Sigma=1$ and $\mathbb{E}[\Pi(0)]=(2\gamma-1)(\Delta\theta)F\sigma_u/4$.

We will show that $\mathbb{E}[\Pi(k)]\leq\mathbb{E}[\Pi(0)]$ for all $k\in[0,1]$, with strict inequality for $k\in(0,1)$. Because both sides are positive, this is equivalent to:

$$(1-k)^2 \;\leq\; 1-k^2(2\hat{e}(k)-1)^2 \;\;\Leftrightarrow\;\; 2k-k^2 \;\geq\; k^2(2\hat{e}(k)-1)^2 \;\;\Leftrightarrow\;\; 2-k \;\geq\; k(2\hat{e}(k)-1)^2,$$

where the last step divides by $k>0$. Since $\hat{e}(k)\in[0,1]$ implies $(2\hat{e}(k)-1)^2\leq 1$ and $k\in(0,1]$ implies $2-k\geq 1\geq k\geq k(2\hat{e}(k)-1)^2$, the inequality holds, and is strict whenever $k\in(0,1)$ and $\hat{e}(k)\in(0,1)$. Therefore $k=0$ maximizes the speculator's expected profit, and the investor-paid CRA optimally sets $k=0$.

Proof of Proposition 11: The insider's equilibrium payoff is $(1-\delta)(\hat{e}+\bar{\theta})F-C(\hat{e})$. His willingness to pay for the rating, relative to the baseline of no rating (where $\hat{e}=0$ and the refinancing price is $\bar{\theta}F$), is $\frac{1}{1-\delta}\left((1-\delta)\hat{e}F-C(\hat{e})\right)$, which is increasing in equilibrium effort $\hat{e}$ up to the full-information level e^{FI}. The CRA maximizes the insider's willingness to pay by choosing k to maximize effort.

I Additional proofs

Proof of Proposition 7: We derive the Kyle equilibrium when a public rating $R_1=k_1S_1+(1-k_1)S_2$ and a purely informational private rating $R_2=S_2$ (i.e. $k_2=0$) coexist. This is the only two-rating configuration that arises in equilibrium: by the Corollary to Lemma 3, two ratings with $k_1 \neq k_2$ reveal both S_1 and S_2 to the speculator; since $A = \hat{A}$ is known in equilibrium, the signal S_1 has zero value for trading, so a second private rating with $k_2 \neq 0$ earns zero profit and is never chosen. We derive the Kyle equilibrium when a public rating R_1 and a private rating R_2 coexist. We derive all coefficients (α, β, γ, μ, λ, ν).

The market maker observes the public rating $R_1 = k_1 S_1 + (1 - k_1) S_2$ and the total order flow $y = q + \tilde{u}$. The speculator observes both ratings:

$$\begin{aligned} R_1 &= k_1 S_1 + (1 - k_1) S_2 = k_1(A + \varepsilon_1) + (1 - k_1)(\theta + \varepsilon_2), \\ R_2 &= S_2 = \theta + \varepsilon_2. \end{aligned}$$

Since the speculator observes two independent linear combinations of S_1 and S_2, she can recover both signals: she behaves as if observing $S_2 = \theta + \varepsilon_2$ directly, regardless of k_2.

We conjecture linear strategies:

$$q(R_1, R_2) = \alpha + \beta(R_2 - \mathbb{E}[R_2]) + \gamma(R_1 - \mathbb{E}[R_1]), \tag{A.39}$$

$$p(y, R_1) = \mu + \lambda(y - \mathbb{E}[y]) + \nu(R_1 - \mathbb{E}[R_1]). \tag{A.40}$$

Assumptions: $\tilde{\theta}, \tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \tilde{u}$ are mutually independent with:

$$\mathrm{Var}(\theta) = \sigma_\theta^2, \quad \mathrm{Var}(\varepsilon_1) = \mathrm{Var}(\varepsilon_2) = 1, \quad \mathrm{Var}(u) = \sigma_u^2.$$

Key variance formulas:

$$\sigma_{R_1}^2 \equiv \mathrm{Var}(R_1) = k_1^2 + (1 - k_1)^2(1 + \sigma_\theta^2), \tag{A.41}$$

$$\sigma_{R_2}^2 \equiv \mathrm{Var}(R_2) = 1 + \sigma_\theta^2. \tag{A.42}$$

The speculator maximizes expected profit:

$$\max_q \mathbb{E}[(X - p(y, R_1))q | R_1, R_2].$$

Expanding:

$$\max_q \mathbb{E}[X | R_1, R_2] \cdot q - \mathbb{E}[p(y, R_1) | R_1, R_2] \cdot q.$$

Substituting the conjectured price function (A.40) with $y = q + u$:

$$\begin{aligned} \mathbb{E}[p(q + u, R_1) | R_1, R_2] &= \mu + \lambda \mathbb{E}[q + u | R_1, R_2] + \nu(R_1 - \mathbb{E}[R_1]) \\ &= \mu + \lambda q + \nu(R_1 - \mathbb{E}[R_1]). \end{aligned} \tag{A.43}$$

The FOC is:

$$\frac{\partial}{\partial q}\left\{\mathbb{E}[X|R_1,R_2]\cdot q - [\mu + \lambda q + \nu(R_1 - \mathbb{E}[R_1])]\cdot q\right\} = 0.$$

Taking the derivative:

$$\mathbb{E}[X|R_1,R_2] - \mu - 2\lambda q - \nu(R_1 - \mathbb{E}[R_1]) = 0.$$

Solving for q:

$$q = \frac{\mathbb{E}[X|R_1,R_2] - \mu - \nu(R_1 - \mathbb{E}[R_1])}{2\lambda}. \tag{A.44}$$

Since the speculator recovers S_2 from (R_1, R_2):

$$\begin{aligned}
\mathbb{E}[X|R_1,R_2] &= \mathbb{E}[A+\theta|S_2] \\
&= \hat{A} + \mathbb{E}[\theta|\theta+\varepsilon_2] \\
&= \hat{A} + \bar{\theta} + \frac{\mathrm{Cov}(\theta,\theta+\varepsilon_2)}{\mathrm{Var}(\theta+\varepsilon_2)}(\theta+\varepsilon_2-\bar{\theta}) \\
&= \hat{A} + \bar{\theta} + \frac{\sigma_\theta^2}{\sigma_\theta^2+1}(S_2-\bar{\theta}). \qquad (A.45)
\end{aligned}$$

Substituting (A.45) into (A.44):

$$q = \frac{\hat{A} + \bar{\theta} + \frac{\sigma_\theta^2}{1+\sigma_\theta^2}(S_2-\bar{\theta}) - \mu - \nu(R_1 - \mathbb{E}[R_1])}{2\lambda}.$$

Since $S_2 = R_2$ (recovered signal) and $\mathbb{E}[R_2] = \bar{\theta}$ (effort correctly anticipated):

$$q = \frac{\hat{A}+\bar{\theta}-\mu}{2\lambda} + \frac{\sigma_\theta^2}{2(1+\sigma_\theta^2)\lambda}(R_2 - \mathbb{E}[R_2]) - \frac{\nu}{2\lambda}(R_1 - \mathbb{E}[R_1]).$$

Matching coefficients with the conjectured form (A.39):

$$\alpha = \frac{\hat{A}+\bar{\theta}-\mu}{2\lambda}, \tag{A.46}$$

$$\beta = \frac{\sigma_\theta^2}{2(1+\sigma_\theta^2)\lambda}, \tag{A.47}$$

$$\gamma = -\frac{\nu}{2\lambda}. \tag{A.48}$$

The market maker sets price equal to expected firm value given available information:

$$p(y, R_1) = \mathbb{E}[X|y, R_1]. \tag{A.49}$$

With multivariate normal variables, the conditional expectation is:

$$\mathbb{E}[X|y, R_1] = \mathbb{E}[X] + \text{Cov}(X, [y, R_1])' \cdot \text{Var}([y, R_1])^{-1} \cdot \begin{pmatrix} y - \mathbb{E}[y] \\ R_1 - \mathbb{E}[R_1] \end{pmatrix}.$$

(i) We have:

$$\mathbb{E}[X] = \mathbb{E}[A + \theta] = \hat{A} + \bar{\theta}. \tag{A.50}$$

(ii) $\text{Cov}(X, R_1)$: Since $X = A + \theta = \hat{A} + \theta$ in equilibrium (A correctly anticipated):

$$\begin{aligned} \text{Cov}(X, R_1) &= \text{Cov}(\theta, k_1(A + \varepsilon_1) + (1 - k_1)(\theta + \varepsilon_2)) \\ &= \text{Cov}(\theta, (1 - k_1)\theta) \\ &= (1 - k_1)\sigma_\theta^2. \end{aligned} \tag{A.51}$$

(iii) Order flow: From (A.46), the order flow is:

$$\begin{aligned} y &= q + u \\ &= \alpha + \beta(R_2 - \mathbb{E}[R_2]) + \gamma(R_1 - \mathbb{E}[R_1]) + u \\ &= \beta(\theta + \varepsilon_2 - \bar{\theta}) + \gamma(k_1\varepsilon_1 + (1 - k_1)(\theta + \varepsilon_2) - (1 - k_1)\bar{\theta}) + u + \alpha \\ &= [\beta + \gamma(1 - k_1)](\theta - \bar{\theta}) + \beta\varepsilon_2 + \gamma k_1\varepsilon_1 + \gamma(1 - k_1)\varepsilon_2 + u + \alpha. \end{aligned} \tag{A.52}$$

(iv) $\text{Cov}(X, y)$: From (A.52):

$$\begin{aligned} \text{Cov}(X, y) &= \text{Cov}(\theta, y) \\ &= \text{Cov}(\theta, [\beta + \gamma(1 - k_1)](\theta - \bar{\theta})) \\ &= [\beta + \gamma(1 - k_1)]\sigma_\theta^2. \end{aligned} \tag{A.53}$$

(v) $\text{Var}(y)$: From (A.52) and independence:

$$\text{Var}(y) = [\beta + \gamma(1 - k_1)]^2(1 + \sigma_\theta^2) + \gamma^2 k_1^2 + \sigma_u^2. \tag{A.54}$$

(vi) $\text{Cov}(y, R_1)$: From (A.52):

$$\text{Cov}(y, R_1) = (1 - k_1)\beta(1 + \sigma_\theta^2) + \gamma\sigma_{R_1}^2. \tag{A.55}$$

From (A.50), the price coefficients are:

$$\lambda = \frac{1}{D}\left[\mathrm{Cov}(X,y)\cdot\sigma_{R_1}^2 - \mathrm{Cov}(X,R_1)\cdot\mathrm{Cov}(y,R_1)\right], \tag{A.56}$$

$$\nu = \frac{1}{D}\left[\mathrm{Cov}(X,R_1)\cdot\mathrm{Var}(y) - \mathrm{Cov}(X,y)\cdot\mathrm{Cov}(y,R_1)\right], \tag{A.57}$$

where

$$D = \mathrm{Var}(y)\cdot\sigma_{R_1}^2 - [\mathrm{Cov}(y,R_1)]^2. \tag{A.58}$$

Substituting covariances into (A.56):

$$\begin{aligned}\lambda &= \frac{\sigma_\theta^2}{D}\left\{[\beta+\gamma(1-k_1)]\sigma_{R_1}^2 - (1-k_1)[(1-k_1)\beta(1+\sigma_\theta^2)+\gamma\sigma_{R_1}^2]\right\}\\ &= \frac{\sigma_\theta^2}{D}\left\{\beta[\sigma_{R_1}^2-(1-k_1)^2(1+\sigma_\theta^2)]\right\}\\ &= \frac{\sigma_\theta^2\beta k_1^2}{D}.\end{aligned} \tag{A.59}$$

Similarly, from (A.57), after substantial algebra:

$$\nu = \frac{\sigma_\theta^2}{D}\left[-\gamma\beta k_1^2 + (1-k_1)\sigma_u^2\right]. \tag{A.60}$$

We conjecture (based on Kyle model structure):

$$\beta = \frac{\sigma_{R_1}\sigma_u}{k_1\sigma_{R_2}},$$

$$\lambda = \frac{\sigma_\theta^2 k_1}{2\sigma_u\sigma_{R_1}\sigma_{R_2}}.$$

Verifying $\beta = \frac{\sigma_\theta^2}{2(1+\sigma_\theta^2)\lambda}$ with $k_2 = 0$:

$$\begin{aligned}\mathrm{RHS} &= \frac{\sigma_\theta^2}{2(1+\sigma_\theta^2)}\frac{2\sigma_u\sigma_{R_1}\sigma_{R_2}}{\sigma_\theta^2 k_1}\\ &= \frac{\sigma_u\sigma_{R_1}\sigma_{R_2}}{(1+\sigma_\theta^2)k_1} = \frac{\sigma_u\sigma_{R_1}}{k_1\sigma_{R_2}} = \beta.\end{aligned}$$

From (A.59):

$$\lambda = \frac{\sigma_\theta^2\beta k_1^2}{D} \quad\Rightarrow\quad D = \frac{\sigma_\theta^2\beta k_1^2}{\lambda} = 2\sigma_{R_1}^2\sigma_u^2. \tag{A.61}$$

From (A.60) with $\gamma = -\frac{\nu}{2\lambda}$:

$$\nu = \frac{\sigma_\theta^2}{D}\left(\frac{\nu\beta k_1^2}{2\lambda} + (1-k_1)\sigma_u^2\right).$$

Rearranging:

$$\nu \left(1 - \frac{\sigma_\theta^2 \beta k_1^2}{2\lambda D}\right) = \frac{\sigma_\theta^2(1-k_1)\sigma_u^2}{D}.$$

Computing the bracketed term using $D = 2\sigma_{R_1}^2\sigma_u^2$:

$$\frac{\sigma_\theta^2 \beta k_1^2}{2\lambda D} = \frac{1}{2}.$$

Therefore:

$$\nu = \frac{2\sigma_\theta^2(1-k_1)\sigma_u^2}{D} = \frac{2\sigma_\theta^2(1-k_1)\sigma_u^2}{2\sigma_{R_1}^2\sigma_u^2} = \frac{\sigma_\theta^2(1-k_1)}{\sigma_{R_1}^2}. \tag{A.62}$$

From (A.48):

$$\gamma = -\frac{\nu}{2\lambda} = -\frac{\sigma_\theta^2(1-k_1)/\sigma_{R_1}^2}{2\sigma_\theta^2 k_1/(2\sigma_u\sigma_{R_1}\sigma_{R_2})} \tag{A.63}$$

$$= -\frac{(1-k_1)\sigma_u\sigma_{R_2}}{k_1\sigma_{R_1}}. \tag{A.64}$$

The expected price conditional on effort is:

$$\begin{aligned}\mathbb{E}[p|A,\hat{A}] &= \mu + \lambda\mathbb{E}[q|A,\hat{A}] + \nu\mathbb{E}[R_1|A,\hat{A}] \\ &= \hat{A} + \bar{\theta} + (\lambda\gamma + \nu)k_1(A - \hat{A}).\end{aligned}$$

Therefore:

$$\lambda\gamma + \nu = -\frac{\sigma_\theta^2(1-k_1)}{2\sigma_{R_1}^2} + \frac{\sigma_\theta^2(1-k_1)}{\sigma_{R_1}^2} = \frac{\sigma_\theta^2(1-k_1)}{2\sigma_{R_1}^2}.$$

So:

$$\mathbb{E}[p|A,\hat{A}] = \hat{A} + \bar{\theta} + \frac{\sigma_\theta^2(1-k_1)k_1}{2\sigma_{R_1}^2}(A - \hat{A}).$$

For the speculator's expected profit:

$$\mathbb{E}[\Pi] = \lambda\sigma_u^2 = \frac{\sigma_\theta^2 k_1}{2\sigma_u\sigma_{R_1}\sigma_{R_2}}\sigma_u^2 = \frac{k_1\sigma_\theta^2\sigma_u}{2\sigma_{R_1}\sigma_{R_2}}.$$

This completes the proof.

Proof of Lemma 4: We need to show that $\frac{\partial}{\partial k_1}\left\{\frac{k_1}{\sigma_{R_1}}\right\} > 0$. Let $f(k_1) = \frac{k_1}{\sigma_{R_1}}$ where

$\sigma_{R_1} = \sqrt{k_1^2 + (1-k_1)^2(1+\sigma_\theta^2)}$. Taking the derivative:

$$f'(k_1) = \frac{\sigma_{R_1} - k_1 \frac{1}{2\sigma_{R_1}} \frac{\partial \sigma_{R_1}^2}{\partial k_1}}{\sigma_{R_1}^2} = \frac{\sigma_{R_1}^2 - k_1(k_1 - (1-k_1)(1+\sigma_\theta^2))}{\sigma_{R_1}^3}.$$

The numerator simplifies to:

$$\begin{aligned}
&k_1^2 + (1-k_1)^2(1+\sigma_\theta^2) - k_1^2 + k_1(1-k_1)(1+\sigma_\theta^2) \\
&= (1-k_1)(1+\sigma_\theta^2) > 0.
\end{aligned}$$

Therefore $f'(k_1) > 0$ for all $k_1 \in (0,1)$.

Proof of Corollary 2: The socially responsible speculator maximizes expected wealth plus social impact. Since social impact equals the insider's effort A, the speculator's ex-ante expected utility from observing a private rating R and trading is:

$$\mathbb{E}\left[(\tilde{X} - p(\tilde{y}))q \middle| R\right] + A$$

The social impact term A does not depend on the speculator's trade q. Therefore, the speculator's optimal trading strategy $q^*(R)$ and the market maker's pricing rule $p(y) = \mu + \lambda y$ are identical to those in Proposition 1. In particular, the speculator's expected trading profit is $f(k) \equiv \frac{\sigma_u(1-k)\sigma_\theta^2}{2\sigma_R}$, the expected stock price satisfies $\mathbb{E}[p|A,\hat{A}] = \hat{A} + \bar{\theta} + \frac{(1-k)k\sigma_\theta^2}{2\sigma_R^2}(A - \hat{A})$, and equilibrium effort $\hat{A}(k)$ is determined by the insider's FOC as in Lemma 1(i).

The speculator's total expected utility, evaluated at the equilibrium effort $\hat{A}(k)$, is therefore:

$$\frac{\sigma_u(1-k)\sigma_\theta^2}{2\sigma_R} + \hat{A}(k)$$

Denote the first term by $f(x)$ and the second by $g(k)$. We show that the derivative of $f(k) + g(k)$ is strictly positive at $k = 0$, so the optimum is at some $k > 0$.

For f, the derivative is:

$$f'(k) = -\frac{\sigma_u \sigma_\theta^2}{2} \frac{k}{\left(k^2 + (1-k)^2(1+\sigma_\theta^2)\right)^{3/2}}$$

which satisfies $f'(0) = 0$.

For $g(k) = \hat{A}(k)$, the implicit function theorem applied to the insider's FOC $C'(\hat{A}) = (1-\delta)\left(1 - \pi + \pi \frac{k(1-k)\sigma_\theta^2}{2\sigma_R^2}\right)$ gives:

$$g'(k) = \frac{d\hat{A}}{dk} = \frac{(1-\delta)\pi}{C''(\hat{A})} \frac{d}{dk}\left[\frac{k(1-k)\sigma_\theta^2}{2\sigma_R^2}\right]$$

At $k = 0$, we have $\frac{d}{dk}\left\{\frac{k(1-k)}{\sigma_R^2}\right\} = \frac{1}{1+\sigma_\theta^2}$ at $k = 0$, which is strictly positive, so $g'(0) > 0$.

Combining these two intermediary results:

$$f'(0) + g'(0) > 0.$$

Since $f + g$ is continuous on $[0, 1]$ and has a strictly positive derivative at $k = 0$, the maximum is not at $k = 0$. Therefore, the rating that maximizes the socially responsible speculator's willingness to pay involves a strictly positive weight k.

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