# When Is (Performance-Sensitive) Debt Optimal?

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## Abstract

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Keywords: informativeness principle, limited liability, performance-sensitive debt

JEL Classifications: D86, G32, G34, J33

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**Abstract**

Existing theories of debt consider a single contractible performance measure ("output"). In reality, other performance signals are available. It may seem that debt is now inefficient: if the signals are sufficiently positive, the manager should be paid even if output is low. This paper shows that debt remains optimal – additional signals do not affect the form of the contract, but only the debt repayment, leading to performance-sensitive debt. However, some informative signals are unused, in contrast to the informativeness principle. We show which signals are valuable when the contract is driven by contracting constraints rather than the standard risk-incentives trade-off.

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The vast majority of firms issue debt. In some cases, such as most start-ups, debt is the only external source of financing (Robb and Robinson, 2014). A large theoretical literature has therefore aimed to understand when debt contracts are optimal. Most justifications are based on moral hazard. In a costly state verification framework, Townsend (1979) and Gale and Hellwig (1985) show that debt contracts minimize audit costs while inducing truthful reporting of the firm's output. Hart and Moore (1998) find that debt allows for external funding even when output is not contractible and can be diverted by the manager. When output is contractible, Innes (1990) demonstrates that debt is optimal if the manager is protected by limited liability and a monotonicity constraint ensures that investors' payoff is non-decreasing in output. Intuitively, limited liability prevents investors from punishing the manager for low output, so they instead incentivize him by maximizing his rewards for high output. Monotonicity means that the manager cannot gain more than one-for-one. He is thus the residual claimant, receiving equity; investors receive debt.

These frameworks assume that output is the only signal of the manager's effort and thus the only possible determinant of pay. This assumption seems to be critical in generating debt as the optimal contract. When output $q$ is below the contractual repayment $q^*$, this is sufficiently bad news about effort that the manager is paid zero; otherwise, he receives the residual $q-q^*$. In reality, investors have access to multiple additional signals of performance, such as revenues, costs, market share, credit ratings, environmental, social and governance ("ESG") ratings, and peer performance. If these signals are sufficiently good news, it may seem optimal to pay the manager a strictly positive amount even if $q < q^*$; a negative signal may mean that the manager should receive less than the residual even if $q > q^*$. A common result in the contracting literature is that optimal contracts are often highly sensitive to small changes in the contracting environment, such as the addition of a signal. For example, Tirole (2006) writes: "a fair amount of structure must be imposed on the agency relationship in order to generate a standard debt contract for the lenders. Consequently, the theories ... are often criticized for their lack of robustness."

This paper studies whether and how the optimal contract changes if the principal has access to a signal $s$ of effort in addition to output $q$. The signal could affect the optimal contract in two ways. First, debt might no longer be the optimal contract. Debt is "bang-bang" in that the manager receives the lowest possible amount (zero) below a threshold, and the highest possible amount (the residual) above. Intuition might suggest that any informative signal will perturb the optimal contract so that the manager's payoff optimally lies between the extremes.

In contrast, we show that debt remains the optimal contract even under strictly informative signals – and even if the signals are informative everywhere, i.e. provide information about effort regardless of the output level. This is because contracting constraints restrict the use of signals. If $q < q^*$ and the signal indicates that the manager has shirked (i.e. suggests that $q < q^*$ is due to low effort rather than bad luck), the principal cannot use the signal to reduce the payment since the manager is receiving zero anyway: the limited liability constraint binds. Likewise, for $q > q^*$, the principal cannot use the signal to increase the payment since the manager is the residual claimant anyway: the monotonicity constraint binds. As a result, the signal does not change the form of the contract, which remains debt.

Second, the signal could affect the optimal contract by changing the threshold $q^*$. In this case, the contract becomes performance-sensitive debt, where the contractual debt repayment depends on the signal and so it is denoted $q_s^*$. For example, a signal that indicates high effort (such as a high credit rating) could lower $q_s^*$ and increase the manager's payoff. In short, if the signal is valuable, the principal will only use it to change $q_s^*$, not any other aspect of the contract.

However, it is not always the case that debt becomes performance-sensitive. This result is in contrast to the informativeness principle, which states that any informative signal affects the contract (Holmström (1979)).[1] The informativeness principle was derived in a model without contracting constraints, and so the principal can always make use of a signal by changing the contract in response. In our model, contracting constraints bind almost everywhere, and so the principal can only use the signal to change $q_s^*$, which significantly narrows down the set of signals that are valuable.

We derive a necessary and sufficient condition for a signal to affect the contract under contracting constraints: that the signal is informative given that output $q$ exceeds the threshold $q^*$ – i.e. provides incremental information about effort over and above the knowledge that $q \geq q^*$. This is because the likelihood ratio that $q \geq q^*$ determines the optimal debt repayment. This condition differs from the informativeness criterion in Holmström (1979), where a signal affects the contract if it is informative at any individual output realization, i.e. provides information about effort over and above observing the level of output. Intuitively, with a binding monotonicity constraint, changing the debt repayment changes the payoff for all $q \geq q^*$ rather than at an individual output realization. Thus, a signal only affects the contract if it is informative conditional on $q \geq q^*$.

As a result, the relevant statistic is an average effect over $q > q^*$ and not a marginal effect at $q^*$. This is potentially interesting because average effects are easier to test empirically, and the economics of average revenues and costs (and thus output) are different from the economics of marginal revenues and costs – leading to different predictions from a standard framework. For example, a credit rating upgrade may leave the output distribution unchanged around the threshold $q^*$, yet still raise the overall probability that output

[1] Kim (1995) shows how to compare information systems when the sufficient statistic criterion in Holmström (1979) does not apply. By contrast, we are determining whether a signal has value for contracting.

exceeds $q^*$. This may explain why loan terms often adjust to ratings even when short-run operating forecasts are unchanged.

The above condition is stronger than that in Holmström (1979): even if a signal is informative almost everywhere, it does not affect the contract if it is not informative conditional on output exceeding the debt repayment. For example, a signal that indicates that high output is due to high effort rather than good luck is informative, but not valuable since the manager is the residual claimant anyway. A signal that indicates that low output is due to low effort is not valuable since the manager receives zero anyway. For example, investors only noticed that Enron was adopting misleading accounting practices when it was already going bankrupt.

Beyond performance-sensitive debt, this result sharpens our understanding of the value of information in settings with contracting constraints. Research on the value of information typically considers an unconstrained setting, where the optimal contract is a trade-off between incentives and risk-sharing (e.g. Holmström (1979), Gjesdal (1982), Amershi and Hughes (1989) and Kim (1995)). A separate literature analyzes the optimal contract when it is driven by constraints such as limited liability and monotonicity (e.g. Innes (1990), Jewitt, Kadan, and Swinkels (2008)). We show that, in these cases, the informativeness principle does not hold and the set of valuable signals is smaller. In particular, when the contract is described by only one parameter (a result of our model), a signal is valuable if and only if it affects this parameter.

Our findings have implications for the applicability of the informativeness principle. The textbook of Bolton and Dewatripont (2005) notes that contract theory has few general results, but the informativeness principle is one of the few that is general: "Very few general results can be obtained about the form of optimal contracts ... Among the main general predictions of the [moral hazard] model is the informativeness principle, which says that the incentive contract should be based on all variables that provide information about the agent's actions." However, Holmström (2017), in a retrospective on his Nobel-winning work, wrote that the informativeness principle provides "useful insights about the value of information at the same time as it made clear that the basic model cannot explain the shape of common incentive schemes." We provide a potential explanation: binding contracting constraints prevent the use of informative signals.

This paper is related to other research on the optimality of debt contracts in richer agency settings than Innes (1990). Hébert (2018) analyzes the case in which a risk-neutral manager can affect the dispersion of output in addition to its mean, and shows that debt remains optimal because it is the least risky security. Georgiadis, Ravid, and Szentes (2024) relax assumptions on the cost of choosing output distributions and find that debt is generally not optimal. In a principal-agent model with a risk-neutral agent, DeMarzo and Fishman

(2007) show that the optimal contract involves debt. Debt can also be optimal in models of financing that involve adverse selection (Myers and Majluf (1984), Nachman and Noe (1994), Fulghieri, García, and Hackbarth (2020), Malenko and Tsoy (2026)).

Our rationale for performance-sensitive debt – that the signal is informative about effort in a moral hazard model – complements existing explanations. Manso, Strulovici, and Tchistyi (2010) model performance-sensitive debt as a mechanism to signal the firm's growth rate in an adverse selection model; there is no moral hazard. Bhanot and Mello (2006) and Koziol and Lawrenz (2010) show that performance-sensitive debt deters risk shifting. Bensoussan, Chevalier-Roignant, and Rivera (2021) model performance-sensitive debt as a solution to debt overhang, and Fu and Zhu (2026) rationalize it in a costly state verification model where the agent chooses an information structure but does not exert effort. Empirically, Adam and Streitz (2016) test whether performance-sensitive debt is used to reduce hold-up problems, which arise from the information the lender acquires over the course of the lending relationship, Adam et al. (2020) show that overconfident managers issue performance-sensitive debt, and Asquith, Beatty, and Weber (2005) contrast the settings in which debt involves interest-decreasing versus interest-increasing provisions.

Innes (1993) derives the optimal contract when profits (which correspond to $q$ in our setting) can be decomposed into output and the output price, i.e. the price is an additional signal. He shows that the optimal contract is a price-contingent commodity bond, which has similarities to performance-sensitive debt; however, the only signal that he analyzes is price (i.e. one component of output). We instead consider a broad class of signals and show that debt remains the optimal contract. Moreover, unlike in Innes (1993) where the price always affects the contract, we show that some informative signals have no value. We derive a necessary and sufficient condition for a signal to be valuable, and show that if a signal is used, it only affects the debt repayment and not any other aspect of the contract.

More broadly, our paper is related to a literature on robust and "detail-free" contracts, e.g. Chassang (2013) and Carroll (2015). In standard agency models, the contract depends on many specific features of the setting. This poses practical difficulties, as some of the important determinants are difficult for the principal to observe and thus use to guide the contract. This literature explains why the structure of real-world contracts does not seem to be as complicated and contingent on as many details of the environment as standard contract theories would suggest. However, those papers do not study how to incorporate additional signals into a contract; for example, Chassang (2013) instead shows that the contracts he proposes are detail-free because they remain approximately optimal even under very different contracting environments, not because they do not use some available information.

In our paper, the optimal contract remains debt, and even if there are informative signals that can be used in the contract because contracting constraints do not bind, they only affect the debt repayment and not the form of the contract. The contracting literature shows that it is difficult to obtain general results on the nature of optimal contracts, and information is typically incorporated in an "anything goes" way (e.g. Holmström (1979); Grossman and Hart (1983)). We show that additional signals on agent performance are incorporated in a specific way: through changing the debt repayment.

# 1 The Model

There are two risk-neutral parties, a principal (board acting on behalf of investors), and an agent (manager). The manager exerts an unobservable effort $e \in [0, \bar{e}]$. As is standard, effort can be interpreted as any action that improves output but is costly to the manager, such as working rather than shirking, choosing projects that generate cash flows rather than private benefits, or not extracting rents. The manager's cost of effort $C(\cdot)$ is strictly increasing, strictly convex, twice continuously differentiable in $[0, \bar{e})$, with $C(0) = C'(0) = 0$ and $\lim_{e \nearrow \bar{e}} C'(e) = +\infty$.

Effort affects the probability distribution of output $q$ and a signal $s$, which are both observable and contractible. Output is distributed with a support that may depend on the signal (but not on effort). For a signal realization $s$, the conditional distribution of output $q$ given $s$ has support on $\mathcal{Q}_s = [0, \overline{q}_s)$, where $\overline{q}_s$ may be $\infty$. To ensure that an optimal contract exists, we assume that the signal is discrete, $s \in \{s_1, ..., s_S\}$. The signal can have one or multiple dimensions (i.e., be a vector).

The signal is distributed according to the probability mass function $\phi_e^s := \Pr(\tilde{s} = s | \tilde{e} = e)$, which is strictly positive and twice continuously differentiable in $e$. Output is distributed according to the cumulative distribution function $F(q|e,s)$. It has density $f(q|e,s)$, which is strictly positive on $\mathcal{Q}_s$ and twice continuously differentiable in $q$ and $e$. The joint distribution of output and the signal is $\mathbf{f}(q, s|e) = \phi_e^s f(q|e,s)$. We assume that the likelihood ratio of output, $\frac{\frac{\partial f}{\partial e}(q|e,s)}{f(q|e,s)}$, is strictly increasing in output $q$ ("MLRP"). For a given effort $e$, signal $s$, and $q \in \mathcal{Q}_s$, the likelihood ratio associated with the event $(\tilde{q} = q, \tilde{s} = s)$ is:

$$LR_s(q,e) := \frac{\frac{\partial \mathbf{f}}{\partial e}(q,s|e)}{\mathbf{f}(q,s|e)} = \frac{\partial \phi_e^s / \partial e}{\phi_e^s} + \frac{\frac{\partial f}{\partial e}(q|e,s)}{f(q|e,s)} \tag{1}$$

Consistent with standard distributions, we assume that $\lim_{q \nearrow +\infty} \frac{\partial \mathbf{f}}{\partial e}(q,s|e) = 0$ for each $s$ such that the distribution has unbounded support, which implies that debt with arbitrarily high contractual debt repayment has low effort incentives. We assume that $\lim_{q \nearrow +\infty} LR_s(q,e) = \infty$ for all $e$ and $s$ with unbounded support. We also assume that expected output is finite and differentiable in effort:

$$\int_0^{\overline{q}_s} q \left| \frac{\partial \mathbf{f}}{\partial e}(q,s|e) \right| dq < \infty \quad \text{for all } s \text{ and } e \in [0,\bar{e}). \tag{2}$$

The principal has full bargaining power and offers the manager a schedule of payments $\{w_s(q)\}$ conditional on each realization of $(q,s)$. We follow Grossman and Hart (1983) and separate the principal's problem into two stages. The first stage determines the optimal contract and the associated cost of implementing each effort. Given this cost, the second stage determines which effort to implement. To induce effort $\hat{e}$, she solves the following program:

$$\max_{w_s(q)} \sum_s \int_0^{\overline{q}_s} (q - w_s(q))\, \mathbf{f}(q,s|\hat{e}) dq \tag{3}$$

subject to

$$\sum_s \int_0^{\overline{q}_s} w_s(q) \mathbf{f}(q,s|\hat{e}) dq - C(\hat{e}) \geq 0, \tag{4}$$

$$\hat{e} \in \arg\max_e \sum_s \int_0^{\overline{q}_s} w_s(q) \mathbf{f}(q,s|e) dq - C(e), \tag{5}$$

$$0 \leq w_s(q) \leq q, \tag{6}$$

$$q - w_s(q) \text{ non-decreasing in } q. \tag{7}$$

In the first stage, the principal maximizes her expected payoff (3) subject to the manager's individual rationality constraint ("IR") (4) and incentive compatibility constraint ("IC") (5). The limited liability constraints (6) stipulate that both investors and the manager are protected by limited liability, and so payments can neither be negative nor exceed the entire output. The monotonicity constraint (7) states that a dollar increase in output cannot increase the wage by more than a dollar (else the manager would inject his own money into the firm to increase output), or equivalently the payoff to the principal cannot decrease in output (else she would exercise her control rights to "burn" output).[2] In the second stage, the principal chooses the optimal effort level. Our analysis focuses on the first stage, and the results extend to the overall optimum since the conditions for signal-independence can be evaluated at the implemented effort.

Given that $C(0) = 0$, the IC (5) and LL (6) imply that the IR (4) is automatically satisfied, and so we ignore it in the analysis that follows. To ensure that an incentive

[2]The monotonicity constraint implies that the principal and agent have some control over output. We do not assume such a constraint over the signal since it may be outside their control, such as economic conditions or peer performance. The signal need not be a cardinal number (such as the firm's credit rating) and need not be ordered (such as the political party in power).

compatible contract exists, we assume:

$$\sum_s \int_0^{\overline{q}_s} q \frac{\partial \mathbf{f}}{\partial e}(q, s|\hat{e}) dq > C'(\hat{e}). \tag{8}$$

The best contract that can be offered to the manager pays the entire output whenever it is positive. The previous condition states that offering this contract is enough to incentivize the manager to choose an effort of at least $\hat{e}$. When this condition fails, there is no contract that induces the manager to choose $\hat{e}$. As in Grossman and Hart (1983), effort levels for which this condition fails can be treated as having infinite cost. Since the optimal contract that induces $\hat{e} = 0$ simply pays zero in all states, we henceforth restrict attention to strictly positive induced effort levels: $\hat{e} > 0$.

# 2 Debt Contracts

## 2.1 When Is Debt Optimal?

As a preliminary result, Lemma 1 below presents a condition for the validity of the First-Order Approach ("FOA") to the effort choice problem in the above program.

Let $K_e$ be defined as:[3]

$$K_e := \sum_s \int_0^{\overline{q}_s} q \max \left\{ \frac{\partial^2 \mathbf{f}}{\partial e^2}(q, s|e), 0 \right\} dq.$$

**Lemma 1** *Suppose that $K_e < C''(e)$ $\forall e \in (0, \bar{e})$. Then the FOA is valid.*

The condition in Lemma 1 relies on the contracting constraints, and the associated bounds on the payment $w_s(q)$ to the manager, to derive an upper bound $K_e$ on the convexity of the expected payment with respect to effort. The FOA is valid if the cost of effort is more convex than this upper bound. We henceforth assume that the condition in Lemma 1 holds. This provides a new sufficient condition for applying the FOA in the setting of a risk-neutral agent and contracting constraints.[4] Given the use of this framework in applied contract theory, this condition may be valuable for future research. Some alternative conditions that can be applied under contracting constraints (e.g. Chaigneau, Edmans, Gottlieb (2022)) rely on bounded utility, which can arise under risk aversion but not risk neutrality. While Rogerson's (1985) sufficient condition can hold under risk neutrality and contracting

[3]For example, with a lognormally distributed output with parameters $e \in [0, 1]$ and $\sigma = 1$ and no signal, we find that $\max_e K_e \approx 5.92$.

[4]The constraints needed for the condition are agent limited liability, plus either principal limited liability and/or monotonicity.

constraints, it rules out many standard distributions such as the normal, lognormal, logistic, and gamma.[5]

Let

$$\overline{LR}_s(q,e) := \frac{\partial \phi_e^s/\partial e}{\phi_e^s} + \frac{\int_q^{\overline{q}_s} \frac{\partial f}{\partial e}(z|e,s)dz}{\int_q^{\overline{q}_s} f(z|e,s)dz} \tag{9}$$

denote the likelihood ratio associated with the event $(\tilde{q} \geq q, \tilde{s} = s)$. The likelihood ratio comprises two terms. The first, $\frac{\partial \phi_e^s/\partial e}{\phi_e^s}$, captures how individually informative the signal is about effort. For example, if $s$ is profits, high profits indicate high effort. The second, $\frac{\int_q^{\overline{q}_s} \frac{\partial f}{\partial e}(z|e,s)dz}{\int_q^{\overline{q}_s} f(z|e,s)dz} = \frac{\frac{\partial}{\partial e}Pr(\tilde{q}>q|e,s)}{Pr(\tilde{q}>q|e,s)}$ captures, for each given signal $s$, how effort affects the probability that output exceeds $q$. For example, if the signal $s$ is peer firm performance, this likelihood ratio and thus the inference about effort will be lower if peer performance is strong, because this shifts the output distribution to the right.

MLRP concerns the standard likelihood ratio, $LR_s(q,e)$. Lemma 2 shows that this property extends to the likelihood ratio $\overline{LR}_s(q,e)$.

**Lemma 2** *The likelihood ratio $\overline{LR}_s(q,e)$ is strictly increasing in $q$.*

In Innes (1990), without an additional signal $s$, investors receive debt and the manager receives equity. The manager receives zero if output is less than the contractual debt repayment $q^*$ (which we will henceforth refer to as the "debt repayment" for brevity), and the residual $q - q^*$ otherwise. The intuition is as follows. Due to MLRP, output is most informative about effort in the tails of the distribution of $q$. The principal cannot incentivize the manager in the left tail by giving negative payments (due to limited liability), so she incentivizes him in the right tail by giving high payments. Under the monotonicity constraint, the maximum possible incentives involve the manager gaining one-for-one from any increase in output, so he receives the residual.

With an additional signal $s$, it is not clear that the optimal contract remains debt. It may be that, for low outputs, if the signal is sufficiently individually indicative of effort (e.g. $\frac{\partial \phi_{\hat{e}}^s/\partial e}{\phi_{\hat{e}}^s}$ is high), it becomes optimal to pay the manager a strictly positive amount, rather than zero as under a debt contract. Conversely, it may be that, for high outputs, if the signal is sufficiently individually indicative of shirking, it becomes optimal to pay the manager less than the residual. However, Proposition 1 below shows that the contract that induces any given effort $\hat{e} > 0$ remains debt.

**Proposition 1** *The optimal contract that induces $\hat{e}$ is $w_s(q) = \max\{q - q_s^*, 0\}$, for some signal-contingent debt repayment $q_s^*$. For interior solutions, ($q_s^* \in (0, \overline{q}_s)$ for each $s$), debt repayments $\{q_s^*\}$ are such that $\overline{LR}_{s_i}(q_{s_i}^*, \hat{e}) = \overline{LR}_{s_j}(q_{s_j}^*, \hat{e})$.*

[5]Innes (1990) assumes that the FOA is valid and cites Rogerson's (1985) conditions.

Proposition 1 shows that, instead of affecting the form of the optimal contract, which remains debt, the signal realization affects the debt repayment. The intuition is as follows. A negative signal means that it is optimal to pay the manager less, but this reduction can only occur for high output levels where the payment is strictly positive. Conceptually, this decrease could be achieved by lowering the slope of the manager's pay, but it turns out to be optimal instead to raise the debt repayment. Due to MLRP, it is more efficient to provide strong incentives for only high output levels than moderate incentives for a larger range of output levels.

Conversely, if output is low, a positive signal only leads to a strictly positive payment if it raises the likelihood ratio in equation (9) above a minimum threshold. Due to MLRP, it is efficient to provide the manager with the minimum possible payment (zero) over a wide range of output levels; thus, a positive signal should lead to a positive payment only at the top end of this range. Overall, the "incentive zone" – the subset of outputs where the manager receives a strictly positive payment – depends on the signal realization. Intuitively, the signal allows the principal to concentrate incentives in states of the world that are stronger positive signals of effort.

Unlike in the standard principal-agent model, the agent's pay is not determined by the standard likelihood ratio in equation (1),[6] but instead by the modified likelihood ratio described in equation (9). This is the likelihood ratio of the event $\tilde{q} \geq q$ conditional on signal $s$, which is over a range of outputs rather than at a single output level $\tilde{q} = q$. This likelihood ratio matters because the firm cannot increase the payment at a specific output level in isolation without increasing it at all lower outputs, as this would violate the monotonicity constraint. One might think that the firm should give the manager a bonus if $s$ is high, but it can only increase $w$ at one $q$ if it does so at all lower $q$'s, and so the bonus can only be implemented by lowering the threshold $q_s^*$. Similarly, the firm cannot decrease the payment at a specific output level in isolation without decreasing it at all higher outputs, which involves increasing the threshold $q_s^*$.

The debt repayment $q_s^*$ is the level of output at which the likelihood ratio in equation (9) equals a cutoff (see equation (19)); this cutoff is the inverse of the Lagrange multiplier associated with the IC. This implies that the likelihood ratios associated with different signal realizations are equal at the debt repayments $q_s^*$. As a result, a signal realization that increases the likelihood ratio is associated with a lower debt repayment. This is because such a signal means that the event $(\tilde{q} \geq q, s)$ is better news about the agent's effort, in the sense that it is more likely to be generated under higher effort.[7]

[6]The agent's optimal payoff depends on this likelihood ratio with a risk neutral agent protected by limited liability as well as in the standard model of Holmström (1979).

[7]In equilibrium, the principal knows the agent's effort, but the optimal contract still depends on the likelihood ratio "as if" it were the solution to a statistical inference problem, as noted by Bolton and

In sum, rather than affecting the form of the optimal contract, which remains debt, the signal only changes the optimal debt repayment $q_s^*$, in which case the contract becomes performance-sensitive debt.

## 2.2 When Is Performance-Sensitive Debt Optimal?

With the debt contract that induces effort $\hat{e}$ derived in Proposition 1, the principal's only relevant choice is the debt repayment $q_s^*$. Part (i) of Proposition 2 gives a necessary and sufficient condition for the contract to be independent of the signal, i.e. $q_s^* = q^* \; \forall s$. The signal-independent debt repayment $q^*$ is determined by the incentive compatibility condition for a contract that does not depend on the signal, i.e. $q^*$ solves the following equation:

$$\sum_s \int_{q^*}^{\overline{q}_s} (q - q^*) \frac{\partial \mathbf{f}}{\partial e}(q, s|\hat{e}) dq = C'(\hat{e}). \tag{10}$$

Part (ii) gives a sufficient condition for the payment to be independent of the signal.

**Proposition 2** *Let $w_s(q)$ be an optimal contract.*

*(i) The optimal contract is independent of the signal ($w_s(q) = w(q)$ for all $s, q$) if and only if $\overline{LR}_{s_i}(q^*, \hat{e}) = \overline{LR}_{s_j}(q^*, \hat{e})$ for all $s_i$, $s_j$, with $q^* \in \mathcal{Q}_s \; \forall s$.*

*(ii) Given output $q$, the payment $w_s(q)$ is independent of the signal if $q \leq \min_s \{q_s^*\}$.*

Part (i) of Proposition 2 asks whether a signal is valuable ex ante – before observing output, would the principal like to make the contract contingent on the signal? It shows that contracting constraints require us to refine the informativeness principle. From Proposition 1, a signal only affects the contract if it changes the principal's optimal choice of the debt repayment. She cannot change the contract for outputs below the debt repayment because the manager is already paid zero, nor for outputs above the debt repayment because the manager is already paid the residual. In turn, the principal will choose not to make the debt repayment depend on the signal, i.e. set $q_s^* = q^* \; \forall s$, if and only if the likelihood ratio that $q \geq q^*$ is the same across signals. If the condition is satisfied for every $\hat{e}$, then the optimal contract that implements the optimal effort is independent from the signal.

The condition in part (i) is different from Holmström (1979). In his paper, a signal affects the contract if and only if it affects the likelihood $LR_s(q, \hat{e})$ of the event $(\tilde{q} = q, \tilde{s} = s)$ at any output $q$, i.e. it is informative about effort conditional on the event that output equals any $q$. Here, it does so if and only if it affects the likelihood $\overline{LR}_s(q^*, \hat{e})$ of the event $(\tilde{q} \geq q^*, \tilde{s} = s)$, i.e. is informative about effort conditional on the event that output exceeds

Dewatripont (2005). Relatedly, the availability of an additional signal improves statistical inference, which is reflected in the effect of the signal on the optimal contract.

the debt repayment $q^*$. As a result, a signal can be informative almost everywhere, yet have no value.

In particular, an individually informative signal has no value for contracting if it is only informative conditional on output being sufficiently low that the manager is paid zero, or sufficiently high that the manager is the residual claimant. For example, if output is high, and high output is only possible following high effort, then observing the number of hours worked is individually informative about effort, but has no contractual value: high output already reveals that the manager has exerted high effort and so he is the residual claimant anyway. An individually uninformative signal can still affect the output distribution, and thus what output implies about effort, as in the peer performance example earlier. However, part (i) highlights that such a signal affects the contract only if it shifts probability mass across the threshold $q^*$, i.e. from below $q^*$ to above $q^*$ (or vice-versa). A signal that redistributes mass within the left tail, or within the right tail, has zero value (we will provide such an example in Example 2 later).

Part (i) thus has implications for when debt contracts should be performance-sensitive. In theory, the debt repayment could depend on many signals, but in practice it is often signal-independent. Part (i) helps rationalize this practice – even if signals are informative about effort, they should not enter the contract if they are only informative in the tails. In addition, part (i) provides conditions under which the repayment *should* depend on additional signals, as in performance-sensitive debt – if and only if the signal is informative about effort conditional on output exceeding the promised repayment. For example, a credit rating downgrade or a decrease in the interest coverage ratio should typically increase the repayment because they are negative signals about managerial effort.

Proposition 2, part (i) also has implications for the generalizability of the informativeness principle beyond the original setting of Holmström (1979). Informative signals allow for superior effort inference, which in turn allows for contracts that better reflect the agent's effort rather than noise. In the model of Holmström (1979) with a risk-averse agent, this reduces the agent's risk exposure and thus the cost of the contract. One may think that the same logic applies to the case of risk neutrality and limited liability: here, the cost of incentives takes the form of limited liability rents rather than a risk premium, and signals might similarly reduce those rents. Indeed, risk neutrality and limited liability are often viewed as an alternative modelling approach to risk aversion. Our paper shows that the same logic does not automatically apply. When the agent is risk neutral, incentives are only costly if there are contracting constraints, otherwise the principal would be able to achieve first-best (e.g., by selling the firm to the agent). However, these same constraints restrict how signals can be used. Informative signals are valuable only if they affect effort inference at output levels where the contracting constraints do not bind.

While part (i) of Proposition 2 asks whether the optimal *contract* depends on the signal, part (ii) asks whether the optimal *payment* depends on the signal. It explores whether a signal is valuable ex post – after observing output, will the payment to the manager depend on the signal? If output is sufficiently low, the signal has no value since the manager will be paid zero even under the most favorable signal realization. Thus, even if the signal realization reduced the optimal debt repayment – i.e. changed the optimal contract – it would not change the payment as it remains zero. Part (ii) is particularly relevant if signals are costly and the principal can observe output before deciding whether to gather the signal.

We now draw out additional implications of Proposition 2 which we illustrate with examples that apply them to concrete settings. Part (ii) of Proposition 2 implies that the payment is zero regardless of the signal whenever output falls below $\min_s\{q_s^*\}$. If a signal realization only occurs for outputs in this range, it therefore has no contractual value even if it is informative about effort. Example 1 illustrates this.

**Example 1** *A state is defined by* $(q, s)$ *where* $q$ *is output and* $s$ *is a signal which is either bad (*$B$*) or good (*$G$*). The signal is informative about effort in a sufficient statistic sense. However, a bad signal is only possible if* $q < q_0$ *(*$\mathcal{Q}_B = [0, q_0)$*), whereas the good signal can arise for any output (*$\mathcal{Q}_G = [0, \infty)$*). Thus, when* $s = B$*, we have* $q < q_0$*. Then, if* $q_0 < \min\{q_B^*, q_G^*\}$*, the manager is paid zero whenever* $s = B$*, so that the signal has no value for the contract.*

This example considers signals such as a failed audit or the loss of investment grade status which might suggest that poor performance is due to low managerial effort rather than bad luck. However, the bad signal only arises when performance is below a given level $q_0$. If that level is below the threshold required for the manager to be paid ($q_0 < \min\{q_B^*, q_G^*\}$), the manager will be paid zero anyway, and so the signal has no value for the contract. Intuitively, low output contains all the information about effort required to determine the manager's payment, and so observing the signal does not change anything. For instance, investors only noticed Enron's accounting irregularities when it was already going bankrupt and its leadership team was already going to be dismissed.

Corollary 1 draws additional implications from part (i) of Proposition 2. While part (i) gave a condition on the likelihood ratio for a signal to have no value for contracting, recall that the likelihood ratio (9) contained two terms. This corollary gives a sufficient condition for an individually uninformative signal (one in which the first term is zero) not to affect the contract.

**Corollary 1** *Consider a signal* $s$ *such that* $\frac{\partial \phi_e^s/\partial e}{\phi_e^s} = 0$ *for any* $s$*. If the support of output does not depend on* $s$ *and* $Pr(\tilde{q} > q^*|e, s) = \int_{q^*}^{\bar{q}_s} f(z|e, s)dz$ *is independent of* $s$ *for* $e \in (\hat{e} - \varepsilon, \hat{e} + \varepsilon)$ *for some* $\varepsilon > 0$*, then the signal has no value for contracting.*

If an individually uninformative signal does not affect the probability that $q > q^*$ for $e$ in a neighborhood of $\hat{e}$, then the signal has no value for contracting – even if the signal affects the second term in the likelihood ratio (9) and is thus informative overall. Example 2 describes a real-life setting where the condition in Corollary 1 is satisfied, and thus an informative signal is not used in the contract.

**Example 2** *Firm output depends in part on the revenues from a new product, which depend on a signal $s$ of market conditions. There are two cases:*

- *If the product is not successfully developed, it does not generate revenues, so that output does not depend on market conditions.*
- *If the product is successfully developed, the probability $Pr(\tilde{q} > q|e, s) = 1$ for $e$ in a neighborhood of $\hat{e}$ and for $q$ sufficiently low – formally, for $q \in \widehat{\mathcal{Q}}$.*

*If $q^* \in \widehat{\mathcal{Q}}$, then the contract does not depend on market conditions.*[8]

In Example 2, $s$ is a signal of market conditions. Market conditions are not affected by effort and so the signal does not affect the first part of the likelihood ratio (9). However, it can still affect the second part by influencing the distribution of output and thus the informativeness of output about effort. For example, for any given level of output, a high $s$ suggests that the output was due to good market conditions rather than effort. This is the intuition behind relative performance evaluation: one might think that good market conditions should always raise the threshold $q^*$.

However, Corollary 1 shows that market conditions $s$ should only affect the contract if they affect the probability that $q \geq q^*$ under the equilibrium effort. This will fail to hold if they affect the level of output but not the probability that output exceeds $q^*$.[9] For example, consider a pharmaceutical company developing a blockbuster new drug; the manager's effort affects the probability that a clinical trial is successful. If it is, $q \geq q^*$ (regardless of market conditions); if it is not, $q < q^*$ (again, regardless of market conditions). Economic conditions affect the actual level of $q$ within each region,[10] but if they do not affect the probability that

[8]For any given $s$, the probability $Pr(\tilde{q} > q|e, s)$ is a weighted sum of this probability when the product is unsuccessfully developed and of this probability when the product is successfully developed, where the weights are given by the probability that the product is successfully developed. When these two probabilities do not depend on $s$, the probability $Pr(\tilde{q} > q|e, s)$ does not depend on $s$ either. Note that the output distribution conditional on $s$ has full support on $\mathcal{Q}_s$ since with some probability the product is unsuccessfully developed.

[9]It will also hold if they affect the probabilities (that $q \geq q^*$ under high and low effort) by the same proportion.

[10]For example, the company has additional minor drugs that also generate sales, but these sales alone are insufficient to increase $q$ above $q^*$.

$q \geq q^*$, because they do not affect the likelihood that the trial will be successful, then they should not be included in the contract. Example 2 shows that this is possible in cases which are not knife-edge (formally, the condition is $q^* \in \widehat{\mathcal{Q}}$). In contrast, for an "everyday" drug, where the probability that $q \geq q^*$ depends on market conditions as well as the manager's effort, then the debt repayment should depend on market conditions.

Since debt being performance sensitive leads to equity being performance sensitive, the model allows us to study not only the optimality of performance-sensitive debt but also the conditions under which the manager's equity claim should depend on additional signals, as studied empirically by Kaplan and Strömberg (2003) for venture capital contracts. Kaplan and Strömberg (2004) find that the debt and equity contracts used in venture capital are determined primarily by agency problems, not risk-sharing considerations. In the example of a pharmaceutical company developing a blockbuster drug, the manager's equity claim should not depend on economic conditions, and the same may be true for some start-ups. For example, Facebook's early revenues depended more on whether people used the platform than economic conditions. Indeed, Kaplan and Strömberg (2003) find that the manager's equity claim depends on performance milestones that are direct signals of managerial effort, rather than economic conditions.[11]

## 2.3 When Signals Can Be Manipulated

Thus far, we have assumed that the signal $\tilde{s}$ could not be manipulated. However, in practice, managers can underreport certain signals, for example by "burning" profits (Innes, 1990), engaging in unnecessary expenditure, or concealing successes. In most contracts, the payment is increasing in all signals so there is no incentive to engage in underreporting, but in our model, the payment can be decreasing in the signal – thus, the manager may wish to lower the signal to make output a more positive indicator of effort. We now consider the case of nonpecuniary signals that can be underreported by the manager at no cost.

Let $\tilde{s}$ denote the actual signal realization and $\hat{s}$ the manager's report. To simplify the exposition, we assume that the signal is unidimensional and can be ordered; a manager who is indifferent between underreporting or not underreporting a signal does not underreport, i.e. $\hat{s} = \tilde{s}$ for any $s$; and a principal who is indifferent between incorporating the signal in the contract or not does not do so. Proposition 3 shows that, in the presence of misreporting, the contract can be independent of a signal even if it is informative about the event ($\tilde{q} \geq q, \tilde{s} = s$), i.e. even if the condition in part (i) of Proposition 2 is violated. Condition (b) of Proposition 3 extends the MLRP to the pooled likelihood ratio.[12]

[11]While the original informativeness principle in Holmström (1979) would suggest that contracts should depend on performance milestones, it does not generally deliver debt and equity as optimal contracts.

[12]When the no-underreporting constraint binds, signals within a group $G$ must have the same payoff

**Proposition 3** *When the manager can underreport the signal, the optimal contract is independent of the signal if (a) $\overline{LR}_{s_i}(q,\hat{e}) \leq \overline{LR}_{s_j}(q,\hat{e})$ for any $s_i > s_j$ and all $q$, and (b) for every consecutive group $G \subseteq \{s_1,\ldots,s_S\}$, the pooled likelihood ratio*

$$\overline{LR}_G(q,\hat{e}) := \frac{\sum_{s\in G}\int_q^{\overline{q}_s}\frac{\partial \mathbf{f}}{\partial e}(z,s|\hat{e})\,dz}{\sum_{s\in G}\int_q^{\overline{q}_s}\mathbf{f}(z,s|\hat{e})\,dz}$$

*is strictly increasing in $q$.*

If a higher signal realization $s_i$ is bad news about effort in the sense of a lower likelihood ratio $\overline{LR}_{s_i}(q,\hat{e})$, then it should be associated with a higher debt repayment. However, the manager would then have incentives to underreport the signal. As a result, the contract is independent of the signal: the signal is informative in equilibrium but has no value for contracting because it can be underreported. Example 3 provides a setting in which this is the case.

**Example 3** *Let $s \in \{0,1\}$ be a binary signal of product development: $s=1$ if a product has been successfully developed, and $s=0$ if it has not. Successful product development ($s=1$) individually indicates higher effort which increases $\frac{\partial\phi_{\hat{e}}^s/\partial e}{\phi_{\hat{e}}^s}$ and thus $\overline{LR}_1(q,\hat{e})$ in equation (9), but it also shifts the distribution of output rightward, which decreases $\overline{LR}_1(q,\hat{e})$.*

- *If the first effect dominates, i.e. $\overline{LR}_1(q,\hat{e}) > \overline{LR}_0(q,\hat{e})\ \forall q$, then the debt repayment should be lower upon successful development. While successful development means that a given output level is less indicative of effort (because output may result from the new product rather than managerial effort), this is outweighed by the fact that the product itself is good news about effort.*

- *If the second effect dominates, i.e. $\overline{LR}_1(q,\hat{e}) < \overline{LR}_0(q,\hat{e})\ \forall q$, the debt repayment should be higher upon successful development if underreporting is impossible. However, if the manager can underreport product development (report $s=0$ even if development has been successful), and the pooled likelihood ratio $\overline{LR}_{\{0,1\}}(q,\hat{e})$ is strictly increasing in $q$, then the debt contract is independent of the signal.*

The fact that debt repayments in practice are independent of product development (and other signals that are individually good news about effort) suggests that the positive effect on $\frac{\partial\phi_{\hat{e}}^s/\partial e}{\phi_{\hat{e}}^s}$ is outweighed by the shift in the output distribution, and also that the signal can

schedule, so the principal minimizes cost under the pooled distribution $\sum_{s\in G}\mathbf{f}(\cdot,s|\hat{e})$. The dominance argument then requires the cumulative likelihood ratio of this pooled distribution, $\overline{LR}_G(q,\hat{e})$, to be increasing in $q$, which is the natural counterpart of MLRP of $\overline{LR}(q,\hat{e})$ at the group level.

be underreported. If the positive effect on $\frac{\partial \phi_{\hat{e}}^s / \partial e}{\phi_{\hat{e}}^s}$ were greater, then the debt repayment would be decreasing in the signal. If the effect on the output distribution were greater, then the debt repayment would be increasing in the signal. However, the manager would then underreport the signal, and so it is optimal for the debt repayment to be independent of it.

Thus, the value of a signal for contracting depends not only on its informativeness about effort, but also on its manipulability. Even when a signal is informative in the sense of Proposition 2, it may optimally be excluded from the contract if it can be underreported. Thus, the absence of certain signals in observed contracts need not reflect a lack of informativeness, but rather the inability to elicit truthful reporting.

# 3 Conclusion

This paper shows that, in a standard moral hazard model with limited liability and monotonicity constraints, the optimal contract remains debt for the principal, even if she has access to additional performance signals. It may seem intuitive that a good signal should lead to the manager being paid even if output is low, and a bad signal should lead to him not being the residual claimant even if output is high. However, we show that the signal does not affect the form of the contract, but only the debt repayment.

As a result, Holmström's (1979) informativeness principle needs to be refined. A signal is only valuable if it is informative conditional on output exceeding the debt repayment, rather than being informative at any possible output level. If this condition is satisfied, then the debt repayment should depend on the signal, as in performance-sensitive debt. If not – such as an audit that uncovers whether low output is due to low effort or bad luck – then the debt contract does not depend on the signal. As a result, a signal can be informative almost everywhere, yet not be incorporated into the contract. Our results have broader implications for the value of information in settings with contracting constraints, and for the robustness of contracts to different features of the contracting environment.

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# Proofs

**Proof of Lemma 1:** The FOA is valid if the following objective function is concave in $e$:

$$\sum_s \int_0^{\overline{q}_s} w_s(q)\mathbf{f}(q,s|e)dq - C(e).$$

A sufficient condition is:

$$\sum_s \int_0^{\overline{q}_s} w_s(q)\frac{\partial^2 \mathbf{f}}{\partial e^2}(q,s|e)dq < C''(e) \qquad \forall e. \tag{11}$$

From equation (6), $w_s(q) \in [0,q]$ for all $q, s$. Then, a sufficient condition for equation (11) is:

$$\sum_s \int_0^{\overline{q}_s} \max\left\{q\frac{\partial^2 \mathbf{f}}{\partial e^2}(q,s|e), 0\right\} dq = \sum_s \int_0^{\overline{q}_s} q \max\left\{\frac{\partial^2 \mathbf{f}}{\partial e^2}(q,s|e), 0\right\} dq < C''(e) \quad \forall e. \blacksquare$$

**Proof of Lemma 2:** We have:

$$\frac{d}{dq}\left\{\frac{\partial \phi^s_{\hat{e}}/\partial e}{\phi^s_{\hat{e}}} + \frac{\int_q^{\overline{q}_s} \frac{\partial f}{\partial e}(z|\hat{e},s)dz}{\int_q^{\overline{q}_s} f(z|\hat{e},s)dz}\right\} = \frac{-\frac{\partial f}{\partial e}(q|\hat{e},s)\int_q^{\overline{q}_s} f(z|\hat{e},s)dz + f(q|\hat{e},s)\int_q^{\overline{q}_s} \frac{\partial f}{\partial e}(z|\hat{e},s)dz}{\left(\int_q^{\overline{q}_s} f(z|\hat{e},s)dz\right)^2}. \tag{12}$$

For $\frac{\partial f}{\partial e}(q|\hat{e},s) \leq 0$, we have $-\frac{\partial f}{\partial e}(q|\hat{e},s)\int_q^{\overline{q}_s} f(z|\hat{e},s)dz \geq 0$ . Moreover, $f(q|\hat{e},s)\int_q^{\overline{q}_s} \frac{\partial f}{\partial e}(z|\hat{e},s)dz > 0$ because of MLRP and $\int_0^{\overline{q}_s} \frac{\partial f}{\partial e}(z|\hat{e},s)dz = 0$. In sum, the right-hand side ("RHS") of equation (12) is positive. For $\frac{\partial f}{\partial e}(q|\hat{e},s) > 0$, the RHS of equation (12) is positive if and only if:

$$\begin{aligned}
& f(q|\hat{e},s)\int_q^{\overline{q}_s} \frac{\partial f}{\partial e}(z|\hat{e},s)dz \geq \frac{\partial f}{\partial e}(q|\hat{e},s)\int_q^{\overline{q}_s} f(z|\hat{e},s)dz \\
\Leftrightarrow \quad & \int_q^{\overline{q}_s} \frac{\frac{\partial f}{\partial e}(z|\hat{e},s)}{\frac{\partial f}{\partial e}(q|\hat{e},s)}dz \geq \int_q^{\overline{q}_s} \frac{f(z|\hat{e},s)}{f(q|\hat{e},s)}dz \\
\Leftrightarrow \quad & \int_q^{\overline{q}_s} \left[\frac{\frac{\partial f}{\partial e}(z|\hat{e},s)}{\frac{\partial f}{\partial e}(q|\hat{e},s)} - \frac{f(z|\hat{e},s)}{f(q|\hat{e},s)}\right] dz \geq 0,
\end{aligned}$$

which holds because by MLRP we have $\frac{\frac{\partial f}{\partial e}(z|\hat{e},s)}{f(z|\hat{e},s)} \geq \frac{\frac{\partial f}{\partial e}(q|\hat{e},s)}{f(q|\hat{e},s)}$ for any $z \geq q$. $\blacksquare$

**Proof of Proposition 1:** We consider the optimal contract that induces $\hat{e} > 0$. The proof is divided into two parts:

*Step 1. Conditional on each signal realization, the optimal contract is debt.*

*Step 1.a.* This part of the proof adapts the proof technique from Lemma 1 in Matthews (2001) to a setting with continuous output and an additional signal. Let $(W_s^*)_{s\in\{1,\ldots,S\}}$ (henceforth denoted by $(W_s^*)$ for brevity) be a feasible payment schedule that induces effort $\hat{e}$. For a given signal realization $s'$, consider an alternative payment schedule which is the same as $(W_s^*)$ for any signal other than $s'$, and $W_{s'}^{q_{s'}} = \max\{0, q - q_{s'}\}$ for a given $s'$. The contractual debt repayment $q_{s'}$ is chosen so that the payment schedules contingent on signal $s'$, $W_{s'}^*$ and $W_{s'}^{q_{s'}}$, have the same expected payment under effort $\hat{e}$:

$$\int_0^{\overline{q}_{s'}} W_{s'}^*(q)\mathbf{f}(q, s'|\hat{e})dq = \int_0^{\overline{q}_{s'}} W_{s'}^{q_{s'}}(q)\mathbf{f}(q, s'|\hat{e})dq. \tag{13}$$

It is straightforward to show that $W_{s'}^{q_{s'}}$ exists and is unique. We will first show that, for a given $s'$, replacing $W_{s'}^*$ by $W_{s'}^{q_{s'}}$ increases effort.

For a given $s'$, define:

$$W_{s,s'}^{**}(q) := \begin{cases} W_s^*(q) & \text{for } s \neq s' \\ W_s^{q_s}(q) & \text{for } s = s' \end{cases}. \tag{14}$$

In what follows we will compare the original payment schedule $(W_s^*)$ to the payment schedule $\left(W_{s,s'}^{**}\right)$ as defined in equation (14). Let $e_{s'}^D$ be an optimal effort for the agent when the payment schedule is $\left(W_{s,s'}^{**}\right)$ instead of $(W_s^*)$:

$$e_{s'}^D \in \arg\max_{e\in[0,\bar{e}]} \sum_s \int_0^{\overline{q}_s} W_{s,s'}^{**}\mathbf{f}(q, s|e)dq - C(e).$$

Since the agent chooses $\hat{e}$ when the payment schedule is $(W_s^*)$ and $e_{s'}^D$ when it is $(W_{s,s'}^{**})$, we must have:

$$\sum_s \int_0^{\overline{q}_s} W_{s,s'}^{**}(q)\mathbf{f}(q, s|e_{s'}^D)dq - C(e_{s'}^D) \geq \sum_s \int_0^{\overline{q}_s} W_{s,s'}^{**}(q)\mathbf{f}(q, s|\hat{e})dq - C(\hat{e}),$$

and

$$\sum_s \int_0^{\overline{q}_s} W_s^*(q)\mathbf{f}(q, s|\hat{e})dq - C(\hat{e}) \geq \sum_s \int_0^{\overline{q}_s} W_s^*(q)\mathbf{f}(q, s|e_{s'}^D)dq - C(e_{s'}^D).$$

Combining these two inequalities, we obtain

$$\sum_s \int_0^{\overline{q}_s} \left[W_{s,s'}^{**}(q) - W_s^*(q)\right] \left[\mathbf{f}(q, s|e_{s'}^D) - \mathbf{f}(q, s|\hat{e})\right] dq \geq 0.$$

Using equation (14), this rewrites simply as:

$$\int_0^{\overline{q}_{s'}} \left[W_{s'}^{q_{s'}}(q) - W_{s'}^*(q)\right]\left[\mathbf{f}(q,s'|e_{s'}^D) - \mathbf{f}(q,s'|\hat{e})\right] dq \geq 0. \tag{15}$$

Since both contracts have the same expected value under effort $\hat{e}$ by construction, and $W_{s'}^{q_{s'}}$ pays the lowest feasible amount for $q < q_{s'}$ and has the highest possible slope for $q > q_{s'}$, there exists $\widehat{q}_{s'} \geq q_{s'}$ such that

$$W_{s'}^{q_{s'}}(q)\left\{\begin{array}{c}\leq\\ \geq\end{array}\right\}W_{s'}^*(q) \text{ for all } q\left\{\begin{array}{c}\leq\\ \geq\end{array}\right\}\widehat{q}_{s'}. \tag{16}$$

We will now show by contradiction that $\hat{e} \leq e_{s'}^D$. Suppose that $\hat{e} > e_{s'}^D$. Then:

$$\begin{aligned}
0 &\leq \textstyle\int_0^{\overline{q}_{s'}} \left[W_{s'}^{q_{s'}}(q) - W_{s'}^*(q)\right]\left[\frac{\mathbf{f}(q,s'|e_{s'}^D)}{\mathbf{f}(q,s'|\hat{e})} - 1\right]\mathbf{f}(q,s'|\hat{e})dq \\
&= \textstyle\int_0^{\overline{q}_{s'}} \left[W_{s'}^{q_{s'}}(q) - W_{s'}^*(q)\right]\frac{\mathbf{f}(q,s'|e_{s'}^D)}{\mathbf{f}(q,s'|\hat{e})}\mathbf{f}(q,s'|\hat{e})dq - \underbrace{\int_0^{\overline{q}_{s'}} \left[W_{s'}^{q_{s'}}(q) - W_{s'}^*(q)\right]\mathbf{f}(q,s'|\hat{e})dq}_{=0} \\
&= \textstyle\int_0^{\widehat{q}_{s'}} \left[W_{s'}^{q_{s'}}(q) - W_{s'}^*(q)\right]\frac{\mathbf{f}(q,s'|e^D)}{\mathbf{f}(q,s'|\hat{e})}\mathbf{f}(q,s'|\hat{e})dq + \int_{\widehat{q}_{s'}}^{\overline{q}_{s'}} \left[W_{s'}^{q_{s'}}(q) - W_{s'}^*(q)\right]\frac{\mathbf{f}(q,s'|e_{s'}^D)}{\mathbf{f}(q,s'|\hat{e})}\mathbf{f}(q,s'|\hat{e})dq \\
&< \textstyle\int_0^{\widehat{q}_{s'}} \left[W_{s'}^{q_{s'}}(q) - W_{s'}^*(q)\right]\frac{\mathbf{f}(\widehat{q}_{s'},s'|e_{s'}^D)}{\mathbf{f}(\widehat{q}_{s'},s'|\hat{e})}\mathbf{f}(q,s'|\hat{e})dq + \int_{\widehat{q}_{s'}}^{\overline{q}_{s'}} \left[W_{s'}^{q_{s'}}(q) - W_{s'}^*(q)\right]\frac{\mathbf{f}(\widehat{q}_{s'},s'|e_{s'}^D)}{\mathbf{f}(\widehat{q}_{s'},s'|\hat{e})}\mathbf{f}(q,s'|\hat{e})dq \\
&= \textstyle\frac{\mathbf{f}(\widehat{q}_{s'},s'|e_{s'}^D)}{\mathbf{f}(\widehat{q}_{s'},s'|\hat{e})}\int_0^{\overline{q}_{s'}} \left[W_{s'}^{q_{s'}}(q) - W_{s'}^*(q)\right]\mathbf{f}(q,s'|\hat{e})dq = 0,
\end{aligned}$$

where, for every $s$, the first line divides and multiplies the expression inside the integral in equation (15) by $\mathbf{f}(q,s'|\hat{e})$; the second line adds a term that equals zero (due to equation (13)); the third line splits the integral between outputs lower and higher than $\widehat{q}_{s'}$; the fourth line uses MLRP supposing that $\hat{e} > e_{s'}^D$ and equation (16); the fifth line uses equation (13). These inequalities give us a contradiction ($0 < 0$), showing that $\hat{e} \leq e_{s'}^D$.

*Step 1.b.* For a given initial contract $(W_s^*)$, repeat the same procedure for every $s \in \{s_1,\ldots,s_S\}$ which is such that the payment schedule under this signal realization does not take the form of debt. The resulting contract, which we denote by $(W_s^D)$, is a debt contract, i.e. the payment schedule takes the form of debt for every $s$. Since the procedure weakly increased the implemented effort for every $s$, the effort implemented by this debt contract, denoted by $e^D$, is weakly larger than the effort $\hat{e}$ to be induced (this directly follows from the fact that the left-hand side ("LHS") of the IC is additive across signals). Online Appendix A shows how to modify this contract to implement the same effort as the initial contract, $\hat{e}$, at a lower cost. Since the resulting contract will still be a debt contract, it satisfies the contracting constraints in equations (6) and (7).

*Step 2. Determining the optimal debt repayment.*

Since any debt contract satisfies bilateral LL and monotonicity, and since we assumed that the condition for the FOA in Lemma 1 holds, the firm's program becomes:

$$\min_{\{q_s\}_{s=1,\dots,S}} \sum_s \int_{q_s}^{\overline{q}_s} (q - q_s)\, \mathbf{f}(q,s|\hat{e})dq \tag{17}$$

subject to

$$\sum_s \int_{q_s}^{\overline{q}_s} (q - q_s)\, \frac{\partial \mathbf{f}}{\partial e}(q,s|\hat{e})dq = C'(\hat{e}), \tag{18}$$

where $\frac{\partial \mathbf{f}}{\partial e}(q,s|\hat{e}) = \frac{\partial \phi_{\hat{e}}^s}{\partial e} f(q|\hat{e},s) + \phi_{\hat{e}}^s \frac{\partial f}{\partial e}(q|\hat{e},s)$. The likelihood ratio can be rewritten as follows:

$$\begin{aligned}
\overline{LR}_s(q,\hat{e}) &= \frac{\int_q^{\overline{q}_s} \left[\frac{\partial \phi_{\hat{e}}^s}{\partial e} f(z|\hat{e},s) + \phi_{\hat{e}}^s \frac{\partial f}{\partial e}(z|\hat{e},s)\right] dz}{\int_q^{\overline{q}_s} \phi_{\hat{e}}^s f(z|\hat{e},s)dz} \\
&= \frac{\int_q^{\overline{q}_s} \frac{\partial \phi_{\hat{e}}^s}{\partial e} f(z|\hat{e},s)dz}{\int_q^{\overline{q}_s} \phi_{\hat{e}}^s f(z|\hat{e},s)dz} + \frac{\int_q^{\overline{q}_s} \phi_{\hat{e}}^s \frac{\partial f}{\partial e}(z|\hat{e},s)dz}{\int_q^{\overline{q}_s} \phi_{\hat{e}}^s f(z|\hat{e},s)dz} \\
&= \frac{\partial \phi_{\hat{e}}^s / \partial e}{\phi_{\hat{e}}^s} + \frac{\int_q^{\overline{q}_s} \frac{\partial f}{\partial e}(z|\hat{e},s)dz}{\int_q^{\overline{q}_s} f(z|\hat{e},s)dz}
\end{aligned}$$

For each fixed $\kappa$ and signal realization $s$, construct the threshold $q_s^*(\kappa)$ as follows:

$$\begin{cases} q_s^*(\kappa) = 0 & \text{if } \overline{LR}_s(0,\hat{e}) > \kappa \\ \overline{LR}_s(q_s^*(\kappa),\hat{e}) = \kappa & \text{if } \overline{LR}_s(0,\hat{e}) \le \kappa \le \lim_{q \nearrow \overline{q}_s} \overline{LR}_s(q,\hat{e}) \\ q_s^*(\kappa) = \overline{q}_s & \text{if } \lim_{q \nearrow \overline{q}_s} \overline{LR}_s(q,\hat{e}) < \kappa \end{cases}. \tag{19}$$

where $\overline{LR}_s(q,\hat{e})$ is strictly increasing in $q$, as already established. The necessary first-order conditions associated with the program in equations (17) and (18) are equation (19) and the binding IC in equation (20), which determine implicitly the cutoff $\kappa$:

$$\sum_s \int_{q_s^*(\kappa)}^{\overline{q}_s} (q - q_s^*(\kappa))\, \frac{\partial \mathbf{f}}{\partial e}(q,s|\hat{e})dq = C'(\hat{e}). \tag{20}$$

where $\kappa := \frac{1}{\mu}$ and $\mu$ is the Lagrange multiplier associated with the IC.

Each $\kappa$ determines $q_s^*(\kappa)$ according to equation (19). From the Intermediate Value Theorem, there exists $\kappa$ that solves equation (20): as $\kappa \searrow -\infty$, the LHS of (20) exceeds $C'(\hat{e})$ since then $q_s^*(\kappa) = 0\ \forall s$ and

$$\sum_s \int_0^{\overline{q}_s} q \frac{\partial \mathbf{f}}{\partial e}(q,s|\hat{e})dq \ge C'(\hat{e})$$

by the assumption in equation (8), and it converges to $0 < C'(\hat{e})$ as $\kappa \nearrow +\infty$.

Finally, the optimal debt repayment schedule $(q_s^*)$ is unique. Indeed, by (19) and Lemma 2, each $q_s^*(\kappa)$ is continuous and non-decreasing in $\kappa$, and strictly increasing for interior solutions. For interior solutions, we have $\frac{\partial}{\partial q_s^*}\int_{q_s^*}^{\overline{q}_s}(q-q_s^*)\frac{\partial \mathbf{f}}{\partial e}(q,s|\hat{e})\,dq = -\int_{q_s^*}^{\overline{q}_s}\frac{\partial \mathbf{f}}{\partial e}(q,s|\hat{e})\,dq = -\kappa\int_{q_s^*}^{\overline{q}_s}\mathbf{f}(q,s|\hat{e})\,dq < 0$, using $\overline{LR}_s(q_s^*,\hat{e}) = \kappa$ and $\kappa > 0$ ($\kappa = 1/\mu > 0$ since relaxing the IC strictly decreases cost). Thus, for each $s$, $\int_{q_s^*(\kappa)}^{\overline{q}_s}(q-q_s^*(\kappa))\frac{\partial \mathbf{f}}{\partial e}(q,s|\hat{e})\,dq$ is continuous and non-increasing in $\kappa$, and strictly decreasing in $\kappa$ at any $\kappa$ for which $q_s^*$ is an interior solution. So the LHS of (20) is non-increasing in $\kappa$. If it were flat on an interval $[\kappa_1,\kappa_2]$, there could not be an interior solution for any $s$ anywhere on that interval, so $q_s^*(\kappa)$ would be constant in $\kappa$ on $[\kappa_1,\kappa_2]$ for every $s$; then, any two solutions $\kappa'$ and $\kappa''$ of (20) would be associated with the same debt repayment schedule $(q_s^*)$. Thus, the cost-minimizing contract that induces $\hat{e}$ is unique. ■

**Proof of Proposition 2:** Start with part (i) of the Proposition. From Proposition 1, there are two possible cases in which the optimal contract does not depend on the signal ($q_{s_1}^* = ... = q_{s_S}^* = q^*$, where $q^*$ is defined in equation (10)): an interior solution $q^* \in (0,\infty)$ and a boundary solution. Using the conditions from equation (19) for interior solutions (i.e. $q_s^* \in (0,\overline{q}_s)\ \forall s$) establishes:

$$\overline{LR}_{s_i}(q^*,\hat{e}) = \overline{LR}_{s_j}(q^*,\hat{e}) = \kappa\ \ \forall s_i, s_j. \tag{21}$$

where $\kappa$ is determined by (20).

We now verify that the solution cannot be at the boundary. For a boundary solution we need either $\overline{LR}_s(0,\hat{e}) > \kappa$ for all $s$ or $\lim_{x\to\overline{q}_s}\overline{LR}_s(x,\hat{e}) < \kappa$ for all $s$. The first case is impossible: if $q_s^* = 0$ for all $s$, the LHS of (20) is $\sum_s\int_0^{\overline{q}_s} q\,\frac{\partial \mathbf{f}}{\partial e}(q,s|\hat{e})\,dq$, which strictly exceeds $C'(\hat{e})$ by assumption (8), contradicting (20). In the second case, the manager always receives zero, violating equation (20) as the IC is not satisfied.

For sufficiency, if $\overline{LR}_{s_i}(q^*,\hat{e}) = \overline{LR}_{s_j}(q^*,\hat{e})$ for all $i,j$, then $q_{s_i}^* = q_{s_j}^* = q^*$ satisfies the FOC from Proposition 1, and since $\overline{LR}_s$ is strictly increasing in $q$, the FOC together with the binding IC uniquely determine the solution.

For part (ii) of the Proposition, if $q \leq \min_s\{q_s^*\}$ then $w_s(q) = 0\ \forall s$, i.e., $w_s(q)$ is independent of $s$. ■

**Proof of Corollary 1:** $Pr(\tilde{q} > q^*|e,s) = \int_{q^*}^{\overline{q}_s} f(z|e,s)dz$ being independent of $s$ for $e \in (\hat{e}-\varepsilon,\hat{e}+\varepsilon)$ for some $\varepsilon > 0$ implies that $\frac{\partial}{\partial e}Pr(\tilde{q} > q^*|\hat{e},s) = \int_{q^*}^{\overline{q}_s}\frac{\partial}{\partial e}f(z|e,s)dz$ is independent of $s$. If in addition $\frac{\partial\phi_e^s/\partial e}{\phi_e^s} = 0$ for any $s$, then $\overline{LR}_s(q^*,\hat{e})$ is independent of $s$, so that condition (i) in Proposition 2 holds. ■

**Proof of Proposition 3:** The proof uses notation from Proposition 1.

Since the manager observes both $q$ and $s$ before reporting and can report $\hat{s} \le \tilde{s}$, no-underreporting (NUR) requires $W_s(q) \ge W_{s'}(q)$ for all $s > s'$ and all $q$: otherwise a manager with signal $s$ would report $s'$. For a debt contract $w_s(q) = \max\{q - q_s, 0\}$ (which is optimal without the NUR constraint, see Proposition 1), $W_s(q) \ge W_{s'}(q)$ for all $q$ if and only if $q_s \le q_{s'}$, so NUR reduces to $q_{s_1} \ge q_{s_2} \ge \cdots \ge q_{s_S}$ (where $s_1 < s_2 < \cdots < s_S$).

We now prove by contradiction that every NUR constraint binds at the optimum. Suppose the optimal NUR-feasible contract is not signal-independent, so that for some adjacent pair $s_k$, $s_{k+1}$ the NUR constraint is slack, i.e. $W_{s_k}(q)$ and $W_{s_{k+1}}(q)$ differ. The binding NUR constraints partition $\{s_1, \ldots, s_S\}$ into consecutive groups, where all signals within a group have the same contract, i.e. $W_{s_k}(q) = W_{s_{k+1}}(q)$. Let the lower-signal group $G^-$ (which includes $s_k$) and the higher-signal group $G^+$ (which includes $s_{k+1}$) denote the groups on either side of the slack constraint.

Because NUR is slack between $G^-$ and $G^+$, each group's common contract can be optimized independently (a small perturbation of one group's contract does not violate NUR between the groups). Within each group, every signal shares the same contract, denoted by $W_G(q)$; replacing $W_G(q)$ with a debt contract $w(q) = \max\{q - q^G, 0\}$ leaves all signals in the group with the same contract, so NUR within the group is preserved. Therefore, debt is feasible for each group. Moreover, debt is optimal. Let $\bar{q}_G := \max_{s\in G} \bar{q}_s$; for $q \in [0, \bar{q}_G]$, let $\bar{G}(q, e) := \sum_{s\in G} \int_{\min\{q,\bar{q}_s\}}^{\bar{q}_s} \mathbf{f}(z, s|e)\, dz$. Since $W_G(0) = 0$ and $\bar{G}(\bar{q}_G, \hat{e}) = 0$, integration by parts rewrites the within-group program as choosing the slope $W_G'(q) \le 1$ (with $W_G'(q) \ge 0$ because condition (b) makes any decreasing portion strictly dominated) to minimize $\int_0^{\bar{q}_G} W_G'(q)\, \bar{G}(q, \hat{e})\, dq$ subject to $\int_0^{\bar{q}_G} W_G'(q)\, \overline{LR}_G(q, \hat{e})\, \bar{G}(q, \hat{e})\, dq = \tau^G$. By condition (b), $\overline{LR}_G(q, \hat{e})$ is strictly increasing in $q$, so the bang-bang solution is $W_G(q) = \max\{q - q^G, 0\}$. Let $q^-$ and $q^+$ be the optimal debt repayments of $G^-$ and $G^+$, respectively. NUR between the groups requires $q^- \ge q^+$, and the assumption that NUR is slack implies $q^- > q^+$.

Let $\kappa := 1/\mu > 0$, where $\mu$ is the Lagrange multiplier associated with the incentive constraint, as in the proof of Proposition 1. For each signal $s$, let $\hat{q}_s$ denote the debt repayment that would be optimal for signal $s$ alone under the same multiplier: $\hat{q}_s$ is the unique solution to $\overline{LR}_s(\hat{q}_s, \hat{e}) = \kappa$ when such a solution exists in $(0, \overline{q}_s)$, and $\hat{q}_s = 0$ or $\hat{q}_s = \overline{q}_s$ at the corresponding corner (as in Proposition 1). The condition of the corollary gives $\overline{LR}_{s_i}(q, \hat{e}) \le \overline{LR}_{s_j}(q, \hat{e})$ for $s_i > s_j$ and all $q$. Evaluating at $\hat{q}_{s_j}$:

$$\overline{LR}_{s_i}(\hat{q}_{s_j}, \hat{e}) \le \overline{LR}_{s_j}(\hat{q}_{s_j}, \hat{e}) = \kappa = \overline{LR}_{s_i}(\hat{q}_{s_i}, \hat{e}).$$

Since $\overline{LR}_{s_i}(q, \hat{e})$ is strictly increasing, this implies $\hat{q}_{s_j} \le \hat{q}_{s_i}$ (the same ordering holds at corner solutions since $\overline{LR}_{s_i}(q, \hat{e})$ is strictly increasing in $q$). That is, higher-indexed signals have weakly higher unconstrained debt repayments.

The first-order condition for the common debt repayment $q^G$ of a group $G$ is:

$$\sum_{s\in G} A_s(q^G)\left(\overline{LR}_s(q^G,\hat{e})-\kappa\right)=0,$$

where $A_s(q) := \int_q^{\overline{q}_s} \mathbf{f}(z,s|\hat{e})\,dz \geq 0$ (with $A_s(q)>0$ for $q<\overline{q}_s$). For $q<\min_{s\in G}\hat{q}_s$, each $\overline{LR}_s(q,\hat{e})<\kappa$, so the LHS is strictly negative; for $q>\max_{s\in G}\hat{q}_s$, every signal with $A_s(q)>0$ satisfies $\overline{LR}_s(q,\hat{e})>\kappa$, so the LHS is strictly positive. So every root, in particular $q^G$, is in the interval $[\min_{s\in G}\hat{q}_s,\, \max_{s\in G}\hat{q}_s]$. Since every signal in $G^-$ has a lower index than every signal in $G^+$,

$$q^- \;\leq\; \max_{s\in G^-}\hat{q}_s \;\leq\; \min_{s'\in G^+}\hat{q}_{s'} \;\leq\; q^+,$$

contradicting $q^->q^+$. In sum, every NUR constraint binds and the contract is independent from the signal. ■

Online Appendix

When Is (Performance-Sensitive) Debt Optimal?

# A Step 1.b. of the proof of Proposition 1

By assumption, the contract $(W_s^*)$ is incentive compatible and the FOA holds, so that:

$$\sum_s \int_0^{\overline{q}_s} W_s^*(q) \frac{\partial \mathbf{f}}{\partial e}(q, s|\hat{e}) dq \;\; = \;\; C'(\hat{e}). \tag{22}$$

Let $\varepsilon$ be an arbitrarily large constant which satisfies the following two conditions: (i) $\varepsilon > \max\{q_1, \dots, q_S\}$, and (ii):

$$\sum_s \int_{\min\{\varepsilon, \overline{q}_s\}}^{\overline{q}_s} (q - \varepsilon) \frac{\partial \mathbf{f}}{\partial e}(q, s|\hat{e})\, dq < C'(\hat{e}). \tag{23}$$

There exists such $\varepsilon$: as $\varepsilon$ gets large enough, each integral vanishes. Consider the subset of $\{s_1, \dots, s_S\}$ such that:

$$\int_{\varepsilon}^{\overline{q}_s} (q - \varepsilon) \frac{\partial \mathbf{f}}{\partial e}(q, s|\hat{e}) dq < \int_0^{\overline{q}_s} W_s^*(q) \frac{\partial \mathbf{f}}{\partial e}(q, s|\hat{e}) dq, \tag{24}$$

and denote this subset by $\mathcal{S}$. $\mathcal{S}$ is nonempty (if it were, summing over signals in equation (24) and comparing with equation (23) would yield the contradiction that equation (22) does not hold).

For any $s \in \mathcal{S}$, we claim and establish below that there exists $\hat{q}_s \geq q_s$ which solves:

$$\int_{\hat{q}_s}^{\overline{q}_s} (q - \hat{q}_s) \frac{\partial \mathbf{f}}{\partial e}(q, s|\hat{e}) dq = \int_0^{\overline{q}_s} W_s^*(q) \frac{\partial \mathbf{f}}{\partial e}(q, s|\hat{e}) dq. \tag{25}$$

For a given $s \in \mathcal{S}$, using the IC with the FOA and the results on effort under the two payment schedules $W_s^*$ and $W_s^{q_s}$ established in Step 1.a. gives the following equation:

$$\int_{q_s}^{\overline{q}_s} (q - q_s) \frac{\partial \mathbf{f}}{\partial e}(q, s|\hat{e}) dq \;\; \geq \;\; \int_0^{\overline{q}_s} W_s^*(q) \frac{\partial \mathbf{f}}{\partial e}(q, s|\hat{e}) dq \tag{26}$$

For each signal $s \in \mathcal{S}$, there are two cases. If, for a given $s$, equation (26) holds as an equality, then set $\hat{q}_s = q_s$, so that equation (25) holds. If, for a given $s$, equation (26) holds as a strict inequality, then for this $s$, there is $\hat{q}_s \in (q_s, \varepsilon)$ such that equation (25) holds because of the intermediate value theorem, which for a given $s$ we apply on the interval $[q_s, \varepsilon]$. The theorem applies because of equation (24), equation (26) as a strict inequality, and $\int_z^{\overline{q}_s} (q-z)\, \frac{\partial \mathbf{f}}{\partial e}(q, s|\hat{e})dq$ is a continuous function of $z$.

First, if $\mathcal{S} = \{s_1, \ldots, s_S\}$ or if

$$\sum_{\tilde{s} \notin \mathcal{S}} \int_{\varepsilon}^{\overline{q}_{\tilde{s}}} (q-\varepsilon)\, \frac{\partial \mathbf{f}}{\partial e}(q, \tilde{s}|\hat{e})dq + \sum_{\tilde{s} \in \mathcal{S}} \int_{\hat{q}_{\tilde{s}}}^{\overline{q}_{\tilde{s}}} (q-\hat{q}_{\tilde{s}})\, \frac{\partial \mathbf{f}}{\partial e}(q, \tilde{s}|\hat{e})dq = C'(\hat{e}), \tag{27}$$

where for each $s \in \mathcal{S}$, the contractual debt repayment $\hat{q}_s$ is implicitly defined in equation (25), then for any $s \in \mathcal{S}$ use the payment schedule:

$$W_s^{\hat{q}_s}(q) = \max\{0, q - \hat{q}_s\}, \tag{28}$$

and for any $s \notin \mathcal{S}$ the contractual debt repayment is set at $\varepsilon$.

Second, if $\mathcal{S} \subset \{s_1, \ldots, s_S\}$ and the condition in equation (27) does not hold, then let the signals in $\mathcal{S}$ be ordered such that $\mathcal{S} = \{s_1^{\mathcal{S}}, \ldots, s_N^{\mathcal{S}}\}$, with $N \geq 1$ (since $\mathcal{S}$ is nonempty). Denote by $\mathcal{S}^c$ the complement of $\mathcal{S}$. For any $s \in \mathcal{S}^c$, set the contractual debt repayment at $\varepsilon$. If

$$\sum_{\tilde{s} \in \mathcal{S}^c \cup \{s_1^{\mathcal{S}}\}} \int_{\varepsilon}^{\overline{q}_{\tilde{s}}} (q-\varepsilon)\, \frac{\partial \mathbf{f}}{\partial e}(q, \tilde{s}|\hat{e})dq + \sum_{\tilde{s} \in \mathcal{S} \setminus \{s_1^{\mathcal{S}}\}} \int_{\hat{q}_{\tilde{s}}}^{\overline{q}_{\tilde{s}}} (q-\hat{q}_{\tilde{s}})\, \frac{\partial \mathbf{f}}{\partial e}(q, \tilde{s}|\hat{e})dq < C'(\hat{e}), \tag{29}$$

then let $\check{q}_{s_1^{\mathcal{S}}}$ be implicitly defined by:

$$\sum_{\tilde{s} \in \mathcal{S}^c} \int_{\varepsilon}^{\overline{q}_{\tilde{s}}} (q-\varepsilon)\, \frac{\partial \mathbf{f}}{\partial e}(q, \tilde{s}|\hat{e})dq + \sum_{\tilde{s} \in \mathcal{S} \setminus \{s_1^{\mathcal{S}}\}} \int_{\hat{q}_{\tilde{s}}}^{\overline{q}_{\tilde{s}}} (q-\hat{q}_{\tilde{s}})\, \frac{\partial \mathbf{f}}{\partial e}(q, \tilde{s}|\hat{e})dq + \int_{\check{q}_{s_1^{\mathcal{S}}}}^{\overline{q}_{s_1^{\mathcal{S}}}} \left(q-\check{q}_{s_1^{\mathcal{S}}}\right) \frac{\partial \mathbf{f}}{\partial e}(q, s_1^{\mathcal{S}}|\hat{e})dq = C'(\hat{e}).$$

$\check{q}_{s_1^{\mathcal{S}}}$ exists and is larger than $\hat{q}_{s_1^{\mathcal{S}}}$ by application of the intermediate value theorem to the interval $[\hat{q}_{s_1^{\mathcal{S}}}, \varepsilon]$, with equations (29) and (30):

$$\sum_{\tilde{s} \in \mathcal{S}^c} \int_{\varepsilon}^{\overline{q}_{\tilde{s}}} (q-\varepsilon)\, \frac{\partial \mathbf{f}}{\partial e}(q, \tilde{s}|\hat{e})dq + \sum_{\tilde{s} \in \mathcal{S}} \int_{\hat{q}_{\tilde{s}}}^{\overline{q}_{\tilde{s}}} (q-\hat{q}_{\tilde{s}})\, \frac{\partial \mathbf{f}}{\partial e}(q, \tilde{s}|\hat{e})dq > C'(\hat{e}). \tag{30}$$

In turn, we get equation (30) because of equation (22) on the one hand, and on the other hand because for signals in $\mathcal{S}$, the contractual debt repayment $\hat{q}_s$ satisfies equation (25), for signals in $\mathcal{S}^c$ the condition in equation (24) does not hold, and equation (27) does not hold here (see above). If condition (29) holds, then set the contractual debt repayment of signal $s_1^{\mathcal{S}}$ at $\check{q}_{s_1^{\mathcal{S}}}$, and set the contractual debt repayment at $\hat{q}_s$ for other signals in $\mathcal{S}$. If condition (29) does not hold, then set $\hat{q}_{s_1^{\mathcal{S}}} = \varepsilon$, repeat the same steps with signal $s_2^{\mathcal{S}}$ (we omit explicit formulation of these steps for brevity), and continue repeating these steps to additional signals in $\mathcal{S}$ until, for a signal $s_i^{\mathcal{S}}$, with $i \leq N$, condition

$$\sum_{\tilde{s} \in \mathcal{S}^c \cup \{s_1^{\mathcal{S}}, \ldots, s_i^{\mathcal{S}}\}} \int_{\varepsilon}^{\overline{q}_{\tilde{s}}} (q - \varepsilon) \frac{\partial \mathbf{f}}{\partial e}(q, \tilde{s}|\hat{e}) dq + \sum_{\tilde{s} \in \mathcal{S} \setminus \{s_1^{\mathcal{S}}, \ldots, s_i^{\mathcal{S}}\}} \int_{\hat{q}_{\tilde{s}}}^{\overline{q}_{\tilde{s}}} (q - \hat{q}_{\tilde{s}}) \frac{\partial \mathbf{f}}{\partial e}(q, \tilde{s}|\hat{e}) dq < C'(\hat{e}) \quad (31)$$

is satisfied, in which case set the contractual debt repayment of signal $s_i^{\mathcal{S}}$ at $\check{q}_{s_i^{\mathcal{S}}}$, which is implicitly defined by:

$$\sum_{\tilde{s} \in \mathcal{S}^c \cup \{s_1^{\mathcal{S}}, \ldots, s_{i-1}^{\mathcal{S}}\}} \int_{\varepsilon}^{\overline{q}_{\tilde{s}}} (q - \varepsilon) \frac{\partial \mathbf{f}}{\partial e}(q, \tilde{s}|\hat{e}) dq + \sum_{\tilde{s} \in \mathcal{S} \setminus \{s_1^{\mathcal{S}}, \ldots, s_i^{\mathcal{S}}\}} \int_{\hat{q}_{\tilde{s}}}^{\overline{q}_{\tilde{s}}} (q - \hat{q}_{\tilde{s}}) \frac{\partial \mathbf{f}}{\partial e}(q, \tilde{s}|\hat{e}) dq$$
$$+ \int_{\check{q}_{s_i^{\mathcal{S}}}}^{\overline{q}_{s_i^{\mathcal{S}}}} \left(q - \check{q}_{s_i^{\mathcal{S}}}\right) \frac{\partial \mathbf{f}}{\partial e}(q, s_i^{\mathcal{S}}|\hat{e}) dq = C'(\hat{e}).$$

$\check{q}_{s_i^{\mathcal{S}}}$ exists and is larger than $\hat{q}_{s_i^{\mathcal{S}}}$ because of the same arguments used above. Because of equation (23), condition (31) will be satisfied for a signal $s_i^{\mathcal{S}}$, with $i \leq N$. If $i < N$, for signals $s \in \{s_{i+1}^{\mathcal{S}}, \ldots, s_N^{\mathcal{S}}\}$ in $\mathcal{S}$, set the contractual debt repayment to $\hat{q}_s$ as in equation (25).

In sum, for each given $s$, the new contract is a debt contract with a repayment of either $\hat{q}_s$ or $\check{q}_s$ or $\varepsilon$, such that $\hat{q}_s \geq q_s$ if $\hat{q}_s$ exists, $\check{q}_s > \hat{q}_s \geq q_s$ if $\check{q}_s$ and $\hat{q}_s$ exist, and $\varepsilon > q_s$. Since by construction the debt contract $(W_s^D)$ with contractual debt repayments $q_s$ has the same cost as the initial contract $(W_s^*)$, and the cost of a debt contract for the principal at a given $s$ is decreasing in the contractual debt repayment at this signal $s$, the new debt contract achieves the same effort $\hat{e}$ as the initial contract $(W_s^*)$ at a lower cost.

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The ECGI will produce and disseminate high quality research while remaining close to the concerns and interests of corporate, financial and public policy makers. It will draw on the expertise of scholars from numerous countries and bring together a critical mass of expertise and interest to bear on this important subject.

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