Socially Responsible Divestment

Finance Working Paper N° 823/2022
November 2024

Alex Edmans
London Business School, CEPR and ECGI

Doron Levit
University of Washington, CEPR and ECGI

Jan Schneemeier
Michigan State University

ECGI Working Paper Series in Finance

Socially Responsible Divestment

Working Paper N° 823/2022
November 2024

Alex Edmans
Doron Levit
Jan Schneemeier

We thank Andres Almazan, Ryan Bubb, Cecilia Bustamante, Davide Cianciaruso, Eduardo Dávila, Shaun Davies, Thomas Geelen, Tom Gosling, Daniel Green, Oliver Hart, Jeong Ho Kim, Abraham Lioui, Evgeny Lyandres, Andrey Malenko, Giorgia Piacentino, Alejandro Rivera, Lin Shen, Tracy Wang, Ed Van Wesep, Haotian Xiang and seminar/conference participants at the Accounting Research Workshop, AFA, Alberta, CFEA, CICF, ECGI conference on Corporate Purpose, Stakeholderism and ESG, EFA, Emory, Financial Reporting Council, FMA, HEC-HKUST Sustainable Finance Seminar, Indiana, Luxembourg Conference in Sustainable Finance, MIT Asia Conference in Accounting, NBER, NYU/Penn Conference on Law and Finance, Paris December Finance Meeting, RCFS Winter Conference, SFS Cavalcade, UN PRI Academic Network Week, UNC, Vienna, Weinberg Center Corporate Governance Symposium, Young Scholars Finance Consortium, and Zurich for valued input.

Abstract

We study the optimal investment strategy for a responsible investor concerned with financial returns and societal impact. Unconditional exclusion of “brown” stocks starves them of capital, reducing externalities. A conditional strategy -- buying a brown firm if it has taken a corrective action -- allows it to expand, but incentivizes reform and yields the investor profits. A lower concern for profits makes an investor less likely to condition; thus, a greater concern for externalities may reduce effectiveness in curbing externalities. We derive novel implications for how responsible investing strategies are affected by ES disclosure, ES ratings, arbitrageurs, and fiduciary duty.

Keywords: Socially responsible investing, sustainable investing, externalities, exclusion, divestment, tilting, exit, governance.

JEL Classifications: D62, G11, G34

Alex Edmans*
Professor of Finance
London Business School
Regent’s Park
London, NW1 4SA, United Kingdom
phone: +44 20 7000 8258
e-mail: aedmans@london.edu

Doron Levit
Professor of Business Economics
University of Washington
434 Paccar Hall, 4273 E Stevens Way NE
Seattle, WA 98195, United States
phone: +1 206-543-3021
e-mail: dlevit@uw.edu

Jan Schneemeier
Assistant Professor of Finance
Michigan State University
667 N Shaw Ln, East Lansing
MI 48824, United States
e-mail: schneem8@msu.edu

*Corresponding Author

Socially Responsible Divestment*

Alex Edmans
LBS, CEPR, and ECGI

Doron Levit
University of Washington, CEPR, and ECGI

Jan Schneemeier
Michigan State University

November 4, 2024

Abstract

We study the optimal investment strategy for a responsible investor concerned with financial returns and societal impact. Unconditional exclusion of "brown" stocks starves them of capital, reducing externalities. A conditional strategy – buying a brown firm if it has taken a corrective action – allows it to expand, but incentivizes reform and yields the investor profits. A lower concern for profits makes an investor less likely to condition; thus, a greater concern for externalities may reduce effectiveness in curbing externalities. We derive novel implications for how responsible investing strategies are affected by ES disclosure, ES ratings, arbitrageurs, and fiduciary duty.

Keywords: Responsible investing, exclusion, divestment, tilting, governance.

JEL Classification: D62, G11, G34

*aedmans@london.edu, dlevit@uw.edu, schneem8@msu.edu. We thank Andres Almazan, Ryan Bubb, Cecilia Bustamante, Davide Cianciaruso, Eduardo Dávila, Shaun Davies, Thomas Geelen, Tom Gosling, Daniel Green, Oliver Hart, Jeong Ho Kim, Abraham Lioui, Evgeny Lyandres, Andrey Malenko, Giorgia Piacentino, Alejandro Rivera, Lin Shen, Tracy Wang, Ed Van Wesep, Haotian Xiang and seminar/conference participants at the Accounting Research Workshop, AFA, Alberta, CFEA, CICF, ECGI conference on Corporate Purpose, Stakeholderism and ESG, EFA, Emory, Financial Reporting Council, FMA, HEC-HKUST Sustainable Finance Seminar, Indiana, Luxembourg Conference in Sustainable Finance, MIT Asia Conference in Accounting, NBER, NYU/Penn Conference on Law and Finance, Paris December Finance Meeting, RCFS Winter Conference, SFS Cavalcade, UN PRI Academic Network Week, UNC, Vienna, Weinberg Center Corporate Governance Symposium, Young Scholars Finance Consortium, and Zurich for valued input.

Responsible investing – the incorporation of environmental, social, and governance ("ESG") factors into investment decisions – is becoming increasingly mainstream. In 2006, the United Nations established the Principles for Responsible Investment, which was signed by 63 investors managing a total of $6.5 trillion. By March 2023, this had grown to 4,841 investors, representing $121 trillion.

A particular goal of environmental and social ("ES") investing is to reduce a company's externalities, which can be achieved through two channels. The first is engagement, which even when effective is often costly. For example, Engine No. 1 spent $30 million placing three climate-friendly directors onto Exxon's board, compared to its stake of $40 million. The second is divestment: selling "brown" companies that exert negative externalities, increasing their cost of capital and hindering their expansion. This paper studies the optimal divestment strategy to mitigate externalities.

Under the cost of capital channel, the most powerful divestment strategy is blanket exclusion of externality-producing industries, a strategy pursued by many investors. The Net Zero Asset Managers Initiative commits members to net-zero aligned portfolios, which are most easily achieved through divesting emitting companies. Beyond climate, $4 trillion of assets worldwide is invested in exclusionary screening strategies.[1] Turning to asset owners, 1,600, collectively managing $40 trillion, have publicly committed to divest from fossil fuels. Many asset owners expect asset managers to exclude at least some sectors, and ask about such exclusions in their due diligence questionnaires. The media and the public sometimes protest against investors who own certain industries: in 2020, Extinction Rebellion members dug up a lawn outside Trinity College, Cambridge due to its investment in fossil fuels.

However, this argument considers only one channel through which divestment can affect a company's real actions – the primary markets channel, whereby divestment affects new capital raising. As the survey of Bond, Edmans, and Goldstein (2012) points out, investors can also have real effects through a secondary markets channel. Specifically, buying or selling shares leads to the stock price reflecting a manager's real actions, thus rewarding or punishing him for taking them. Even if a firm is in an irremediably brown sector, where externalities are always negative, the manager may be able to take corrective actions to mitigate these externalities. Blanket exclusion is an unconditional strategy that fails to reward such actions because the firm is divested no matter what. Thus, it may be optimal for a responsible investor to pursue a

[1] Global Sustainable Investment Review (2022).

conditional strategy that invests in brown firms that have reduced their per-unit externalities – even though doing so allows such firms to expand and emit externalities over a larger scale. Practitioners sometimes refer to the latter as a "tilting" strategy, where the investor tilts away from a brown industry but does not exclude it entirely.

We build a model in which responsible investment affects firm behavior through both above channels. There is a brown firm that emits negative externalities. The firm's manager can take a non-contractible corrective action that reduces both externalities and firm value. The firm also raises capital which it uses to fund an expansion, increasing both firm value and externalities. The firm's manager is concerned with both firm value and the stock price, although the results continue to hold if he is concerned with firm value alone.

The firm is owned by a continuum of risk-averse, profit-motivated, atomistic investors ("households"), and a risk-neutral responsible investor who is able to take large positions and have price impact; we thus refer to her as a blockholder. The blockholder's objective consists of both trading profits and externalities. Indeed, fiduciary duty requires asset managers of even sustainable funds to place significant weight on financial returns. It may seem that the blockholder will only incentivize the externality-reducing action if her concern for externalities (relative to profits) is sufficiently high, but we show that the opposite is typically true.

The blockholder announces an investment strategy that depends on whether the firm takes a corrective action. Without loss of generality, the strategy is either "conditional" where she invests more if it takes the action, or "unconditional" where she invests the same amount irrespective of the action. If this same amount is zero, the strategy is "full exclusion;" if it is positive it is "partial exclusion." We initially assume that the blockholder can commit to her strategy. Some fund mandates prohibit investment in certain industries; deviation will lead to client withdrawals and potentially regulatory action. Other funds announce a conditional strategy which involves investing in leaders in brown industries, such as funds with the "Sustainability Improvers" label in the U.K. Deviating and excluding entire industries may be inconsistent with the fund mandate, increase tracking error, reduce risk-adjusted returns, or lead to investors withdrawing to cheaper passive funds that pursue exclusion.

We show that under the optimal investment strategy, the blockholder conditions if and only if the effectiveness of the corrective action exceeds a threshold and excludes otherwise. The threshold depends on the weight the blockholder places on trading profits. This is because, when choosing her strategy, the blockholder balances three forces. First, since the brown firm continues to produce negative externalities even under the corrective action, the blockholder's concern for externalities leads her to want to minimize its size. She does so through full exclusion – by holding none of the brown firm's shares, they have to be held entirely by risk-averse households, who require a risk premium for doing so. This minimizes the stock price, similar to Heinkel, Kraus, and Zechner (2001), and thus the new funds the firm can raise. Second, the blockholder's concern for externalities leads her to also want to incentivize the action. Exclusion provides no such incentives, since the firm is always divested. A conditional strategy rewards the action – by buying shares, the blockholder reduces the number that must be held by households, thus increasing the stock price. Third, the blockholder's concern for profits leads her to want to buy more shares, up to a point, because doing so allows her to make trading profits: there are gains from trade when the risk-neutral blockholder buys shares from risk-averse households.

The first two forces explain why the blockholder follows a threshold strategy. If the action is effective at reducing the externality, the blockholder reduces externalities more effectively by inducing the action (a conditional strategy) than by starving the company of capital (an unconditional strategy). Exclusion may thus be optimal for industries such as controversial weapons, where it is difficult to reduce the harm produced. In contrast, a conditional strategy may be preferred for fossil fuels, where managers can take corrective actions such as developing clean energy.[2] In addition, incentivizing managers to take corrective actions requires fewer purchases, and thus less expansion, if their stock price concerns are high. As a result, externalities may fall if the manager is more short-term oriented, since the blockholder is more likely to condition and induce the action. This observation contrasts common concerns that managerial short-termism is socially undesirable.

The blockholder's choice of strategy, however, is complicated by the fact that the blockholder is not only concerned with externalities but also profits, and this implies that the threshold depends on the blockholder's profit motive. Moreover, the way that it does so depends on the other parameters in the model. For some parameter values, the effectiveness threshold is monotonically decreasing in the blockholder's profit concerns. Conditioning involves the manager buying shares if the action is taken, earning trading profits. Thus, if the blockholder cares more about profits, the action does not need to be as effective to induce her to condi-

[2]Cohen, Gurun, and Nguyen (2024) show that the fossil fuel industry produces more green patents than nearly any other sector, suggesting that companies within this industry can take corrective actions.

tion: indeed, even if conditioning is less effective at reducing externalities than exclusion, the blockholder's profit motive may lead her to nonetheless condition. This contrasts the intuition that the blockholder will induce externality reduction only if her concern for externalities is sufficiently high.

However, this monotonic result only holds if the cost of the action is low, the firm is raising little new capital, or the manager's stock price concerns, firm risk, and household risk aversion are high. If this is not the case, the blockholder needs to buy a large number of shares to incentivize the action. This conflicts with the blockholder's profit motive because buying more shares increases profits only up to a point: after that point, her price impact becomes too large, lowering her profits overall. Thus, if the blockholder needs to buy more than the profit-maximizing amount to induce the action, conditioning requires her to sacrifice trading profits and so the effectiveness threshold is U-shaped in profit concerns. If profit concerns are sufficiently high, the blockholder buys the optimal number of shares to maximize profits, which is too small to induce the action.

These results have nuanced implications for policymakers. In most cases, they imply that ensuring that funds fulfill their fiduciary duty to generate financial returns may support, rather than hinder, social returns as often believed. However, if corrective actions are particularly costly to firm value, then too strict a definition of fiduciary duty may deter investors from inducing them. This is not because investor profits are eroded by costly actions: since they are publicly observable, they are already reflected in the stock price and do not affect trading profits. Instead, it is because costly actions can only be induced by purchasing a large number of shares. More broadly, by analyzing the trade-offs between conditional and unconditional divestment strategies, we demonstrate when each approach is most effective at addressing externalities and which is more likely to be adopted by responsible investors.

We then consider the case in which the blockholder cannot commit to a trading strategy, perhaps because holdings can be window-dressed just before reporting periods. Then, a blockholder concerned only with externalities cannot induce the action. Once the action has been taken, she has no incentive to buy shares because doing so will help the firm expand. Thus, any promise to buy shares upon the reform is non-credible, and full exclusion is the only equilibrium divestment strategy. However, a sufficiently profit-motivated investor will buy shares to earn trading profits. She is more willing to do so if the action has been taken, as the action minimizes the additional externalities created by her share purchases. Therefore, a profit motivation makes it credible for the blockholder to condition. Indeed, even when conditioning is the most effective strategy at reducing externalities, an investor with a sufficiently high weight on externalities is unable to commit to it: a greater concern with externalities may, paradoxically, lead to more externalities. Clients and the media often view exclusion as the most responsible strategy, but funds may pursue it not becasue it is effective but because they are unable to commit to the more effective conditional strategy.

Next, we extend the model to the case in which the corrective action is unobservable, and so the investor can only condition her investment on a noisy public signal of the action. The noise may result from imperfect ES ratings, poor disclosure standards, or attempts by the brown firm to greenwash. The noisier the signal, the greater the reward the investor needs to offer the manager to induce the action, and the more likely she is to choose exclusion. This result highlights a new benefit of ES disclosure – it allows investors to induce corrective actions without having to promise large amounts of capital.

Finally, a common criticism of divestment is that arbitrageurs can buy divested stocks, attenuating the price impact. In our final extension, we introduce an arbitrageur who is purely profit motivated, like households, but has price impact. He optimally buys half the shares that are not purchased by the blockholder, lessening the impact of the blockholder's trading decisions. This makes a conditional strategy less effective (since the arbitrageur partially offsets the blockholder's trades, she needs to promise an even larger purchase to induce the action), and an unconditional strategy also less effective (since he buys up underpriced stock and reduces the impact on the cost of capital). Since the arbitrageur buys half of the free float, his impact is greater on the unconditional strategy, where the blockholder's trade is lower and the free float is higher, than on the conditional strategy. Therefore, conditioning is more likely to be optimal in the presence of arbitrageurs.

1 Related Literature

This paper is related to responsible investment theories in which conditioning capital on a corrective action induces the action. Heinkel, Kraus, and Zechner (2001) feature a set of responsible investors that refuse to hold brown stocks. Divestment by them requires financial investors to hold more stocks, increasing the risk they bear and lowering the stock price. As a result, some firms take corrective actions to obtain a higher price. In Oehmke and Opp

(2024), a responsible investor can induce a clean technology by buying a green bond that stipulates this technology at a loss, and so the firm issues it despite the stipulation. In our paper, the blockholder always makes a profit from investing, and so a conditional strategy does not require investors willing to make losses.[3] Neither paper considers unconditional strategies – starving a company of capital to hinder its expansion and reduce its externalities. In Heinkel et al. (2001), responsible investors' demands are deterministic – they automatically buy stocks of a reformed brown company – while we study the choice between divestment strategies by a strategic investor. If they extended their model to allow investors to choose a strategy, conditioning would always be optimal because there is no cost; in our paper, it allows the firm to expand and generate more externalities. Oehmke and Opp (2024) intentionally shut off exclusion by considering risk-neutral investors and a perfectly elastic supply of capital from financial investors.[4]

In Landier and Lovo (2024), a responsible fund sets a cap to externalities which caps output and thus firm profits, and withholds financing from any firm that exceeds this cap – a conditional strategy. As a result, the firm may limit externalities to safeguard financing. They also show that unconditional exclusion has no effect on externalities unless the responsible fund is very large, in which case exclusion leads to significantly lower returns than conditional strategies. Thus, when they allow clients to choose funds, they consider only conditional strategies. In our model, exclusion is sometimes optimal, consistent with the common usage of this strategy in reality, and we derive predictions for the settings in which responsible investors should pursue exclusion – how this depends on the firm (the effectiveness and cost of corrective actions, firm risk, and the manager's contract), the risk aversion of other investors, and the responsible investor's profit motives. Another difference is that the papers making primary capital conditional on a corrective action require the action to be both known by the investor ex ante and also contractible. In reality, investors may not know what actions a company can take ex ante, even if they can evaluate whether it has done so ex post. In the language of Dow and

[3] In a similar vein, in Chowdhry, Davies, and Waters (2018), the impact investor makes a donation in return for the entrepreneur committing to pursue social goals. Financial returns are relevant since fiduciary duty limits asset managers' ability to sacrifice profits to achieve social objectives (see, e.g., Gosling and MacNeil (2023); Edmans, Gosling, and Jenter (2024)); BlackRock started opposing many climate resolutions because they did not "consider them to be consistent with our clients' long-term financial interests."

[4] Models of governance through exit, such as Admati and Pfleiderer (2009), Edmans (2009), and Edmans and Manso (2011), also feature the idea that making divestment conditional upon an action induces the action. These papers do not feature externalities and thus there is no role for unconditional exclusion.

Gorton (1997), they may have retrospective but not prospective information – their expertise is evaluating not running companies, or stock selection rather than activism.[5] We show that when corrective actions are neither contractible nor easily observable by regulators and clients, fund managers' profit motives—rather than concerns about externalities—drive managers to take externality-reducing actions, effectively substituting for the inability to commit to conditional strategies. While we study which strategy maximizes impact for a given firm, Green and Roth (2024) study which firms to invest in to maximize impact. They find that, if responsible investors buy companies that create positive externalities, their impact may be limited as they displace traditional investors.

Some empirical studies examine the effectiveness of divestment (either conditional or unconditional), finding mixed results. Teoh, Welch, and Wazzan (1999) show that the South Africa exclusion campaign had a negligible effect on company valuations. The model and calibration of Berk and van Binsbergen (2024) show that ES-motivated exclusion has little effect on the cost of capital, because arbitrageurs can buy the underpriced stocks. In our model, the investor can pursue conditional as well as unconditional strategies and we find that arbitrageurs increase the relative effectiveness of the former. Berk and van Binsbergen's empirical estimation studies the effect of being added to the FTSE USA 4 Good Select Index, i.e. the price impact of positive screening; in our model, exclusion involves negative screening. It is thus closer to Zerbib (2022), which shows that exclusion leads to a return premium of 2.79% per year. Green and Vallée (2024) find that bank divestment policies reduce total assets for coal firms they have relationships with, beause there is limited substitution from lenders without divestment policies. Pastor, Stambaugh, and Taylor (2022) document a significant return

[5] While we consider the relative effectiveness of different divestment strategies, other papers compare divestment with other mechanisms to reduce externalities. Broccado, Hart, and Zingales (2022) and Gollier and Pouget (2022) show that engagement is more effective than divestment if investors have a binding vote on whether firms are green or brown. They provide the normative implication that such a vote may be useful; however, no such vote currently exists. Other papers on responsible investment study different questions. Davies and Van Wesep (2018) demonstrate that the low price from divestment raises the number of shares granted to the manager if his equity-based pay is fixed in dollar terms, paradoxically rewarding him. Pedersen, Fitzgibbons, and Pomorski (2021) focus on the asset pricing implications of responsible investing. Goldstein et al. (2022) show that responsible investors can increase the cost of capital, because their trades reflect ES rather than financial performance, thus making the stock price less informative about financials. Pastor, Stambaugh, and Taylor (2021) model how greater taste for green companies increases their valuation and reduces expected returns. The stock price does not affect investment because it is financed by internal cash flow. These papers do not study different strategies pursued by responsible investors – instead, investors' demands are automatic given their tastes.

differential between German green bonds and otherwise-equivalent non-green twins, also implying limited arbitrage. More broadly, despite the presence of arbitrageurs, Koijen and Yogo (2019) find a significant effect of institutional investor demand on asset prices.

2 The Model

2.1 Players and Timing

We consider a single firm with a risk-neutral manager ("M"). The firm is in a brown industry and thus emits negative externalities to be specified later. The initial number of shares is normalized to one. The financial market consists of a continuum of risk-averse, profit-motivated, atomistic investors ("households"), indexed by $i \in [0,1]$, and a risk-neutral investor that cares about both trading profits and externalities. We refer to this investor as a blockholder ("B").[6]

There are four dates, $t \in \{0,1,2,3\}$. At $t=0$, B announces an investment strategy $x(a)$ that depends on a publicly-observable action $a \in \{0,1\}$ taken by the firm. We sometimes refer to $a=1$ as the "corrective action", or simply the "action", such as a fossil fuel company investing in clean energy, an alcohol company reducing the alcohol content of its drinks and youth-targeted advertising, or a hard-to-abate sector (such as steel, cement, and ammonia) reducing its carbon emissions, pollution, and biodiversity impact. This action is non-contractible. It is very difficult to specify ex ante what corrective actions a company can take, particularly because it depends on factors such as technological feasibility (e.g. the affordability of renewable energy or carbon capture). In addition, a company's ES performance depends on many qualitative and quantitative dimensions, and it is impossible to measure all of them and put them into a contract.

The "conditional" strategy $x(1) > x(0)$ involves B holding more of the stock if $a=1$. The "unconditional" strategy $x(1) = x(0)$ involves B holding the same stake in the firm regardless of its action. We distinguish between two types: $x(1) = x(0) = 0$ represents "full exclusion" and $x(1) = x(0) > 0$ represents "partial exclusion".[7] The key distinction between the conditional

[6] The results would be qualitatively unchanged if B were also risk-averse, since lower investment by the blockholder would mean that each household has to hold more shares, leading to inefficient risk-sharing as in Heinkel, Kraus, and Zechner (2001).

[7] We refer to the latter strategy as "partial exclusion" since B buys fewer shares than a purely profit motivated investor, and thus partially excludes the firm from her portfolio.

and unconditional strategy is that the former provides incentives to take corrective actions. Under the unconditional strategy, the only force is increasing the firm's cost of capital and preventing its expansion. The model analyzes the tradeoff between these two forces.

The blockholder's initial stake is zero, but all of our results would be unchanged if she had a positive initial stake, in which case "exclusion" would involve divestment, not just non-investment. Practitioners sometimes refer to a conditional strategy as "tilting" or "best-in-class". We build a parsimonious model with only one firm so the best-in-class concept does not apply literally, but if the model were extended to multiple firms, buying more of those that take corrective actions involves investing in those that are best-in-class.[8]

Initially, we assume that B can commit to the strategy. An asset manager can launch a fund with a mandate to exclude particular sectors, such as the Vanguard FTSE Social Index fund, or have a firmwide policy to do so; deviating will lead to client withdrawals, reputational damage, and possibly legal action. Alternatively, it can launch a fund with a conditional strategy, such as the UBS Sustainable Improvers fund. Such funds claim to add value through active management and company-specific analysis; not holding any firms in a brown industry may lead to withdrawals as it would be cheaper to hold a passive exclusionary fund, and not holding ES leaders in that industry would violate the fund mandate. Edmans, Gosling, and Jenter (2024) survey 509 active equity portfolio managers (of both traditional and sustainable funds) and find that "to be consistent with our fund's mandate" is the most common reason for incorporating ES performance into stock selection, even more so than to increase returns or reduce risk. Deviating from a conditional strategy may also lead to regulatory action: the UK's Financial Conduct Authority has "sustainability improvers" (a conditional strategy) as one of three permissible sustainability labels, and has historically forced funds to remove labels for deviating from their stated strategy. In Section 4 we consider the case in which B is unable to commit to her investment strategy.

At $t = 1$, M takes action $a \in \{0, 1\}$. Action $a = 1$ reduces the firm's externalities and decreases firm value by $c > 0$, net of any benefit, otherwise it would automatically be taken without the need for responsible investment. At $t = 2$, the firm issues $q \in (0, 1)$ additional shares to finance an investment project and investors, including the blockholder, trade claims

[8]Note that "best-in-class" would refer to buying the companies within an industry that have experienced the greatest decline in externalities, rather than those with the lowest level of externalities, as the strategy rewards companies that have taken corrective actions.

to the firm's terminal value. At $t = 3$, the firm generates both a terminal cash flow and externalities.

2.2 Firm Value and Externalities

The firm's terminal value is specified as:

$$V = \theta + rI - ca, \tag{1}$$

where $\theta \sim N(\mu, \sigma)$ represents the random return generated by the firm's assets in place. The cost of the action is ca and the gross return from the new investment is rI with $r > 1$.[9] The firm finances the investment by issuing new shares so that $I = pq$. To focus on the main economic mechanism – the blockholder's trade-off between incentivizing the action and hindering expansion – we take the firm's issuance decision q as given. For example, q may be limited by the amount of equity that can be raised without the agency costs of outside equity becoming too severe. With fixed q, lower demand for the firm's stock by the blockholder reduces the stock price p and thus the level of investment $I = pq$. This setting involves constant returns to scale – the firm can invest any amount I, with a constant gross return of r. An alternative assumption would be to have decreasing returns to scale and endogenous q, which would substantially complicate the model without qualitatively changing the results.[10] A decrease in the stock price p would reduce the optimal level of q, lowering investment as in the current setup. The parameter q allows for variation on how much external capital firms require; q will be low in firms that finance their investments with retained earnings.

The firm's operations generate a negative externality of f to society:

$$f(\theta, rI, a) = \lambda(\theta + rI)(1 - \xi a). \tag{2}$$

[9] We assume the cost c does not reduce the funds available for investment. For example, it could represent the choice of a greener technology, which requires no up-front investment but reduces future profits. If the cost c did reduce investment I, then this would create an additional benefit to conditioning – the firm uses some capital to reduce its externalities rather than to expand.

[10] The per-share value of the firm is given by $v \equiv \frac{V}{1+q}$, and so q affects both the numerator (through affecting investment and thus aggregate value) and denominator. Thus, if q were endogenous, it would not be solvable in closed form. Similarly, if the return on investment r were stochastic, the risk premium in equation (9) would depend on the stock price p, causing the market-clearing condition to become non-linear in p. Thus, for tractability, we assume r is deterministic.

The externality depends on the firm's assets in place θ, the payoff from investment rI, and the action a. The parameter $\lambda > 0$ scales the externality and $\xi \in (0, 1)$ determines the efficacy of the action. Importantly, the action can never fully eliminate the externality ($\xi < 1$): a fossil fuel company will still retain its fossil fuel assets even if it also develops clean energy, an alcohol company will still have negative health effects even if it introduces low-alcohol drinks, and "hard-to-abate" industries will still emit carbon and pollution even if they invest in reduction technologies.[11] For simplicity, ξ is deterministic, but the analysis is unchanged if ξ were instead a random variable (independent of θ), in which case B's optimal divestment strategy will be determined by the action's expected efficacy, $\mathbb{E}[\xi]$.

The functional form for f implies that the action reduces the externality in a multiplicative way. For example, if the action involves developing a less polluting technology, this is implemented firm-wide and thus has a larger effect on larger firms. In Appendix B.1 we show that the main results continue to hold if the action has an additive effect on the externality that is independent of firm size. This extension also captures the case where the action has a multiplicative effect, as in (2), as well as a cost that is multiplicative in firm size.

As mentioned in the introduction, exclusion is ineffective in other papers, for example because they intentionally shut it down by featuring risk neutrality and externalities that do not scale with the firm. Our paper models exclusion by containing an externality that scales with firm size and risk-averse households so that divestment by the blockholder has a price impact. This captures the main argument for exclusion used in practice – to starve the firm of capital and hinder its expansion – and allows us to study the choice between different divestment strategies. Despite modelling all the features needed for the unconditional strategy to be effective, we show that a conditional strategy can be optimal.

2.3 Manager's Problem

The manager's utility depends on the equilibrium stock price p and the per-share firm value v:

$$U_m = \omega p + (1 - \omega)v, \tag{3}$$

[11] This would also be true in a multi-period model: certain industries will always extert externalities no matter how many corrective actions they take and over how many periods. Thus, in a multi-period model, just as in a single-period model, the unconditional and conditional strategy remain distinct: full exclusion involves never holding a brown firm, under the argument that it will always produce externalities; conditioning involves investing more in a brown firm as long as it continues to take corrective actions.

with $\omega \in [0,1]$. The concern for the short-term stock price ω is standard in the literature and can arise from a number of sources introduced by prior research, such as takeover threat (Stein, 1988) or reputational concerns (Narayanan, 1985; Scharfstein and Stein, 1990). All our results continue to hold if $\omega = 0$, because a manager fully aligned with firm value v will still care about the stock price p as it will affect the terms at which he will raise equity; we feature the parameter ω as it affects the relative effectiveness of conditional and unconditional strategies. Heinkel, Kraus, and Zechner (2001), Pastor, Stambaugh, and Taylor (2021), and Broccado, Hart, and Zingales (2022) feature $\omega = 1$, i.e. the manager is only concerned about the stock price.

At $t = 1$, the manager solves:

$$\max_{a \in \{0,1\}} \mathbb{E}[U_m], \tag{4}$$

where the expectation is taken over θ. Importantly, the manager takes the blockholder's investment policy $x(a)$ as given when choosing a.

2.4 Investors' Problem

Households maximize a standard mean-variance objective with constant absolute risk aversion parameter $\gamma > 0$. When submitting their demands, households condition on the action a and the stock price p:

$$\max_{x_i} \mathbb{E}[x_i(v-p)|a,p] - \frac{\gamma}{2} Var(x_i(v-p)|a,p). \tag{5}$$

The blockholder commits to a demand schedule $x(a)$. She chooses the investment strategy $x(a)$ to maximize her objective function U_B which is a weighted sum of trading profits and externalities:

$$U_B = \varphi x(v-p) - (1-\varphi) f \tag{6}$$

where $\varphi \in [0,1]$ parametrizes the blockholder's concern for profits.

We assume that $0 \leq x\,(a) \leq 1+q$. The assumption $x(a) \geq 0$ results from short-sale constraints, which are standard in the blockholder exit literature (e.g. Admati and Pfleiderer, 2009; Edmans, 2009); otherwise, blockholders have no special role as any investor can exit, regardless of her initial stake. However, this assumption is not necessary for our results. If short-sales are possible, all the results continue to apply except that B need not be a blockholder

– she can be any large investor.[12] Similarly, $x(a) \leq 1+q$ means that the blockholder cannot buy more than the entire firm, i.e. households cannot short sell. If this assumption is relaxed, our results become stronger as the blockholder has a greater ability to reward the action.

3 Optimal Investment Strategies

We solve the model by backwards induction. We first solve for the market-clearing stock price. Households maximize their objective function (5) leading to demand of:

$$x_i = \frac{\mathbb{E}[v|a,p] - p}{\gamma Var(v|a,p)}. \tag{7}$$

Market clearing requires that total demand equals supply:

$$x(a) + \int_0^1 x_i di = 1 + q. \tag{8}$$

Solving for p yields:

$$p = \mathbb{E}[v|a,p] - \gamma Var(v|a,p)\,(1 + q - x(a)) \tag{9}$$

with $\mathbb{E}[v|a,p] = \frac{\mu + rI - ca}{1+q}$, $Var(v|a,p) = \frac{\sigma^2}{(1+q)^2}$, and $I = qp$.

The stock price p is the certainty equivalent per-share value of the firm. The second term represents the risk discount, which is increasing in risk $Var(\cdot)$, household risk aversion γ, and the number of shares held by households $1+q-x(a)$. An increase in the blockholder's demand raises the stock price by reducing the number of shares that risk-averse investors need to hold.

We next re-write the equilibrium stock price as a function of B's trade and M's action. We take the stock price in equation (9), plug in $I = pq$ and solve for p:

$$p(x,a) = \frac{\mu - ca - \left(1 - \frac{x}{1+q}\right)\gamma\sigma^2}{1 + q - rq}. \tag{10}$$

The intuition is as follows. Absent an investment decision, the stock price is the certainty equivalent firm value divided by the number of shares $(1+q)$. One may think that investment

[12] We would only need a limit on the maximum possible short-sales to prevent the stock price in equation (9) from turning negative.

should add an additional term to firm value in the numerator. However, since the value of the investment is $rqp(x,a)$, it effectively reduces the number of shares by rq in the denominator.[13]

To ensure that $p(x,a)$ is positive, we assume that $\mu > \gamma\sigma^2 + c$ so that expected firm value is not outweighed by the risk premium and the cost of the action, and that $1+q-rq>0$ so the effective number of shares does not turn negative. The second condition can be rewritten $r < \frac{1+q}{q}$. Intuitively, if r is sufficiently large, then households demand more shares when the price is higher, since their funds will be invested in a very profitable investment opportunity, leading to an upward-sloping demand curve.

Firm value, given B's trade and M's action, is:

$$v(x,a) = \frac{\mu - ca + rqp(x,a)}{1+q}. \tag{11}$$

The short-term stock price affects long-term firm value by increasing the funds raised.

Lemma 1 solves for the blockholder's trading strategy, taking action a as given. All proofs are given in the Appendix.

Lemma 1 *The blockholder's optimal trade given action a is*

$$\hat{x}(a) \equiv \frac{1+q}{2} \max\left\{1 - \frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda(1-a\xi), 0\right\}, \tag{12}$$

where

$$z \equiv \frac{rq}{1+q} \in (0,1). \tag{13}$$

Intuitively, if the blockholder is purely profit-motivated ($\varphi = 1$), the optimal trade would be $\arg\max_x v(x,a) - p(x,a) = \frac{1+q}{2}$, i.e., the solution of a monopolist in the financial market. She buys shares because there are gains from trade between the risk-neutral blockholder and the risk-averse households. If the blockholder also has social motives (i.e., $\varphi < 1$), then she buys fewer shares because she partially internalizes how her purchases increase the stock price,

[13] To see this, we have:

$$\begin{aligned} \text{Market value of firm} &= \text{Certainty equivalent fundamental value of firm} \\ p(1+q) &= \text{Certainty equivalent assets in place} + rqp \\ p(1+q-rq) &= \text{Certainty equivalent of assets in place} \end{aligned}$$

the funds raised, and thus the externalities emitted. Indeed, $\hat{x}(a)$ is increasing in φ. If the blockholder's social motives are sufficiently strong, then she engages in full exclusion, i.e., $\hat{x}(1) = \hat{x}(0) = 0$.

Note that $\hat{x}(1) \geq \hat{x}(0)$: the blockholder buys more shares when the manager takes the action, even though the action reduces firm value. The intuition is as follows. The action's impact on firm value does not affect trading profits, because the action is public and thus fully reflected in the stock price. However, the action means that buying shares, and thus helping the company expand, increases externalities by less. Thus, buying shares has the same benefit (trading profits are unchanged) but a lower cost (externalities are smaller), and so the blockholder buys more shares if $a = 1$.

The above analysis takes a as given: we have $\hat{x}(1) \geq \hat{x}(0)$ even though the blockholder's strategy has no effect on managerial incentives. In reality, the manager's action will be influenced by the blockholder's strategy, and so the blockholder will choose her strategy taking into account its effect on the action. To incorporate these feedback effects, consider first the manager's incentives to take the action, given the blockholder's strategy $x(a)$ (which could differ from $\hat{x}(a)$). Plugging (10) and (11) into (3) shows that the manager chooses $a = 1$ if and only if

$$x(1) - x(0) \geq \Delta \equiv \frac{c(1+q)}{\gamma\sigma^2 [\omega + (1-\omega)z]}. \tag{14}$$

The action $a = 1$ has two effects on M's objective function. First, it incurs a cost c which reduces firm value (to which M attaches weight $1 - \omega$) and thus the stock price (to which M attaches weight ω); the latter in turn lowers investment and further reduces v. Second, it increases the blockholder's demand from $x(0)$ to $x(1)$, raising the stock price p and thus investment and firm value. Thus, M takes the action if the second force is sufficiently strong, i.e. B conditions with a high $x(1) - x(0)$.

We assume that

$$c \leq \gamma\sigma^2 [\omega + (1-\omega)z] \tag{15}$$

so that the constraint $x(1) \leq 1 + q$ does not bind.[14] Proposition 1 gives the blockholder's optimal strategy, now taking into account how her trades affect the manager's action.

Proposition 1 *(Blockholder's strategy): There exists $\Xi(\varphi)$, where $\Xi(0) \in (0, 1)$, such that in*

[14] In the extensions, condition (15) will differ, and we will state the new required condition.

equilibrium, if $\xi < \Xi(\varphi)$, then the manager chooses $a^* = 0$ and the blockholder chooses an unconditional strategy: $x^*(0) = x^*(1) = \hat{x}(0)$. If $\xi > \Xi(\varphi)$, then the manager chooses $a^* = 1$ and the blockholder chooses a conditional strategy: $x^*(1) = \max\{\hat{x}(1), \Delta\}$ and $x^*(0) = 0$. Moreover,

(i) *If $\Delta < \frac{1+q}{2}$ then there exists $\underline{\varphi} \in (0,1)$ such that $\Xi(\varphi) > 0 \Leftrightarrow \varphi < \underline{\varphi}$, and $\Xi(\varphi)$ is decreasing in $[0, \underline{\varphi}]$.*

(ii) *If $\Delta \geq \frac{1+q}{2}$ then $\Xi(\varphi)$ is U-shaped, and there exists $\overline{\varphi} \in (0,1)$ such that $\Xi(\varphi) < 1 \Leftrightarrow \varphi > \overline{\varphi}$.*

(iv) *If $\Xi(\varphi) \in (0,1)$, then $\Xi(\varphi)$ is increasing in (c, γ, σ) and decreasing in (μ, ω); it may be increasing or decreasing in (r, q).*

The blockholder's optimal strategy is represented by a function $\Xi(\varphi)$, which gives the minimum effectiveness of the action ξ, for a given profit motivation φ, required to induce the blockholder to condition. If $\xi > \Xi(0)$, the blockholder reduces externalities more by inducing the action than hindering expansion. However, this trade-off is complicated because the blockholder is also concerned with trading profits, and as a result, the threshold depends on φ. Moreover, this dependence is driven by other parameters in the model.

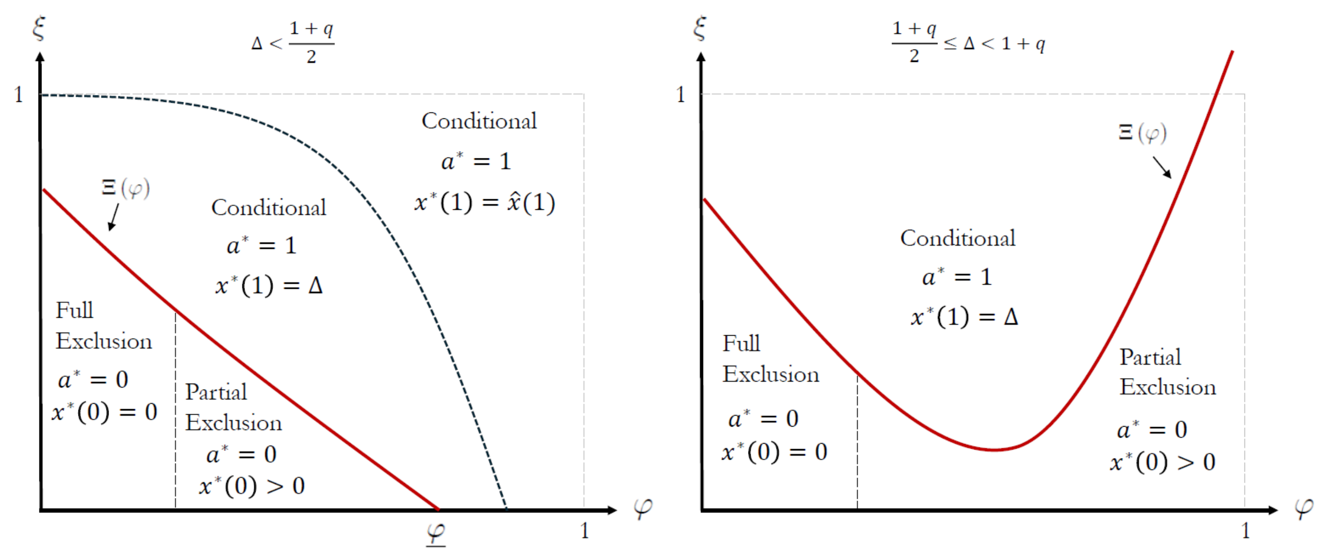

Figure 1. Conditional vs. Unconditional (exclusion) strategies in equilibrium.

Part (i) applies when $\Delta < \frac{1+q}{2}$, and so few additional share purchases are needed to induce the action. This case is illustrated in the left-hand side of Figure 1. The function $\Xi(\varphi)$ is the lower red curve, and the cutoff $\underline{\varphi}$ is its intersection with the φ-axis. If the action is sufficiently effective (i.e., $\xi \geq \Xi(\varphi)$), the blockholder conditions. As shown in equation (14), the manager chooses $a = 1$ if and only if $x^*(1) - x^*(0) \geq \Delta$. If the blockholder's profit motivation φ is high, this constraint is slack. Her profit motivation induces her to buy so many more shares if $a = 1$ that doing so automatically induces the action as a by-product. The blockholder thus sets $x^*(1) = \hat{x}(1) > \Delta$: she buys the same number of shares as if there were no incentive concerns. This is the region above the upper dashed curve in the left panel of Figure 1. If φ is low, the manager's incentive constraint binds (i.e., $\hat{x}(1) < \Delta$) and so $x^*(1) = \Delta \geq \hat{x}(1)$: to induce the action, the blockholder has to buy more than the profit-maximizing level of shares. This is the region in between the two curves in the left panel.

If the action is not sufficiently effective (i.e., $\xi < \Xi(\varphi)$), then the blockholder optimally chooses not to induce it. If her profit motivation φ is high (while remaining in the region where $\xi < \Xi(\varphi)$), she still buys some shares ($x^*(1) = x^*(0) = \hat{x}(0) > 0$) to earn trading profits. If her profit motivation φ is low, she fully excludes the firm ($x^*(1) = x^*(0) = 0$) to minimize externalities.

In part (i), the function $\Xi(\varphi)$ is downward sloping. Intuitively, the less effective the action, the less willing the blockholder to induce it by purchasing additional shares if $a = 1$. However, the greater the blockholder's profit motive, the greater her incentive to buy additional shares and earn trading profits. Thus, profit motives substitute for action effectiveness in inducing the blockholder to condition. If a social planner is concerned only with externality reduction and not risk sharing, then the blockholder's incentives coincide with the social planner's when $\varphi = 0$. Then, the social optimum is to condition if and only if $\xi \geq \Xi(0)$. The negative slope of $\Xi(\varphi)$ implies that a partially profit-motivated blockholder conditions too often and induces the corrective action even when it is socially suboptimal to do so.

The right-hand side of Figure 1 illustrates part (ii) of Proposition 1, which applies when $\Delta \geq \frac{1+q}{2}$. Recall that, taking the manager's action as given, the blockholder buys no more than $\frac{1+q}{2}$ shares. When $\Delta \geq \frac{1+q}{2}$, the manager's incentive constraint always binds: to induce the action, the blockholder always needs to buy more shares than optimal for profits. As a result, unlike in part (i), the function $\Xi(\varphi)$ is U-shaped: in particular, it is increasing after a point. Inducing the action requires the blockholder to buy too many shares; thus, if her profit motivation φ is sufficiently high, she switches to exclusion.[15] Overall, as in part (i), increasing φ from zero encourages the blockholder to condition as doing so earns trading profits. However, unlike in part (i), a conditional strategy always requires the blockholder to buy more shares than she would like, and so when φ becomes too high, the blockholder switches back to an unconditional strategy.

Part (iii) describes the comparative statics of the threshold $\Xi(\varphi)$. Since they hold for any value of φ, they yield both positive results with respect to the blockholder's willingness to induce the corrective action, and normative results for when inducing the action is optimal to reduce externalities.

A conditional strategy is more likely to be optimal if stock price concerns ω are high and the cost of the action c is low. This reduces the number of shares that the blockholder needs to purchase to induce the action. As a result, conditional on $\xi > \Xi(\varphi)$ (and thus $a = 1$), higher stock price concerns ω are socially optimal as they allow the action to be induced with fewer purchases and thus externalities. This result contrasts standard "governance through exit" models. In such models, the blockholder induces the action by reflecting it in the stock price through her trading, and so higher ω increases the blockholder's effectiveness. Despite this, the optimal ω is zero – the action improves firm value, and so if the manager cared exclusively about firm value, he would take the action even without blockholder trading. In our setting, the action reduces firm value due to the cost c, and so the only motive for $a = 1$ is to increase p; the magnitude of this motive depends on ω. Thus, a higher ω can be optimal for inducing the action, in contrast to common concerns that managerial short-termism is necessarily detrimental for society. In Appendix B.2, we show that managerial short-termism may lead to lower expected externalities.

Conditioning is also preferred if the firm is large (high μ) and risk σ or risk aversion γ are small, because these factors increase the stock price and thus the amount of investment. In addition, high μ increases assets in place. Both factors lead to greater firm value and thus a greater benefit from the multiplicative action a. Lower risk and risk aversion also mean that unconditional exclusion has a less negative effect on the stock price and is less effective.

Finally, the capital raised by the firm (q) and the profitability of the investment (r) have non-monotonic effects on $\Xi(\varphi)$. One might think that they should have an unambiguous effect

[15] In the proof of Proposition 1 we show that if $c < \gamma\sigma^2 \frac{\omega+(1-\omega)z}{2-\omega+(1-\omega)z}$ then the minimum of $\Xi(\varphi)$ is negative, and hence for intermediate values of φ, the action is induced regardless of how effective it is.

– the greater the capital raised, the more important the cost of capital channel, and thus the more valuable exclusion is to increase the cost of capital. However, there is a force in the opposite direction – the greater the capital raised, the more important the action is to reduce the externalities from the new investment. If q and r are sufficiently high, such a large amount of capital is raised that this second force dominates and further increases in q and r make a conditional strategy more effective.

All of these comparative statics continue to apply as long as the effect of the corrective action is increasing in firm size; it does not need to be multiplicative. In Appendix B.1, we consider the case in which the action has an additive effect on the externality that is independent of firm size, for example if the cost c involves donating money to a charity which reduces the externality by $\lambda\xi$. This extension also captures the case in which the action has a multiplicative effect on the externality, as in (2), and the cost of the action is also multiplicative in expected firm size. In the core model, the action has a fixed cost c, such as the cost of developing a greener technology which, once invented, can be implemented costlessly across the firm. However, if the technology is already known (and thus does not require a fixed development cost) but its implementation involves a higher per-unit cost of production, then this cost will scale with firm size. The key result that a conditional strategy is preferred if and only if ξ exceeds a cutoff continues to hold, as do the comparative statics with respect to c, ω, q, and r. μ no longer affects the optimal strategy. Notably, increasing γ or σ^2 makes a conditional strategy *more* likely in this case because the stock price is more sensitive to B's demand so she can induce the action at lower cost.

Corollary 1 describes how the expected externality is affected by the blockholder's profit motivation.

Corollary 1 *(Comparative statics for the externality): In the equilibrium described in Proposition 1, the expected externality $E[f]$ increases in φ.*

Externalities are always increasing in φ. Intuitively, the blockholder buys more shares when she has stronger profit motives, thus enabling the firm to expand. More surprisingly, this result continues to hold despite a countervailing force: the endogenous response of the blockholder's trading strategy. If φ rises sufficiently, the blockholder switches to a conditional strategy which induces the action and thus reduces externalities. To understand why the first effect dominates, consider a blockholder with profit motives φ such that $\xi = \Xi(\varphi)$, where

$\Xi'(\varphi) < 0$. This blockholder is indifferent between inducing the corrective action and not inducing it. Consider another blockholder with profit motives $\varphi' = \varphi + \varepsilon$, where $\varepsilon > 0$ is arbitrarily small. Blockholder φ' will only condition if doing so increases externalities as well as profits. If conditioning instead reduced externalities, then blockholder φ will strictly prefer the same strategy as blockholder φ', and enjoy lower externalities, higher profits and thus a higher payoff. This would contradict blockholder φ's indifference.

A key force behind Corollary 1 is the blockholder's ability to commit to the investment strategy. As we show in the next section, without commitment, the expected externality generated by the firm in equilibrium can decrease with the blockholder's profit's motive, φ.

4 No Commitment

In this section, we relax the assumption that the blockholder can commit to an investment strategy. Fund holdings may not be easily observed by regulators or clients; for example, funds may be able to window-dress and change their portfolio holdings just before they are reported (e.g., Muñoz, Ortiz, and Vicente (2022) and Tang et al. (2024)). In addition, as explained at the start of Section 2, many corrective actions are non-contractible.[16] Specifically, we assume the blockholder chooses her trade at $t = 2$ optimally after the action a has become public at $t = 1$. As in the baseline model, given action a, the blockholder maximizes her expected utility by choosing $\hat{x}(a)$, as given by (12). The manager has rational expectations about the blockholder's trade $\hat{x}(a)$. He takes the action if and only if

$$\hat{x}(1) - \hat{x}(0) \geq \Delta, \tag{16}$$

where Δ is given by (14). The equilibrium is given by Proposition 2.

Proposition 2 *(No commitment): Suppose the blockholder cannot commit to an investment strategy. In equilibrium:*

(i) If $\xi \leq \frac{2\Delta}{1+q}$, the blockholder buys $\hat{x}(0)$ shares and $a^* = 0$.

[16] Even if the action is non-contractible, B may be able to commit to it due to reputational concerns: there may be client withdrawals or regulatory action if the B announces a strategy and does not follow through with it. B will be unable to commit to the strategy if she has no reputational concerns: for example if the fund manager is close to retirement, or can undertake other actions to avoid withdrawals or regulatory action such as voting for shareholder proposals.

(ii) *If* $\xi > \frac{2\Delta}{1+q}$, *there exist* $0 < \underline{\varphi} < \overline{\varphi} < 1$ *such that if* $\varphi \in \left[\underline{\varphi}, \overline{\varphi}\right]$, *then the blockholder buys* $\hat{x}(1)$ *shares and* $a^* = 1$, *and if* $\varphi \notin \left[\underline{\varphi}, \overline{\varphi}\right]$, *the blockholder buys* $\hat{x}(0)$ *shares and* $a^* = 0$.

(iii) *Let* $f^*(\varphi)$ *be the externalities in equilibrium when blockholder's profit motive is* φ. *Then, there exist* $\Xi_{nc} \in (0,1)$ *and* $\varphi_{nc} > \underline{\varphi}$ *such that* $f^*(\varphi) < f^*(0)$ *if and only if* $\xi > \max\left\{\frac{2\Delta}{1+q}, \Xi_{nc}\right\}$ *and* $\varphi \in \left(\underline{\varphi}, \varphi_{nc}\right)$.

Part (i) states that, as in the baseline model, the blockholder cannot induce the action if it is sufficiently ineffective. Part (ii) demonstrates that, if the action is sufficiently effective, the blockholder can induce it if her profit motives fall within an intermediate range. On the one hand, they need to be sufficiently high to induce the blockholder to buy shares even though doing so also increases externalities. On the other hand, the blockholder's concern for externalities also has to be sufficiently high that she buys significantly more shares when $a = 1$ than when $a = 0$. If φ is low, the manager knows that the blockholder will not buy more shares if he takes the action, as doing so will allow the firm to expand. If φ is high, the manager knows that the blockholder will buy many shares even if he does not take action, to make trading profits. In both cases, the manager has insufficient incentives to take the action.

These results demonstrate the robustness of the core model to the no-commitment case. In particular, for investors to induce managers to take externality-reducing actions, they need to be concerned with profits, not just externalities. Part (ii) is qualitatively similar to the baseline model for the case of $\Delta \geq \frac{1+q}{2}$ in which the cutoff $\Xi(\varphi)$ is U-shaped, and so the blockholder only conditions when her profit concerns are in an intermediate range.

Even if profit motives allow the blockholder to induce the action, this does not automatically mean that externalities fall, since profit motives also lead the blockholder to buy shares which helps the firm expand. Part (iii) shows that, for an intermediate range of profit motives and a sufficiently effective action, the former effect outweighs the latter and so externalities do fall – in contrast to Corollary 1 of the baseline model where externalities are always increasing in profit motives. Figure 2 illustrates this case.

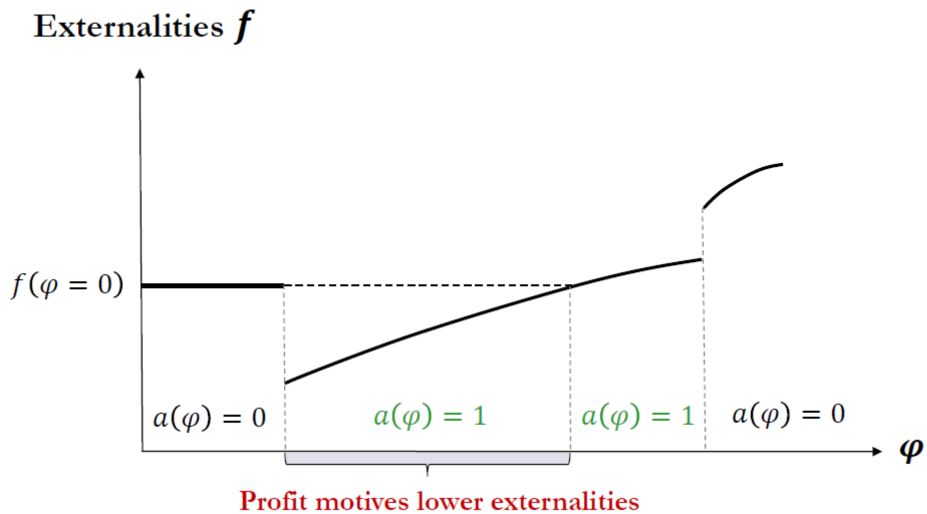

Figure 2. Comparative statics of f with respect to φ

The intuition is as follows. In the baseline model, the blockholder is able to commit to buying more shares when the action is taken. If the blockholder cannot commit, and is only concerned with externalities, she will not buy shares, regardless of the manager's action, as doing so increases externalities. Profit motives substitute for the ability to commit – they make it individually rational for the blockholder to buy more shares when $a = 1$, effectively committing her to reward the action.

Our analysis shows that, without commitment, investors with strong externality concerns (i.e., low φ) will adopt full exclusion irrespective of the effectiveness of the corrective action. This is consistent with the prevalence of exclusion among self-proclaimed responsible investors, and some clients and the media evaluating funds according to their holdings of brown stocks under the assumption that exclusion is the most "responsible" investment strategy. However, responsible funds do not pursue exclusion because it is more effective at reducing externalities, but because they are unable to commit to a conditional strategy even when it is more effective. As a result, strong externality concerns can actually weaken an investor's effectiveness at curbing externalities.

While a profit motivation makes it credible for the blockholder to reward the action, it may lead to the blockholder buying more shares than necessary to induce the action. In Appendix B.3, we show that a regulation that caps the stake that an investor may hold in brown firms can induce the action while also limiting firm expansion, thus lowering externalities overall. The blockholder will not voluntarily impose such a cap as it reduces her trading profits, hence the role of regulation in doing so.

Finally, when portfolio holdings are transparent and window-dressing opportunities are limited, asset managers may find it easier to commit to full exclusion than conditional strategies – for example, by having a firmwide policy preventing investing in a particular sector. In such cases, investors face a binary choice: either commit to full exclusion or make no commitment at all (which results in purchasing $\hat{x}(a)$ shares). Online Appendix B.4 demonstrates that this partial commitment framework maintains the same prevalence of full exclusion and yields an equilibrium identical to Proposition 2. Intuitively, without commitment, the blockholder's social motives inherently favor full exclusion. Once the action has been taken, only the cost of capital channel remains active, and negative externalities are minimized by minimizing investment in the firm. The blockholder only deviates from full exclusion when profit motives are significant enough to overcome externality concerns.

5 Unobservable Corrective Action

In this section, we consider the case in which the action is not publicly observable. As a result, the blockholder cannot condition her holdings on a, since she does not observe it directly. Instead, there is a public signal $s \in \{0,1\}$ which is correlated with the action, such as an ES rating. The signal precision is given by

$$\tau \equiv \Pr[s = a|a] \in [0.5, 1). \tag{17}$$

Superior disclosure standards, more trusted third-party verification of disclosures, or greater consensus between ES rating agencies (which Berg, Kölbel, and Rigobon (2022) show to disagree significantly) will increase precision. Alternatively, if the manager is able to greenwash, i.e. give the impression of taking the action without actually doing so, then τ is lower. The signal is publicly observed at $t = 2$, before trade in the secondary market takes place. The blockholder conditions her holdings on this signal, $x(s) \geq 0$. Households have rational expectations and correctly conjecture the manager's equilibrium action.

We proceed in two steps. First, we take the signal precision as given and analyze how τ

affects the optimal investment strategy. Second, we endogenize signal precision and allow the manager to choose τ ex ante.

5.1 Optimal Investment Strategies

Following the same steps as in the baseline model, the manager takes the action if and only if:

$$x(1) - x(0) \geq \frac{1}{2\tau - 1} \frac{(1-\omega)c(1+q)(1-z)}{\gamma\sigma^2(\omega + (1-\omega)z)} \equiv \Delta_{\text{unob}}(\tau). \tag{18}$$

The threshold $\Delta_{\text{unob}}(\tau)$ is decreasing in signal precision τ: $\frac{\partial \Delta_{\text{unob}}}{\partial \tau} < 0$. Intuitively, the blockholder has to provide the manager stronger incentives to take the action when the signal is less precise. If $\tau = \frac{1}{2}$, the signal is uninformative about the action. Since the blockholder is unable to reward the action, the manager always chooses $a = 0$.[17]

The blockholder's optimal investment strategy is given by Proposition 3:[18]

Proposition 3 *(Unobservable action): There exists $\Xi_{unob}(\tau)$ such that $\Xi_{unob}(\tau)$ is decreasing in τ. In equilibrium, if $\xi < \Xi_{unob}(\tau)$ then the manager chooses $a^* = 0$ and the blockholder chooses an unconditional strategy: $x^*(s, \tau) = \hat{x}(0)\ \forall\ s$. If $\xi > \Xi_{unob}(\tau)$ then the manager chooses $a^* = 1$ and the blockholder chooses a conditional strategy:*

$$x^*(s, \tau) = \max\{\hat{x}(1) - \tau\Delta_{unob}(\tau), 0\} + \Delta_{unob}(\tau) \cdot s. \tag{19}$$

Moreover, $\lim_{\varphi \to 0} \Xi_{unob}(\tau) < 1$ *for all* $\tau \in \left(\frac{1}{2}, 1\right)$.

As in the baseline model, the blockholder conditions if the action is sufficiently effective, $\xi > \Xi_{\text{unob}}(\tau)$. This threshold is decreasing in signal precision τ. The noiser the signal, the less likely that $a = 1$ will lead to $s = 1$, and so the blockholder needs to buy relatively more upon $s = 1$ to induce the action: $x^*(1, \tau) - x^*(0, \tau) = \Delta_{\text{unob}}(\tau)$, which is decreasing in τ. Since this leads to more externalities, conditioning is more costly and so the threshold is higher. If the signal is pure noise, then the blockholder never conditions. In this sense, a unconditional

[17] Note that there is a discontinuity in $\Delta_{\text{unob}}(\tau)$ around $\tau = 1$, i.e. $\lim_{\tau \to 1} \Delta_{\text{unob}}(\tau)$ differs from the threshold in Section 3. For any $\tau < 1$, households ignore the signal and rely on their equilibrium conjecture $\widehat{a}$ to predict the corrective action. However, if the signal is perfectly informative ($\tau = 1$), they rely exclusively on the signal.

[18] The equivalent of condition (15), to ensure $x^*(s) \leq 1 + q$, is $c \leq \frac{\gamma\sigma^2[\omega + (1-\omega)z](2\tau - 1)}{(1-\omega)(1-z)}$.

strategy is more "robust" than a conditional one – it does not rely on the blockholder's ability to observe the action. However, this robustness comes at a cost of providing no incentives to take it.

Proposition 3 thus highlights a new benefit of superior ES disclosure. Common arguments are that ES disclosure allows investors to allocate capital according to ES performance, and to hold managers to account. Both of these channels operate here, but there is an additional force – by allowing investors to allocate capital according to ES performance, they can induce corrective actions without having to commit to a significant investment in a brown firm.

5.2 Optimal Disclosure

In this section, we allow the manager to choose τ ex ante by engaging in disclosure. We assume that if the manager is indifferent between different values of τ, he chooses the lowest possible value of τ, which is $\frac{1}{2}$. This would be strictly optimal if disclosure were costly. We also assume the choice of τ is made public, so that B can condition her investment strategy on τ.

Proposition 4 *(Optimal disclosure): There exists $\Xi_{disc} > 0$ such that if $\Delta_{unob}(1) \geq 1 + q$ or $\xi < \Xi_{disc}$, the manager chooses $\tau = \frac{1}{2}$ and $a^* = 0$, and the blockholder chooses an unconditional strategy: $x^*\left(s, \frac{1}{2}\right) = \hat{x}(0)$. If $\Delta_{unob}(1) < 1 + q$ and $\xi \geq \Xi_{disc}$ then the manager chooses $\tau = \tau^*(\xi) \in \left(\frac{1}{2}, 1\right)$ and $a^* = 1$, and the blockholder chooses a conditional strategy, as given by (19) in Proposition 3. Moreover, $\tau^*(\xi)$ is decreasing in ξ.*

The manager discloses information (i.e. chooses $\tau > \frac{1}{2}$) and the blockholder conditions if and only if the action is sufficiently effective, i.e., $\xi \geq \Xi_{\text{disc}}$. One might think that the manager should choose full disclosure ($\tau = 1$) so that his action is always reflected in the signal ($s = 1$). In contrast, the manager deliberately discloses noisy signals (i.e., $\tau^*(\xi) < 1$), so that the blockholder has to promise a high investment in order to induce the action. Indeed, $\tau^*(\xi)$ is the minimum disclosure that persuades the blockholder to condition and induce the action.

The model considers a blockholder who chooses optimally between conditional and unconditional strategies. Stepping outside the model, if there were a probability that the blockholder only implements unconditional strategies (e.g. due to a probability that the fund family introduces a firmwide exclusion policy, a lack of sophistication, its clients believing that unconditional exclusion is the best way to have impact, or clients demanding an exclusion policy due to their tastes), then the greater this probability, the more likely it is for the manager to choose minimal disclosure ($\tau = \frac{1}{2}$). Thus, if the economy contains more investors that are open to adopting a conditional strategy, this would encourage firms to disclose more information about their ES activities, in turn reinforcing investors' incentives to condition.

Relatedly, if the blockholder's investment strategy and the manager's disclosure choice were made simultaneously, rather than sequentially, then a conditional strategy and disclosure would be strategic complements – the manager will disclose more if he expects that the blockholder will condition. Thus, there would be multiple equilibria where a conditional strategy is self-fulfilling. As a result, regulation that encourages either disclosure or a conditional strategy (e.g. not punishing responsible firms for holding brown stocks) could help coordinate on the conditional, high-disclosure equilibrium.

6 Presence of Arbitrageur

A common criticism of divestment strategies is they allow arbitrageurs to buy brown firms at depressed prices, attenuating the impact of divestment on prices. This section extends the model to incorporating an arbitrageur who is purely profit-motivated like households, and is risk-neutral and can take large stakes and have price impact like the blockholder. We return to the case in which the action a is publicly observable; this simplifies the analysis as it means that firm value (which is net of c, if $a = 1$) is publicly observable.

With probability $\eta \in (0, 1]$, the arbitrageur arrives after the blockholder has announced her investment strategy and the manager has taken action a. The presence of the arbitrageur is public information. He trades an amount y at $t = 2$ to maximize $\Pi_A(y) = y(v - p)$.[19] The solution is given in Proposition 5:

Proposition 5 *(Arbitrageur). If the arbitrageur is present, his trading volume and profit are given by*

$$y^*(x) = \arg\max_y \Pi_A(y) = \frac{1 + q - x}{2} \tag{20}$$

$$\Pi_A(y^*(x)) = \left(\frac{1}{2}\frac{1 + q - x}{1 + q}\right)^2 \gamma\sigma^2 \tag{21}$$

[19] The equivalent of condition (15), to ensure $x(1) \leq 1 + q$, is $c \leq \gamma\sigma^2[\omega + (1 - \omega)z]\left(1 - \frac{\eta}{2}\right)$.

and, conditional on the blockholder's trade x, the stock price is given by

$$p\left(x, a, y^{*}\left(x\right)\right)=\frac{\mu-ca-\left(1-\frac{\eta}{2}\right)\left(1-\frac{x}{1+q}\right)\gamma\sigma^{2}}{1+q-rq}. \tag{22}$$

There exists $\Xi_{arb}\left(\eta\right)$ such that if $\xi<\Xi_{arb}\left(\eta\right)$, then the manager chooses $a^{*}=0$ and the blockholder chooses an unconditional strategy: $x^{*}(0)=x^{*}(1)=\hat{x}(0)$. If $\xi>\Xi_{arb}\left(\eta\right)$, then the manager chooses $a^{*}=1$ and the blockholder chooses a conditional strategy: $x^{*}\left(1\right)=\max\left\{\hat{x}\left(1\right), \frac{\Delta}{1-\frac{\eta}{2}}\right\}$ and $x^{*}\left(0\right)=0$. Moreover, $\Xi_{arb}\left(\eta\right)$ decreases with η.

As is standard, the arbitrageur buys half of the free float not acquired by the blockholder, as shown in (20). Comparing (22) with (10), there is an additional $(1-\frac{\eta}{2})$ term in the numerator, which multiplies the term containing x and means that the blockholder's trade has a lower effect on the stock price. Intuitively, if the arbitrageur is present, she buys half of the free float, so the blockholder's impact is halved. As a consequence, $x^{*}\left(1\right)$ contains an additional $(1-\frac{\eta}{2})$ term in the denominator – since the blockholder has smaller price impact, she must promise a higher purchase to induce the action, which makes a conditional strategy more expensive to implement. Exclusion also becomes less effective because the arbitrageur partially reverses the impact of exclusion on the stock price and the cost of capital. Since the arbitrageur buys half of the free float, his impact is decreasing in the blockholder's trade. Thus, while the arbitrageur makes both conditional and unconditional strategies less effective, the impact is greater on the unconditional strategy as the blockholder's trade is zero. As a result, the threshold Ξ_{arb} is decreasing in η – the greater the probability of the arbitrageur appearing, the more likely the blockholder is to condition.

7 Conclusion

This paper has analyzed the optimal investment strategy of a responsible investor who aims to reduce the externalities emitted by a brown firm. While unconditional exclusion minimizes the stock price and thus the amount of externality-enhancing investment the firm can undertake, it provides no incentives for the firm to undertake a corrective action. A conditional strategy encourages the action at the cost of providing capital to a brown firm and allowing it to expand. However, these externality-reduction considerations must also be balanced against the investor's concern for trading profits. Up to a point, this concern encourages the investor to condition and earn profits from risk bearing. As a result, the blockholder's strategy is given by a threshold. If the effectiveness of the action in reducing externalities exceeds the threshold, the blockholder conditions to induce it; otherwise she unconditionally excludes. The threshold is in turn decreasing in the blockholder's profit concerns, as they encourage her to buy more shares if the manager has taken the action, and these concerns thus substitute for action effectiveness in inducing a conditional strategy.

However, if the action is sufficiently costly, the amount of new capital raised is high, and the manager's stock price concerns, firm risk, and household risk aversion are low, then inducing the action requires the blockholder to buy so many shares that she exceeds the level that maximizes trading profits. Then, the threshold is U-shaped in profit concerns: if they are too high, the blockholder limits her purchases to maximize her profits even though this fails to induce the action. This result has nuanced implications for policymakers' enforcement of fiduciary duty. In most cases, ensuring that funds place sufficient weight on financial returns may support, rather than conflict with, inducing the externality-reducing action. However, too strict an enforcement may lead to funds focusing purely on maximizing trading profits.

While our paper considers secondary market trading by equity investors, our results will continue to hold for secondary market trading by debt investors. Loan sales decrease the price of debt and hinder future debt capital raising. Many prior divestment theories require the manager to care about the stock price (e.g. due to equity-based pay or concerns with takeover threat), because divestment operates through lowering the stock price. They would not apply to debt since some managers may be unconcerned by the debt price. Here, all results hold even if the manager cares only about firm value, and thus continue to apply to debt investors. Our results also continue to hold if the blockholder provides primary debt or equity capital, such as a bank loan or venture capital investment to a start-up. A policy to never finance such companies may seem optimal by hindering them from expanding; in October 2022, Lloyds became the first UK bank to announce that it would not finance new oil and gas projects. However, being willing to provide primary capital to brown firms that are best-in-class may encourage such companies to reform.

Our results have a number of potential implications for policymakers. Most obviously, it highlights that regulators should not automatically punish sustainable funds that hold brown stocks as a measure of an investor's sustainability. Under the EU's Sustainable Finance Disclosure Regulation, Article 9 funds are viewed as the most sustainable, and are preferred by many clients, but the common interpretation of the requirements is that such funds cannot hold fossil fuel stocks even if they are best-in-class. Similarly, the EU's Non-Financial Reporting Directive requires institutional investors to report the percentage of their portfolio that is environmentally sustainable, as defined by their alignment with the EU Taxonomy. This also encourages investors to choose unconditional over conditional strategies. In contrast, regulators should police the opposite behavior – sustainable funds that claim to be actively managed but unconditionally exclude certain sectors. Just as regulators are scrutinizing actively-managed mainstream funds that act like closet indexers, they could also scrutinize actively-managed sustainable funds that engage in unconditional exclusion.

While our analysis has focused on brown industries, a similar result likely applies for green industries. Some sustainable funds will be significantly more likely to own a company if it is in a green sector. However, such an unconditional policy provides few incentives for companies within this sector to increase their positive externalities, since they will likely be held anyway. A conditional strategy, which overweights green industries but avoids "worst-in-class" companies within such sectors, may be more effective than unconditional inclusion. An investment strategy that tilts away from brown sectors and towards green sectors will require similar amounts of capital to one that automatically excludes the former and includes the latter. While a conditional strategy requires capital to hold best-in-class brown firms, it also saves capital by avoiding worst-in-class green firms.

The model suggests potential avenues for future research. One extension is to multiple firms. In addition to demonstrating that the optimal divestment strategy might involve buying firms that are literally best-in-class, featuring these firms as competing with each other in the same industry would have implications for how the investor's divestment strategy affects product market interactions. A second extension is to multiple responsible investors. They may increase the power of a conditional strategy if multiple investors are able to reward the manager for the action. In contrast, if they compete for client flows and unsophisticated clients view funds that unconditionally exclude as being more responsible, such competition may discourage a conditional strategy. A third is to a dynamic setting, where past actions may affect the effectiveness of future ones and so the optimal strategy may change over time – for example, if there are diminishing returns to corrective actions, the blockholder may start off with a conditional strategy but later switch to an unconditional one. Note that changes to parameters such as the effectiveness of the action can already be studied by our comparative statics, but new results may emerge that are unique to a dynamic setting such as an increased ability to commit.

References

[1] Admati, Anat R. and Paul Pfleiderer (2009): "The "Wall Street Walk" and Shareholder Activism: Exit as a Form of Voice." *Review of Financial Studies* 22, 2645–2685.

[2] Berk, Jonathan and Jules H. van Binsbergen (2024): "The Impact of Impact Investing." Working Paper, Stanford University.

[3] Berg, Florian, Julian F. Kölbel, and Roberto Rigobon (2022): "Aggregate Confusion: The Divergence of ESG Ratings." *Review of Finance* 26, 1315–1344.

[4] Bond, Philip, Alex Edmans, and Itay Goldstein (2012): "The Real Effects of Financial Markets." *Annual Review of Financial Economics* 4, 339–360.

[5] Broccado, Eleonora, Oliver Hart, and Luigi Zingales (2022): "Exit versus Voice." *Journal of Political Economy* 130, 3101–3145.

[6] Chowdhry, Bhagwan, Shawn William Davies, and Brian Waters (2018): "Investing for Impact." *Review of Financial Studies* 32, 864–904.

[7] Cohen, Lauren, Umit G. Gurun, and Quoc Nguyen (2024): "The ESG-Innovation Disconnect: Evidence from Green Patenting." Working Paper, Harvard University.

[8] Davies, Shaun William and Edward Dickersin Van Wesep (2018): "The Unintended Consequences of Divestment." *Journal of Financial Economics* 128, 558–575.

[9] Dow, James, and Gary Gorton (1997): "Stock Market Efficiency and Economic Efficiency: Is There a Connection?" *Journal of Finance* 52, 1087–1129.

[10] Edmans, Alex (2009): "Blockholder Trading, Market Efficiency, and Managerial Myopia." *Journal of Finance* 64, 2481–2513.

[11] Edmans, Alex and Gustavo Manso (2011): "Governance Through Trading and Intervention: A Theory of Multiple Blockholders." *Review of Financial Studies* 24, 2395–2428.

[12] Edmans, Alex, Tom Gosling, and Dirk Jenter (2024): "Sustainable Investing: Evidence From the Field." Working Paper, London Business School.

[13] Goldstein, Itay, Alexandr Kopytov, Lin Shen, and Haotian Xiang (2022): "On ESG Investing: Heterogeneous Preferences, Information, and Asset Prices." Working Paper, University of Pennsylvania.

[14] Gollier, Christian and Sébastien Pouget (2022): "Investment Strategies and Corporate Behaviour with Socially Responsible Investors: A Theory of Active Ownership." *Economica* 89, 997–1023.

[15] Gosling, Tom and Iain MacNeil (2023): "Can Investors Save the Planet? NZAMI and Fiduciary Duty." *Capital Markets Law Journal*, forthcoming.

[16] Green, Daniel and Benjamin Roth (2024): "The Allocation of Socially Responsible Capital." *Journal of Finance*, forthcoming.

[17] Green, Daniel and Boris Vallée (2024): "Measurement and Effects of Bank Exit Policies." Working Paper, Harvard Business School.

[18] Heinkel, Robert, Alan Kraus and Josef Zechner (2001): "The Effect of Green Investment on Corporate Behavior." *Journal of Financial and Quantitative Analysis* 36, 431–449.

[19] Koijen, Ralph S.J. and Motohiro Yogo (2019): "A Demand System Approach to Asset Pricing." *Journal of Political Economy* 127, 1475–1515.

[20] Landier, Augustin and Stefano Lovo (2024): "Socially Responsible Finance: How to Optimize Impact?" *Review of Financial Studies*, forthcoming.

[21] Muñoz, Fernando, Cristina Ortiz, and Luis Vicente (2024): "Ethical Window Dressing: SRI Funds Are as Good as Their Word." *Finance Research Letters* 49, 103109.

[22] Narayanan, M. P. (1985): "Managerial Incentives for Short-term Results." *Journal of Finance* 40, 1469–1484.

[23] Oehmke, Martin and Marcus Opp (2024): "A Theory of Socially Responsible Investment." *Review of Economic Studies*, forthcoming.

[24] Pastor, Lubos, Robert F. Stambaugh, and Lucian A. Taylor (2021): "Sustainable Investing in Equilibrium." *Journal of Financial Economics* 142, 550–571.

[25] Pastor, Lubos, Robert F. Stambaugh, and Lucian A. Taylor (2022): "Dissecting Green Returns." *Journal of Financial Economics* 146, 403–424.

[26] Pedersen, Lasse Heje, Shaun Fitzgibbons, and Lukas Pomorski (2021): "Responsible Investing: The ESG-Efficient Frontier." *Journal of Financial Economics* 142, 572–597.

[27] Scharfstein, David and Jeremy Stein (1990): "Herd Behavior and Investment." *American Economic Review* 80, 465–479.

[28] Stein, Jeremy C. (1988): "Takeover Threats and Managerial Myopia." *Journal of Political Economy* 46, 61–80.

[29] Stein, Jeremy C. (1989): "Efficient Capital Markets, Inefficient Firms: A Model of Myopic Corporate Behavior." *Quarterly Journal of Economics* 104, 655–669.

[30] Tang, Huiqiong, Bart Frijns, Aaron Gilbert, and Ayesha Scott (2024): "Window Dressing Among Responsible Investment Funds." Working Paper, Auckland University of Technology.

[31] Yeoh, Siew Hong, Ivo Welch, and C. Paul Wazzan (1999): "The Effect of Socially Activist Investment Policies on the Financial Markets: Evidence from the South African Boycott." *Journal of Business* 72, 35–89.

[32] Zerbib, Olivier David (2022): "A Sustainable Capital Asset Pricing Model (S-CAPM): Evidence from Environmental Integration and Sin Stock Exclusion." *Review of Finance* 26, 1345–1388.

A For Online Publication: Proofs

Proof of Lemma 1. Plugging (2), (10), and (11) into (6), gives

$$\begin{aligned} E\left[U_B\left(x,a\right)\right] &= \varphi x\left(v\left(x,a\right)-p\left(x,a\right)\right)-\left(1-\varphi\right)\lambda\left(\mu+qrp\left(x,a\right)\right)\left(1-a\xi\right) \qquad (23) \\ &= \varphi\frac{x}{1+q}\left(1-\frac{x}{1+q}\right)\gamma\sigma^2-\left(1-\varphi\right)\lambda\left(\mu+qr\frac{\mu-ca-(1-\frac{x}{1+q})\gamma\sigma^2}{1+q-rq}\right)\left(1-a\xi\right), \end{aligned}$$

which is concave in x. The FOC with respect to x is

$$\varphi\frac{1}{1+q}\left(1-\frac{2x}{1+q}\right)\gamma\sigma^2-\left(1-\varphi\right)\lambda qr\frac{\frac{1}{1+q}\gamma\sigma^2}{1+q-rq}\left(1-a\xi\right)=0,$$

and rearranging terms gives (12). ■

Proof of Proposition 1. Plugging (10) and (11) into (3), gives

$$\begin{aligned} E\left[U_M\left(x\left(a\right),a\right)\right] &= \omega p\left(x\left(a\right),a\right)+(1-\omega)v\left(x\left(a\right),a\right) \\ &= \omega\frac{\mu-ca-\left(1-\frac{x}{1+q}\right)\gamma\sigma^2}{1+q-rq}+(1-\omega)\frac{\mu-ca+rqp\left(x,a\right)}{1+q} \end{aligned}$$

and it is straight forward to see that $E\left[U_M\left(x\left(1\right),1\right)\right]>E\left[U_M\left(x\left(0\right),0\right)\right]$ if and only if (14) holds.

If B wants to induce $a=1$, she can set $x(0)=0$ to maximize the set of $x(1)$ satisfying $x(1)\geq\Delta$. If she wants to induce $a=0$, she can set $x(1)=x(0)$ without loss of generality. As a result, B chooses among the following two feasible strategies:

1. $x^*(1)=\max\{\hat{x}(1),\Delta\}$ and $x^*(0)=0$; the manager chooses $a^*=1$ and B's expected payoff is $E\left[U_B\left(x^*(1),1\right)\right]$.

2. $x^*(0)=\hat{x}(0)$ and $x^*(1)=x^*(0)$; the manager chooses $a^*=0$ and B's expected payoff is $E\left[U_B\left(x^*(0),0\right)\right]$

Notice

$$\hat{x}(1)\geq\Delta\Leftrightarrow\xi\geq\Xi_H\left(\varphi\right)\equiv1-\frac{\varphi}{1-\varphi}\frac{\frac{1+q}{2}-\Delta}{z\lambda}\frac{2\left(1-z\right)}{1+q}.$$

Let

$$\bar{\varphi}_H \equiv \frac{\frac{z}{1-z}\lambda}{\frac{2}{1+q}\left(\frac{1+q}{2}-\Delta\right)+\frac{z}{1-z}\lambda}.$$

If $\Delta < \frac{1+q}{2}$ then $\bar{\varphi}_H < 1$ and $\Xi_H(\varphi) > 0 \Leftrightarrow \varphi < \bar{\varphi}_H$. Moreover, $\Xi_H(0) = 1$ and $\Xi'_H(\varphi) < 0$. Thus, if $\varphi > \bar{\varphi}_H$ or $\xi \geq \Xi_H(\varphi)$ then $\hat{x}(1) \geq \Delta$. In this case, B prefers inducing $a^* = 1$. Indeed,

$$E\left[U_B\left(\hat{x}(1), 1\right)\right] \geq E\left[U_B\left(\hat{x}(0), 1\right)\right] > E\left[U_B\left(\hat{x}(0), 0\right)\right].$$

The first inequality follows from the fact that $\hat{x}(1) = \arg\max_x E\left[U_B(x, 1)\right]$ and the second inequality follows from the assumption $\mu > \gamma\sigma^2 + c$ and the observation that $E\left[U_B(x, 1)\right] > E\left[U_B(x, 0)\right]$ for any $x \geq 0$. In this case, $x^*(1) = \hat{x}(1) > 0$.

If $\Delta > \frac{1+q}{2}$ then $\bar{\varphi}_H > 1$ and $\Xi_H(\varphi) > 1$, and hence $\hat{x}(1) < \Delta$.

Suppose $\varphi \in [0, \bar{\varphi}_H)$ and $\xi < \Xi_H(\varphi)$. Notice that if $\varphi = \bar{\varphi}_H$ then $\hat{x}(0) = \Delta$, and recall $\hat{x}(0)$ is increasing in φ. Therefore, if $\varphi \in [0, \bar{\varphi}_H)$ then $\hat{x}(0) < \Delta$. We compare B's utility from (i) $x^*(1) = \Delta$, $x^*(0) = 0$, $a^* = 0$ and (ii) $x^*(1) = x^*(0) = \hat{x}(0)$, $a^* = 0$. Then, B prefers inducing $a^* = 1$ if and only if

$$E\left[U_B(\Delta, 1)\right] \geq E\left[U_B\left(\hat{x}(0), 0\right)\right],$$

which holds if and only if

$$\begin{aligned}
&\varphi\frac{\Delta}{1+q}\left(1-\frac{\Delta}{1+q}\right)\gamma\sigma^2 - (1-\varphi)\lambda(1-\xi)z\frac{\frac{\mu}{z} - c - \left(1-\frac{\Delta}{1+q}\right)\gamma\sigma^2}{1-z} \\
&\geq \varphi\frac{\hat{x}(0)}{1+q}\left(1-\frac{\hat{x}(0)}{1+q}\right)\gamma\sigma^2 - (1-\varphi)\lambda z\frac{\frac{\mu}{z} - \left(1-\frac{\hat{x}(0)}{1+q}\right)\gamma\sigma^2}{1-z} \Leftrightarrow \\
&\xi \geq \Xi_L(\varphi),
\end{aligned}$$

where

$$\Xi_L(\varphi) \equiv 1 - \frac{\frac{\varphi}{1-\varphi}\frac{1-z}{z}\frac{\gamma\sigma^2}{\lambda}\left[\frac{\Delta}{1+q}\left(1-\frac{\Delta}{1+q}\right) - \frac{\hat{x}(0)}{1+q}\left(1-\frac{\hat{x}(0)}{1+q}\right)\right] + \frac{\mu}{z} - \left(1-\frac{\hat{x}(0)}{1+q}\right)\gamma\sigma^2}{\frac{\mu}{z} - c - \left(1-\frac{\Delta}{1+q}\right)\gamma\sigma^2}$$

Notice, if $\varphi = 0$ then $\hat{x}(0) = 0$ and hence

$$\begin{aligned}\Xi_L(0) &\equiv 1 - \frac{\frac{\mu}{z} - \gamma\sigma^2}{\frac{\mu}{z} - c - \left(1 - \frac{\Delta}{1+q}\right)\gamma\sigma^2} \\ &= \frac{-c + \frac{\Delta}{1+q}\gamma\sigma^2}{\frac{\mu}{z} - c - \left(1 - \frac{\Delta}{1+q}\right)\gamma\sigma^2} \\ &= c\frac{\frac{1}{\omega+(1-\omega)z} - 1}{\frac{\mu}{z} - c - \left(1 - \frac{\Delta}{1+q}\right)\gamma\sigma^2} > 0\end{aligned}$$

Last, notice $\hat{x}(0) = \frac{1+q}{2}\left(1 - \frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda\right)$ and $\frac{\partial \hat{x}(0)}{\partial \varphi} = \frac{1+q}{2}\frac{1}{\varphi^2}\frac{z}{1-z}\lambda > 0$. Therefore,

$$\begin{aligned}\frac{\partial \Xi_L(\varphi)}{\partial \varphi} &= -\frac{\begin{array}{c}\frac{1}{(1-\varphi)^2}\frac{1-z}{z}\frac{\gamma\sigma^2}{\lambda}\left[\frac{\Delta}{1+q}(1 - \frac{\Delta}{1+q}) - \frac{\tilde{x}(0)}{1+q}(1 - \frac{\tilde{x}(0)}{1+q})\right] \\ -\frac{\varphi}{1-\varphi}\frac{1-z}{z}\frac{\gamma\sigma^2}{\lambda}\left[1 - \frac{2\tilde{x}(0)}{1+q}\right]\frac{\lambda}{2}\frac{1}{\varphi^2}\frac{z}{1-z} + \frac{1}{\varphi^2}\frac{z}{1-z}\frac{\lambda}{2}\gamma\sigma^2\end{array}}{\frac{\mu}{z} - c - \left(1 - \frac{\Delta}{1+q}\right)\gamma\sigma^2} \\ &= -\frac{\frac{1}{(1-\varphi)^2}\frac{1-z}{z}\frac{\gamma\sigma^2}{\lambda}\left[\frac{\Delta}{1+q}(1 - \frac{\Delta}{1+q}) - \frac{\tilde{x}(0)}{1+q}(1 - \frac{\tilde{x}(0)}{1+q})\right]}{\frac{\mu}{z} - c - \left(1 - \frac{\Delta}{1+q}\right)\gamma\sigma^2}\end{aligned}$$

Thus, $\frac{\partial \Xi_L(\varphi)}{\partial \varphi} < 0$ if and only if

$$\begin{aligned}\frac{\Delta}{1+q}(1 - \frac{\Delta}{1+q}) - \frac{\tilde{x}(0)}{1+q}(1 - \frac{\tilde{x}(0)}{1+q}) > 0 \Leftrightarrow \\ \Delta - \frac{1+q}{2} < \frac{1+q}{2} - \hat{x}(0) \Leftrightarrow \\ \hat{x}(0) + \Delta < 1 + q.\end{aligned}$$

Therefore, there are two cases:

1. If $\Delta \leq \frac{1+q}{2}$ then $\bar{\varphi}_H < 1$ and $\varphi < \bar{\varphi}_H \Rightarrow \hat{x}(0) < \Delta$. Therfore, $\hat{x}(0) + \Delta < 1 + q$ and $\Xi_L(\varphi)$ decreases in φ. Recall that if $\varphi = \bar{\varphi}_H \leq 1$ then $\hat{x}(0) = \Delta$, and therefore,

$$\Xi_L(\bar{\varphi}_H) \equiv 1 - \frac{\frac{\mu}{z} - \left(1 - \frac{\Delta}{1+q}\right)\gamma\sigma^2}{\frac{\mu}{z} - c - \left(1 - \frac{\Delta}{1+q}\right)\gamma\sigma^2} < 0.$$

Since $\Xi_L(0) > 0 > \Xi_L(\bar{\varphi}_H)$, there is $\bar{\varphi}_L \in (0, \bar{\varphi}_H)$ such that $\Xi_L(\varphi) > 0 \Leftrightarrow \varphi \in [0, \bar{\varphi}_L]$.

2. Suppose $\Delta > \frac{1+q}{2}$. In this case, $\bar{\varphi}_H > 1$ and $\tilde{x}(0) \to \frac{1+q}{2}$ and $\Xi_L(1) \to +\infty$. Notice,

$$\hat{x}(0) > 0 \Leftrightarrow \varphi > \frac{z\lambda}{z\lambda + 1 - z} \in (0, \bar{\varphi}_H).$$

If $\varphi < \frac{z\lambda}{z\lambda+1-z}$ then $\hat{x}(0) = 0$ and $\hat{x}(0) + \Delta < 1 + q$. In this range, $\Xi_L(\varphi)$ decreases in φ. If $\varphi > \frac{z\lambda}{z\lambda+1-z}$ then

$$\begin{aligned} \hat{x}(0) + \Delta < 1 + q \Leftrightarrow \\ \frac{1+q}{2}\left(1 - \frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda\right) + \Delta < 1 + q \Leftrightarrow \\ \varphi < \varphi_{\min} \equiv \frac{z\lambda}{\frac{2}{1+q}\left(\Delta - \frac{1+q}{2}\right)(1-z) + z\lambda} \in \left(\frac{z\lambda}{z\lambda + 1 - z}, 1\right). \end{aligned}$$

Therefore $\frac{\partial \Xi_L(\varphi)}{\partial \varphi} < 0 \Leftrightarrow \varphi < \frac{\frac{z}{1-z}\lambda}{\frac{2}{1+q}\left(\Delta - \frac{1+q}{2}\right) + \frac{z}{1-z}\lambda}$, i.e., it is U-shaped, with the minimum at $\varphi_{\min}$. Notice, if $\varphi = \varphi_{\min}$ then $\hat{x}(0) \equiv 1 + q - \Delta > 0$, and hence $\varphi_{\min} > \frac{z\lambda}{z\lambda+1-z}$. Also,

$$\Xi_L(\varphi_{\min}) = \frac{\left(\frac{2}{1+q}\Delta - 1\right)\gamma\sigma^2 - c}{\frac{\mu}{z} - c - \left(1 - \frac{\Delta}{1+q}\right)\gamma\sigma^2},$$

and $\Xi_L(\varphi_{\min}) < 0$ if and only if $\Delta < \left(1 + \frac{c}{\gamma\sigma^2}\right)\frac{1+q}{2} \Leftrightarrow c < \gamma\sigma^2 \frac{\omega + (1-\omega)z}{2 - \omega + (1-\omega)z}$. Since

The function in the statement is $\Xi_L(\varphi)$ and the cutoff is $\bar{\varphi}_L$.

Finally, we derive comparative statics on the threshold $\Xi_L(\varphi)$. We focus on the case $\Xi_L \in (0, 1)$. First, we show that Ξ_L increases with $\frac{\Delta}{1+q}$. Note that $E[U_B(x, 0)]$ does not depend on $\frac{\Delta}{1+q}$. Moreover,

$$\frac{\partial E[U_B(\Delta, 1)]}{\partial\left[\frac{\Delta}{1+q}\right]} = \varphi\gamma\sigma^2\left(1 - 2\frac{\Delta}{1+q}\right) - (1-\varphi)\frac{z}{1-z}\lambda\gamma\sigma^2(1-\xi) \tag{24}$$

which is negative if $\frac{\Delta}{1+q} > \frac{\hat{x}(1)}{1+q}$. Recall that the blockholder always chooses $a = 1$ if $\frac{\Delta}{1+q} \leq \frac{\hat{x}(1)}{1+q}$. Since $E[U_B(\Delta, 1)]$ decreases with $\frac{\Delta}{1+q}$, it follows that Ξ_L, which sets $E[U_B(\Delta, 1)] = E[U_B(\hat{x}(0), 0)]$, increases with $\frac{\Delta}{1+q}$.

1. $\frac{\partial \Xi_L}{\partial \mu}$: Note that Δ and $\hat{x}(0)$ do not depend on μ. Thus, Ξ_L decreases with μ.

2. $\frac{\partial \Xi_L}{\partial \omega}$: Since ω only affects Ξ_L through its impact on Δ and Δ decreases with ω, it follows that Ξ_L decreases with ω.

3. $\frac{\partial \Xi_L}{\partial \varphi}$: We have shown above that Ξ_L can increase or decrease with φ.

4. $\frac{\partial \Xi_L}{\partial z}$: Consider $\varphi = 0$ so that $\hat{x}(0) = 0$ and $\Xi_L(0) = \frac{\frac{\Delta}{1+q}\gamma\sigma^2 - c}{\frac{\mu}{z} + \frac{\Delta}{1+q}\gamma\sigma^2 - c}$. It follows that

$$\frac{\partial \Xi_L(0)}{\partial z} = \frac{c\mu(1-\omega)\left(\omega(1-2z) - (1-\omega)z^2\right)}{(c(1-\omega)(1-z)z + \mu(\omega + (1-\omega)z))^2} \tag{25}$$

which is positive if and only if $z < \frac{\sqrt{\omega} - \omega}{1-\omega}$. Recall that $z = \frac{rq}{1+q}$ increases with r and decreases with q. Hence, Ξ_L can either increase or decrease with q and r.

5. $\frac{\partial \Xi_L}{\partial \lambda}$: if $\hat{x}(0) = 0$, then Ξ_L increases with λ. If $\hat{x}(0) > 0$, then we can re-write Ξ_L as:

$$\Xi_L = \frac{\left(\frac{\Delta}{1+q} - \frac{\hat{x}(0)}{1+q}\right)\gamma\sigma^2 - \frac{\gamma\sigma^2}{1-2\frac{\hat{x}(0)}{1+q}}\left[\frac{\Delta}{1+q}(1 - \frac{\Delta}{1+q}) - \frac{\hat{x}(0)}{1+q}(1 - \frac{\hat{x}(0)}{1+q})\right] - c}{\frac{\mu}{z} - c - \left(1 - \frac{\Delta}{1+q}\right)\gamma\sigma^2}. \tag{26}$$

Note that $\frac{\partial \Xi_L}{\partial \lambda} = \frac{\partial \Xi_L}{\partial \hat{x}(0)} \frac{\partial \frac{\hat{x}(0)}{1+q}}{\partial \lambda}$ with $\frac{\partial \frac{\hat{x}(0)}{1+q}}{\partial \lambda} = -\frac{1-\varphi}{2\varphi}\frac{z}{1-z}$ and

$$\frac{\partial \Xi_L}{\partial \frac{\hat{x}(0)}{1+q}} = -\frac{2\gamma\sigma^2\left(\frac{\Delta}{1+q} - \frac{\hat{x}(0)}{1+q}\right)\left(1 - \frac{\Delta}{1+q} - \frac{\hat{x}(0)}{1+q}\right)}{\left(1 - \frac{\hat{x}(0)}{1+q}\right)^2\left(\frac{\mu}{z} - c - (1 - \frac{\Delta}{1+q})\gamma\sigma^2\right)} \tag{27}$$

Therefore, Ξ_L increases with λ if $1 > \frac{\Delta}{1+q} + \frac{\hat{x}(0)}{1+q}$ and it decreases with λ otherwise.

6. $\frac{\partial \Xi_L}{\partial \gamma\sigma^2}$: we re-write Ξ_L as

$$\Xi_L = \frac{f(\gamma\sigma^2)}{g(\gamma\sigma^2)} \tag{28}$$

so that $\frac{\partial \Xi_L}{\partial \gamma\sigma^2} > 0 \Leftrightarrow \frac{\partial f}{\partial \gamma\sigma^2} - \Xi_L \frac{\partial g}{\partial \gamma\sigma^2} > 0$. Let $k = \frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda$. It follows that

$$\frac{\partial f}{\partial \gamma\sigma^2} = \frac{1}{k\gamma\sigma^2}\left(\frac{\Delta}{1+q}\right)^2 + \frac{\hat{x}(0)}{1+q}\frac{1}{k}\left(1-(k+1)\frac{\hat{x}(0)}{1+q}\right) \tag{29}$$

$$\frac{\partial g}{\partial \gamma\sigma^2} = -1. \tag{30}$$

Note that (i) $1 > k > 0$ if $\hat{x}(0) > 0$ and (ii) $\frac{\hat{x}(0)}{1+q} < \frac{1}{2}$. Hence, Ξ_L increases with $\gamma\sigma^2$.

7. $\frac{\partial \Xi_L}{\partial c}$: we re-write $1-\Xi_L$ as

$$1 - \Xi_L = \frac{f(c)}{g(c)} \tag{31}$$

so that $\frac{\partial \Xi_L}{\partial c} > 0 \Leftrightarrow \frac{\partial f}{\partial c} - (1-\Xi_L)\frac{\partial g}{\partial c} < 0$. Let $k = \frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda$. It follows that

$$\frac{\partial f}{\partial c} = \frac{\gamma\sigma^2}{k}\left(1-2\frac{\Delta}{1+q}\right)\frac{\Delta}{c} \tag{32}$$

$$\frac{\partial g}{\partial c} = \frac{\Delta}{(1+q)}\frac{\gamma\sigma^2}{c} - 1. \tag{33}$$

Hence, $\frac{\partial \Xi_L}{\partial c} > 0$ is equivalent to:

$$\frac{\gamma\sigma^2}{k}\left(1-\frac{2\Delta}{1+q}\right)\frac{\Delta}{(1+q)} - (1-\Xi_L)\left(\frac{\Delta\gamma\sigma^2}{(1+q)} - c\right) < 0 \Leftrightarrow 1-\Xi_L > \frac{\frac{\gamma\sigma^2\Delta}{k(1+q)}\left(1-\frac{2\Delta}{1+q}\right)}{\frac{\gamma\sigma^2\Delta}{1+q} - c} \tag{34}$$

If $\frac{\Delta}{1+q} > \frac{1}{2}$, then the condition always holds. If $\frac{\Delta}{1+q} < \frac{1}{2}$, we require $\Delta > \frac{1}{2}\left(1 - k(1-\Xi_L)\right)$ because otherwise the blockholder always chooses $a = 1$. We can re-write this condition as follows:

$$1 - \Xi_L > \frac{1}{k}(1-2\Delta) > \frac{1}{k}\left(1-2\frac{\Delta}{1+q}\right) > \frac{\gamma\sigma^2\Delta}{\gamma\sigma^2\Delta - c}\frac{1}{k}\left(1-2\frac{\Delta}{1+q}\right). \tag{35}$$

We conclude that Ξ_L increases in c.

■

Proof of Corollary 1. We show that the firm's externalities in equilibrium increase with φ. Let $x^*(a,\varphi)$ be the optimal trade of B if her type is φ and M takes action a. We let $f(\varphi)$

and $\pi(\varphi)$ be the externalities and trading profits in equilibrium that are induced by strategies $x^*(a,\varphi)$, respectively. The equilibrium utility of type φ is $\varphi\pi(\varphi) - (1-\varphi) f(\varphi)$. Suppose to the contrary there are $\varphi' < \varphi''$ such that $f(\varphi') > f(\varphi'')$, that is, a blockholder with a greater profits motive induces less externalities in equilibrium. This implies either $x^*(0,\varphi') \neq x^*(0,\varphi'')$ or $x^*(1,\varphi') \neq x^*(1,\varphi'')$. Notice $f(\varphi') > f^*(\varphi'')$ implies $\pi(\varphi') > \pi(\varphi'')$. Otherwise,

$$\varphi'\pi(\varphi') - (1-\varphi') f(\varphi') < \varphi'\pi(\varphi'') - (1-\varphi') f(\varphi'')$$

and type φ' has a profitable deviation from $x^*(a,\varphi')$ to $x^*(a,\varphi'')$, a contradiction. Suppose $f(\varphi') > f^*(\varphi'')$ and $\pi(\varphi') > \pi(\varphi'')$. By revealed preferences of types φ' and φ'' we have

$$\varphi'\pi(\varphi') - (1-\varphi') f(\varphi') > \varphi'\pi(\varphi'') - (1-\varphi') f(\varphi'') \Leftrightarrow \varphi' > \frac{\frac{f(\varphi')-f(\varphi'')}{\pi(\varphi')-\pi(\varphi'')}}{1+\frac{f(\varphi')-f(\varphi'')}{\pi(\varphi')-\pi(\varphi'')}}$$
$$\varphi''\pi(\varphi'') - (1-\varphi'') f(\varphi'') > \varphi''\pi(\varphi') - (1-\varphi'') f(\varphi') \Leftrightarrow \varphi'' < \frac{\frac{f(\varphi')-f(\varphi'')}{\pi(\varphi')-\pi(\varphi'')}}{1+\frac{f(\varphi')-f(\varphi'')}{\pi(\varphi')-\pi(\varphi'')}}$$

Since $\varphi'' > \varphi'$, we have a contradiction. ■

Proof of Proposition 2. Without commitment, the blockholder chooses $\hat{x} = \hat{x}(a)$ given action a. B's expected utility is $E[U(a,\hat{x}(a))]$. The manager chooses $a = 1$ if and only if $\hat{x}(1) - \hat{x}(0) > \Delta$, where Δ is given by (14). Let

$$\Lambda \equiv \frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda, \tag{36}$$

then

$$\hat{x}(1) - \hat{x}(0) = \frac{1+q}{2} \times \begin{cases} \Lambda\xi & \text{if } \Lambda \leq 1 \\ 1-\Lambda(1-\xi) & \text{if } 1 < \Lambda < \frac{1}{1-\xi} \\ 0 & \text{if } \frac{1}{1-\xi} \leq \Lambda \end{cases} \tag{37}$$

Thus, $\hat{x}(1) - \hat{x}(0) > \Delta$ if and only if $\Lambda \leq 1$ and $\frac{1+q}{2}\Lambda\xi > \Delta$, or $1 < \Lambda < \frac{1}{1-\xi}$ and $\frac{1+q}{2}(1-\Lambda(1-\xi)) > \Delta$. These conditions can be rewritten as

$$\frac{2\Delta}{1+q}\frac{1}{\xi} < \Lambda \leq 1 \text{ or } 1 < \Lambda < \frac{1}{1-\xi}\left(1-\frac{2\Delta}{1+q}\right). \tag{38}$$

Notice

$$\frac{2\Delta}{1+q}\frac{1}{\xi}<\frac{1}{1-\xi}\left(1-\frac{2\Delta}{1+q}\right)\Leftrightarrow\xi>\frac{2\Delta}{1+q}.$$

Thus, if $\xi \leq \frac{2\Delta}{1+q}$ then condition (38) is empty. If $\xi > \frac{2\Delta}{1+q}$ then condition (38) is reduced to

$$\frac{2\Delta}{1+q}\frac{1}{\xi}<\Lambda<\frac{1}{1-\xi}\left(1-\frac{2\Delta}{1+q}\right)\Leftrightarrow\varphi\in\left(\underline{\varphi},\overline{\varphi}\right)$$

where

$$\overline{\varphi}\equiv\frac{1}{1+\frac{1-z}{z}\frac{1}{\lambda}\frac{1}{\xi}\frac{2\Delta}{1+q}}\text{ and }\underline{\varphi}\equiv\frac{1}{1+\frac{1-z}{z}\frac{1}{\lambda}\frac{1}{1-\xi}\left(1-\frac{2\Delta}{1+q}\right)}.$$

Thus, if $\xi > \frac{2\Delta}{1+q}$ and $\varphi \in \left(\underline{\varphi},\overline{\varphi}\right)$ then $\hat{x}(1)-\hat{x}(0)>\Delta$, $a^*=1$, and B buys $\hat{x}\left(1\right)$ shares. If $\xi \leq \frac{2\Delta}{1+q}$ or $\varphi \notin \left(\underline{\varphi},\overline{\varphi}\right)$ then $\hat{x}(1)-\hat{x}(0)\leq\Delta$, $a^*=0$, and B buys $\hat{x}(0)$ shares.

Consider part (iii). Notice $\hat{x}\left(a\right)$ weakly increases in φ, and that given action a and trade x, the externalities are

$$f\left(a,x\right)=\left(\mu+z\frac{\mu-ac-(1-\frac{x}{1+q})\gamma\sigma^2}{1-z}\right)\left(1-a\xi\right).$$

If $\xi \leq \frac{2\Delta}{1+q}$ then $a=0$ and $x=\hat{x}\left(0\right)$. $f\left(0,\hat{x}\left(0\right)\right)$ strictly increases in φ if $\hat{x}\left(0\right)>0\Leftrightarrow\varphi>\frac{\frac{z}{1-z}\lambda}{1+\frac{z}{1-z}\lambda}$, and is invariant to φ otherwise. Thus, $f^*\left(\varphi\right)$ weakly increases in φ.

Suppose $\xi > \frac{2\Delta}{1+q}$. If $\varphi \notin \left(\underline{\varphi},\overline{\varphi}\right)$ then $a=0$ and $x=\hat{x}\left(0\right)$. Note that $\xi > \frac{2\Delta}{1+q}$ implies $\frac{1}{\xi}\frac{2\Delta}{1+q}<1<\frac{1}{1-\xi}\left(1-\frac{2\Delta}{1+q}\right)$ and hence $\frac{\frac{z}{1-z}\lambda}{1+\frac{z}{1-z}\lambda}\in\left(\underline{\varphi},\overline{\varphi}\right)$. That is, if $\varphi<\underline{\varphi}$ then $\varphi<\frac{\frac{z}{1-z}\lambda}{1+\frac{z}{1-z}\lambda}$, and hence, $\hat{x}\left(0\right)=0$. Therefore, $\varphi<\underline{\varphi}\Rightarrow f^*\left(\varphi\right)=f\left(0,0\right)$. If $\varphi>\overline{\varphi}\Rightarrow f^*\left(\varphi\right)=f\left(0,\hat{x}\left(0\right)\right)$, and it is strictly increases in φ.

Suppose $\xi > \frac{2\Delta}{1+q}$ and $\varphi \in \left(\underline{\varphi},\overline{\varphi}\right)$. Then, it must be $\hat{x}\left(1\right)>0$. Notice $f\left(1,\hat{x}\left(1\right)\right)$ increases in φ, and

$$
\begin{aligned}
& f\left(1, \hat{x}\left(1\right)\right) < f\left(0,0\right) \Leftrightarrow \\
\lambda\left(\mu + z\frac{\mu - c - \frac{1+\frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda(1-\xi)}{2}\gamma\sigma^2}{1-z}\right)\left(1-\xi\right) &< \lambda\left(\mu + z\frac{\mu-\gamma\sigma^2}{1-z}\right) \Leftrightarrow \\
\frac{\left(\frac{\mu}{z} - c - \frac{1}{2}\gamma\sigma^2\right)\left(1-\xi\right) - \left(\frac{\mu}{z} - \gamma\sigma^2\right)}{\frac{1}{2}\left(1-\xi\right)^2\gamma\sigma^2} &< \frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda \Leftrightarrow \\
\varphi < \varphi_{\mathrm{nc}} &\equiv \frac{1}{1 + \frac{1-z}{z}\frac{1}{\lambda}\frac{\left(\frac{\mu}{z}-c-\frac{1}{2}\gamma\sigma^2\right)(1-\xi)-\left(\frac{\mu}{z}-\gamma\sigma^2\right)}{\frac{1}{2}(1-\xi)^2\gamma\sigma^2}}
\end{aligned}
$$

and

$$
\begin{aligned}
& \varphi_{\mathrm{nc}} > \underline{\varphi} \Leftrightarrow \\
\frac{1}{1 + \frac{1-z}{z}\frac{1}{\lambda}\frac{\left(\frac{\mu}{z}-c-\frac{1}{2}\gamma\sigma^2\right)(1-\xi)-\left(\frac{\mu}{z}-\gamma\sigma^2\right)}{\frac{1}{2}(1-\xi)^2\gamma\sigma^2}} &> \frac{1}{1 + \frac{1-z}{z}\frac{1}{\lambda}\frac{1}{1-\xi}\left(1 - \frac{2\Delta}{1+q}\right)} \Leftrightarrow \\
\frac{1}{1-\xi}\left(1 - \frac{2\Delta}{1+q}\right) &> \frac{\left(\frac{\mu}{z} - c - \frac{1}{2}\gamma\sigma^2\right)\left(1-\xi\right) - \left(\frac{\mu}{z} - \gamma\sigma^2\right)}{\frac{1}{2}\left(1-\xi\right)^2\gamma\sigma^2} \Leftrightarrow \\
\xi > \Xi_{\mathrm{nc}} &\equiv \frac{\gamma\sigma^2\frac{\Delta}{1+q} - c}{\frac{\mu}{z} - \gamma\sigma^2 + \gamma\sigma^2\frac{\Delta}{1+q} - c}.
\end{aligned}
$$

Thus, if $\frac{2\Delta}{1+q} < \xi < \Xi_{\mathrm{nc}}$ then $\varphi_{\mathrm{nc}} \leq \underline{\varphi}$. Hence, if $\varphi \in \left(\underline{\varphi}, \overline{\varphi}\right)$ then $\varphi > \varphi_{\mathrm{nc}}$ and $f\left(1, \hat{x}\left(1\right)\right) > f\left(0,0\right)$. In this case, $f^*\left(\varphi\right)$ weakly increases in φ. If $\xi > \max\left\{\frac{2\Delta}{1+q}, \Xi_{\mathrm{nc}}\right\}$ then $\varphi_{\mathrm{nc}} > \underline{\varphi}$. Thus, if $\varphi \in (0, \underline{\varphi})$ then $f^*\left(\varphi\right) = f\left(0,0\right)$, if $\varphi \in \left(\underline{\varphi}, \varphi_{\mathrm{nc}}\right)$ then $f^*\left(\varphi\right) = f\left(1, \hat{x}\left(1\right)\right) < f^*\left(0\right)$, and if $\varphi \in \left(\varphi_{\mathrm{nc}}, 1\right)$ then $f^*\left(\varphi\right) = f\left(0, \hat{x}\left(0\right)\right)$. Overall, if and only if $\xi > \max\left\{\frac{2\Delta}{1+q}, \Xi_{\mathrm{nc}}\right\}$ and $\varphi \in \left(\underline{\varphi}, \varphi_{\mathrm{nc}}\right)$ we have $f^*\left(\varphi\right) < f^*\left(0\right)$. ■

Proof of Proposition 3. First, we prove Equation (18). The equilibrium stock price given s is given by:

$$
p(\widehat{a}, s) = \frac{\mu - c\widehat{a} - \left(1 - \frac{x(s)}{1+q}\right)\gamma\sigma^2}{1 + q - rq}, \tag{39}
$$

where $\widehat{a}$ denotes the action conjectured by households.

If M chooses $a = 1$, his expected utility is given by:

$$\mathbb{E}[U_m|a=1] = \omega\left[\tau p(\widehat{a},1) + (1-\tau)p(\widehat{a},0)\right] + (1-\omega)\frac{\mu + rq\left[\tau p(\widehat{a},1) + (1-\tau)p(\widehat{a},0)\right] - c}{1+q}.$$

If he chooses $a = 0$, his expected utility is given by:

$$\mathbb{E}[U_m|a=0] = \omega\left[\tau p(\widehat{a},0) + (1-\tau)p(\widehat{a},1)\right] + (1-\omega)\frac{\mu + rq\left[\tau p(\widehat{a},0) + (1-\tau)p(\widehat{a},1)\right]}{1+q}.$$

Conditional on a conditional strategy, M chooses $a = 1$ if and only if $\mathbb{E}[U_m|a = 1] \geq \mathbb{E}[U_m|a = 0]$, which is equivalent to the condition in equation (18).

Next, let $E\left[U_B\left(x,a\right)\right]$, which is given in (23) is concave in x. If B expects $a = 0$ then he chooses $\arg\max_x E\left[U_B\left(x,0\right)\right] = \hat{x}\left(0\right)$ regardless of signal s. If B expects $a = 1$, then $\Pr\left[s = 1|a = 1\right] = \tau$ and B solves

$$\max_{x\geq 0,\ \Delta\geq\Delta_{\text{unob}}(\tau)} \tau E\left[U_B\left(x+\Delta,1\right)\right] + (1-\tau)\,E\left[U_B\left(x,1\right)\right].$$

Given x, and without any constraints, the optimal $\Delta\left(x\right)$ solves

$$\frac{\partial E\left[U_B\left(x+\Delta,1\right)\right]}{\partial\Delta} = 0 \Leftrightarrow \Delta\left(x\right) = \hat{x}\left(1\right) - x.$$

However. requiring $\Delta \geq \Delta_{\text{unob}}\left(\tau\right)$ implies

$$\Delta\left(x\right) = \max\left\{\hat{x}\left(1\right) - x, \Delta_{\text{unob}}\left(\tau\right)\right\}$$

Thus, the optimal x solves

$$\max_{x\geq 0} \tau E\left[U_B\left(x + \max\left\{\hat{x}\left(1\right) - x, \Delta_{\text{unob}}\left(\tau\right)\right\},1\right)\right] + (1-\tau)\,E\left[U_B\left(x,1\right)\right].$$

If $x < \hat{x}\left(1\right) - \Delta_{\text{unob}}\left(\tau\right)$ then B chooses

$$\begin{aligned}&\arg\max_{x\geq 0}\left\{\tau E\left[U_B\left(\hat{x}\left(1\right),1\right)\right] + (1-\tau)\,E\left[U_B\left(x,1\right)\right]\right\}\\&= \arg\max_{x\geq 0} E\left[U_B\left(x,1\right)\right] = \hat{x}\left(1\right)\end{aligned}$$

Since $\Delta_{\text{unob}}(\tau) > 0$, the best B can do in this case is choosing $(x, \Delta) = (\hat{x}(1) - \Delta_{\text{unob}}(\tau), \Delta_{\text{unob}}(\tau))$, and get

$$\tau E[U_B(\hat{x}(1), 1)] + (1-\tau) E[U_B(\hat{x}(1) - \Delta_{\text{unob}}(\tau), 1)].$$

Notice this case requires $\hat{x}(1) \geq \Delta_{\text{unob}}(\tau)$.

If $x \geq \hat{x}(1) - \Delta_{\text{unob}}(\tau)$ then B chooses

$$\arg\max_{x \geq 0} \tau E[U_B(x + \Delta_{\text{unob}}(\tau), 1)] + (1-\tau) E[U_B(x, 1)].$$

The FOC is

$$\left[\begin{array}{c} \tau \left(\varphi \frac{1}{1+q}\left(1 - \frac{2(x+\Delta_{\text{unob}}(\tau))}{1+q}\right)\gamma\sigma^2 - (1-\varphi)\lambda q r \frac{\frac{1}{1+q}\gamma\sigma^2}{1+q-rq}(1-\xi)\right) \\ +(1-\tau)\left(\varphi \frac{1}{1+q}\left(1 - \frac{2x}{1+q}\right)\gamma\sigma^2 - (1-\varphi)\lambda q r \frac{\frac{1}{1+q}\gamma\sigma^2}{1+q-rq}(1-\xi)\right) \end{array}\right] = 0 \Leftrightarrow$$

$$x = \frac{1+q}{2}\left(1 - \frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda(1-\xi)\right) - \tau\Delta_{\text{unob}}(\tau).$$

We conclude, it must

$$x = \begin{cases} \hat{x}(1) - \Delta_{\text{unob}}(\tau) & \text{if } x < \hat{x}(1) - \Delta_{\text{unob}}(\tau) \\ \max\left\{\frac{1+q}{2}(1 - \frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda(1-\xi)) - \tau\Delta_{\text{unob}}(\tau), 0\right\} & \text{if } x \geq \hat{x}(1) - \Delta_{\text{unob}}(\tau), \end{cases}$$

and there are there cases:

1. If $\hat{x}(1) \leq \tau\Delta_{\text{unob}}(\tau)$ then it must be $x \geq \hat{x}(1) - \Delta_{\text{unob}}(\tau)$. Also, if $\hat{x}(1) \leq \tau\Delta_{\text{unob}}$ then $\frac{1+q}{2}(1 - \frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda(1-\xi)) \leq \tau\Delta_{\text{unob}}$, and therefore, $x = 0$.
2. If $\hat{x}(1) \in (\tau\Delta_{\text{unob}}(\tau), \Delta_{\text{unob}}(\tau))$ then it must be $x \geq \hat{x}(1) - \Delta_{\text{unob}}(\tau)$. Also, if $\hat{x}(1) \in (\tau\Delta_{\text{unob}}(\tau), \Delta_{\text{unob}}(\tau))$ then $\frac{1+q}{2}(1 - \frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda(1-\xi)) = \hat{x}(1) \in (\tau\Delta_{\text{unob}}(\tau), \Delta_{\text{unob}}(\tau))$ and $x = \hat{x}(1) - \tau\Delta_{\text{unob}}(\tau)$.
3. If $\hat{x}(1) \geq \Delta_{\text{unob}}$ then $\frac{1+q}{2}\left(1 - \frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda(1-\xi)\right) = \hat{x}(1) \geq \Delta_{\text{unob}}$. And, either $x = \hat{x}(1) - \tau\Delta_{\text{unob}}$, or $x = \hat{x}(1) - \Delta_{\text{unob}}(\tau)$. Since in both cases the difference is $\Delta_{\text{unob}}(\tau)$, and the former is the unconstrained optimum, the optimal strategy is $x = \hat{x}(1) - \tau\Delta_{\text{unob}}(\tau)$.

As argued above, in all cases, $\Delta = \max\{\hat{x}(1) - x, \Delta_{\text{unob}}(\tau)\}$. We conclude:

- To induce $a^* = 0$, B chooses $x^*(s) = \hat{x}(0)$ regardless of signal, and her expected payoff is
$$U_{\text{unob}}(0) \equiv E\left[U_B\left(\hat{x}(0), 0\right)\right],$$
which is independent of τ.

- To induce $a = 1$, B chooses
$$x^*(s) = \max\left\{\hat{x}(1) - \tau\Delta_{\text{unob}}(\tau), 0\right\} + \Delta_{\text{unob}}(\tau) \cdot \mathbf{1}_{s=1}, \tag{40}$$
and her expected payoff is
$$\begin{aligned} U_{\text{unob}}(1,\tau) \equiv\, & \tau E[U_B(\max\{\hat{x}(1) - \tau\Delta_{\text{unob}}(\tau), 0\} + \Delta_{\text{unob}}(\tau), 1)] \\ & + (1-\tau) E[U_B(\max\{\hat{x}(1) - \tau\Delta_{\text{unob}}(\tau), 0\}, 1)]. \end{aligned}$$

We require $\max\left\{\hat{x}(1) - \tau\Delta_{\text{unob}}(\tau), 0\right\} + \Delta_{\text{unob}}(\tau) < 1 + q$. Since $\hat{x}(1) \le \frac{1+q}{2}$, a sufficient condition is $\Delta_{\text{unob}}(\tau) < \frac{1+q}{2}$, which holds if and only if
$$c < \frac{\gamma\sigma^2(\omega + (1-\omega)z)}{2(1-\omega)(1-z)}\left(2\tau - 1\right).$$

Next, we prove there is $\Xi_{\text{unob}}(\tau)$ such that $a^* = 1 \Leftrightarrow \xi > \Xi(\varphi)$. Notice that B prefers inducing $a^* = 1$ if and only if
$$U_{\text{unob}}(1,\tau) \ge E\left[U_B\left(\hat{x}(0), 0\right)\right].$$

Notice
$$U_{\text{unob}}(1,\tau) = \begin{cases} \tau E[U_B(\Delta_{\text{unob}}(\tau), 1)] + (1-\tau) E[U_B(0,1)] & \text{if } \hat{x}(1) \le \tau\Delta_{\text{unob}}(\tau) \\ \tau E[U_B(\hat{x}(1) + (1-\tau)\Delta_{\text{unob}}(\tau), 1)] & \\ + (1-\tau) E[U_B(\hat{x}(1) - \tau\Delta_{\text{unob}}(\tau), 1)] & \text{if } \hat{x}(1) > \tau\Delta_{\text{unob}}(\tau), \end{cases}$$
and the two terms are identical when $\hat{x}(1) = \tau\Delta_{\text{unob}}(\tau)$. Also, recall $E\left[U_B\left(\hat{x}(0), 0\right)\right]$ and $\Delta_{\text{unob}}(\tau)$ do not depend on ξ, $\hat{x}(1)$ increases in ξ, and for a given x, $E[U_B(x, 1)]$ strictly increases in ξ. Therefore, if ξ is small such that $\hat{x}(1) \le \tau\Delta_{\text{unob}}(\tau)$, then $U_{\text{unob}}(1,\tau)$ increases

in ξ since the trades do not depend on ξ. If ξ is large such that $\hat{x}(1) > \tau\Delta_{\text{unob}}$ then the trades do depend on ξ. However, since

$$U_{\text{unob}}(1,\tau) = \max_{x\geq 0,\ \Delta\geq\Delta_{\text{unob}}(\tau)} \tau E\left[U_B(x+\Delta,1)\right] + (1-\tau)E\left[U_B(x,1)\right],$$

it must be increasing in ξ in this region as well. Overall, $U_{\text{unob}}(1,\tau)$ strictly increases in ξ whereas $U_{\text{unob}}(0)$ is invariant to ξ, there is $\Xi_{\text{unob}}(\tau)$ such that $a^* = 1 \Leftrightarrow \xi > \Xi_{\text{unob}}(\tau)$.

Next, we prove $\lim_{\varphi\to 1}\Xi_{\text{unob}}(\tau) > 1$ and $\lim_{\varphi\to 0}\Xi_{\text{unob}}(\tau) < 1$. If $\xi = 1$ then $\hat{x}(1) = \frac{1+q}{2}$ and

$$E\left[U_B(x,1)\right] = \varphi\frac{x}{1+q}\left(1-\frac{x}{1+q}\right)\gamma\sigma^2.$$

If $\frac{1+q}{2} - \tau\Delta_{\text{unob}} < 0$ then

$$U_{\text{unob}}(1,\tau) = \tau E[U_B(\Delta_{\text{unob}}(\tau),1)] + (1-\tau)E\left[U_B(0,1)\right] = \tau\varphi\frac{\Delta_{\text{unob}}(\tau)}{1+q}\left(1-\frac{\Delta_{\text{unob}}(\tau)}{1+q}\right)\gamma\sigma^2$$

If $\frac{1+q}{2} - \tau\Delta_{\text{unob}} > 0$ then

$$\begin{aligned}
U_{\text{unob}}(1,\tau) &= \tau E\left[U_B\left(\frac{1+q}{2} + (1-\tau)\Delta_{\text{unob}}(\tau),1\right)\right] + (1-\tau)E\left[U_B\left(\frac{1+q}{2} - \tau\Delta_{\text{unob}}(\tau),1\right)\right] \\
&= \varphi\left[\begin{array}{c} \tau\frac{\frac{1+q}{2}+(1-\tau)\Delta_{\text{unob}}(\tau)}{1+q}\left(1-\frac{\frac{1+q}{2}+(1-\tau)\Delta_{\text{unob}}(\tau)}{1+q}\right) \\ +(1-\tau)\frac{\frac{1+q}{2}-\tau\Delta_{\text{unob}}(\tau)}{1+q}\left(1-\frac{\frac{1+q}{2}-\tau\Delta_{\text{unob}}(\tau)}{1+q}\right) \end{array}\right]\gamma\sigma^2 \\
&= \varphi\left[\frac{1}{4} - (1-\tau)\tau\left(\frac{\Delta_{\text{unob}}(\tau)}{1+q}\right)^2\right]\gamma\sigma^2
\end{aligned}$$

Either way, $U_{\text{unob}}(1,\tau) < \max_x\left\{\varphi\frac{x}{1+q}\left(1-\frac{x}{1+q}\right)\gamma\sigma^2\right\} = \varphi\frac{1}{4}\gamma\sigma^2$. Moreover,

$$E\left[U_B(\hat{x}(0),0)\right] = \varphi\frac{\hat{x}(0)}{1+q}\left(1-\frac{\hat{x}(0)}{1+q}\right)\gamma\sigma^2 - (1-\varphi)\lambda\left(\mu + qr\frac{\mu-(1-\frac{\hat{x}(0)}{1+q})\gamma\sigma^2}{1+q-rq}\right).$$

Thus, if $\varphi\to 1$ then $\hat{x}(0)\to\frac{1+q}{2}$ and $E\left[U_B(\hat{x}(0),0)\right]\to\varphi\frac{1}{4}\gamma\sigma^2 > U_{\text{unob}}(1,\tau)$. Therefore $a^* = 0$, that is, $\lim_{\varphi\to 1}\Xi_{\text{unob}}(\tau) > 1$. If $\varphi\to 0$ then $\hat{x}(0)\to 0$, $U_{\text{unob}}(1,\tau)\to 0$, but $E\left[U_B(\hat{x}(0),0)\right]$ $\to -\lambda\left(\mu + qr\frac{\mu-\gamma\sigma^2}{1+q-rq}\right) < 0$. Therefore $a^* = 1$, that is $\lim_{\varphi\to 0}\Xi_{\text{unob}}(\tau) < 1$.

Last, we prove that $\Xi_{\text{unob}}(\tau)$ decreases in τ. If $\hat{x}(1) \leq \tau\Delta_{\text{unob}}$, then

$$\frac{\partial U_{\text{unob}}(1,\tau)}{\partial\tau} = E[U_B(\Delta_{\text{unob}}(\tau),1)] - E[U_B(0,1)] + \tau\frac{\partial E[U_B(\Delta_{\text{unob}}(\tau),1)]}{\partial x}|_{x=\Delta_{\text{unob}}(\tau)}\frac{\partial\Delta_{\text{unob}}(\tau)}{\partial\tau}$$

notice $\frac{\partial\Delta_{\text{unob}}(\tau)}{\partial\tau} = -\frac{2}{2\tau-1}\Delta_{\text{unob}}(\tau) < 0$, and since $\Delta_{\text{unob}}(\tau) > \tau\Delta_{\text{unob}}(\tau) \geq \hat{x}(1)$, we have $\frac{\partial E[U_B(\Delta_{\text{unob}}(\tau),1)]}{\partial x}|_{x=\Delta_{\text{unob}}(\tau)} < 0$. Therefore, $\frac{\partial E[U_B(\Delta_{\text{unob}}(\tau),1)]}{\partial x}|_{x=\Delta_{\text{unob}}(\tau)}\frac{\partial\Delta_{\text{unob}}(\tau)}{\partial\tau} > 0$. Also notice $E[U_B(\Delta_{\text{unob}}(\tau),1)] - E[U_B(0,1)]$ if and only if $\Delta_{\text{unob}}(\tau) - \hat{x}(1) < \hat{x}(1) \Leftrightarrow 0.5\Delta_{\text{unob}}(\tau) < \hat{x}(1)$. There are two subcases:

1. If $0.5\Delta_{\text{unob}}(\tau) < \hat{x}(1) \leq \tau\Delta_{\text{unob}}(\tau)$ then $E[U_B(\Delta_{\text{unob}}(\tau),1)] - E[U_B(0,1)] > 0$ and hence $\frac{\partial U_{\text{unob}}(1,\tau)}{\partial\tau} > 0$.

2. If $\hat{x}(1) < 0.5\Delta_{\text{unob}}$ the

$$\frac{\partial E[U_B(\Delta_{\text{unob}}(\tau),1)]}{\partial x} = \left[\varphi\left(1 - 2\frac{\Delta_{\text{unob}}(\tau)}{1+q}\right) - (1-\varphi)\lambda\frac{z}{1-z}(1-\xi)\right]\frac{1}{1+q}\gamma\sigma^2$$
$$E[U_B(\Delta_{\text{unob}}(\tau),1)] - E[U_B(0,1)] = \left[\varphi\left(1 - \frac{\Delta_{\text{unob}}(\tau)}{1+q}\right) - (1-\varphi)\lambda\frac{z}{1-z}(1-\xi)\right]\frac{\Delta_{\text{unob}}(\tau)}{1+q}\gamma\sigma^2$$

Thus,

$$\begin{aligned}\frac{\partial U_{\text{unob}}(1,\tau)}{\partial\tau} &= \left[\varphi\left(1 - \frac{\Delta_{\text{unob}}(\tau)}{1+q}\right) - (1-\varphi)\lambda\frac{z}{1-z}(1-\xi)\right]\frac{\Delta_{\text{unob}}(\tau)}{1+q}\gamma\sigma^2 \\ &+ \tau\left[\varphi\left(1 - 2\frac{\Delta_{\text{unob}}(\tau)}{1+q}\right) - (1-\varphi)\lambda\frac{z}{1-z}(1-\xi)\right]\frac{1}{1+q}\gamma\sigma^2\left(-\frac{2}{2\tau-1}\Delta_{\text{unob}}(\tau)\right)\end{aligned}$$

and

$$\frac{\partial U_{\text{unob}}(1,\tau)}{\partial\tau} > 0 \Leftrightarrow (\tau+0.5)\Delta_{\text{unob}}(\tau) > \frac{1+q}{2}\left(1 - \frac{1-\varphi}{\varphi}\lambda\frac{z}{1-z}(1-\xi)\right).$$

If $\hat{x}(1) = 0$ then $\frac{1+q}{2}\left(1 - \frac{1-\varphi}{\varphi}\lambda\frac{z}{1-z}(1-\xi)\right) < 0$ and the above inequality holds. If $\hat{x}(1) > 0$ the above inequality is equivalent to $(\tau+0.5)\Delta_{\text{unob}}(\tau) > \hat{x}(1)$. Therefore, if $\hat{x}(1) < 0.5\Delta_{\text{unob}}(\tau)$ then $\frac{\partial U_{\text{unob}}(1,\tau)}{\partial\tau} > 0$.

If $\hat{x}(1) > \tau\Delta_{\text{unob}}$ then

$$\begin{aligned}\frac{\partial U_{\text{unob}}(1,\tau)}{\partial\tau} &= E[U_B(\hat{x}(1)+(1-\tau)\Delta_{\text{unob}}(\tau),1)] - E[U_B(\hat{x}(1)-\tau\Delta_{\text{unob}}(\tau),1)] \\ &+ \tau\frac{\partial E[U_B(\hat{x}(1)+(1-\tau)\Delta_{\text{unob}}(\tau),1)]}{\partial x}|_{x=\hat{x}(1)+(1-\tau)\Delta_{\text{unob}}(\tau)}\left(-\Delta_{\text{unob}}(\tau)+(1-\tau)\frac{\partial\Delta_{\text{unob}}(\tau)}{\partial\tau}\right) \\ &+ (1-\tau)\frac{\partial E[U_B(\hat{x}(1)-\tau\Delta_{\text{unob}}(\tau),1)]}{\partial x}|_{x=\hat{x}(1)-\tau\Delta_{\text{unob}}(\tau)}\left(-\Delta_{\text{unob}}(\tau)-\tau\frac{\partial\Delta_{\text{unob}}}{\partial\tau}(\tau)\right)\end{aligned}$$

Since $(1-\tau)\Delta_{\text{unob}}(\tau) < \tau\Delta_{\text{unob}}(\tau)$ the term in the first line above is positive. Since $\hat{x}(1) + (1-\tau)\Delta_{\text{unob}}(\tau) > \hat{x}(1)$ we have $\frac{\partial E[U_B(\hat{x}(1)+(1-\tau)\Delta_{\text{unob}}(\tau),1)]}{\partial x}|_{x=\hat{x}(1)+(1-\tau)\Delta_{\text{unob}}(\tau)} < 0$. Since $-\Delta_{\text{unob}} + (1-\tau)\frac{\partial\Delta_{\text{unob}}}{\partial\tau} < 0$, the term in the second line above is also positive. Finally, since $\hat{x}(1) - \tau\Delta_{\text{unob}}(\tau) < \hat{x}(1)$ we have $\frac{\partial E[U_B(\hat{x}(1)-\tau\Delta_{\text{unob}}(\tau),1)]}{\partial x}|_{x=\hat{x}(1)-\tau\Delta_{\text{unob}}(\tau)} > 0$ and notice $-\Delta_{\text{unob}}(\tau) - \tau\frac{\partial\Delta_{\text{unob}}(\tau)}{\partial\tau} = \frac{1}{2\tau-1}\Delta_{\text{unob}}(\tau) > 0$. Therefore, the term in the third line above is positive as well. Overall, $\frac{\partial U_{\text{unob}}(1,\tau)}{\partial\tau} > 0$. Since $\frac{\partial U_{\text{unob}}(0)}{\partial\tau} = 0$, $\Xi_{\text{unob}}(\tau)$ is decreasing in τ. ■

Proof of Proposition 4. As a preliminary, if $\Delta_{\text{unob}}(1) \geq 1+q$ then $x^*(s,1) \geq 1+q$, where $x^*(s,\tau)$ is given by (19), and according to Proposition 3, a conditional strategy is not feasible. Therefore, M chooses $\tau^* = \frac{1}{2}$ and $a^* = 0$.

Suppose $\Delta_{\text{unob}}(1) < 1+q$. Let $\tau_{\min} \in \left(\frac{1}{2},1\right)$ be the unique solution of

$$\max\{\hat{x}(1) - \tau\Delta_{\text{unob}}(\tau), 0\} + \Delta_{\text{unob}}(\tau) = 1+q \tag{41}$$

Indeed, since $\Delta_{\text{unob}}(\tau)$ and $\tau\Delta_{\text{unob}}(\tau)$ decrease in τ, if a solution exists it is unique. Since $\lim_{\tau\to\frac{1}{2}}\Delta_{\text{unob}}(\tau) = \infty$, it must be $\tau_{\min} > \frac{1}{2}$. Since $\max\{\hat{x}(1), \Delta_{\text{unob}}(1)\} < 1+q$ [indeed, $\Delta_{\text{unob}}(1) < 1+q$ and $\hat{x}(1) \leq \frac{1+q}{2}$ imply $\max\{\hat{x}(1), \Delta_{\text{unob}}(1)\} \leq 1+q$] it must be $\tau_{\min} < \frac{1}{2}$. Notice, since $\hat{x}(1)$ increases in ξ, $\tau_{\min}(\xi)$ weakly increases in ξ. Thus, a conditional strategy is feasible if and only if $\tau \geq \tau_{\min}(\xi)$.

Next, we have shown in the proof of Proposition 3 that $\Xi_{\text{unob}}(\tau)$ is a decreasing function of τ. If $\Xi_{\text{unob}}(1) \geq \xi$, then B chooses an unconditional strategy regardless of τ. In this case, M chooses $\tau^* = \frac{1}{2}$ and $a^* = 0$. Suppose $\Xi_{\text{unob}}(1) < \xi$. Since $\Xi_{\text{unob}}\left(\frac{1}{2}\right) > 1$ there exists $\widehat{\tau}(\xi) \in (\frac{1}{2}, 1)$ such that, $\xi \geq \Xi_{\text{unob}}(\tau) \Leftrightarrow \tau \geq \widehat{\tau}(\xi)$. Notice $\widehat{\tau}(\xi)$ decreases in ξ. Overall, a conditional strategy requires $\tau \geq \tau_{\text{con}}$ where

$$\tau_{\text{con}} \equiv \max\left\{\widehat{\tau}\left(\xi\right), \tau_{\min}\left(\xi\right)\right\}.$$

Suppose $\Delta_{\text{unob}}\left(1\right) < 1+q$ and $\Xi_{\text{unob}}\left(1\right) < \xi$. $B's$ expected trade from an unconditional strategy is $\hat{x}\left(0\right)$, and from a conditional strategy is

$$\begin{aligned}
\tau x^{*}\left(1,\tau\right) + \left(1-\tau\right)x^{*}\left(0,\tau\right) &= \tau\left[\max\left\{\hat{x}\left(1\right) - \tau\Delta_{\text{unob}}\left(\tau\right), 0\right\} + \Delta_{\text{unob}}\left(\tau\right)\right] \\
&\quad + \left(1-\tau\right)\max\left\{\hat{x}\left(1\right) - \tau\Delta_{\text{unob}}\left(\tau\right), 0\right\} \\
&= \max\left\{\hat{x}\left(1\right), \tau\Delta_{\text{unob}}\left(\tau\right)\right\}
\end{aligned}$$

The expected payoff and stock price are

$$\mathbb{E}[p\left(\tau\right)] = \frac{\mu - \left(1 - \frac{\hat{x}(0)}{1+q}\right)\gamma\sigma^{2} + \left(\frac{\max\{\hat{x}(1),\tau\Delta_{\text{unob}}(\tau)\} - \hat{x}(0)}{1+q}\gamma\sigma^{2} - c\right)\mathbf{1}_{\tau\geq\tau_{\text{con}}}}{1+q-rq}$$

and

$$\mathbb{E}[v\left(\tau\right)] = \frac{\mu + rq\mathbb{E}[p\left(\tau\right)] - c\mathbf{1}_{\tau\geq\tau_{\text{con}}}}{1+q}.$$

M's expected utility can be rewritten as:

$$\begin{aligned}
\mathbb{E}[U_{m}\left(\tau\right)] &= \omega\mathbb{E}[p\left(\tau\right)] + (1-\omega)\mathbb{E}[v\left(\tau\right)] \\
&= \left(\omega + (1-\omega)z\right)\mathbb{E}[p\left(\tau\right)] + (1-\omega)\frac{\mu - c\cdot\mathbf{1}_{\tau\geq\tau_{\text{con}}}}{1+q} \\
&= \left(\omega + (1-\omega)z\right)\left(\frac{\mu - \left(1-\frac{\hat{x}(0)}{1+q}\right)\gamma\sigma^{2}}{1+q-rq} + \frac{\frac{\max\{\hat{x}(1),\tau\Delta_{\text{unob}}(\tau)\}-\hat{x}(0)}{1+q}\gamma\sigma^{2} - c}{1+q-rq}\cdot\mathbf{1}_{\tau\geq\tau_{\text{con}}}\right) \\
&\quad + (1-\omega)\frac{\mu - c\cdot\mathbf{1}_{\tau\geq\tau_{\text{con}}}}{1+q} \\
&= \left(\omega + (1-\omega)z\right)\left(\frac{\mu - \left(1-\frac{\hat{x}(0)}{1+q}\right)\gamma\sigma^{2}}{1+q-rq}\right) + (1-\omega)\frac{\mu}{1+q} \\
&\quad + \left(\left(\omega + (1-\omega)z\right)\frac{\frac{\max\{\hat{x}(1),\tau\Delta_{\text{unob}}(\tau)\}-\hat{x}(0)}{1+q}\gamma\sigma^{2} - c}{1+q-rq} - (1-\omega)\frac{c}{1+q}\right)\cdot\mathbf{1}_{\tau\geq\tau_{\text{con}}}
\end{aligned}$$

If $\tau < \tau_{\text{con}}$ then $\mathbb{E}[U_m(\tau)]$ is invariant to τ. Suppose $\tau \geq \tau_{\text{con}}$. Recall $\tau\Delta_{\text{unob}}(\tau) = \frac{\tau}{2\tau-1}\frac{(1-\omega)c(1+q)(1-z)}{\gamma\sigma^2(\omega+(1-\omega)z)}$ which is decreasing in τ. Therefore, if τ is such that $\hat{x}(1) \geq \tau\Delta_{\text{unob}}(\tau)$ then $\mathbb{E}[U_m(\tau)]$ is invariant to τ, and otherwise, $\mathbb{E}[U_m(\tau)]$ is weakly decreasing in τ. Therefore, the two relevant choices of M are $\tau \in \left\{\frac{1}{2}, \tau_{\text{con}}\right\}$. Thus, M chooses $\tau = \tau_{\text{con}}$ if and only if

$$(\omega + (1-\omega)z)\frac{\frac{\max\{\hat{x}(1),\tau_{\text{con}}\Delta_{\text{unob}}(\tau_{\text{con}})\}-\hat{x}(0)}{1+q}\gamma\sigma^2 - c}{1+q-rq} > (1-\omega)\frac{c}{1+q} \Leftrightarrow$$

$$\max\{\hat{x}(1), \tau_{\text{con}}\Delta_{\text{unob}}(\tau_{\text{con}})\} > \hat{x}(0) + \Delta, \tag{42}$$

where Δ is given by (14). Condition (42) holds if and only if:

- $\hat{x}(1) < \tau_{\text{con}}\Delta_{\text{unob}}(\tau_{\text{con}})$ and $\tau_{\text{con}}\Delta_{\text{unob}}(\tau_{\text{con}}) > \hat{x}(0) + \Delta$, or
- $\hat{x}(1) \geq \tau_{\text{con}}\Delta_{\text{unob}}(\tau_{\text{con}})$ and $\hat{x}(1) > \hat{x}(0) + \Delta$

This is equivalent to $\hat{x}(1) > \hat{x}(0) + \Delta$ or $\hat{x}(1) \leq \hat{x}(0) + \Delta < \tau_{\text{con}}\Delta_{\text{unob}}(\tau_{\text{con}})$. The latter can be written as

$$\hat{x}(1) \leq \hat{x}(0) + \Delta \text{ and } \tau_{\text{con}}[2\hat{x}(0) + 2\Delta - \Delta_{\text{unob}}(1)] < \hat{x}(0) + \Delta.$$

Notice

$$\begin{aligned}\Delta - \Delta_{\text{unob}}(1) &= \frac{c(1+q)}{\gamma\sigma^2[\omega + (1-\omega)z]} - \frac{(1-\omega)c(1+q)(1-z)}{\gamma\sigma^2(\omega + (1-\omega)z)} \\ &= c\frac{1+q}{\gamma\sigma^2}\left[1 + \frac{1}{\omega + (1-\omega)z}\right] > 0.\end{aligned}$$

The manager chooses $\tau^* = \tau_{\text{con}}$ and $a^* = 1$ if $\Delta_{\text{unob}}(1) < 1 + q$, $\Xi_{\text{unob}}(1) < \xi$, and

- either $\hat{x}(1) > \hat{x}(0) + \Delta$
- or, $\hat{x}(1) \leq \hat{x}(0) + \Delta$ and $\tau_{\text{con}} < \frac{\hat{x}(0)+\Delta}{2\hat{x}(0)+2\Delta-\Delta_{\text{unob}}(1)}$. Notice $\frac{\hat{x}(0)+\Delta}{2\hat{x}(0)+2\Delta-\Delta_{\text{unob}}(1)} \in \left(\frac{1}{2}, 1\right)$ and it is independent of ξ. Recall $\tau_{\text{con}} \equiv \max\{\widehat{\tau}(\xi), \tau_{\min}(\xi)\}$, $\widehat{\tau}(\xi)$ decreases in ξ, and $\tau_{\min}(\xi)$ weakly increases in ξ. If $\widehat{\tau}(\xi) \geq \tau_{\min}(\xi)$ then τ_{con} decreases in ξ. If $\widehat{\tau}(\xi) < \tau_{\min}(\xi)$, then (42) requires $\tau_{\text{con}}\Delta_{\text{unob}}(\tau_{\text{con}}) > \hat{x}(0) + \Delta$, combined with $\hat{x}(0) + \Delta \geq \hat{x}(1)$, we have $\tau_{\min}(\xi)\Delta_{\text{unob}}(\tau_{\min}(\xi)) > \hat{x}(1)$. According to (41), $\tau_{\min}$ solves $\Delta_{\text{unob}}(\tau_{\min}) = 1 + q$, which is independent of ξ. Overall, in this case, τ_{con} weakly decreases in ξ.

In all other cases, the manager chooses $\tau^* = \frac{1}{2}$ and $a^* = 0$. Therefore, letting $\tau^*(\xi) = \tau_{\text{con}}$ and $\overline{\xi}_{\text{disc}}$ be the lowest such that $\Xi_{\text{unob}}(1) < \xi$, and either $\hat{x}(1) > \hat{x}(0) + \Delta$ [recall $\hat{x}(1)$ increases in ξ] or $\tau_{\text{con}} < \frac{\hat{x}(0)+\Delta}{2\hat{x}(0)+2\Delta-\Delta_{\text{unob}}(1)}$ [recall τ_{con} decreases in ξ], completes the proof. ■

Proof of Proposition 5. Given x, a, and y, the stock price is given by:

$$p(x,a,y) = \frac{\mu - ca - \left(1 - \frac{x+y}{1+q}\right)\gamma\sigma^2}{1+q-rq}. \tag{43}$$

Thus, A's profit is given by:

$$\begin{aligned}
\Pi_A(y) &= y\left(v(x,a,y) - p(x,a,y)\right) \\
&= y\left(\frac{\mu + rqp(x,a,y) - ac}{1+q} - p(x,a,y)\right) \\
&= y\left(\frac{\mu - (1-q-rq)\,p(x,a,y) - ac}{1+q}\right) \\
&= y\left(\frac{\mu - \left[\mu - ca - \left(1 - \frac{x+y}{1+q}\right)\gamma\sigma^2\right] - ac}{1+q}\right) \\
&= y\frac{\left(1 - \frac{x+y}{1+q}\right)\gamma\sigma^2}{1+q}
\end{aligned}$$

and so his trade is given by:

$$y^*(x) = \arg\max_y \Pi_A(y) = \frac{1+q-x}{2}$$

which yields a profit of

$$\Pi_A(y^*(x)) = \left(\frac{1}{2}\frac{1+q-x}{1+q}\right)^2 \gamma\sigma^2.$$

Thus, B expects the stock price to be

$$
\begin{aligned}
p\left(x, a, y^*\left(x\right)\right) &= \left(1-\eta\right) \frac{\mu - ca - \left(1 - \frac{x}{1+q}\right)\gamma\sigma^2}{1+q-rq} + \eta \frac{\mu - ca - \left(1 - \frac{x+y^*(x)}{1+q}\right)\gamma\sigma^2}{1+q-rq} \\
&= \frac{\mu - ca - \left(1 - \frac{x}{1+q} - \eta\frac{y^*(x)}{1+q}\right)\gamma\sigma^2}{1+q-rq} \\
&= \frac{\mu - ca - \left(1 - \frac{x}{1+q} - \eta\frac{\frac{1+q-x}{2}}{1+q}\right)\gamma\sigma^2}{1+q-rq} \\
&= \frac{\mu - ca - \left(1 - \frac{\eta}{2}\right)\left(1 - \frac{x}{1+q}\right)\gamma\sigma^2}{1+q-rq}
\end{aligned}
$$

M chooses $a = 1$ if and only if

$$
\begin{aligned}
\omega p\left(x\left(1\right), 1\right) + \left(1-\omega\right)\frac{\mu + rqp\left(x\left(1\right), 1\right) - c}{1+q} &> \omega p\left(x\left(0\right), 0\right) + \left(1-\omega\right)\frac{\mu + rqp\left(x\left(0\right), 0\right)}{1+q} \\
\left[\omega + (1-\omega)z\right]\left[p\left(x\left(1\right), 1\right) - p\left(x\left(0\right), 0\right)\right] &> \left(1-\omega\right)\frac{c}{1+q} \\
x\left(1\right) - x\left(0\right) &> \frac{\left(1+q\right)c}{\left(1 - \frac{\eta}{2}\right)\gamma\sigma^2\left[\omega + (1-\omega)z\right]}.
\end{aligned}
$$

The blockholder's expected utility becomes:

$$
E[U_B] = \varphi\frac{x}{1+q}\left(1 - \frac{x}{1+q}\right)(1-\frac{\eta}{2})\gamma\sigma^2 - (1-\varphi)\lambda\left(\mu + \frac{\mu - ca - (1-\frac{x}{1+q})(1-\frac{\eta}{2})\gamma\sigma^2}{\frac{1+q}{rq} - 1}\right)(1-a\xi)
$$

This expression is identical to that in the baseline model, except that $\gamma\sigma^2$ is scaled by $(1-\frac{\eta}{2})$. It follows that we can replace $\gamma\sigma^2$ in $\Xi(\varphi)$ by $(1-\frac{\eta}{2})\gamma\sigma^2$ to obtain Ξ_{arb}. Since Ξ_L increases with $\gamma\sigma^2$, Ξ_{arb} decreases with η. ■

B Supplemental Analyses

B.1 Additive Externality

This Appendix considers the case in which the action a has an additive effect on the externality, i.e. $f(\widetilde{A}, rI, a) = \lambda\left(\widetilde{A} + rI - \xi a\right)$. Proceeding as in the main model leads to Proposition 6.

Proposition 6 *(Blockholder's strategy, additive externality): There exists $\Xi_{add}(\varphi)$, where $\Xi_{add}(0) = \frac{c}{\frac{\omega}{(1-\omega)z}+1}$, such that in equilibrium, if $\xi < \Xi_{add}(\varphi)$, then the manager chooses $a^* = 0$ and the blockholder's investment strategy satisfies: $x^*(0) = x^*(1) = \hat{x}(0)$. If $\xi > \Xi_{add}(\varphi)$, then the manager chooses $a^* = 1$ and the blockholder's investment strategy satisfies $x^*(1) = \max\{\hat{x}(0), \Delta\}$ and $x^*(0) = 0$.*

Proof of Proposition 6. The blockholder's expected utility is given by:

$$E[U_B] = \varphi\frac{x}{1+q}\left(1-\frac{x}{1+q}\right)\gamma\sigma^2 - (1-\varphi)\lambda\left(\frac{\mu}{z} - \frac{ca + \left(1-\frac{x}{1+q}\right)\gamma\sigma^2}{\frac{1}{z}-1} - a\xi\right) \tag{44}$$

which is concave in x. It follows that, for a given a, the optimal position is

$$\hat{x}(0) = \frac{1+q}{2}\max\{1 - \frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda, 0\}. \tag{45}$$

Different from the baseline model, the action a does not enter this expression. The manager continues to take the action if and only if $x(1) - x(0) \geq \Delta$.

The blockholder chooses between the following two strategies: (i) $x^*(1) = \max\{\hat{x}(0), \Delta\}$ and $x^*(0) = 0$; (ii) $x^*(0) = \hat{x}(0)$ and $x^*(1) = \hat{x}(0)$.

If $\hat{x}(0) \geq \Delta$, then the blockholder always chooses a conditional strategy because (i) and (ii) feature the same trading profits but (ii) leads to a more negative externality.

If $\hat{x}(0) < \Delta$, then the blockholder chooses a conditional strategy if and only if $E[U_B(\Delta, 1)] > E[U_B(\hat{x}(0), 0)]$. Plugging in the expressions for U_B leads to the condition $\xi \geq \Xi_{\text{add}}(\varphi)$ where

$$\Xi_{\text{add}}(\varphi) \equiv \frac{\varphi\gamma\sigma^2}{(1-\varphi)\lambda}\left[\frac{\hat{x}(0)}{1+q}\left(1-\frac{\hat{x}(0)}{1+q}\right) - \frac{\Delta}{1+q}\left(1-\frac{\Delta}{1+q}\right)\right] + \frac{\left(\frac{\Delta}{1+q} - \frac{\hat{x}(0)}{1+q}\right)\gamma\sigma^2 - c}{\frac{1}{z}-1} \tag{46}$$

and $\Xi_{\text{add}}(0) = \frac{\frac{\Delta}{1+q}\gamma\sigma^2 - c}{\frac{1}{z}-1} = \frac{c}{\frac{\omega}{(1-\omega)z}+1}$.

Note that Ξ_{add} does not depend on μ. Next, we consider the impact of $\frac{\Delta}{1+q}$ on Ξ_{add}:

$$\frac{\partial \Xi_{\text{add}}}{\partial \frac{\Delta}{1+q}} = \frac{\varphi\gamma\sigma^2}{(1-\varphi)\lambda}\left(2\frac{\Delta}{1+q} - 1\right) + \frac{z}{1-z}\gamma\sigma^2. \tag{47}$$

This expression is positive if $\Delta > \hat{x}(0) = \frac{1+q}{2}\max\{1 - \frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda, 0\}$. Because Δ decreases with ω, we conclude that Ξ_{add} decreases with ω. Moreover,

$$\frac{d\Xi_{\text{add}}}{dc} = \frac{1}{c}\left[-\frac{\varphi\gamma\sigma^2}{(1-\varphi)\lambda}\left(1 - \frac{\Delta}{1+q}\right)\Delta + \frac{\Delta\gamma\sigma^2 - c}{\frac{1}{z} - 1} + \frac{\varphi\gamma\sigma^2}{(1-\varphi)\lambda}\Delta^2\right] \tag{48}$$

Note that $\Xi_{\text{add}} > 0$ is equivalent to

$$-\frac{\varphi\gamma\sigma^2}{(1-\varphi)\lambda}\left(1 - \frac{\Delta}{1+q}\right)\Delta + \frac{\Delta\gamma\sigma^2 - c}{\frac{1}{z} - 1} + \frac{\varphi\gamma\sigma^2}{(1-\varphi)\lambda}\frac{\hat{x}(0)}{1+q}\left(1 - \frac{\hat{x}(0)}{1+q}\right) - \frac{\gamma\sigma^2\frac{\hat{x}(0)}{1+q}}{\frac{1}{z} - 1} > 0 \tag{49}$$

So, if $\frac{\hat{x}(0)}{1+q} = 0$, it immediately follows that $\frac{d\Xi_{\text{add}}}{dc} > 0$. If instead, $\frac{\hat{x}(0)}{1+q} = \frac{1}{2}\left(1 - \frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda\right)$, we have that $\Xi_{\text{add}} > 0$ is equivalent to

$$-\frac{\varphi\gamma\sigma^2}{(1-\varphi)\lambda}\left(1 - \frac{\Delta}{1+q}\right)\Delta + \frac{\Delta\gamma\sigma^2 - c}{\frac{1}{z} - 1} + \frac{\varphi\gamma\sigma^2}{(1-\varphi)\lambda}\left(\frac{\hat{x}(0)}{1+q}\right)^2 \tag{50}$$

which again implies that $\frac{d\Xi_{\text{add}}}{dc} > 0$ because $\Delta > \hat{x}(0)$.

We continue with the effect of $\gamma\sigma^2$:

$$\begin{aligned}
\frac{d\Xi_{\text{add}}}{d\gamma\sigma^2} &= \frac{\varphi}{(1-\varphi)\lambda}\left[\frac{\hat{x}(0)}{1+q}\left(1 - \frac{\hat{x}(0)}{1+q}\right) - \frac{\Delta}{1+q}\left(1 - \frac{\Delta}{1+q}\right)\right] - \frac{\frac{\hat{x}(0)}{1+q}}{\frac{1}{z} - 1} \qquad (51)\\
&+ \frac{\varphi}{(1-\varphi)\lambda}\left(1 - \frac{2\Delta}{1+q}\right)\frac{\Delta}{(1+q)} \\
&= \frac{\varphi}{(1-\varphi)\lambda}\left[\frac{\hat{x}(0)}{1+q}\left(1 - \frac{\hat{x}(0)}{1+q}\right) - \left(\frac{\Delta}{1+q}\right)^2\right] - \frac{\frac{\hat{x}(0)}{1+q}}{\frac{1}{z} - 1}.
\end{aligned}$$

If $\hat{x}(0) = 0$, we can see immediately that $\frac{d\Xi_{\text{add}}}{d\gamma\sigma^2} < 0$. If $\hat{x}(0) = \frac{1}{2}\left(1 - \frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda\right)$, we find that:

$$\begin{aligned}\frac{d\Xi_{\text{add}}}{d\gamma\sigma^2} &= \frac{\varphi}{(1-\varphi)\lambda}\left[\frac{\hat{x}(0)}{1+q}\left(1 - \frac{\hat{x}(0)}{1+q}\right) - \left(\frac{\Delta}{1+q}\right)^2 + \frac{\hat{x}(0)}{1+q}\left(1 - \frac{2\hat{x}(0)}{1+q}\right)\right] \\ &= \frac{\varphi}{(1-\varphi)\lambda}\left[\left(\frac{\hat{x}(0)}{1+q}\right)^2 - \left(\frac{\Delta}{1+q}\right)^2\right] < 0.\end{aligned} \tag{52}$$

We conclude that Ξ_{add} decreases with $\gamma\sigma^2$.

Next, consider the comparative statics with respect to φ (or equivalently $\frac{\varphi}{1-\varphi}$). First, suppose $\hat{x}(0) = 0$:

$$\frac{d\Xi_{\text{add}}}{d\frac{\varphi}{1-\varphi}} = -\frac{\gamma\sigma^2}{\lambda}\frac{\Delta}{1+q}\left(1 - \frac{\Delta}{1+q}\right) < 0. \tag{53}$$

Second, consider $\hat{x}(0) = \frac{1}{2}\left(1 - \frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda\right) > 0$:

$$\begin{aligned}\frac{d\Xi_{\text{add}}}{d\frac{\varphi}{1-\varphi}} &= -\frac{\gamma\sigma^2}{\lambda}\frac{\Delta}{1+q}\left(1 - \frac{\Delta}{1+q}\right) + \frac{(1-\varphi)\gamma\sigma^2}{\varphi}\frac{z}{1-z}\left(\frac{1}{2} - \frac{\hat{x}(0)}{1+q}\right) + \frac{\lambda\gamma\sigma^2}{2}\left(\frac{1-\varphi}{\varphi}\frac{z}{1-z}\right)^2 \\ &= \frac{\gamma\sigma^2}{\lambda}\left[\left(1 - \frac{2\hat{x}(0)}{1+q}\right)^2 - \frac{\Delta}{1+q}\left(1 - \frac{\Delta}{1+q}\right)\right]\end{aligned} \tag{54}$$

which can be positive (e.g., if $\frac{\hat{x}(0)}{1+q} \to 0$) or negative (e.g., if $\frac{\hat{x}(0)}{1+q} \to \frac{1}{2}$).

Finally, Ξ_{add} continues to be monotone in z and, thus, in (r, q). To see this consider $\omega = 0$ and $\hat{x}(0) = 0$. In this case, we find that:

$$\frac{d\Xi_{\text{add}}}{dz} = \frac{c\varphi\left(z\gamma\sigma^2 - 2c\right)}{\gamma\sigma^2 z^3\lambda(1-\varphi)} \tag{55}$$

which can be negative or positive.

The analysis is identical for the case in which the action has multiplicative effect on the externality, as in the baseline model, and the cost of the action is also multiplicative in firm size so that firm value is given by $V = \theta + rI - ca(\mu + rI)$ rather than $V = \theta + rI - ca$. Since firm size affects both the benefits and costs of the action, it drops out and reduces to the additive model. ■

B.2 Comparative statics for the expected externality

We have shown in Corollary 1 that the expected externality always increases in the blockholder's weight on profits φ. Below, we formally analyze how $E[f]$ depends on the remaining model parameters.

Corollary 2 *(Comparative statics for the externality): In the equilibrium described in Proposition 1, the expected externality $E[f]$ may increase or decrease in $(c, z, \mu, \gamma, \sigma, \omega)$.*

Proof. The expected externality given corrective action $a \in \{0, 1\}$ and B's position $x(a) \in [0, 1+q]$ equals:

$$E[f] = \frac{\mu z^{-1} - ca - \left(1 - \frac{x(a)}{1+q}\right)\gamma\sigma^2}{\frac{1}{z} - 1}(1 - \xi a)$$

where $z = \frac{rq}{1+q}$, so $\frac{dz}{dr} > 0$ and $\frac{dz}{dq} > 0$. If the blockholder's equilibrium strategy is conditional, then the manager takes the corrective action and the blockholder demands $x^*(1) = \max\{\Delta, \hat{x}(1)\}$. If it is unconditional, then the corrective action is not taken and $x^*(0) = \hat{x}(0)$. Note that:

$$\frac{\Delta}{1+q} = \frac{c}{\gamma\sigma^2\left[\omega + (1-\omega)z\right]} \tag{56}$$

$$\frac{\hat{x}(a)}{1+q} = \frac{1}{2}\max\{1 - \frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda(1 - a\xi), 0\}. \tag{57}$$

First, we analyze the comparative statics of $E[f]$ for a given divestment strategy. Then, we focus on the threshold $\xi = \Xi(\varphi)$, i.e., when the blockholder is indifferent between conditional and unconditional strategies.

1. $\xi > \Xi(\varphi)$**: blockholder prefers a conditional strategy.** Under a conditional strategy, the manager takes the corrective action and the expected externality is given by:

$$E[f] = \frac{\mu z^{-1} - c - \left(1 - \frac{\max\{\Delta, \hat{x}(1)\}}{1+q}\right)\gamma\sigma^2}{\frac{1}{z} - 1}(1 - \xi). \tag{58}$$

It immediately follows from the expressions for $E[f]$ and $x^*(1)$ that $\frac{dE[f]}{d\mu} > 0$, $\frac{dE[f]}{d\omega} \leq 0$, and $\frac{dE[f]}{d\gamma\sigma^2} < 0$. If $\max\{\Delta, \hat{x}(1)\} = \hat{x}(1)$, then $\frac{dE[f]}{dc} < 0$. If $\max\{\Delta, \hat{x}(1)\} = \Delta$, then $\frac{dE[f]}{dc} > 0$.

2. $\xi < \Xi(\varphi)$**: blockholder prefers a unconditional strategy.** Under an unconditional strategy, the corrective action is not taken and the expected externality is given by:

$$E[f] = \frac{\mu z^{-1} - \left(1 - \frac{\hat{x}(0)}{1+q}\right)\gamma\sigma^2}{\frac{1}{z} - 1}. \tag{59}$$

It immediately follows from the expressions for $E[f]$ and $x^*(0)$ that $\frac{dE[f]}{d\mu} > 0$, $\frac{dE[f]}{d\omega} = 0$, $\frac{dE[f]}{d\gamma\sigma^2} < 0$, and $\frac{dE[f]}{dc} = 0$.

3. $\xi = \Xi(\varphi)$**: blockholder is indifferent between conditional and unconditional strategies.** $\xi = \Xi$ makes the blockholder indifferent between a conditional strategy, i.e., $x^*(1) = \Delta$ (because at $x^*(1) = \hat{x}(1)$ the blockholder always prefers a conditional strategy) and an unconditional strategy, i.e. $x^*(0) = \hat{x}(0)$. We have shown above that the threshold increases in $(c, \gamma\sigma^2)$ and that it decreases in (μ, ω). By definition, the blockholder's expected utility at the threshold satisfies:

$$\varphi\frac{\Delta}{1+q}\left(1 - \frac{\Delta}{1+q}\right)\gamma\sigma^2 - (1-\varphi)E[f|x = \Delta] = \varphi\frac{\hat{x}(0)}{1+q}\left(1 - \frac{\hat{x}(0)}{1+q}\right)\gamma\sigma^2 - (1-\varphi)E[f|x = \hat{x}(0)]. \tag{60}$$

So, the difference in expected externalities at the threshold equals:

$$E[f|x = \Delta] - E[f|x = \hat{x}(0)] = \frac{\varphi\gamma\sigma^2}{1-\varphi}\left[\frac{\Delta}{1+q}\left(1 - \frac{\Delta}{1+q}\right) - \frac{\hat{x}(0)}{1+q}\left(1 - \frac{\hat{x}(0)}{1+q}\right)\right]. \tag{61}$$

It follows that transitioning from an unconditional to a conditional strategy increases (decreases) the expected externality if and only if it leads to higher (lower) expected trading profits, i.e., the term in square brackets. We obtain the following condition:

(a) If $\frac{\Delta}{1+q}\left(1 - \frac{\Delta}{1+q}\right) - \frac{\hat{x}(0)}{1+q}\left(1 - \frac{\hat{x}(0)}{1+q}\right) > 0$, then an increase in (μ, ω) and a decrease in $(c, \gamma\sigma^2)$ reduce the threshold Ξ and, for a given ξ, lead to a transition from a unconditional to a conditional strategy. In this case, $E[f]$ increases.

(b) If $\frac{\Delta}{1+q}\left(1 - \frac{\Delta}{1+q}\right) - \frac{\hat{x}(0)}{1+q}\left(1 - \frac{\hat{x}(0)}{1+q}\right) < 0$, then an increase in (μ, ω) and a decrease in $(c, \gamma\sigma^2)$ reduce the threshold Ξ and, for a given ξ, lead to a transition from a unconditional to a conditional strategy. In this case, $E[f]$ decreases.

We have also shown that the threshold Ξ may increase or decrease with z. As a result, an increase in z can either increase or decrease $E[f]$.

Summary.

1. $\frac{dE[f]}{d\mu}$: an increase in μ increases the expected externality if $\xi \neq \Xi(\varphi)$. At the threshold, an increase in μ decreases $E[f]$ if and only if $\Delta > \hat{x}(1)$ and $\frac{\Delta}{1+q}\left(1-\frac{\Delta}{1+q}\right) < \frac{\hat{x}(0)}{1+q}\left(1-\frac{\hat{x}(0)}{1+q}\right)$. As a result, $\frac{dE[f]}{d\mu} < 0$ requires that $\frac{\Delta}{1+q} \geq \frac{1}{2}$ and $\varphi > \varphi_{\min}$ (such that $\Xi(\varphi) < 1$ as shown in the Proof of Proposition 1).

2. $\frac{dE[f]}{d\gamma\sigma^2}$: an increase in $\gamma\sigma^2$ decreases the expected externality if $\xi \neq \Xi(\varphi)$. At the threshold, a decrease in $\gamma\sigma^2$ decreases $E[f]$ if and only if $\Delta > \hat{x}(1)$ and $\frac{\Delta}{1+q}\left(1-\frac{\Delta}{1+q}\right) < \frac{\hat{x}(0)}{1+q}\left(1-\frac{\hat{x}(0)}{1+q}\right)$. So, as above, $\frac{dE[f]}{d\gamma\sigma^2} < 0$ requires that $\frac{\Delta}{1+q} \geq \frac{1}{2}$ and $\varphi > \varphi_{\min}$ (such that $\Xi(\varphi) < 1$ as shown in the Proof of Proposition 1).

3. $\frac{dE[f]}{d\omega}$: as shown above, $E[f]$ only depends on ω in two cases, (i) if $\xi > \Xi(\varphi)$ and $x^*(1) = \Delta$ and (ii) if $\xi = \Xi(\varphi)$. In case (ii), an increase in ω reduces $E[f]$ if $\Delta > \hat{x}(1)$ and $\frac{\Delta}{1+q}\left(1-\frac{\Delta}{1+q}\right) < \frac{\hat{x}(0)}{1+q}\left(1-\frac{\hat{x}(0)}{1+q}\right)$, which requires that $\frac{\Delta}{1+q} \geq \frac{1}{2}$ and $\varphi > \varphi_{\min}$ (such that $\Xi(\varphi) < 1$ as shown in the Proof of Proposition 1).

4. $\frac{dE[f]}{dc}$ and $\frac{dE[f]}{dz}$: an increase in c and z may lead to an increase or a decrease in the expected externality, as shown above.

■

B.3 Cap on Holdings

In this Appendix, we study a variation of the setting without commitment (see Section 4). We consider a cap on the blockholder's position $\overline{x} \geq 0$ such that $x(a) \leq \overline{x}$. We proceed in two steps. First, we show that B never wants to commit to such a cap.

Proposition 7 *The blockholder never commits to a cap on her trades.*

Proof. Note that $\overline{x}$ can only make B better off if it changes M's equilibrium choice of a. If a is unchanged, then, by definition, $x^*(a)$, maximizes B's expected utility and any $\overline{x} \neq x^*(a)$ makes B (weakly) worse off. Since B's position is weakly smaller with a cap, we only need to consider the case in which the cap changes M's choice from $a^* = 1$ to $a^* = 0$. This, in turn, requires $\overline{x}$ such that $\overline{x} - x^*(0) < \Delta$. It is sufficient to consider the case $\overline{x} = x^*(0)$ because, again, any cap $\overline{x} < x^*(0)$ would make B worse off, and any $x^*(0) < \overline{x} < x^*(0) + \Delta$ is observationally equivalent to $\overline{x} = x^*(0)$ because $a^* = 0$ in this case.

In other words, the question is whether $E\left[U\left(1, x^*\left(1\right)\right)\right] \geq E\left[U\left(0, x^*\left(0\right)\right)\right]$. Notice the optimality of $x^*\left(1\right)$ when $a = 1$ implies

$$E\left[U\left(1, x^*\left(1\right)\right)\right] \geq E\left[U\left(1, x^*\left(0\right)\right)\right]$$

We argue $E\left[U\left(1, x\right)\right] > E\left[U\left(0, x\right)\right]$ for every $x \geq 0$. Indeed,

$$\begin{aligned}
&E\left[U\left(1, x\right)\right] - E\left[U\left(0, x\right)\right] = \varphi x \begin{pmatrix} v\left(x,1\right) - v\left(x,0\right) \\ +p\left(x,0\right) - p\left(x,1\right) \end{pmatrix} - \begin{pmatrix} \left(1-\varphi\right)\lambda\left(\mu + qrp\left(x,1\right)\right)\left(1-\xi\right) \\ -\left(1-\varphi\right)\lambda\left(\mu + qrp\left(x,0\right)\right) \end{pmatrix} \\
&= \varphi x \left(\left(\frac{rq-1-q}{1+q}\right)\left(p\left(x,1\right) - p\left(x,0\right)\right) - \frac{c}{1+q}\right) - \left(1-\varphi\right)\lambda \begin{pmatrix} \left(\mu + qrp\left(x,1\right)\right)\left(1-\xi\right) \\ -\left(\mu + qrp\left(x,0\right)\right) \end{pmatrix} \\
&= \varphi x \left(\left(\frac{rq-1-q}{1+q}\right)\left(\frac{-c}{1+q-rq}\right) - \frac{c}{1+q}\right) - \left(1-\varphi\right)\lambda \begin{pmatrix} \left(\mu + qrp\left(x,1\right)\right)\left(1-\xi\right) \\ -\left(\mu + qrp\left(x,0\right)\right) \end{pmatrix} \\
&= 0 - \left(1-\varphi\right)\lambda \begin{pmatrix} \left(\mu + qrp\left(x,1\right)\right)\left(1-\xi\right) \\ -\left(\mu + qrp\left(x,0\right)\right) \end{pmatrix} \\
&= -\left(1-\varphi\right)\lambda \begin{pmatrix} \left(\mu + qrp\left(x,0\right) - c\frac{qr}{1+q-rq}\right)\left(1-\xi\right) \\ -\left(\mu + qrp\left(x,0\right)\right) \end{pmatrix} \\
&= \left(1-\varphi\right)\lambda \left(\xi\left[\mu + qrp\left(x,0\right)\right] + c\frac{qr}{1+q-rq}\left(1-\xi\right)\right) \\
&> 0
\end{aligned}$$

Therefore,

$$E\left[U\left(1, x^*\left(1\right)\right)\right] \geq E\left[U\left(1, x^*\left(0\right)\right)\right] > E\left[U\left(0, x^*\left(0\right)\right)\right],$$

as required.

Intuitively, given x, action a does not affect B's trading profits since it is fully priced in by households. However, it does reduce externalities, both directly and also indirectly by reducing the stock price and thus new capital raising. Therefore, given trade x, B is better off inducing the corrective action. ■

Next, we analyze whether a cap can lower the expected externality. If the cap has no effect then we assume it is infinity.

Proposition 8 *There is $\bar{\xi} \in (0,1]$ such that:*

1. *If $\xi < \bar{\xi}$ then absent a cap $a^* = 0$ and $x^* = x^*(0)$. The optimal cap is $\overline{x}^* = 0$ and under the optimal cap $a^* = 0$.*

2. *If $\xi \geq \bar{\xi}$ then absent a cap $a^* = 1$ and $x^* = x^*(1)$. There is $\widehat{\xi}_1$ such that:*

   (a) If $\xi > \widehat{\xi}_1$ then the optimal cap is $\overline{x}^* = x^*(0) + \Delta$ and under the optimal cap $a^* = 1$.

   (b) If $\xi < \widehat{\xi}_1$ then the optimal cap is $\overline{x}^* = 0$ and under the optimal cap $a^* = 0$.

Proof. We distinguish between the two cases $x^*(1) - x^*(0) \geq \Delta$ (so that $a^* = 1$) and $x^*(1) - x^*(0) < \Delta$ (so that $a^* = 0$).

1. If $x^*(1) - x^*(0) < \Delta$, then M does not take the corrective action without a cap. Since a cap $\overline{x}$ can only reduce B's position, we have that $a^* = 0$ with any cap. It follows that for $\overline{x} > x^*(0)$, the cap is not binding and the expected externality is given by $-\lambda\left(\mu + qrp\left(x^*(0), 0\right)\right)$. If $\overline{x} \leq x^*(0)$, then the cap is binding and the expected externality equals $-\lambda\left(\mu + qrp\left(\overline{x}, 0\right)\right)$. Since p strictly increases in x, it follows that the optimal cap is equal to $\overline{x}^* = 0$. Note that

$$x^*(1) - x^*(0) < \Delta \Leftrightarrow$$

$$\frac{1+q}{2}\left(\max\left\{1 - \frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda\left(1-\xi\right), 0\right\} - \max\left\{1 - \frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda, 0\right\}\right) < \frac{c(1+q)}{\gamma\sigma^2\left[\omega + (1-\omega)z\right]}$$

   The left-hand side increases in ξ, equals 0 if $\xi = 0$, and is positive for $\xi = 1$. Thus, there is $\bar{\xi} \in (0,1]$ such that $x^*(1) - x^*(0) < \Delta \Leftrightarrow \xi < \bar{\xi}$.

2. If $x^*(1) - x^*(0) \geq \Delta$, then M takes the corrective action without a cap. In this case, the expected externality is equal to $-\lambda\left(\mu + qrp\left(x^*(1), 1\right)\right)(1-\xi)$. There are three cases to consider:

(a) If $\overline{x} > x^*(1)$, then the cap does not bind and the expected externality is the same as without the cap.

(b) If $\overline{x} \leq x^*(1)$ and $\overline{x} - x^*(0) \geq \Delta$, then the manager takes the corrective action and B holds $\overline{x}$. As before, since the expected externality increases in x it is optimal to set $\overline{x} = x^*(0) + \Delta$.

(c) If $\overline{x} \leq x^*(1)$ and $\overline{x} - x^*(0) < \Delta$, then the manager does not take the action. In this case, it is optimal to set $\overline{x} = 0$.

Hence, we have to compare the expected externality under $\overline{x} = x^*(0) + \Delta$ and $\overline{x} = 0$:

$$E[f|\overline{x} = 0] = -\lambda\left(\mu + qrp\left(\overline{x} = 0, 0\right)\right) = -\lambda \frac{z^{-1}\mu - \gamma\sigma^2}{z^{-1} - 1}$$

and

$$\begin{aligned} E[f|\overline{x} = x^*(0) + \Delta] &= -\lambda\left(\mu + qrp\left(\overline{x} = x^*(0) + \Delta, 1\right)\right)(1 - \xi) \\ &= -\lambda(1 - \xi)\frac{z^{-1}\mu - \gamma\sigma^2 + \frac{x^*(0)+\Delta}{1+q}\gamma\sigma^2 - c}{z^{-1} - 1} \end{aligned} \tag{62}$$

It follows that the expected externality is lower with the cap at $x^*(0) + \Delta$ if and only if

$$\begin{aligned} &(1 - \xi)\left(z^{-1}\mu - \gamma\sigma^2 + \frac{x^*(0) + \Delta}{1 + q}\gamma\sigma^2 - c\right) < z^{-1}\mu - \gamma\sigma^2 \\ &\qquad \Leftrightarrow \xi > 1 - \frac{z^{-1}\mu - \gamma\sigma^2}{z^{-1}\mu - \gamma\sigma^2 + \frac{x^*(0)+\Delta}{1+q}\gamma\sigma^2 - c} \equiv \widehat{\xi}_1 \end{aligned}$$

Let $K_1 \equiv \frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda$. Note that $x^*(1) - x^*(0) \geq \Delta$ implies that:

$$x^*(1) - x^*(0) \geq \Delta \Leftrightarrow \begin{cases} \xi \geq \frac{2c}{K_1\gamma\sigma^2[\omega + (1-\omega)z]} \equiv \widehat{\xi}_2 & \text{if } K_1 \leq 1 \\ \xi \geq \frac{2c}{K_1\gamma\sigma^2[\omega + (1-\omega)z]} + \frac{K_1 - 1}{K_1} \equiv \widehat{\xi}_3 & \text{if } 1 < K_1 < \frac{1}{1-\xi} \end{cases} \tag{63}$$

If $K_1 \leq 1$, then $\widehat{\xi}_2$ can be greater or less than $\widehat{\xi}_1$. However, if $1 < K_1 < \frac{1}{1-\xi}$, then $\widehat{\xi}_3 < \widehat{\xi}_1$.

Given $\xi > \widehat{\xi}_1$, the optimal cap that minimizes expected externalities is given as follows:

$$\overline{x}^* = \begin{cases} x^*(0) + \Delta & \text{if } K_1 \leq 1 \text{ and } \xi \geq \widehat{\xi}_2 \text{ and if } 1 < K_1 < \frac{1}{1-\xi}; \\ 0 & \text{otherwise.} \end{cases}$$

If $\xi \leq \widehat{\xi}_1$, then $\overline{x}^* = 0$, as discussed before.

■

B.4 Partial Commitment

In this Appendix, we study a variation of the setting without commitment (see Section 4). We consider the possibility that the blockholder can commit to full exclusion, but not to other divestment strategies. We show that the ability to commit to full exclusion only does not change the conclusions from Section 4.

Proposition 9 *(partial commitment): Suppose the blockholder can commit to full exclusion, but not any other divestment strategy. Then, the equilibrium is characterized as in Proposition 2. Moreover, the blockholder is weakly better off without committing to full exclusion.*

Proof. If B commits to full exclusion, then he buys $\hat{x}(0)$ shares, the manager chooses $a^* = 0$, and B's expected utility is $E\left[U_B\left(0,0\right)\right]$. Based on Proposition 2 parts (i) and (ii):

- If $\xi \leq \frac{2\Delta}{1+q}$ or $\varphi \notin \left(\underline{\varphi}, \overline{\varphi}\right)$, then without commitment, $a^* = 0$ and B buys $\hat{x}(0)$ shares. Since $\hat{x}\left(0\right) = \arg\max_x E\left[U_B\left(x,0\right)\right]$, B is weakly better off without commitment.

- If $\xi > \frac{2\Delta}{1+q}$ and $\varphi \in \left(\underline{\varphi}, \overline{\varphi}\right)$, then without commitment, $a^* = 1$ and B buys $\hat{x}(1) > 0$ shares. B's expected utility is $E\left[U_B\left(\hat{x}(1),1\right)\right]$. Notice that

$$E\left[U_B\left(\hat{x}(1),1\right)\right] > E\left[U_B\left(0,1\right)\right] > E\left[U_B\left(0,0\right)\right].$$

  The first inequality stems from $\hat{x}\left(1\right) = \arg\max_x E\left[U_B\left(x,1\right)\right]$ and $\hat{x}\left(1\right) > 0$. The second inequality stems from the fact that given $x = 0$, B has no trading profits and his utility is solely determined by the level of externalities. Given $x = 0$, externalities are lower when $a = 1$. Overall, B is strictly better off without commitment.

■

about ECGI

The European Corporate Governance Institute has been established to improve *corporate governance through fostering independent scientific research and related activities.*

The ECGI will produce and disseminate high quality research while remaining close to the concerns and interests of corporate, financial and public policy makers. It will draw on the expertise of scholars from numerous countries and bring together a critical mass of expertise and interest to bear on this important subject.

The views expressed in this working paper are those of the authors, not those of the ECGI or its members.

Electronic Access to the Working Paper Series

The full set of ECGI working papers can be accessed through the Institute's Web-site (www.ecgi.global/content/working-papers) or SSRN:

Finance Paper Series	http://www.ssrn.com/link/ECGI-Fin.html
Law Paper Series	http://www.ssrn.com/link/ECGI-Law.html